Name - Ankit Roll No. - 11706 Subject - LPP

Assignment - 4

Q.1

- No. of machines available in the 4 categories are 25, 30, 20 & 30.
No. of jobs in the 5 tasks are → 30, 10, 20, 25 & 20

Task Type.

		1	2	3	4	5
Machine	1	10	2	3	15	9
Category	2	5	10	15	2	4
	3	15	5	14	7	15
	4	20	15	13	-	8
	5	-	-	-	-	-

Subtract the lowest value from each row to make min. 1 zero.

	1	2	3	4	5
1	8	0	1	13	7
2	3	8	13	0	12
3	10	0	9	2	10
4	12	7	5	-	0
5	-	-	-	-	-

Now,
Subtract smallest value from each column to make min. 1 zero in each column.

	1	2	3	4	5
1	5	0	0	13	7
2	0	8	12	0	2
3	7	0	8	2	10
4	9	7	4	-	0
5	-	-	-	-	-

Here, we have drawn a line to show which row or column has min. zero. If no. of line is equal to no. of colums or row then we can proceed further

	1	2	3	4	5
1	5	~~0~~	$\boxed{0}$	13	7
2	~~0~~	8	12	$\boxed{0}$	2
3	7	$\boxed{0}$	8	2	10
4	9	7	4	-	$\boxed{0}$
5	-	-	-	-	-

Task assigned to machines are →

∴ Machine 1 → Task 3.

Machine 2 → Task 4.

Machine 3 → Task 2.

Machine 4 → Task 5

So, minimum cost is = 10X2 + 15X3+25X5+ 5X2 +20X7 + 5X20+5X13+20X8.

= \$605

Q:-2. Demand for next 5 quarters → 200 +150+300+250+400=1300

Production capacities → 180+230+430+300+300=1440.

⇒ 1440-1300 = 140

So, there is 140 units extra capacity available. Max portion of this surplus will be used for satisfying next Q excess demand.

	1	2	3	4	5	Surplus	
R1	100	104	108	112	116	0	180
O1	150 20	154	158	162	166	0 70	90
R2		96 150	100 80	104	108	0	230
O2		144	148	152	156	0 115	115
R3			116 220	120	124 50	0 160	430
O3			174	178	182	0 215	215
R4				102 250	106 50	0	300
O4				153	157	0 150	150
R5					106 300	0	300
O5					159	0 150	150
	200	150	300	250	400	150	860.

∴ Total cost of satisfying demand =
$21000 + $14400 + $32 360 + $26400 + $42400
= $ 136,560.

Total production at normal capacity = 1280 units.
Overtime Production quality = 20 units.

Q:-4

Depature date from Dallas	Return date.
Monday, June 3	Friday, June 7
" , June 10	Wednesday, June 12
" , June 17	Friday, June 21
Tuesday, June 25	" , June 28.

Make a table to easily solve the problem.
Notation ⇒ Dallas → D ; Atlanta → A

	A_7	A_{12}	A_{21}	A_{28}
D_3	400	300	300	280
D_{10}	300	400	300	300
D_{17}	300	300	400	300
D_{25}	300	300	300	400

Now,
Try to make min. one zero in each row & column.

	A_7	A_{12}	A_{21}	A_{28}
D_3	120	20	20	0
D_{10}	0	100	0	0
D_{17}	0	0	100	0
D_{25}	0	0	0	100

Now, do row & column operation by assigning zero, by making square on it & cancelling the column.

	A7	A12	A21	A28
D_3	120	20	20	0
D_{10}	0	100	0	0
D_{17}	0	0	100	0
D_{25}	0	0	0	100

Optimal solⁿ is possible when no. of squared zero = no. of row / Column.

$D_3 - A28 = 280$.
$D_{10} - A21 = 300$
$D_{17} - A12 = 300$
$D_{25} - A21 = 300$.

$\therefore Z_{min} = 280 + 300 + 300 + 300$
$= \$1180$

Q.5

Destination

Source	D1	D2	D3	
S1	\$3	\$4	\$5	40
S2	\$1	\$2	\$1	20
S3	\$2	\$3	\$3	30
	30	20	20	

For source 1, ~~which~~ the destination with least cost is D_1 which cost \$3 but demand is 30 units.

STEP1 →

	D_1	D_2	D_3	
S_1	3 (30)	4	5	40 10
S_2	1	2	1	20
S_3	2	3	3	30
	30 0	20	20	

STEP2 → For D_2 because it has now 2nd least cost in source 1.

