RELATIONS

A & B are 2 non-empty sets, then a binary relation from A to B is a subset of $A \times B$.
$\therefore$ R is a relation from A to B iff $R \subseteq A \times B$.

$\boxed{xRy}$ $\rightarrow$ x is related to y by R.

- Total no. of ordered pairs possible in $A \times B = mn$
  - Total subsets in $A \times B = 2^{mn}$
  - So total relations $= 2^{mn}$

- Set of all first coordinates of ordered pairs belonging to R = domain of R.
- Set of all second coordinates of ordered pairs belonging to R = range of R.

- Inverse relation. for R from $A \to B$
  ↳ R^{-1} = so for $R = \{(a,b) : a \in A, b \in B\}$
  then $R^{-1} = \{(b,a) : b \in B, a \in A\}$

  Domain of R^{-1} = Range of R
  Range of R^{-1} = Domain of R.

Types

Smallest ① Empty (void) - for set A, $\phi \subseteq A \times A$ is true

Largest ② Universal - for set A, $A \times A \subseteq A \times A$ is true

③ Identity - $R = \{(x,x) : x \in A\}$ on A is identity / diagonal relation, denoted by Δ_A or Δ

If $A = \{a,b,c\}$, $R_1 = \{(a,a), (b,b), (c,c)\}$

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Reflexive - If every element is related to itself
$(x,x) \in R$ ← Identity & Universal relations are reflexive
, but if $(x,x) \notin R$, not reflexive.

(ii) Irreflexive - $(a,a) \notin R$
if there exists atleast one $(a,a) \in R$, it is not irreflexive.

(iii) Symmetric

$$(x,y) \in R \Rightarrow (y,x) \in R \ \forall x,y \in A$$

$A = \{1,2,3,4\}$, $R = \{(1,3),(1,4),(3,1),(2,2),(4,1)\}$

$(1,3) \in R \Rightarrow (3,1) \in R$
$(1,4) \in R \Rightarrow (4,1) \in R$
$(2,2) \in R \Rightarrow (2,2) \in R$.

(iv) Transitive

$(x,y) \in R$, $(y,z) \in R \Rightarrow (x,z) \in R \ \forall x,y,z \in A$.

Eg - A is set of all parallel lines.
$l_1 \parallel l_2$, $l_2 \parallel l_3 \Rightarrow l_1 \parallel l_3$.

(V) Asymmetric

$(x,y) \in R$ and $(y,x) \in R \Rightarrow x=y$
OR xRy and yRx, $x=y$

Eg $a \leq b$ and $b \leq a \Rightarrow a=b$.

Equivalence Relation
└ iff it is reflexive, symmetric & transitive.

Compatible
└ if R is reflexive & symmetric

Partial Order - reflexive, anti-symmetric & transitive

Ternary -

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Closure Properties

1) Reflexive closure – R_R is ref. closure of R if R_R is smallest relation containing R having reflexive property.

2) Symmetric closure – R_S is sym. closure of R if R_S is smallest relation containing R having symmetric property. $R_S = R \cup R^{-1}$

3) Transitive closure – R_T. - -

Composition of Relation

Let R and S be 2 relations from A to B & B to C. Then RoS is composition relation from A to C where $(a,c) \in RoS$ iff we can find $b \in B$ s.t. $(a,b) \in R$ and $(b,c) \in S$.

So, RoS is composition of R & S.

or

$R \subseteq A \times B$ and $S \subseteq B \times C$,

$RoS = \{(a,c) : \text{There exists } b \in B \text{ for } (a,b) \in R \text{ and } (b,c) \in S\}$

$A = \{1,2,3\}$, $B = \{a,b,c\}$, $C = \{x,y,z\}$

$R = \{(1,a), (2,b), (2,c)\}$

$S = \{(a,y), (b,x), (c,y), (c,z)\}$

$RoS \Rightarrow \{(2,x), (2,z)\} \in RoS$

- $RoS \neq SoR$ (generally)

$(RoS)^{-1} = S^{-1} o R^{-1}$

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