Periodic Signals

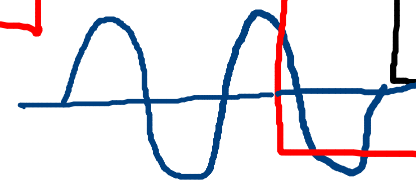
3 Aug 2017

$$x(t) = x(t+T) \quad \boxed{\forall\, t}$$

Every portion of the signal must repeat (for all t)

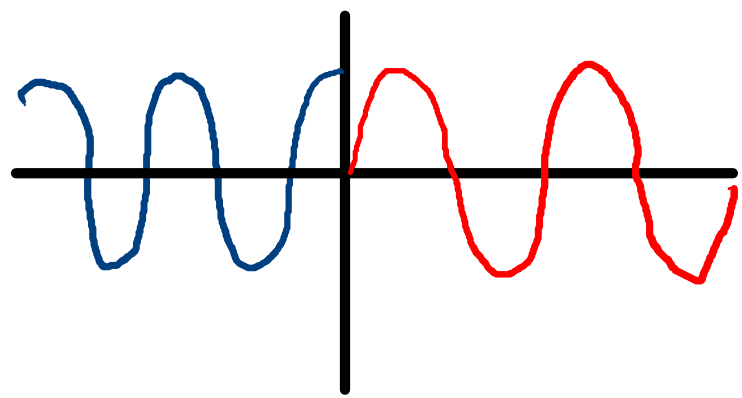
* Continuous-time sinusoids are always periodic with period $\boxed{T = \frac{2\pi}{|\omega|}}$

e.g.: $x(t) = \sin(t)$ or $x(t) = \cos(t)$ $\forall\, t$



$T = \frac{2\pi}{\omega} = 2\pi$ Periodic

e.g.: $x(t) = \begin{cases} \sin(t) ; & t \geq 0 \\ \cos(t) ; & t < 0 \end{cases}$



Not periodic

Discrete-time Sinusoids :

If , $x[n] = x[n+N] \quad \forall\, n$

then signals are said to be periodic

$x[n] = \cos(\omega n)$

$x[n+N] = \cos(\omega(n+N)) = \cos(\omega n + \omega N)$

No factors in common

for $\omega = 0$

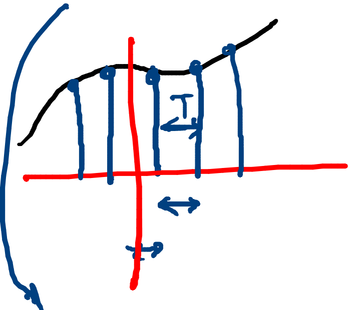
for $\omega \neq 0 \quad \omega N = K(2\pi)$

For discrete-time sinusoids to be periodic $\Rightarrow$ $\boxed{\dfrac{2\pi}{\omega} = \dfrac{N}{K}}$

$\left.\dfrac{N}{K}\right\}$ rational number

ratio of two integers

$x(t) \rightarrow$



$x(nT)$

$x[n]$

$$x(t) = x(t+T)$$

b

π

1.23

$$x[n] = x[n+N]$$

$x[2+1.5]$

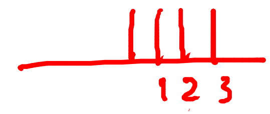
$x[3.5]$

$$\frac{2\pi}{\omega} = \frac{N}{k}$$

$\frac{2\pi}{\omega} = T$ = real value

2.57	1.33	3	

1 2



$n = 0, \pm 1, \pm 2$ .....

$x[\cdot]$

sample number

e.g $x[n] = \cos\left(\frac{\pi}{6} n\right)$

$\omega = \pi/6 \qquad \frac{2\pi}{|\omega|} = \frac{N}{k} = \frac{12}{1}$

$$\boxed{N = 12}$$

e.g $x[n] = \cos\left(\frac{8\pi}{31} n\right)$

$$\frac{2\pi}{\omega} = \frac{\cancel{2}\cancel{\pi}}{{}_4\cancel{8}\cancel{\pi}/31}$$

