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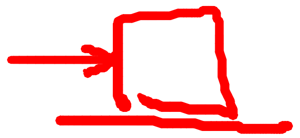
Signal : mapping of independent → dependent variable(s)

function of one or more independent variable

System:

$$\begin{tikzcd} x(t) \arrow[r] \arrow[d] & \boxed{} \arrow[r] & y(t) \arrow[d] \\ \text{mapping} \arrow[rr] & & \text{mapping} \end{tikzcd}$$



$$f(t) = m\frac{dv(t)}{dt}$$

$$i(t) = C\frac{dv(t)}{dt}$$

$$|x(t)| < B < \infty$$

④ Time invariance :

Check the input output relationship for all time ($\forall t$)

$$x(t) \longrightarrow y(t)$$

$$x(t-t_0) \longrightarrow y(t-t_0)$$

$$\forall\, x, t_0$$

Then this system is time-invariable

e.g $$y(t) = t\,x(t)$$

$$x(t) \longrightarrow t\,x(t)$$

Shifting the input by t_0 ; $x(t-t_0) \longrightarrow t\,x(t-t_0)$

Lets shift the output by t_0 ; $y(t-t_0) = (t-t_0)\,x(t-t_0)$

NO

Not Time-Invariant

Time Varying

e.g : $y(t) = \sin(x(t))$

$x(t) \longrightarrow \sin(x(t))$

Shifting the i/p $x(t-t_0) \longrightarrow \sin(x(t-t_0))$

Shifting the o/p $y(t-t_0) \longrightarrow \sin(x(t-t_0))$

YES / SAME

Yes TIME INVARIANT

e.g.

$$y(t) = x(2t)$$

$$x(t) \longrightarrow x(2t)$$

Shifting I/p : $x(t-t_0) \longrightarrow x(2t-t_0)$

NOT same.

Shifting O/p : $y(t-t_0) \longrightarrow x(2(t-t_0)) = x(2t-2t_0)$

Replace every t by $(t-t_0)$

$\therefore$ System is NOT time-invariant

Time varying

e.g. $y(t) = x(t) + 5$

Step ① : $x(t) \longrightarrow x(t) + 5$

Step ② : Shift the input by t_0

$x(t - t_0) \longrightarrow x(t - t_0) + 5$

Step ③ : Shift the output by t_0 (Replace each t by $(t - t_0)$)

$y(t - t_0) = x(t - t_0) + 5$

YES

YES $\boxed{\text{TIME Invariant}}$

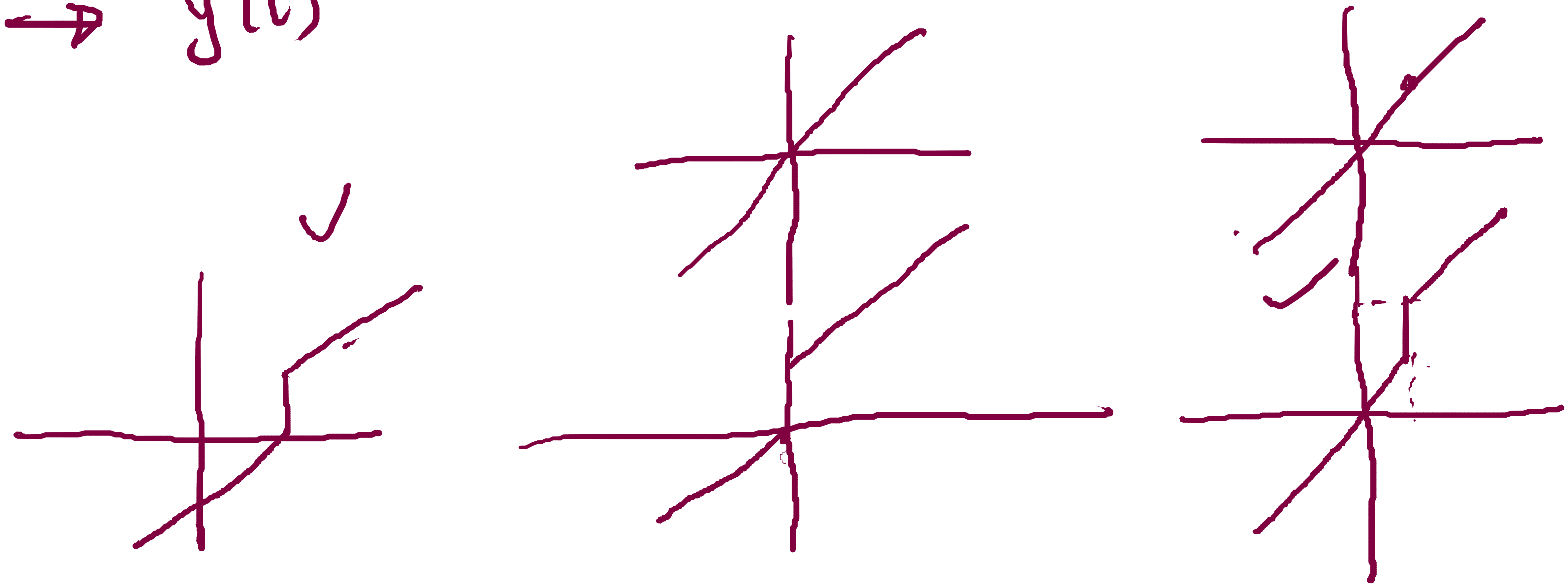
e.g $\quad y[n] = [x[n]]^2 \quad$ Time - invariant

