REGULAR EXPRESSION

Algebraic rep. of string. Operands = Alphabet or NULL. Operator = $\cup$, +, Null closure ϵ, concatenation & R_1R_2

Union : $R_1 \cup R_2$, $R_1 | R_2$, $R_1 + R_2$

$L(R_1)^* = \Lambda, R_1, R_1R_1, R_1R_1R_1 \ldots$
ϵ

Application : Data validation
Data Abstraction
Data transformation [ex. row to col]
Data wrapping.

Q. Set of strings over 0,1 that end in 3 consecutive 1.

$L = \{ (0+1)^* 111 \}$

Q. Set of string over 0,1 that have atleast 1 one

$RE = \{ (0+1)^* 1 (0+1)^* \}$

$RE = 0^* 1 (0+1)^*$

Q. set of string over 0,1 that have atmost 1,

~~$0^*1^*0^*$~~

$0^* 1 0^* \cup 0^*$

$\boxed{\text{Doubt}}$; $(0^*+1)0^*$

Regular set : $(0+1)^*$ – we write all combination of this expression, which is called Regular set.

$a = \{a\}$

$a+b = \{a,b\}$

$(a+b)^* = \{\Lambda, \{a,b\}^*$

$= \{\Lambda, a, b, ab, ba \text{----}\}$

$(0+10^*) \Rightarrow L = \{0, 1, 10, 100, 1000 \ldots\}$

$(0^*10^*) \Rightarrow L = \{1, 01, 10, 010 \text{----}\}$

$(0+\Lambda)(1+\Lambda) \Rightarrow L = \{\Lambda, 0, 1, 01\}$..

$(a+b)^* \Rightarrow L = \{\Lambda, a, b, ab, ba \text{---}\}$

$(a+b)^*abb = \{abb, aabb, babb, ababb, baabb\}$

$(11)^* = L = \{\Lambda, 11, 1111, 111111, \ldots\}$

$(aa)^*(bb)^*b = \{b, aab, bbb, aabbb \text{---}\}$

$(aa+ab+ba+bb)^* = \{\Lambda, aa, ab, ba, bb, aaab,$
aaba, aabb, abaa,
abba, abbb, baaa, baab,
babb, bbaa, bbab, bbba..
---- }

016

ab
ba

Imp
$(0+1)^*$.

$c^* a (a+c)^*$ …
$c^* b(b+c)^* a (a+b+c)^*$

$(a+b)^* = \{\Lambda, a, b, ab, ba, \dots\}$
$(a+b)^+ = \{a+b\}^* - \Lambda$
$= \{a, b, ab, ba, aab, \dots\}$

<u>from set to Regular expression</u>

Q. $\{1, 11, 111, 1111 \dots\} = 1^+$
$= 11^*$

Q Describe the following sets by Regular expression.

1) set of all string of 0, 1 ending 00 $= (0+1)^* 00$

2) set of all st. 0, 1 begining 0 & ending 1
$0(0+1)^* 1$

3) write $\{a, b, c\}$ containing atleast 1 a and at least 1 b.

$(a+b+c)^* a (a+b+c)^* b (a+b+c)^*$
$\cup (a+b+c)^* b (a+b+c)^* a (a+b+c)^*$

$c^* a (a+c)^* b (a+b+c)^* +$
$c^* b (b+c)^* a (a+b+c)^*$.

⇒ A declarative way to express the strings we want to accept.

OPERATORS

1. Union:- $L \cup M$, set of strings either L or M or both.

$L = \{001, 10, 111\}$ & $M = \{\epsilon, 001\}$

$L \cup M = \{\epsilon, 001, 10, 111\}$

2. Concatenation: LM, can be formed by taking any string in L and concatenating it with any string in M.

3. Closure (star or kleene closure): L^* any no. of strings from L, possibly with repetitions, & concatenating all of them.

$L = \{0, 11\}$

$L^0 = \{\epsilon\}$ - selection of 0 string from L

$L^1 = L$ - choice of one string from L.

L^2 = pick any two strings from L, with repetitions allowed, so there are 4 choices: $L^2 = \{00, 011, 110, 1111\}$.

L^3 = set of strings that may be formed by making three choices of two strings in L.

$\{000, 0011, 0110, 1100, 01111, 11011, 11110, 11111\}$.

(*) Σ^*: The operator (*) formed all strings whose symbols are chosen from the alphabet Σ.

Q. $\Sigma = \{a, b\}$ containing aab

$(a+b)^* aab (a+b)^*$.

PRECEDENCE

1. $*$: Applies only to the smallest sequence of symbols to its left that is a well-formed regular expression.
2. '.' : After grouping all the $*$, we group '.'.
3. Union

Ex: Group 01^*+1

$\Rightarrow (0(1^*))+1$

Q. set of all string begin with 110

$110(0+1)^*$

Q. containing 1011

$(1+0)^* 1011 (1+0)^*$

Q. containing exactly 3 1's.

$0^*10^*10^*10^*$

Q. No. of 0's is odd.

$$1^*0(1+01^*0)^*$$

Imp

Q. String that donot contain $\boxed{1101}$

wrong $r = 0^*(11^*00^+)^* 1^* 0^*$

Q. String except 110 as substring

$$(0+10)^*1^*$$

$$(0+11^*00)^*(\lambda+11^*+11^*0)$$

Q. String except 101

$$\cancel{(0+1+}\ (0^*1^*00)^* \cancel{0^*1^*}$$

$$0^*(1^*000^*)^* 1^*0^*$$

$$\cancel{(0+(11^*))^* + (0+(11^*))^* 11^* + (0+(11^*))^* 11^* 0}$$

Q. set of all strings with an equal no. of 0's & 1's, such that no prefix has two more 0's than 1's nor two more 1's than 0's.

$$(10+01)^*$$

Q. No. of 0's is divisible by five and no. of 1's is even.

→ make dfa using union of two dfa & then make Regex

Q. tenth symbol from right is 1.

$(0+1)^* 1 (1+0)(1+0)(1+0)(1+0)(1+0)(1+0)$
$(1+0)(1+0)(1+0)$

$= (0+1)^* 1 (1+0)^9$

Q. $L = \{a^{2n} b^{2m+1} : n \geq 0, m \geq 0\}$

$r = (aa)^* (bb)^* b.$

Q. $L = \{w \in (0,1)^* :$ w has no pair of consecutive zeros $\}$.

$(1+01)^* (0+\epsilon)$

Q. $L = \{a^n b^m \; n \geq 4, m \leq 3\}$

$(aaaa)(a^*)(\epsilon + b + bb + bbb)$

Q. $L = \{a^n b^m : (n+m)$ is even $\}$.

$a(aa)^* b(bb)^* + (aa)^*(bb)^*$

Q. $L = \{ab^n, (a) \; n \geq 3 \; w \in (a+b)^* \}$..

$r = a(bbb)(b)^*$

Q. $L = \{ab^n w : n \geq 3$ and $w \in (a,b)^+$

$r = ab^3 b^* (a+b)^+$

Q. Let $L = \{w : |w| \bmod 3 = 0\}$ $w \in (a,b)^*$

$r = ((a+b)^3)^*$

Q. $L = \{w \in (a,b)^* : n_a(w) \bmod 3 = 0\}$ –

$r = \{b^* a\, b^* a\, b^* a\, b^*\}^*$

Q. for even binary numbers –

$r = (0+1)^* 0$

Q. Atleast 2 a's and ends with even b's

$\cancel{b^* a b^* a (bb)^*}$

$\cancel{b^* a b^* a b^* a^* (bb)^*}$

$b^* a b^* a (a+b)^* (bb)^*$

Q. begins with atleast 2a's and ends with even b's.

$aa(a)^* (bb)^*$. | $aa(a+b)^*(bb)^*$

Q. Begins with a, ends with b and total length is even.

$a(aa)^*(bb)^* + (aa)^*(bb)^*$ / $a((a+b)(a+b))^* b$

↳ case when nothing in b/w.

Q. Only one pair of $\boxed{00}$

$(1+01)^* (00)(1+10)^*$

Scanned with CamScanner

Q. Fifth symbol as zero.

$(0+1)(0+1)(0+1)(0+1)0(0+1)^*$

Q. No more than two 0's.

$1^* + 1^*01^* + 1^*01^*01^*$

Q. $L = \{uvw \mid u, w \in \Sigma^* \text{ \& } |v|=2\}$

$\Sigma = \{a, b\}$

$r = (a+b)^* (a+b)(a+b)(a+b)^*$

Q. Binary string that are $2n$ where n is odd no.

$\left((00+11+10+01)^*(0+1) + (0+1)(00+11+10+01)^*\right)^2$

$(0+1)^*01$

Q. containing sequence 011

$\cancel{0^*1^*(0)^*}$ $\cancel{1^*0^*11(0+1)^*}$

$\boxed{(0+1)^*011(0+1)^*}$ ya phir

pura dfa se convert.

Q. atleast one 00 and one 11.

$\cancel{r = 1^*0}$

$r = (1+0)^*00(1+0)^*11(1+0)^* + (1+0)^*11(1+0)^*00(1+0)^*$

Ans diff

Q. 2 occurrences of at least 2 consecutive 1's & two occurrences not being adjacent
i.e, 011011 ✓ but 011111 X

Sol: Min. string = 1111 or 11011

~~$0^*(11+00)$~~

i) $11(0+1)^+11$

Ans $(0+1)^*11(0+1)^+11(0+1)^*$

$(0+10)^*11(0+1)(0+10)^*11(0+1)^*$

Q. atleast 2 consecutive 0's

~~$(0+1)^*00(0+1)^*$~~

$\Rightarrow (1+01)^*00(0+\phi\epsilon)^*$

Q. $\Sigma=\{0,1,2\}$ such that at least one 0, 0 is followed by at least one 1, which is followed by at least one 2

$r = 0^+1^+2^+$

Q. Starting & ending with a.

~~$a^*b^*a^+$~~

$\boxed{a^+(a+b)^*a^+}$ $\boxed{a(b+aa^*b)^*aa^*}$

Q. ending with 011

$(0+1)^*011$

$1^*0(1111^*0+0+10+110)^*11$

Q- All combination of 0 & 1 but not 2 consecutive 0.

$\cancel{0^*(1+10)^* 1^* (0+1)^*}$

$\cancel{0^*(1+10)^* 1^*}$

$(1+01)^* (0+\wedge)$

Q. $\Sigma=\{a,b\}$ atleast one combination of double letter

× $r=(a+b)^* (aa+bb)^+ (a+b)^*$ not ×

solve w d dfa

Q. $\Sigma=\{a,b\}$ without any combination of double letters.

$r=(\epsilon+b)(ab)^*(\epsilon+a)$

Q. even no. of 0's.

$\cancel{(1^* 0 1^* 0 1^*)^*} 1^*$

$1^* 00(1+00)^*$

Q. Fourth symbol from beginning is c. $\Sigma\{a,b,c\}$

$\Rightarrow (a+b+c)(a+b+c)(a+b+c)\,c\,(a+b+c)^*$

Q. $\Sigma=\{a,b\}$ even no. of a's.

$(b^* a b^* a b^*)^*$ $\boxed{b^* a a (b+aa)^*}$

Q. $\Sigma=\{a,b\}$ at most 3 a's.

$b^* + b^* a b^* + b^* a b^* a b^*$
$+ b^* a b^* a b^* a b^*$

Q- $a^{2n} b^{2m}$ n & m are +ve integers

$r = (aa)^* (bb)^*$

Q = length divisible by 2 but not 3

$\Sigma = \{a, b\}$

$r = (a+b)((a+b)^6)^* (a+b)^3 + (a+b)((a+b)^6)^* (a+b)$

Ed

$\Rightarrow (a+b)((a+b)^6)(a+b)(\wedge + (a+b)^2)$.

Q. Binary no., if they are even length, the no. rep. by them is even, otherwise the no. is odd if length is odd.

~~(K+0010)*~~

~~(00+01+~~

$r = (00+01+11+10)^* 1 + (00+01+11+10)^* (1+0)^*$
(0)

Q. String over {a, b, c} with a single c before which an even no. of a's and any no of b's in any order, after the c, there must be an odd no. of a's & any no. of bs in any order.

(*) even no. of a' and any b's.

$b^* (a b^* a b^*)^*$

(*) odd no. of a's.

$b^*(ab^*ab^*)^*ab^*$

$\Rightarrow$ $b^*(ab^*ab^*)^* c\ b^*(ab^*ab^*)^* ab^*$

Q. $a^n b^m c^k$ $\quad n+m = \text{odd}$ $\quad$ c is even.

a = odd $\quad$ b = even

b = odd $\quad$ a = even

$(aa)^* a (bb)^* (cc)^* + (aa)^* b (bb)^* (cc)^*$

Q. $a^n b^m c^k$ $\quad n \le 4 \quad m \ge 2 \quad k \le 2$

(*) Many conditions are possible

(*) How many to rep.

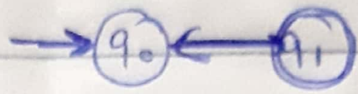
Q. String {a, b, c} in which at least one a appears in the first 3 symbols and at least one b appears in last four symbols.

$r = ((a+b+c)^+ (a+b+c)^+ a + (a+b+c)^+ a (a+b+c)^+$
$+ (a)(a+b+c)^+ (a+b+c)^+)(a+b+c)^*$
$((a+b+c)^+ (a+b+c)^+ (a+b+c)^+ b +$
$(a+b+c)^+ (a+b+c)^* b (a+b+c)^+)$
$(a+b+c)^+ b (a+b+c)^+ (a+b+c)^+ +$
$b (a+b+c)^+ (a+b+c)^+ (a+b+c)^+)$

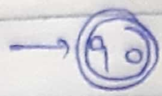
Regular expression to FA

I> $r = \phi$.

