25 Sept 2017

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

CT FT

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega)\, e^{j\omega t}\, d\omega$$

Relationship between FS & FT :

$\delta(\omega)$

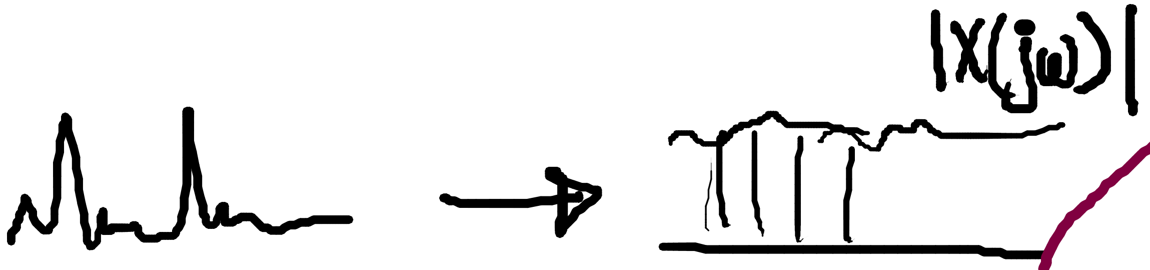
$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$$

$$X(j\omega) = \delta(\omega)$$

$$x(t) = \frac{1}{2\pi}$$

$$X(j\omega) = \delta(\omega - \omega_0)$$

$$x(t) = \left(e^{j\omega_0 t}\right)\frac{1}{2\pi}$$



$\cos(\omega_0 n)$

$$\underbrace{\sum_{\langle k \rangle} 2\pi a_k \delta(\omega - k\omega_0)}_{\text{CT Fourier Transform}} \xrightarrow{\text{IFT}} \underbrace{\sum_{\langle k \rangle} a_k e^{jk\omega_0 t}}_{\text{CT Fourier Series}}$$

If we have a continuous-time periodic signal

$$x(t) \to a_k$$

CT, Periodic $\longrightarrow$ Fourier Series

CT, Aperiodic $\longrightarrow$ Fourier Transform.

Both CT, Periodic & CT, Aperiodic signals can be represented by Fourier Transform if we allow impulses.

If a CT, periodic signal $x(t)$ is represented by Fourier series coefficients a_k, then the Fourier Transform of $x(t)$ is scaled (by 2π) and shifted impulses (by integer multiple of ω_0)

Example : $x(t) = \cos(\omega_0 t)$

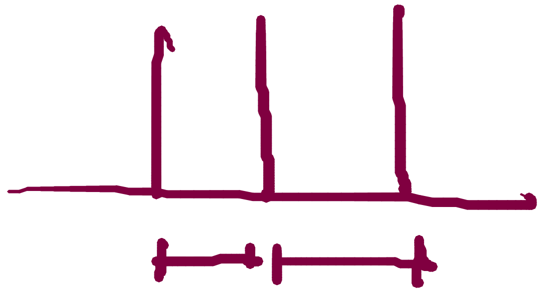
$$x(t) = \underbrace{\frac{1}{2} e^{j\omega_0 t}}_{k=1} + \underbrace{\frac{1}{2} e^{-j\omega_0 t}}_{k=-1}$$

