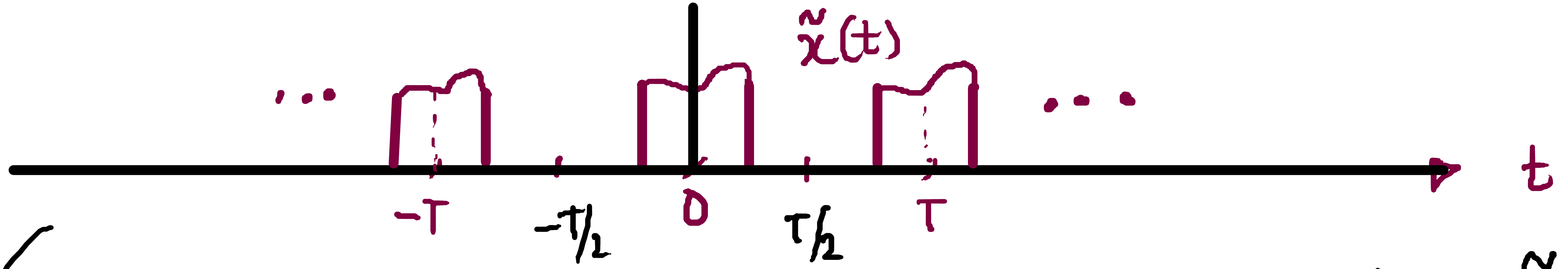
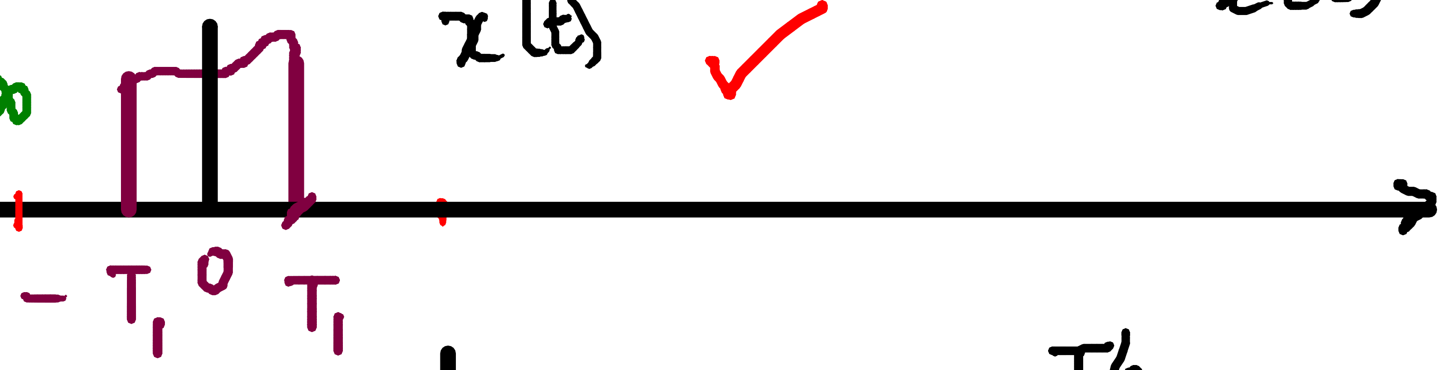
20 Sept 2017



$$x(t) = \tilde{x}(t) \quad ; \quad |t| < T/2$$

Periodic with $T \to \infty$



$T = \frac{2\pi}{\omega}$ $\therefore$ $\omega \to 0$

$$\tilde{x}(t) = \sum_{\langle k \rangle} a_k e^{j\omega_0 k t}$$

$$a_k = \frac{1}{T} \int_{-T/2}^{T/2} \tilde{x}(t) e^{-j\omega_0 k t} \, dt$$

$$= \frac{1}{T} \int_{-T/2}^{T/2} x(t) e^{-j\omega_0 k t} \, dt$$

$$= \frac{1}{T} \underbrace{\int_{-\infty}^{\infty} x(t) e^{-j\omega_0 k t} \, dt}_{X(j\omega_0 k)}$$

$$X(jk\omega_0) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega_0 k t}\, dt \Rightarrow \boxed{X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt}$$

