Chapter 4
Knowledge representation, inference & reasoning

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Reference: Associate Professor ***Bibek Ropakheti*** notes on Discrete Structure

Chapter 4

Knowledge representation, inference & reasoning

4.1. Approaches to Knowledge Representation

4.2. Issues in Knowledge Representation

4.3. Propositional logic, predicate logic, FOPL

4.4. Rules of inference, resolution refutation system (RRS), answer extraction from RRS

4.5. Statistical Reasoning : Probability and Bayes theorem and casual networks, reasoning in belief network

4.6 Semantic nets and frames

4.1. Approaches to Knowledge Representation

4.1.1. Knowledge

- Knowledge is an abstract term that attempts to capture an individual's understanding of a given subject.
- Knowledge is more than just data, it consist of: facts, ideas, beliefs, heuristics, associations, rules, abstractions, relationships, customs.

Generally knowledge can be classified as follows:

1. **Procedural Knowledge** : It is compiled or processed form of information. *Example*: Sequence of steps to solve a problem is procedural knowledge.
2. **Declarative Knowledge** : It is passive knowledge in the form of statements of facts about the world. *Example*: Mark statement of a student is declarative knowledge.
3. **Heuristic Knowledge**: Heuristic knowledge is used to make judgements & also to simplify solution of problems. It is acquired through experience.

4.1.2. Knowledge Representation

Knowledge is more than just data, it consist of: facts, ideas, beliefs, heuristics, associations, rules, abstractions, relationships, customs.

“ It can be represented in different forms as mental images in one’s thought , as spoken or written words in some language, as graphical or other pictures and as characters, strings or collection of magnetic spots stored in a computer.”

Knowledge Acquisition:

It is the process of acquiring knowledge from a human expert for an expert system, which must be carefully organized into IF-THEN rules or some other form of knowledge representation.

4.1.2.1. Knowledge Representation Schemes

Logical Schemes • Predicate Calculus • Propositional Calculus	**Procedural schemes** • IF…THEN rules
Networked schemes • Semantic nets • Conceptual graphs	**Structured schemes** • Scripts • Frames

- **Logical schemes** represent knowledge, using mathematical or orthographic symbols, inference rules and are based on precisely defined syntax and semantics.
- **Procedural schemes** knowledge is represented as a set of instructions for problem solving. That allows to modify a knowledge base easily.

4.1.2.1. Knowledge Representation Schemes

Logical Schemes • Predicate Calculus • Propositional Calculus	**Procedural schemes** • IF…THEN rules
Networked schemes • Semantic nets • Conceptual graphs	**Structured schemes** • Scripts • Frames

- **Networked schemes** use a graph to represent knowledge. Nodes of a graph display objects or concepts in a domain, but arcs define relationships between objects, their attributes and values of attributes.
- **Structured schemes** extend networked representation by displaying each node in a graph as a complex data structure.

4.2. Issues in Knowledge Representation

4.2. Issues in Knowledge Representation

1. What is the nature of knowledge & how do we represent it ?
2. Are there are any attributes that occur in many different types of problem ?
3. What about the relationship between the attributes of an object, such as, inverses, existence, techniques for reasoning about values and single valued attributes ?
4. At what level should the knowledge be represented and what are the primitives ?
5. How should sets of objects be represented?
6. Given a large amount of knowledge stored in a database, how can relevant parts be accessed when they are needed ?

4.3. Propositional logic, predicate logic, FOPL

First Session

- Logic and Proof
- Proposition and Truth function
- Propositional Logic
- Expressing statements in Logic Propositional Logic

Logic

- Study of reasoning
- Specify the meaning of mathematical statements
- Basis of all mathematical reasoning
- Basis of all automated reasoning
- Practical application to the design of computing machines
- To specification of systems, AI, Programming, Programming languages and other areas of computer science as well as other fields

Logic

- Concerned with whether the reasoning is correct
- It focuses on the relationship among the statements as opposed to the content of any particular statement
- Example:
    - All Nepali love politics
    - Anyone who loves politics is an politician
    - Hence, all Nepali are politician
- Logic doesn’t help in determining if the statements are true
- However if the first two statements are true, logic helps to generalize

Proof

- What makes up a correct mathematical argument
- Once we prove a mathematical statement, its theorem
- Demonstration of theorem being true is also proof
- Used in computer programming to show the output of the given set of inputs and execution of algorithm
- Automated reasoning systems have been created to allow computers to construct their own proof

Proposition

- A sentence that is either true or false but not both is a proposition
- Let us examine if the given statements are proposition or not

1. The only positive integer that divide 2 are 1 and 2 itself.
2. Mahabir Pun won Magsaysay award in 2007.
3. The sum of 2 and 3 is 6.
4. Hetauda is the capital of Bagmati province.
5. What is your name?
6. The difference between x and y is z.
7. Raju, please go to the library.
8. Its warm day today 31st May 2020 at Kathmandu.

