5	33	1		$x_{11} = 1$	
	$x_3 = 5$		1	28	1
1	28	1		$x_{12} = 1$	
	$x_4 = 1$		1	53	0
1	93	0	2	12	1
2	58	0		$x_{13} = 2$	
3	18	1	1	89	0
	$x_5 = 3$		2	87	0
			3	67	0
			4	30	1
				$x_{14} = 4$	

Thus random values of x are 8, 4, 5, 1, 3, 2, 1, 1, 3, 5, 1, 1, 2, 4

$$\text{Observed mean } \bar{x} = \frac{1}{n}\sum x_i = \frac{41}{14} = 2.9286$$

$$\text{Standard deviation S.D.} = \frac{1}{n-1}\sum (x_i - \bar{x})^2$$

$$\sum (x_i - \bar{x})^2 = 25.719 + 1.148 + 4.291 + 3.719 + .0051 + .8623 + 3.7195 + 3.7195 + .0051 + 4.291 + 3.7195 + 3.7195 + .8623 + 1.1479 = 57.9268$$

$$\text{S.D.} = \sqrt{\frac{57.9268}{13}} = 2.1108$$

$$\text{True mean} = \frac{1}{p} = \frac{1}{.35} = 2.857$$

$$\text{True S.D.} = \sqrt{\frac{1-p}{p^2}} = \sqrt{\frac{1-.35}{.35 \times .35}} = \sqrt{5.306} = 2.304$$

Example 5.16. Generate an Exponential distribution with mean value equal to 3.

Solution : An exponential variable x is given by

$$x = -\text{Mean} \times \ln(r)$$

where r is a random number between 0 and 1. The probability density function can be drawn by generating a large number of random observations. Larger the number of observations, more accurate will be the distribution. Fifty values of x have been generated in Table 5.14. The random numbers used have been taken from the table given in Appendix.

Based on these 50 values of x, a histogram showing the frequency distribution is plotted in Fig. 5.15. If the number of observations is increased, and the interval is further shortened, the histogram will result into a smooth curve like the one shown in Fig. 5.15.

The observed mean of the distribution,

$$\bar{x} = \frac{1}{\sum f}\sum f \cdot x = \frac{1}{50}\,[12 \times 0.5 + 11 \times 1.5 + 6 \times 2.5 + 5 \times 3.5 + 5 \times 4.5 + 3 \times 5.5 + 5 \times 6.5 + 1 \times 7.5 + 2 \times 8.5 + 0 \times 9.5]$$

$$= \frac{1}{50}\,[151] = 3.02$$

Obtained standard deviation is

$$= \left[\frac{1}{49}\ [12\,(3.02-0.5)^2 + 11\,(3.02-1.5)^2 + 6\,(3.02-2.5)^2 + 5\,(3.02-3.5)^2 + 5\,(3.02-4.5)^2 + 3\,(3.02-5.5)^2 + 5\,(3.02-6.5)^2 + (3.02-7.5)^2 + 2\,(3.02-8.5)^2\,]\right]^{\frac{1}{2}}$$

$$= \sqrt{\frac{274.48}{49}} = 2.367$$

Table 15.14

Rand	*x*	*Rand*	*x*	*Rand*	*x*
2182	4.57	7615	0.82	1955	4.90
1128	6.55	8508	0.48	2177	4.66
7112	1.02	6970	1.08	7471	0.87
6557	1.27	5799	1.63	8674	0.43
4199	2.60	6364	1.36	1092	6.64
3544	3.11	4165	2.63	9061	0.30
1749	5.23	8354	0.54	6438	1.32
1903	0.28	9130	0.27	1834	5.09
0764	7.72	5826	1.62	1884	5.01
3492	3.16	6285	1.39	6791	1.16
1292	6.14	7527	0.85	2068	4.73
4397	2.46	8976	0.32	7295	0.95
3807	2.90	2327	4.37	3440	3.20
4984	2.09	1182	6.41	5435	1.83
1340	6.03	3659	3.02	3090	3.52
0590	8.49	5924	1.57	0607	8.41
9566	0.13	3941	2.79	—	—

