Theory of Computation

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Pushdown Automata

Definition of a PDA

A pushdown automaton is $(Q, \Sigma, \Gamma, \delta, q_0, F)$ where:

- Q is a finite set of states;
- Σ is the input alphabet;
- Γ is the stack alphabet
- q_0 in Q is the initial state;
- $F \subseteq Q$ is a set of final states;
- δ is the transition function

Z_0)

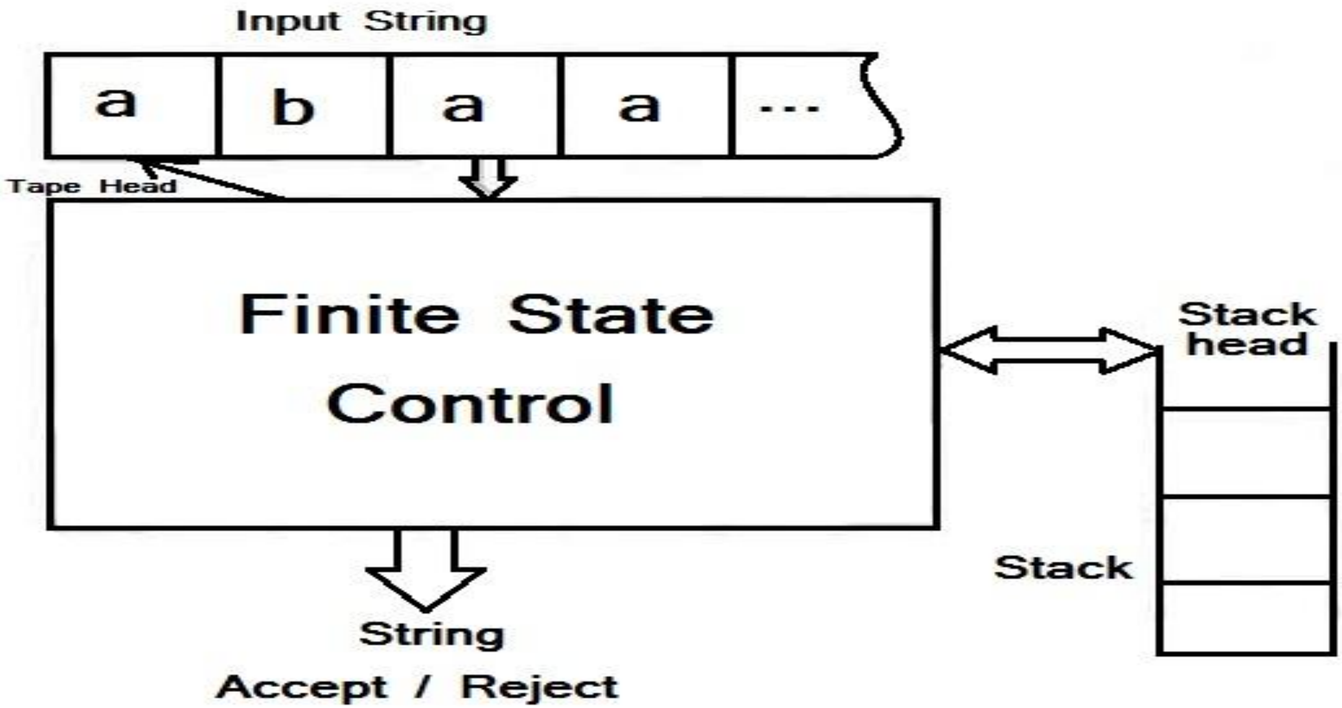
inital stack symbol

$$\boldsymbol{\delta: Q \times (\Sigma \cup \{\varepsilon\}) \times (\Gamma \cup \{\varepsilon\}) \text{ subsets of } Q \times (\Gamma \cup \{\varepsilon\})}$$

