Probability and Random Processes

Problems

Gaussian random vectors

1. $M_{ZS}(\omega_1, \omega_2) = e^{i(\omega_1+\omega_2)m_S - \frac{1}{2}\sigma_S^2(\omega_1+\omega_2)^2} e^{-\frac{1}{2}\omega_1^2\sigma_N^2}$
2.
3. $f_Y(y) = \begin{cases} \dfrac{1}{\sqrt{2\pi y}} e^{-\frac{1}{2}y}, & y > 0 \\ 0, & y < 0 \end{cases}$; $\quad M_Z(\omega) = (1-2i\omega)^{-n/2}$
4. $M_Y(\omega_1, \omega_2) = e^{-\frac{1}{2}3\omega_1^2} e^{-\frac{1}{2}\omega_2^2} = M_{Y_1}(\omega_1) M_{Y_2}(\omega_2)$, $\quad Y_1$ and Y_2 are independent.
   $f_{Y_1}(y) = \dfrac{1}{\sqrt{2\pi}\sqrt{3}} e^{-\frac{1}{2}\frac{y^2}{3}}$
5. $\begin{pmatrix} 2 & 1 \\ 1 & 5 \end{pmatrix}$
6. $K_Y = \begin{pmatrix} 4 & 4 \\ 4 & 7 \end{pmatrix}$, $\quad f_{Y_1}(y) = \frac{1}{\sqrt{8\pi}} e^{-(y-1)^2/8}$, $f_{Y_2}(y) = \frac{1}{\sqrt{14\pi}} e^{-(y+1)^2/14}$, $\quad \mathrm{P}(X_3 < X_1 + X_2) = F_{N(0,1)}\left(\frac{1}{2}\right)$
7.
8. $\mathrm{E}(X) = 2, \quad \mathrm{Var}(X) = 5, \quad \mu_{11;XY_1} = 3$
9. $M_{SR}(\omega_1, \omega_2) = e^{-\omega_1^2 - \omega_2^2}, \quad M_S(\omega_1) = e^{-\omega_1^2}$, $M_R(\omega_2) = e^{-\omega_2^2}$
10. $1 - F_{\mathsf{N}(0,1)}\left(\dfrac{n-1}{\sqrt{2(n+1)}}\right)$
11. 2
12.
13.