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Date __/__/__
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STUDY BUDDIES

Assignment LPP-2.

Q: 1.

P_{xA} = Amount invested in year "x" with plan A ($1000).
P_{xB} = " " " " B ($1000)
x = (1, 2, 3)

Objective Function is :-

$$Max\ Z = 3P_{2B} + 1.7P_{3A}.$$

Constraints :-

$$P_{1A} + P_{1B} \leq 100$$
$$-1.7P_{1A} + P_{2A} + P_{2B} = 0.$$
$$-3P_{1B} - 1.7P_{2A} + P_{3A} = 0.$$

$(P_{xA}, P_{xB} \geq 0$ for $x = 1, 2, 3)$

Using LP Solver →.

Max Z = 510,000

P_{xA} = $100000 P_{xB} = $170000

or P_{xB} = $100000 P_{xA} = $300000.

Q: 2 (i) Variables →

A = Set of suppliers

B = Set of Months

C_{ij} = Price per carton ∈ Months of supplier j ∀ i ∈ B ∧ j ∈ A [$/carton]

D_i = Demand ∈ Month ∀ i ∈ B.

R = Refrigerating cost per unit per month

M = Supplier Max. Capacity.

(ii) Decision Variables →

X_{ij} = No. of cartons ∈ Month of Supplier j ∀ i ∈ B ∧ ∀ j ∈ A.

U_{ij} = Unused Capacity ∈ Month by Supplier

I_i = Ending Inventory ∈ Month.

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Objective Function →

Minimize Costs $= \sum_{i \in B} \sum_{j \in A} C_{ij} X_{ij} + \sum_{i \in 1}^{3} \sum_{j=1}^{2} X_{ij} + I_{i-1} + I_i$

Constraints

Max. Capacity → $X_{ij} + U_{ij} = M$

Monthly Demand → $\sum_{j=1}^{2} X_{ij} + I_{i-1} - I_i = D$

Non Negativity: $X_{ij}, U_{ij}, I_i \geq 0$.

Optimal Solution → (Using LPP Solver).

i	X_{i1}	X_{i2}	I
1	400	100	0
2	400	400	200
3	200	0	0

Min $Z = \$167,450$

Q: 3.

$P_o(A,B,C)$ = oats in cereal A, B, C.

$P_r(A,C)$ = Raisins in cereals A, C.

$P_c(B,C)$ = Coconuts in cereals B, C

$P_a(A,B,C)$ = Almonds in " A, B, C.

$Q_o = P_{oA} + P_{oB} + P_{oC}$

$Q_r = P_{rA} + P_{rC}$.

$Q_c = P_{cB} + P_{cC}$.

$Q_a = P_{aA} + P_{aB} + P_{aC}$.

$R_A = P_{oA} + P_{rA} + P_{aA}$.

$R_B = P_{oB} + P_{cB} + P_{aB}$

$R_C = P_{oC} + P_{rC} + P_{cC} + P_{aC}$

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Objective function →

$$Max.\ Z = \frac{1}{5}\left[2R_A + 2.5R_B + 3R_C\right] - \frac{1}{2000}\left[100Q_0 + 120Q_r + 110Q_c + 200Q_a\right].$$

Constraints →

$R_A \le 2500$, $R_B \le 3000$, $R_C \le 4000$.

$Q_0 \le 10000$, $Q_r \le 4000$, $Q_c \le 2000$, $Q_a \le 2000$

$P_{OA} = \frac{50}{5} P_{0A}$, $P_{OA} = \frac{50}{2} P_{aA}$, $P_B = \frac{60}{2} P_{cB}$, $P_{OB} = \frac{60}{3} P_{aB}$, $P_{OC} = \frac{60}{3} P_{rC}$,

$P_{OC} = \frac{60}{4} P_{cC}$, $P_{OC} = \frac{60}{2} P_{aC}$.

all variables ≥ 0.