ω_0

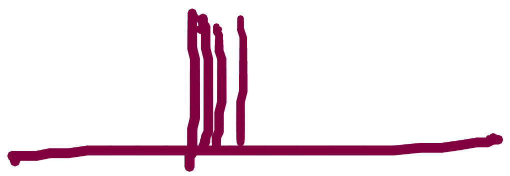
$k\omega_0$

$T_0 \to \infty$

$\omega_0 \to 0$



ω



Fourier Series $\quad a_k \Big|_{k=\pm 1} = \frac{1}{2}$

Fourier Transform

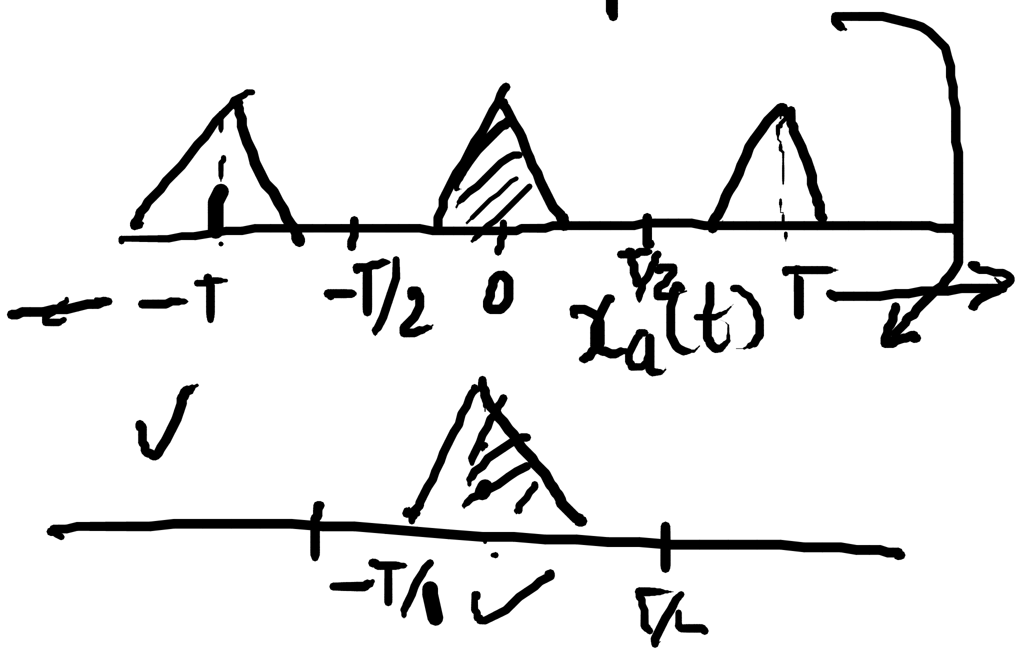
$$X(j\omega) = \pi\,\delta(\omega - \omega_0) + \pi\,\delta(\omega + \omega_0)$$

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$\delta(\omega)$

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega) e^{j\omega t}\, d\omega$$

$x_p(t)$



Continuous time Periodic signal $\left.\right\}$ $x_p(t) \longrightarrow a_k$

$$x_a(t)$$

$= \frac{1}{2\pi}$

$$x_p(t) = x_a(t) \Big|_{t \leq |T/2|}$$

$$a_k = \frac{1}{T} \int_{-T/2}^{T/2} x_p(t)\, e^{-j\omega_0 k t}\, dt$$

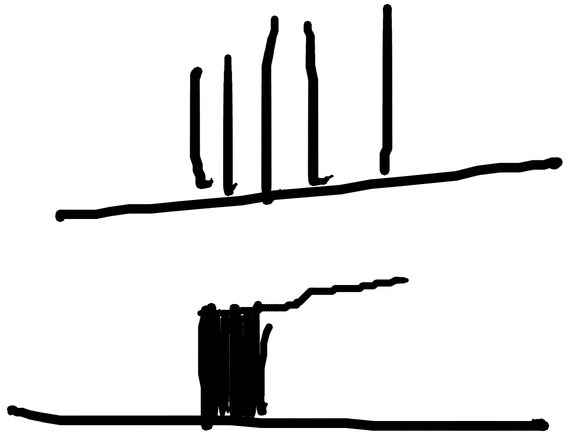
$$= \frac{1}{T} \int_{-T/2}^{T/2} x_a(t)\, e^{-j\omega_0 k t}\, dt$$

$$= \frac{1}{T} \int_{-\infty}^{\infty} x_a(t)\, e^{-j\omega_0 k t}\, dt$$

$T \to \infty$

$\omega_0 \to 0$

$k\omega_0 \to \omega$



$$x(t) = \sum_{\langle k \rangle} a_k e^{j\omega_0 k t}$$

$$= \frac{\omega}{2\pi} \sum_k X(j\omega) e^{j\omega_0 k t}$$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega) e^{j\omega t} \, d\omega$$