$\omega = \frac{8\pi}{31} \qquad \frac{2\pi}{|\omega|} = \frac{N}{k} = \frac{31}{4} \qquad = \frac{31}{4}$

$$\boxed{N = 31}$$

e.g $x[n] = \cos\left(\frac{n}{4}\right)$

$\omega = \frac{1}{4}$ $\qquad \frac{2\pi}{|\omega|} = \frac{N}{k} = \frac{2\pi}{1/4} = 8\pi$

~~irrational~~

Not ratio of integers
(Not rational)

$\therefore$ signal is NOT periodic

e.g $x[n] = \cos(\pi/4\, n) \rightarrow N = 8$

$x[n] = \cos(\pi/3\, n) \rightarrow N = 6$

$x[n] = \cos(\pi/4\, n) + \cos(\pi/3\, n)$?

$N = \text{LCM}(8,6) = 24$

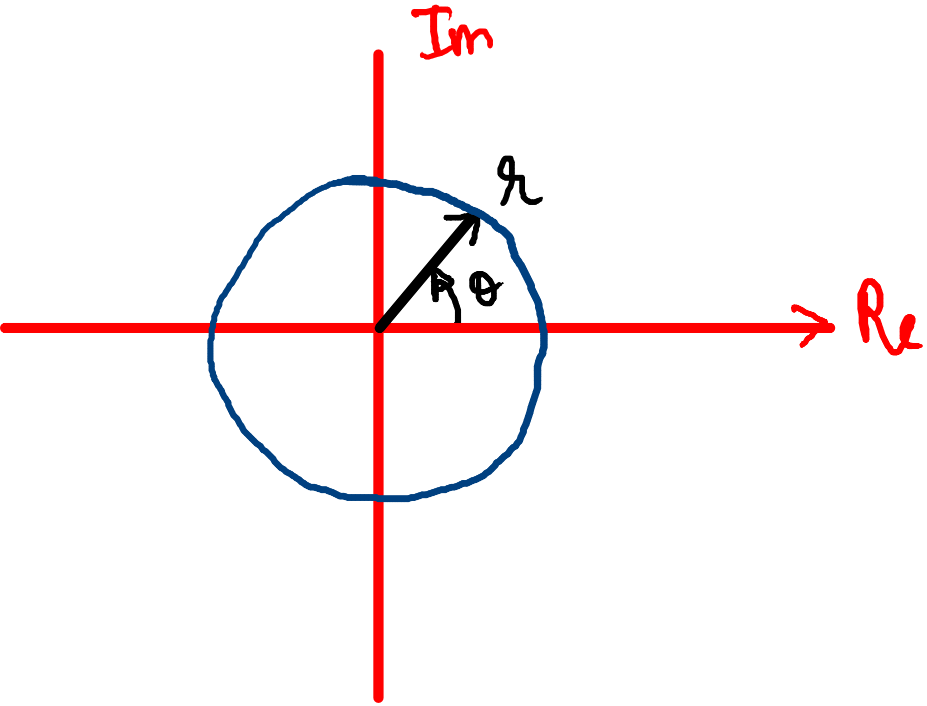
$$\cos(\pi/3) + \cos(\pi/4)$$

$$2 \cos\left(\frac{\pi/3 + \pi/4}{2}\right) \cos\left(\frac{\pi/3 - \pi/4}{2}\right)$$

Complex Exponentials :

Continuous-time	Discrete-time
$x(t) = e^{j\Omega t}$	$x[n] = e^{j\omega n}$
$x(t+T) = e^{j\Omega(t+T)}$	$x[n+N] = e^{j\omega[n+N]}$
$= e^{j\Omega t} e^{j\Omega T}$	$= e^{j\omega n} e^{j\omega N}$
$e^{j\Omega T} = 1$	$e^{j\omega N} = 1$
$\Omega T = 2\pi$	$\Rightarrow \omega N = k(2\pi)$
$\boxed{T = \frac{2\pi}{\Omega}}$	$\Rightarrow \boxed{\frac{\omega}{2\pi} = \frac{k}{N}}$

This signal is always Periodic with period



$re^{j\theta}$

e.g $x_1[n] = e^{j\left(\frac{2\pi}{3}n\right)}$

e.g $x_2[n] = e^{j\left(\frac{3\pi}{4}n\right)}$

e.g $x[n] = x_1[n] + x_2[n]$