Chapter 7

# SIMULATION OF INVENTORY SYSTEMS

## 7.1 Introduction

Inventory management is one of the important areas of operations research and management science, which have been modelled, analyzed and simulated extensively. Though simple problems are amenable to the analytical techniques, but the real life inventory problems are generally so complex that Simulation is the only tool, which provides reasonably accurate solutions.

There is a wide range of situations where one or the other type of inventory is required for the smooth functioning of the system. The need of inventory arises out of difference in the timing or location of demand and supply. This applies whether one is dealing with the raw materials for a production process or finished goods stocked by a manufacturer, wholesaler, or retailer of any sort of goods. From a customer's perspective inventory should be such that the demand is always fulfilled. This is generally true in case of many grocery items, but rarely true for items like cars and special purpose machine tools. Inventories represent idle capital and thus cost money. In simple form inventory management, calls for maintaining adequate supply of specified items so as to meet the expected pattern of their demand and at the same time keeping the costs involved in maintaining the inventory and that of shortages at the minimum.

## 7.2 Classification of Inventory Systems

The inventory systems are generally classified into three types:

- Repetitive order-independent demand
- Single order-independent demand
- Repetitive order-dependent demand.

The first type is the most common situation faced in business. The second type, single-order systems like orders for Christmas trees and Diwali gifts occur but not very frequently. Systems with dependent demand are generally witnessed in manufacturing, where the parts procured from outside are used in the assembly of the final product. The demand for parts from outside vendors is dependent on the demand for finished items. The analysis of such situations is called the Material Requirements Planning (MRP).

Another way of classification of inventory, which is more relevant to the manufacturing systems, is according to their relation to the overall sequence of production operations.

- Supplies.
- Materials.
- In-process goods.
- Finished goods.

Supplies constitute of such staple items like office stationery, computer paper, floppy disks, compact discs, programming tapes, drawing office materials and various forms used in production planning. Such items generally have independent demand. Materials constitute all those items, which are worked upon or consumed in the manufacturing process. The raw materials, oils, lubricants, fuels, chemicals and paints etc. are materials. In-process goods are incomplete products held between various workstations, while the finished goods are the end product of the production system.

A large number of inventory models are available to deal with a wide variety of inventory situations. These can though be classified in a number of ways, but commonly these are grouped into two broad categories.

- Fixed-order quantity models
- Fixed-order period models

In a fixed-order quantity model the demand is met out of the inventory held in stock and the stock position is continuously (or perpetually) updated. If a particular demand is not fulfilled or is partially met, either a back order is taken or the sale is lost. As soon as the stock level touches the reorder point order for fixed quantity is placed. In case of fixed-order period model, there is no perpetual updating of the stock; rather periodic reviews are made at fixed intervals of time. The amount balance in stock plus amount on order minus the backlog is compared with the desired maximum level and the order is placed for the difference. Thus in fixed-order quantity model the order quantity is fixed and the period between orders is variable, while reverse is true for the fixed order period model.

## 7.3 Inventory Costs

The costs involved in inventory systems are classified into three types.

1. ***Reorder cost*** or ***Set up cost***: It is the administrative cost associated with the placing of the order.
2. ***Inventory carrying cost*** or ***inventory holding cost***: It is the cost of maintaining the inventory, which includes storage charges, insurance, interest on tied capital, etc.
3. ***Shortage cost***: This cost is incurred when the system runs out of stock. This results in loss of profits as well as loss of goodwill and reputation.

***Price of an Item***: The purchase cost or price of an item includes the basic price of an item plus the taxes and transportation costs if any. In case of items produced by the firm itself, the full production cost is called the price.

***Lead-Time***: This is the time lag between the placing of order and obtaining the delivery of items. It is also called ***delivery lag***.

***Service level***: The service level for an inventory system is a measure of the customer satisfaction and may be defined as the ratio of units furnished to customers to the total number of units demanded by the customers. If the shortages are more the service level will naturally be low. No management will like to have an inventory policy with poor service level, even if it is the most economical operating policy. The service level is generally expressed as a percentage.

