Functions

Let A & B are 2 non-empty sets, then a function or mapping from A to B, $f : A \to B$ or $f(a) = b$

"b is an image of a"

"a is pre-image of b"

each element of A has only 1 image in B.

Different elements of A can have same image in B.

Total functions from A to B $= n^m$

$[n \times n \times n \times \dots m \text{ times} = n^m]$

Domain of function — $f : A \to B$, set A is domain

Co-domain — $f : A \to B$, set B is co-domain.

Image — $f(x) = y$, y is image of x under f.

Pre-image or Inverse image — $f^{-1}(T) = \{a \in A, f(a) \in T\}$

Range of Fn or $Im(f)$

Set of all images of its domain.

Everywhere defined function — if $dom(f) = A$.

Function as a Relation

$aRb \in A \times B$, this relation is called function if —

(i) for each $a \in A$, there exist $b \in B$ s.t. $(a,b) \in f$.

(ii) If $(a,b) \in f$ & $(a,c) \in f$, then $b = c$.

i.e. no 2 ordered pairs in f have same first element.

Eg (i) $f_1 = (1,2), (2,3), (3,4)$ —

$f_2 = (1,2), (1,3), (2,3), (3,4)$ ×

$f_3 = (1,3), (2,4)$ ×

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Types of functions

(a) <u>Injective (One-to-one)</u> - Every element of domain X has a unique image in co-domain Y & no element of Y has more than one pre-image in X.

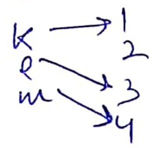
(b) <u>Surjective (Onto)</u> $f: X \to Y$

If each element in Y is ~~atleast~~ the image of atleast one element in X.

(c) <u>Bijective</u> (One-to-one onto) - Both Inj & Surj.

(d) <u>Into</u> - If range of function is not equal to co-domain Y. Therefore, there must be an element of Y which is not the image of any element in X.

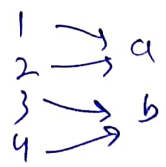
(e) <u>One-one Into</u> — Its one-one but not onto.
(Y can be bigger than X)



(f) <u>Many one</u> - More than one elements in X having same image in Y.

(g) <u>Many one into</u> -

(h) <u>many one onto</u> -



<u>Equal functions</u> - f and g are 2 fns from X→Y

$f(a) = g(a)$

(i) Domain (f) = domain (g)
(ii) Co-dom (f) = co-dom(g)
(iii) $f(x) = g(x) \ \forall \ x \in D$ where $D = dom(f) \cap dom(g)$

Identity fn - $f: A \to A$ (image on itself)

Constant fn - $f \to$ function with domain A.
for every $x \in A$, $f(x) = c$ is some constant.

Invertible (Inverse) fn - $f: X \to Y$ iff it is bijective fn (one 2 one onto)

$X = \{1, 2, 3\}$ $Y = \{k, l, m\}$
$f = \{(1, k), (2, m), (3, l)\} \Rightarrow f^{-1} = \{(k, 1), (m, 2), (l, 3)\}$

~~Hashing function~~ Composition of functions

let $f: A \to B$ & $g: B \to C$, then $gof: A \to C$

$gof(x) = g(f(x)) \ \forall x \in A$.

Thus for finding image of a under gof, we first find image of a under f and image of f(a) under g.

gof is defined iff. $R_f \subseteq D_g$.

Eg $f: \{1, 2, 3\} \to \{a, b\} \Rightarrow f(1) = a, f(2) = a, f(3) = b$
$g: \{a, b\} \to (5, 6, 7) \Rightarrow g(a) = 5, \ g(b) = 7$.

Find gof
$R_f = \{a, b\}$, $D_g = \{a, b\}$
$R_f \subseteq D_g$, so defined $\Rightarrow \{1, 2, 3\} \to \{5, 6, 7\}$

$gof(1) = g(f(1)) = g(a) = 5$
$gof(2) = g(f(2)) = g(a) = 5$
$gof(3) = g(f(3)) = g(b) = 7$

fog $R_g = \{5, 6, 7\}$, $D_f = \{1, 2, 3\}$

$R_g \not\subseteq D_f$; fog not defined

Eg $f(x) = x^2$; $g(x) = x^3$

find $(f \circ g \circ f \circ g)(x)$ $\left[\begin{array}{l} f(x) = x^2 \\ f(x^3) = (x^3)^2 \end{array} \right]$

$= f(g(f(g(x)))) = f(g(f(x^3)))$

$= f(g(x^6)) = f(x^{18}) = x^{36}$.

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