INDIAN INSTITUTE OF INFORMATION TECHNOLOGY, NAGPUR (IIITN)

Department of Electronics and Communication

Signals and Systems (Code: ECL201)

Semester-end exam || Semester-3

Slot: E
Date: Nov 24, 2017
Time: 09:00–12:00 A.M.
Maximum Marks: 50
Weightage: 50 %

Time: 3 hours Roll No. ____________

Important instructions:

- *All questions are compulsory, except BONUS questions which are all optional. In any case, the maximum marks that can be obtained is 50. Bonus questions will be evaluated only if the total marks obtained in main questions are less than 50.*
- Non-programmable calculators are permitted for use during the examination.
- Please indicate the important steps of reasoning/ calculations clearly.
- Course Outcomes (CO) Mapping: (Q1,Q3,Q4)→ CO2 & CO5, Q2→ CO3, (Q5,Q6)→ CO2 & CO4.

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1. (a) Find the Fourier series of the periodic signal $x(t) = e^{-t}$ with period $T = 1$ sec. Draw its magnitude and phase spectra. **[4]**

   <u>*BONUS*</u> *Q1 (2 Marks):* What are Dirichlet's conditions.

   (b) A message signal $x(t)$, say human voice with maximum frequency of ω_m is unable to travel longer distance. In order to propagate it for a longer distance, the message signal has to be multiplied in time domain by a cosinusoidal signal with a frequency $\omega_c >> \omega_m$. This process is called *modulation*. Explain the effect of modulation in frequency domain. **[3]**

2. Answer the following questions.

   (a) Calculate the energy of the signal $x(t) = \frac{\sin(\pi t)}{\pi t}$. **[3]**

   <u>*BONUS*</u> *Q2a (2 Marks):* Explain Gibbs's phenomenon.

   (b) Calculate the bound of $\sin(t) * (2e^{-2t}u(t))$, where $*$ denotes convolution. **[3]**

   (c) Let $h[n] = a^n u[bn+c]$ be an impulse response of a system, where (a, b, c) are real valued constants. For which of the following values of (a, b, c) is the system stable? Justify your answer. **[4]**

   1. $(-2, -3, -6)$
   2. $(1, 1, 0)$
   3. $(0.5, -2, 3)$
   4. $(2, -4, 4)$

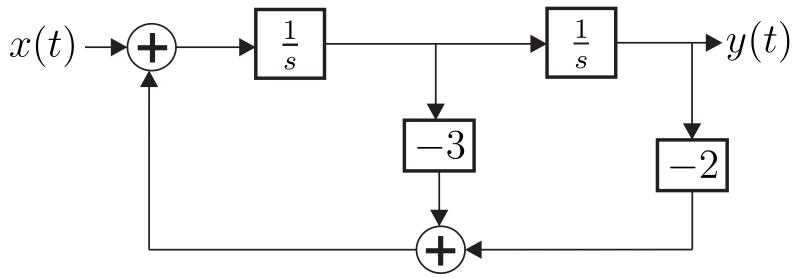
   <u>*BONUS*</u> *Q2c (2 Marks):* Prove that, for a discrete time LTI system to be stable, its impulse response must be absolutely summable.

   (d) If an LTI system represented by $h(t) = e^{-2t}u(t)$ gives an output $y(t) = 2e^{-2t}u(t) + e^{-3t}u(t)$, determine the input $x(t)$. **[4]**

3. (a) Consider a signal with Fourier transform pair $x(t) \leftrightarrow X(j\omega)$. Let $X(j\omega) * X(j\omega) = 0$ for $|\omega| > 16000\pi$. If $x(t)$ is sampled with sampling duration $T = 0.1$ mSec, can $x(t)$ be recovered exactly? Illustrate your answer with neatly sketched and labeled Frequency-domain representation. **[3]**

   (b) Consider two signals with Fourier transform pairs $x_1(t) \leftrightarrow X_1(j\omega)$ and $x_2(t) \leftrightarrow X_2(j\omega)$. Let $X_1(j\omega) = 0$ for $|\omega| > 500\pi$ and $X_2(j\omega) = 0$ for $|\omega| > 1000\pi$. If we sample $y(t) = x_1(t) * x_2(t)$, specify the range of sampling period T that guarantee the recovery of $y(t)$. **[3]**

4. For system shown in figure below, answer the following questions.



   (a) Determine the system function $H(s)$. **[2]**

   (b) For the system function obtained in Q4a, determine all possible ROCs and corresponding $h(t)$. **[3]**

   (c) Assuming that the system function obtained in Q4a is an LTI system. If the input is $x(t) = e^{-t}$ (please note the absence of $u(t)$), determine the output of the system. **[2]**

5. Describe all the steps in conversion of continuous-time signal $x(t)$ to discrete sequence starting from $x(t) \to x(t).p(t) \to x(nT) \to x[n]$. Illustrate each step with equivalent time and Frequency domain expressions and corresponding diagrams. **[4]**

