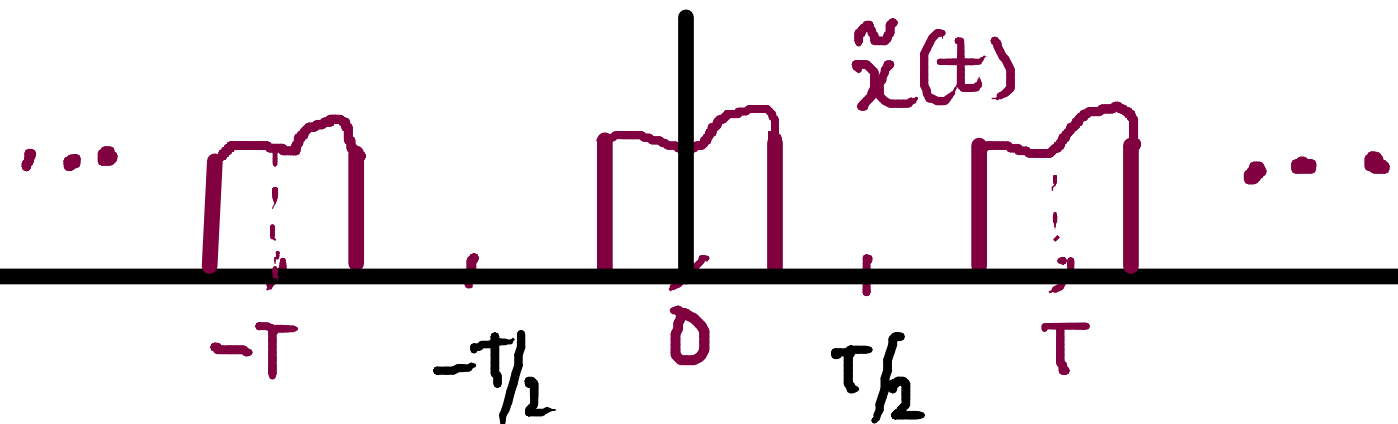
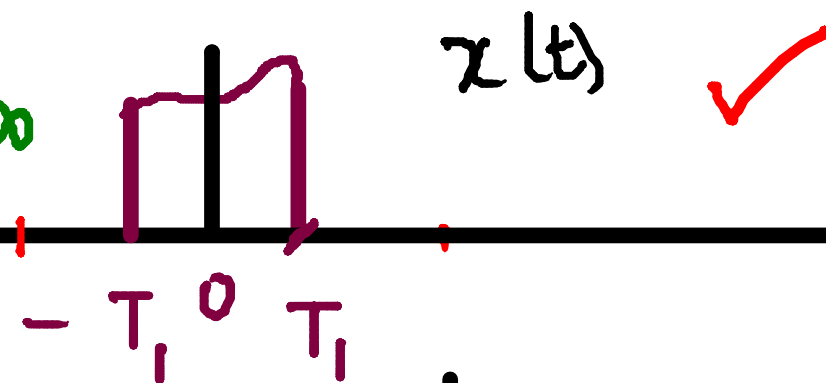


20 Sept 2017



$$x(t) = \tilde{x}(t) \quad ; \quad |t| < T/2$$

Periodic with $T \rightarrow \infty$



$$T = \frac{2\pi}{\omega} \quad \therefore \omega \rightarrow 0$$

$$\tilde{x}(t) = \sum_{\langle k \rangle} a_k e^{j\omega_k t}$$

$$\begin{aligned} a_k &= \frac{1}{T} \int_{-T/2}^{T/2} \tilde{x}(t) e^{-j\omega_k t} dt \\ &= \frac{1}{T} \int_{-T/2}^{T/2} x(t) e^{-j\omega_k t} dt \\ &= \frac{1}{T} \int_{-\infty}^{\infty} x(t) e^{-j\omega_k t} dt \end{aligned}$$

$X(j\omega_k)$

$$X(jk\omega_0) = \int_{-\infty}^{\infty} x(t) e^{-jk\omega_0 t} dt \Rightarrow \boxed{X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt}$$

Analytic
FT

$$a_k = \frac{X(jk\omega_0)}{T}$$

Substituting this in synthesis eqn

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$$

$$\Rightarrow x(t) = \sum_{k=-\infty}^{\infty} \left(\frac{X(jk\omega_0)}{T} \right) e^{jk\omega_0 t}$$

$$\Rightarrow x(t) = \frac{1}{2\pi} \sum_{k=-\infty}^{\infty} X(jk\omega_0) e^{jk\omega_0 t}$$

