Sets

∟ collection of distinct objects of same type or class.

$A = \{1, 2, 3, 4, 5\}$

Formation – Tabular / Roster $P = \{a, b, c\}$

– Builder $P = \{x : x \in N,$ x is multiple of $3\}$

(Property method) $R = \{x : x > 1$ and $x < 10\}$

$x \in A$, $x \notin A$, ϕ, U, N, I, I_0 (non-zero integers), I_+ (+ve integers), C, C_0, R, R_0, R_+ (real)

Kinds of Sets

1. Finite
2. Infinite $E = \{x : x \in N, x$ is multiple of $2\}$
3. Equality $A = B$, same elements
   (# Order doesn't matter)
   (# repitition " ")
4. Disjoint $R = \{a, b\}$, $S = \{q, r\}$
5. Family of sets – sets of sets $A = \{\{1, 2\}, \{3, 4\}, \{5\}, \phi\}$
6. Subsets – $A \subseteq B$ – when every element of A lies in B.
   ∟ Proper subset $A = \{2, 3\}$, $B = \{2, 3, 4\}$ (ϕ is improper subset of every set)
   ∟ Improper $A = \{2, 3\}$, $B = \{2, 3\}$
   (Every set is improper subset of itself.)
7. Null set / Empty – no element
   $A = \{x : x^2 = 4, x$ is odd$\}$
   $A = \{x : x^2 = 9, 2x = 4\}$
   $A = \{0\}$ ← not null set.
   $A = \{\phi\}$ ← not " "
8. Power Set – Set of all its subsets, $P(A)$. If A has n elements, then $P(A)$ has 2^n elements.
9. Universal set – All sets under investigation are subsets of U.

Comparability - If $A \subseteq B$ or $B \subseteq A$, they are comparable.
ϕ is comparable to all. Every set is comparable to U.

~~Abstraction~~ =

Operations

1 Union. $A \cup B$

2 Intersection - $A \cap B$ (common)

3 Difference - $A - B$ or A/B [All elements which belong to A but not B]

$A = \{a, b, c, d\}$, $B = \{d, l, m, n\}$

$A - B = d$

4 Complement w.r.t. U $A^c = U - A$

$A = \{1, 2, 3\}$

A^c = {all natural numbers except 1,2,3}

5 Symmetric difference

$A \oplus B = (A \cup B) - (A \cap B)$

Algebra of Sets

Law's

(i) Idempotent Law

(a) $A \cup A = A$

(b) $A \cap A = A$

(ii) Associative

(a) $(A \cup B) \cup C = A \cup (B \cup C)$

(b) $(A \cap B) \cap C = A \cap (B \cap C)$

(iii) Commutative

(a) $A \cup B = B \cup A$

(b) $A \cap B = B \cap A$

(iv) Distributive

(a) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(b) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

