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Date _____

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STUDY BUDDIES

Assignment LPP

Q:-1

Let day 'i' be 1, 2, 3, 4, ... starting from Monday = day 1, ...
There are 3 shifts.

Night shift - 1, Day shift - 2, Late shift - 3.

Also, let N_{ij} denote the number of workers who are starting their work on day i & shift j.

Constraints:-

Total number of workers ≤ 60 .

$$\sum_{i=1}^7 \sum_{j=1}^3 N_{ij} \leq 60 \quad - (1)$$

Secondly, we have a lower bound on number of workers in a single day & particular shift.

Let W_{ij} denote the minimum number of workers required on day i & shift j.

Workers who work on Monday -

- either they start on Monday.
- or, they had started on Friday, Saturday or Sunday.

∴ For Monday i.e. $i=1$;

$$\text{Monday} \quad N_{1j} + N_{7j} + N_{6j} + N_{5j} \geq W_{1j} \quad (j=1, 2, 3)$$

$$\text{Tuesday} \quad N_{2j} + N_{1j} + N_{7j} + N_{6j} \geq W_{2j}$$

$$\text{Wednesday} \quad N_{3j} + N_{2j} + N_{1j} + N_{7j} \geq W_{3j}$$

$$\text{Thursday} \quad N_{4j} + N_{3j} + N_{2j} + N_{1j} \geq W_{4j}$$

$$\text{Friday} \quad N_{5j} + N_{4j} + N_{3j} + N_{2j} \geq W_{5j}$$

$$\text{Saturday} \quad N_{6j} + N_{5j} + N_{4j} + N_{3j} \geq W_{6j}$$

$$\text{Sunday} \quad N_{7j} + N_{6j} + N_{5j} + N_{4j} \geq W_{7j}$$

• The objective function is to minimize

$$\text{Min} \quad \sum_{i=1}^7 \sum_{j=1}^3 N_{ij}$$

Advantages of solving this problem as LPP →

- (i) Ability to convert work problem systematically into a LPP.
- (ii) We can optimize the problem in almost linear time to reduce workforce.

Disadvantages →

- (i) The required solution must be integer which is not always TRUE.
- (ii) The optimal solⁿ doesn't take the learning or arrival of new workers (holiday taken by worker will be considered).

Q:-2.

Let x_1 denote no. of units of A
 x_2 " " " " " " B

Objective Function →

$$\text{Max } Z = 20x_1 + 50x_2$$

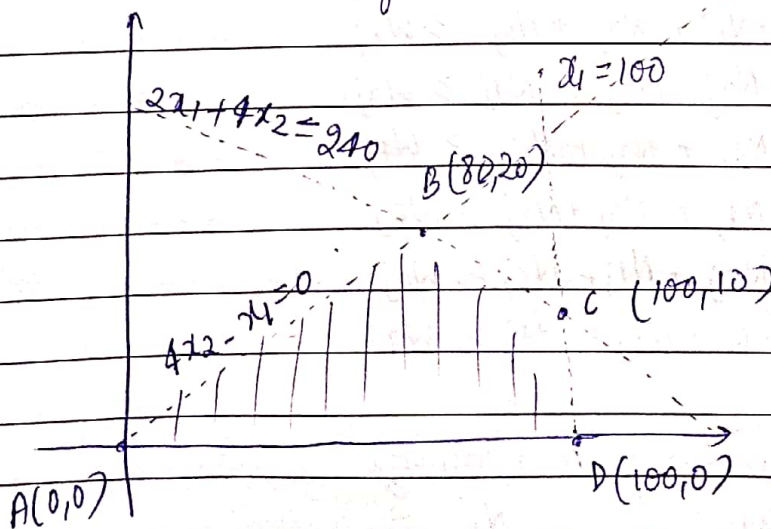
Constraints :-

$$(1) \frac{x_1}{x_1 + x_2} \geq \frac{80}{100} \quad \text{or} \quad 4x_2 - x_1 \leq 0$$

$$(2) x_1 \leq 100$$

$$(3) 2x_1 + 4x_2 \leq 240; \quad x_1, x_2 \geq 0$$

Now we have to find the intersection point by graphical method.