6. Let the Fourier transform of discrete sequence $x[n]$ be $X(j\omega) = 1 - \frac{9}{2\pi}\omega$ for $0 \leq \omega \leq \frac{2\pi}{p}$ and 0 otherwise. Neatly sketch and label the Fourier transform for,

   (a) $x_a[n] = x[3n]$. **[3]**

   (b) $x_b[n] = x_a[\frac{n}{3}]$, where $x_a[n]$ is defined in Q6a. **[2]**

   (c) $x_c[n] = x[5n]$. **[3]**

   (d) $x_d[n] = x_c[\frac{n}{5}]$, where $x_c[n]$ is defined in Q6c. **[2]**

   (e) Determine the maximum value of N at with $x[n]$ can be sampled to obtain $x[Nn]$ without the possibility of aliasing. **[2]**

BONUS Q6 (4 Marks): Write the analysis and synthesis equations of Continuous-time Fourier Series (CTFS), Discrete-time Fourier Series (DTFS), Continuous-time Fourier Transform (CTFT), and Discrete-time Fourier Transform (DTFT)?

Q.1

a]

$$x(t) = e^{-t} \quad ; \quad 0 \le t \le 1$$

$$a_k = \frac{1}{T}\int_0^T x(t)\, e^{-jk\omega_0 t}\, dt \qquad \left\{ \begin{aligned} \omega_0 &= \frac{2\pi}{T} \\ &= 2\pi \end{aligned} \right.$$

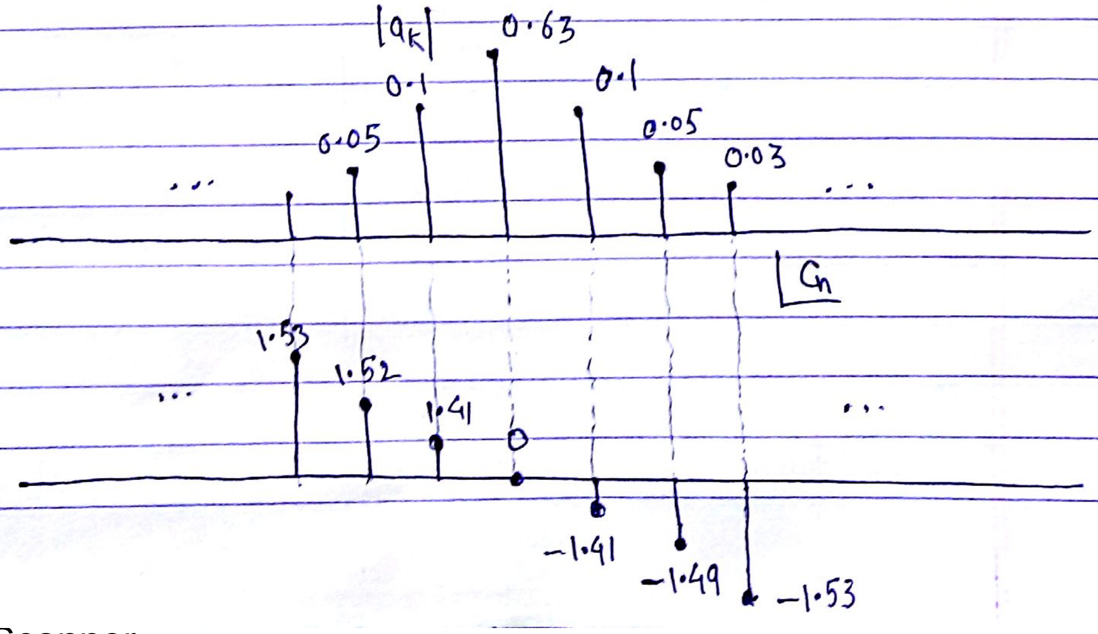
$$= \frac{1}{1}\int_0^1 e^{-t}\, e^{-jk2\pi t}\, dt$$

