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Q:-1. A cube has its vertices located at \rightarrow .

$$A(0,0,10)$$

$$B(10,0,10)$$

$$C(10,10,10)$$

$$D(0,10,10)$$

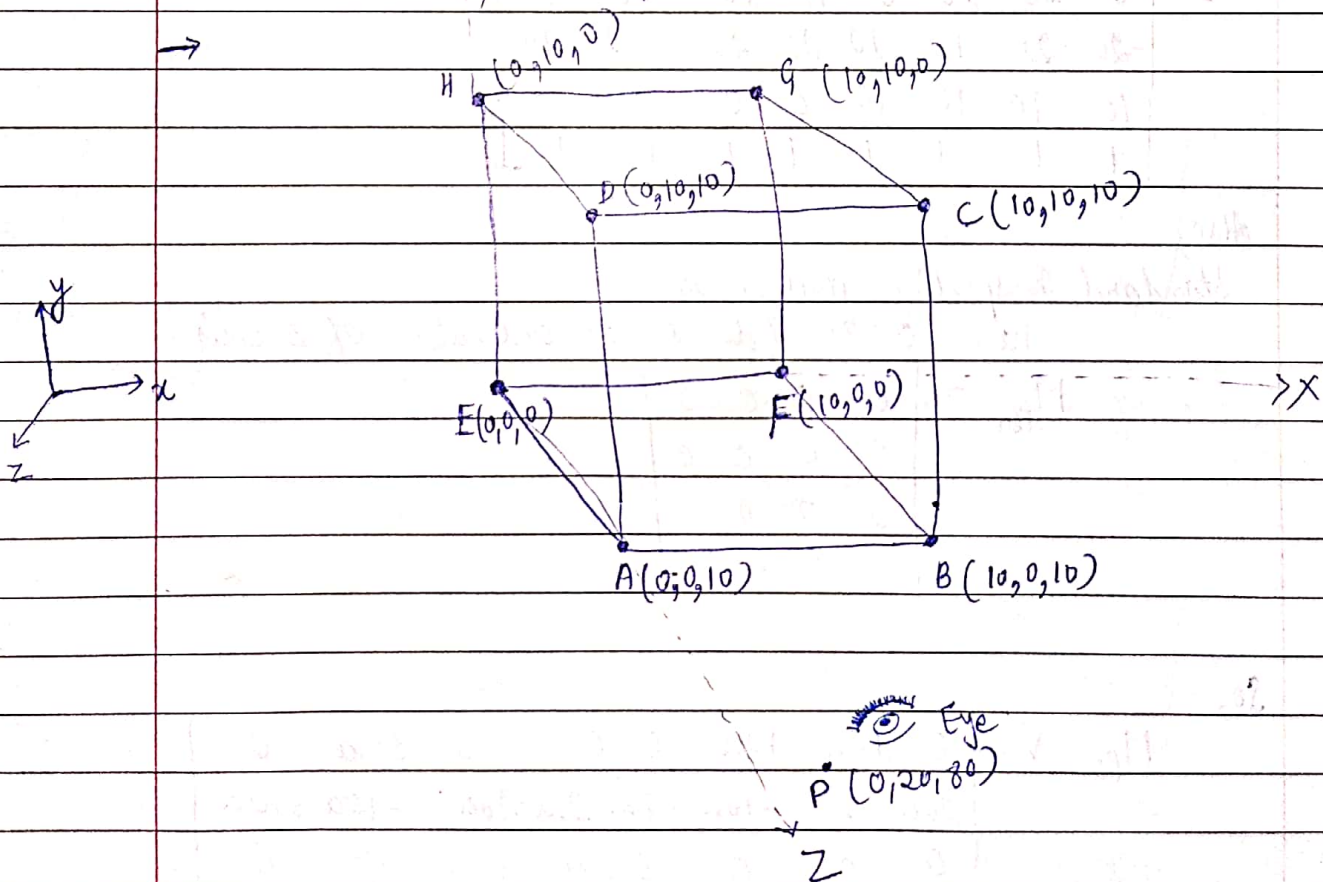
$$E(0,0,0)$$

$$F(10,0,0)$$

$$G(10,10,0)$$

$$H(0,10,0)$$

The Y-axis is vertical & Z-axis is oriented towards the viewer. The cube is being viewed from point $(0,20,80)$. Calculate the perspective view of the cube on XY-plane.