(v) De Morgan's law

$(A \cup B)^c = A^c \cap B^c$

$(A \cap B)^c = A^c \cup B^c$

(vi) Identity

(a) $A \cup \phi = A$

(b) $A \cap U = A$

(c) $A \cup U = U$

(d) $A \cap \phi = \phi$

(vii) Complement

$A \cup A^c = U$

$A \cap A^c = \phi$

$U^c = \phi$

$\phi^c = U$

(viii) Involution

$(A^c)^c = A$

$\cup A = A$

$A \& B$, $B \subseteq A \cup B$, so $A \subset A \cup A$

$x \in A \cup A \Rightarrow x \in A$ or $x \in A$

$\Rightarrow x \in A$

$A \cup A \subseteq A$

$A \subset A \cup A$, $A \cup A \subset A$

$\Rightarrow A = A \cup A$

$A \cap A = A$

Sets $A \& B$, $A \cap B \subset B$ so $A \cap A \subset A$

$x \in A \Rightarrow x \in A$ and $x \in A$

$x \in A \cap A$

$A \cap A \subset A$, $A \subset A \cap A$

$A = A \cap A$

$(A \cup B) \cup C = A \cup (B \cup C)$

$x \in (A \cup B) \cup C$

$\Rightarrow (x \in A$ or $x \in B)$ or $x \in C$

$\Rightarrow x \in A$ or $x \in B$ or $x \in C$

$\Rightarrow x \in A$ or $(x \in B$ or $x \in C)$

$x \in A$ or $(x \in B \cup C)$

$x \in A \cup (B \cup C)$

$(A \cap B) \cap C = A \cap (B \cap C)$

$x \in A$ and $x \in B \cap C$

$x \in A$ and $(x \in B$ and $x \in C)$

$A \cup B = B \cup A$

$A \cup B = \{x : x \in A$ or $x \in B\}$

$= \{x : x \in B$ or $x \in A\}$

$= B \cup A$

$A \cap B = B \cap A$

$A \cap B = \{x : x \in B$ and $x \in A\}$

$=$

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

[Intersection of sets is distributive w.r.t. union of sets.]

$x \in A \cap (B \cup C) \Rightarrow x \in A$ and $x \in B \cup C$

$\Rightarrow (x \in A$ and $x \in A)$ and $(x \in B$ or $x \in C)$

$\Rightarrow (x \in A$ and $x \in B)$ or $(x \in A$ and $x \in C)$

$\Rightarrow (x \in A \cap B)$ or $(x \in A \cap C)$

$\Rightarrow (A \cap B) \cup (A \cap C)$

So $A \cap (B \cup C) \subset (A \cap B) \cup (A \cap C)$

Now $y \in (A \cap B) \cup (A \cap C)$

$(y \in A$ and $y \in B)$ or $(y \in A$ and $y \in C)$

$\Rightarrow (y \in A$ or $y \in A)$ and $(y \in B$ or $y \in C)$

$\Rightarrow (y \in A)$ and $y \in (B \cup C)$

$\Rightarrow A \cap (B \cup C)$

$\Rightarrow A \cap B \cup (A \cap C) \subset A \cap (B \cup C)$

$(A \cup B)^c = A^c \cap B^c$

$x \in (A \cup B)^c$

$\Rightarrow x \notin (A \cup B)$

$\Rightarrow x \notin A$ and $x \notin B$

$x \in A^c$ and $x \in B^c$

$x \in A^c \cap B^c$

$\therefore (A \cup B)^c \subset A^c \cap B^c$

Let $x \in A^c \cap B^c$

$x \in A^c$ and $x \in B^c$

$x \notin A$ and $x \notin B$

$x \notin (A \cup B)$

$x \in (A \cup B)^c$

$A^c \cap B^c \subset (A \cup B)^c$

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$\phi = A$

$x \in A \cup \phi \Rightarrow x \in A$ or $x \in \phi$

$\Rightarrow x \in A$ ($\because x \notin \phi$ as ϕ is null set)

$A \cup \phi \subset A$

Now $A \subset A \cup B$ for any set B

for $B = \phi$, $A \subset A \cup \phi$

$A \subset A \cup \phi$, $A \cup \phi \subset A \Rightarrow A = A \cup \phi$

$A \cap \phi = \phi$

$x \in A$, then $x \notin \phi$, so $A \cap \phi = \phi$

$A \cup U = U$

Every set is subset of U

$A \cup U \subseteq U$

$U \subseteq A \cup U$

So $A \cup U = U$

$A \cap U = A$

$x \in A \Rightarrow x \in A$ and $x \in U$

$\Rightarrow x \in A \cap U$ $\therefore$ $A \subset A \cap U$, so $A \cap U = A$

$A \cup A^c = U$

we have to show $U \subseteq A \cup A^c$

$x \in A$ or $x \notin A$

$x \in U \Rightarrow x \in A$ or $x \in A^c \Rightarrow x \in A \cup A^c$

$\Rightarrow x \in A$ or $x \in A^c$

$\Rightarrow U \subseteq A \cup A^c$

So $A \cup A^c = U$

$A \cap A^c = \phi$

ϕ is subset of every set

$x \in A \cap A^c \Rightarrow x \in A$ and $x \in A^c$

$\Rightarrow x \in A$ and $x \notin A$

$\Rightarrow x \in \phi$

$A \cap A^c \subset \phi$

$U^c = \phi$

$x \in U^c = x \notin U \Rightarrow x \in \phi$

$\phi^c = U$

$x \notin \phi \Rightarrow x \in U$

$x \in (A^c)^c$

$x \notin A^c \Rightarrow x \in A$

$(A^c)^c = A$.

ountable Sets.
- Uncountable Sets.

