Continuous-time Fourier Transform: $x(t) \longleftrightarrow X(j\omega)$

21 Sept 2017

$$\boxed{\begin{aligned} X(j\omega) &= \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt \\ x(t) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega)\, e^{j\omega t}\, d\omega \end{aligned}}$$

$T \to \infty$

$\omega_0 \to 0$

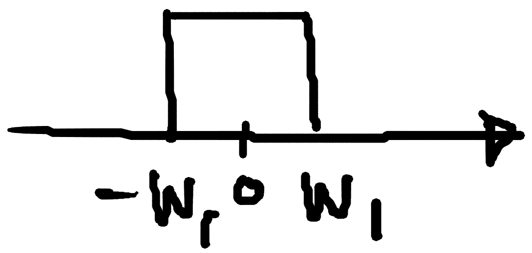
$\underbrace{k\omega_0}_{\text{discrete}} \to \underbrace{\omega}_{\text{continuous}}$

e.g. $x(t) = \begin{cases} 1 & ; t < |T_1| \\ 0 & ; t > |T_1| \end{cases}$

$$X(j\omega) = \int_{-T_1}^{T_1} e^{-j\omega t}\, dt$$

$$= \left. \frac{e^{-j\omega t}}{-j\omega} \right|_{-T_1}^{T_1} = \frac{2\sin(\omega T_1)}{\omega}$$

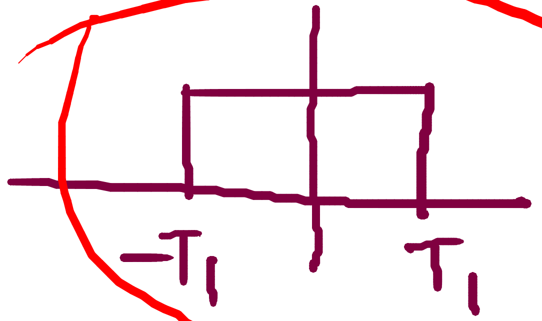
$$X(j\omega) = \begin{cases} 1 & ; \ \omega < |W_1| \\ 0 & ; \ \omega > |W_1| \end{cases}$$



$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega) e^{j\omega t} \, d\omega$$

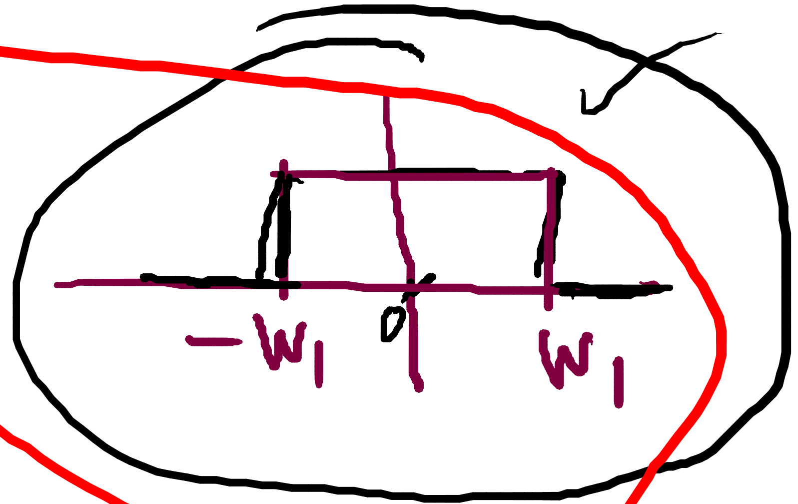
$$= \frac{1}{2\pi} \int_{-W_1}^{W_1} e^{j\omega t} \, d\omega = \frac{1}{2\pi} \left. \frac{e^{j\omega t}}{jt} \right|_{-W_1}^{W_1}$$

$$= \frac{1}{2\pi j t} \left[e^{jW_1 t} - e^{-jW_1 t} \right] = \frac{\sin(W_1 t)}{\pi t}$$