After solving using LP Solver the optimum solution is →

Max Z = \$ 5344.26 / day.

R_A : 2500 lb

R_B : 3000 lb.

R_C : 5794 lb.

P_0 = 10000 lb

P_r = 471.19 lb.

P_c = 428.16 lb.

P_a = 394.11 lb.

Q:- 4. let X_i be the no. of volunteers starting from hour i.

Objective →

$$Min\ Z = \sum_{i=1}^{14} X_i$$

Constraints →

Time	Equation	
8:00AM	X_1	≥ 4
9AM	$X_1 + X_2$	≥ 4.
10AM	$X_1 + X_2 + X_3$	≥ 6
11AM	$X_2 + X_3 + X_4$	≥ 6
12PM	$X_3 + X_4 + X_5$	≥ 8
1PM	$\cancel{X_3} + X_4 + X_5 + X_6$	≥ 8.

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2PM	$x_5+x_6+x_7$	≥ 6
3PM	$x_6+x_7+x_8$	≥ 6.
4PM	$x_7+x_8+x_9$	≥ 4.
5PM	$x_8+x_9+x_{10}$	≥ 4.
6PM	$x_9+x_{10}+x_{11}$	≥ 6.
7PM	$x_{10}+x_{11}+x_{12}$	≥ 6.
8PM	$x_{11}+x_{12}+x_{13}$	≥ 8
9PM	$x_{12}+x_{13}$	≥ 8.

Using LPP Solver, we get the minimum as →.

$$\text{Min } Z = 32 \quad \text{at}$$

$x_1=4$, $x_2=0$, $x_3=2$, $x_4=6$, $x_5=0$, $x_6=2$, $x_7=4$, $x_8=0$, $x_9=0$, $x_{10}=6$, $x_{11}=0$, $x_{12}=8$, $x_{13}=0$.

Q:-5.

Let x_i = No. of starting at hour i.

$i=1$ (8:01) to $i=9$ (4:01).

No student is starting at 12 PM so x_5 has to be 0.

Objective Function →

$$\text{Min } Z = \sum_{i=1}^{9} x_i .$$

Constraints.

Time	Equations	
8:01 AM	x_1	≥ 2
9:01 AM	x_1+x_2	≥ 2.
10:01 AM	$x_1+x_2+x_3$	≥ 3
11:01 AM	$x_2+x_3+x_4$	≥ 4.
12:01 PM	x_3+x_4	≥ 4.
1:01 PM	$\cancel{x_3}+x_4+x_6$	≥ 3.
2:01 PM	x_6+x_7	≥ 3.
3:01 PM	x_7+x_8	≥ 3.
4:01 PM	$x_7+x_8+x_9$	≥ 3.

$x_5=0$, $x_i \geq 0$ $i=1,2,3 \ldots 9$.

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Using LP solver, we get:

Min $Z = 9$ students at:

$X_1 = 2, X_2 = 0, X_3 = 1, X_4 = 3, X_5 = 0, X_6 = 0, X_7 = 3, X_8 = 0, X_9 = 0.$

Q:-6.

Objective Function →.

$$Max\ Z = 2X_1 + 5X_2 - MR_1.$$

Constraints →.

$$3X_1 + 2X_2 - S_1 + R_1 = 6.$$

$$2X_1 + X_2 + S_2 = 2.$$

$$X_1, X_2, S_1, R_1, S_2 \geq 0.$$

Using M- Method.

