Theory of Computation

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CNF & GNF

Context-Free Grammar

- A grammar is context-free if every production is of the form

$$A \rightarrow \alpha$$

where $A \, \mathcal{E} \, V_N$ *and* $\alpha \, \mathcal{E} \, (V_N \, U \sum)^*$.

Chomsky Normal Form: CNF

- A CFG is said to be in *Chomsky Normal Form* if every production is of one of these two forms:
  1. A → BC (right side is two variables).
  2. A → a (right side is a single terminal).

Example 1: Reduce the following grammar G to CNF. G is S→ aAD, A → aB | bAB, B →b, D → d.

null or Unit production

Step 1: Elimination of ~~terminals~~ on RHS

As there are no null or unit productions.

We can proceed to Step 2.

Step 2: Eliminations of terminals on RHS

$B \to b$, $D \to d$

$S \to aAD$

$S \to C_a AD$

$C_a \to a$

$A \to C_a B$

$A \to C_b AB$

$C_b \to b$

Step 3: Restricting the number of variables on RHS

$B \to b$, $D \to d$

$S \to C_a A D$, $C_a \to a$

$A \to C_b A B$, $C_b \to b$

$A \to C_a B$

$S \to C_a A D$

✓ $S \to C_a C_1$

✓ $C_1 \to A D$

$A \to C_b A B$

✓ $A \to C_b C_2$

✓ $C_2 \to A B$

CNF $G = (\{S, A, B, D, C_a, C_b, C_1, C_2\}, \{a, b\}, \{P'\}, \{S\})$

Example 2: Find a grammar in CNF equivalent to $S \rightarrow aAbB$, $A \rightarrow aA \mid a$, $B \rightarrow bB \mid b$.

Step 1: Elimination of ~~terminals~~ null & unit productions on RHS

As there are no null or unit productions.

We can proceed to Step 2.

Step 2: Eliminations of terminals on RHS

$A \rightarrow a$, $B \rightarrow b$

$S \rightarrow aAbB$

$S \rightarrow C_a A C_b B$

$C_a \rightarrow a$

$C_b \rightarrow b$

$A \rightarrow aA$

$A \rightarrow C_a A$

$B \rightarrow C_b B$

Step 3: Restricting the number of variables on RHS

$S \rightarrow C_a \underline{A\, C_b B}$

$S \rightarrow C_a C_1$ ✓

$C_1 \rightarrow A \underline{C_b B}$

$C_1 \rightarrow A C_2$ ✓

$C_2 \rightarrow C_b B$ ✓

P':

$A \rightarrow a$

$B \rightarrow b$

$A \rightarrow C_a A$

$B \rightarrow C_b B$

$C_a \rightarrow a$

$C_b \rightarrow b$

$S \rightarrow C_a C_1$

$C_1 \rightarrow A C_2$

$C_2 \rightarrow C_b B$

Greibach Normal Form

A context-free grammar is in Greibach normal form if every production is of the form

$$A \rightarrow a\cancel{\alpha}.$$

$A \rightarrow a\Sigma$

$A \rightarrow a$

$A \rightarrow aABCD \ldots$

where $\alpha \in V_N$ and $a \in \Sigma$ (α may be $\wedge$).

For example, G given by

$S \rightarrow aAB \mid \cancel{\wedge}$, $A \rightarrow bC$, $B \rightarrow b$, $C \rightarrow c$ is in GNF.

Example 3: Construct a grammar in Greibach normal form equivalent to the grammar $S \rightarrow AA \mid a,\ A \rightarrow SS \mid b$.

$S \rightarrow A_1$

$A \rightarrow A_2$

Solution:

Step 1: The given grammar is in CNF. S and *A are renamed as A1 and A2,* respectively. So the productions are $A1 \rightarrow A1A2 \mid a$ *and* $A2 \rightarrow A1A1 \mid b$. As the given grammar has no null productions and is in CNF we need not carry out step 1. So we proceed to step 2.