Propositions

- Building blocks of logic
- Declarative sentence
  - Declares the fact
- Propositions are either true or false at any instance
- But not both
- In previous slide, statement 1-4 are propositions as 1, 2 and 4 are always true and 3 is always false
- In previous slide, statement 5-7 are not propositions

Propositional Variables and Truth Value

- Letters are used to denote propositional variables
- A variable represent a proposition just as letters used to denote numerical variables
- The truth value of a proposition is true, denoted by T, if the proposition is true
- The truth value of a proposition is false, denoted by F, if the proposition is false
- Conventional letters are used for propositional variables like p, q, r...

Propositional Calculus

- The area of logic
- Deals with proposition
- Also called Propositional Logic

Compound Proposition

- The law of thoughts discussed the methods for producing new propositions from the existing ones
- Mathematical statements are constructed by combining one or more propositions
- New propositions are called compound propositions
- Formed from existing propositions using logical operators

Logical Operators

- Negation
- Conjunction
- Disjunction
- Exclusive OR
- Conditional Statements
  - Converse, Contrapositive and Inverse
- Biconditional Statements

Negation

- Let P be a proposition
- The negation of P denoted by ¬P(~P 0r |P) is the statement “It is not the case that P”
- The proposition ¬P is read “not P”
- The truth value of ¬P is opposite of the truth value of P
- Example:
  - P: Today is Sunday
  - ¬P: Today is not Sunday
  - ¬P will be true on all other days than Sunday

Negation – Truth Table

P	¬P
T	F
F	T

P: Today is Sunday

¬P: Today is not Sunday

Conjunction

- Let P and Q be propositions
- The conjunction of P and Q is denoted by $P \wedge Q$
- $P \wedge Q$ is the proposition “P and Q”
- The proposition $P \wedge Q$ is true when both P and Q are true and false otherwise

Conjunction – Example and Truth Table

- Let
  - P: Today is Sunday
  - Q: Today is Sunny
  - P∧Q: Today is Sunny and Sunday
  - P∧Q will be true on Sunny Sundays and false on non Sundays or non Sunny days

P	Q	P∧Q
T	T	T
T	F	F
F	T	F
F	F	F

Disjunction

- Let P and Q be propositions
- The disjunction of P and Q is denoted by PvQ
- PvQ is the proposition “P or Q”
- The proposition PvQ is true when either P or Q is true and false when both P and Q are false

Disjunction – Example and Truth Table

P	Q	PvQ
T	T	T
T	F	T
F	T	T
F	F	F

- Let
  - P: Today is Sunday
  - Q: Today is Sunny
  - PvQ : Today is Sunny or Sunday
  - PvQ will be true on any Sunday or any day that is Sunny

Exclusive OR

- Let P and Q be propositions
- The exclusive or of P and Q is denoted by $P \oplus Q$
- $P \oplus Q$ is the proposition “P exclusive or Q”
- The proposition $P \oplus Q$ is true when exactly one of P and Q is true otherwise false

Exclusive OR – Example and Truth Table

P	Q	P⊕Q
T	T	F
T	F	T
F	T	T
F	F	F

- Let
  - P: Today is Sunday
  - Q: Today is Sunny
  - P⊕Q will be true on any Sunday which is not sunny or any sunny day that is not Sunday

Conditional Statements

- Let P and Q be propositions
- The conditional statement $P \to Q$ is the proposition “if P then Q”
- The conditional statement $P \to Q$ is false when P is true and Q is false, and is true otherwise
- In conditional statement $P \to Q$, P is called hypothesis (premise or antecedent) and Q is called conclusion or consequence

Conditional Statement - Example and Truth Table

P	Q	P→Q
T	T	T
T	F	F
F	T	T
F	F	T

- Let
    - P: You get 100% on the exam
    - Q: You get an A
    - P→Q: If you get 100% on the exam, then you will get an A
    - If you do not get 100%, you may get an A Or you may not get an A, which depends on other factors
    - There wont be problem if you get 100% and an A
    - If 100% is scored but you do not get an A then you may feel disappointed

Conditional Statements: Clauses

"if p, then q"
"if p, q"
"p is sufficient for q"
"q if p"
"q when p"
"a necessary condition for p is q"
"q unless ¬p"

$P \rightarrow Q$

"p implies q"
"p only if q"
"a sufficient condition for q is p" "q whenever p"
"q is necessary for p"
"q follows from p"

Converse, Contrapositive and Inverse

- For a proposition $P \rightarrow Q$, its
- Converse is $Q \rightarrow P$,
- Contrapositive is $\neg Q \rightarrow \neg P$, and
- Inverse is $\neg P \rightarrow \neg Q$

Converse: If $p \rightarrow q$ is an implication, then its converse is $q \rightarrow p$.

Inverse: If $p \rightarrow q$ is an implication, then its inverse is $\neg p \rightarrow \neg q$.

Contrapositive: If $p \rightarrow q$ is an implication, then its contrapositive is $\neg q \rightarrow \neg p$

Example

- For a conditional statement,

“ If you score 100%, then you get an A ”

p = you score 100%
q = you get an A

Converse is, “If you get an A, then you scored 100%”

Contrapositive is, “If you don’t get an A, then you don’t score 100%”, and

Inverse is, “If you don’t score 100%, then you don’t get an A”

Example

Q. Write converse, inverse & contrapositive of the following integration.

1. The program is readable only if it is well structured.

implication: if the program is readable then it is well structured.

p = the program is readable

q = it is well structured

Converse: if it is well structured, then the program is readable.

