

# Artificial Intelligence

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**Chapter 2**

**What we will  
discussed  
today ?**

- Constraint Satisfaction Problem

## Chapter 2: Constraint Satisfaction Problem

- CSPs are mathematical problems defined as a set of objects whose state must satisfy a number of constraints.

### **CSPs consists of**

- A set of variables
- A domain for each variable
- A set of constraints
- *Examples:* Eight queens puzzle, Map coloring problem, Sudoku, **Crypt Arithmetic Problem**

- Crypt Arithmetic Problem (i)

$$\begin{array}{rcccc} & S & E & N & D \\ + & M & O & R & E \\ \hline - & - & - & - & - & - & - \\ M & O & N & E & Y \end{array}$$

**Constraints are:**

- No two digits can be assigned to same letter.
- Only single digit number can be assign to a letter.
- No two letters can be assigned to same digit.

**Initial State of Problem:**

D = ?, E = ?, Y = ?, N = ?, R = ?, O = ?, S = ?,  
M = ?

**Goal State of Problem:** The digits to the letters must be assigned in such a manner so that the sum is satisfied.

- Crypt Arithmetic Problem (i)

$$\begin{array}{cccc}
 C4 & C3 & C2 & C1 \\
 & S & E & N & D \\
 + & M & O & R & E \\
 \hline
 M & O & N & E & Y
 \end{array}$$

Let us suppose,

C1, C2, C3 & C4 indicates the carry bits out of the columns numbering from the right.

	M								
0	1	2	3	4	5	6	7	8	9

$$C4 = 1$$

- i. Initial guess  $M=1$ , since two single digit numbers plus a carry cannot have total more than 19  
i.e.  $M = 1$

# Crypt Arithmetic

## Problem (ii)

$C4 \ C3 \ C2 \ C1$

	S	E	N	D	
+	M	O	R	E	
-	-	-	-	-	-
M	O	N	E	Y	

ii. When  $M=1$ , the largest single digit number added to  $M (=1)$  can generate the sum of either 10 or 11 depend on the carry received from the carry sum i.e. on the value of  $C3$ . So the value of  $O$  is either 0 or 1. Because  $M$  is already 1 so the value of  $O$  is 0.

O	M								
0	1	2	3	4	5	6	7	8	9

$$C4 = 1$$

## Problem (iii)

# SEND

# + 1 M O R E

[illegible]

# MONEY

1 0

**iii.** That means the value of S is either 8 or 9, depending on the value of C3.

O	M								
0	1	2	3	4	5	6	7	8	9

$$C4 = 1$$

# Crypt Arithmetic

## Problem (iv)

*C4 C3 C2 C1*  
S E N D  
+ M O R E  
- - - -  
M O N E Y

iv. Now  $E + O + C2 = N$

Here  $O = 0$  and  $C2$  at most 1.

So  $E = N$  or  $E + 1 = N$  depending on the value of  $C2$ .

$E = N$  is not possible, so  $E + 1 = N$  and  $C2 = 1$ .

O	M								
0	1	2	3	4	5	6	7	8	9

$$C4 = 1$$

$$C2 = 1$$



# Crypt Arithmetic

## Problem (v)

$$\begin{array}{r} \text{SEN D} \\ + \text{M O R E} \\ \hline \text{M O N E Y} \end{array}$$

*C4 C3 C2 C1*

v.  $E + 1 = N$  means E and N are in pairs and this implies that carry is not possible. So, the value of C3 is 0.

Thus,

The value of **S** is **9**.

O	M								S
0	1	2	3	4	5	6	7	8	9

$$C4 = 1$$

$$C2 = 1$$

$$C3 = 0$$

# Crypt Arithmetic

## Problem (vi)

$$\begin{array}{r}
 \begin{array}{cccc} C4 & C3 & C2 & C1 \end{array} \\
 \begin{array}{r}
 \text{SEND} \\
 + \text{MORE} \\
 \hline
 \text{MONEY}
 \end{array}
 \end{array}$$

$$C4 = 1, C2 = 1, C3 = 0$$

vi. Since  $C2 = 1$ , we have

$$N + R + C1 = 10 + E$$

From case iv, we have  $E + 1 = N$

$$\text{So, } E + 1 + R + C1 = 10 + E$$

$$\text{or, } 1 + R + C1 = 10$$

(R can be either 9 or 8, since  $S = 9$ . So, **R = 8** and **C1 = 1**)

O	M							R	S
0	1	2	3	4	5	6	7	8	9

$$C4 = 1, C2 = 1, C3 = 0, C1 = 1$$

# Crypt Arithmetic

## Problem (vii)

$C_4 \ C_3 \ C_2 \ C_1$

S E N D

+ M O R E

- - - - -

M O N E Y

$C_4 = 1, C_3 = 0, C_2 = 1, C_1 = 1$

O	M							R	S
0	1	2	3	4	5	6	7	8	9

vii. Now  $D + E = 10 + Y$  (Since,  $C_1 = 1$ )

- Here, the maximum possibility of  $D + E$  is  $6 + 7$  i.e. 13.
- Other possibilities are 12, 11 and 10.
- If the value of  $D$  &  $E$  are either 6 or 7. Then, the value of  $E$  is either 6 or 7.

*If  $E = 6$  then  $D = 7$*

( which is not possible because  $E + 1 = N$  )

*If  $E = 7$  then  $D = 8$*

(but  $R = 8$ , so not possible)

# Crypt Arithmetic

## Problem (vii)

*C 4 C 3 C 2 C 1*

S E N D

+ M O R E

- - - - -

M O N E Y

*C4 = 1, C3 = 0, C2 = 1, C1 = 1*

O	M							R	S
0	1	2	3	4	5	6	7	8	9

$$D + E = 10 + Y \text{ (Since, } C1 = 1 \text{)}$$

- Again, if  $D + E = 11$  or  $10$  then  $Y = 1$  or  $0$  which is not possible.

- So,  $D + E = 12$  i.e.  $Y = 2$

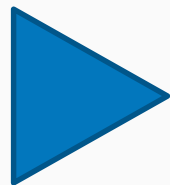
and the combination of  $D$  and  $E$  is  $5$  and  $7$ .

i.e.  $D = 7$  and  $E = 5$ .

Also,  $N = E + 1 = 6$

O	M	Y			E	N	D	R	S
0	1	2	3	4	5	6	7	8	9

- Crypt Arithmetic Problem (viii)

$$\begin{array}{r}
 \text{S E N D} \\
 + \text{M O R E} \\
 \hline
 \text{M O N E Y}
 \end{array}$$


$$\begin{array}{r}
 9567 \\
 + 1085 \\
 \hline
 10652
 \end{array}$$

O	M	Y			E	N	D	R	S
0	1	2	3	4	5	6	7	8	9

THANK YOU

Any Queries ?