	D_1	D_2	D_3	
S_1	3 (30)	4 (10)	5	40^{0}
S_2	1	2	1	20
S_3	2	3	3	30
	30^{0}	20^{10}	20	

Now, in Source 2.

STEP 3 →

	D_1	D_2	D_3	
S_1	3	4 (10)	5	40^{0}
S_2	1	2	1 (20)	20^{0}
S_3	2	3	3	30
	30^{0}	20^{10}	20^{0}	

Again but for source 3.

STEP 4 →

	D_1	D_2	D_3	
S_1	3 (30)	4 (10)	5	40^{0}
S_2	1	2	1 (20)	20^{0}
S_3	2	3 (10)	3	30^{20}
	30^{0}	20^{10}	20^{0}	

Shipping Schedule

STEP 1 ⇒ Source 1 → D_1, 30 units at \$3 = \$90.
STEP 2 ⇒ S_1 → D_2, 10 units at \$4 = \$40.
STEP 3 ⇒ S_2 → D_3, 20 units at \$1 = \$20.
STEP 4 ⇒ S_3 → D_2, 10 units at \$3 = \$30.

Storage Cost for 20 units at S_3 → $20 \times 3 = \$60$.

∴ Total cost ⇒ $90 + 40 + 20 + 30 + 60$
$= \$240$.

Also,
$S_1 D_3 = \$5$ is the largest cost for the Source 1. So, ~~on~~ this

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basis remains same for any value greater than \$5.

$\therefore$ Range $S_1D_3 > \$5$

Q:-6.

	Easy S_1	Moderate S_2	Tough S_3
a_1	85	60	40
a_2	92	85	81
a_3	100	88	82

Course action for Hank →

In S_3 → $a_3 = 82$ is ~~the~~ for exam to be toughest for which he have to study all night.

In S_1 → $a_3 = 100$ is for exam to be easy with all night study.

Also, Subtract each row with its max value.

	S_1	S_2	S_3	Max. Regret
a_1	0	0	0	0
a_2	8	3	1	8
a_3	15	28	42	42

With the help of max regret we can find the regret corresponding course of action

$\therefore$ decision $= a_3 =$ Study all night.

Therefore, laplace criteria →

Probability of occurence $= \frac{1}{3}$.

$\therefore$ Expected payoff for each decision is

$E(a_1) = 61.7$, $E(a_2) = 86$. $E(a_3) = 90$.

Hence, Expected pay-off for a_3 is maximum.

$\therefore$ decision $= a_3 =$ Study all night.

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Q: 7

In games (a) & (b), Each game has a pure strategy.

(a)

	B_1	B_2	B_3	B_4	Row minima
A_1	9	6	2	8	2
A_2	8	9	4	5	4
A_3	7	5	2	5	2
Column Maxima	9	9	4	8	

Max(Minimum) = Max (2, 4, 2)
= 4.

Min (Maximum) = Min (9, 9, 4, 8)
= 4

Maximin value = 4 = Minimax value

∴ Saddle point exists. The value of the game is the Saddle point, which is 4

also,

The optimal Strategy is the position of the saddle point & is given by (A_2, B_3)

(b)

	B_1	B_2	B_3	B_4	Row minima
A_1	5	-4	-5	6	-5
A_2	-3	-4	-8	-2	-8
A_3	6	8	-8	-9	-9
A_4	7	3	-9	6	-9
Column maxima	7	8	-5	6	

∴ Maxi (Minimum) = Max (-5, -8, -9, -9)
= -5

Mini (Maximum) = Min (7, 8, -5, 6)
= -5

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Maximin = Minimax = −5.

∴ Saddle point exists. The value of the game is the saddle point which is −5. The optimal strategy is the position of the saddle point & is given by (A_1, B_3)

Q:- 8.
=

Company B.

Company A Strategy	Share	1 (0%)	2 (50%)	3 (30%)	4 (20%)	5 (80%)	6 (70%)	7 (50%)	8 (100%)	Row Min
1	0%	0	−50	−30	−20	−80	−70	−50	−100	−100
2	50%	50	0	20	30	−30	−20	0	−50	−50
3	30%	30	−20	0	20	−50	−40	−20	−70	−70
4	20%	20	−30	−20	0	−60	−50	−30	−80	−80
5	80%	80	30	50	60	0	10	30	−20	−20
6	70%	70	20	40	50	−10	0	20	−30	−30
7	50%	50	0	20	30	−30	−20	0	−50	−50
8	100%	100	50	70	80	20	30	50	0	0
Column Max		100	50	70	80	20	30	50	0	

∴ Max (Minimum) = (−100, −50, −70, −80, −20, −30, −50, 0).
= 0

&,

Min (Maximum) = (100, 50, 70, 80, 20, 30, 50, 0).
= 0

∴ Max Min = Min Max = 0
↳ Saddle Point.