state input symbol pop symbol state push symbol

Model of pushdown automata

$a^n b^n \mid n \geq 1$



Example 1: Construct a PDA that accepts $L = \{a^n b^n \mid n \geq 1\}$

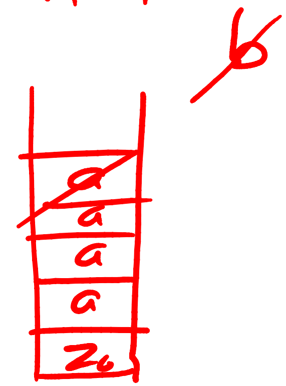
Transition function

Sol^n

1) $\delta(q_0, a, Z_0) = \{(q_0, aZ_0)\}$
2) $\delta(q_0, a, a) = \{(q_0, aa)\}$
3) $\delta(q_0, b, a) = \{(q_1, \wedge)\}$
4) $\delta(q_1, b, a) = \{(q_1, \wedge)\}$
5) $\delta(q_1, \wedge, Z_0) = \{(q_f, \wedge)\}$ OR $\{(q_f, Z_0)\}$

aaba

n=4



$(q_0, aaabbb, Z_0) \vdash (q_0, aabbb, aZ_0) \vdash (q_0, abbb, aaZ_0) \vdash$

$(q_0, bbb, aaaZ_0) \vdash (q_1, \cancel{b}b, \cancel{a}aZ_0) \vdash (q_1, b, aZ_0) \vdash$

$(q_1, \wedge, Z_0) \vdash (q_f, \wedge)$

Example 2: Construct a PDA that accepts L = { wcw^R | w = $(a+b)^*$ }

Sol^n: δ: Transition function:

$\delta(q_0, a, Z_0) = \{(q_0, aZ_0)\}$

$\delta(q_0, a, a) = \{(q_0, a)\}$

$\delta(q_0, b, Z_0) = \{(q_0, bZ_0)\}$

$\delta(q_0, b, b) = \{(q_0, bb)\}$

$\delta(q_0, a, b) = \{(q_0, ab)\}$

$\delta(q_0, b, a) = \{(q_0, ba)\}$

$\delta(q_0, c, a) = \{(q_1, a)\}$

$\delta(q_0, c, b) = \{(q_1, b)\}$

$\delta(q_0, c, Z_0) = \{(q_1, Z_0)\}$

$\delta(q_1, a, a) = \{(q_1, \Lambda)\}$

$\delta(q_1, b, b) = \{(q_1, \Lambda)\}$

$\delta(q_1, \Lambda, Z_0) = \{(q_f, \Lambda)\}$

$Q = \{q_0, q_1, q_f\}$

$\Sigma = \{a, b, c\}$

$\Gamma = \{a, b\}$

$q_0 = \{q_0\}$

$F = \{q_f\}$

δ :

$Z_0 = \{Z_0\}$

Example 3: Construct a pda ***A accepting the set of all strings over {a, b} with equal*** **number of a's and b's.**

δ:

$$\delta(q_0, a, Z_0) = \{(q_0, aZ_0)\}$$
$$\delta(q_0, a, a) = \{(q_0, aa)\}$$
$$\delta(q_0, b, Z_0) = \{(q_0, bZ_0)\}$$
$$\delta(q_0, b, b) = \{(q_0, bb)\}$$
$$\delta(q_0, a, b) = \{(q_0, \Lambda)\}$$
$$\delta(q_0, b, a) = \{(q_0, \Lambda)\}$$
$$\delta(q_0, \Lambda, Z_0) = \{(q_f, \Lambda)\}$$

OR

$$\{(q_f, Z_0)\}$$

Example 4: Construct a PDA that accepts $L = \{a^n b^{2n} \mid n \geq 1\}$

δ :

$$\delta(q_0, a, Z_0) = \{(q_0, aZ_0)\}$$

$$\delta(q_0, a, a) = \{(q_0, aa)\}$$

$$\delta(q_0, b, a) = \{(q_1, a)\}$$

$$\delta(q_1, b, a) = \{(q_2, \lambda)\}$$

$$\delta(q_2, \underline{b}, a) = \{(q_1, a)\}$$

$$\delta(q_2, \lambda, Z_0) = \{(q_f, Z_0)\}$$