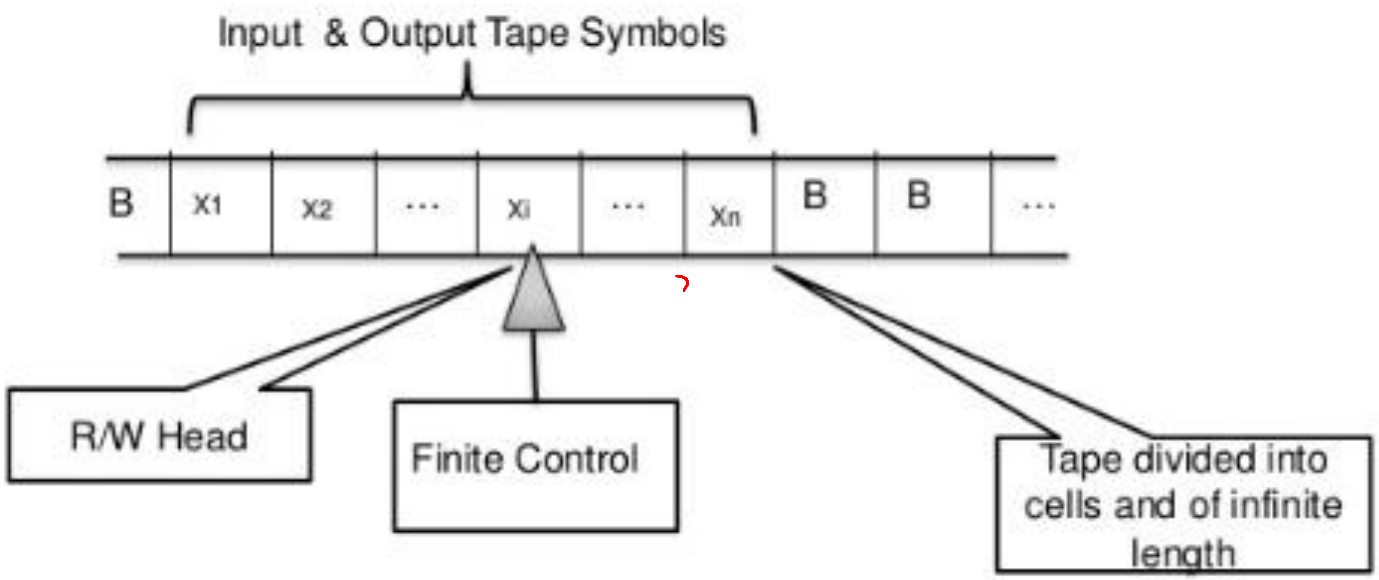
Theory of Computation

Dr Samayveer Singh

Turing Machine

Turing Machine Model



Turing Machine

Definition A Turing machine M is a 7-tuple, namely $(Q, \Sigma, \Gamma, \delta, q_0, b, F)$, where

1. Q is a finite nonempty set of states,
2. Γ is a finite nonempty set of tape symbols,
3. $b \in \Gamma$ is the blank,
4. Σ is a nonempty set of input symbols and is a subset of Γ and $b \notin \Sigma$,
5. δ is the transition function mapping (q, x) onto (q', y, D) where D denotes the direction of movement of R/W head; $D = L$ or R according as the movement is to the left or right.
6. $q_0 \in Q$ is the initial state, and
7. $F \subseteq Q$ is the set of final states.

Example 1: Construct a TM that accepts $L = \{a^n b^n \mid n \geq 1\}$

$\delta:$

δ	a	b	x	y	B
q_0	q_1, X, R	–	–	q_3, Y, R	
q_1	q_1, a, R	q_2, Y, L	–	q_1, Y, R	
q_2	q_2, a, L	–	q_0, X, R	q_2, Y, L	
q_3	–	–		q_3, Y, R	q_f, B, L
q_f	–	–			

$q_0\, aaabbb$

$X\, q_1\, aabbb$

$X a\, q_1\, abbb$

$X a a\, q_1\, bbb$

$X a a\, q_2\, Y bb$

$X a\, q_2\, a Y bb$

$X\, q_2\, a a Y bb$

$X\, q_0\, a a Y bb$

$X X\, q_1\, a Y bb$

$X X a\, q_1\, Y bb$

$X X a Y\, q_1\, bb$

$X X a Y\, q_2\, Y b$

$X X a\, q_2\, Y Y b$

$X X\, q_2\, a Y Y b$

$X X\, q_0\, a Y Y b$

$X Y X\, q_1\, Y Y b$

$X X X Y Y\, q_1\, b$

$X X X Y Y\, q_2\, Y$

$X X X\, q_2\, Y Y Y$

$X X X\, q_0\, Y Y Y$

$X X X Y\, q_3\, Y Y$

$X X X Y Y Y\, q_3\, B$

Example 2: Construct a TM that accepts $L = \{a^n b^n c^n \mid n \geq 1\}$

δ	a	b	c	X	Y	Z	B
q_0	q_1, X, R				q_4, Y, R		
q_1	q_1, a, R	q_2, Y, R			q_1, Y, R		
q_2		q_2, b, R	q_3, Z, L			q_2, Z, R	
q_3	q_3, a, L	q_3, b, L		q_0, X, R	q_3, Y, L	q_3, Z, L	
q_4					q_4, Y, R	q_4, Z, R	q_f, B, L

$q_0\,aabbcc$
$X\,q_1\,abbcc$
$Xa\,q_1\,bbcc$
$XaY\,q_2\,bcc$

$XaYb\,q_2\,cc$
$XaYb\,q_3\,Zc$
$XaY\,q_3\,bZc$
$Xa\,q_3\,YbZc$
$X\,q_3\,aYbZc$

$X\,q_0\,aYbZc$
$XX\,q_1\,YbZc$
$XXY\,q_1\,bZc$
$XXYY\,q_2\,Zc$
$XXYYZ\,q_2\,c$

$XXYYZ\,q_3\,Z$
$XXYY\,q_3\,ZZ$
$XX\,q_3\,YYZZ$
$XX\,q_0\,YYZZ$
$XXY\,q_4\,YZZ$

Example 3: Construct a TM that accept palindrome numbers.

$0 \to X$

$1 \to Y$

q_0 0110

q_0 1001

State	Symbol				
	0	1	X	Y	B
q_0	(q_1, X, R)	(q_2, Y, R)	(q_6, X, R)	(q_6, Y, R)	(q_6, B, R)
q_1	$(q_1, 0, R)$	$(q_1, 1, R)$	(q_3, X, L)	(q_3, Y, L)	(q_3, B, L)
q_2	$(q_2, 0, R)$	$(q_2, 1, R)$	(q_4, X, L)	(q_4, Y, L)	(q_4, B, L)
q_3	(q_5, X, L)	–	(q_6, X, R)	(q_6, Y, R)	–
q_4	–	(q_5, Y, L)	(q_6, X, R)	(q_6, Y, R)	–
q_5	$(q_5, 0, L)$	$(q_5, 1, L)$	(q_0, X, R)	(q_0, Y, R)	–