$$= \frac{1}{1}\int_0^1 e^{-t(1+jk2\pi)}\, dt$$

$$= \left.\frac{e^{-t(1+jk2\pi)}}{-(1+k2\pi)}\right|_0^1$$

$$= \frac{1-e^{-(1+jk2\pi)}}{(1+jk2\pi)} = \frac{1-e^{-1}}{1+jk2\pi}$$

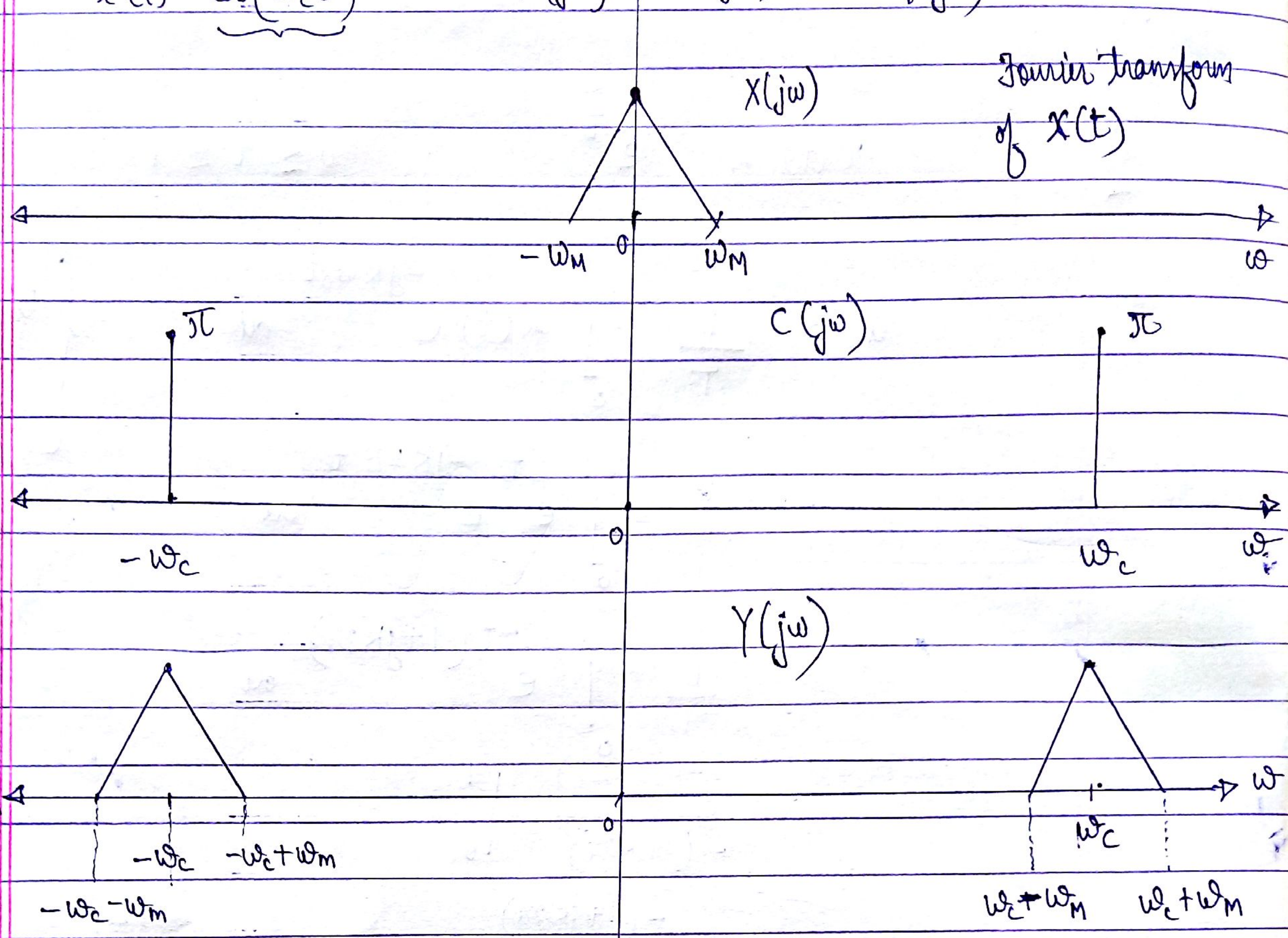
$$\Rightarrow \boxed{x(t) = \sum_{n=-\infty}^{\infty} \left(\frac{1-e^{-1}}{1+jk2\pi}\right) e^{jk2\pi t}}$$



Q.1 b]

$$x(t)\cdot \underbrace{\cos(\omega_c t)}_{C(t)} \longleftrightarrow X(j\omega) * C(j\omega) = Y(j\omega)$$

Fourier transform of $x(t)$



$\boxed{\text{The spectrum of original signal gets shifted (~~replicated~~) at } \pm\omega_c}$

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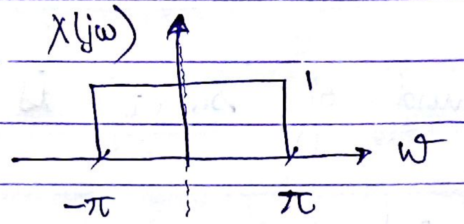
Q.2

a] $x(t) = \dfrac{\sin(\pi t)}{\pi t}$



$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{+j\omega t}\, d\omega$$

$$x(t) = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{j\omega t}\, d\omega$$

$$x(t) = \frac{1}{2\pi}\left[\frac{e^{j\pi t} - e^{-j\pi t}}{jt}\right] = \frac{\sin(\pi t)}{\pi t}$$

