16 AUG 2017

1. Time/shift invariance:
2. Homogeneity/Scaling ✓
3. Additivity

Linearity → Additivity + Homogeneity } Superposition

e.g $y(t) =$

complex number.

$x_1(t) \longrightarrow y_1(t)$ $\qquad$ $c\,x(t) \longrightarrow c\,y(t)$

$x_2(t) \longrightarrow y_2(t)$

$$\alpha x_1(t) + \beta x_2(t) \longrightarrow \alpha y_1(t) + \beta y_2(t)$$

e.g $\quad y(t) = x(t) + 5$

$$\alpha x_1(t) + \beta x_2(t) \longrightarrow \overbrace{\left[\alpha x_1(t) + \beta x_2(t)\right]}^{x(t)} + 5$$

$$\neq \underbrace{\left[\alpha x_1(t) + \alpha 5\right]}_{y_1(t)} + \underbrace{\left[\beta x_2(t) + \beta 5\right]}_{y_2(t)}$$

e.g $\quad y(t) = t^2 x(t)$

Linear system

e.g $\quad y(t) = x^2(t)$

NonLinear

Homework

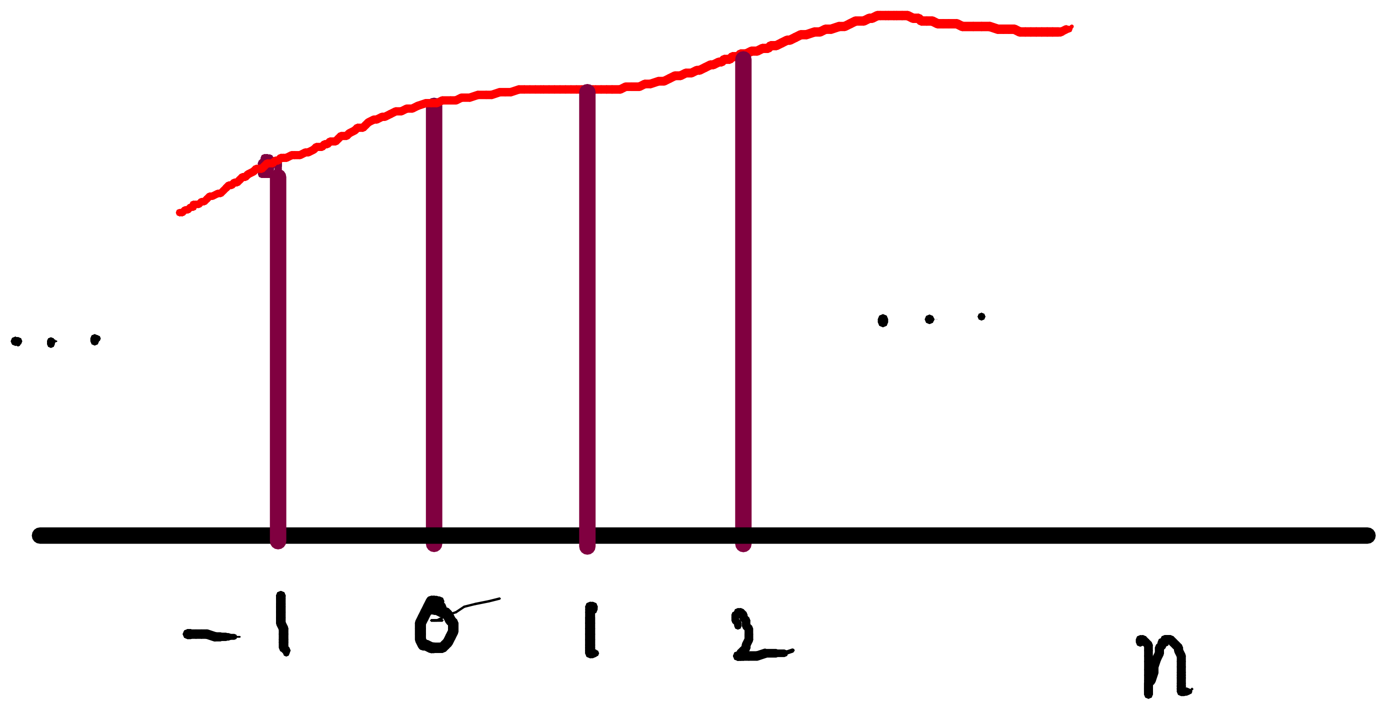
Come-up with examples of different systems