$A_1 \rightarrow A_2A_2 \mid a$ & $A_2 \rightarrow A_1A_1 \mid b$

i j

Step 2: (i)To get the productions in the form $Ai \rightarrow a\alpha$ or $Ai \rightarrow Aj\alpha$, where $j > i$, convert the Ai-Productions ($i = 1, 2, \ldots, n - 1$) to the form $Ai \rightarrow Ai\alpha$ such that $j > i$.

$A_1 \rightarrow A_2A_2 \mid a$, $A_2 \rightarrow b$

(ii) The construction given in Lemma1 is simple. To eliminate B in $A \rightarrow B\alpha$, we simply replace B by the right-hand side of every B-production.

$A_2 \rightarrow A_1A_1$

$A_2 \rightarrow A_2A_2A_1 \mid aA_1$

$A_1 \rightarrow A_2A_2 \mid a$,

$A_2 \rightarrow A_2A_2A_1 \mid aA_1 \mid b$

Step 3: (Lemma 2) Let the set of A-productions be $A \rightarrow A\alpha 1 \mid \ldots A\alpha r,\ \beta 1 \mid \beta 2 \mid \ldots. \mid \beta s.$

Let Z be a new variable. Let G1, *where P1 is defined* as follows:

(i) The set of A-productions in *P1 are* $A \rightarrow \beta 1 \mid \beta 2 \mid \ldots. \mid \beta s$

$$A \rightarrow \beta 1Z \mid \beta 2Z \mid \ldots. \mid \beta sZ$$

(ii) The set of Z-productions in *P1 are* $Z \rightarrow \alpha 1 \mid \alpha 2 \mid \ldots \mid \alpha r$

$$Z \rightarrow \alpha 1Z \mid \alpha 2Z \mid .. \mid \alpha rZ$$

(iii) The productions for the other variables are as in *P. Then G1 is a CFG* and equivalent to G.

$A_2 \rightarrow A_2 A_2 A_1 \mid aA_1 \mid b$

$$\begin{cases} A_2 \rightarrow aA_1, & A_2 \rightarrow b \\ A_2 \rightarrow aA_1Z_2, & A_2 \rightarrow bZ_2 \\ Z_2 \rightarrow A_2A_1, & Z_2 \rightarrow A_2A_1Z_2 \end{cases}$$

$\{A_1 \rightarrow A_2 A_2 \mid a$

Step 4: Modify Ai productions:

✓ $A_2 \rightarrow aA_1 \mid aA_1Z_2 \mid b \mid bZ_2$

$A_1 \rightarrow \underline{A_2}A_2 \mid a$

✓ $A_1 \rightarrow aA_1A_2 \mid aA_1Z_2A_2 \mid bA_2 \mid bZ_2A_2 \mid a$

Step 5: Modify Zi productions:

$Z_2 \to \underline{A_2} A_1$ $\quad\quad$ $Z_2 \to \underline{A_2} A_1 Z_2$

✓ $Z_2 \to \underline{a A_1} A_1 \mid a A_1 Z_2 A_1 \mid b A_1 \mid b Z_2 A_1$

✓ $Z_2 \to a A_1 A_1 Z_2 \mid a A_1 Z_2 A_1 Z_2 \mid b A_1 Z_2 \mid b Z_2 A_1 Z_2$

P':

$A_1 \to a \mid a A_1 A_2 \mid a A_1 Z_2 A_2 \mid b A_2 \mid b Z_2 A_2$

$A_2 \to a A_1 \mid a A_1 Z_2 \mid b \mid b Z_2$

$Z_2 \to a A_1 A_1 \mid a A_1 Z_2 A_1 \mid b A_1 \mid b Z_2 A_1$

$Z_2 \to a A_1 A_1 Z_2 \mid a A_1 Z_2 A_1 Z_2 \mid b A_1 Z_2 \mid b Z_2 A_1 Z_2$

Example: Convert the grammar $S \rightarrow AB$, $A \rightarrow BS \mid b$, $B \rightarrow SA \mid a$ into GNF.