GROUPS

Algebraic Structure

$(A, op1, op2, \ldots opn)$

where A → non-empty set.

Opn → operations on A.

Binary operation

$f: A \to A$, f is a binary operation, whose domain is set of ordered pairs.

$*$ is a binary opn, $*$ can be $+, -, \oplus$, etc.

Properties

1) Closure – Set A, op → $*$, then A is closed under operation $*$, if $a * b \in A$

Eg $A = \{-1, 0, 1\}$ – check for addition / multiplication

Eg $A = \{1, 3, 5, 7, 9 \ldots\}$ " " " "

2) Associative

$(a * b) * c = a * (b * c)$

Eg $a * b = a + b - ab \quad \forall\, a, b \in Q$

let a, b, c

$(a*b)*c = (a+b-ab) * c$

$= (a+b-ab) + c - (a+b-ab)c$

$= a+b-ab+c-ac-bc+abc$

$= a+b+c-ab-ac-bc+abc$

Similarly $a*(b*c) = a+b+c - - -$

So, Yes.

Eg $a * b = a^b$

let a, b, c $a = 2, b = 2, c = 3$

$a*(b*c) = 2*(2*3) = 2*2^3 = 2*8$

$= 2^8 = 256$

$(a*b)*c = 2^2 * 3 = 4*3 = 4^3 = 64$

Commutative - $a * b = b * a$

Eg $a * b = a^2 + b^2$
$= b^2 + a^2 = b * a$ ✓

Eg $a * b = \frac{ab}{2}$ → check for associativity & commutative

4 Identity

$a * e$ (right identity) $= e * a$ (left identity)

Eg $a * b = \frac{ab}{2}$, determine identity element.

$e * a = a$

$\frac{ea}{2} = a \Rightarrow e = 2$

Similarly $a * e = a$

$\frac{ae}{2} = a \Rightarrow e = 2$.

5 Inverse

$a * b$ (right inverse) $= b * a$ (left inverse)

6 Idempotent

$a * a = a \; \forall \; a \in A$

7 Distributive

$a * (b + c) = (a * b) + (a * c)$ [L.D.]

$(b + c) * a = (b * a) + (c * a)$ [R.D.]

8 Cancellation

$a * b = a * c \Rightarrow b = c$ [L.C.]

$b * a = c * a \Rightarrow b = c$ [R.C.]

Semi-group

Let there be an algebraic structure $(A, *)$, then it is semi-group if it satisfies –

(i) operation $*$ is closed operation on set A.

(ii) " " associative

Eg – $A = \{1, 3, 5, 7 - - -\}$
$* \rightarrow$ multiplication]

Eg