## 7.4 Single Item Constant Demand Inventory Model

Let us consider a very simple inventory system to have a feel of the basic inventory theory. Fig. 7.1 shows the variation in stock over time for a single item with constant demand. To start with the inventory level is Q, which goes down at a constant rate to zero, where it again jumps back to level Q. The cycle goes on repeating with average inventory level as Q/2 items.

In this model as soon as the inventory becomes zero it is replenished without any lag of time and the system never runs out of stock. Thus there is no shortage cost.

Let $k$ be the carrying cost per item per day,
R be the order cost per order,
D be the volume of daily sale,

Then total inventory cost per day is

$$\boxed{C = (Q/2)\cdot k + (D/Q)\cdot R}$$

Q/D is the number of days between two consecutive deliveries.

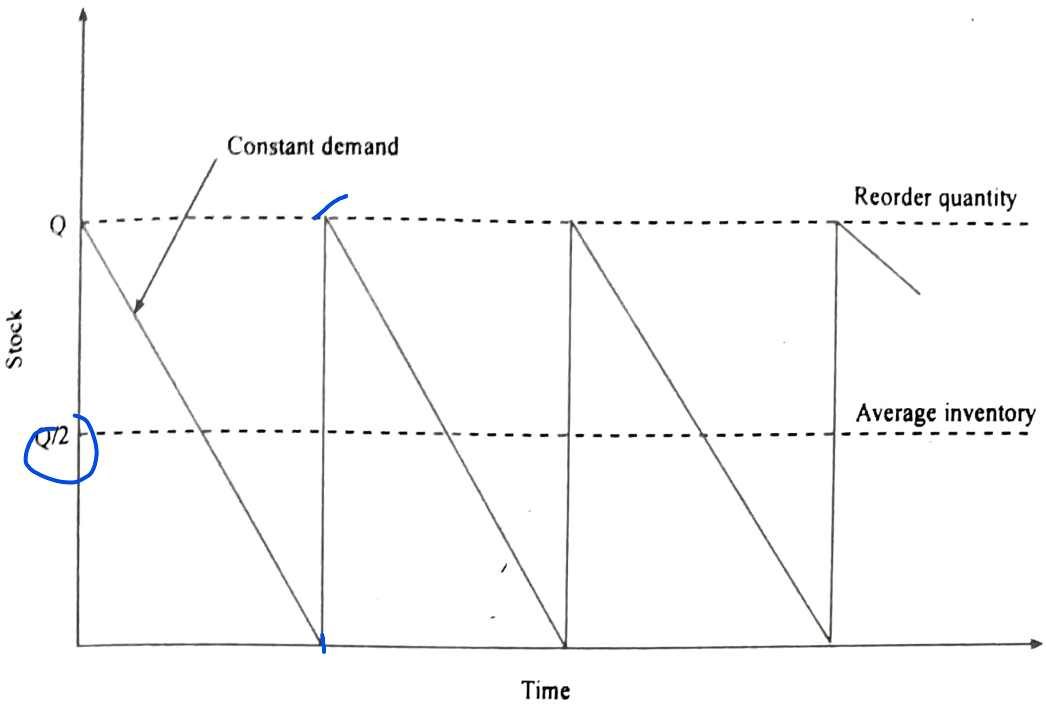


**Fig. 7.1**

On differentiating C with respect to Q and equating $dC/dQ$ to zero, we get

$$\frac{dc}{dQ} = \frac{k}{2} - \frac{DR}{Q^2} = 0$$

$$Q^* = \sqrt{\frac{2DR}{k}}$$

The value of Q so obtained is also referred to as the Economic Order Quantity (EOQ).

Thus $$\text{EOQ} = \sqrt{\frac{2DR}{k}}$$

The EOQ gives the optimal reorder quantity, which minimizes the inventory cost C. Thus an order of $Q^*$ items should be placed after fixed regular intervals of $Q^*/D$ days.