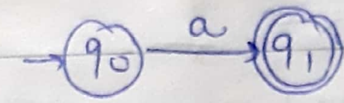
Not reaching final s...



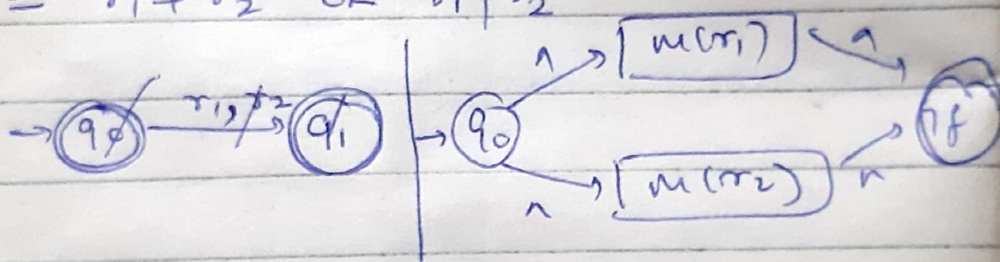
II> $r = \wedge$ OR ε OR λ



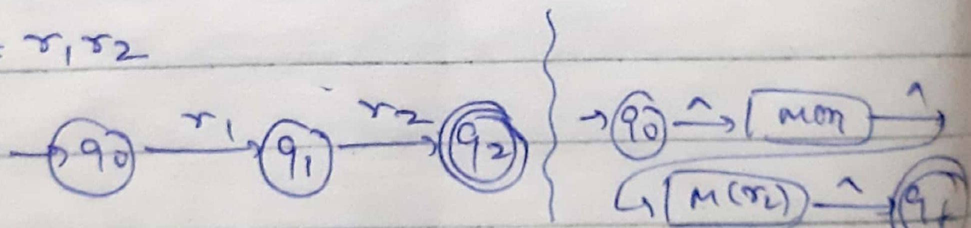
III> $r = a$



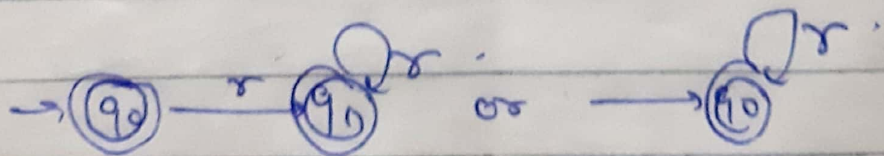
IV> $r = r_1 + r_2$ OR $r_1 | r_2$



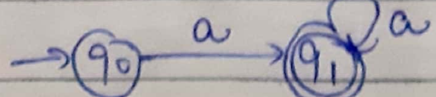
V> $r = r_1 r_2$



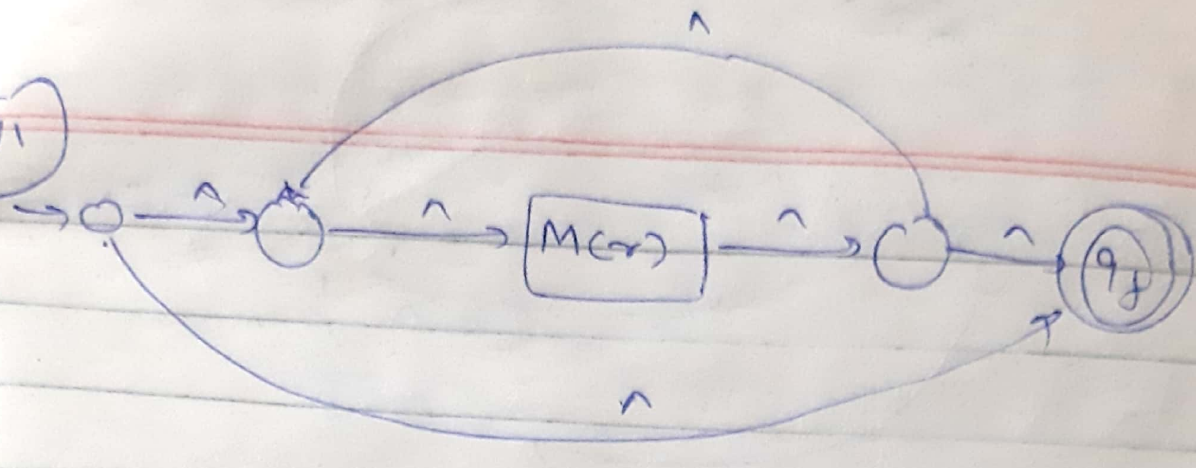
VI> $r = r^*$



VII> $r = a^+$

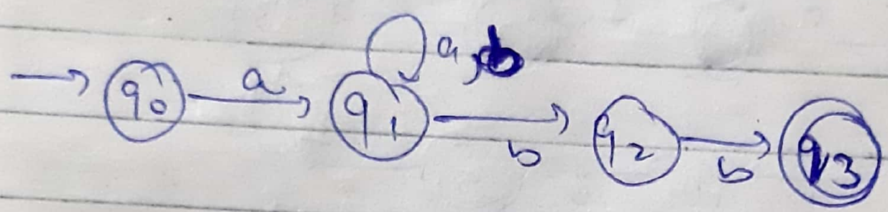


(vi)

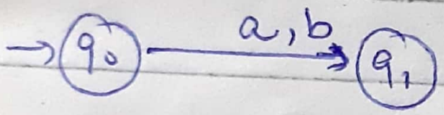


(vii) $r = a^+$

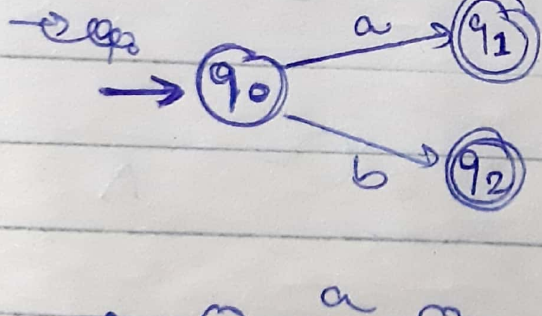
Q- $r = a \cdot (a+b)^* bb$. construct fa.

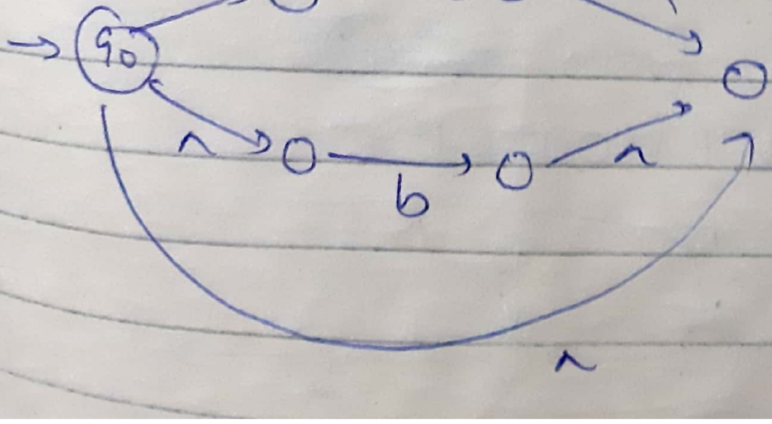


(1)

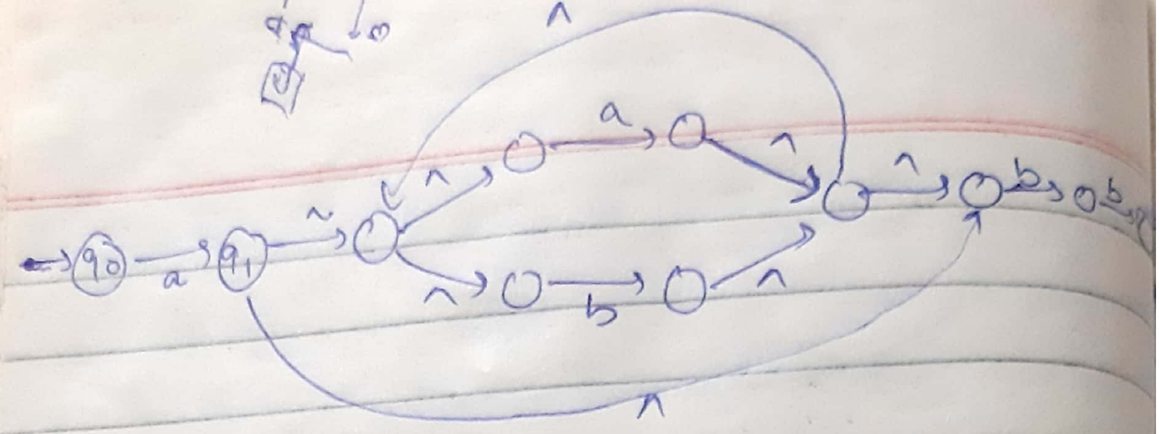


$(a+b)$.

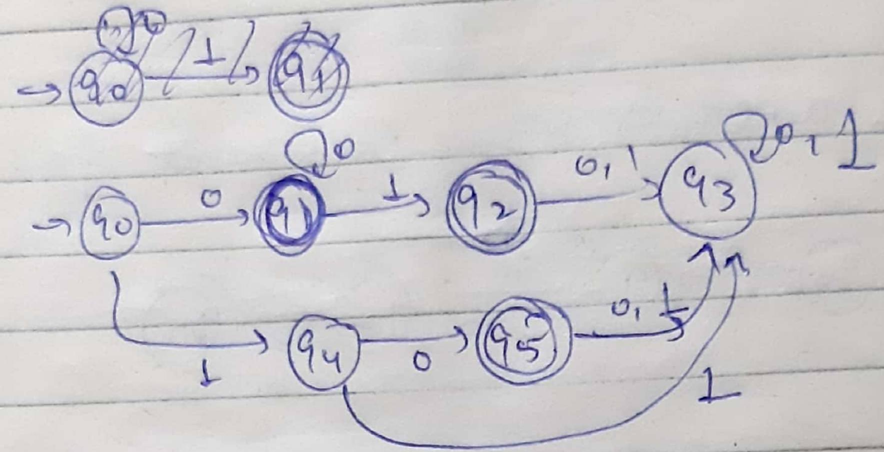


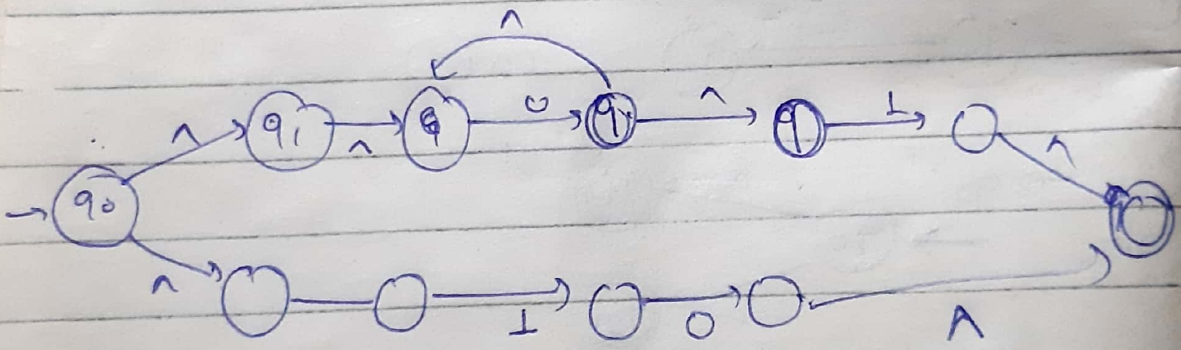


$(a+b)^*$

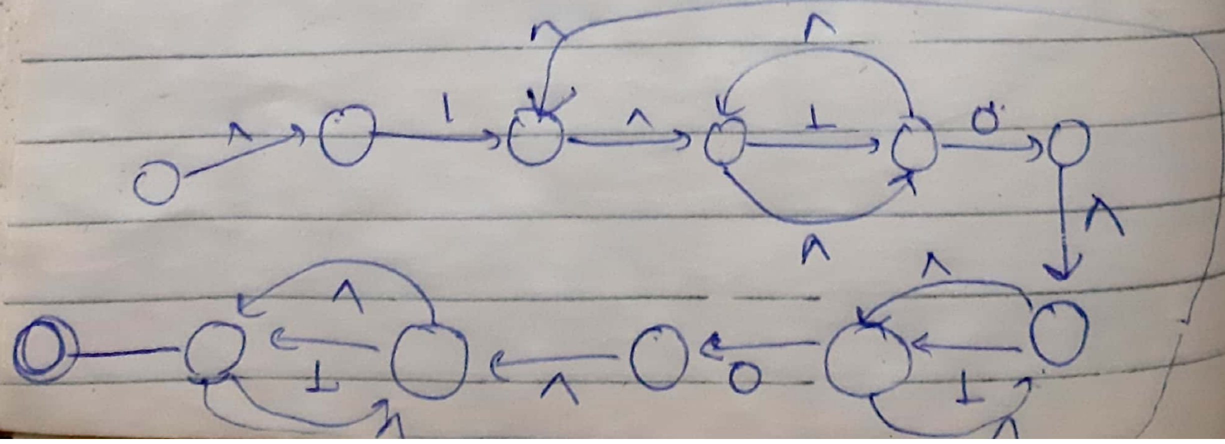


Q. Construct FA $r = 0^*1 + 10$



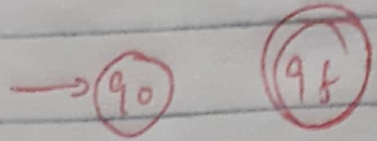


Q. Design NFA from
$r = 1(1^*01^*01^*)^*$

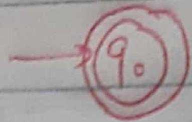


Comparative study of Regular expression.