Contrapositive: If it is not well structured, then the program is not readable.

Inverse: If the program is not readable, then it is not well structured.

Example

Q. Write converse, inverse & contrapositive of the following integration.

1. **If two angles are congruent, then they have the same measure**.

p = Two angles are congruent

q = they have the same measure

Converse: If two angles have same measure then they are congruent.

Contrapositive: If two angles don't have same measure then they are not congruent.

Inverse: If two angles are not congruent, then they don't have the same measure.

Biconditional Statement

- Let P and Q be propositions
- The biconditional statement $P \leftrightarrow Q$ is the proposition “P if and only if Q”
- The biconditional statement $P \leftrightarrow Q$ is true when P and Q have same truth values and is false otherwise
- These are also called bi-implications

Biconditional Statement - Example and Truth Table

P	Q	P↔Q
T	T	T
T	F	F
F	T	F
F	F	T

- Let
  - P: You can take the flight
  - Q: You buy a ticket
  - P↔Q : You can take the flight if and only if you buy a ticket

Biconditional Statements: Clauses

- “p is necessary and sufficient for q”
- “if p then q, and conversely”
- “p iff q”
- “p if and only if q”

Precedence of Logical Operators

1. $\neg$
2. $\land$
3. $\lor$
4. $\rightarrow$
5. $\leftrightarrow$

Truth table of Compound Propositions

- **Construct the truth table of compound proposition $(P \lor \neg Q) \rightarrow (P \land Q)$**

Truth table of Compound Propositions

P	Q	¬Q	P∨¬Q	P∧Q	(P∨¬Q)→(P∧Q)
T	T	F	T	T	T
T	F	T	T	F	F
F	T	F	F	F	T
F	F	T	T	F	F

- Construct the truth table of compound proposition (P∨¬Q)→(P∧Q)

Application

- Boolean Searches
- Logic Puzzles
- Logic Circuits

Today’s Class

- Propositional Equivalences
  - Tautology
  - Contradiction
  - Logical Equivalences
    - Using Truth Table
    - Important Equivalences (as Laws)
    - Arguments

Propositional Equivalences

- Replacement of a mathematical statement with another statement with the same truth value
- Mathematical statements with same set of truth values are propositionally equivalent
- Classification of compound propositions on the basis of their truth value
  - Tautology
  - Contradiction
  - Contingency

Tautology

- Proposition that is always true, no matter what the truth values of the propositions that occur in it
- Example:
  - $P \vee \neg P$

P	$\neg P$	$P \vee \neg P$
T	F	T
F	T	T

Contradiction

- Proposition that is always false, no matter what the truth values of the propositions that occur in it
- Example:
  - $P \wedge \neg P$

P	$\neg P$	$P \wedge \neg P$
T	F	F
F	T	F

Contingency

- A compound proposition that is neither a tautology nor a contradiction is called contingency
- Both tautology and contradiction are important in mathematical reasoning

Logical Equivalences

- Propositions that have the same truth values in all possible cases are called logically equivalent
- The compound propositions p and q are logically equivalent if $p \leftrightarrow q$ is a tautology
- The notation $p \equiv q$ denotes that p and q are logically equivalent
- $\equiv$ is not a connective
- $p \equiv q$ is not a compound proposition
- $\Leftrightarrow$ is also used instead of $\equiv$

Example

- Show that $\neg(p \vee q) \equiv \neg p \wedge \neg q$
- Show that $(\neg(p \vee q)) \leftrightarrow (\neg p \wedge \neg q)$ is a tautology

Example

- Show that $\neg(p \lor q) \equiv \neg p \land \neg q$
- Show that $(\neg(p \lor q)) \leftrightarrow (\neg p \land \neg q)$ is a tautology

p	q	$p \lor q$	$\neg(p \lor q)$	$\neg p$	$\neg q$	$\neg p \land \neg q$	$(\neg(p \lor q)) \leftrightarrow (\neg p \land \neg q)$
T	T	T	F	F	F	F	T
T	F	T	F	F	T	F	T
F	T	T	F	T	F	F	T
F	F	F	T	T	T	T	T

Practice

- $P \rightarrow Q$ and $\neg P \vee Q$ are logically equivalent
- $P \vee (Q \wedge R)$ AND $(P \vee Q) \wedge (P \vee R)$ are logically equivalent

Logical Equivalences

Equivalences	Laws
$p \wedge \mathbf{T} \equiv p$ $p \vee \mathbf{F} \equiv p$	Identity laws
$p \vee \mathbf{T} \equiv \mathbf{T}$ $p \wedge \mathbf{F} \equiv \mathbf{F}$	Domination laws
$p \vee p \equiv p$ $p \wedge p \equiv p$	Idempotent laws
$\neg(\neg p) \equiv p$	Double negation law
$p \vee q \equiv q \vee p$ $p \wedge q \equiv q \wedge p$	Commutative laws
$(p \vee q) \vee r \equiv p \vee (q \vee r)$ $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	Associative laws
$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	Distributive laws
$\neg(p \wedge q) \equiv \neg p \vee \neg q$ $\neg(p \vee q) \equiv \neg p \wedge \neg q$	De Morgan's laws
$p \vee (p \wedge q) \equiv p$ $p \wedge (p \vee q) \equiv p$	Absorption laws
$p \vee \neg p \equiv \mathbf{T}$ $p \wedge \neg p \equiv \mathbf{F}$	Negation laws