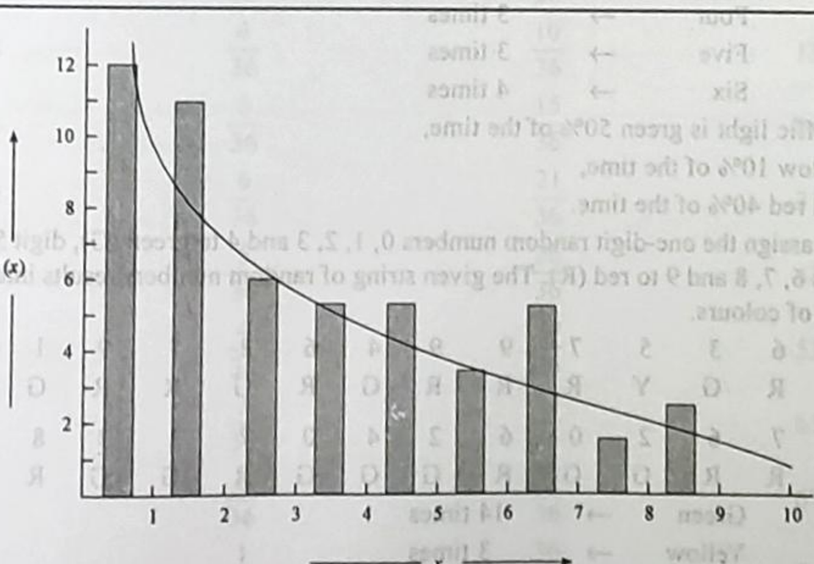

Fig. 5.15

Example 5.17. Use the one-digit random numbers 6, 3, 5, 7, 9, 9, 4, 6, 2, 7, 9, 1, 5, 8, 0, 7, 6, 2, 0, 6, 2, 4, 0, 7, 1, 3, 8, 5, 2, 4 ; to generate random observations for each of the following situations.

(*a*) Throwing an unbiased coin

(*b*) Throwing a dice

(*c*) The colour of a traffic light found by a randomly arriving car, when green is 50% of the time, yellow is 10% of the time, and red is 40% of the time. **[PTU. B.Tech Dec. 2005]**

Solution: (*a*) **Throwing an unbiased coin:** When an unbiased coin is tossed, the chances of getting the head (H) and tail (T) are equal that is 50 : 50. Using the one-digit random numbers between 0 and 9, let the first five digits 0, 1, 2, 3 and 4 stand for H and the remaining five digits 5, 6, 7, 8 and 9 stand for T. For the given string of random numbers, the observations are given below in Table 5.15.

Table 5.15

Rand. No. :	6	3	5	7	9	9	4	6	2	7	9	1	5	8	0
H or T :	T	H	T	T	T	T	H	T	H	T	T	H	T	T	H
Rand. No. :	7	6	2	0	6	2	4	0	7	1	3	8	5	2	4
H or T :	T	T	H	H	T	H	H	H	T	H	H	T	T	H	H

Number of heads = 14

Number of tails = 16

Thus out of 30 throws head comes 14 times, while tail comes 16 times.

(*b*) **Throwing a Dice:** A dice has six faces, and each face has same chance of coming up. Out of the given one-digit random numbers, let us use the numbers 1, 2, 3, 4, 5 and 6 to represent the six faces of the dice, and reject the remaining. Using the given string of random numbers the first throw results into six, while the second gives 3 and third through gives 5 etc.

We have;

One	→	2 times
Two	→	4 times
Three	→	2 times
Four	→	3 times
Five	→	3 times
Six	→	4 times

(*c*) Traffic light is green 50% of the time,
yellow 10% of the time,
and red 40% of the time.

We can assign the one-digit random numbers 0, 1, 2, 3 and 4 to green (G), digit 5 to yellow (Y) and the digits 6, 7, 8 and 9 to red (R). The given string of random numbers results into the following observations of colours.

Rand. No. :	6	3	5	7	9	9	4	6	2	7	9	1	5	8	0
Colour :	R	G	Y	R	R	R	G	R	G	R	R	G	Y	R	G
Rand. No. :	7	6	2	0	6	2	4	0	7	1	3	8	5	2	4
Colour :	R	R	G	G	R	G	G	G	R	G	G	R	Y	G	G

Green	→	14 times
Yellow	→	3 times
Red	→	13 times

Example 5.18. The game of craps requires the players to throw two dice one or more times until a decision has been reached. The player wins the game if the first throw results in a sum of 7 or 11 or if the first sum is 4, 5, 6, 8, 9 or 10 and the same repeats before a sum of 7 has appeared.