✓ $x_p(t) \longrightarrow a_k$ FS

$T_0 \to \infty$

$\omega_0 \to 0$

✓ $x_a(t) \longrightarrow X(j\omega)$ FT

p & a $\longrightarrow$ FT

CTFS DTFC

CTFT DTFT

$\omega_0 = \frac{2\pi}{T}$

$$x(t) = \sum_{\langle k \rangle} a_k e^{jk\omega_0 t}$$

$$a_k = \frac{1}{T} \int_{\langle T \rangle} x(t) e^{-jk\omega_0 t} dt$$

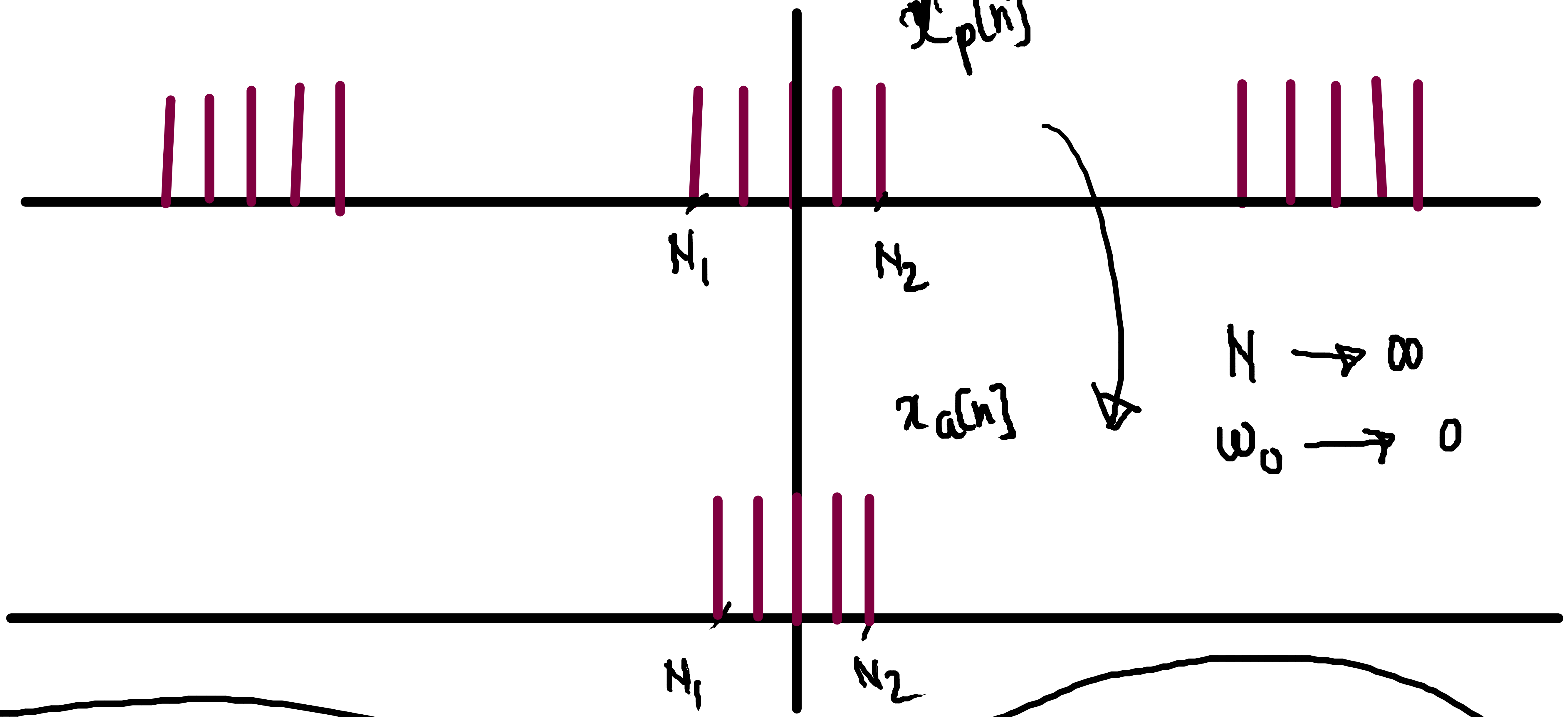
CTFS

$\omega_0 = \frac{2\pi}{N}$

$$x[n] = \sum_{\langle k \rangle} a_k e^{jk\omega_0 n}$$

$$a_k = \frac{1}{N} \sum_{\langle n \rangle} x[n] e^{-jk\omega_0 n}$$

DTFS



$N \to \infty$

$\omega_0 \to 0$

$$X(j\omega) = \sum_{k=-\infty}^{\infty} x[n]\, e^{-j\omega n} \qquad x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(j\omega)\, e^{j\omega n}\, d\omega$$