Optimum occurs at B (80, 20).

$$x_1 = 80, \quad y = 20$$

$$Z = 20x_1 + 50x_2 = 20 \times 80 + 50 \times 20 \\ = 2600$$

Q:-3

Let x_1 = no. of hour Jack plays

x_2 = " " " works

Objective Function \rightarrow

$$\text{Minimize } Z = 2x_1 + x_2$$

Constraints \rightarrow

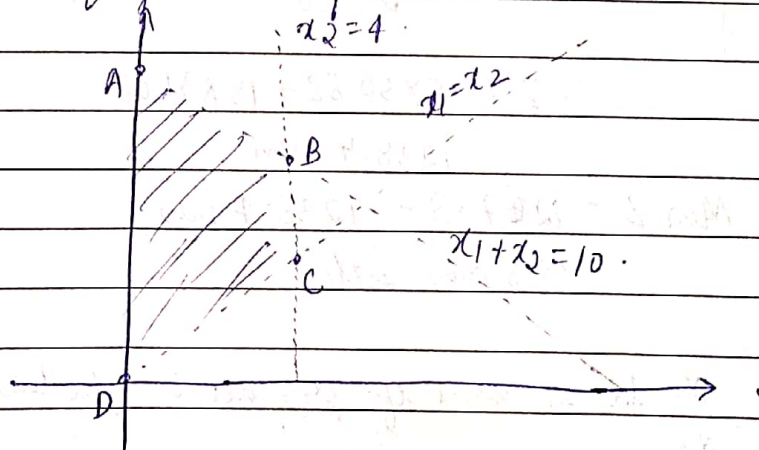
$$x_1 + x_2 \leq 10$$

$$x_1 \geq x_2 \Rightarrow x_2 - x_1 \leq 0$$

$$x_2 \leq 4$$

$$x_1, x_2 \geq 0$$

Now, Min of $Z = 2x_1 + x_2$ is equivalent to 0 because the graph of above eqⁿ give the feasible area.



In this the point D satisfies the least criteria & so;
 $\text{Min } Z = 0$ at $x_1 = 0$ & $x_2 = 0$.

Q:-4

x_1 = no. of HiFi-1 unit

x_2 = " " HiFi-2 "

Constraints -

$$6x_1 + 4x_2 \leq 432$$

$$5x_1 + 5x_2 \leq 412.8$$

$$4x_1 + 6x_2 \leq 422.4$$

Let I_1 be the time machine 1 sits idle, same for I_2 & I_3 .