(*a*) Simulate 15 plays of the game, in the form of a table.

(*b*) Develop a simulation program in any computer language and run the simulation for 100, 200 and 300 games. **[PTU. B.E. 4th Sem. Elect. Dec. 2005]**

Solution: There are two dice, which are thrown at a time, and a throw can result into a sum of 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 or 12. Sum 2 can appear only in one way *i.e.*, 1 + 1, the probability of the same is $\frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$. Sum 3 can occur in two ways 1 + 2 and 2 + 1, and thus the probability of sum 3 is $2\left(\frac{1}{6} \times \frac{1}{6}\right) = \frac{2}{36}$. The ways in which a sum can occur and the corresponding probabilities were computed in Example 5.1, and the same are given below:

Sum (s) :	2	3	4	5	6	7	8	9	10	11	12
$p(s)$:	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

For simulating the game, let us take the random numbers from 00 to 72, and assign these to different outcomes of the throws that is sums, as in Table 5.16.

Table 5.16

Sum	*Probability*	*Cumulative probability*	*Random numbers*
2	$\frac{1}{36}$	$\frac{1}{36}$	01 to 02
3	$\frac{2}{36}$	$\frac{3}{36}$	03 to 06
4	$\frac{3}{36}$	$\frac{6}{36}$	07 to 12
5	$\frac{4}{36}$	$\frac{10}{36}$	13 to 20
6	$\frac{5}{36}$	$\frac{15}{36}$	21 to 30
7	$\frac{6}{36}$	$\frac{21}{36}$	31 to 42
8	$\frac{5}{36}$	$\frac{26}{36}$	43 to 52
9	$\frac{4}{36}$	$\frac{30}{36}$	53 to 60
10	$\frac{3}{36}$	$\frac{33}{36}$	61 to 66
11	$\frac{2}{36}$	$\frac{36}{36}$	67 to 70
12	$\frac{1}{36}$	$\frac{36}{36}$	71 to 72

The random numbers may be drawn from a random number table, or generated by some random number generation technique. The random numbers used in this simulation have been generated by a scientific calculator. In each number, the first two digits were retained. Taking the following string of random numbers 12, 62, 60, 33, 04, 33, 44, 64, 50, 72, 08, 60, 51, ..., etc. simulation is done as follows.

First game: The first throw of two dice, simulated by first random number 12, gives the sum as 4. It is neither 7 nor 11, hence take the 2nd throw, a random number 62 gives sum of 10. Third throw, random number 60, gives sum of 9, while 4th throw, random number 33, gives sum of 7. Since first sum '4' does not reappear before the occurrence of sum 7, player loses the first game.

Second game: Next random number is 04, which gives a sum of 3. Since, it is neither 7 or 11, nor any of 4, 5, 6, 8, 9 and 10, the player loses the second game too.

Third game: Next random number 33 results in a sum of 7. Player wins the game in the very first throw.

The simulation of the process of 15 games is given in Table 5.17. Seven times the player wins and eight times loses the game. Based on this small sample, the probability of a win is thus $\frac{7}{15} \times 100 = 46.6\%$.

However, such a small length of simulation run, cannot give very reliable results.

Table 5.17

Game No.	*Random numbers*	*Sum*	*Result*
1	12	4	
	62	10	
	60	9	
	33	7	Loses
2	04	3	Loses
3	33	7	Wins
4	44	8	
	64	10	
	50	8	Wins
5	72	12	Loses
6	08	4	
	60	9	
	51	8	
	01	2	
	17	5	
	11	4	Wins
7	30	6	
	44	8	
	71	12	
	04	3	
	20	5	
	34	7	Loses

8	46	8	
	18	5	
	42	7	Loses
9	56	9	
	12	4	
	02	2	
	44	8	
	48	8	
	36	7	Loses
10	59	9	
	24	6	
	65	10	
	44	8	
	57	9	Wins
11	05	3	Loses
12	37	7	Wins
13	49	8	
	66	10	
	06	3	
	54	9	
	07	4	
	43	8	Wins
14	11	4	
	55	9	
	51	8	
	16	5	
	39	7	Loses
15	67	11	Wins