Logical Equivalences involving Conditional statements

- $p \rightarrow q \equiv \neg p \vee q$
- $p \rightarrow q \equiv \neg q \rightarrow \neg p$
- $p \vee q \equiv \neg p \rightarrow q$
- $p \wedge q \equiv \neg(p \rightarrow \neg q)$
- $\neg(p \rightarrow q) \equiv p \wedge \neg q$
- $(p \rightarrow q) \wedge (p \rightarrow r) \equiv p \rightarrow (q \wedge r)$
- $(p \rightarrow r) \wedge (q \rightarrow r) \equiv (p \vee q) \rightarrow r$
- $(p \rightarrow q) \vee (p \rightarrow r) \equiv p \rightarrow (q \vee r)$
- $(p \rightarrow r) \vee (q \rightarrow r) \equiv (p \wedge q) \rightarrow r$

Logical Equivalences involving Biconditionals

- $p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$
- $p \leftrightarrow q \equiv \neg p \leftrightarrow \neg q$
- $p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$
- $\neg(p \leftrightarrow q) \equiv p \leftrightarrow \neg q$

Practice Questions

Q. There are two restaurants next to each other. One has a sign board as: "**Good food is not cheap**". The other has a sign board as "**Cheap food is not good**". Are both the sign board saying the same thing?

- C= Cheap food
- G= Good food

First statement: $G \rightarrow \neg C$
Second statement: $C \rightarrow \neg G$

G	C	$\neg G$	$\neg C$	$G \rightarrow \neg C$	$C \rightarrow \neg G$
T	T	F	F	F	F
T	F	F	T	T	T
F	T	T	F	T	T
F	F	T	T	T	T

We need to find out whether they are logically equivalent ?? Using truth table.

Argument

- Argument is a sequence of propositions written

P1

P2

.

.

.

<u>Pn</u>

∴ Q

Where, P1, P2 … Pn are hypothesis or premises and Q is the conclusion.

Evaluating Logical Equivalences

- Example
- Show that the given conditional statement is a tautology without using truth tables

$p \rightarrow (p \vee q)$

L.H.S.	$= p \rightarrow (p \vee q)$	
	$\equiv \neg p \vee (p \vee q)$	Law of Implication
	$\equiv (\neg p \vee p) \vee q$	Associative Law
	$\equiv T \vee q$	Negation Law
	$\equiv T$	Domination law

Constructing new Logical Equivalences

- Show that $\neg(P \to Q)$ and $P \wedge \neg Q$ are logically equivalent
- L.H.S. $\equiv \neg(P \to Q)$

$\equiv \neg(\neg P \vee Q)$ {since, $p \to q \equiv \neg p \vee q$}

$\equiv \neg(\neg P) \wedge \neg Q$ {De Morgan's law, $\neg(p \vee q) \equiv \neg p \wedge \neg q$}

$\equiv P \wedge \neg Q$ {Double Negation law, $\neg(\neg p) \equiv p$}

$\equiv$ R.H.S.

hence, proved

Practice

- Show that $(P \wedge Q) \rightarrow (P \vee Q)$ is a tautology
- Show that $\neg(P \vee (\neg P \wedge Q))$ and $\neg P \wedge \neg Q$ are logically equivalent