Analytic FT

$$a_k = \frac{X(jk\omega_0)}{T}$$

Substituting this in synthesis eqn

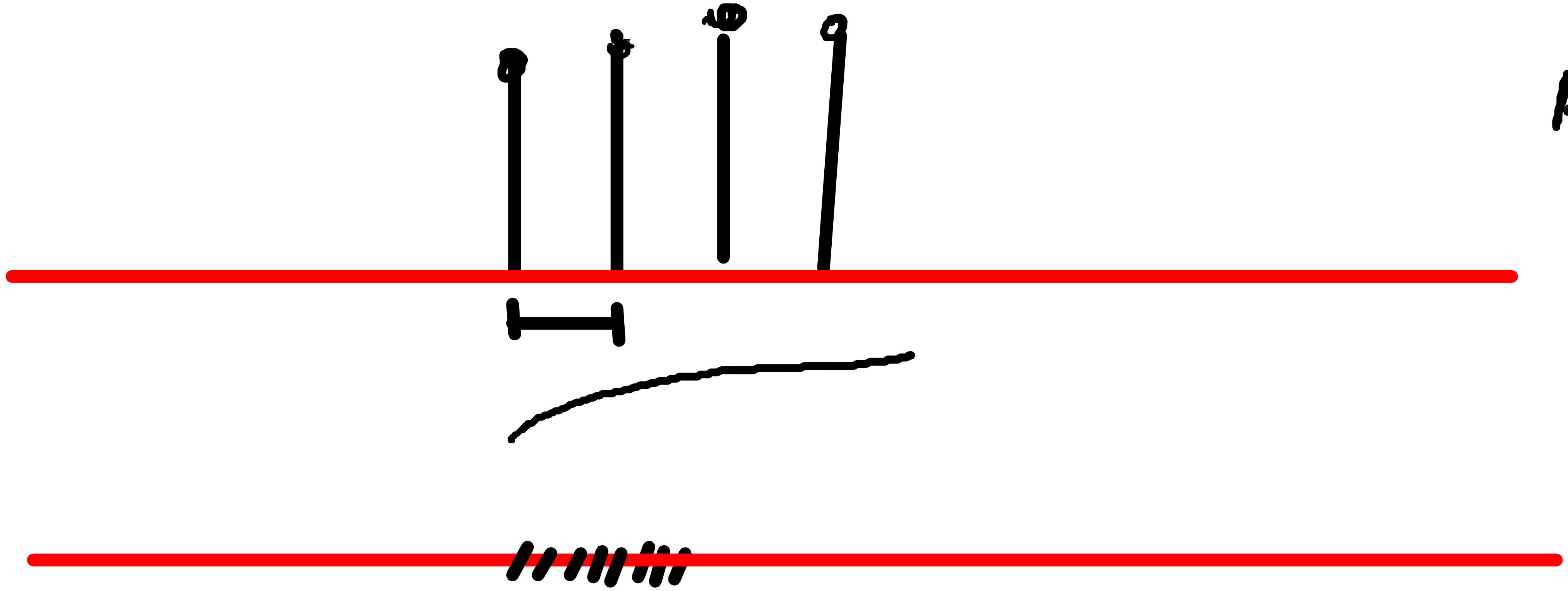
$$x(t) = \sum_{k=-\infty}^{\infty} a_k\, e^{j\omega_0 k t} \Rightarrow x(t) = \sum_{k=-\infty}^{\infty} \frac{X(jk\omega_0)}{T}\, e^{j\omega_0 k t}$$

$$\Rightarrow x(t) = \frac{1}{2\pi} \sum_{k=-\infty}^{\infty} X(jk\omega_0)\, e^{jk\omega_0 t}\, \omega_0$$

$\omega = k\omega_0$ $T = \frac{2\pi}{\omega}$

$\int$ ω $d\omega$

Integer multiple of fundamental frequency



$\omega = k\omega_0$

As $T_0 \to \infty$

or $\omega \to 0$

$\downarrow$

$k\omega_0$ discrete

$\downarrow$

Continuous ω

$$\boxed{x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)\, e^{j\omega t}\, d\omega}$$

Time Domain $x(t) \xleftrightarrow{FT} X(j\omega)$ Fourier Domain

Synthesis $\leftarrow$ $$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)\, e^{j\omega t}\, d\omega$$

Analysis $\leftarrow$ $$X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

Continuous-time Fourier Transform Pair

Time	Periodic/Aperiodic	Representation
CT	P	CTFS
DT	P	DTFS
CT	A	CTFT
DT	A	DTFT

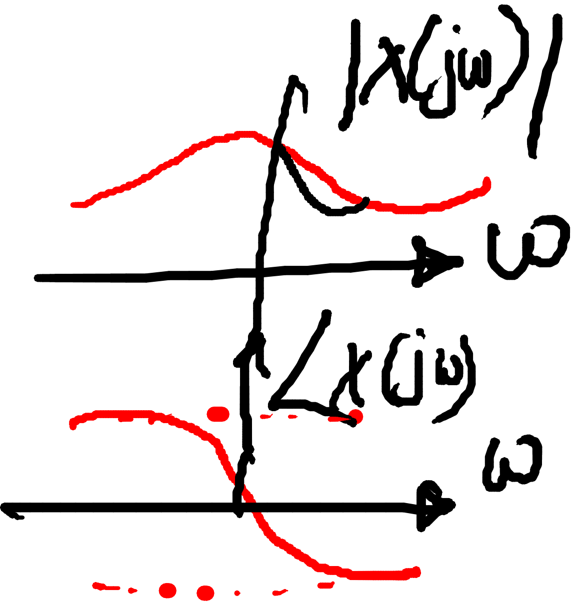
Example: $x(t) = e^{-at} u(t); \quad a > 0$