Number of games won = 7

Number of games lost = 8

A computer program for simulating the game of craps, developed in C language is given below.

```
#include<stdio.h>
#include<stdlib.h>
#include<ctype.h>
int throw(void);
main()
   {  /* Simulation of Game of Craps*/
      /* n is the number of games */

      float x;
      int i,k,n,n1,n2,score1,score2,win=0,loss=0;
      printf("\n The number of games to play n=");
```

```
scanf("%d",&n);
for(i=0;i<=n;++i) {
score1=throw();
printf("\n i=%d  score=%d",i,score1);
if(score1==7 || score1==11) { win=win+1;
        printf("       It is a Win"); }
 else if(score1==2 || score1==3 || score1==12) { loss+=1;
        printf("       It is a loss"); }
 else {
        do  {
        score2=throw();
        printf(" score2=%d",score2);
          } while(score2 != score1 && score2 != 7);
        if(score2== score1) { win+=1;
        printf("       Win by matching first score");}
        else  { loss+=1;
        printf("       Lose due to failingto match first score");}
      }
}
printf("\n i=%d Wins=%d Losses=%d",i,win,loss);
/* printf("\n Any digit");
 scanf("%d",k); */
}
/* Function throw to generate the sum of two throws of dice*/
int throw (void) {
int n1,n2;
float x;
x=rand()/32768.;
n1=1+(int)(6*x);
x=rand()/32768.;
n2=1+(int)(6*x);
return(n1+n2);    }
```

Some results obtained from this program are as under.

Games	*Wins*	*Loses*
50	25	25
100	51	49
200	96	104
300	148	152

Example 5.19. The distribution of inter arrival times in a single server model is,

T :	1	2	3
$f(t)$:	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

and the distribution of service times is

S :	1	2	3
$f(s)$:	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$

Complete the following table, using the two-digit random numbers 12, 40, 48, 93, 61, 17, 55, 21, 85, 88 to generate arrivals and 54, 90, 18, 38, 16, 87, 91, 41, 54, 11 to generate the corresponding service times.

Arrival number	*Arrival time*	*Time service begins*	*Time service ends*	*Waiting time in Queue*
1				
2				
3				
4				
etc.				

Solution : It is a simple case of a single server queuing simulation. Both the inter arrival and service times of the customers are random and follow the given discrete distributions.

The distribution of inter arrival times is

T	*f (t)*	*F (t)*	*Random numbers*
1	.25	.25	00 to 24
2	.50	.75	25 to 74
3	.25	1.00	75 to 99

Inter arrival times corresponding to the given random numbers are,

Random number :	12	40	48	93	61	17	55	21	85	88
Inter arrival time :	1	2	2	3	2	1	2	1	3	3

The distribution of service times is,

S	*f (s)*	*F (s)*	*Random numbers*
1	.50	.50	00 to 49
2	.25	.75	50 to 74
3	.25	1.00	75 to 99

Service times corresponding to given random numbers are;

Random number :	54	90	18	38	16	87	91	41	54	11
Service time :	2	3	1	1	1	3	3	1	2	1

Now the required table can be completed (Table 5.18). As soon as an arrival takes place, it will go into service, if the facility is idle that is the service on the previous arrival has been completed, otherwise it will wait till the facility becomes available.

Table 5.18

Arrival number	*Arrival time* $AT + T = AT$	*Time service begins SB*	*Time service ends* $SB + S = SE$	*Waiting time in Queue*
1	$0 + 1 = 1$	1	$1 + 2 = 3$	0
2	$1 + 2 = 3$	3	$3 + 3 = 6$	0
3	$3 + 2 = 5$	6	$6 + 1 = 7$	1
4	$5 + 3 = 8$	8	$8 + 1 = 9$	0
5	$8 + 2 = 10$	10	$10 + 1 = 11$	0
6	$10 + 1 = 11$	11	$11 + 3 = 14$	0
7	$11 + 2 = 13$	14	$14 + 3 = 17$	1
8	$13 + 1 = 14$	17	$17 + 1 = 18$	3
9	$14 + 3 = 17$	18	$18 + 2 = 20$	1
10	$17 + 3 = 20$	20	$20 + 1 = 21$	0

5.20 Central Limit Theorem

The central limit theorem is the most important theorem in statistical inferences. It sates that the average of sample of observations drawn from some population with any shape distribution approaches the normal distribution as the sample size increases. There are theoretical situations, where the central limit theorem fails, but these are rarely encountered in practice. In theoretical statistics there are many versions of the central limit theorem. These versions mainly depend upon the conditions under which the theorem is used and upon the assumptions about the distribution of parent population and the sampling techniques employed.

