Theory of Computation

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Pushdown Automata

CFG to PDA Conversion

Let $L = L(G)$, where $G = (V_N, \Sigma, P, S)$ is a context-free grammar. We construct a PDA A as

$$A = \{q, \Sigma, \Gamma = V_N \cup \Sigma, \delta, q, Z_0 = S, \Phi\}$$

where δ is defined by the following rules:

$R1: \delta(q, \wedge, A) = \{(q, \alpha) \mid A \rightarrow \alpha \text{ is in } P\}$

$R2: \delta(q, a, a) = \{(q, \wedge)\}$ for every a in Σ.

Construct a PDA *equivalent to the following context-free grammar:* $S \to 0BB$, $B \to 0S \mid 1S \mid 0$. *Test whether 010000 is accepted by PDA.*

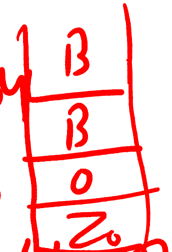
$R_1 \quad \delta(q, \Lambda, S) = \{(q, 0BB)\}$

$R_2 \quad \delta(q, \Lambda, B) = \{(q, 0S), (q, 1S), (q, 0)\}$

$R_3 \quad \delta(q, 0, 0) = \{(q, \Lambda)\}$

$R_4 \quad \delta(q, 1, 1) = \{(q, \Lambda)\}$

$(q, 010000, \underline{S}) \Rightarrow (q, 010000, \underline{0BB})$ Apply R_3



$(q, 10000, BB) \Rightarrow (q, 10000, 1SB)$ Apply R_4

$(q, 0000, SB) \Rightarrow (q, 0000, 0BBB)$

Construct a PDA *equivalent to the following context-free grammar:* $S \rightarrow a \mid aS \mid Sa \mid bSS \mid SbS \mid SSb$.

$$\delta(q, \lambda, S) = \{(q, a), (q, aS), (q, Sa), (q, bSS), (q, SbS), (q, SSb)\}$$

PDA to CFG Conversion

If $A = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$ is a pda, then there exists a context-free grammar G such that $L(G) = N(A)$.

We first give the construction of G and then prove that $N(A) = L(G)$.

(Construction of G). We define $G = (V_N, \Sigma, P, S)$, where

$$V_N = \{S\} \cup \{[q, Z, q'] \mid q, q' \in Q, Z \in \Gamma\}$$

i.e. any element of V_N is either the new symbol S acting as the start symbol for G or an ordered triple whose first and third elements are states and the second element is a pushdown symbol.

The productions in P are induced by moves of pda as follows:

R_1: S-productions are given by $S \to [q_0, Z_0, q]$ for every q in Q.

R_2: Each move erasing a pushdown symbol given by $(q', \Lambda) \in \delta(q, a, Z)$ induces the production $[q, Z, q'] \to a$.

$\delta(q, a, Z) = \{(q', \Lambda)\}$

R_3: Each move not erasing a pushdown symbol given by $(q_1, Z_1Z_2 \ldots Z_m) \in \delta(q, a, Z)$ induces many productions of the form

$$[q, Z, q'] \to a[q_1, Z_1, q_2][q_2, Z_2, q_3] \ldots [q_m, Z_m, q']$$

where each of the states $q', q_2, \ldots, q_m$ can be any state in Q. Each move yields many productions because of R_3. We apply this construction to an example before proving that $L(G) = N(A)$.