Cardinality - $n(A)$

Partition

Let S be a non-empty set, partition P is ~~a~~ a finite collection $\{A_i\}_{j=1}^{n}$ of non-empty subsets of S.

s.t. (i) $A_i \cap A_j = \phi \ \forall i \neq j$, A_i are mutually disjoint.

(ii) $\bigcup_{i=1}^{n} A_i = S$

The subsets in partition are called cells.

Cross partition

If $[A_1, A_2 \ldots A_m]$ and $[B_1, B_2 \ldots B_n]$ are partitions of set S, set $P = \{A_i \cap B_i\}$ is called C.P.

Venn-diagrams

Singleton

Fundamental Product.

Let $A_1, A_2 \ldots A_n$ be disjoint sets. F.P. of these sets is a set of form $A_1^* \cap A_2^* \cap A_3^* \cap \ldots \cap A_n^*$ where A_i^* is eith A_i or A_i^c. 2^n F.P. can be there.

All Sets of F.P. are disjoint and union of all makes U.

~~MP~~

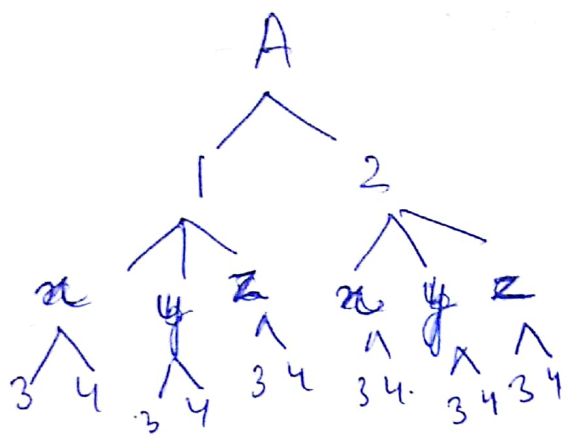
...rected Graph (Digraph of Relation)

$A = \{1,2,3,4\}$

$R = \{(1,2), (2,2), (2,4), (3,2), (3,4), (4,1), (4,3)\}$

Show $\underline{n(A \times B) = \underset{\substack{\downarrow \\ \text{choices for } A}}{n(A)} \cdot \underset{\substack{\downarrow \\ \text{choices for } B}}{n(B)}}$

Ex $A = \{1, 2\}$, $B = \{x, y, z\}$, $C = \{3, 4\}$



$A \times B \times C$

Ex (a) $X = \{0, 1, 2, 3 \ldots\}$
R is defined by $x^2 + y^2 = 25$. find ordered pairs.

(b) $S \Rightarrow 3x + 4y = 17$, find o.p.

(c) $R \Rightarrow x^2 + 2y = 100$
find domain & range

Ex $A = \{1, 2, 3, 4\}$
$R = \{(1,1), (3,1), (3,4), (4,2), (4,3)\}$
$S = \{(1,3), (2,1), (3,1), (3,2), (4,4)\}$
(a) find $R \circ S$, $S \circ R$, $R^2 = R \circ R$, $R^3 = R \circ R \circ R$

Ex $A = \{1, 2, 3 \ldots 8\}$, $R =$ 'y is divisible by x'

Minterms or Minsets

A be set, then $D_1 \cap D_2 \cap D_3 \ldots D_n$ make minterms.

B_i or B_i^c ← subsets of A.

Maxterms

$D_1 \cup D_2 \cup D_3 \cup \ldots D_n$

Cartesian Product.

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