The most commonly used version of the central limit theorem states that, if there is a random sample of size n, n being larger than 30, drawn from an infinite population having a unit standard deviation, the standardized sample mean converges to a standard normal distribution or equivalently the sample mean approaches a normal distribution with mean equal to the population mean and standard deviation equal to standard deviation of population divided by the square root of sample size n.

The significance of the central limit theorem is that it enables us to use the sample statistics to draw inferences about the population parameters, without knowing anything about the probability distribution of the population.

Example 5.20: Fig. 5.16 (*a*) shows the distribution of annual commission earnings of all salesmen in a large corporation. It has a mean of Rs. 12500 and a standard deviation of Rs. 1500. The distribution is skewed to the right. A random sample of 30 salesmen has been drawn from population. What is the probability that their average earnings will be more than Rs. 13100?

According to the central limit theorem, the sampling distribution of the mean approaches normal distribution irrespective of the shape of the parent population distribution. The sampling distribution is shown on Fig. 5.16 (*b*).

Further, the central limit theorem states, that the sampling distribution has a standard deviation, which we also call a standard error, equal to the population standard deviation divided by the square root of sample size.

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

where $\sigma_{\bar{x}}$ is the standard error of the mean and σ is the standard deviation of the population, and n is sample size.

$$\sigma_{\bar{x}} = \frac{1500}{\sqrt{30}} = 273.87$$

Now, we determine the z statistics, the standard deviation from the mean of a standard normal probability distribution

$$z = \frac{\bar{x} - \mu}{\sigma_{\bar{x}}}$$

where, μ is the mean of the population distribution. μ is also the mean of the sample distribution. Z gives the distance of the given point from the mean.

$$z = \frac{13100 - 12500}{273.87} = 2.2$$

The area under the curve between the mean and a point 2.2 standard deviations away to the right of the mean is 0.4861 (Appendix Table A-7) the area beyond the point 2.2 is $0.5 - .4861 = 0.0139$, the shaded area on Fig. 5.16 (*b*).

Thus, we can say that there are hardly 1.4% chances that the average annual income from commission earnings of all the salesman exceeds Rs. 13100.

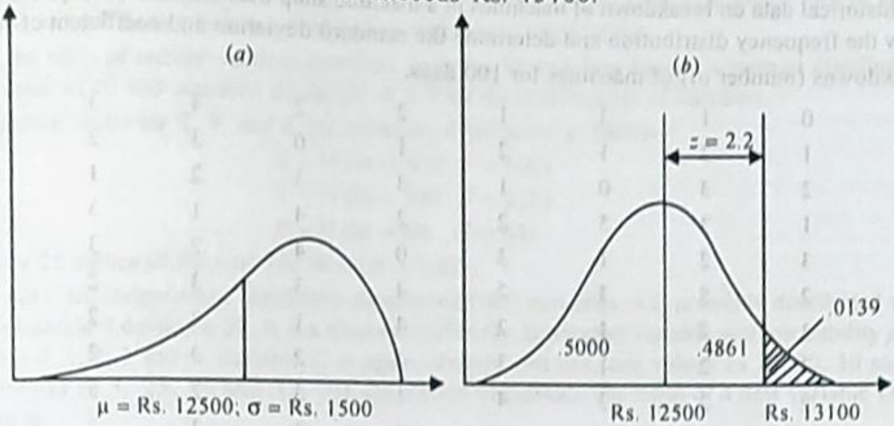

Fig. 5.16

Example 5.21: A mainframe computer includes a backup circuit to minimize the downtimes. After running the simulation of the working of a particular component, it has been found that its average service life is 4300 hours, with a standard deviation of 730 hours. The backup circuit contains two duplicate components so that in case of malfunction of one, the other component automatically gets switched on.