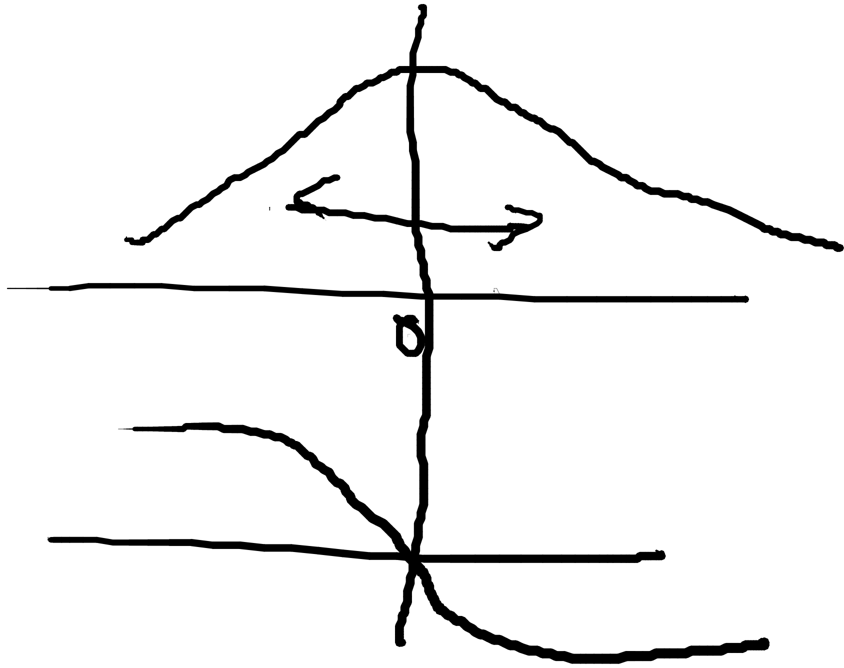
$$\longleftrightarrow \quad \frac{2\sin(T_1\omega)}{\omega}$$

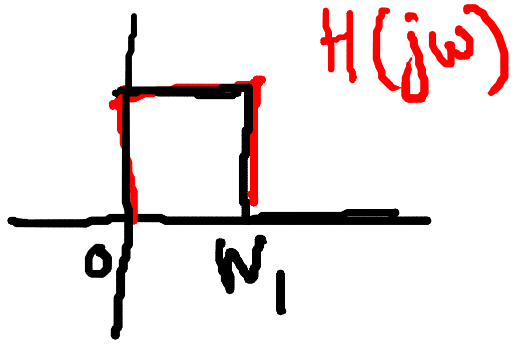
$$\frac{\sin(W_1 t)}{\pi t} \longleftrightarrow$$



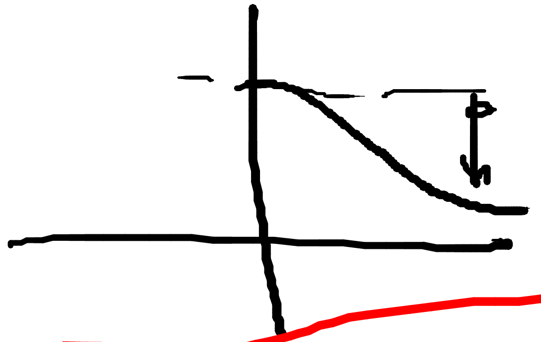
$$\boxed{\mathrm{sinc}(\theta) = \frac{\sin(\pi\theta)}{\pi\theta}}$$

$$e^{-at}$$





$\longrightarrow$ LPF



$\longrightarrow$ HPF

$\longrightarrow$ BPF

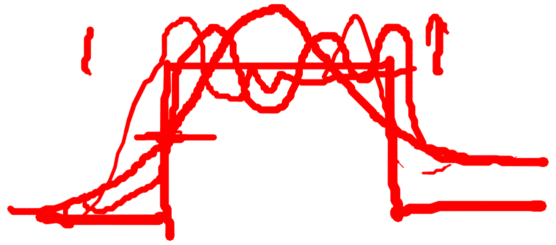
$$H(s) = \int_{-\infty}^{\infty} h(t)\, e^{-j\omega t}\, dt$$

$$e^{st} \longrightarrow H(s)\, e^{st}$$

$$\sum_{\langle k \rangle} a_k e^{s_k t} \longrightarrow \sum a_k H(s_k)\, e^{s_k t}$$

Relationship between Fourier Series & Fourier Transform:

$$\int_{-\infty}^{\infty} \delta(t) \, dt = 1$$



Gibbs

$$X(j\omega) = \int_{-\infty}^{\infty} \delta(t) e^{-j\omega t} \, dt$$

Michelson

$$= \int_{-\infty}^{\infty} \delta(t) \, dt$$

$$X(j\omega) = 1$$

$$X(j\omega) = \delta(\omega)$$

$$x(t) = \frac{1}{2\pi}$$