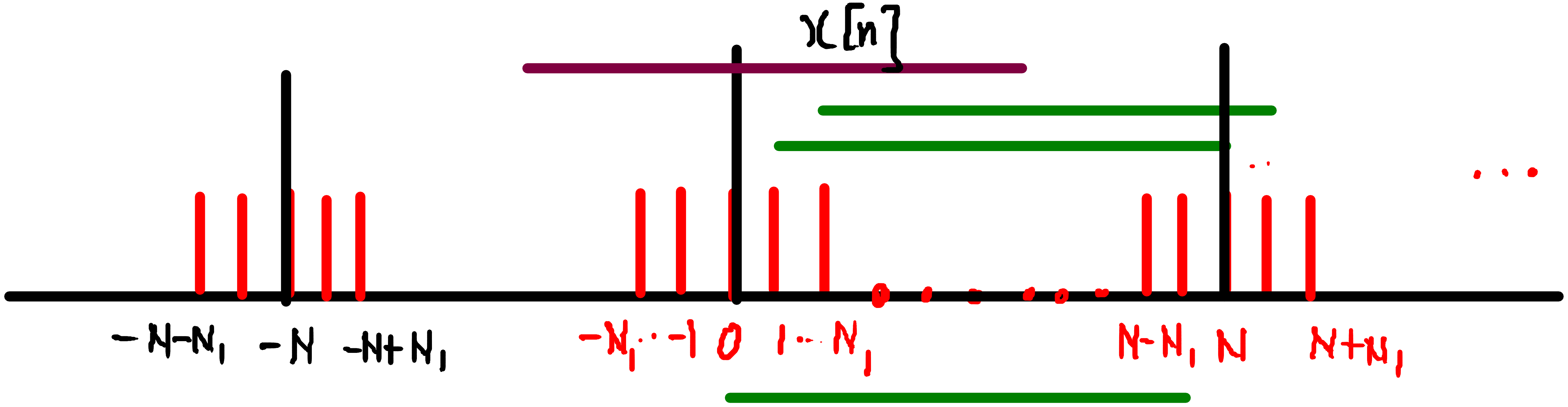
Example

14/9/17

$$x[n] = 1 \quad ; \quad -N_1 \leq n \leq N_1$$
$$= 0 \quad ; \quad \text{otherwise}$$

This signal is periodic with period N and $N_1 < N$



$$C_k = \frac{1}{N} \sum_{n=\langle N \rangle} x[n]\, e^{-j\omega_0 k n}$$

$$C_k = \frac{1}{N} \sum_{n=-N_1}^{N_1} x[n]\, e^{-j\omega_0 k n} \qquad \omega_0 = \frac{2\pi}{N}$$

$$C_k = \frac{1}{N} \sum_{n=-N_1}^{N_1} e^{-j\omega_0 k n} = \frac{1}{N} \sum_{m=0}^{2N_1} e^{-j\omega_0 k (m-N_1)} \quad \left\{ \begin{array}{l} e^{-j\omega_0 k m} \\ \times\, e^{+j\omega_0 k N_1} \end{array} \right.$$

$n + N_1 = m$

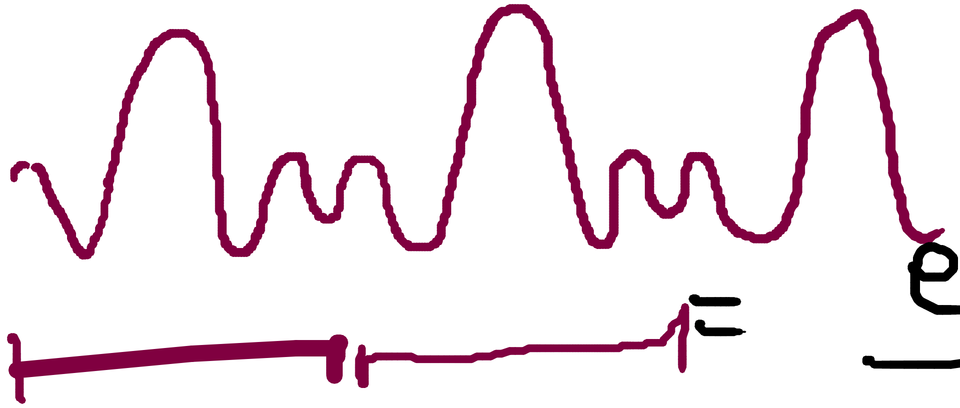
$n \rightarrow -N_1$ to N_1

$m \rightarrow 0$ to $2N_1$

$$= \frac{1}{N} e^{j\omega_0 k N_1} \sum_{m=0}^{2N_1} \left(e^{-j\omega_0 k m} \right)$$

