DTFT :

27 Sept 2017

$$\boxed{\begin{aligned} X(j\omega) &= \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\omega n} \\ x[n] &= \frac{1}{2\pi}\int_{-\pi}^{\pi} X(j\omega)\, e^{j\omega n}\, d\omega \end{aligned}}$$

e.g : $x[n] = \delta[n]$

$$\boxed{X(j\omega) = 1}$$

e.g : $x[n] = a^n u(n)$

$$X(j\omega) = \frac{1}{1 - a e^{-j\omega}}$$

e.g $x[n] = u[n]$

$$X(j\omega) = \frac{1}{1 - e^{-j\omega}}$$

e.g : $X(j\omega) = e^{-j\omega}$

$x[n] = ?$

e.g : $x[n] = \sin\left(\frac{\pi n}{3}\right) u[n]$

$$x[n] = \sin\left(\frac{\pi}{3}n\right) u[n]$$

$$X(j\omega) = \sum_{n=0}^{\infty} \left[\frac{e^{j\frac{\pi}{3}n} - e^{-j\pi/3 n}}{2j}\right] e^{-j\omega n}$$

$$= \frac{1}{2j}\left[\sum_{n=0}^{\infty} e^{jn\left(\frac{\pi}{3}-\omega\right)} - \sum_{n=0}^{\infty} e^{-jn\left(\frac{\pi}{3}+\omega\right)}\right]$$

$$= \frac{1}{2j}\left[\frac{1}{1-e^{j(\pi/3-\omega)}} - \frac{1}{1-e^{-j(\pi/3+\omega)}}\right]$$

$$= \frac{1}{2j}\left[\frac{\cancel{1} - e^{-j\pi/3} e^{-j\omega} - \cancel{1} + e^{j\pi/3} e^{-j\omega}}{\left(1-e^{j(\pi/3-\omega)}\right)\left(1-e^{-j(\pi/3+\omega)}\right)}\right] = \left[\frac{\sin(\pi/3)}{}\right.$$

1. Convolution :

$$x[n] * h[n] \longrightarrow X(e^{j\omega}) \cdot H(e^{j\omega})$$

2. Multiplication in time

$$x[n] \cdot h[n] \longrightarrow \frac{1}{2\pi} \int_{2\pi} X(j\omega)\, H(e^{j(\omega-\theta)})\, d\omega$$

3. Parseval's Theorem

$$\sum_{n=-\infty}^{\infty} |x[n]|^2 = \frac{1}{2\pi} \int_{\langle 2\pi \rangle} |X(e^{j\omega})|^2\, d\omega$$

CTFS

$$a_k = \frac{1}{T}\int_{\langle T\rangle} x(t)\, e^{-jk\omega_0 t}\, dt$$

$$x(t) = \sum_{k=-\infty}^{\infty} a_k\, e^{jk\omega_0 t}$$

DTFS

Duality

$$a_k = \frac{1}{N}\sum_{n=\langle N\rangle} x[n]\, e^{-jk\omega_0 n}$$

$$x[n] = \sum_{k=\langle N\rangle} a_k\, e^{jk\omega_0 n}$$

$a_k = a_{k+N}$

CTFT

Duality

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)\, e^{j\omega t}\, d\omega$$

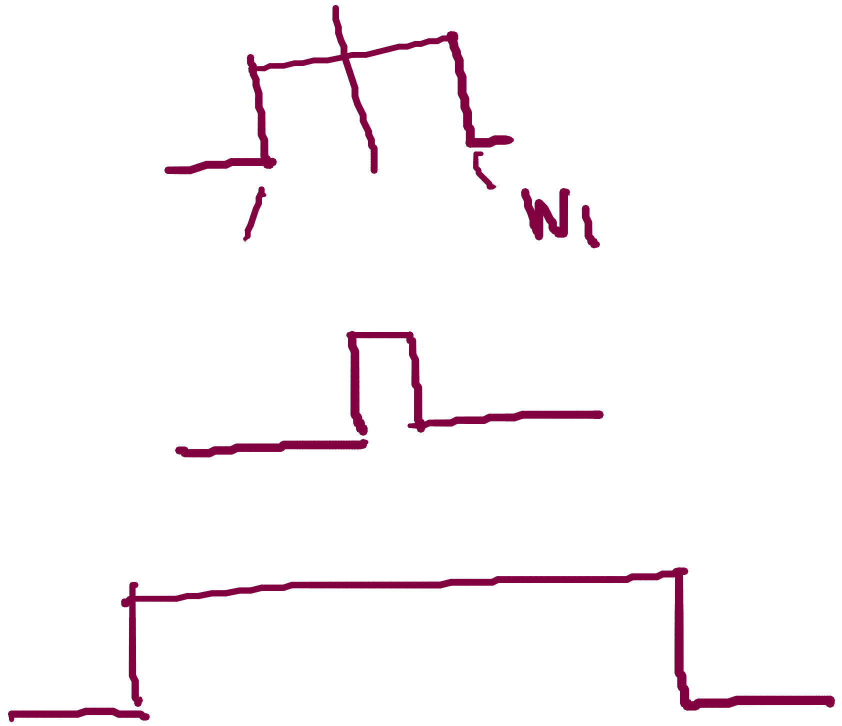
DTFT

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\omega n}$$

$X(e^{j\omega}) = X(e^{j(\omega+2\pi)})$

$$x[n] = \frac{1}{2\pi}\int_{\langle 2\pi\rangle} X(e^{j\omega})\, e^{j\omega n}\, d\omega$$

Fourier



$$\frac{\sin(Wn)}{\pi n}$$

$$Wn = k\pi$$