Using Parseval's theorem, $\displaystyle\int_{-\infty}^{\infty} |x(t)|^2 = \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(j\omega)|^2 d\omega$

Energy is retained in time & Frequency domain

$$\Rightarrow E\{x(t)\} = \int_{-\infty}^{\infty} \left|\frac{\sin(\pi t)}{\pi t}\right|^2 dt = \frac{1}{2\pi}\int_{-\pi}^{\pi} 1\, d\omega$$

$$= \frac{2\pi}{2\pi} = 1$$

$$\Rightarrow \boxed{\text{Energy of } x(t) = 1}$$

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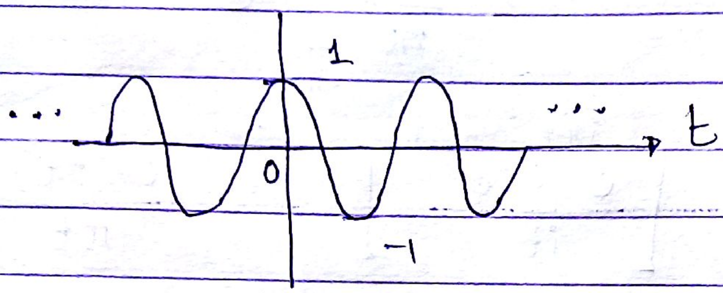
Q.2

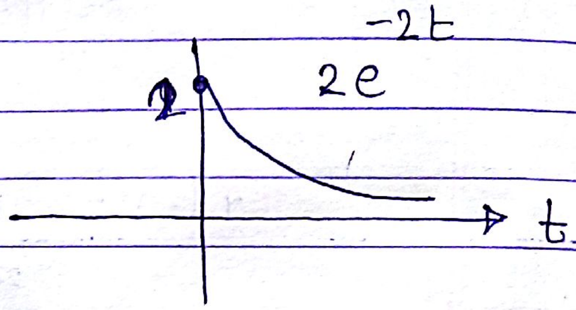
b]

$$x(t) = \sin(t) * \left(2e^{-2t} u(t)\right)$$

The maximum bound of $\sin(t)$ is 1

As, $\max |\sin(t)| = 1$





$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

$$\leq 1 \int_{-\infty}^{\infty} h(t-\tau)\, d\tau \leq 2\int_{0}^{\infty} e^{-2t}\, dt$$

$$\leq 2 \left.\frac{e^{-2t}}{-2}\right|_0^{\infty} = -[0-1] = 1$$

The maximum value of bound is therefore determined by $2e^{-2t}u(t)$ which is $\boxed{1}$

Q.2 c]

$$h[n] = a^n u[bn + c]$$

1. $a = -2$; $b = -3$; $c = -6$

$$h[n] = (-2)^n u[-3n - 6]$$

This is left-hand sided signal ~~growing beyond bound~~. decaying with integer powers of $(-2)^{-1}$

This sequence is bounded by $(-2, -3, -6)$

2. $a = 1$; $b = 1$; $c = 0$

$$h[n] = u[n]$$

This ~~signal~~ System is clearly unstable. as $\sum_{n=-\infty}^{\infty} u[n] = \infty$

3. $(0.5, -2, 3)$

$$h[n] = (0.5)^n u[-2n + 3]$$

This is left-hand sided ~~exponential~~ sequence which grows out of bound.

$$\sum_{n=-\infty}^{\infty} h[n] = \infty$$

$\Big\{$ Integer to the Power of integers $|a| > 1$

4. $(2, -4, 4)$

$$h[n] = (2)^n u[-4n + 4]$$

This sequence is bounded as $\sum_{n=-\infty}^{\infty} h[n] < \infty$ $\Big\{$ Fraction to the Power of Integer $|a| < 1$

$\therefore$ ANSWER IS 1 & 4

Q.2

d] For, $h(t) = e^{-2t} u(t)$

$H(j\omega)$ ~~=~~

$$y(t) = 2e^{-2t} u(t) + e^{-3t} u(t)$$

Taking Fourier transform, $H(j\omega) = \int_{-\infty}^{\infty} \{ e^{-2t} u(t) \} e^{-j\omega t} dt$

$$= \frac{1}{2+j\omega}$$

$$Y(j\omega) = \frac{2}{2+j\omega} + \frac{1}{3+j\omega}$$

$$X(j\omega) = \frac{Y(j\omega)}{H(j\omega)} = \frac{\frac{6+2j\omega+2+j\omega}{(2+j\omega)(3+j\omega)}}{\frac{1}{(2+j\omega)}} = \frac{\frac{8+3j\omega}{(2+j\omega)(3+j\omega)}}{\frac{1}{(2+j\omega)}}$$