(*a*) What is the probability that the set of components will last 14000 hours ?

(*b*) At the most 12000 hours ?

Solution: The three identical components combined will have a life of $4300 \times 3 = 12900$ hours with the standard deviation of 730 hours.

$$\mu = 12900, \quad \sigma = 730$$

When $\bar{x} = 14000$

$$Z = (14000 - 12900)/730 = 1100/730 = 1.5.$$

The area of the tail beyond 14000 = 0.4332 from Appendix Table A-7.

Thus the possibility of life of components being more than 14000 $= .5000 - .4332$
$= .0668 = 6.7\%$

When $\bar{x} = 12000$,

$$Z = \frac{12000 - 12900}{730} = -1.23.$$

From Appendix Table A-7, the area under the normal curve between -1.23 and mean, μ, is 0.39. Thus the area under the curve up to the right tail is 0.89 as shown in Fig. 5.17. The probability of the life of components being 12000 hours or more is thus 89%.

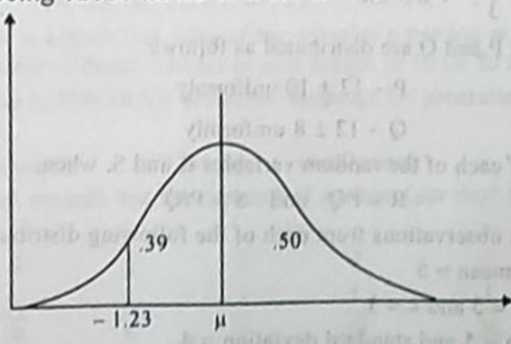

Fig. 5.17

5.21 Exercises

1. The historical data on breakdown of machines in a machine shop over the last 100 days is given below. Draw the frequency distribution and determine the standard deviation and coefficient of variation. Breakdowns (number of) of machines for 100 days.

2	0	1	3	1	2	2	4	3	2
3	1	2	1	2	1	0	3	2	5
4	2	3	0	1	3	3	2	1	2
0	1	3	5	2	2	4	1	3	4
1	3	2	1	3	0	4	2	3	2
4	2	5	3	2	4	3	3	2	1
2	3	2	1	2	4	1	3	2	5
0	3	2	1	3	4	2	3	2	1
1	2	2	3	4	1	3	2	5	0
3	4	0	1	3	2	3	3	2	1

2. Draw a frequency distribution from the following data, grouping the data in steps of 200. Find the mean and standard deviation of the data. Also determine the mode and median of the distribution.

196	565	3229	1575	288
411	224	420	1720	723
1128	945	519	508	36
852	380	321	3158	598
2056	1180	417	479	1350
1106	99	717	1530	518
417	599	662	2961	93
544	245	420	16	275
1268	133	715	149	3071
174	870	1500	1968	1217

3. Generate 5 random observations from,
   (*a*) a Uniform distribution between –5 and 40
   (*b*) a Uniform distribution between 20 and 60
   (*c*) the distribution having the p.d.f. as,

   $$(i)\ f(x) = \begin{cases} \dfrac{1}{20}(x^2 - 5) & \text{if } 5 \le x \le 20 \\ 0, & \text{otherwise} \end{cases}$$

   $$(ii)\ f(x) = \frac{7}{8}x^2 + \frac{x}{3}, \quad 1 \le x \le 2$$

4. The random variables P and Q are distributed as follows:

   $$P \sim 12 \pm 10 \text{ uniformly}$$
   $$Q \sim 12 \pm 8 \text{ uniformly}$$

   Simulate 50 values of each of the random variables R and S, when,

   $$R = PQ \quad \text{and} \quad S = P/Q$$

5. Generate five random observations from each of the following distributions.
   (*a*) Exponential with mean = 5
   (*b*) Erlang with mean = 5 and $k = 3$
   (*c*) Normal with mean = 5 and standard deviation = 4.