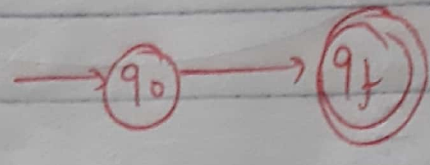
ϕ



ϵ



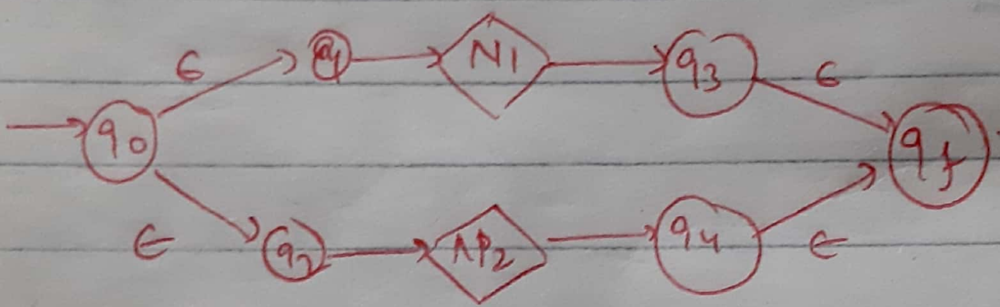
$\{a\}$



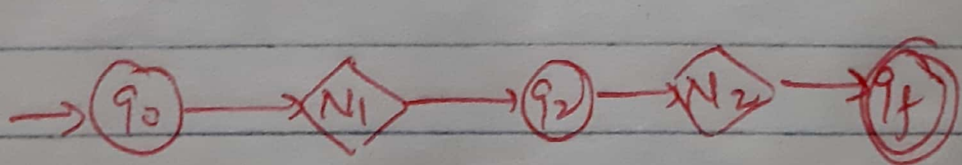
Every a in set in accep

$r_1 + r_2$

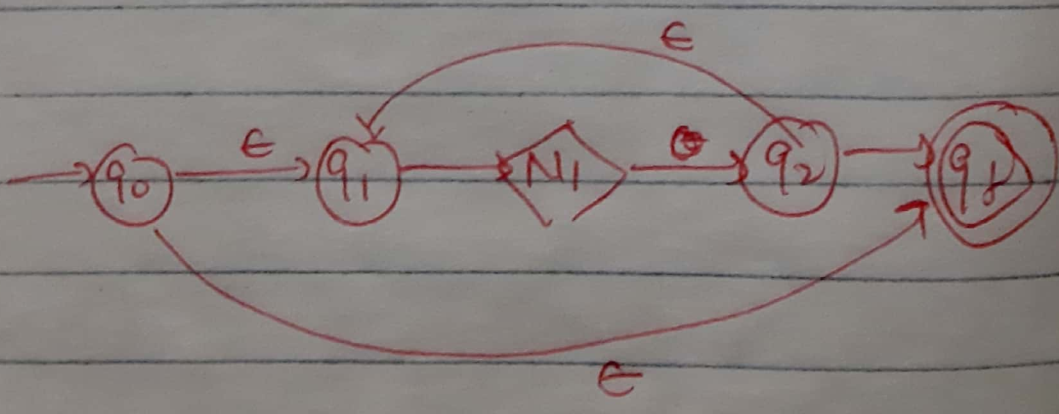
N_1 = FA accepting r_1
N_2 = FA accepting r_2



$r_1 r_2$



r^*



How to solve

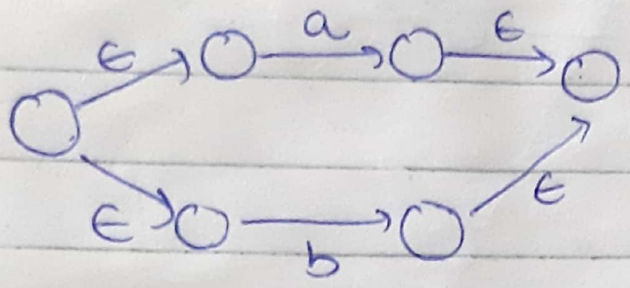
i) Make of +

ii) Make for ·

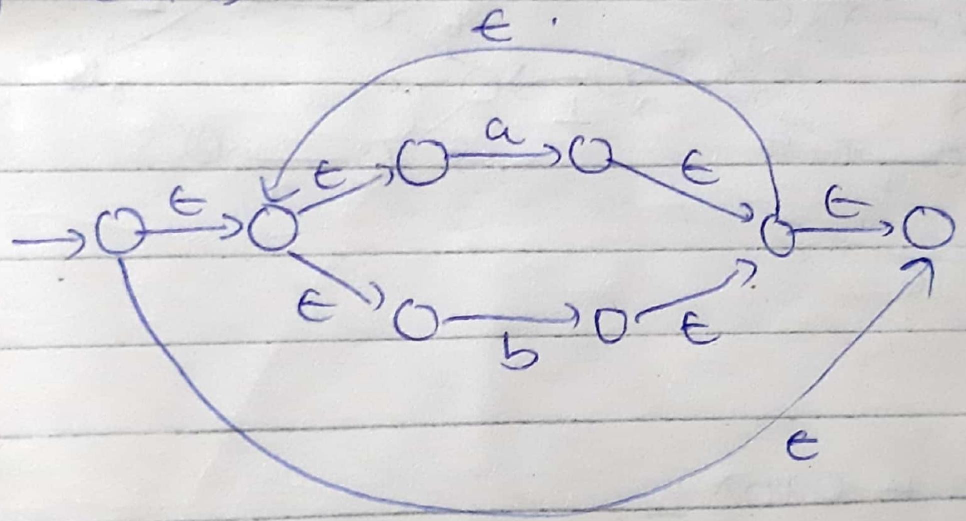
iii) Make for *

Q. $a \cdot (a+b)^* \cdot b \cdot b$

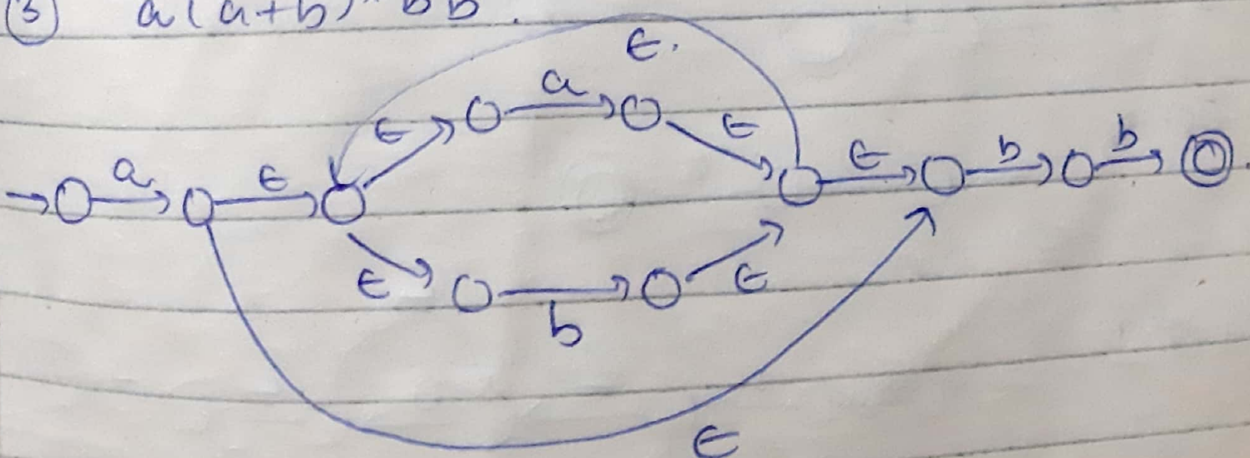
① (a+b)



② $(a+b)^*$

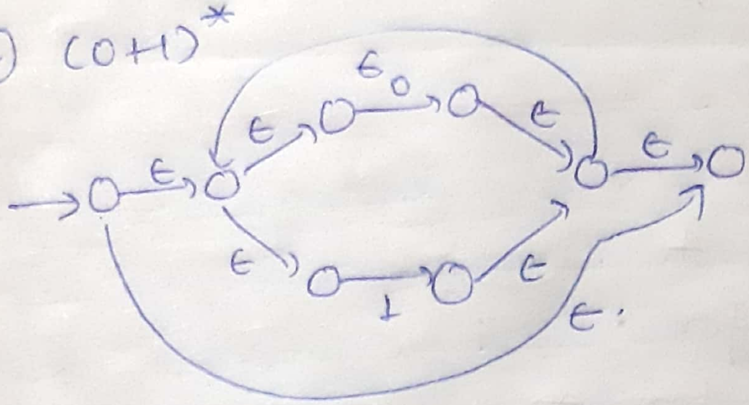


③ $a(a+b)^*bb$.

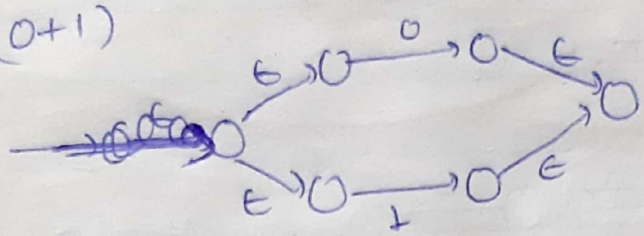


Q. $(0+1)^*1(0+1)$

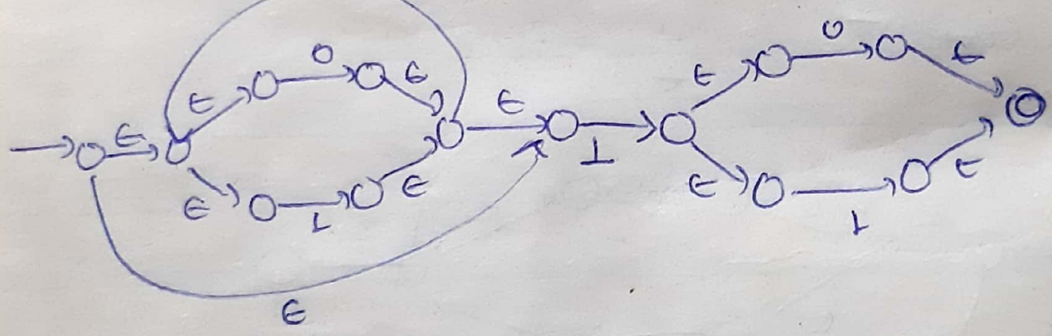
(a) $(0+1)^*$



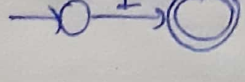
(b) $(0+1)$

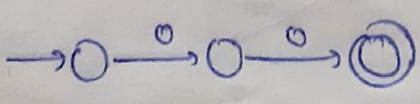


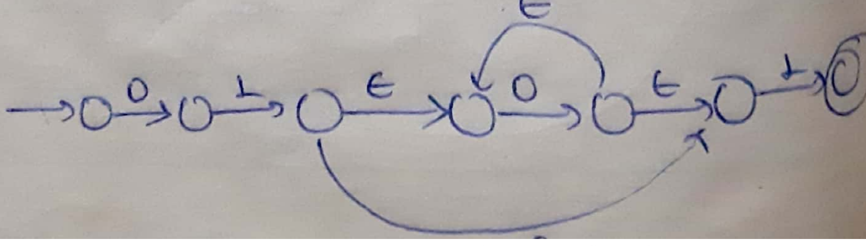
(c) $(0+1)^*1(0+1)$.



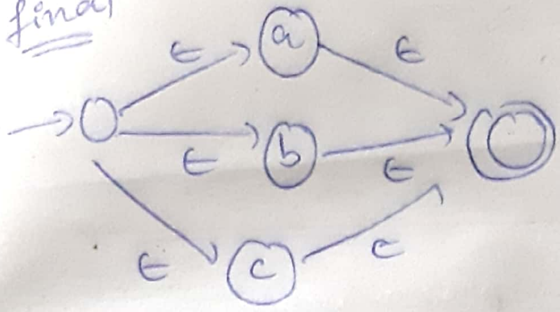
Q. $1+00+010^*1$.