This lot size formula is known as Wilson's Formula (named after R.H. Wilson) and has been extensively used in inventory theory.

To illustrate the use of this formula let us consider an inventory problem where;

$k$, the carrying cost per item per day is Re. 0.01,

D, the volume of daily sales is 100 units, and

R, the reorder cost per order is Rs. 200.

$$\text{EOQ} = \sqrt{\frac{2DR}{k}} = 2000 \text{ units}$$

$Q/D = 2000/100 = 20$

$C = (2000/2)\cdot 0.01 + (100 \times 200)/2000 =$ Rs. 20 per day.

Thus, 2000 units should be ordered after every 20 days for a minimum inventory cost of Rs. 20.00 per day.

## 7.5 EOQ with Constant Lead-Time

In the above model it was assumed that the stock was replenished immediately when it touched zero level, which is not practically feasible. There is always some *delivery lag*. If this delivery lag is constant say *d* days, it does not make any difference to the EOQ model. The order is placed exactly *d* days before the items desired arrival day. These reorder days are shown as reorder points in the Fig. 7.2. However in case the delivery lag is not constant and is a random variable the analysis becomes complicated and the situation can best be handled by simulation.

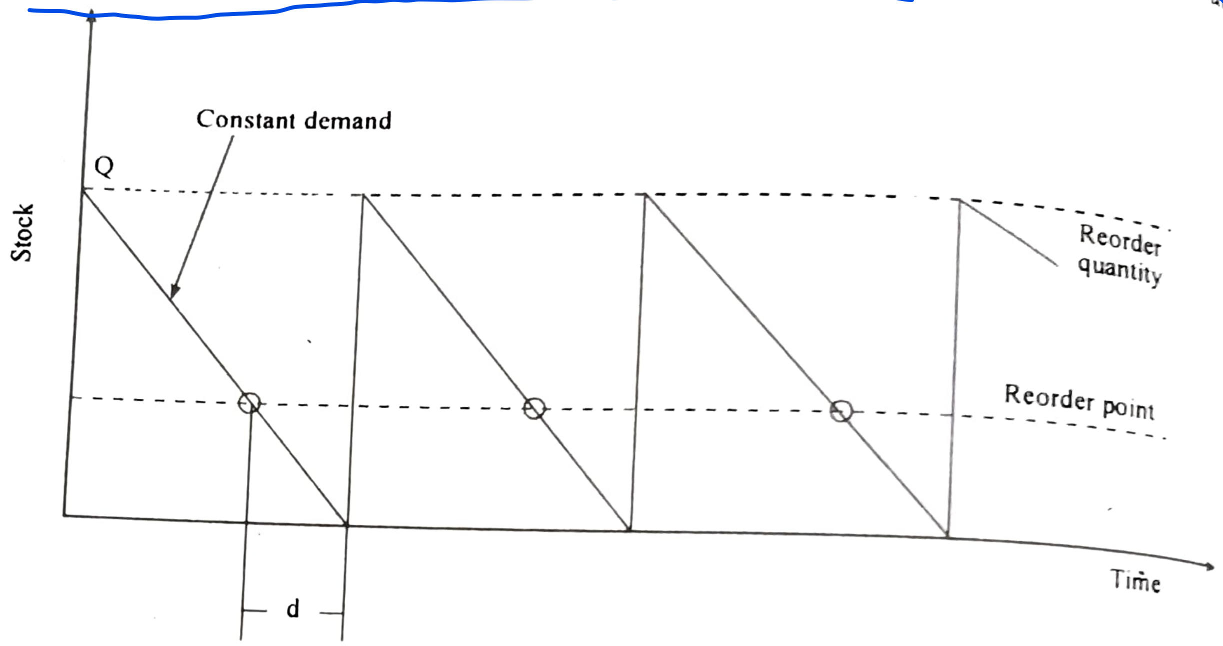


**Fig. 7.2**

## 7.6 EOQ with Shortage

The Wilson's EOQ Formula derived above assumed that the stock was replenished as soon as the stock touched zero level and that there was never any shortage that is system works at 100% service level. However in practice the 100% service level sometimes becomes uneconomical, as comparatively large inventory has to be maintained.