$$= \frac{8+3j\omega}{3+j\omega}$$

$$= \frac{8}{3+j\omega} + \frac{3j\omega}{3+j\omega}$$

$$= 8e^{-3t} u(t) - 3\frac{d}{dt}\{ e^{-3t} u(t) \}$$

$$= 8e^{-3t} u(t) - 3\left[u(t) e^{-3t}(-3) + e^{-3t} \delta(t) \right]$$

$$= 8e^{-3t} u(t) - 3\left[-3e^{-3t} u(t) + \delta(t) \right]$$

$$\boxed{x(t) = 3\delta(t) - e^{-3t} u(t)}$$

Q.3

a] If $X(j\omega) * X(j\omega) = 0$ for $|\omega| > 16000\pi$

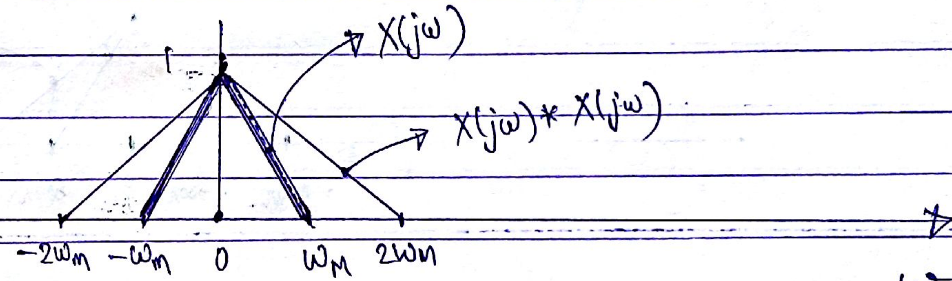
$\Rightarrow X(j\omega) = 0$ for $|\omega| > 8000\pi$

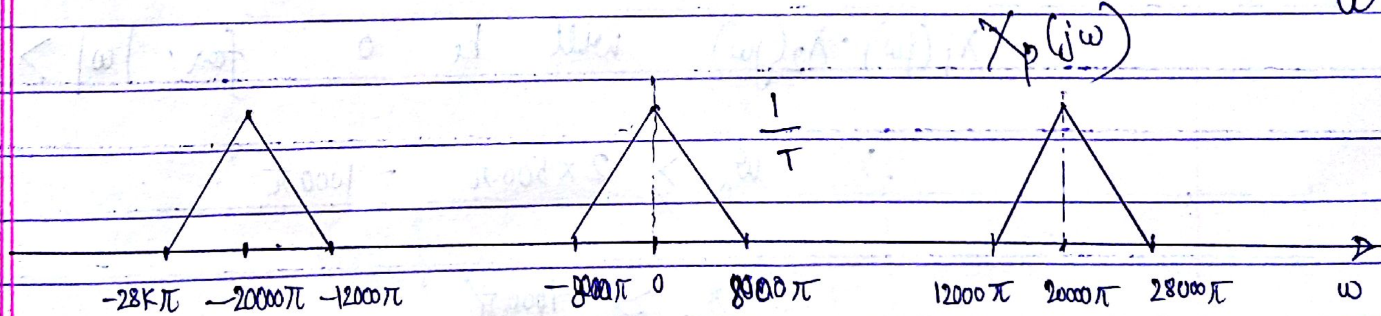
$T = 0.1\ ms$ $F = 10{,}000\ Hz \Rightarrow \omega_s = 20000\pi$