6. Generate five random observations from a Poisson distribution, if the mean is 4.0.
7. Determine the probability of their being n arrivals ($n = 0, 1, \ldots, 10$) in an inter arrival time of 10 seconds, when the arrivals have a Poisson distribution with mean value of 0.4.
8. Using the table of normal random numbers generate 50 numbers having a normal distribution, with mean value of 10 and standard deviation of 3. Plot the distribution of numbers.
9. The random variables X, Y, and Z are normally distributed as follows:

$$X \sim N\ (m = 100,\ s^2 = 100)$$
$$Y \sim N\ (m = 300,\ s^2 = 225)$$
$$Z \sim N\ (m = 40,\ \ s^2 = 64)$$

Simulate 25 values of the variable $W = (X + Y)/Z$

10. A, B, and C are independent identically distributed (IID) variables. A is normally distributed with mean 100 and standard deviation 20. B is a discrete uniformly distributed variable with probability $p(b) = 0.2$ with $b = 0, 1, 2, 3$ and 4. Variable C is again discrete and can take values as 10, 20, 30 and 40 with probabilities of .5, .25, .45 and .15. Use simulation to estimate the mean of a new variable D, which is defined as

$$D = (A - 25\ B)\ /\ (2C).$$

Use a sample of size 20.

11. The number of customers, who arrive at a repair shop can be described by a Poisson distribution that has a mean of 4 per hour. Generate arrivals for the first 100 hours. Plot the frequency distribution.
12. The lifetime, in years, of a satellite placed in orbit is given by the following pdf

$$f(x) = \begin{cases} 0.4e^{-0.4x}, & x \geq 0 \\ 0, & otherwise \end{cases}$$

(*a*) What is the probability of the life of satellite being more than 5 years?

(*b*) What is the probability of the life of the satellite being between 3 and 6 years?

13. Data have been collected on service times at a drive in bank window at the Ludhiana Bank. This data are summarized into intervals as follows:

Interval (Seconds)	*Frequency y*
10 – 20	10
20 – 35	20
35 – 50	30
50 – 75	40
75 – 100	50
100 – 140	20
140 – 200	10

Generate 10 values of service time using the following random number string.

74, 50, 03, 31, 21, 09, 23, 50, 97, 39.

14. From the past records, it is known that 30% of the vehicles reporting at a filling station have diesel engines. Generate the number of diesel vehicles in each sample of 10 for 50 samples. Plot the distribution.
15. Write a computer program in FORTRAN or BASIC language for generating and plotting the following probability distribution :

(*i*) Normal (*ii*) Exponential (*iii*) Erlang (*iv*) Poisson.

16. It is known from the past records that the demand of an item (per day) takes place according to the following distribution:

Demand	0	2	3	4	5
Probability	$\frac{1}{10}$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{10}$	$\frac{1}{20}$

Simulate the demand for 15 days. **[PTU, B.Tech. (Prod.) May 2006]**

17. Collect the data on the inter arrival times and service times at a petrol pump. Approximate a probability density function for the inter arrival times and service times. How do the pdf vary over different time intervals of the day, like 7.00 AM to 10.00 AM, from 12.00 noon to 3.00 PM and from 5.00 PM to 8.00 PM.
18. Go to a major traffic intersection and determine the inter arrival times distribution, for the vehicles from each direction. Some arrivals want to go straight, some turn left, some turn right and some may take a U-turn. Based on this data design traffic lights system and develop its simulation model.
19. The electrical resistance of a copper wire increases with the increase in temperature. The experimental observations are given below :

Resistance (R)	19.10	25.00	30.10	36.00	40.00	45.10	50.00
Temperature (T)	76.30	77.80	79.75	80.80	82.35	89.90	85.10

By plotting a graph it can be seen that the relationship between R and T is almost linear. Develop the regression equation. **[Ans.** $R = 71.14 + 0.2797$**]**

20. Generate 10 random observations from the following probability density function assuming a set of 10 random numbers.

$$f(x) = \begin{cases} 2x, & \text{if } 0 \le x \le \\ 0, & \textit{otherwise} \end{cases}$$ **[P.U. ME (Mech.) 1988]**

21. Use the one-digit random numbers 2, 7, 9, 5, 8, 0, 7, 6, 2, 5, 6, 3, 5, 7, 8 to generate random observations for the colour of a traffic light found by a randomly arriving car, when green is 60% of the time, yellow is 10% and red 30% of the time. **[PTU B.Tech. (Prod.) Dec 2006]**