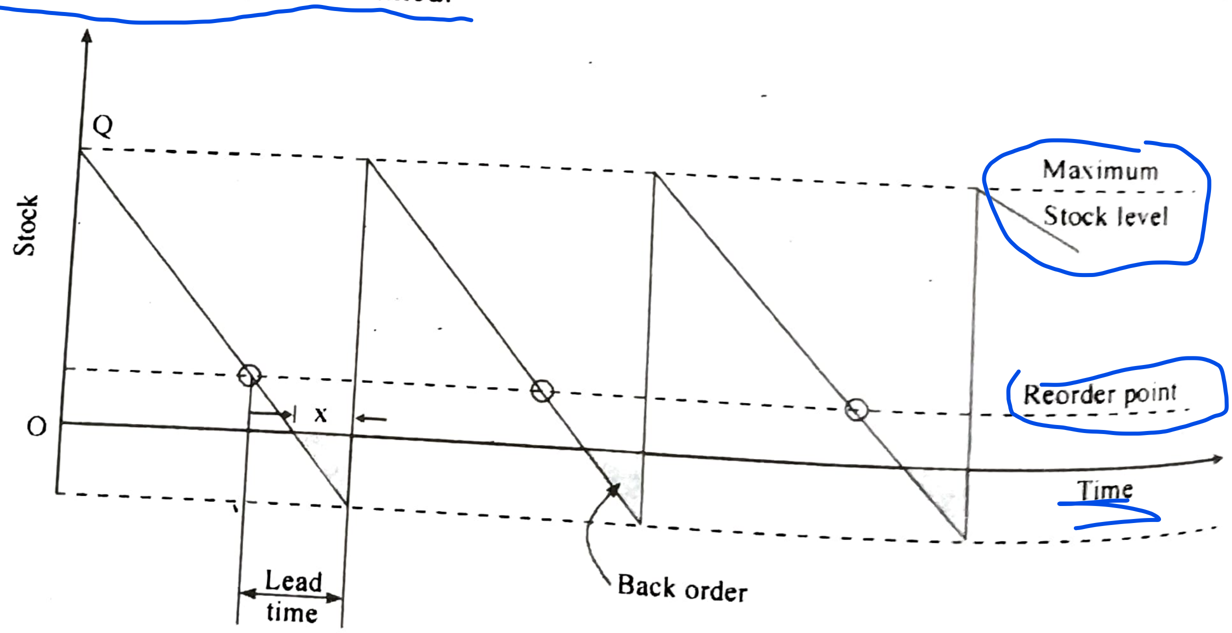


**Fig. 7.3**

Two types of situation occur when the system runs out of stock. First, the customer goes to some other supplier and the sale is permanently lost. Second, the customer leaves his order and the same is replenished as soon as the stock is received. This is called back order or backlog. In the case of back orders, though the sale is not lost but some penalty cost per item per unit time may occur. Penalty may occur in the form of cost of maintaining back orders, cost of shipping the stock to the customer, rebate in price and loss in goodwill, etc.

Let us modify the Wilson's model for the back ordering situation. Fig. 7.3 shows a system with constant daily demand D, a reorder quantity Q, a constant lead-time $d$. For a fraction of time $x$ the system remains out of stock, while for the remaining time $(1 - x)$ the demand is met. The backlog is maintained which is filled as soon as the next delivery is received.

$$EOQ = \sqrt{\frac{2DR}{k}} \cdot \sqrt{\frac{k+b}{b}} \qquad \text{where } b = \frac{kx}{1-x}$$

## 7.7 Why Simulation of Inventory Systems?

In the previous section very simple inventory situations have been analyzed and simple inventory models derived. The real situations are much more complex than these situations. In a production environment we come across different types of inventories as raw materials, finished products and in-process inventories. Many complicated formulas are available in mathematical inventory theory to deal with problems of various types. However, no analytical formula is sufficiently versatile to solve the reasonably complex real life inventory problems. The real life inventory problems are generally so complex that simulation is the only tool, which can be employed to find the optimal operating policies. The following are some of the factors, which add to the complexities of inventory problems.