(a) $1 \Rightarrow$ 

(b) $00 \Rightarrow$ 

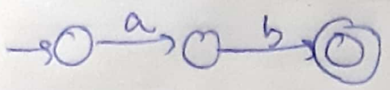
(c) 010^*1 

final

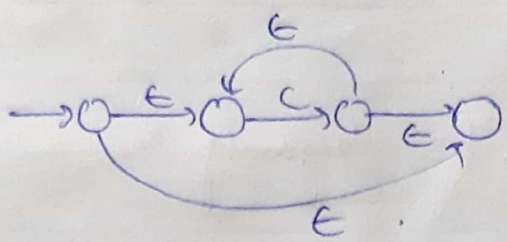


Q $r = (ab + c^*)^* b$

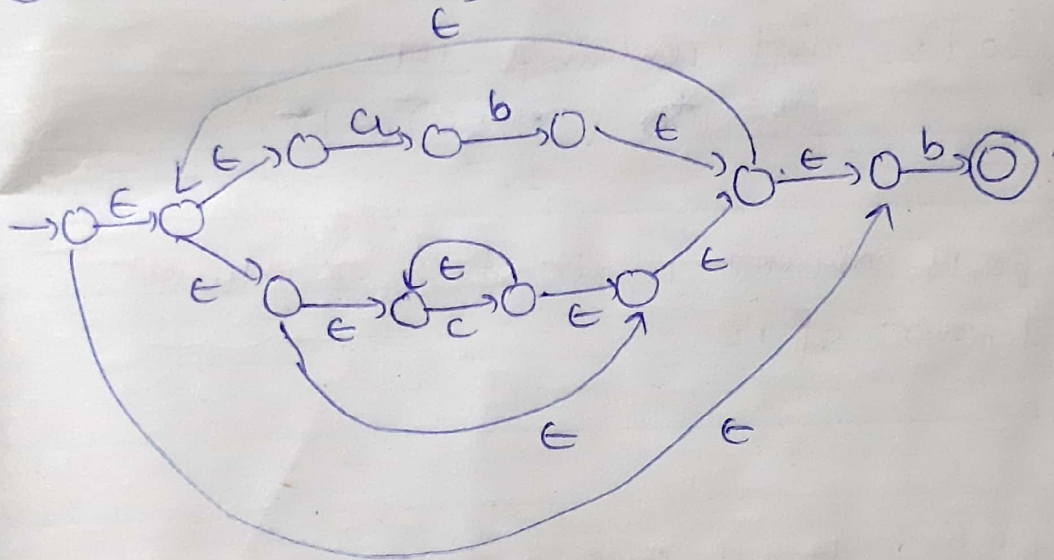
(a) ab



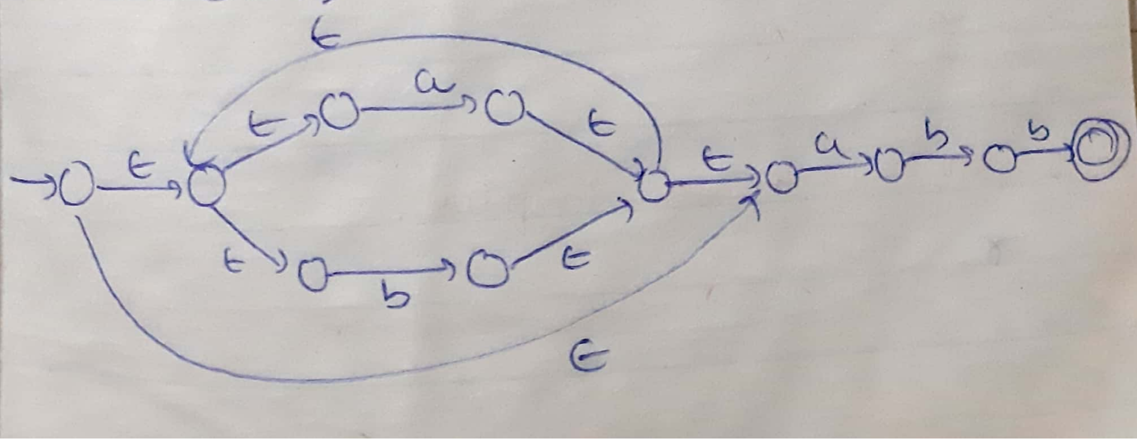
(b) c^*



(c) ~~(a+b~~ $(ab + c^*)^* b$



Q. $r = (a+b)^* abb$.



Questions of regular expression.

1. {0,1} atleast 2 occurrence of 1 b/w any two occurrence of 0

2. {a,b} ending bb begining a

$r = a^{+}(a+b)^{*}bb$.

3. {0,1} not containing 101

$0^{*}\cancel{(11^{*}0)}\ (0^{*}1^{*}00)^{*}0^{*}1^{*}$

4. {0,1} atmost 1 pair of 0's or atmost 1 pair of 1's.

5. {a,b} do not end cod double letter

$r = (a+b)^{*}(ab+ba)$

6. {a,b} b never triples.

$r =$

Scanned with CamScanner

7. {a, b} a is tripped or b is tripled.

8. Even a's & odd b's.

9. Odd a's & odd b's.

10. String not ending wd 01

$r = (0+1)^* (11 + 10 + 00)$

11. containing even 0's.

$r = (1^* 0 1^* 0 1^*)^*$

12. atmost two Occurrence of 00.

$r = (1 + 10)^* (\lambda + 00 + 000 + 001^* 00)(1 + 10)^*$

13. Not containing (10).

$re = 0^* \; 0^* 11^*$

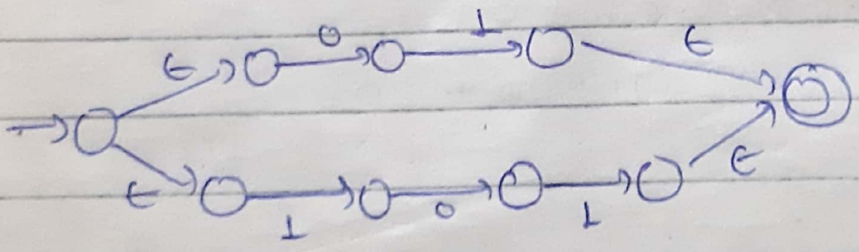
16. Binary string not containing 011

14. $L = \{a^n b^m, n \geq 1, m \geq 1, nm \geq 3\}$

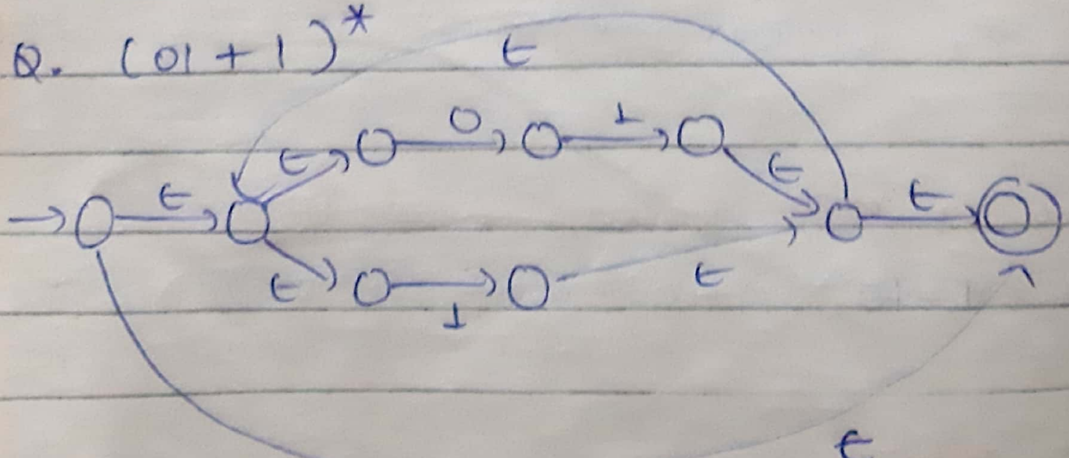
15. $L = \{ab^n w : n \geq 3, w \in \{a,b\}^+\}$

Regular exp. to NFA:

Q. 01 + 101.



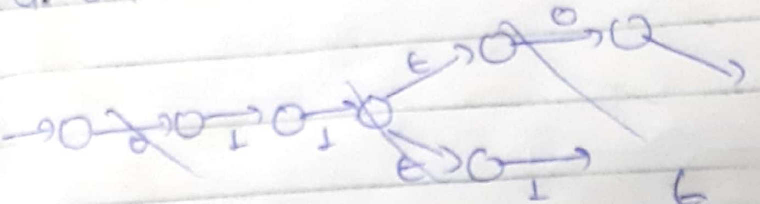
Q. $(01+1)^*$

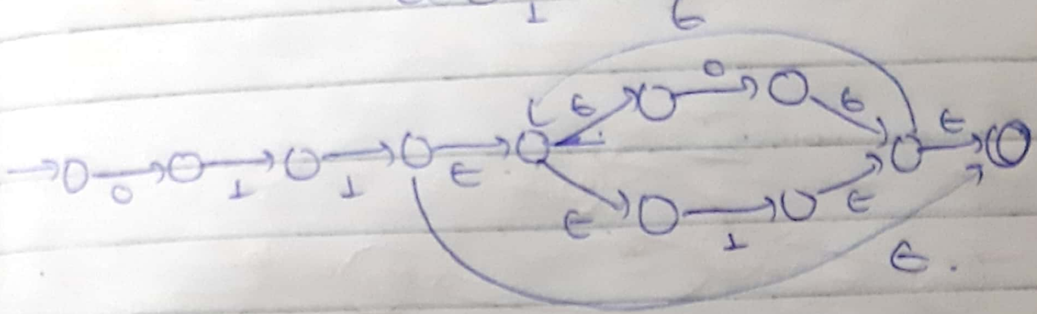


17. every 0 is followed by 11.

$r = (1^* (011)^* 0 1^*)^*$

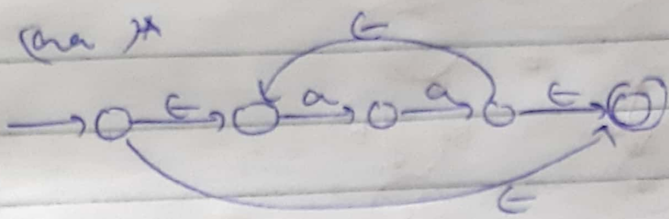
Q. $011(0+1)^*$



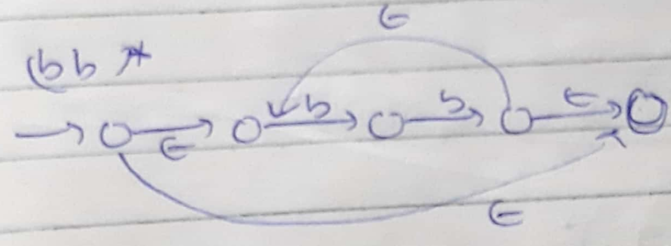


Q. $(aa)^*(bb)^* + a(aa)^* b(bb)^*$

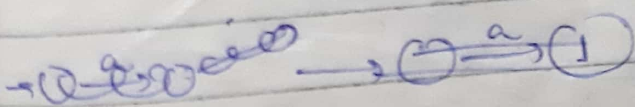
(1) $(aa)^*$

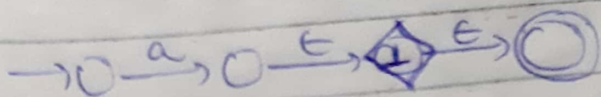


(2) $(bb)^*$

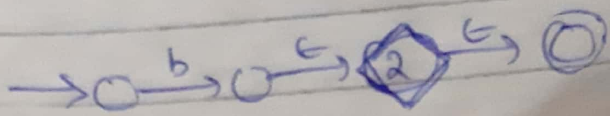


(3) $a(aa)^*$

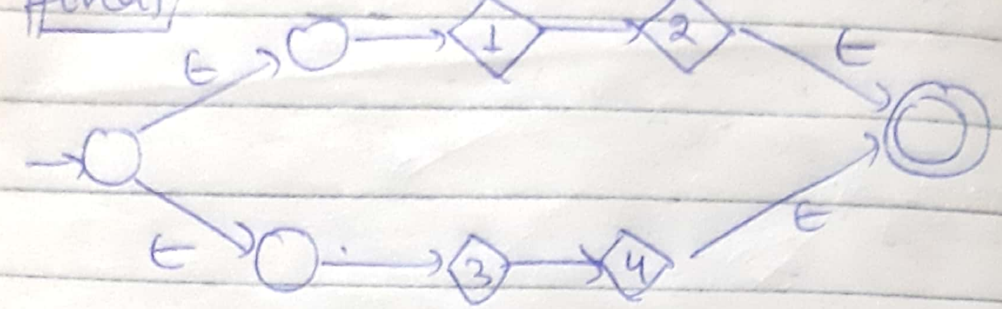




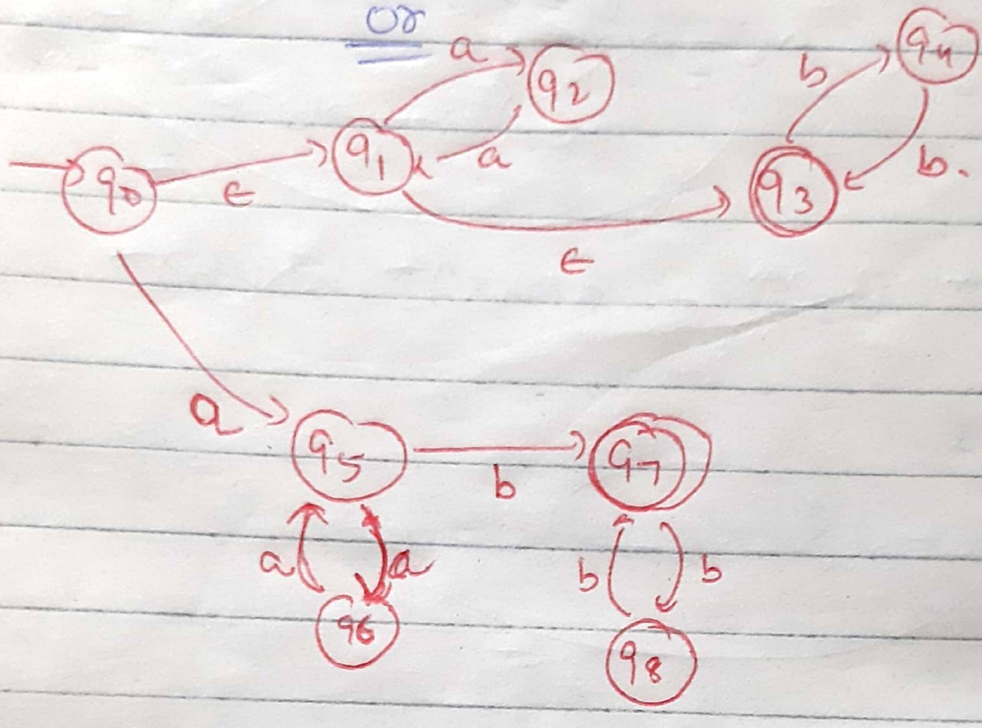
(4)



