# Properties of FS

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| CT | DT |
| --- | --- |
| $T \begin{cases} x(t) \rightarrow a_k \\ x_1(t) \rightarrow b_k \\ x_2(t) \rightarrow c_k \\ h(t) \rightarrow d_k \end{cases}$ Not periodic | $N \begin{cases} x[n] \rightarrow a_k \\ x_1[n] \rightarrow b_k \\ x_2[n] \rightarrow c_k \\ h[n] \rightarrow d_k \end{cases}$ Periodic with period N |
| ① Linearity: $A x_1(t) + B x_2(t) \rightarrow A b_k + B c_k$ | ① Linearity: $A x_1[n] + B x_2[n] \rightarrow A b_k + B c_k$ |
| ② Time shift: $x(t - t_0) \rightarrow \left(e^{-j\omega_0 k t_0}\right) a_k$ | ② $x[n-n_0] \rightarrow \left(e^{-j\omega_0 k n_0}\right) a_k$ |

③ Time reversal

$$x(-t) \longrightarrow a_{-k}$$

④ Conjugate

$$x^*(t) \longrightarrow a^*_{-k}$$

⑤ Convolution property

$$\underbrace{x(t) \circledast h(t)}_{\text{Periodic convolution } T} \longrightarrow T\, a_k \cdot d_k$$

③ Time reversal

$$x[-n] \longrightarrow a_{-k}$$

④ Conjugate

$$x^*[n] \longrightarrow a^*_{-k}$$

⑤ Convolution

$$\underbrace{x[n] \circledast h[n]}_{\text{Periodic convolution } N} \longrightarrow N\, a_k \cdot d_k$$

6 $x(t) \cdot h(t) \longrightarrow a_k \circledast d_k$

7 <u>Parseval's Theorem:</u>

$$\frac{1}{T}\int_T |x(t)|^2 dt = \sum_{\langle k \rangle} |a_k|^2$$

8 <u>Derivative</u>

$$\frac{dx(t)}{dt} \longrightarrow (jw_0 k)\, a_k$$

6 $x[n] \cdot h[n] \longrightarrow a_k \circledast d_k$

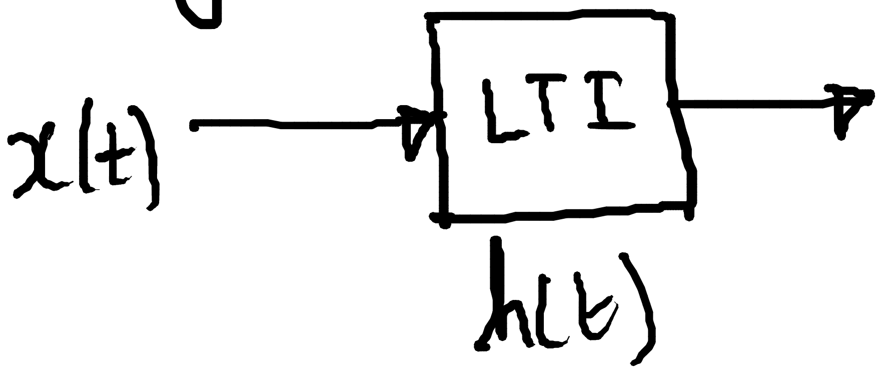
7 <u>Parseval's Theorem</u> :

$$\frac{1}{N}\sum_{\langle n \rangle} |x[n]|^2 = \sum_{\langle k \rangle} |a_k|^2$$

8 First order difference

$$x[n] - x[n-1]$$

# Fourier series & LTI Systems




$$y(t) = x(t) \circledast h(t)$$

$$a e^{st} \longrightarrow a H(s) e^{st}$$

$$\boxed{H(s) = \int_{-\infty}^{\infty} h(\tau) e^{-s\tau} d\tau}$$

System function

$$\underbrace{\sum_{\langle k \rangle} a_k e^{s_k t}}_{x(t)} \longrightarrow \sum_{\langle k \rangle} a_k H(s_k) e^{s_k t}$$

$j\omega_0 k$

$$\longrightarrow y(t) = \sum_{\langle k \rangle} a_k H(s_k) e^{s_k t}$$

$$H(s_k) = \int_{-\infty}^{\infty} h(t) e^{-s_k \tau} d\tau$$

$$s = \cancel{\sigma} + j\omega$$

Frequency response

Example : $x(t) = \cos\left(\frac{2\pi}{5}t\right)$ ; $h(t) = e^{-t}u(t)$

$$a_1 = \frac{1}{2} \; ; \; a_{-1} = \frac{1}{2}$$

$$x(t) = \frac{1}{2} e^{j\left(\frac{2\pi}{5}\right)1t} + \frac{1}{2} e^{-j\left(\frac{2\pi}{5}\right)t}$$

$$\sum a_k e^{jw_0kt}$$

$$H(s) = \int_{-\infty}^{\infty} h(t)\, e^{-st}\, dt$$

$$H(s) = \int_{0}^{\infty} e^{-t} e^{-st}\, dt$$

$$= \int_{0}^{\infty} e^{-(1+s)t}\, dt = \frac{1}{+(1+s)}$$

$$H(jw) = \frac{1}{1+jw}$$

For $k=1$ ; $H(jw_0) = \left[\frac{1}{1+j\left(\frac{2\pi}{5}\right)}\right)$

For $k=-1$ ; $H(-jw_0) = \frac{1}{1-j\left(\frac{2\pi}{5}\right)}$

$$H(jw_0k) = \frac{1}{1+jw_0k}$$

$$\underbrace{\underbrace{\frac{1}{2}e^{j\left(\frac{2\pi}{5}\right)t}}_{k=1} + \underbrace{\frac{1}{2}e^{-j\left(\frac{2\pi}{5}\right)t}}_{k=-1}}_{x(t)=}$$

$$x(t) = \frac{1}{2}e^{j\left(\frac{2\pi}{5}\right)t} + \frac{1}{2}e^{-j\left(\frac{2\pi}{5}\right)t}$$

$$\downarrow$$

$$y(t) = \underbrace{\frac{1}{2}}_{a_1}\underbrace{\left(\frac{1}{1+j\left(\frac{2\pi}{5}\right)}\right)}_{H(j\omega_0)}e^{j\left(\frac{2\pi}{5}\right)t} + \underbrace{\frac{1}{2}}_{a_{-1}}\underbrace{\left(\frac{1}{1-j\left(\frac{2\pi}{5}\right)}\right)}_{H(-j\omega_0)}e^{-j\left(\frac{2\pi}{5}\right)t}$$

$$
\begin{tikzcd}
x(t) \arrow[r] \arrow[dd] & \boxed{\mathrm{LTI}} \arrow[r] & y(t) \arrow[dd] \\
 & h(t) \arrow[d] & \\
a_k & d_k & T\, a_k \cdot d_k
\end{tikzcd}
$$