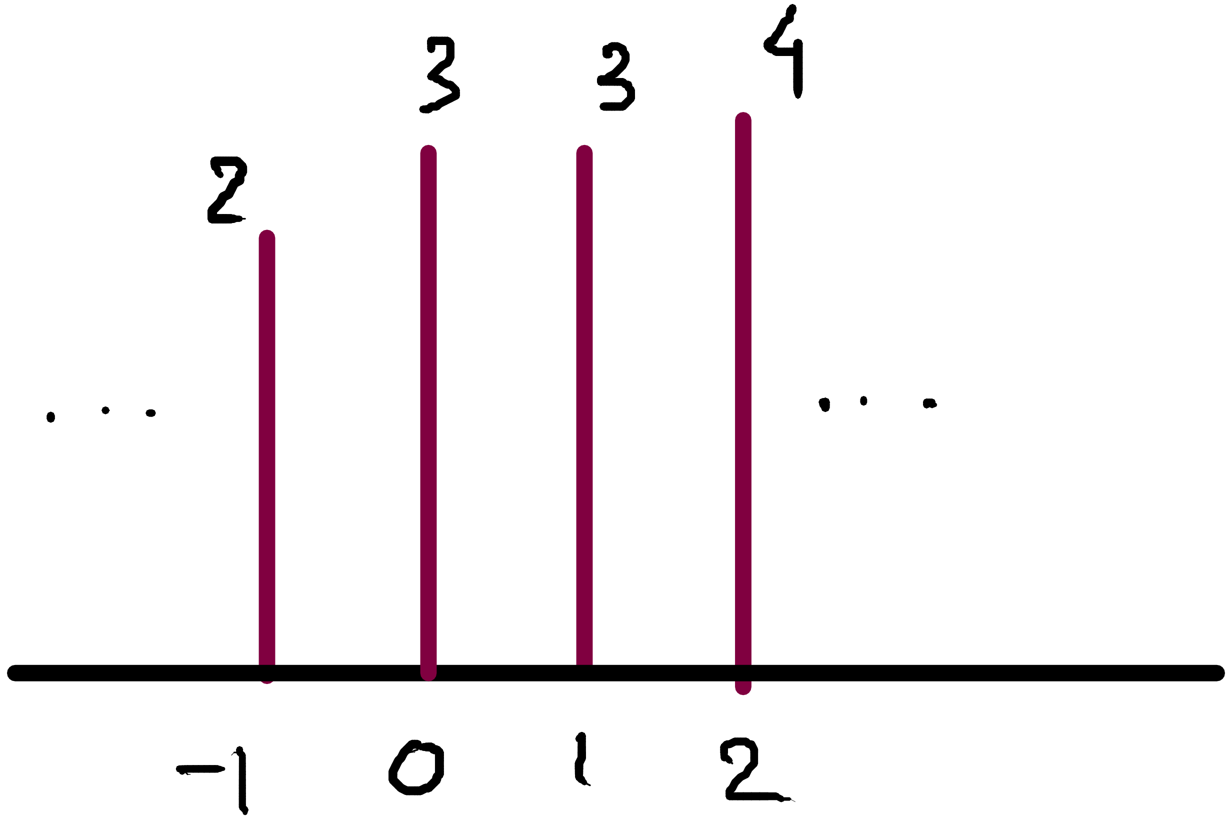
	A	H	C	S	TI
	0	0	0	0	0
	1	1	1	1	1
	0	1	0	1	1