As $\omega_s \geq 2\omega_m$

$20000\pi \geq 2 \times 8000\pi$

The signal can be recovered exactly



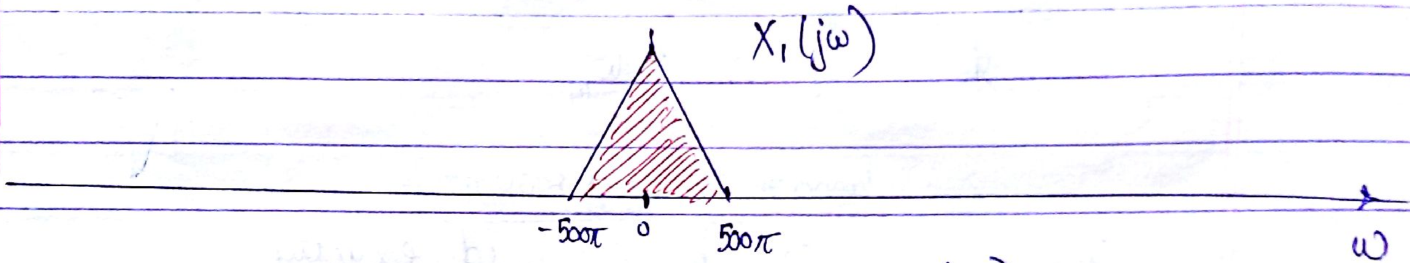


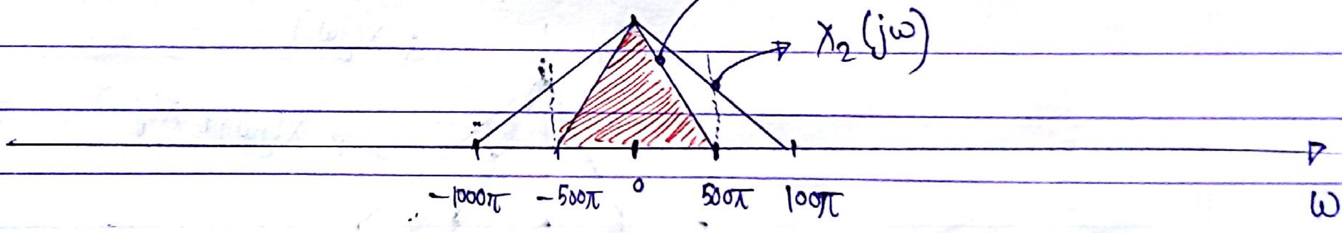
Q.3

b] $X_1(j\omega) = 0 \quad \text{for} \quad |\omega| > 500\pi$

$X_2(j\omega) = 0 \quad \text{for} \quad |\omega| > 1000\pi$

$x_1(t) * x_2(t) \longleftrightarrow X_1(j\omega) \cdot X_2(j\omega)$





$\therefore X_1(j\omega) \cdot X_2(j\omega)$ will be 0 for $|\omega| > 500\pi$

$\therefore \quad \omega_s \geq 2 \times 500\pi = 1000\pi$

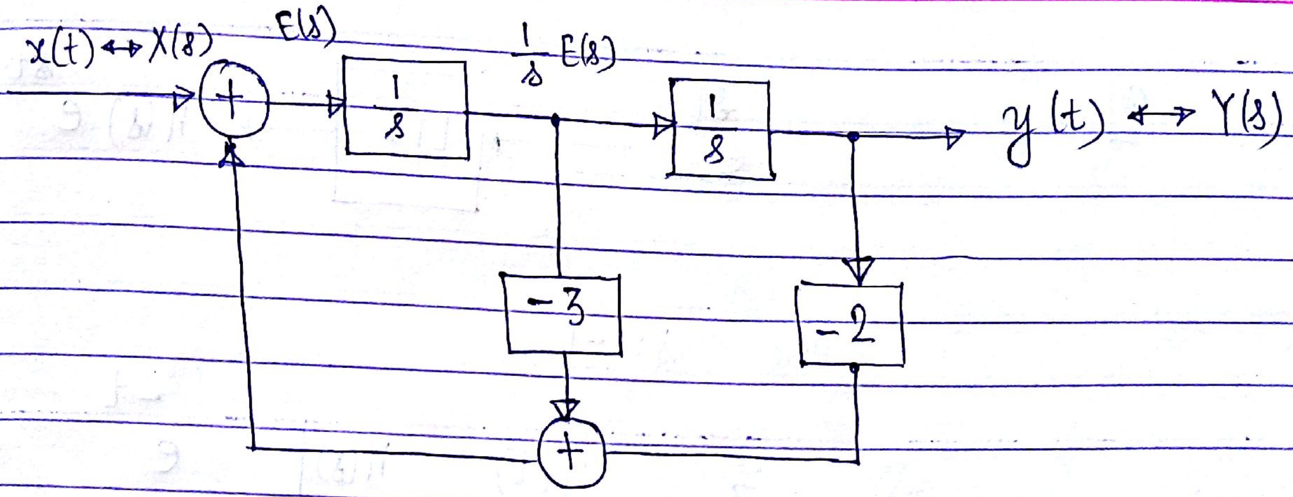
$$\frac{2\cancel{\pi}}{T_s} \geq 1000\cancel{\pi}$$

$$T_s \leq \frac{2}{1000} = \frac{1}{500}$$

$$\boxed{T_s \leq 2\,\text{msec}}$$

Q.4

a]



$$Y(s) = \frac{1}{s^2} E(s)$$

$$E(s) = X(s) + \left(-\frac{3}{s} E(s) - 2\,Y(s) \right)$$

$$s^2 Y(s) = X(s) - \left(\frac{3}{s} s^2 \cancel{E}Y(s) + 2\,Y(s) \right)$$

$$s^2 Y(s) = X(s) - 3s\,Y(s) - 2\,Y(s)$$

$$\left(s^2 + 3s + 2 \right) Y(s) = X(s)$$

$$H(s) = \boxed{\frac{Y(s)}{X(s)} = \frac{1}{s^2+3s+2}}$$

b]

$$H(s) = \frac{1}{(s+2)(s+1)} = \frac{A}{(s+2)} + \frac{B}{(s+1)} = \frac{1}{s+1} - \frac{1}{s+2}$$