Example

Construct a context-free grammar G which accepts $N(A)$, where

$$A = (\{q_0, q_1\}, \{a, b\}, \{Z_0, Z\}, \delta, q_0, Z_0, \emptyset)$$

and δ is given by

$$\delta(q_0, b, Z_0) = \{(q_0, ZZ_0)\}$$

$$\delta(q_0, \Lambda, Z_0) = \{(q_0, \Lambda)\}$$

$$\delta(q_0, b, Z) = \{(q_0, ZZ)\}$$

$$\delta(q_0, a, Z) = \{(q_1, Z)\}$$

$$\delta(q_1, b, Z) = \{(q_1, \Lambda)\}$$

$$\delta(q_1, a, Z_0) = \{(q_0, Z_0)\}$$

Sol^n Construction of V_N

$V_N = S, [q_0, Z_0, q_0], [q_0, Z_0, q_1]$
$[q_0, Z, q_0], [q_0, Z, q_1]$
$[q_1, Z_0, q_0], [q_1, Z_0, q_1]$
$[q_1, Z, q_0], [q_1, Z, q_1]$

$P_1 : \quad S \rightarrow [\underline{q_0, Z_0}, \underline{q_0}]$

$P_2 : \quad S \rightarrow [\underline{q_0, Z_0}, \underline{q_1}]$

R3 $\delta(q_0, b, Z_0) = \{(q_0, ZZ_0)\}$

P_3 : $[q_0, Z_0, q_0] \rightarrow b\, [q_0, Z, q_0]\, [q_0, Z_0, q_0]$

P_4 : $[q_0, Z_0, q_0] \rightarrow b\, [q_0, Z, q_1]\, [q_1, Z_0, q_0]$

P_5 : $[q_0, Z_0, q_1] \rightarrow b\, [q_0, Z, q_0]\, [q_0, Z_0, q_1]$

P_6 : $[q_0, Z_0, q_1] \rightarrow b\, [q_0, Z, q_1]\, [q_1, Z_0, q_1]$

R2 $\delta(q_0, \Lambda, Z_0) = \{(q_0, \Lambda)\}$

P_7 : $[q_0, Z_0, q_0] \rightarrow \Lambda$

R3

$\delta(q_0, b, Z) = \{(q_0, ZZ)\}$

$P_8 : [q_0, Z, q_0] \rightarrow b\,[q_0, Z, q_0](q_0, Z, q_0)$

$P_9 : [q_0, Z, q_0] \rightarrow b\,[q_0, Z, q_1]\,[q_1, Z, q_0]$

$P_{10} : [q_0, Z, q_1] \rightarrow b\,[q_0, Z, q_0]\,[q_0, Z, q_1]$

$P_{11} : [q_0, Z, q_1] \rightarrow b\,[q_0, Z, q_1]\,[q_1, Z, q_1]$

R3 $\delta(q_0, a, Z) = \{(q_1, Z)\}$

$R_{12} : [q_0, Z, \underline{q_0}] \rightarrow a\,[q_1, Z, \underline{q_0}]$

$R_{13} : [q_0, Z, \underline{q_1}] \rightarrow a\,[q_1, Z, \underline{q_1}]$

R_2 ✓ $\delta(q_1, b, Z) = \{(q_1, \wedge)\}$

P_{14}: $[q_1, Z, q_1] \longrightarrow b$

$\delta(q_1, a, Z_0) = \{(q_0, Z_0)\}$

P_{15}: $[q_1, Z_0, q_0] \longrightarrow a\,[q_0, Z_0, q_0]$

P_{16}: $[q_1, Z_0, q_1] \longrightarrow a\,[q_0, Z_0, q_1]$

Construct the corresponding context-free grammar accepting the same set of given PDA.

The pda A accepting $\{a^n b^m a^n \mid m, n \geq 1\}$ is defined as follows:

$$A = (\{q_0, q_1\}, \{a, b\}, \{a, Z_0\}, \delta, q_0, Z_0, \emptyset)$$

where δ is defined by

$$R_1: \delta(q_0, a, Zo) = \{(q_0, aZ_0)\}$$

$$R_2: \delta(q_0, a, a) = \{(q_0, aa)\}$$

$$R_3: \delta(q_0, b, a) = \{(q_1, a)\}$$

$$R_4: \delta(q_1, b, a) = \{(q_1, a)\}$$

$$R_5: \delta(q_1, a, a) = \{(q_1, \Lambda)\}$$

$$R_6: \delta(q_1, \Lambda, Z_0) = \{(q_1, \Lambda)\}$$