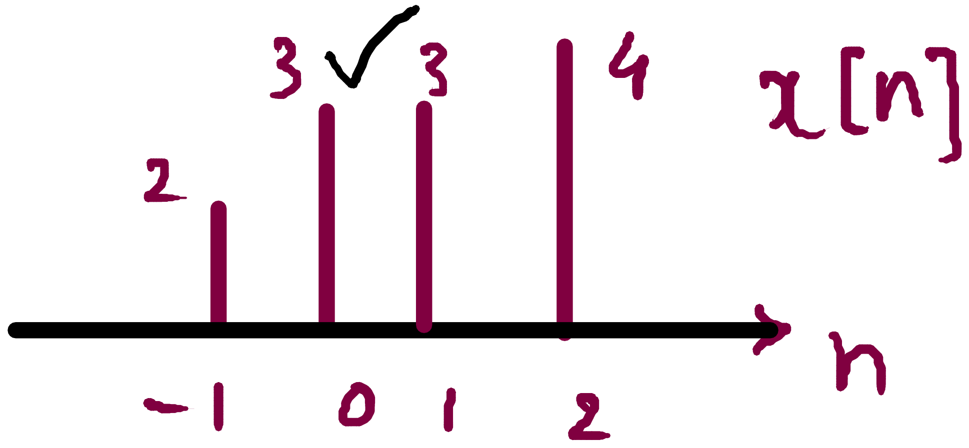
1. Time-invariance
2. Linearity/Superposition

} LTI Systems.

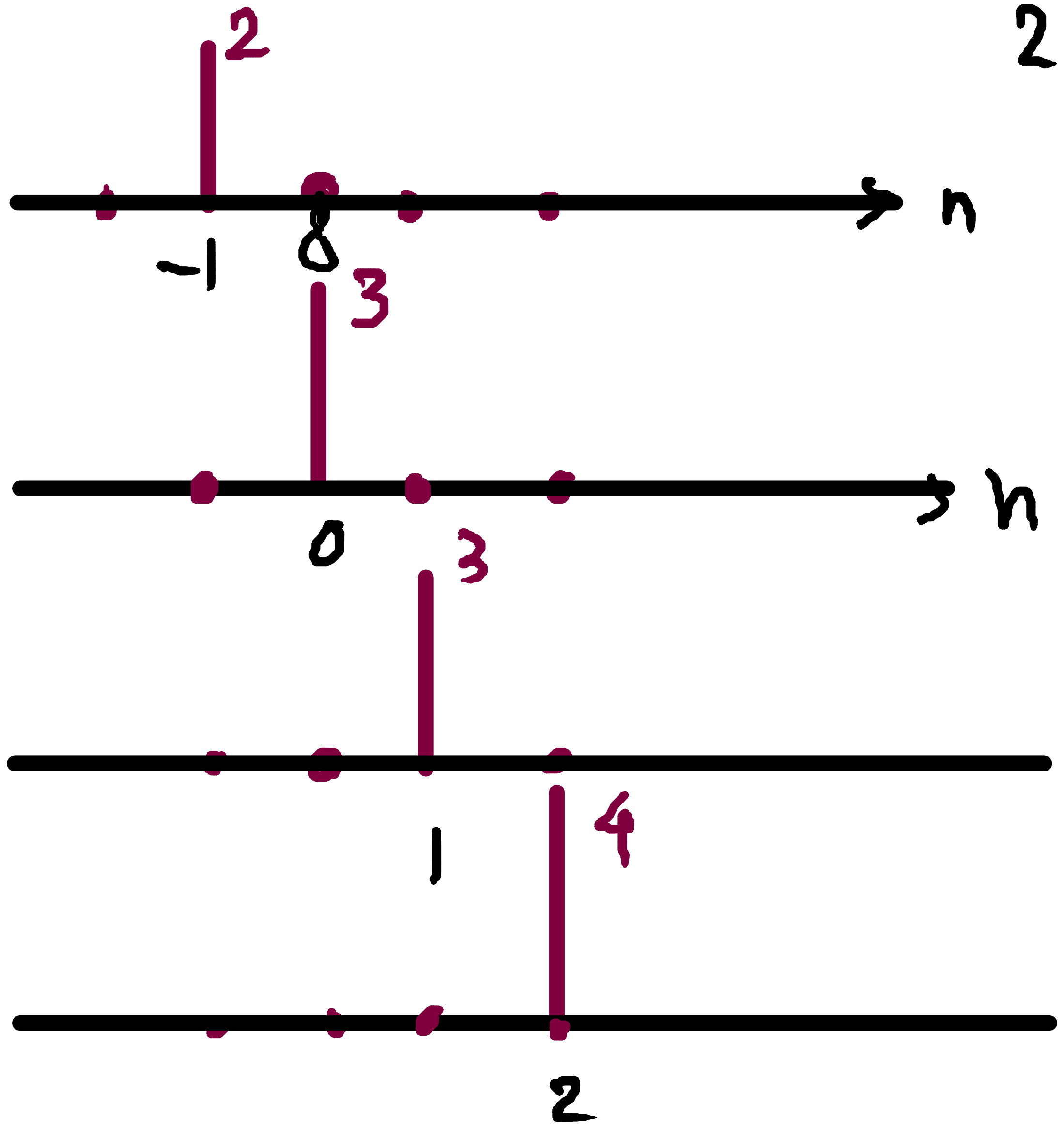
$$x[n] = \{2, \underset{\uparrow}{3}, 3, 4\}$$