CASE ① $h(t) = e^{-t} u(t) - e^{-2t} u(t)$; ROC : $Re\{s\} > -1$

CASE ② $h(t) = -e^{-t} u(-t) + e^{-2t} u(-t)$; ROC : $Re\{s\} < -2$

CASE ③ $h(t) = -e^{-t} u(-t) + e^{-2t} u(t)$; ROC : $-2 \leq Re\{s\} \leq -1$

c]

$$e^{st} \longrightarrow \boxed{\text{LTI}} \longrightarrow H(s)\, e^{st}$$

Here $s = -1$

$$\Rightarrow \quad y(t) = H(s)\Big|_{s=-1} e^{-t}$$

For $s=-1$; $H(s) = \dfrac{1}{s^2+3s+2} = \dfrac{1}{1-3+2} = \infty$

This would go out of bound.

$\therefore$ $s > -1$ for output to be bounded.

Q.5

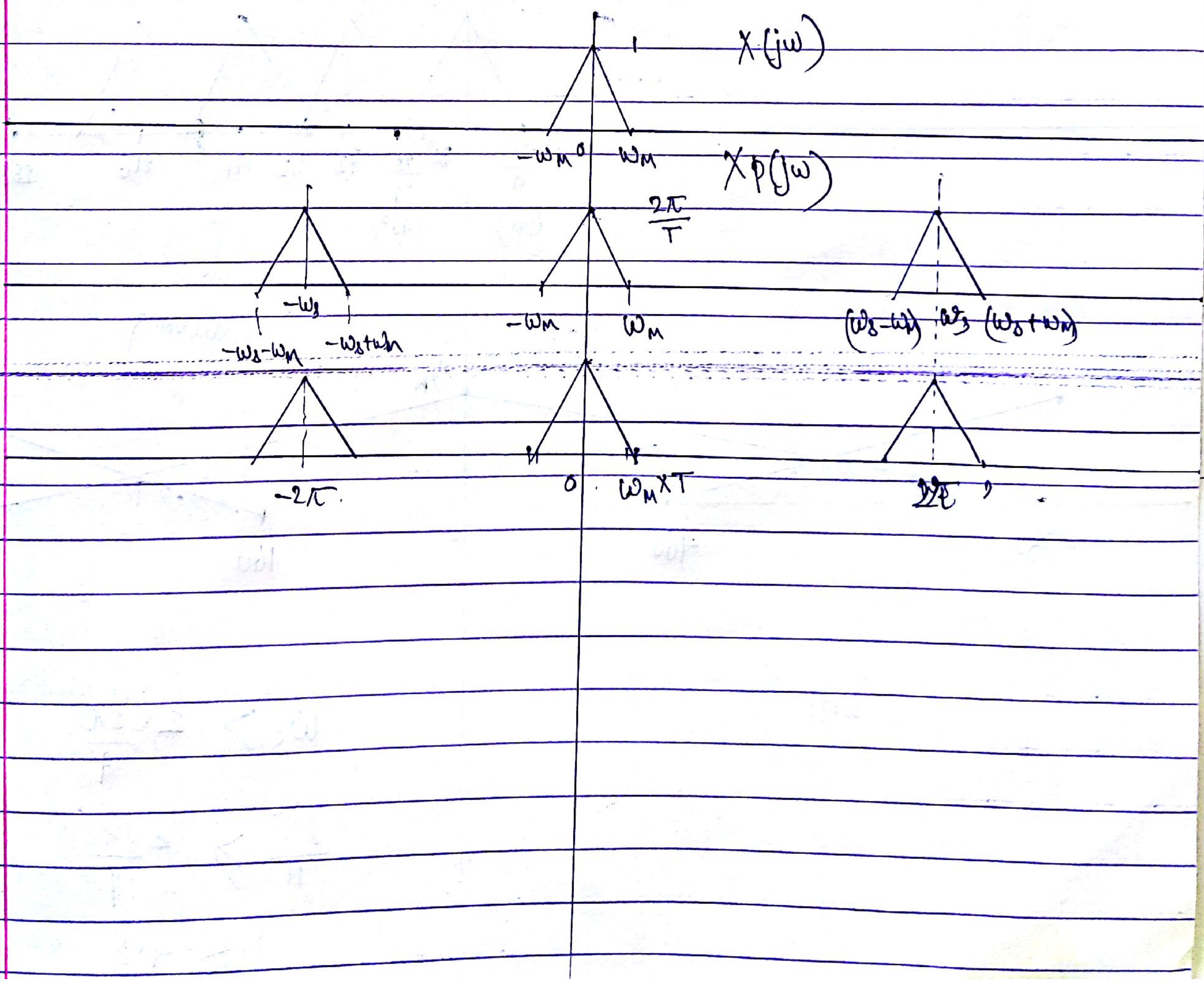
$$x(t) \longleftrightarrow X(j\omega)$$

