Theory of Computation

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CNF & GNF



Context-Free Grammar

> A grammar is context-free if every production is of the form

$$A \rightarrow \alpha$$

where $A \mathcal{E} V_N$ and $\alpha \mathcal{E} (V_N U_{\Sigma})^*$.

Chomsky Normal Form: CNF

- A CFG is said to be in Chomsky Normal Form if every production is of one of these two forms:
- 1. $A \rightarrow BC$ (right side is two variables).
- 2. $A \rightarrow a$ (right side is a single terminal).

Example 1: Reduce the following grammar G to CNF. G is S→ aAD, A → $aB \mid bAB, B \rightarrow b, D \rightarrow d$.

Step 1: Elimination of tempinals on RHS

As there are no null or unit productions.

We can proceed to Step 2.

Step 2: Eliminations of terminals on RHS

8-4, D-d

SJAAD

SIGAD

Ca -29

A 1 68

A J C B A B

Step 3: Restricting the number of variables on RHS

CAF G = ({5, A, B, 1)	Q, cb, c1, (2},	{a,b}, {P'}	(
B-6, D-9	SIGAR, CAIG		SI GAD LALCARS	CAC1	

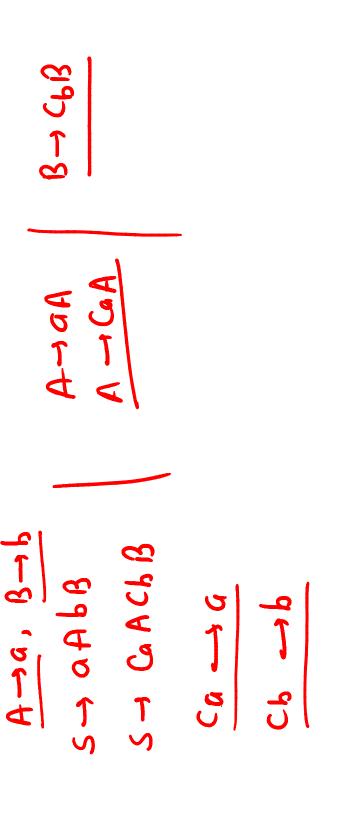
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Step 1: Elimination of terminals on RHS

As there are no null or unit productions.

We can proceed to Step 2.

Step 2: Eliminations of terminals on RHS



Step 3: Restricting the number of variables on RHS

P': A-10	13-16	A - CA	B→ C6B	ره ۱ م	9 - 3	59 TS	27 A2
S-> GACGB	> 1250 T S	C1 > A C68	くらせ トロ	C2→ C6B /			

E→ C6B

Greibach Normal Form

A context-free grammar is in Greibach normal form if every production is of the form

where $\alpha \in V_N$ and $\alpha \in \Sigma$ (α may be \wedge).

For example, G given by

 $S \rightarrow aAB \mid \mathcal{A}, A \rightarrow bC, B \rightarrow b, C \rightarrow c \text{ is in GNF.}$

Example 3: Construct a grammar in Greibach normal form equivalent to the grammar $S \rightarrow AA \mid a, A \rightarrow SS \mid b$.

respectively. So the productions are $A \mid \rightarrow A_2 \mid A_2 \mid a$ and $A2 \rightarrow AIAI \mid b$. As the given grammar has no null productions and is in CNF we need not Step 1: The given grammar is in CNF. S and A are renamed as A1 and A2, carry out step 1. So we proceed to step 2.

Step 2: (i)To get the productions in the form Ai $\rightarrow \alpha x$ or Ai \rightarrow Ajx, where j > i, convert the Ai-Productions (i = 1, 2..., n - 1) to the form Ai \rightarrow Aix such that

AI - AzAz la, Az - 6

(ii) The construction given in Lemma 1 is simple. To eliminate B in $A \rightarrow BA$, we simply replace B by the right-hand side of every B-production.

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Step 3: (Lemma 2) Let the set of A-productions be $A \rightarrow A\alpha I | \widetilde{A}\alpha r, \beta I' | \beta 2 |$

Let Z be a new variable. Let G1, where P1 is defined as follows:

(i) The set of A-productions in PI are
$$A \to \beta I \mid \beta 2 \mid \mid \beta s$$

$$A \to \beta IZ \mid \beta 2Z \mid \mid \beta sZ$$

(ii) The set of Z-productions in PI are
$$Z \rightarrow \alpha I \mid \alpha 2 \mid ... \mid \alpha r$$

$$Z \rightarrow \alpha IZ \mid \alpha 2Z \mid ... \mid \alpha rZ$$

(iii) The productions for the other variables are as in P. Then GI is a CFG and equivalent to G.

 \Box

Step 4: Modify Ai productions:

Step 5: Modify Zi productions:

$$Z_2 \rightarrow A_2 A_1$$
 $Z_2 \rightarrow A_2 A_1$ $Z_2 \rightarrow A_2 A_1$ $Z_2 \rightarrow A_2 A_1$ $Z_2 \rightarrow A_1 A_1 Z_2 A_1 | b A_1 | b A_1 | b A_2 A_2$ $Z_2 \rightarrow a A_1 A_1 Z_2 | a A_1 Z_2 A_1 Z_2 | b A_1 Z_2 | b A_2 Z_2 A_1$
 $P': A_1 \rightarrow a A_1 A_2 | a A_1 Z_2 A_2 | b A_2 | b Z_2 A_1$
 $A_2 \rightarrow a A_1 A_1 Z_2 | a A_1 Z_2 A_2 | b A_2 Z_2 A_1$
 $Z_2 \rightarrow a A_1 A_1 Z_2 | a A_1 Z_2 A_2 Z_2 | b A_2 Z_2$

Example: Convert the grammar $S \rightarrow AB$, $A \rightarrow BS \mid b$, $B \rightarrow SA \mid a$ into