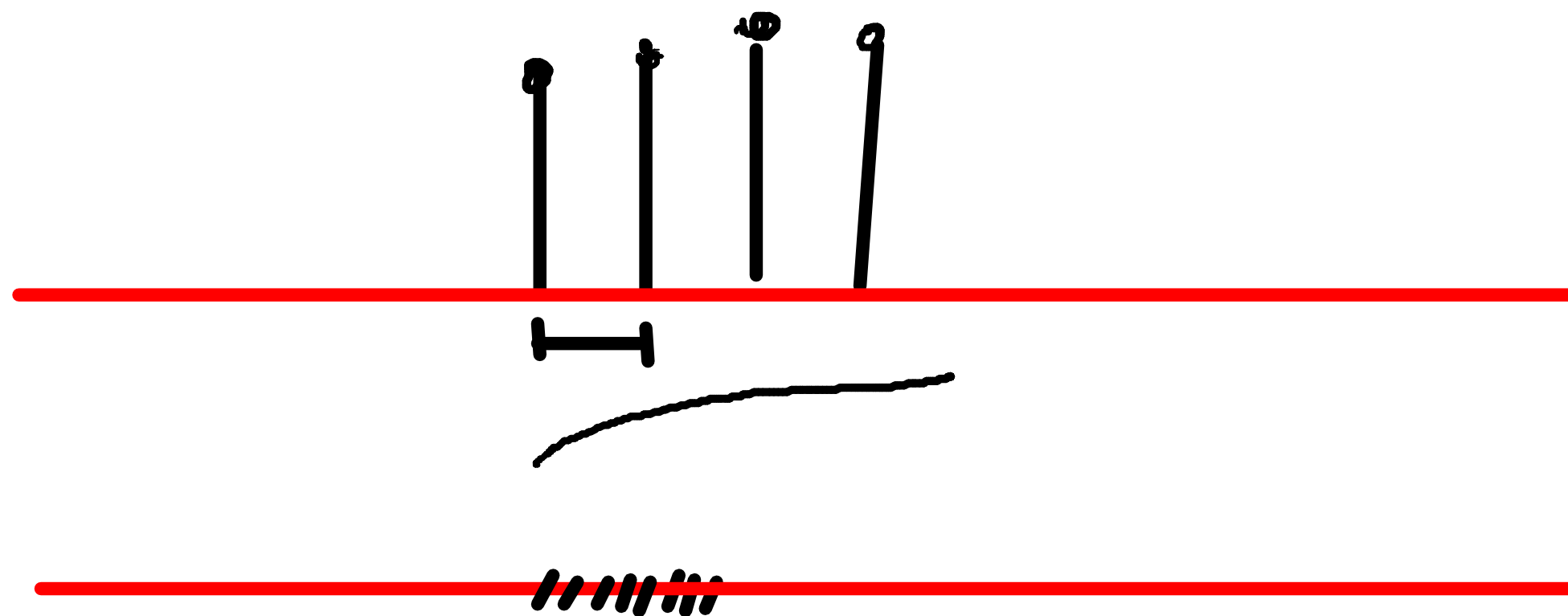
Diagram illustrating the relationship between the summation index k and the frequency ω :

- The summation index k is circled in purple.
- The term $X(jk\omega_0)$ is circled in red.
- The term $e^{jk\omega_0 t}$ is circled in purple.
- The frequency ω_0 is circled in purple.
- Arrows point from the circled k and ω_0 to a circled ω and a circled $d\omega$ respectively.

$$\omega = k\omega_0$$

$$T = \frac{2\pi}{\omega_0}$$

Integer multiple of
fundamental frequency



$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega) e^{j\omega t} d\omega$$

$$\omega = k\omega_0$$

$$As \quad T_0 \rightarrow \infty$$

$$or \quad \omega \rightarrow 0$$



$k\omega_0$ discrete



Continuous

ω

Time Domain $x(t) \xleftrightarrow{FT} X(j\omega)$ Fourier Domain

Synthesis \leftarrow

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega) e^{j\omega t} d\omega$$

Analysis \leftarrow

$$X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

Continuous-time
Fourier
Transform Pair

Time	Periodic/ Aperiodic	Representation
CT	P	CTFS
DT	P	DTFS
CT	A	CTFT
DT	A	DTFT

Example: $x(t) = e^{-at} u(t); \quad a > 0$

$$X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

$$= \int_0^{\infty} e^{-(a+j\omega)t} dt$$

$$X(j\omega) = \frac{1}{a+j\omega}$$

$$= \int_0^{\infty} e^{-at} e^{-j\omega t} dt$$

$$= \frac{1}{a+j\omega}$$

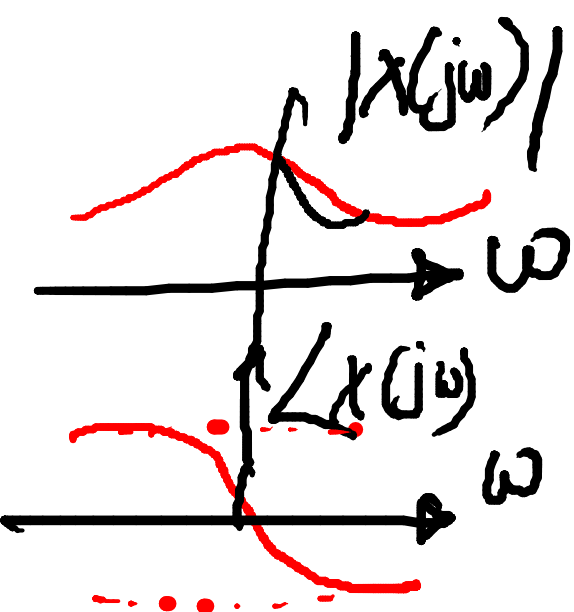
For $a=1$ $X(j\omega) = \frac{1}{1+j\omega}$

