Regression Analysis with Seasonal Data

The demand for a particular product in the past four years on quarterly basis

Year	Period	Demand (in Millions)
1	Spring	15
	Summer	25
	Fall	16
	Winter	8
2	Spring	17
	Summer	29
	Fall	14
	Winter	10
3	Spring	20
	Summer	30
	Fall	18
	Winter	11
4	Spring	18
	Summer	32
	Fall	19
	Winter	12

Forecast the demand for Summer of year 5.

Method: Considering each quarter/season as individual time period.

Period (x)	Demand (y)	x^2	xy
1	15	1	15
2	25	4	50
3	16	9	48
4	8	16	32
5	17	25	85
6	29	36	174
7	14	49	98
8	10	64	80
9	20	81	180
10	30	100	300
11	18	121	198
12	11	144	132
13	18	169	234
14	32	196	448
15	19	225	285
16	12	256	192
$\Sigma x = 136$	$\Sigma y = 294$	$\Sigma x^2 = 1496$	$\Sigma xy = 2551$

$\bar{x} = \frac{\Sigma x}{n} = \frac{136}{16} = 8.5$

$\bar{y} = \frac{\Sigma y}{n} = \frac{294}{16} = 18.38$

$$b = \frac{\Sigma xy - n\bar{x}\bar{y}}{\Sigma x^2 - n(\bar{x})^2} = \frac{2551 - (16 \times 8.5 \times 18.38)}{1496 - (16 \times 8.5 \times 8.5)}$$

$$= \frac{2551 - 2500}{1496 - 1156} = \frac{50}{340} = 0.15$$

$$a = \bar{y} - b\bar{x}$$

$$= 18.38 - (0.15 \times 8.5)$$

$$= 17.11$$

Value of Trend line $F = 17.11 + 0.15(t)$

For year 1, $t = 2$ (Summer) $\quad F_2 = 17.11 + (0.15 \times 2) = 17.11 + 0.30 = 17.41$

Seasonal factor (Year 1, Summer)

$$SF_{1(2)} = \frac{D_2}{F_2} = \frac{25}{17.41} = 1.43$$

Similarly, for year 2, $t = 6$ (Summer)

$$SF_{2(6)} = \frac{D_6}{F_6}$$

$$F_6 = 17.11 + (0.15 \times 6) = 17.11 + 0.90 = 18.01$$

$$SF_{2(6)} = \frac{29}{18.01} = 1.61$$

Similarly, for year 3, $t = 10$ (Summer)

$$F_{10} = 17.11 + (0.15 \times 10) = 17.11 + 1.5 = 18.61$$

$$SF_{3(10)} = \frac{D_{10}}{F_{10}} = \frac{30}{18.61} = 1.61$$

Similarly, for year 4, $t = 14$ (Summer)

$$F_{14} = 17.11 + (0.15 \times 14) = 17.11 + 2.30 = 19.41$$

$$SF_{4(14)} = \frac{D_{14}}{F_{14}} = \frac{32}{19.41} = 1.64$$

$$SF_{(Summer)} = \frac{SF_{1(Summer)} + SF_{2(Summer)} + SF_{3(Summer)} + SF_{4(Summer)}}{4}$$

$$= \frac{1.43 + 1.61 + 1.61 + 1.64}{4} = \frac{6.29}{4} = 1.57$$

$$F_{5(Summer)} = F_{18} \cdot SF_{(Summer)}$$

$$F_{18} = 17.11 + (0.15 \times 18) = 17.11 + 2.70 = 19.81$$

$$F_{5(Summer)} = 19.81 \times 1.57 = 31.10$$

Considering whole year as a time period

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Year (x)	Demand (Y)	x^2	xy
1	64	1	64
2	70	4	140
3	79	9	237
4	81	16	324
$\Sigma x = 10$	$\Sigma y = 294$	$\Sigma x^2 = 30$	$\Sigma xy = 765$

$\bar{x} = 10/4 = 2.5$ $\quad \bar{y} = \frac{294}{4} = 73.5$

$$b = \frac{\Sigma xy - n\bar{x}\bar{y}}{\Sigma x^2 - n(\bar{x})^2}$$

$$= \frac{765 - 4 \times 2.5 \times 73.5}{30 - (4 \times 2.5 \times 2.5)}$$

$$= \frac{765 - 735}{30 - 25} = \frac{30}{5} = 6$$

$$a = \bar{y} - b\bar{x}$$

$$= 73.5 - 6 \times 2.5$$

$$= 58.5$$

$$F = 58.5 + 6x$$

F_5 (Forecast for years) $= 58.5 + (6 \times 5) = 88.5$

$$SF_{(Summer)} = \frac{\sum_{i=1}^{n} D_{(Summer)}}{\sum_{i=1}^{n} D_{Year}} = \frac{116\ (25+29+30+32)}{294} = 0.39$$

$$F_{5(Summer)} = F_5 \cdot SF_{(Summer)}$$

$$= 88.5 \times 0.39 = \underline{34.52}$$