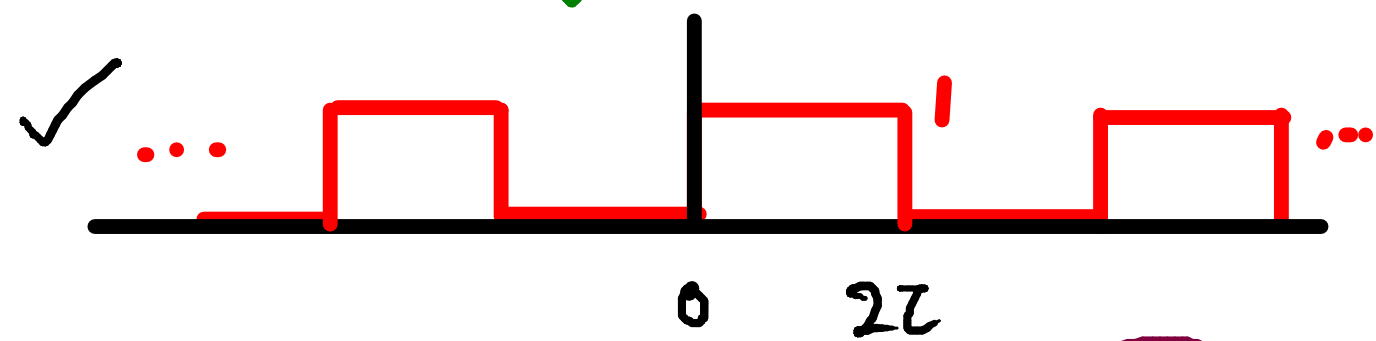


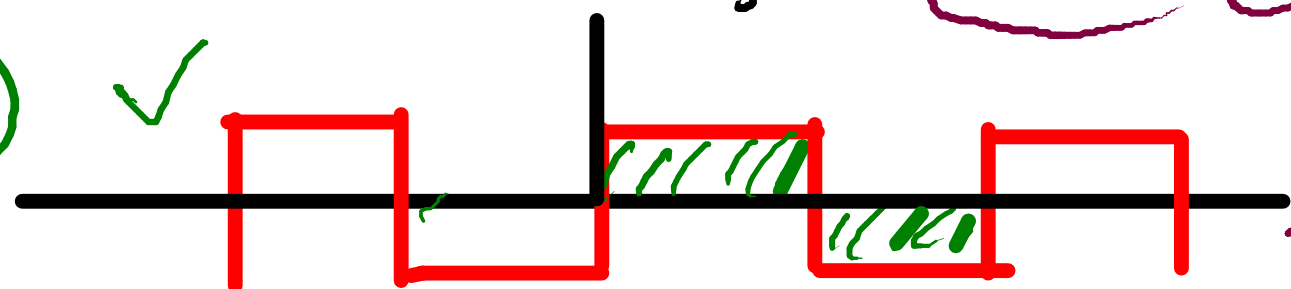
(A)



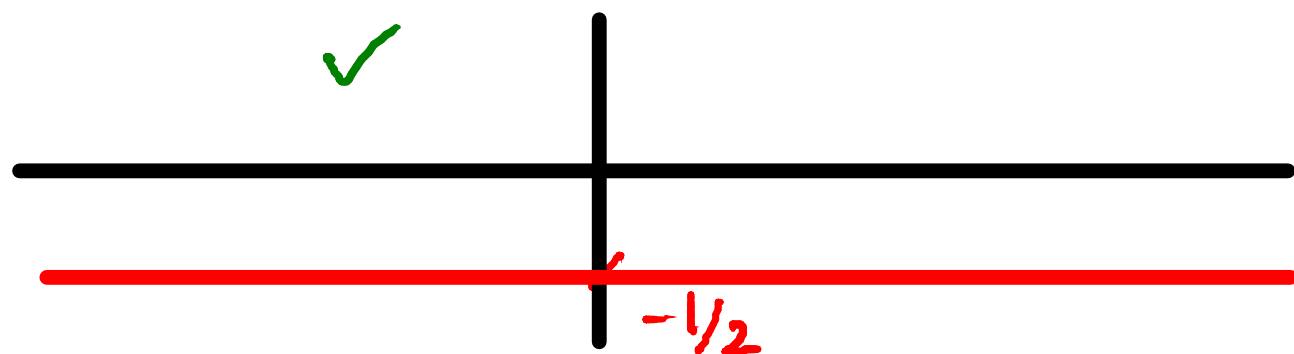
$$x_3(t) = x_1(t - z) - \frac{1}{2}$$

(*)

(C)