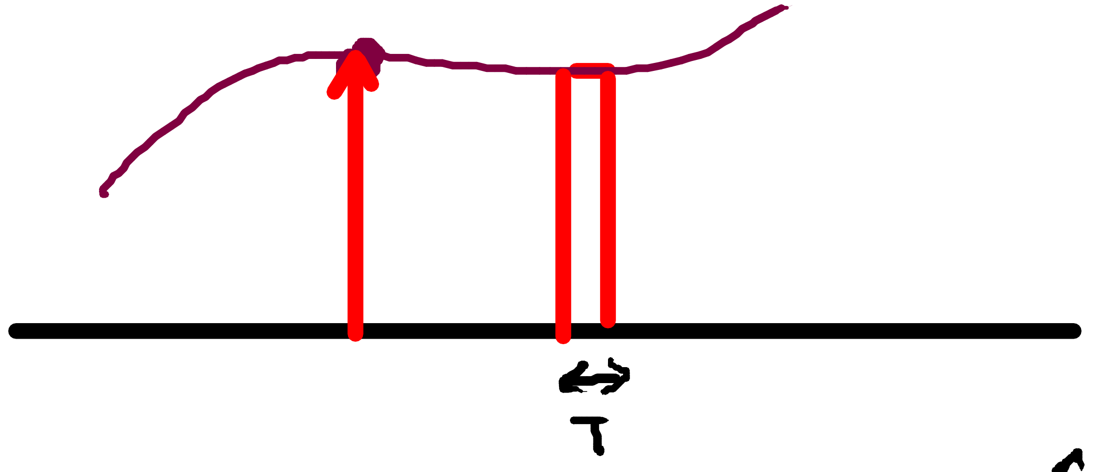
$$\boxed{x[n] = \sum_{k=-\infty}^{\infty} x[k]\,\delta[n-k]}$$



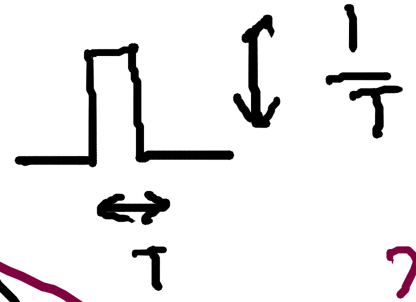
$$\left.\begin{aligned}
2\delta[n+1] &= x[-1]\,\delta[n+1] \\
3\delta[n] &= x[0]\,\delta[n-0] \\
3\delta[n-1] &= x[1]\,\delta[n-1] \\
4\delta[n-2] &= x[2]\,\delta[n-2]
\end{aligned}\right\} x[n]$$

Any signal can be represented by shifted & scaled versions of the unit impulses

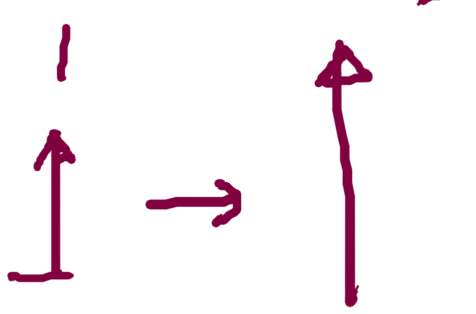
$$x[n] = \sum_{k=-\infty}^{\infty} x[k]\,\delta[n-k]$$



