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Incorporating Prior Information from Engineering Design into Gaussian Process Regression: with applications to Aeronautical Engineering

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Nomenclature

$GP[m(x), k(x, x')]$ Gaussian Process

$\Pr[a \mid b]$ Conditional probability distribution of a given b

$\Pr[a, b]$ Joint probability distribution of a and b

$\Pr[a]$ Probability distribution of a

$\boldsymbol{a}$ Vector $\boldsymbol{a}$, written in bold lowercase

$\boldsymbol{a}^T$ Transpose of vector $\boldsymbol{a}$

$\mathcal{N}[\boldsymbol{\mu}, \boldsymbol{\Sigma}]$ Multivariate Gaussian Distribution

$\mathcal{N}[\mu, \sigma^2]$ Scalar Gaussian distribution

$\mathcal{R}$ Set of all real numbers

$\boldsymbol{A}$ Matrix $\boldsymbol{A}$, written in bold uppercase

a Scalar a, written in lowercase

A_i i^{th} mode shape

$Cov[x, x']$ Covariance between two random variables

D_{inputs} Number of dimensions in input data

$D_{outputs}$ Number of dimensions in the output data

M Number of inducing points

M_x Bending moment on the wing

N Number of points in training dataset

N_* Number of points in test dataset

$N_{experiments}$ Number of experiments

$N_{experts}$ Number of experts in distributed GP

N_{hyp} Number of hyper-parameters

N_{joint} Total number of points for MTGP

N_j Number of points in the j^{th} output for MTGP

N_{nodes} Number of nodes in aerodynamic mesh

P Number of points in an expert

Q Number of Gaussians in a SM kernel

T_z Shear Load on the wing

Ω Full spatial coordinates of the aerodynamic mesh

$\boldsymbol{\theta}$ Vector of hyper-parameters

$\boldsymbol{y_{joint}}$ Joint output dataset for MTGP

$\boldsymbol{y}$ Full training output dataset

$\boldsymbol{y}^j$ Full set of outputs for the j^{th} output for MTGP

δ Dirac-delta function

ϵ Independent white noise

η_{edge} Wing span at the edge of the wing

η_{root} Wing span at the root of the wing

λ_i i^{th} modal frequency

$\mathbf{E}[x]$ Expectation of the random variable x

$\mathcal{D}_i$ i^{th} Data-set

μ Mean value of a Gaussian Distribution

$\boldsymbol{C}$ Damping matrix

$\boldsymbol{H}$ Low-rank matrix

$\boldsymbol{I_N}$ Identity matrix of size N

$\boldsymbol{K(X_1, X_2)}$ Gram matrix between $\boldsymbol{X_1}$ and $\boldsymbol{X_2}$

$\boldsymbol{K_{FITC}}$ Gram matrix using FITC approximation

$\boldsymbol{K_{Nystrom}}$ Gram matrix using Nyström approximation

$\boldsymbol{K_{lower}}$ Lower Cholesky decomposition of Gram matrix

$\boldsymbol{K_{outputs}}$ Covariance matrix between output dimensions

$\boldsymbol{K_{stiffness}}$ Stiffness matrix

$\boldsymbol{M}$ Mass matrix

$\boldsymbol{P}$ Full Pressure data for the aerodynamic mesh

$\boldsymbol{X^M}$ Set of inducing points

$\boldsymbol{X^j}$ Full set of inputs for the j^{th} output for MTGP

$\boldsymbol{X_{joint}}$ Joint input dataset for MTGP

$\boldsymbol{X}$ Full training input dataset

$\boldsymbol{\Sigma_{Prior}}$ Covariance matrix of prior over parameters

$\boldsymbol{x_*}$ Full test input dataset

ν Degrees of freedom of student distribution

ω_i i^{th} spatial coordinate of the aerodynamic node

$\phi(x)$ Basis functions

$\sigma^{(i)}(x)$ Predicted covariance of the i^{th} expert

σ_n^2 Variance of the white noise

τ Time lag between two points

$\theta_{amplitude}$ Amplitude hyper-parameter

$\theta_{changeLocation}$ Change location hyper-parameter

$\theta_{intensity}$ Intensity hyper-parameter for k_{CP}

$\theta_{lengthScale}$ Length scale hyper-parameter

b Wing span

d Distance between two input points

$diag(\boldsymbol{K})$ Diagonal of the matrix

f Function mapping transformation from x to y

$f_{pressure}$ Pressure function mapping transformation from ω_i to p_i

$k(x, x')$ Covariance function a GP

k^{δ} Covariance function of the difference between multi-fidelity GPs

$m(x)$ Mean function a GP

$m^{(i)}(x)$ Predicted mean of the i^{th} expert

p_i Pressure at the i^{th} aerodynamic node

w Parameters of functions

x_* Single test input point

x_i i^{th} observational input point

y_i i^{th} observational output point

$ANOVA$ Analysis Of Variance

ARD Automatic Relevance Determination

BCM Bayesian Committee Machine

BIC Bayesian Information Criterion

CFD Computational Fluid Dynamics

CP Change-Point

CRM Common Research Model

CV Cross Validation

EMA Experimental Modal Analysis

FEM Finite Element Method

$FITC$ Fully Independent Training Conditional

FTF Flap Track Fairing

GP Gaussian Process

HCT Heritage Court Tower

IT Information Technology

LOO Leave One Out

MDO Multi Disciplinary Optimization

$MDOF$ Multi Degree Of Freedom

$NExT$ Natural Excitation Technique

OMA Operational Modal Analysis

PCA Principal Component Analysis

POD Proper Orthogonal Decomposition

POE Product Of Experts

PSD Positive Semi Definite

$RANS$ Reynolds Averaged Navier-Stokes

RFP Rational Fractional Polynomial

$RMSE$ Root Mean Square Error

ROM Reduced Order Models

SE Standard Exponential

SVD Singular Value Decomposition

$gPOE$ generalized Product Of Experts

$rBCM$ robust Bayesian Committee Machine