(B)



$$a_0 = \frac{2z}{T}$$

$$a_k = \frac{\sin(\omega_0 k z)}{k\pi}$$

$$a_k \quad x(t) \xleftrightarrow{FS} a_k \quad x(t - t_0) \xleftrightarrow{\quad} \left(e^{-j\omega_0 k z} \right) a_k$$

$$z = \frac{T}{4} \quad a_0 = \frac{2z}{T}$$

$$c_k = a_k + b_k$$

$$b_k \Rightarrow b_0 = -\frac{1}{2}$$

$$k = 0$$

$$c_0 = a_0 + b_0$$

$$c_k = a_k + b_k$$

$$k \neq 0 \quad c_k = \left(e^{-j\omega_0 k z} \right) \left(\frac{\sin \omega_0 k z}{k\pi} \right)$$

Discrete-time Fourier Series:

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{j\omega_0 k t} \quad \text{CTFS}$$

$$a_k = \frac{1}{T} \int_{\langle T \rangle} x(t) e^{-j\omega_0 k t} dt$$

$$\omega_0 = \frac{2\pi}{T}$$

Not Periodic

$$x[n] = \sum_{k=\langle N \rangle} a_k e^{j\omega_0 k n} \quad \text{DTFS}$$

$$a_k = \frac{1}{N} \sum_{n=\langle N \rangle} x[n] e^{-j\omega_0 k n}$$

Periodic

$$a_k = a_{k+N}$$

eg

CT

$$x(t) = \cos\left(\frac{2\pi}{5}t\right)$$

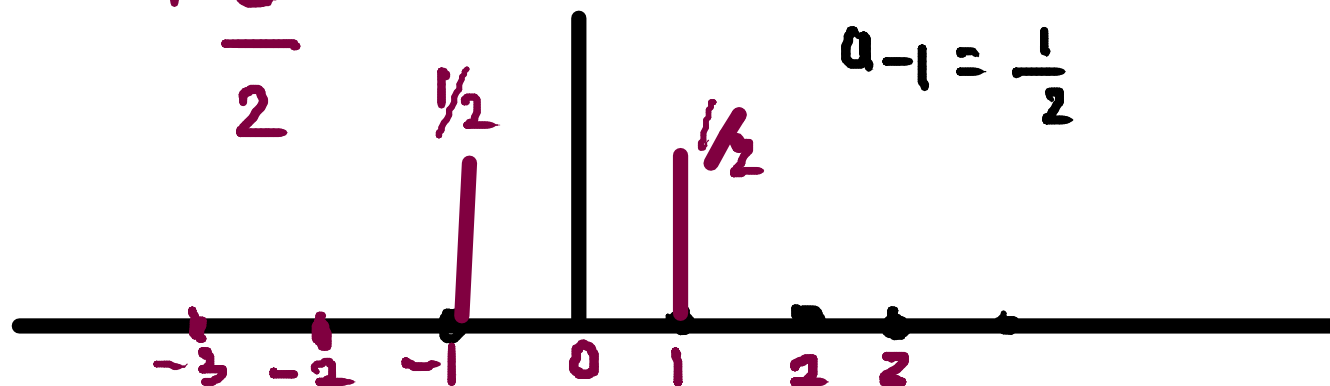
$$x(t) = \sum_{\langle k \rangle} a_k e^{j\omega_0 k t}$$

$$= a_0 e^{j\omega_0 0 t} + a_1 e^{j\omega_0 1 t} + a_{-1} e^{j\omega_0 (-1) t} + a_2 e^{j\omega_0 2 t} + a_{-2} e^{j\omega_0 (-2) t} + \dots$$