Basic	X_1	X_2	S_1	R_1	S_2	
Z	-2	-5	0	M	0	–
R_1	3	2	-1	1	0	6
S_2	2	1	0	0	1	2.
Z	-2-3M	-5-2M	M	0	0	-6M.
R_1	3	2	-1	1	0	6
S_2	2	1	0	0	1	2.
Z	0	$\frac{-4-M}{2}$	M	0	$\frac{1+3M}{2}$	-2+3M
R_1	0	1/2	-1	0	-3/2	3
X_1	1	1/2	0	0	1/2	1
Z	8+M	0	M	0	5+2M	10-2M
R_1	-1	0	-1	1	-2	2
X_2	2	1	0	0	1	2

Here, Z-row show that solution is optimal. The solⁿ is infeasible because the artifical variable R_1 assumes +ve value which violates the constraints of the original eqⁿ →.

$$3X_1 + 2X_2 \geq 6.$$

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Q:- 7

X_1 = No. of hats of type 1 produced daily.
X_2 = " " 2 " "

Objective Function →

Max $Z = 8X_1 + 5X_2$.

Constraints →

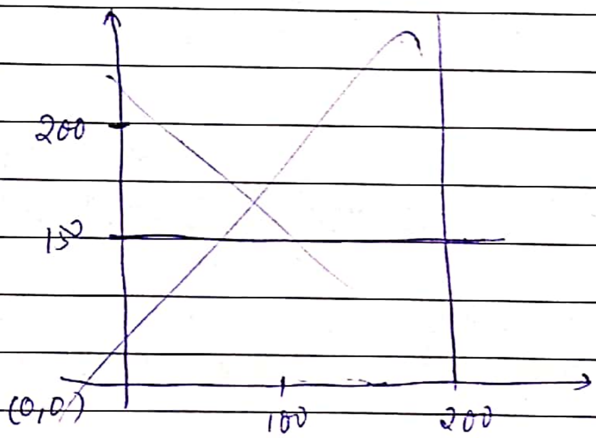
$2X_1 + X_2 \leq 400$

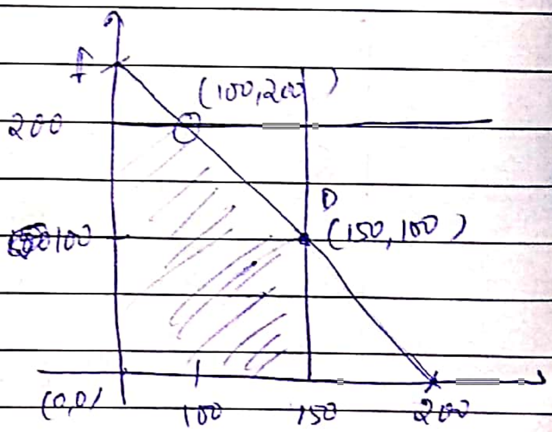
$X_1 \leq 150$

$X_2 \leq 200$

$X_1, X_2 \geq 0$.

(a) Using graphical representation →





Optimum occurs at B (100,200)

$Z = 8 \times 100 + 5 \times 200 = \$1800$

(b) Point A = (0,200), C = (150,200).

	Capacity	Z
A	$2\times0 + 1\times200 = 200$	$8\times0 + 5\times200 = 1000$
C	$2\times150 + 1\times200 = 500$	$8\times150 + 5\times200 = 2200$

Dual price → $\dfrac{2200 - 1000}{500 - 200} = \dfrac{1200}{300}$ 4 = 4 per type-2 hat.

Range → (200 - 500).

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(c) Dual price = 0 in the range (100, ∞)

∴ change from $X_1 \le 150$ to $X_1 \le 120$ has no effect on optimum of Z.

(d) Let

d = Demand limit for type - 2 hat.

	d	Z
D(150, 100)	100	8(150) + 5(100) = \$1700
E (0, 400)	400	8(0) + 5(400) = \$2000

Dual price = \$1

Range (100, 400).

∴ Maximum increase in demand limit for the type-2 hat is = 400 - 200 = 200 hats.

Q:-8 (a) Let X_1 = practical courses.