final

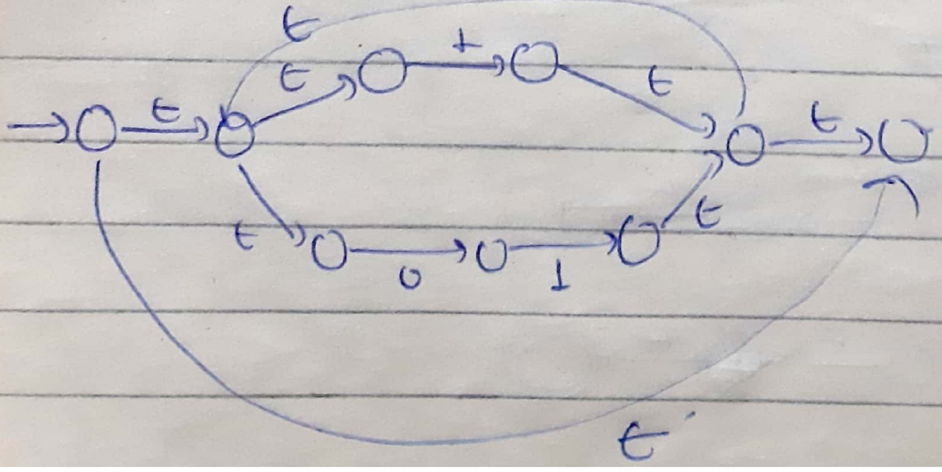


or

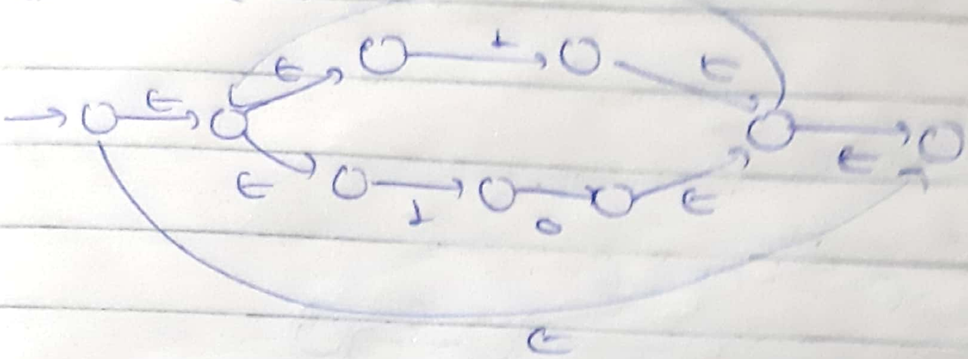


Q. $(1+(01))^* 00 (1+(10))^*$

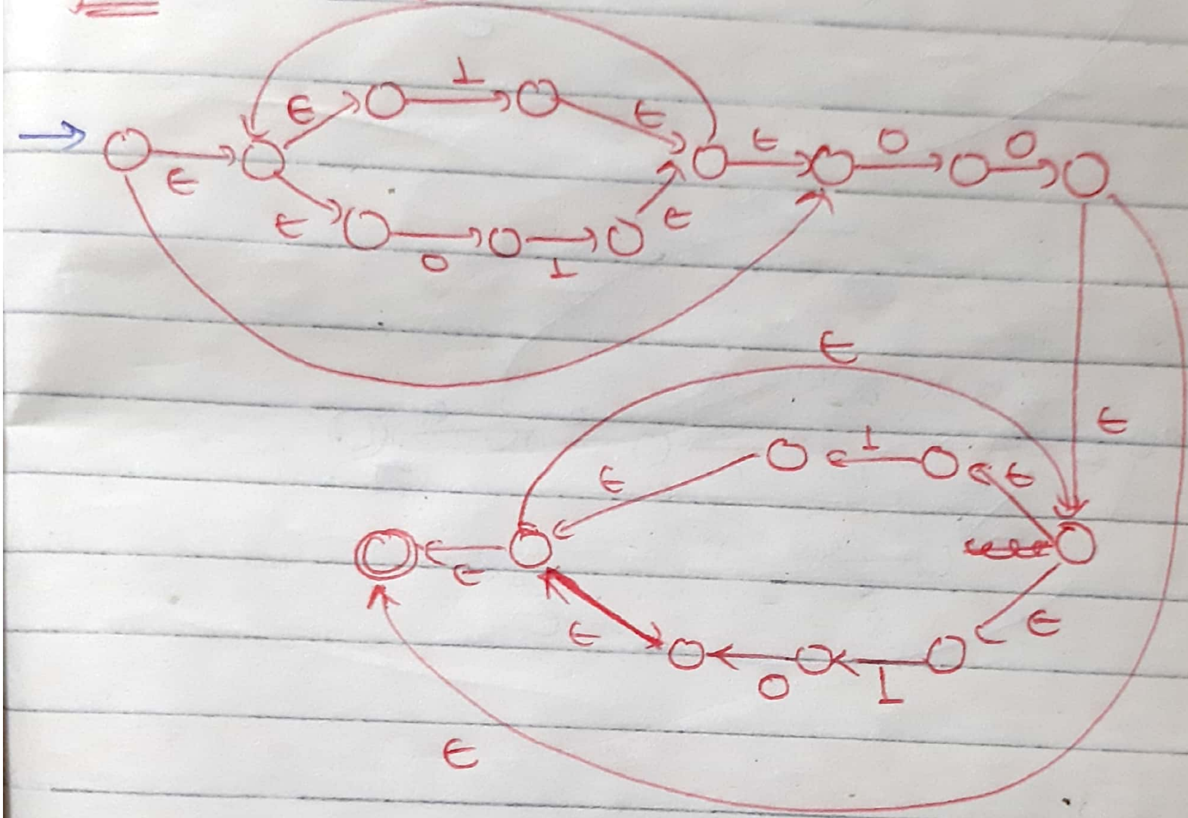
① $(1+(01))^*$



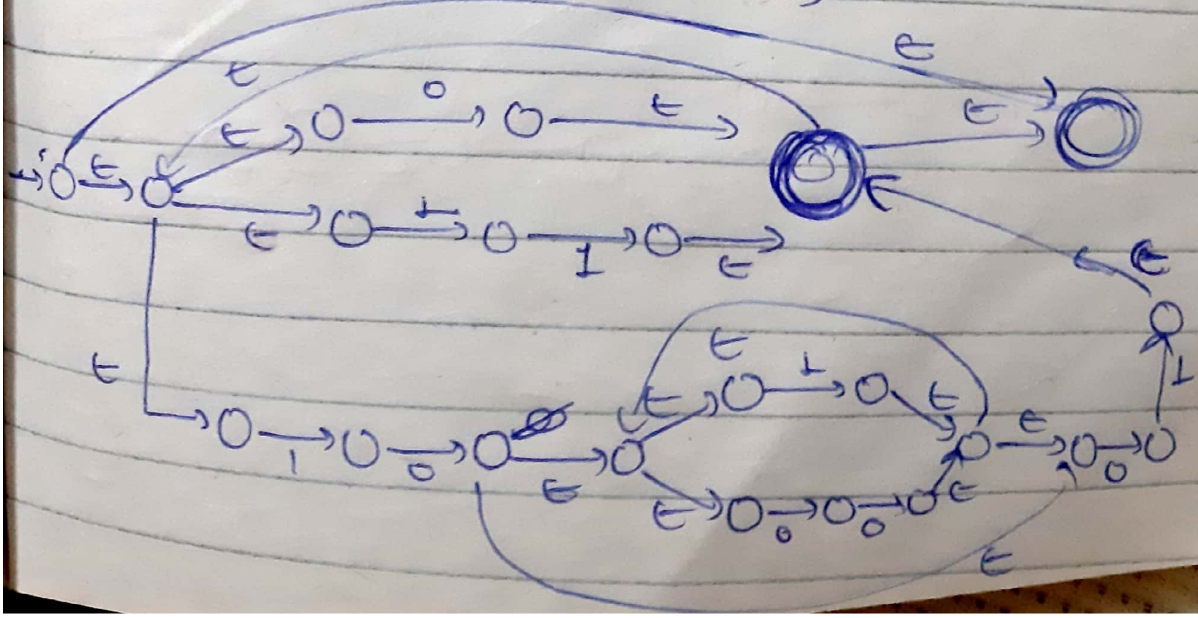
② $(1+(10))^*$



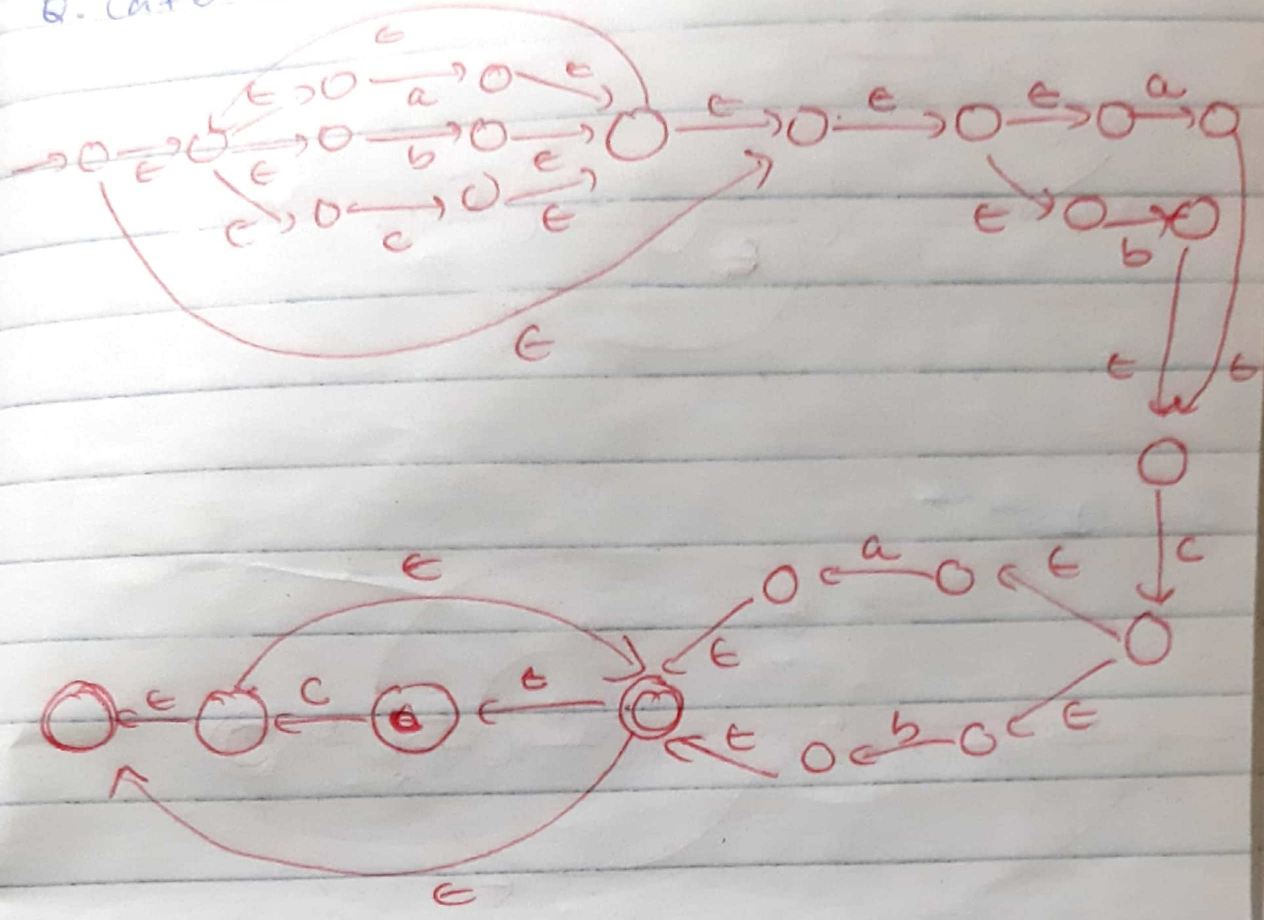
final



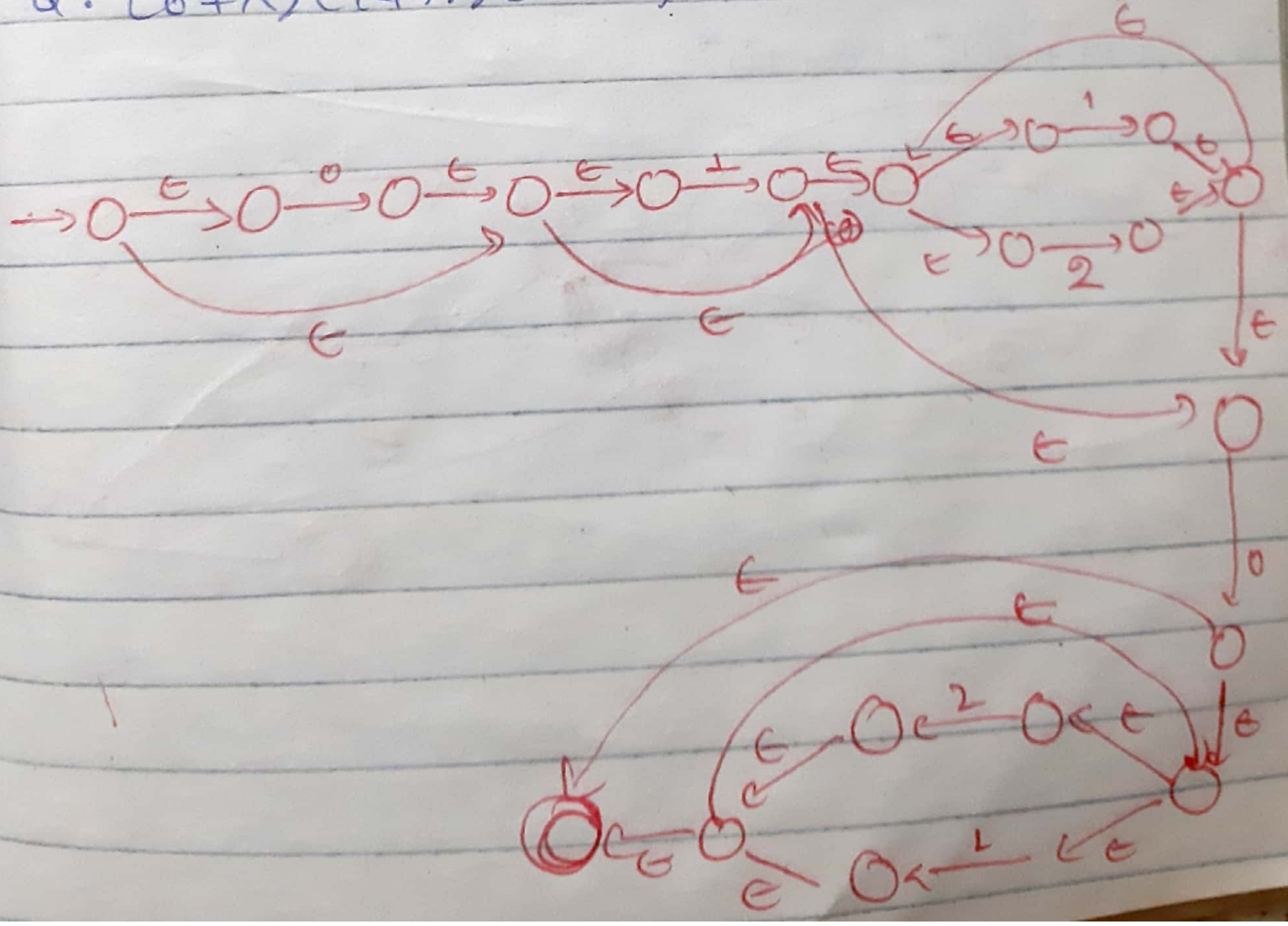
Q. $(0+11+10(1+00)^*01)^*$



Q. $(a+b+c)^* c (a+b) c (a+b)(c+\lambda)^*$



Q. $(0+\lambda)(1+\lambda)(1+2)^* 0 (2+1)^*$



Umang

Simplify RegEx Arden's Theorem

Date 25/02/2020

Convert FA to Regular expression

Identities for R.E (used for simplifying RegEx)

I_1 : $\phi + R = R$

I_2 : $\phi R = R\phi = \phi$

I_3 : $\wedge R = R\wedge = R$

I_4 : $\wedge^* = \wedge$ and $\phi^* = \wedge$

I_5 : $R + R = R$

I_6 : $R^* R^* = R^*$

I_7 : $RR^* = R^* R$

$I_8 = (R^*)^* = R^*$

$I_9 = \wedge + RR^* = R^* = \wedge + R^* R$

I_{10} : $(PQ)^* P = P(QP)^*$

I_{11} : $(P+Q)^* = (P^* Q^*)^* = (P^* + Q^*)^*$

I_{12} : $(P+Q)R = PR + QR$ and

$R(P+Q) = RP + RQ$

Ex: $r = (1 + 00^*1) + (1 + 00^*1)(0 + 10^*1)^*(0 + 10^*1)$

$= 0^*1\,(0 + 10^*1)^*$

RHS $\Rightarrow (1+00^*1) + (1+00^*1)(0+10^*1)^*(0+10^*1)$

$\Rightarrow (1+00^*1)(\wedge + (0+10^*1)^*(0+10^*1))$

$\Rightarrow (1+00^*1)(0+10^*1)^*$

$\Rightarrow (\wedge + 00^*)1\,(0+10^*1)^*$

$\Rightarrow 0^*1\,(0+10^*1)^*$

P 3 ...

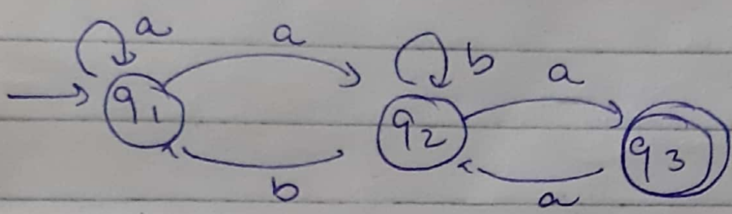
Let P and Q two RegEx over Σ, and P doesn't contain NULL

$R = Q + RP$

Then we have unique sol. to this eq.

$R = QP^*$ Arden's theorem.

Q. Consider the transition system given in figure. Prove that string recognizes R.



$q_3 = (a + a(b+aa)^* b)^* a(b+aa)^* a$

(final state)

Sol. $q_1 = q_1 a + q_2 b + \Lambda$ —(i)

$q_2 = q_2 b + q_1 a + q_3 a$ —(ii)

$q_3 = q_2 a$ —(iii)

$\Rightarrow$ $q_1 = q_1 a + q_2 b + \Lambda$

$q_2 = q_2 b + q_1 a + q_2 aa$

$\Rightarrow$ ~~$q_2 = q_2 b + q$~~