4.3. Propositional logic, predicate logic, FOPL

4.4. Rules of inference, resolution refutation system (RRS), answer extraction from RRS

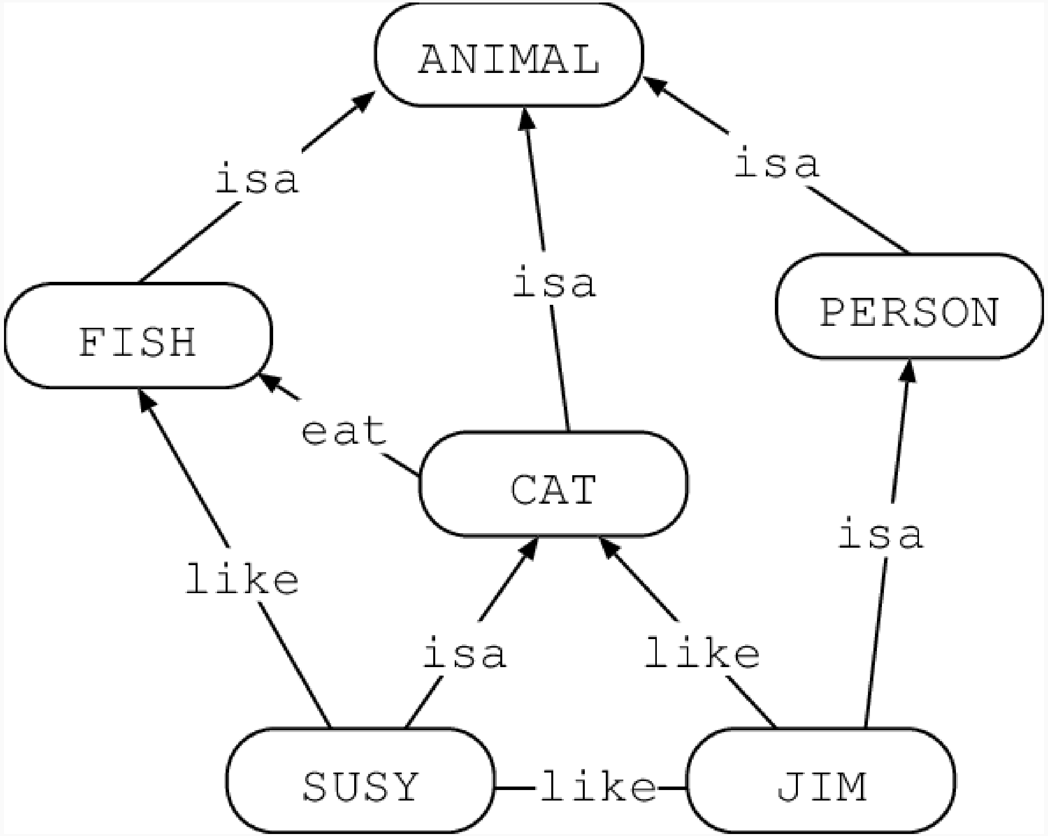
(Class lectured by RRA Sir) : Refer class notes & ppt provided

4.6 Semantic nets and frames

Semantics net

- Study of Meaning
- Semantic networks can
    - Show natural relationships between objects/concepts
    - Be used to represent declarative/descriptive knowledge
- Semantic Network is the graphical representation of the knowledge.
- Semantic networks are constructed using nodes linked by directional lines called arcs.

Example:



Jim is a person
Jim likes Susy.
Susy is cat & so on.