Hence, the saddle point is (8, 8) both the companies will have fair game.

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Q. 9.

Travel Time (min)

District	Addison	Beeks	Canfield	Daly	Student Population
North	12	23	35	17	250
South	26	15	21	27	340
East	18	20	22	31	310
West	29	24	35	10	210
Central	15	10	23	16	290

$$Z = \left[\begin{array}{l} 12x_{NA} + 23x_{NB} + 35x_{NC} + 17x_{ND} \\ + 26x_{SA} + 15x_{SB} + 21x_{SC} + 27x_{SD} \\ 18x_{EA} + 20x_{EB} + 22x_{EC} + 31x_{ED} \\ 29x_{WA} + 24x_{WB} + 35x_{WC} + 10x_{WD} \\ 15x_{CA} + 10x_{CB} + 23x_{CC} + 16x_{CD} \end{array}\right]$$

Total population = $250 + 340 + 310 + 210 + 290 = 1400$

Evenly distribute the no. of students.

i.e. $\frac{1400}{5} = 280$ every group.

For supply,

$x_{NA} + x_{NB} + x_{NC} + x_{ND} = 280$

$x_{SA} + S_{SB} + x_{SC} + x_{SD} = 280$

$x_{EA} + x_{EB} + x_{EC} + x_{ED} = 280$

$x_{WA} + x_{WB} + x_{WC} + x_{WD} = 280$

$x_{CA} + x_{CB} + x_{CC} + x_{CD} = 280$

For Demand; Max. capacity = 400 (given)

$x_{AN} + x_{AS} + x_{AE} + x_{AW} + x_{AC} \le 400$

$x_{BN} + x_{BS} + x_{BE} + x_{BW} + x_{BC} \le 400$

$x_{CN} + x_{CS} + x_{CE} + x_{CN} + x_{CC} \le 400$

$x_{DN} + x_{DS} + x_{DE} + x_{DW} + x_{DC} \le 400$

where,

$x_{ij} \geq 0$

By using QM in Excel →

From District 1 to Dest. A → Value → 280

District 2 to Dest. B Value → 240

" 2 " " C Value → 40

" 3 " " A " → 120

" 3 " " C " → 160

" 4 " " D " → 280

" 5 " " B " → 160

" 5 " " D " → 120

$$Z = \begin{bmatrix} 12(280) + 23(0) + 35(0) + 17(0) \\ 26(0) + 15(240) + 21(40) + 27(0) \\ 18(120) + 20(0) + 22(160) + 31(0) \\ 29(0) + 24(0) + 35(0) + 10(280) \\ 15(0) + 10(160) + 23(0) + 16(120) \end{bmatrix}$$

$= 19,800$

Q:-10 The initial feasible solution can be given by using least cost method. As profit has to be maximized, taking -ve cost values.

Terminal

Warehouse	St. Louis	Atlanta	New York	Supply
Charlotte	-1800	30 -2100	-1600	30
Memphis	30 -1000	-700	-900	30
Louisville	-1400	-800	30 -2200	30
Dummy	10 0	30 0	20 0	60
Demand	40	60	50	

Now using MODI method

	$V_1=-2100$	$V_2=-2100$	$V_3=-2100$
$U_1=0$	-1800	30 -2100	-1600
$U_2=1100$	30 -1000	-700	-900
$U_3=-100$	-1400	-800	30 -2200
$U_4=2100$	10 0	30 0	20 0

$P_{11} = -300$

$P_{13} = -500$

$P_{22} = -300$

$P_{23} = -100$

$P_{31} = -800$

$P_{32} = -1400$

$\therefore$ Optimality has been reached.

Hence, Total profit $= 2100 \times 30 + 1000 \times 30 + 2200 \times 30$
$= \$159{,}000$

Q:- 11.

Balancing the assignment problem by adding dummies & minimizing the problem by subtracting all elements by 630

SALES

EMPLOYEE	1	2	3	4	D_1	D_2
1	290	470	20	340	630	630
2	70	260	110	180	630	630
3	360	–	280	210	630	630
4	270	410	0	480	630	630
5	180	440	60	320	630	630
6	350	310	140	270	630	630

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Row reduction by making row value zero

	1	2	3	4	D_1	D_2
1	270	450	0	320	610	610
2	0	190	40	110	560	560
3	150	–	70	0	420	420
4	270	410	0	480	630	630
5	120	380	0	260	570	570
6	210	170	0	130	490	490

Column Reduction to make column value zero.