$q_2 = \underline{q_1 a} + q_2 \underline{(b+aa)}$.

$R = Q + RP$

$R = QP^*$

$q_2 = q_1 a (b+aa)^*$

$q_1 = q_1 a + q_1 a(b+aa)^* b + \Lambda.$

$q_1 = q_1 (a + a(b+aa)^* b) + \Lambda.$

$R = R \; P + Q$

$\cancel{q_1 = q_1 a(\Lambda + (b+aa)^* b) + \Lambda.}$

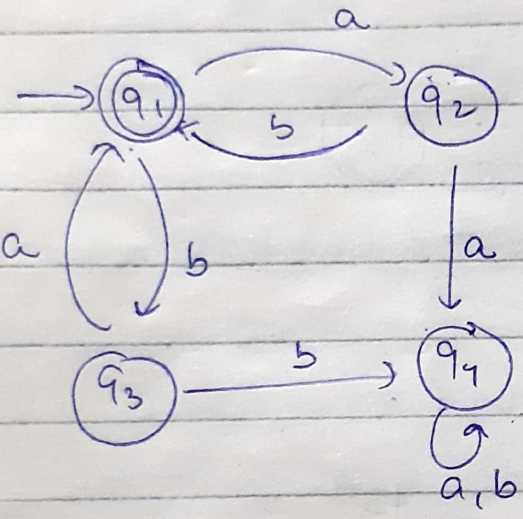
$q_1 = \Lambda(a + a(b+aa)^* b)^*$

$q_1 = (a + a(b+aa)^* b)^*$

$q_2 = (a + a(b+aa)^* b)^* a(b+aa)^*$

$q_3 = (a + a(b+aa)^* b)^* a(b+aa)^* a$

Q.



$q_1 = \Lambda + q_3 a + q_2 b.$

$q_2 = q_1 a$

$q_3 = q_1 b$

$q_4 = q_4 a + q_4 b + q_2 a + q_3 b.$

$q_4 = q_4 (a+b) + q_2 a + q_3 b$

$q_4 = q_4 (a+b) + q_1 aa + q_1 bb.$

$q_4 = q_4 (a+b) + q_1 (aa+bb).$

$R = R \; P + Q.$

$\boxed{q_4 = q_1 (aa+bb)(a+b)^*}$

$q_1 = \wedge + q_1 ba + q_1 ab$.

$q_1 = q_1 (ba + ab) + \wedge$

$R = R \quad P \quad + Q$

$R = QP^*$

$q_1 = \wedge (ba + ab)^*$

$\boxed{q_1 = (ba + ab)^*}$ Ans

$\boxed{q_4 = (ba + ab)^* (aa + bb)(a+b)^*}$

$\boxed{q_2 = (ba + ab)^* a}$

$\boxed{q_3 = (ba + ab)^* b}$.

ARDEN'S THEOREM.

$R = Q + RP$

$= Q + (Q + RP)P$

$= Q + QP + RP^2$

$= Q + QP + (Q + RP)P^2$

$= Q + QP + QP^2 + RP^3$

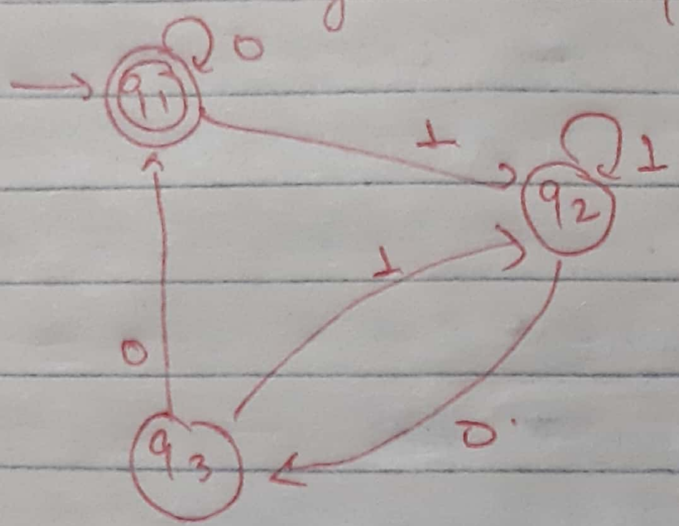
$= Q + QP + QP^2 + QP^3 + \cdots$

$= Q (1 + P + P^2 + P^3 \cdots)$

$= Q (\wedge + P + P^2 + P^3 + \cdots)$

$R = QP^*$

Q. Construct Regular ex. for.



$\Rightarrow q_1 = q_1 0 + q_3 0 + \Lambda$

$q_2 = q_2 1 + q_3 1 + q_1 1$

$q_3 = q_2 0$

$\Rightarrow q_2 = q_2 1 + q_2 0 1 + q_1 1.$

$\Rightarrow q_2 = q_2 (1 + 01) + q_1 1$

$R = RP + Q.$

$R = QP^*$

$q_2 = q_1 1 (1+01)^*$

$q_3 = q_1 1 (1+01)^* 0.$

$\Rightarrow q_1 = q_1 0 + q_1 1 (1+01)^* 0 0 + \Lambda.$

$q_1 = q_1 (0 + 1(1+01)^* 00) + \Lambda$

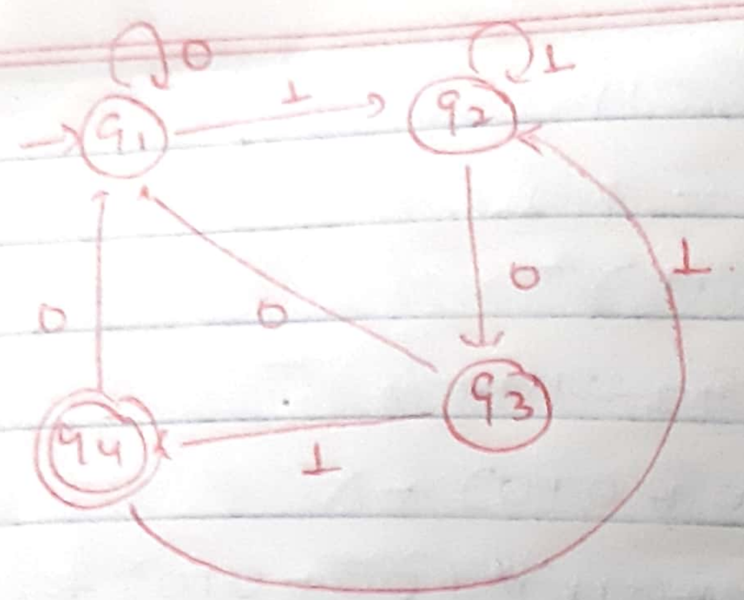
$R = R P + Q.$

$R = QP^*$

$q_1 = \Lambda (0 + 1(1+01)^* 00)^*$

$q_1 = (0 + 1(1+01)^* 00)^*.$

Q.



$q_1 = q_1 0 + q_4 0 + q_3 0 + \Lambda$.

$q_2 = q_1 1 + q_2 1 + q_4 1$

$q_3 = q_2 0$.