Pros/Cons of Semantic Nets

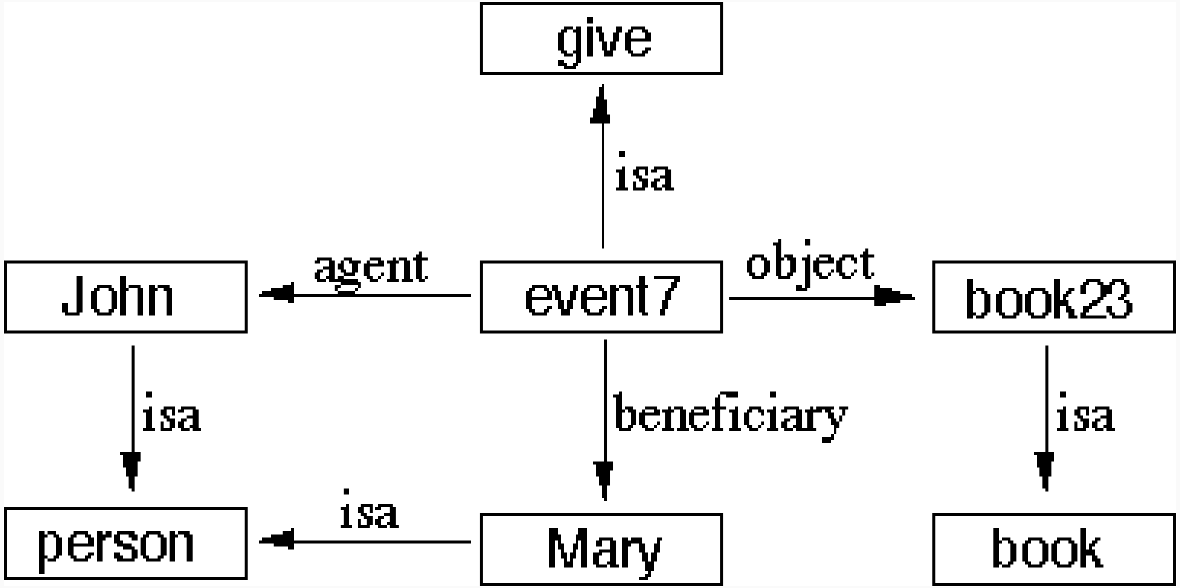
Advantages of a semantic network

1. Easy to understand.
2. Quick inference possible.
3. Supports default reasoning.

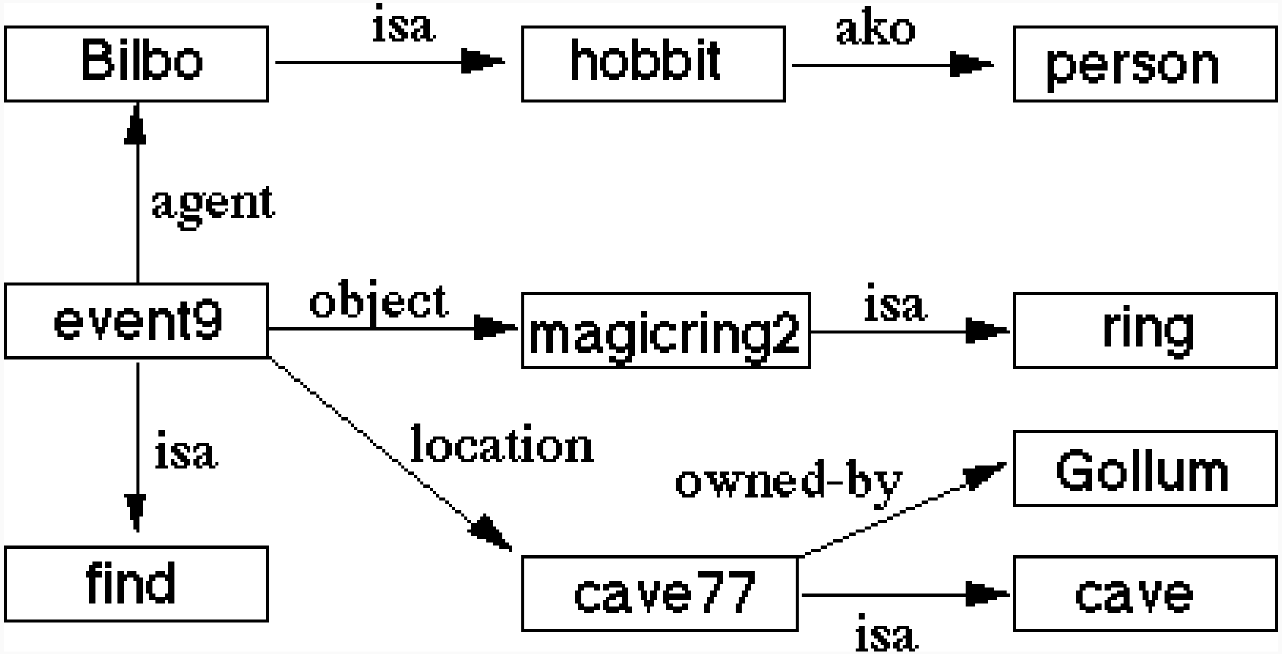
Disadvantages of a semantic network

1. incomplete(no expressed operational/procedural knowledge)
2. no interpretation standard
3. lack of standards, ambiguity in node/link descriptions

Example: John gives the book to Mary

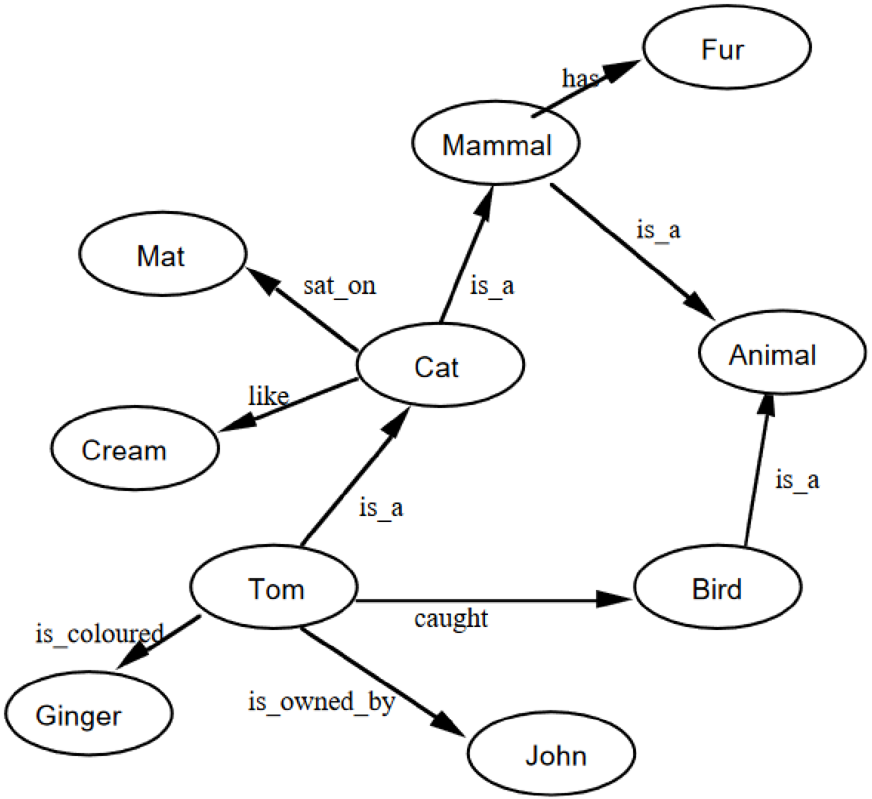


Example: Bilbo finds the magic ring in Gollum's cave



Draw semantic network of following clauses:

Tom is a cat. Tom caught a bird. Tom is owned by John. Tom is ginger in color. Cats like cream. The cat sat on the mat. A cat is a mammal. A bird is an animal. All mammals are animals. Mammals have fur.