	1	2	3	4	D_1	D_2
1	270	280	0	320	190	190
2	0	20	40	110	140	140
3	150	–	70	0	0	0
4	270	240	0	480	210	210
5	120	210	0	260	150	150
6	210	0	0	130	70	70

Zero Assignment : 1

	1	2	3	4	D_1	D_2	
1	270	280	$\boxed{0}$	320	190	190	✓
2	$\boxed{0}$	20	40	110	140	140	
3	150	–	70	$\boxed{0}$	$\cancel{0}$	$\cancel{0}$	
4	270	240	$\cancel{0}$	480	210	210	✓
5	120	210	$\cancel{0}$	260	150	150	✓
6	210	$\boxed{0}$	$\cancel{0}$	130	70	70	
			✓				

Assigned zeros < 6.

The Smallest uncovered element is 20.

Again,

	1	2	3	4	5	6	
1	150	140	$\boxed{0}$	180	50	50	✓
2	~~0~~	~~0~~	160	90	120	120	✓
3	170	–	210	$\boxed{0}$	~~0~~	~~0~~	
4	150	100	~~0~~	340	70	70	✓
5	$\boxed{0}$	70	~~0~~	120	10	10	✓
6	230	$\boxed{0}$	140	130	70	70	✓
	✓	✓	✓				

The smallest element is 10.

	1	2	3	4	5	6	
1	150	140	$\boxed{0}$	170	40	40	✓
2	$\boxed{0}$	~~0~~	160	80	110	110	
3	180	–	220	$\boxed{0}$	~~0~~	~~0~~	
4	150	100	~~0~~	330	60	60	✓
5	0	70	~~0~~	110	$\boxed{0}$	~~0~~	
6	230	$\boxed{0}$	140	120	60	60	
			✓				

The smallest element is 40.

	1	2	3	4	5	6
1	110	100	~~0~~	130	$\boxed{0}$	~~0~~
2	$\boxed{0}$	~~0~~	200	80	110	110
3	180	–	260	$\boxed{0}$	~~0~~	~~0~~
4	110	60	$\boxed{0}$	290	20	20
5	~~0~~	70	40	110	~~0~~	$\boxed{0}$
6	230	$\boxed{0}$	180	120	60	60

Employee 2 → Home Furnishing
" 6 → China
4 → Appliances
3 → Jewellery.

Total expected daily sales = 560+320+630+420
= $1930

Q:-12.

(a) Row Reduction [Using Hungarian Method].

	A	B	C	D	E	F
1	3	0	1	6	1	2
2	0	1	0	8	3	1
3	2	2	3	0	3	0
4	6	1	2	0	4	4
5	0	3	0	5	2	0
6	1	3	1	0	0	0

(b) Column Reduction
will be same as above one

(c) Zero Assignment

	A	B	C	D	E	F
1	3	$\boxed{0}$	1	6	1	2
2	$\boxed{0}$	1	$\cancel{0}$	8	3	1
3	2	2	3	$\cancel{0}$	3	$\boxed{0}$
4	6	1	2	$\boxed{0}$	4	4
5	$\cancel{0}$	3	$\boxed{0}$	5	2	$\cancel{0}$
6	1	3	1	$\cancel{0}$	$\boxed{0}$	$\cancel{0}$

∴ Total no. of nights = 4+4+7+5+5+9
= 34 nights

Q: 3

We want to maximize the total bidding revenue, therefore taking the bids to be -ve. The initial feasible solⁿ can be given by using least cost method →

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Bidders

	1	2	3	4	Supply
1	-520	—	10 -650	-180	20
2	-210	30 -570	—	-430	30
3	-510	-495	-240	-710	10
Dummy	30 0	0 0	10 0	20 0	60
Demand	30	30	30	30	

Note → Supply & demand is 1000

Now, using MODI Method

$V_1=-650$	$V_2=-650$	$V_3=-650$	$V_4=-650$
-520	—	20 -650	-180
-210	-510	—	-430
-570	-495	-240	10 -710
30 0	0	10 0	20 0

$P_{11}=-130$
$P_{12}=-14$
$P_{14}=-470$
$P_{21}=-300$
$P_{23}=-$
$P_{24}=-80$
$P_{31}=-140$
$P_{32}=-215$
$P_{33}=-470$

∴ Optimality has been reached.

Total = $354,000,000

Bidder 1 = 0 acers
" 2 = 30 "
3 = 20 acers
4 = 10 acers

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