### 7.7.1 *Gradual Replenishment*

In deriving Wilson's Formula with or without lead-time, we assumed that the stock was replenished instantaneously as soon as the delivery was received. However this is not true in many situations especially in a production environment where the replenishment occur gradually. The demand is met from stock as well as from production. The particular item may be produced for some period and the same production facilities may then be used for the production of some other item. If the rate of production during the production period is $P$ units a day and the demand is $D$ units a day then during that period the inventory will be replenished at the rate of $(P - D)$ units a day. For such a situation following formula for the EOQ can be derived.

$$EOQ = \sqrt{\frac{2DR}{k\,(1 - D/P)}}$$

### 7.7.2 *Multi-item Production*

We derived the above formula, taking only one item into consideration. Generally a manufacturer uses the same production facility for the production of several different items one after the other. In such situations a batch (quantity of items produced) of one product should be sufficiently large so as to last for the duration when other products are being produced. Thus a production cycle is established such that one batch of each product is produced in one cycle. It can be shown that under such a situation the optimal number of production cycles per unit of time is given by

$$\sqrt{\frac{\Sigma D_j k_j (1 - D_j / P_j)}{2\Sigma r_j}}$$

where $D_j$ = demand for the $j$th product per unit time,

$k_j$ = carrying cost for the $j$th product per unit time,

$r_j$ = set up cost for the $j$th product, and

$p_j$ = production rate for the $j$th product.

### 7.7.3 *Capital Restriction*

In the previous sections it has been assumed that there was no restriction as for as the capital was concerned and as many units as determined by the EOQ formula could be procured. However, in many situations there are restrictions on capital available and limits are imposed, that the value of inventory should not exceed a specified amount.

### 7.7.4 *Quantity Discounts*

In many situations discounts on price are offered on making large purchases. The price reduction may occur at discrete points or it may be continuous. The rate of discount may also vary with the size of the order. The advantages of quantity discount are the direct savings due to reduced price; and saving in the interest on capital tied in inventory, which reduces the per unit carrying cost. On the other hand, to take this advantage large average stock has to be maintained, which results in increased carrying cost. Mathematical models have been developed to take into consideration the quantity discounts or price breaks but their application are limited only to very simple price structures.

### 7.7.5 *Varying Demand*

In deriving the Wilson's Formula it was assumed that the demand of the items was constant which is rarely true in real life situations. Generally the demand is a stochastic variable. The pattern of demand has to be determined by using the historical data and the forecasting techniques, which may give some discrete or continuous probability distribution. The effect of seasonal variation in demand has also to be superimposed. In addition to the number of arrivals the number of items demanded by each arrival also vary randomly. If the customers arrive independently, their inter arrival times generally follow exponential distribution and if each customer demands only one unit then the total demand in any period will be Poisson distributed. However, if the number of units demanded by the customers vary randomly then the probability density function of demand will depend upon the probability distribution of the number of units demanded.

### 7.7.6 *Varying Lead-time*

The lead-time, which we assumed to be a constant in the derivation of Wilson's Formula, is generally a random variable. The frequency distribution of lead-time has also to be established using the past data and the forecasting techniques. Lead-time generally follows Erlang distribution that is gamma distribution with integral value of parameter $k$, which becomes exponential for $k = 1$ and normal when $k$ becomes large.

### 7.7.7 *Multiple Orders Outstanding*

In the derivation of various EOQ Formulae, it was assumed that at any time only one reorder could be outstanding. But in many inventory situations two or more than two reorders could be pending at a time. If, in addition to that, the lead-times are stochastic, the analysis of the situation becomes almost impossible for any analytical technique. In such a situation it is not essential that orders be delivered in the sequence in which they were placed. The reorder placed later can arrive earlier. And if the reorders must always arrive in the same sequence in which they were placed, then their lead times are not random variables.

