Theory of Computation

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The Pumping Lemma for CFL's

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Let L be a context-free language. Then we can find a natural number n, such that

For every string z in L of length $\geq$ n

There exists z = uvwxy such that:

1. $|vwx| \leq n$.
2. $|vx| > 0$.
3. For all $i \geq 0$, uv^iwx^iy is in L.

Example: Show that L={a^nb^nc^n | n>=1} is not context-free.

step 1) Assume L is Context free Language. n be the natural number obtained by the pumping lemma.

Step 2) let $z = a^n b^n c^n$ $\quad n+n+n = 3n$

$|z| = 3n > n$

$z = u\overline{\underline{vwx}}y$ where $|vx| \geq 1$

step 3)

$u\underline{vwx}y = \underline{a^n b^n c^n}$

$= \underbrace{a^n b}_{u} \underbrace{b^{n-1}}_{vwx} \underbrace{c^n}_{y}$

$= \underbrace{a^n b}_{u} \underbrace{b}_{v} . \underbrace{b^{n-3}}_{w} \underbrace{b}_{x} \underbrace{c^n}_{y}$

$\Rightarrow \quad u v^i w x^i y = a^n b (b)^i b^{n-3} (b)^i c^n$

$n=3 \Rightarrow a^3 b b^i b^i c^3$

$i=2 \Rightarrow a^3 b b^2 b^2 c^3$

$\Rightarrow a^3 b^5 c^3 \notin L$

$\left.\begin{matrix} n=4 \\ i=2 \end{matrix}\right\} \Rightarrow a^4 b (b)^2 b (b)^2 c^4$

$\Rightarrow a^4 b^6 c^4 \notin L$

This is Contradiction, so this language is not Context free.