Complex exponentials are Eigenfunctions of LTI systems :

$$\longrightarrow \boxed{LTI} \longrightarrow$$

Eigen → Self

German

$$\boxed{Av = \lambda v}$$

$$|A - \lambda I| = 0$$

v

$$e^{j\omega t}$$

$$\omega = \frac{2\pi}{T}$$

$$e^{2x}$$

$$re^{j\omega t}$$

$$e^{j\omega t} \longrightarrow H(j\omega)\, e^{j\omega t}$$

$$\boxed{e^{st} \longrightarrow H(s)\, e^{st}}$$

$s = \cancel{\sigma} + j\omega$

$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

$$= \int_{-\infty}^{\infty} h(\tau)\, x(t-\tau)\, d\tau$$

$$= \int_{-\infty}^{\infty} h(\tau)\, e^{s(t-\tau)}\, d\tau$$

$$= \underbrace{e^{st}}\ \underbrace{\int_{-\infty}^{\infty} h(\tau)\, e^{-s\tau}\, d\tau}_{H(s)}$$

$$y(t) = H(s)\, e^{st}$$

Eigen value

$$a_1 e^{j\omega_1 t} \longrightarrow a_1 H(j\omega_1)\, e^{j\omega_1 t}$$

$$+ \qquad\qquad\qquad\qquad +$$

$$a_2 e^{j\omega_2 t} \longrightarrow a_2 H(j\omega_2)\, e^{j\omega_2 t}$$

$$\sum_k a_k e^{s_k t} \longrightarrow \sum_k a_k H(s_k)\, e^{s_k t}$$

Euler

Lagrange

JBJ Fourier

$$x(t) = x(t+T)$$

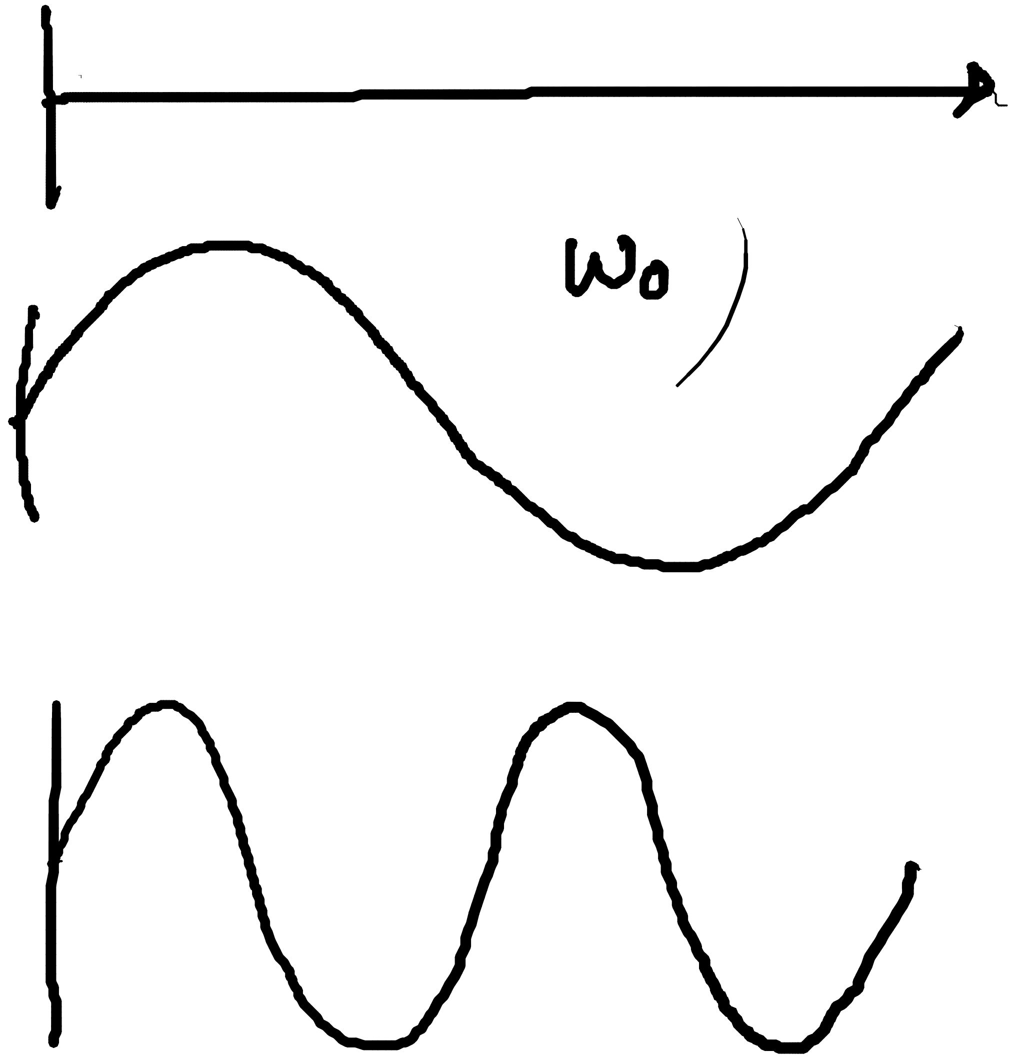
$$\downarrow$$

$$T = \frac{2\pi}{\omega}$$

$$= \frac{2\pi}{2\pi F}$$

$$k\omega_0$$

Bablonyia



Laplace Loeronix Monge Lagrange.

Fourier

$$\cos(\theta) = \frac{e^{j\theta} + e^{-j\theta}}{2}$$

$$= \frac{1}{2} e^{j\theta} + \frac{1}{2} e^{-j\theta}$$

$$x(t) = \sum_{k} a_k e^{jk\omega_0 t}$$

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t} \qquad \omega_0 = \frac{2\pi}{T_0}$$

Multiply both sides $e^{-j\omega_0 n t}$ and then integrate over a period.

$$x(t)\, e^{-j\omega_0 n t} = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t} e^{-jn\omega_0 t}$$

$$x(t)\, e^{-j\omega_0 n t} = \sum_{k=-\infty}^{\infty} a_k e^{j\omega_0 (k-n) t}$$

Integrating both sides over a period

$$\int_0^T x(t)\, e^{-j\omega_0 n t} = \int_0^T \sum_{k=-\infty}^{\infty} a_k e^{j\omega_0 (k-n) t}$$

$$\int_0^T x(t)\,e^{-j\omega_0 n t}\,dt = \sum_{k=-\infty}^{\infty} a_k \int_0^T e^{j\omega_0 (k-n) t}\,dt$$

$$\boxed{\int_0^T e^{j\omega_0 (k-n) t}\,dt = \begin{cases} T & ; \; n = k \\ 0 & ; \; n \neq k \end{cases}}$$

$$\int_0^T x(t)\,e^{-j\omega_0 n t} = a_n T$$

$$\boxed{a_n = \frac{1}{T} \int_0^T x(t)\,e^{-j\omega_0 n t}}$$

$$a_k = \frac{1}{T} \int_0^T x(t)\, e^{-j\omega_0 k t}\, dt$$

Analysis

$$x(t) = \sum_{k=-\infty}^{\infty} a_k\, e^{j\omega_0 k t}$$

Synthesis Equation

e^{st}

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