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

$$X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} \, dt = \int_{0}^{\infty} e^{-at} e^{-j\omega t} \, dt$$

$\pi/2 = \tan^{-1}(\infty)$

$$= \int_{0}^{\infty} e^{-(a+j\omega)t} \, dt = \frac{1}{a+j\omega}$$

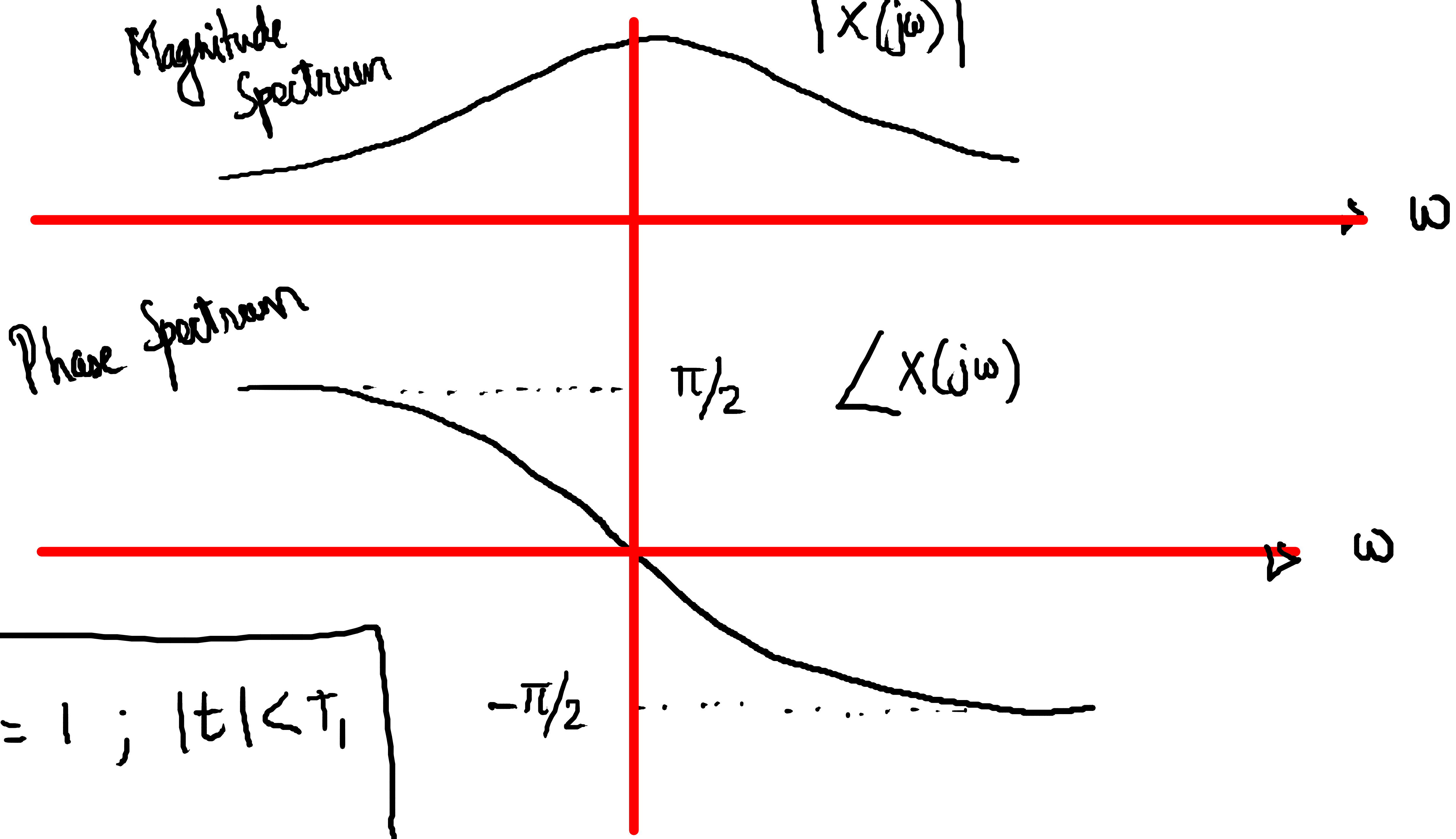


$$X(j\omega) = \frac{1}{a+j\omega}$$

For, $a = 1$ $\quad X(j\omega) = \dfrac{1}{1+j\omega}$

Magnitude $|X(j\omega)|$

Angle $\measuredangle X(j\omega)$



e.g

$$\boxed{x(t) = 1 \; ; \; |t| < T_1}$$

e.g : $x(t) = \delta(t)$