$q_4 = q_3 1$

$q_4 = q_2 0 1$

$q_2 = q_1 1 + q_2 1 + q_2 011.$

$q_2 = q_2(1+011) + q_1 1.$

$q_2 = q_1 1 (1+011)^*$

$q_1 = q_1 0 + q_2 00 + q_2 010 + \Lambda$

$q_1 = q_1 0 + q_2(00+010) + \Lambda.$

$q_1 = q_1 0 + q_1 1 (1+011)^*(00+010) + \Lambda.$

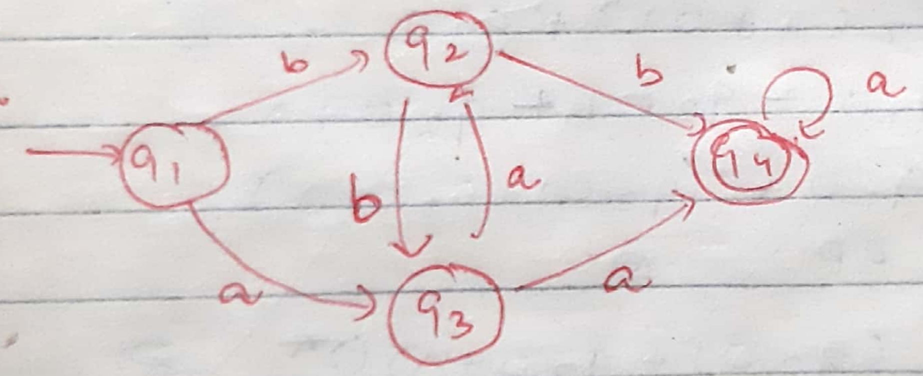
$q_1 = q_1 (0 + 1(1+011)^*(00+010)) + \Lambda$

$q_1 = (0+1(1+011)^*(00+010))^*$

$q_2 = (0+1(1+011)^*(00+010))^* 1 (1+011)^*$

$q_4 = (0+1(1+011)^*(00+010))^* 1 (1+011)^* 0$

Q.



$q_1 = \Lambda$

$q_2 = q_1 b + q_3 a$

$q_3 = q_1 a + q_2 b$

$q_4 = q_4 a + q_3 a + q_2 b.$

$q_2 = b + q_3 a$

$q_3 = a + q_2 b$

$\Rightarrow q_3 = a + (b + q_3 a) b$

$q_3 = q_3 ab + bb + a$

$R = RP + Q$

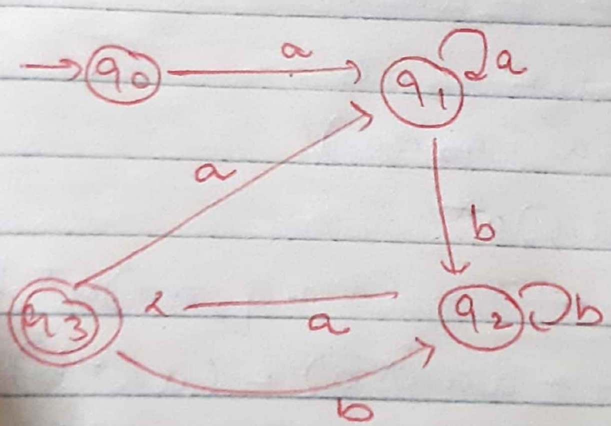
$q_3 = (bb+a)(ab)^*$

$q_2 = b + (bb+a)(ab)^* a.$

$q_4 = q_4 a + (bb+a)(ab)^* a +$
$(b + (bb+a)(ab)^* a) b.$

$q_4 = [(bb+a)(ab)^* a + (b + (bb+a)(ab)^* a) b] a^*$

Q.



$q_0 = \wedge$

$q_1 = \cancel{q_1 a + q_3 a} + q_0 a$

$q_2 = q_2 b + q_1$

$q_0 = \wedge$

$q_1 = q_1 a + q_0 a + q_3 a$

$q_2 = q_2 b + q_1 b + q_3 b$

$q_3 = q_2 a$

$\Rightarrow q_1 = q_1 a + a + q_2 a$

$q_2 = q_2 b + q_1 b + q_2 a.$

$q_1 = (a + q_2 a) a^*$

$q_2 = q_2 b + q_2 a + (a + q_2 a) a^* b$.

$q_2 = q_2 b + q_2 a + a a^* b + q_2 a a^* b$

$q_2 = q_2 (a + b + a a^* b) + a a^* b$

$q_2 = a a^* b (a + b + a a^* b)^*$.

$q_3 = q_2 a$

$q_3 = a a^* b (a + b + a a^* b)^* a$

$q_0 = \Lambda$

$q_1 = q_0 a + q_1 a + q_3 a$

$q_2 = q_1 b + q_3 b + q_2 b$

$q_3 = q_2 a$

$q_1 = a + q_1 a + q_2 a a$.

$q_2 = q_1 b + q_2 b + q_2 a b$.

$q_1 = q_1 a + (a + q_2 a a)$

$q_1 = (a + q_2 a a) a^*$

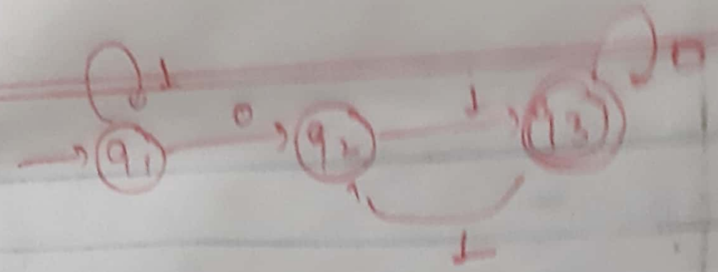
$q_2 = q_2 (b + ab) + (a + q_2 a a) a^* b$.

$q_2 = q_2 (b + ab + a a a^* b) + (a a^* b)$

$q_2 = (a a^* b)(b + ab + a a a^* b)^*$

$q_3 = (a a^* b)(b + ab + a a a^* b)^* a$

Q.



$q_1 = \wedge + q_1 1$

$q_2 = q_1 0 + q_3 1$

$q_3 = q_3 0 + q_2 1$

$q_1 = 1^*$

$q_2 = 1^*0 + q_3 1$

$q_3 = q_3 0 + (1^*0 + q_3 1)1$

$q_3 = q_3 0 + q_3 11 + 1^*01$

$q_3 = q_3(0+11) + 1^*01$

$q_3 = 1^*01(0+11)^*$

Q. $(0+1)^*10(0+1)^* + (0+1)^*11(0+1)^*$

$(0+1)^*(10(0+1)^* + 11(0+1)^*)$

$(0+1)^*(10+11)(0+1)^*$

$(0+1)^*1(0+1)(0+1)^*$.

Q. Prove or disprove each

a) $(R+S)^* = R^* + S^*$

No., this is not true.

b). $(RS + R)^*R = R(SR+R)^*$

$\Rightarrow (RS+R)^*R = (RSR+RR)^*$

$\Rightarrow R(SR+R)^*$

Date:- 29/02/2020

PROVE THAT A LANGUAGE IS REGULAR OR NOT

PUMPING LEMMA FOR REGULAR EXPRESSION

- This theorem can be used to prove that certain sets are not regular

Step 1 - Assume that L is regular. Let n be the no. of states in corresponding FA.

Step 2 - Choose a string w such that length of $|w| \geq n$

Use pumping lemma to write $w = xyz$ with length of $|xy| \leq n$ & $|y| > 0$

Step 3 - Find a suitable integer i such that $xy^iz \notin L$

(we are pumping the value of y)

This contradicts our assumption. Hence L is not regular

Q. Show that $L = \{a^m b^m \mid m \geq 1\}$ is not regular

1. Let L is regular and n is no. of states.

2. $w = a^m b^m$ $\quad |w| = 2m \geq n$

$w = ab$

$x = -$ $\quad y = a$ $\quad z = b$.

$xy^iz \notin L$ $\quad$ Case 1

$a^i b \notin L$

$ab \in L$

$a^2 b \notin L$

If $y = ab$ then $(ab)^0$ ab abab ababab

$(ab)^1$ $(ab)^2$ $(ab)^3$

Case 2: $x = a$ $y = b$ $z = -$

$xy^i z \notin L$

$ab^0 \notin L$

$ab^2 \notin L$

Q. Show that the set $L = \{ a^{i^2} \mid i \geq 1 \}$ is not regular.

1. Let L is regular expression & n is no. of states

2. $w = a^{i^2}$ $|w| = i^2 \geq n$.

$w = aaaa$

$x = aa$ $y = a$ $z = a$

$xy^i z \notin L$

$aa a^0 a \notin L$

$aaaa \in L$

$aaaaa \notin L$

Case $w = a^{j^2}$

$w = a \cdot a^{j^2 - 1}$

$= a^i (a^{j^2 - 1})$

$y = a$ $z = a^{j^2 - 1}$

$w = a\, a^{j^2 - 1} \Rightarrow a^{j^2} \in L$

$i = 2$

$a^2 a^{j^2 - 1} \notin L$

$\boxed{a^{j^2 + 1} \notin L}$

Date: 2/08/2020

GRAMMER

S → <noun><verb><adverb>

S → <noun><verb>

noun → Umang

verb → ate

adverb → quickly

English Grammer

$G = (V_n, \Sigma, P, S)$

V_n = Non terminals / variables

Σ = Finite set of Terminals.

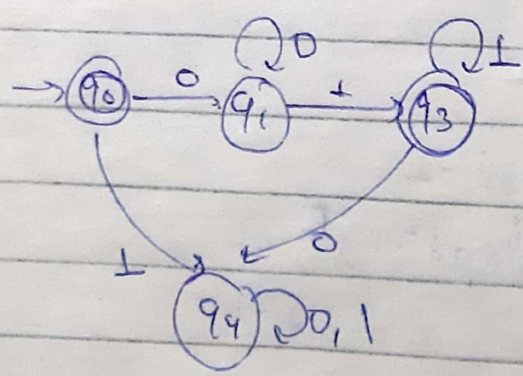
P = set of productions.

S = Starting stage / symbol, special symbol.

Q. if $G = (\{S\}, \{0,1\}, \{S \rightarrow 0S1, S \rightarrow \wedge\}, S)$

Find L(G)

DFA =



$\boxed{0^n 1^m}$

X

cos same no. of 01

∴ non possible

$L(G) = \{0^n 1^n \mid n >= 0\}$

0 S 1

& 0 S 1 — 0 0 S 1 1 — $0^n 1^n$

(01) — 0 0 0 S 1 1 1

Q. if $G = \{\{S\}, \{a\}, \{S \to SS\}, S\}$,

find $L(G)$ — aa

$S \to aa$ — aaa — a

$S \to aSS$ — aaa

$S \to aaSS$ — aaaa

$S \to SSSS$

$S \to SS$ — SS — S^{2^n}

$S \to SSSS$ — SS SS

$S \to SSS$ — SS SS SS SS SS

$L = \{\phi\}$

Q. let $G = \{\{S, C\}, \{a, b\}, P, S\}$

where P consist of

$S \to aCa$

$C \to aCa \mid b$.

find $L(G)$.

$S \to aCa$

$S \to aba$ ✓

$S \to aaCaa \Rightarrow aabaa$

$S \to aaaCaaa \to \boxed{aaabaaa}$

$L = \{a^n b a^n \mid n >= 1\}$

asa aaa
asa abab.
absba (aabaa) as
abasab abb

Q. if G is S → aS | bS | a | b. find L(G)

S → aa

S → ab.

S → ba

S → bb.

S → aaS → aaa → aab

→ aaa

→ abS → aba → abb

$L(G) = \{ (a+b)^+ \}$

$= \{ (a+b)^* - \Lambda \}$

aa bb

Q. Let L be set of all palindrome over (a,b)
Construct a grammar G. generating L.

Sol:

- Λ ✓
- a ✓
- b ✓
- aa
- ab ✗
- bb ✓
- abba

S → Λ

S → ~~CAC~~ CAAAC | ACCA

C → a | C

A → b. | A

$\boxed{S \to \Lambda \mid a \mid b \mid aSa \mid bSb}$

abba
baab.

sab

Q. construct a Grammar Generating

$L = \{ w c w^T \mid w \in (a,b)^* \}$

ex: $w = ab$

$w^T = ba$

$L = c$

$= aca$

$= aacaa$

$= abcba$

$= bcb$

$= bacab$

$S \to c \mid aSa \mid bSa$

$S \to c$

$\to K \to A\,K\,A \mid \ldots \mid c$

asa

aab ...

bSb

$A \to a$

$B \to b$

$\to \boxed{ACA}$

Aca

ACA

$A \to$

Date: 3|3|2020

Q. Find Grammar

$L = \{ a^n b^n c^i \mid n \geq 1,\ i \geq 0 \}$

$S \to aSb \mid ab$

$A \to SK$

$K \to KK \mid c \mid \wedge$

$KK \mid c \mid \wedge$

$S \to A$

$S \to Sc$

$A \to ab$

$A \to aAb$

Q. G. P = $S \rightarrow aSb$

$S \rightarrow ab$

$L = \{a^n b^n \mid n \geq 1\}$

Q. $L = \{wcw^R \mid w \in (a,b)^*\}$

→ $S \rightarrow aSa$

$S \rightarrow bSb$

$S \rightarrow c$

Q. Grammar for pallindrome binary String.

$S \rightarrow 0S0$

$S \rightarrow 1S1$

$S \rightarrow 0 / 1 \mid \wedge$.

Q. Grammar for $r = 0^*1(0+1)^*$.

$S \rightarrow 1 \mid \cancel{A1}\ 0SB$

$B \rightarrow 0B \mid 1B \mid 0 \mid 1 \mid \wedge$.

Q. Grammar that generate equal no. of a's & b's.

$S \rightarrow SaSbS \mid \cancel{b}SaS \mid \wedge$.