Draw semantic network of following clauses:

Subset_of(People, Mammal),

Subset_of(Male, People),

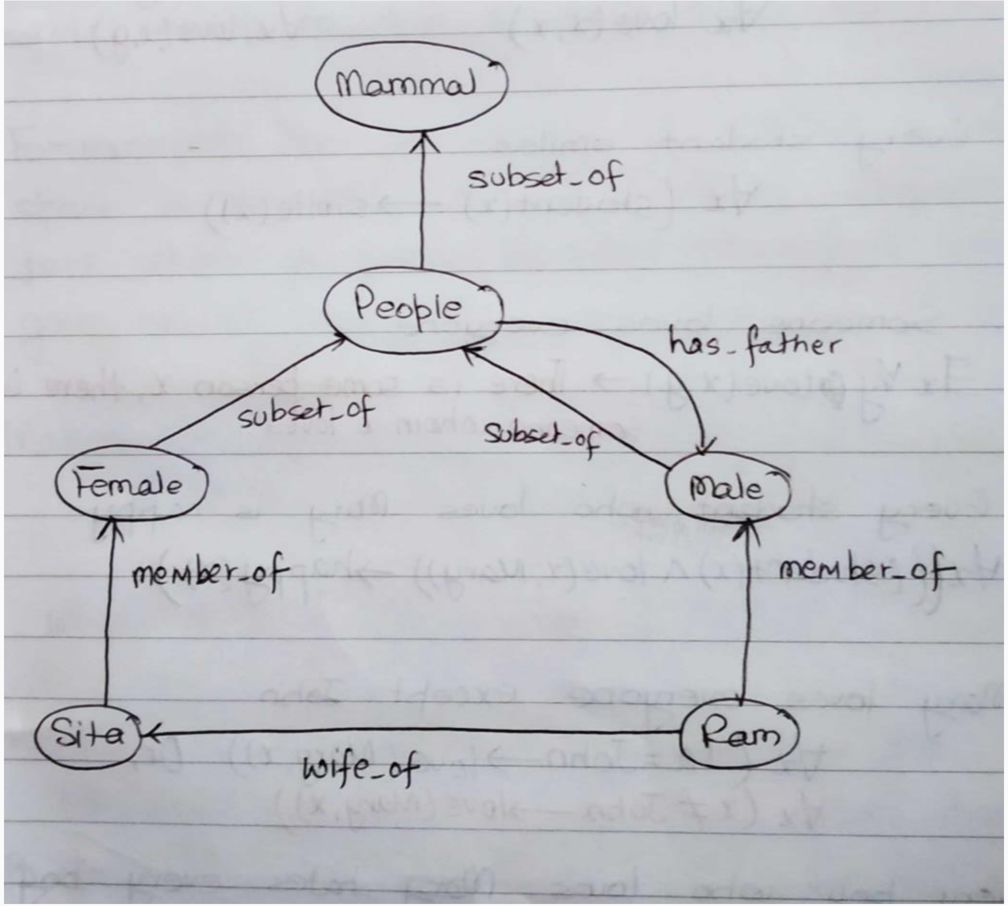
Subset_of(Female, People),

Has_Father(People, Male),

Member_of(Ram, Male),

Member_of(Sita, Female),

Wife_of(Sita,Ram).



Frames

- A frame is a data structure containing typical knowledge about a concept or object.
- The frame contains information on how to use the frame, what to expect next, and what to do when these expectations are not met.
- A frame represents knowledge about real world things (or entities).
- Each frame has a **name** and **slots.**

Frames

- Each piece of information about a particular frame is held in a slot. The information can contain:

- Facts or Data
- Procedures (also called procedural attachments): to find a value
- Default Values: For Data, For Procedures
- Other Frames or Sub frames

Pros/Cons

Disadvantages

- complex
- reasoning (inferencing) is difficult
- explanation is difficult, expressive limitation

Advantages

- knowledge domain can be naturally structured [a similar motivation as for the O-O approach].
- easy to include the idea of default values, detect missing values, include specialised procedures and to add further slots to the frames

Example of Two Frames:

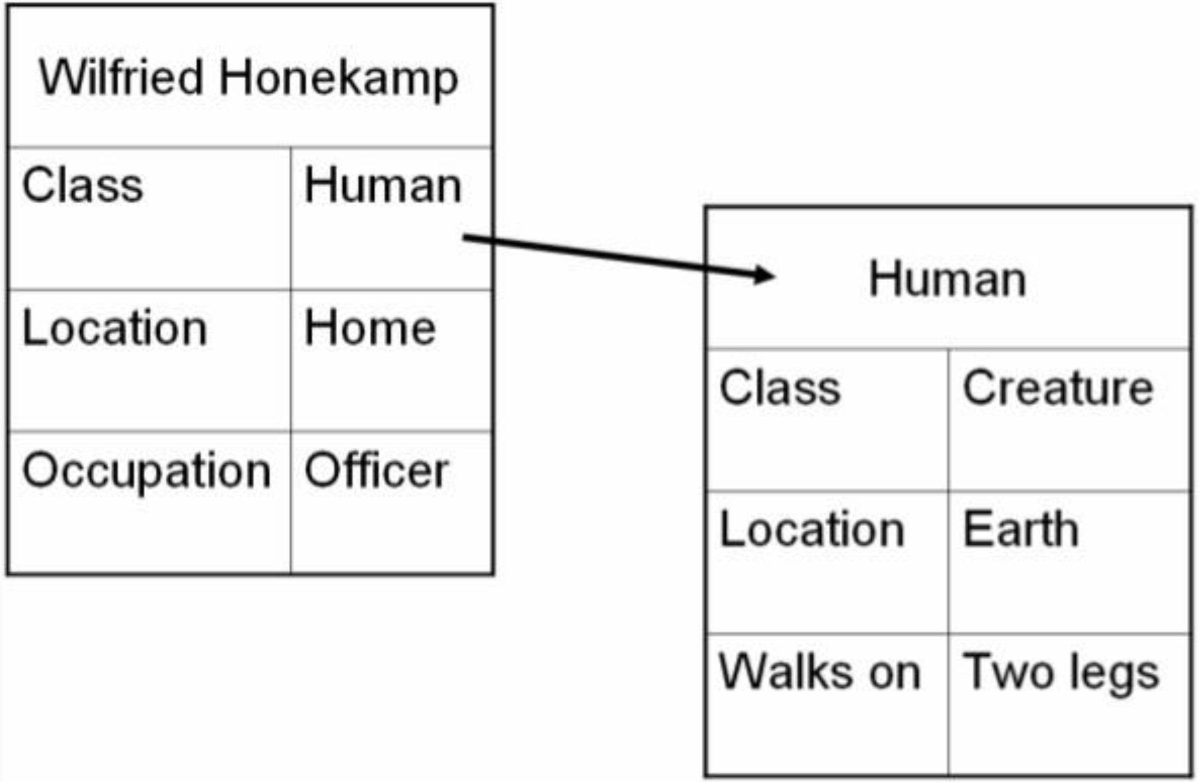
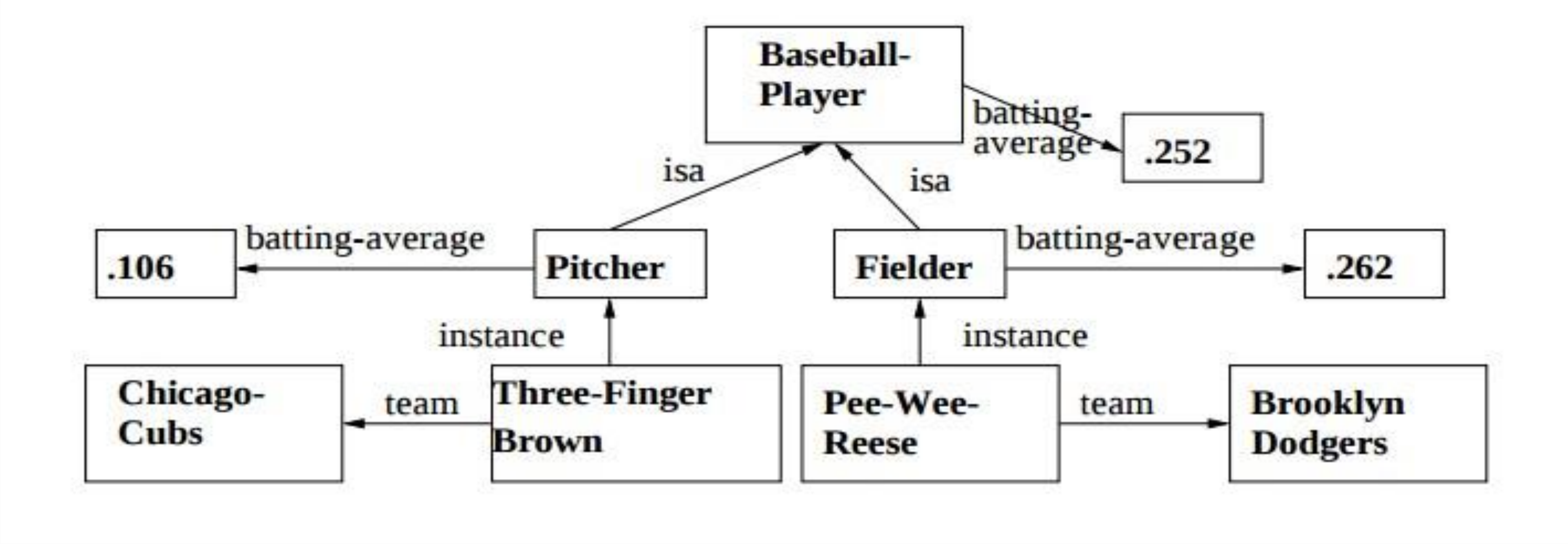


Fig: Two frames describing a human being

Converting Semantic Net into Frame



Sample frame for above semantic network

Baseball Player
is-a: Adult Male
batting average: .252
bats: equal to handed
team:
⋮

Fielder
is-a: Baseball player
batting average: .262

Pee-Wee-Reese
instance: Baseball player
team: Brooklyn Dodgers

THANK YOU

Any Queries ?