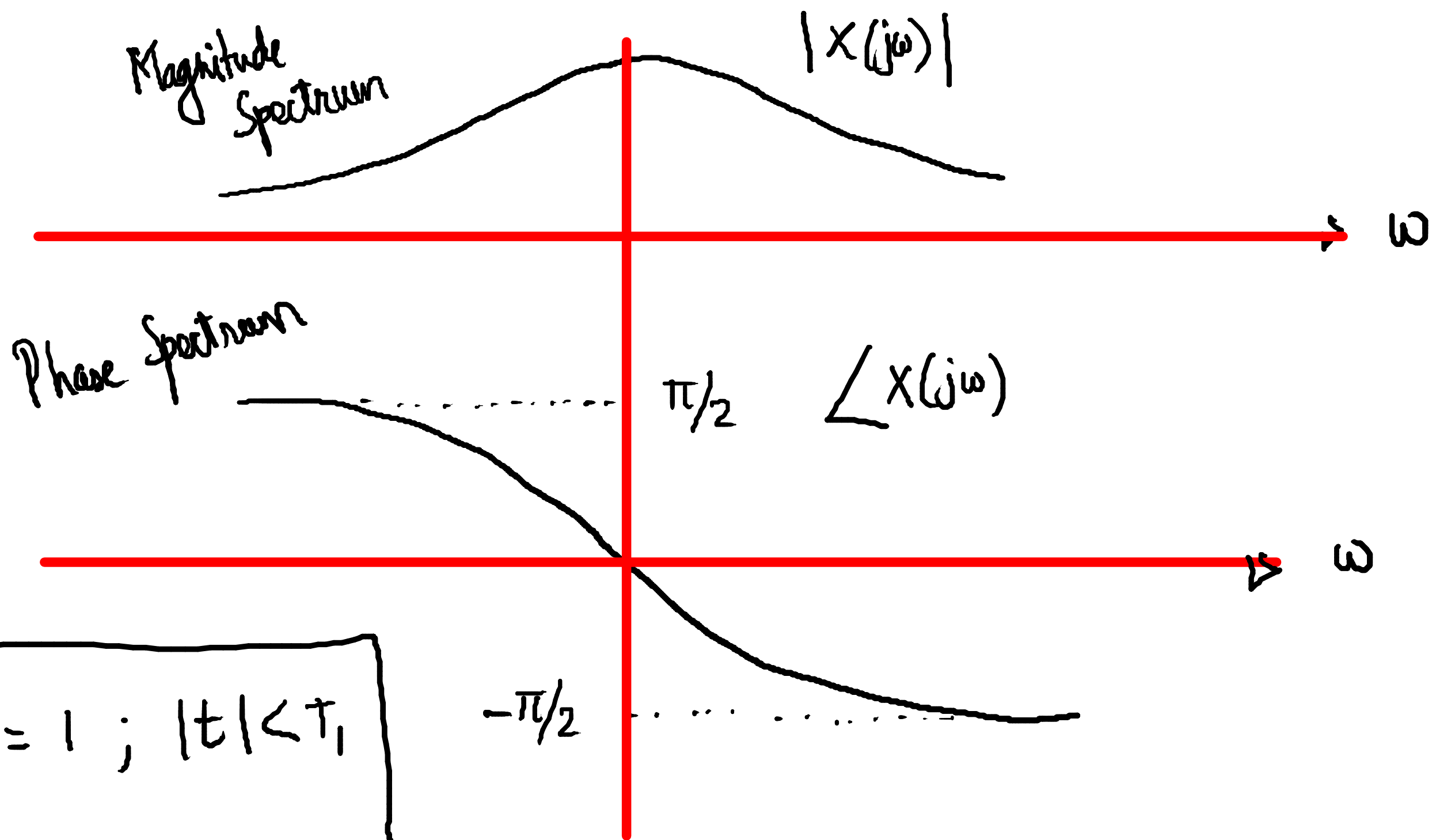
Magnitude $|X(j\omega)|$

Angle $\angle X(j\omega)$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\pi/2 = \tan^{-1}(\infty)$$





eg

$$x(t) = 1 ; |t| < \tau_1$$

eg : $x(t) = \delta(t)$