Q. Any combination of a & b except null

$S \rightarrow aS \mid bS \mid a \mid b \mid$

Q. Grammar for $(a+b)^* aa (a+b)^*$

$S \rightarrow aS \mid bS \mid a \mid b \mid \wedge$

$A \rightarrow SaaS$

Q. Grammar for $L(G) = \{ ww^T \mid w \in (a,b)^* \}$

$S \rightarrow aSa \mid bSb \mid \wedge$

Q. Grammar for $L(G) = \{ ab (bbaa)^n bba (ba)^n$
$n \geq 0$.

~~$S \rightarrow abA\ bba\ C$~~
~~$A \rightarrow bbaaA \mid \wedge$.~~
~~$C = baC \mid \wedge$.~~

$S \rightarrow abK$

$K \rightarrow bbaaKba \mid bba$

Q. Grammar for language

$L = \{ (0^n 1^n \mid n \geq 0) \cup (1^n 0^n \mid n \geq 0 \}$.

$S \rightarrow 0S1 \mid \wedge$

$A \rightarrow 1A0 \mid \wedge$

✓ $K \rightarrow S \mid A$

Q. Grammar, all the string with exactly 1a

$S \rightarrow KaK \mid a$

$K \rightarrow bK \mid \wedge$

Q. Grammar for all strings atleast 1 a

$S \rightarrow kak$

$k \rightarrow \wedge | ak | bk$

Q. Grammar for atleast 3 a's.

$S \rightarrow kakakak$

$k \rightarrow \wedge | ak | bk$.

Q. Language for

$S \rightarrow aA$

$A \rightarrow bS$

$S \rightarrow \epsilon$

$L(G) = \{(ab)^n \mid n \geq 0\}$

Q. Grammar for $L = \{a^n b^n c^m d^m \mid n >= 1\ m \geq 1\}$

$S \rightarrow aSb | ab$

$G \rightarrow cGd | cd$

$A \rightarrow SG$

Q. Grammar for $L = \{a^n b^m c^m d^n \mid n, m \geq 1\}$

$S \rightarrow aSd | aAd$

$A \rightarrow bAc | bc$

Q. Grammar for $L = \{a^n b^{2n} \mid n \geq 0\}$

$S \rightarrow aSbb | \wedge$

Q. Grammar for $L = \{a^{2n} b^m \mid n \geq 0, m \geq 0\}$

$S \rightarrow AB$

$A \rightarrow aaA \mid \wedge$

$B \rightarrow bB \mid \wedge$

Q. $L = \{(a^n b^n c^m d^m \mid n \geq 1\ m \geq 1) \cup$
$(a^n b^m c^m d^n \mid n \geq 1, m \geq 1)\}$.

$S \rightarrow A \mid B$.

$A \rightarrow PQ$

$P \rightarrow aPb \mid ab$

$Q \rightarrow cQd \mid cd$.

$B \rightarrow aBd \mid aRd$

$R \rightarrow bRc \mid bc$

Q. Grammar for $L = \{0^i 1^j 2^k \mid k \leq i \text{ or } k \leq j\}$

- let it be 2 grammars.

$L_1 = \{0^i 1^j 2^k \mid k \leq i\}$

$L_2 = \{0^i 1^j 2^k \mid k \leq j\}$

$\left.\begin{array}{l} S \rightarrow 0S2 \mid K \mid 0S \\ K \rightarrow 1K \mid 1 \end{array}\right] L_1$

$\left.\begin{array}{l} R \rightarrow Q1P2 \mid Q1P \mid Q \\ P \rightarrow 1P2 \mid 1P \mid \wedge \\ Q \rightarrow 0Q \mid \wedge \end{array}\right] L_2$

$M \rightarrow L1 \mid L2$

Q. $L = \{a^n b^{2n} c^m \mid n, m \geq 0\}$.

$S \rightarrow AB$

$A \rightarrow aAbb \mid \Lambda$.

$B \rightarrow cB \mid \Lambda$

Q. $L = \{a^i b^j c^k \mid i = j+k\}$

$$\boxed{\begin{array}{l} S \rightarrow aSc \mid aBc \\ B \rightarrow bBc \mid bc \end{array}}$$

Q. $L = \{a^i b^j \mid i <= 2j\}$

$S \rightarrow aSbb \mid \lambda \mid aKbb$

$K \rightarrow bK \mid K$

Q. Grammar for string where no. of 0's & 1's is not same.

$S \rightarrow S0S1S \mid 1S0S \mid 0 \mid 1$

Q. $a^{2n} b^n$

$S \rightarrow aaSb \mid \Lambda$.

Scanned with CamScanner

Q. $a^n b^m c^k$ $2n = m$ $k \geq 2$

$S \to AK$

$A \to aaAb \mid \wedge$.

~~$K \to ccK \mid \wedge$~~ $K \to cB$.

$B \to cB \mid c$

Q. equal a's or b's or 2x as many bs as as.

$S \to A \mid B$

$A \to aAb \mid \wedge$.

$B \to BaBaBbB \mid BaBbBaB \mid$
$BbBaBbB \mid \wedge$.

Q. $a^n w w^R b^n$ $w = (a+b)^*$

$S \to aKb \mid aSb \mid K$

$K \to$ ~~aeK~~ $aKa \mid bKb \mid \wedge$.

Q. $uvwv^R$ $|v| \geq 1$ $|u|; |w| \geq 2$ $\{a, b\}$.

$S \to XW$

$X \to a$

Q. $S \to pA \mid qB \mid rC$

$A \to Sp$

$B \to Sq$

$C \to \Lambda$

$L(G) = \{ w r w^T \mid w \in (pq^*) \}$

Q. $S \to AB \mid \Lambda \quad A \to aB \quad B \to Bb \mid b$

$L(G) = \{ a^m b^n \mid m = = 0 \ \& \ n = = 0) \mid\mid m = = 1 \mid\mid n \geq 1$

Q. $S \to aS \mid bS \mid SS \mid \Lambda$

$L(G) = \{ (a+b)^* \}$.

Q. $S \to aS \mid bS \mid SSS \mid \Lambda$

$L(G) = ((a+b)^*)$.

Q. $S \to aS \mid Sb \mid bS \mid Sa \mid SS \mid \Lambda$

$L(G) = (a+b)^*$.

Q. $S \to aA \mid Bb\Lambda, \ A \to Aa \mid a, \ B \to bB \mid b$.

$L(G) = \{ a^n \mid\mid b^n \} \ n \geq 1$

Q. $S \to aAb \mid aBb \mid \Lambda$, $A \to aBb \mid a$

$B \to aAb \mid b$.

$L(G) = \{a^n a^m b^k b^n \mid n \geq 1, m == 0 \,\&\&\, k == 1$

$\,||\, m == 1 \,\&\&\, k == 0$

$\}$

Q. $S \to aSa \mid A$, $A \to bAb \mid \Lambda$

$L(G) = \{a^n b^m a^n \mid n \geq 0, m \geq 0\}$

Q. $S \to aS \mid AB$, $A \to bA \mid B$, $B \to AA \mid \Lambda$.

$L(G) = \{a^n b^m \mid n \geq 1, m \leq 1\}$

Q. $S \to a \mid bB \mid ccC$, $B \to bB \mid \Lambda$,

$C \to cC \mid \Lambda$, $D \to d$.

$L(G) = \{a$ OR b^n OR $c^n \mid n > 0)$
$\}$

Q. $S \to AaB \mid aaB$, $A \to \lambda$, $B \to bb \mid \Lambda$.

$L(G) = \{a^n b^k \mid 0 \leq n \leq 2 \,\&\&\, k == 0$

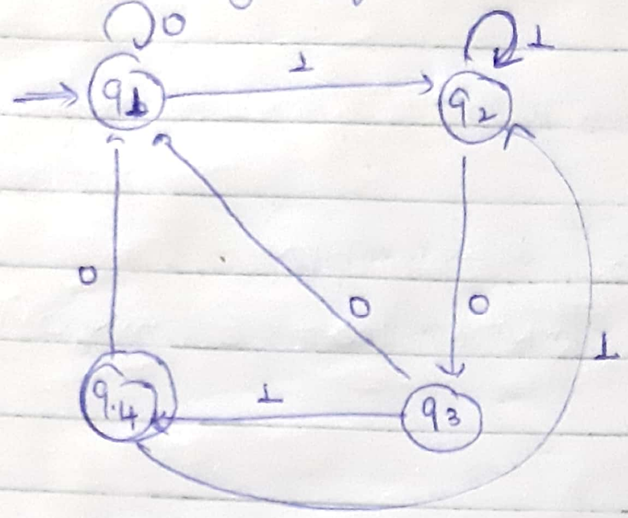
$\,||\, k == 2$

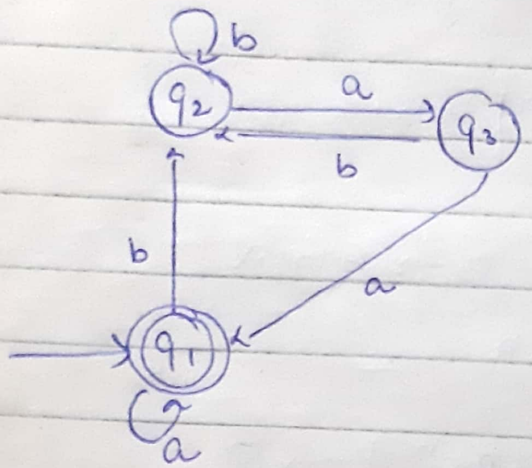
$\}$

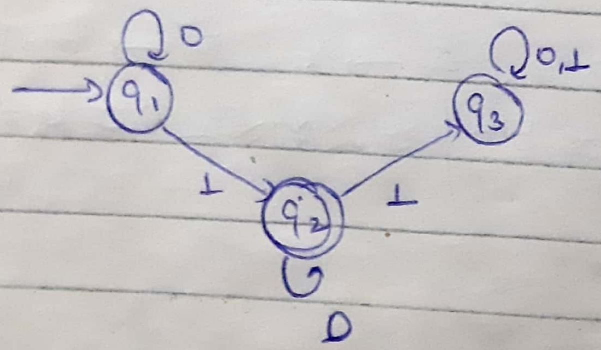
3/3/2000 $R = RQ + P$ $\underline{Q^* P}$

TUT

Q1 Find Regular expression corresponding to the following figure:

i) 

(ii) 

(iii) 

Q2 - Show that $L = \{ww \mid w \in \{a,b\}^*\}$ is not Regular

Q3. Construct a RE accepting

$L = \{ w \in (a,b)^* \mid w$ is a string over $\{a,b\}$ such that no. of b's is 3 mod 4.

$RE = a^* b a^* b a^* b a^*$

~~Soln~~

Ans 2 let $w = ab$

$\therefore ww = abab$

$\therefore w' = abab$

$x = a \quad y = b \quad z = ab$

$w \quad xy^i z$

$= abab \in L$

$i = 2$

$= ab^2ab \notin L$

Hence L is not a regular.

$w = a^n b$

$ww = a^n b a^n b$

$x = a^n$

$y = b$

$z = a^n b$

$\boxed{a^n b b a^n b}$

X

Ans 1 (iii)

$q_1 = q_1 0 + \Lambda \Rightarrow q_1 = 0^*$

$q_2 = q_1 1 + q_2 0$

$q_2 = 0^* 1 + \overset{R}{q_2} \overset{I}{0}$

$q_2 = \cancel{0^* 0^* 1}\ 0^* 1 0^*$

$\boxed{q_2 = 0^* 1 0^*}$

(ii) $q_1 = \Lambda + q_1 a + q_3 a$

$q_2 = q_1 b + q_2 b + q_3 b$

$q_3 = q_2 a$

$\Rightarrow \quad q_1 = \Lambda + q_1 a + q_2 aa$

$q_2 = q_1 b + q_2 b + q_2 ab$

$q_2 = q_1 b + q_2 (b + ab)$

$q_2 = q_1 b (b + ab)^*$

$q_1 = \Lambda + q_1 a + q_1 b(b+ab)^* aa$

$q_1 = \Lambda + q_1 (a + b(b+ab)^* aa)$.

$q_1 = (a + b(b+ab)^* aa)^*$

$q_4 0$

$q_3 0$

(iii) $q_1 = \Lambda + q_1 0 + q_4 0 + q_3 0$

$q_2 = q_1 1 + q_2 1 + q_4 1$

$q_3 = q_2 0$

$\rightarrow q_4 = q_3 1$

$q_4 = q_2 01$.

$q_2 = q_1 1 + q_2 1 + q_2 011$

$q_2 = q_2 (1 + 011) + q_1 1$

$\Rightarrow q_2 = q_1 1 (1 + 011)^*$

$q_1 = \Lambda + q_1 0 + q_1 1 (1+011)^* 010$

$+ q_1 1 (1+011)^* 00$

Scanned with CamScanner

$q_1 = \Lambda + (0 + 1(1+011)^* 010 + 1(1+011)^* 00)$

$q_1 = (0 + 1(1+011)^* 010 + 1(1+011)^* 00)^*$

$q_4 = (0 + 1(1+011)^* 010 + 1(1+011)^* 00)^* 1$
$(1+011)^* \underline{01}$

or

$q_4 = (0 + 1(1+011)^* 0(10+0))^* 1(1+011)^* 01$

1011101001110110 1 01
10010

Q. Pumping lemma for Regular expression. (x Reg Ex)

$L = \{a^n \mid n \text{ is prime no.}\}$

let $w = aa$

$= aa^i \quad i=1$

$= aaa \quad i=2$

$\boxed{aaaa} \quad$ no $\underline{i=3}$

therefore its not regular Expression.

Q. $L = \{a^i b^j c^k \mid j > i+k\}$

$S \to aAbBCc \mid b \mid aAbbB \mid bbBcC \mid$

$A \to aAb \mid \Lambda$

$C \to bCc \mid \Lambda$

$B \to bB \mid \Lambda$

Q. $L = \{a^i b^j c^k \mid i=j \text{ or } j=k\}$