$$x(t)\,\delta(t-t_0) = x(t_0)\,\delta(t-t_0)$$



$$\int_{-\infty}^{\infty} x(t)\,\delta(t-t_0) = x(t_0)$$



Sifting property

$$x[n] = \sum_{k=-\infty}^{\infty} x[k]\,\delta[n-k]$$

$$x(t) = \int_{-\infty}^{\infty} x(\tau)\,\delta(t-\tau)\,d\tau$$

$$\delta[n] \longrightarrow \boxed{S} \longrightarrow h[n]$$

Impulse response

① Invoking Shift Invariance

$$\delta[n-k] \longrightarrow \boxed{S} \longrightarrow h[n-k]$$

Shift-Invariant

② Invoking

$$c\,\delta[n] \longrightarrow \boxed{S} \longrightarrow c\,h[n]$$

Homogeneous

$$\delta[n] \longrightarrow h[n]$$

$$\delta[n-1] \longrightarrow h[n-1]$$

$$\delta[n-2] \longrightarrow h[n-2]$$

$$\delta[n-k] \longrightarrow h[n-k]$$

$$2\,\delta[n] \longrightarrow 2\,h[n]$$

$$c\,\delta[n] \longrightarrow c\,h[n]$$

$$x[k]\,\delta[n-k] \xrightarrow[\substack{\text{Shift invariance}\\ \text{Homogeneity}}]{\boxed{S}} x[k]\,h[n-k]$$

③ Invoking additivity

$$\overbrace{\sum_{k=-\infty}^{\infty} x[k]\,\delta[n-k]}^{x[n]} \longrightarrow \overbrace{\sum_{k=-\infty}^{\infty} x[k]\,h[n-k]}^{y[n]}$$

$$\delta[n] \longrightarrow \boxed{\mathcal{S}} \longrightarrow h[n]$$

$$\delta[n-k] \longrightarrow \boxed{\mathcal{S}} \longrightarrow h[n-k]$$

$$2\,\delta[n-k] + 3\,\delta[n-k] \quad \boxed{\mathcal{S}} \longrightarrow 2\,h[n-k] + 3\,h[n-k]$$

LTI

$$y[n] = \sum_{k=-\infty}^{\infty} x[k] \cdot h[n-k]$$

shift

flip

Add.

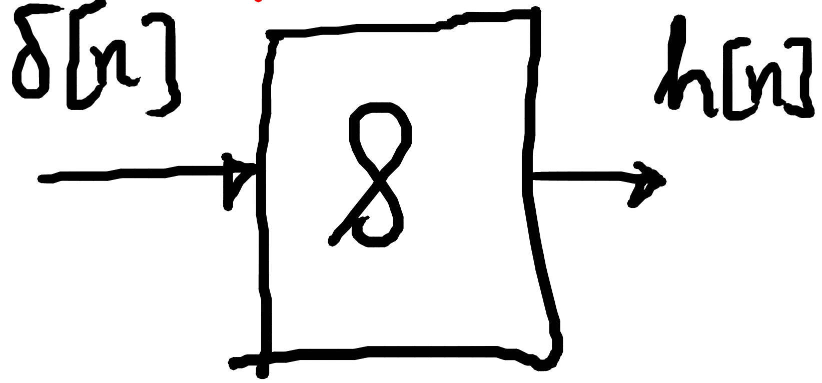
multiply

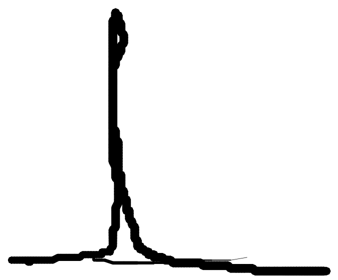
CONVOLUTION

CON — Together

VOLVE — Rolling

$h[n]$: Impulse response





$$h[n-k] = h[-k+n]$$
$$= h[-(k-n)]$$

shift $\rightarrow h[k-n] \xrightarrow{\text{flip}} h[-k-n]$

$h[k] \rightarrow$ flip $\rightarrow h[-k] \xrightarrow{\text{shift}} h[-(k-n)]$

$$\boxed{y[n] = \sum_{k=-\infty}^{\infty} x[k]\, h[n-k]}$$

Superposition

Sum

Convolution

x : input for which the output needs to be calculated

h : impulse response

Repeat for all n

Step (1) : Take h and Flip it (time reverse it)

Step (2) : Shift the signal obtained in step (1) by n towards right

Step (3) : Multiply sample by sample x & h

Step (4) : Add all the values obtained in step (3)

$15 + 15 + 50 + 20$

S1 S2 End