e.g $\quad y(t) = x(t) + x(t-1) - x(t-2)$

$x(t-1) = u(t-1)$

$y(t) = t + x(t)$

$x(t) \rightarrow y(t)$



e.g. $y(t) = t\,x(t)$

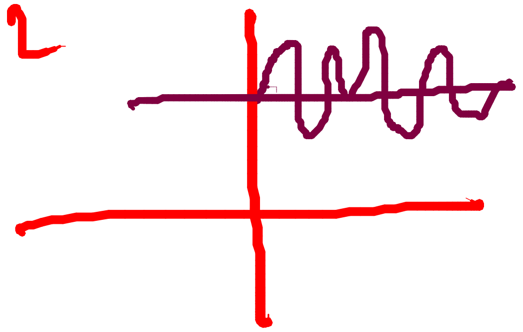
If you want to prove certain property
that property must be proved for all x all t

But, If you want to disprove a property
one counter example is enough

5 Additivity :

Protein bar 1 $\quad x_1(t) \longrightarrow y_1(t) \quad$ Energy 1

Protein bar 2 $\quad x_2(t) \longrightarrow y_2(t) \quad$ Energy 2



$$x_1(t) + x_2(t) \longrightarrow y_1(t) + y_2(t)$$

Then the system is said to be additive

e.g.

$$y(t) = [x(t)]^2$$

Step 1 :

$$x_1(t) \longrightarrow [x_1(t)]^2$$

$$x_2(t) \longrightarrow [x_2(t)]^2$$

$$x_1(t) + x_2(t) \longrightarrow [x_1(t) + x_2(t)]^2$$

$$y_1(t) + y_2(t) = [x_1(t)]^2 + [x_2(t)]^2$$

$$x_1(t) + x_2(t) \longrightarrow [x_1(t) + x_2(t)]^2$$

$$= [x_1(t)]^2 + [x_2(t)]^2 + 2x_1(t)\,x_2(t)$$

when inputs are added & given to the system

$$y_1(t) + y_2(t) = [x_1(t)]^2 + [x_2(t)]^2$$

when individual outputs are added

Not additive

e.g. $y(t) = x(t) + 5$

$$x_1(t) \longrightarrow x_1(t) + 5$$

$$x_2(t) \longrightarrow x_2(t) + 5$$

$$x_1(t) + x_2(t) \longrightarrow x_1(t) + x_2(t) + 5$$

$$\neq y_1(t) + y_2(t)$$

⑥ Homogeneity / Scaling :

e.g $y(t) = x(t) + 5$

e.g $y(t) = t\,x(t)$

e.g $y(t) = t^2 x(t-1)$

$x(t) \longrightarrow y(t)$

$c\,x(t) \longrightarrow c\,y(t)$

Then the system is said to be Homogeneous.

$\forall\ c, x, t$

Independent variable → Real

Dependent variables and Constant → Complex

e.g $y(t) = [x(t)]^2$

① $x(t) \longrightarrow [x(t)]^2$

② $c\,x(t) \longrightarrow c^2 [x(t)]^2$ } scaling the input

③ $c\,y(t) = c\,[x(t)]^2$ } scaling the output

steps ② ≠ ③

Not Homogeneous