$$= \frac{e^{j\frac{2\pi}{5}t}}{2} + \frac{e^{-j\frac{2\pi}{5}t}}{2}$$

$$a_1 = \frac{1}{2}$$

$$a_{-1} = \frac{1}{2}$$



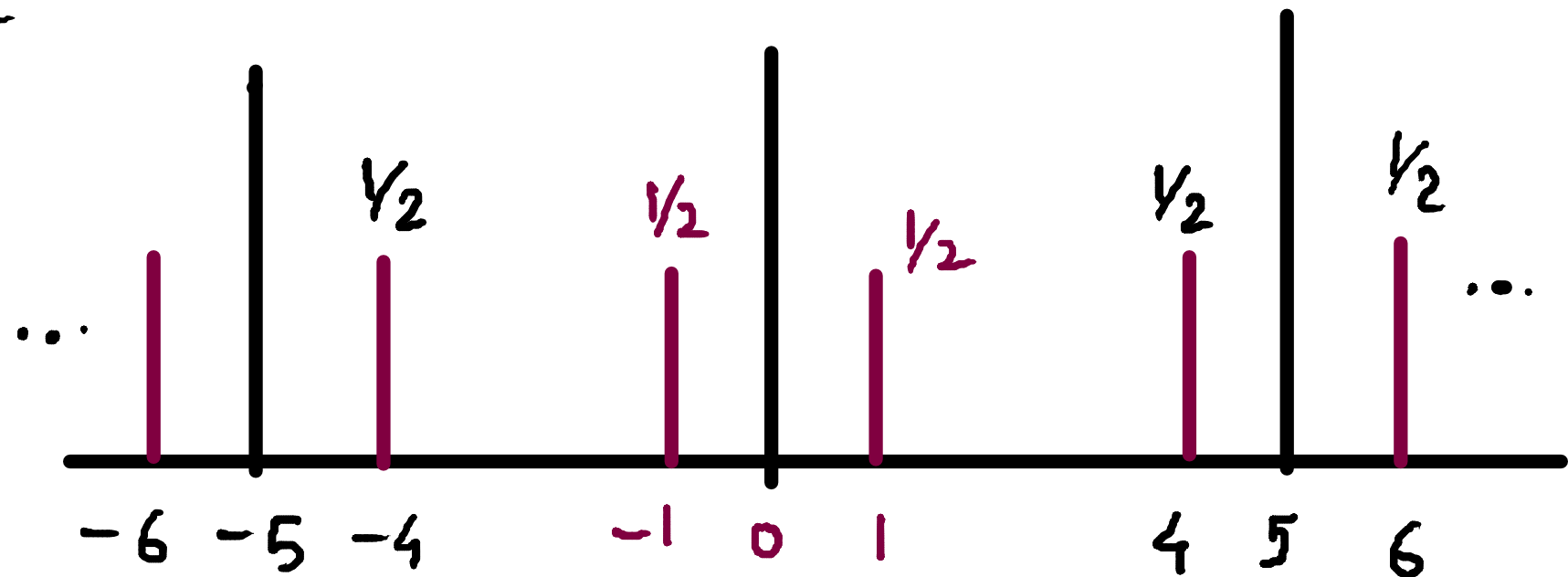
DT

$$\omega_0 = \frac{2\pi}{N}$$

$$N = 5$$

$$x[n] = \cos\left(\frac{2\pi}{5}n\right)$$

$$x[n] = \frac{e^{j\frac{2\pi}{5}n}}{2} + \frac{e^{-j\frac{2\pi}{5}n}}{2}$$



0 to 5

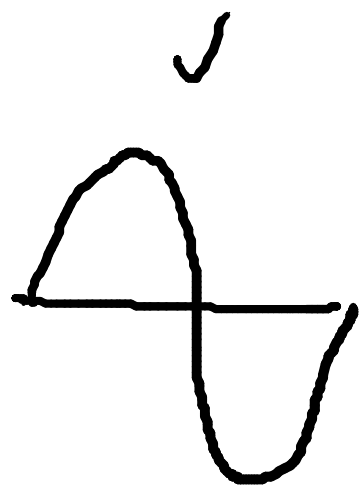
$$e^{j\frac{2\pi}{5}n}$$

$$e^{j\frac{2\pi}{5}6}$$

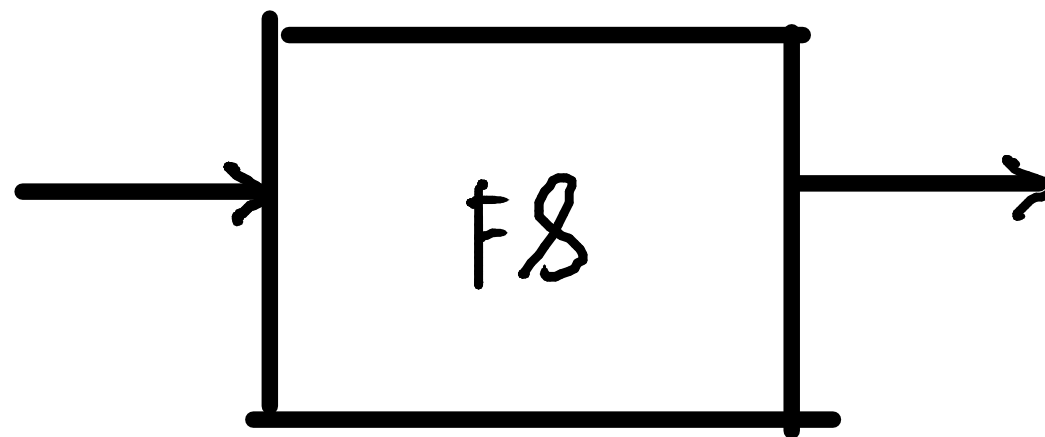
$$e^{j\frac{2\pi}{5}} = e^{j\frac{2\pi}{5}(5+1)} = e^{j\frac{2\pi}{5} + \frac{2\pi}{5}} = e^{j\frac{2\pi}{5}}$$

Parseval's Theorem :

$$\frac{1}{T} \int_{\langle T \rangle} |x(t)|^2 dt = \sum_{\langle k \rangle} |a_k|^2$$



TD



FD

✓
 a_k