Hence, $6x_1 + 4x_2 + I_1 = 432$

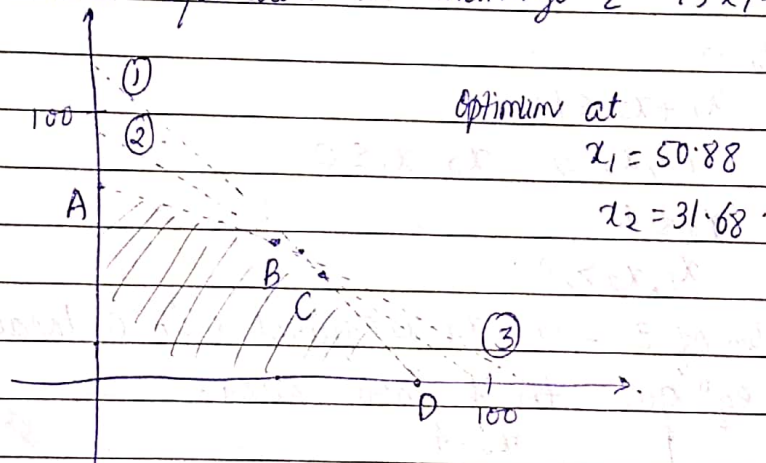
$$5x_1 + 5x_2 + I_2 = 412.8$$

$$4x_1 + 6x_2 + I_3 = 422.8$$

Objective Function \rightarrow

$$\text{Min } Z = I_1 + I_2 + I_3 = 1267.2 - 15x_1 - 15x_2$$

This is equivalent to Maximize $Z' = 15x_1 + 15x_2$



$$\therefore Z' = 15 \times 50.88 + 15 \times 31.68$$

$$= 1238.4 \text{ min}$$

Hence, $\text{Min } Z = 1267.2 - 1238.4 \text{ min}$
 $= 28.8 \text{ min idle}$

Q: 5 Let x_1 be amount of product i, produced on machine X per week
 x_2 " " " " " " " " " " " "

Constraints \rightarrow

$$0.1x_1 + 0.15(x_2 + y_2) + 0.5(x_3 + y_3) + 0.05(x_4 + y_4) \leq 50$$

$$0.95(2(x_3 + y_3)) \leq x_2 + y_2 \leq 1.05(2(x_3 + y_3))$$

$$10x_1 + 12x_2 + 13x_3 + 8x_4 \leq 1995$$

$$27y_1 + 19y_2 + 33y_3 + 23y_4 \leq 1953$$

Objective Function \rightarrow

$$\text{Max } Z = 10x_1 + 12(x_2 + y_2) + 17(x_3 + y_3) + 8(x_4 + y_4)$$

Q:-6.

=

x_1 = Ratio of scrap A alloy

x_2 = " " scrap B alloy

Objective Function \rightarrow

$$\text{Min } Z = 100x_1 + 80x_2$$

Constraints \rightarrow

$$0.03 \leq 0.06x_1 + 0.03x_2 \leq 0.06$$

$$0.03 \leq 0.03x_1 + 0.06x_2 \leq 0.05$$

$$0.03 \leq 0.04x_1 + 0.03x_2 \leq 0.07$$

$$x_1 + x_2 = 1$$

and, $x_1, x_2 \geq 0$

For solving this eqn we have to use LPP solver.

After putting the constraints in the LPP solver then,

$$\text{Optimum } x_1 = 0.33, x_2 = 0.67$$

$$\begin{aligned} \text{Min } Z &= ~~86.67~~ 100 \times 0.33 + 80 \times 0.67 \\ &= 33 + 53.6 \\ &= 86.6 \end{aligned}$$

Q:-7. Let given \rightarrow

x_1 = No. of single family homes

x_2 = " " double " "

x_3 = " " triple " "

x_4 = No. of recreation areas

$$\text{Maximize } Z = 10000x_1 + 12000x_2 + 15000x_3.$$

Constraints \rightarrow

$$2x_1 + 3x_2 + 4x_3 + x_4 \leq 680.$$

$$0.5x_1 - 0.5x_2 - 0.5x_3 \geq 0.$$

$$200x_4 - x_1 - 2x_2 - 3x_3 \geq 0.$$

$$1000x_1 + 1200x_2 + 1400x_3 + 800x_4 \geq 100000$$

$$400x_1 + 600x_2 + 840x_3 + 450x_4 \leq 200000.$$

$$\& \quad x_1, x_2, x_3, x_4 \geq 0.$$

Using LPP Solver \rightarrow

optimum solution is at \rightarrow

$$x_1 = 339.15 \text{ homes.}$$

$$x_2 = 0$$

$$x_3 = 0$$

$$x_4 = 1.69 \text{ recreation areas.}$$

Hence,

$$\begin{aligned} \text{Max } Z &= 10000 \times 339.15 + 0 + 0 + 15000 \times 1.69 \\ &= 3391500 + 25350 \\ &= 3416850. \end{aligned}$$