

I.
①

$$L = \{ a^{2n} b^n \mid n \geq 1 \}$$

$$P: \begin{cases} S \rightarrow aaAb \\ A \rightarrow aaAb \mid \Lambda \end{cases}$$

$$\therefore G \Rightarrow (\{A, S\}, \{a, b\}, P, S)$$

②

$$L = \{ a^n b^n c^m d^m \mid n, m \geq 1 \}$$

$$P: \begin{cases} S \rightarrow aAbcBd \\ A \rightarrow aAb \mid \Lambda \\ B \rightarrow cBd \mid \Lambda \end{cases}$$

$$G \Rightarrow (\{A, B, S\}, \{a, b, c, d\}, P, S)$$

③

$$L = \{ a^n b^m c^m d^n \mid n, m \geq 1 \}$$

$$P \Rightarrow \begin{cases} S \rightarrow aBd \\ B \rightarrow aBd \mid X \\ X \rightarrow ~~a~~ bYc \\ Y \rightarrow bYc \mid \Lambda \end{cases}$$

$$G \Rightarrow (\{S, B, X, Y\}, \{a, b, c, d\}, P, S)$$

④

$$L = \{ a^n b^m c^K \mid \text{where } 2n = m \text{ \& } K \geq 2 \}$$

$$P: \begin{cases} S \rightarrow AccX \\ A \rightarrow aAbb \mid \Lambda \\ X \rightarrow cX \mid \Lambda \end{cases}$$

$$G \Rightarrow (\{S, A, X\}, \{a, b, c\}, P, S)$$

$$5) \quad L = \{ a^n b^m \mid n = 2 + (m \% 3) \}$$

$$P: \begin{cases} S \rightarrow aaA \\ A \rightarrow ab \mid aabb \mid Abbb \mid \Lambda \end{cases}$$

$$G = \{ \{S, A\}, \{a, b\}, P, S \}$$

$$\begin{aligned} \text{ii)} \quad S &\rightarrow pA \mid qB \mid rC \\ A &\rightarrow Sp \\ B &\rightarrow Sq \\ C &\rightarrow \Lambda \end{aligned}$$

$$\text{Let i) } S \rightarrow pA$$

$$\Rightarrow pSp \Rightarrow pqBp \Rightarrow pqSp \Rightarrow \underline{pqrp}$$

$$\text{ii) } S \rightarrow rC \Rightarrow \underline{r}$$

$$\text{iii) } S \rightarrow pA \Rightarrow pSp \Rightarrow \underline{pSp}$$

$$\text{iv) } S \rightarrow qB \Rightarrow qSq \Rightarrow \underline{qSq}$$

$$\text{So, } L(G) = \{ wrw^T : \text{where } w \in (P, q)^* \}$$

$$\begin{aligned} \text{2) } S &\rightarrow AB \mid \Lambda \\ A &\rightarrow aB \\ B &\rightarrow Bb \mid b \end{aligned}$$

$$\text{Let i) } S \rightarrow \Lambda \Rightarrow \underline{\Lambda}$$

$$\begin{aligned} \text{ii) } S &\rightarrow AB \Rightarrow aBB \Rightarrow aBbB \Rightarrow abbbB \\ &\Rightarrow abbbBb \Rightarrow \underline{abbbb} \end{aligned}$$

$$\text{iii) } S \rightarrow AB \Rightarrow aBB \Rightarrow \underline{abb}$$

$$\text{iv) } S \rightarrow AB \Rightarrow qBB \Rightarrow aBbB \Rightarrow abbbB \Rightarrow \underline{abbbb}$$

$$\therefore L(G) = \{ \Lambda \mid ab^n : \text{where } n \geq 2 \}$$

$$(3) \quad s \rightarrow as | bs | ss | \Lambda$$

Let $s \rightarrow as \Rightarrow a$

$s \rightarrow bs \Rightarrow b$

$s \rightarrow \Lambda \Rightarrow \Lambda$

$s \rightarrow as \Rightarrow abs \Rightarrow ab$

$s \rightarrow bs \Rightarrow bas \Rightarrow ba$

$$\therefore L = \{ \text{~~all~~ } (a,b)^* \}$$

$$(4) \quad s \rightarrow as | bs | sss | \Lambda$$

from previous question, with the help of $s \rightarrow as | bs | \Lambda$, we can generate $(a,b)^*$.

~~Since~~ $(a,b)^*$ already includes all the possible strings of a & b so $s \rightarrow sss$ will give no additional contribution.

$$\text{so } L = (a,b)^*$$

$$(5) \quad s \rightarrow as | bs | sb | sa | ss$$

from previous question \Rightarrow

$$L = (a,b)^*$$