$$= \left(\frac{e^{j\omega_0 k N_1}}{N} \right) \frac{1 - e^{-j\omega_0 k (2N_1+1)}}{1 - e^{-j\omega_0 k}}$$

$$= \frac{e^{j\omega_0 k N_1}}{N} \frac{e^{-j\omega_0 k (2N_1+1)/2}}{e^{-j\omega_0 k/2}} \frac{\left[e^{j\omega_0 k (2N_1+1)/2} - e^{-j\omega_0 k (2N_1+1)/2} \right]}{\left[e^{j\omega_0 k/2} - e^{-j\omega_0 k/2} \right]}$$

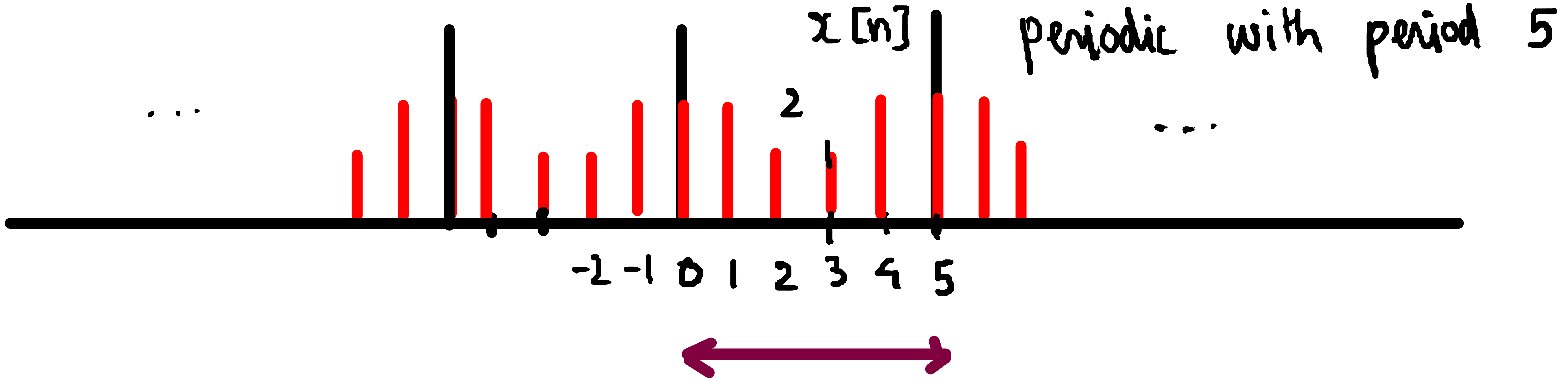


$$= \frac{e^{j\omega_0 k N_1}}{N} \; \frac{e^{-j\omega_0 k\frac{(2N_1+1)}{2}}}{e^{-j\omega_0 k/2}} \; \frac{\sin\left(\frac{\omega_0 k(2N_1+1)}{2}\right)}{\sin\left(\omega_0 k/2\right)}$$

$$= \frac{\cancel{e^{j\omega_0 k N_1}}}{N} \; \frac{\cancel{e^{-j\omega_0 k N_1}} \; \cancel{e^{-j\omega_0 \frac{k}{2}}}}{\cancel{e^{-j\omega_0 k/2}}} \; \frac{\sin\left(\frac{\omega_0 k(2N_1+1)}{2}\right)}{\sin\left(\frac{\omega_0 k}{2}\right)}$$

$$c_k \begin{cases} = \dfrac{1}{N} \dfrac{\sin\left(\frac{\omega_0 k(2N_1+1)}{2}\right)}{\sin\left(\frac{\omega_0 k}{2}\right)} & ; \quad k \neq 0, \pm N, \pm 2N \\[2ex] = \dfrac{1}{N}(2N_1+1) & ; \quad k = 0, \pm N \end{cases}$$

Example :



1890 } Albert Michelson

N= 80

↓

Josiah Gibbs