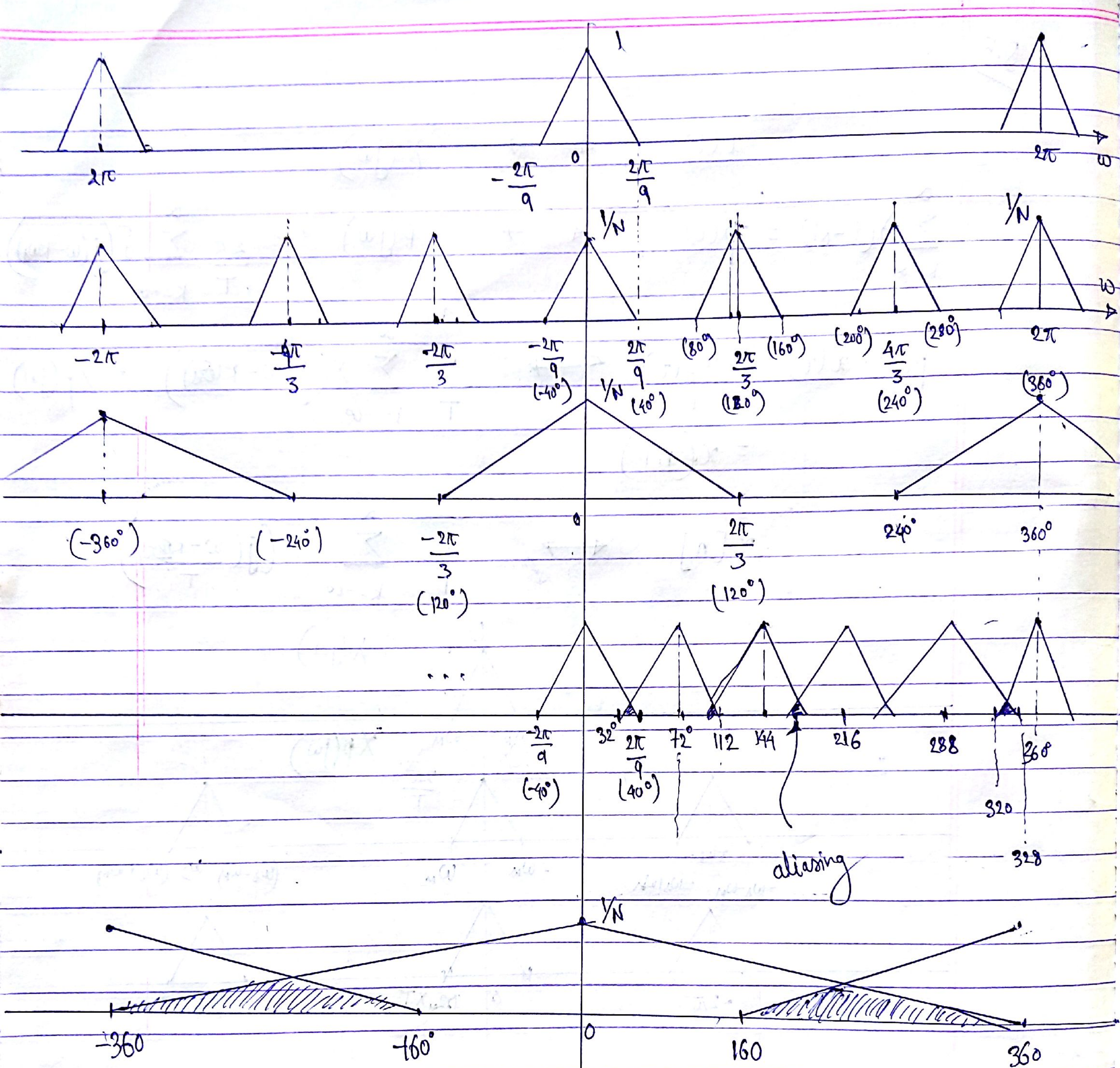
$$\sum_{k=-\infty}^{\infty} \delta(t-kT) = p(t) \longleftrightarrow P(j\omega) = \frac{2\pi}{T} \sum_{k=-\infty}^{\infty} \delta(j(\omega-k\omega_s))$$

$$p(t)\cdot x(t) = x_p(t) \longleftrightarrow \frac{1}{T}\sum_{k=-\infty}^{\infty} X(j(\omega-k\omega_s)) = X_p(j\omega)$$

$$= x(nT)$$

$$x[n] \longleftrightarrow \frac{1}{T}\sum_{k=-\infty}^{\infty} X\left(j\left(\frac{\omega-k2\pi}{T}\right)\right)$$





$$\omega_s > 2 \times \frac{2\pi}{9}$$

$$\frac{2\pi}{N} \geq 2\,\frac{2\pi}{9}$$

$$N \leq 4.5$$

$$\boxed{N = 4}$$

Maximum value of N