$$V = (A B C D E F G H) = \begin{bmatrix} 0 & 10 & 10 & 0 & 0 & 10 & 10 & 0 \\ 0 & 0 & 10 & 10 & 0 & 0 & 10 & 10 \\ 10 & 10 & 10 & 10 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

Translating the co-ordinates about Y-axis, so that projection P is on Z-axis

$$P' = (0, 0, 80)$$

And,

so the change in the matrix of cube due to shift of co-ordinate from Y-axis by 20.

$$= \begin{bmatrix} 0 & 10 & 10 & 0 & 0 & 10 & 10 & 0 \\ -20 & -20 & -10 & -10 & -20 & -20 & -10 & -10 \\ 10 & 10 & 10 & 10 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

Also,

Standard Perspective matrix is →

Here $a = 80$ (due to co-ordinates of Z-axis)

$$M_{\text{Per}} = \begin{bmatrix} a & 0 & 0 & 0 \\ 0 & a & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & a \end{bmatrix}$$

So,

$$M_{\text{Per}} \cdot V = \begin{bmatrix} 0 & 10a & 10a & 0 & 0 & 10a & 10a & 0 \\ -20a & -20a & -10a & -10a & -20a & -20a & -10a & -10a \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 10+a & 10+a & 10a & 10a & a & a & a & a \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 800 & 800 & 0 & 0 & 800 & 800 & 0 \\ -1600 & -1600 & -800 & -800 & -1600 & -1600 & -800 & -800 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 90 & 90 & 90 & 90 & 80 & 80 & 80 & 80 \end{bmatrix}$$

Take common from the matrix.

$$= \begin{bmatrix} 0 & 80/9 & 80/9 & 0 & 0 & 10 & 10 & 0 \\ -160/9 & -160/9 & -80/9 & -80/9 & -20 & -20 & -10 & -10 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

Again,

transform back to its original position by adding +20 in R_2 .

$$= \begin{bmatrix} 0 & 80/9 & 80/9 & 0 & 0 & 10 & 10 & 0 \\ -20/9 & -20/9 & 100/9 & 100/9 & 0 & 0 & 10 & 10 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

∴ New Vertices with Perspective View on XY-plane.

$$V' = (A' B' C' D' E' F' G' H')$$

$$= \begin{bmatrix} 0 & 80/9 & 80/9 & 0 & 0 & 10 & 10 & 0 \\ -20/9 & -20/9 & 100/9 & 100/9 & 0 & 0 & 10 & 10 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

Ans.

Q:2. The reflection along the line $X=Y$ is equivalent to the reflection along the X -axis followed by counter clockwise rotation by θ degree. Find the value of ' θ '.

→ • Reflection along $X=Y$ line

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \quad \text{--- (1)}$$

• Reflection matrix θ (counter clockwise).

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \quad \text{--- (2)}$$

• Reflection along X -axis

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \quad \text{--- (3)}$$

Also, Reflection along X -axis followed by rotation of θ degree counter clockwise.

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \quad \text{--- (4)}$$

Equate eq- (1) & (4).

$$\Rightarrow \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\therefore \cos\theta = 0 \quad ; \quad \sin\theta = 1$$

$$\text{i.e. } \theta = \frac{\pi}{2} \quad \theta = \frac{\pi}{2}$$

Hence,

$$\text{Value of } \theta = \frac{\pi}{2}$$

Ans