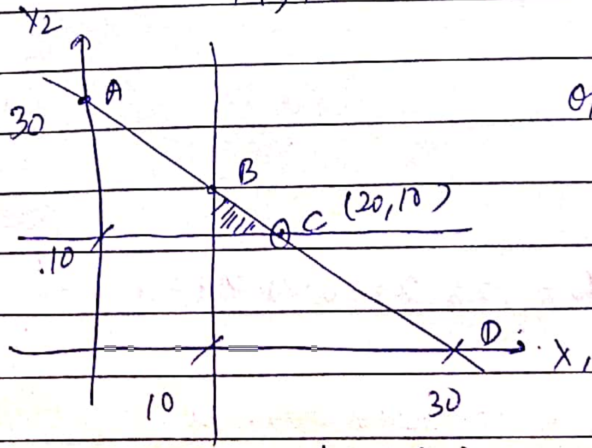
X_2 = theoretical "

Objective Function →

$$\text{Max } Z = 1500 X_1 + 1000 X_2.$$

Constraints →

$$X_1 + X_2 \le 30$$
$$X_1 \ge 10$$
$$X_2 \ge 10$$
$$X_1, X_2 \ge 0$$



Optimal at C (20, 10)

$Z = 1500 \times 20 + 1000 \times 10$

$= 40000$.

The optimal solⁿ for this is Z = \$40000 at $X_1 = 20$, $X_1 = 10$ using graphical method.

(b) Dual prices per addition of practice course is \$1500 per course. As this is equal to the revenue of the practical course, the course must be pratical type.

(c) Using solver for equations →.

S_2 (Surplus for constraint Z).

$S_2 = 10 + D_1 \geq 0$. (D_1 = demand)

$X_1 = 20 + D_1 \geq 0$.

$X_2 = 10$.

$\Rightarrow -10 \leq D_1 < \infty$.

∴ this dual price is applicable for any products ≥ 20.

(d) Dual price in this case is \$500.

To determine range,

$S_1 = 10 - D_3 \geq 0$.

$X_1 = 20 - D_3 \geq 0$

$X_2 = 10 + D_3 \geq 0$.

$\Rightarrow -10 \leq D_3 \leq 10$.

Unit increase in lower limit on humanities course will decrease revenue by \$500.

Q:- 9.

⇒ let

X_1 = Daily units of cable 320.

X_2 = " " " 325

X_3 = " " " 340

X_4 " " " 370

Objective Function →.

$$\text{Max } Z = 9.4x_1 + 10.8x_2 + 8.75x_3 + 7.8x_4.$$

Constraints →

$$10.5x_1 + 9.3x_2 + 11.6x_3 + 8.2x_4 \leq 4800$$

$$20.4x_1 + 24.6x_2 + 17.7x_3 + 26.5x_4 \leq 9600$$

$$3.2x_1 + 2.5x_2 + 3.6x_3 + 5.5x_4 \leq 4700$$

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$5x_1 + 5x_2 + 5x_3 + 5x_4 \leq 4500$.

$x_i \geq 100 \quad (i = 1, 2, 3, 4)$.

Using LP solver, we find the dual prices.

(b) only soldering capacity can be increased bcz its dual > 0.

(c) As dual prices of lower bounds on X_1, X_2, X_3 & X_4 are -ve, the bounds have a bad effect.

(d) As the dual price for soldering is \$0.49 per minute, ie is valid in the range (8420, 10201.7) minutes. Hence \$0.49 profit per minute is guaranteed upto the range 10201.

Q:-10. let

X_j = No. of units of PP_j $j = 1, 2, 3, 4$.

Objective Function →

$$\text{Max } Z = 3x_1 + 6x_2 + 5x_3 + 4x_4$$

Constraints →

$$2x_1 + 5x_2 + 3x_3 + 4x_4 \leq 5300$$

$$3x_1 + 4x_2 + 6x_3 + 4x_4 \leq 5300$$

From the above constraints are fed into solver to get reduced cost of the parts not produced.

Variable	Reduced Cost
X_3	0.1429
X_4	1.1429

∴ Rate of deterioration of Z per unit of X_3 = \$0.14.

and, Rate of " " " X_4 = \$1.14.

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