The various factors discussed above make the inventory problems so complicated that it becomes impossible to find a reasonably good solution by employing the analytical techniques. The mathematical techniques at the most help to obtain the approximate guidelines. Since, inventory occupies an important place in any business or industrial operation, and has a significant bearing on the profits, its analysis and optimization is of utmost importance to any management. As already emphasized, simulation is the only tool to determine the optimal operating policy for a complex situation.

## 7.8 Example — Simulation of an Inventory Problem

In an inventory system the demand as well as the lead-times are random variables defined by discrete distributions as given below.

| | | | | | |
|---|---|---|---|---|---|
| Demand | : | 3 | 4 | 5 | 6 |
| Probability | : | 0.15 | 0.30 | 0.35 | 0.20 |
| Lead-time | : | 2 | 3 | 4 | |
| Probability | : | 0.2 | 0.6 | 0.2 | |

Two reorders each of quantity (Q) of 15 units can be outstanding at a time. There are two reorder points RP1 and RP2 at levels of 10 and 5 units respectively. The shortages are lost forever. The reorder cost is insignificant as compared to the carrying cost, and the objective is to determine the service level and the average stock held for the given reorder points and the reorder quantity. Taking an initial stock of 10 units, let us manually simulate the system for the first 20 days. This manual simulation will be helpful in understanding the mechanism of simulation and will make the computer programming of the model simple.

Though in computer simulation the demand and lead-times will be generated as and when required, here let us generate the demand for all the 20 days and some lead-times before starting the simulation process.

Generation of demand:

| *Demand* | *Probability* | *Commu. Prob* |
|---|---|---|
| 3 | 0.15 | 0.15 |
| 4 | 0.30 | 0.45 |
| 5 | 0.35 | 0.80 |
| 6 | 0.20 | 1.00 |

| | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Day | : | 1, | 2, | 3, | 4, | 5, | 6, | 7, | 8, | 9, | 10, | 11, | 12, | 13, | 14, | 15, | 16, | 17, | 18, | 19, | 20 |
| Random No. | : | 64, | 33, | 18, | 87, | 74, | 41, | 53, | 09, | 67, | 93, | 56, | 35, | 48, | 70, | 32, | 11, | 87, | 12, | 23, | 08 |
| Demand | : | 5, | 4, | 4, | 6, | 5, | 4, | 5, | 3, | 5, | 6, | 5, | 4, | 5, | 5, | 4, | 4, | 6, | 3, | 4, | 3 |

Similarly the lead-times are generated. First 10 lead-times are,

3, 2, 4, 3, 3, 3, 2, 4, 3, 3

The simulation process is shown in Table 7.1. On day 1, the stock is equal to the initial stock of 10 units. Since the reorder point is at 10, on the very first day the reorder is placed for 15 units. The lead-time for this order is 3 days hence the delivery will take place on 1 + 3 = 4th day in the morning. The reorder (ord 1) is assigned 15 and due date (ddate 1) is marked 4. Since the stock is more than the second reorder point hence ord 2 and due date (ddate 2) are marked zeros. Demand on day 1 is 5 units, which is completely met, and hence lost sale (short) on day 1 is zero. On day 2, stock available is 5 units, ord 1 is 15, and ddate 1 is 4. Now since the stock has touched the second reorder point, the second reorder is also placed with a lead-time of 2 days. With this ord 2 becomes 15 and the ddate 2 becomes 2 + 2 = 4. Thus both the reorders will arrive at the same time, on the morning of 4th day. On day 2, demand is 4 units, which is completely met. Lost sale on day 2 again is zero. On day 3 stock available is only one unit while demand is 4 units. There is a shortage of 3 units, which is the lost sale on day 3. On day 4, both the reorders have been delivered and the stock becomes 30 units. Since stock is more than the reorder points hence, ord 1, ddate 1, ord 2 and ddate 2 are marked zeros. This way the simulation continues.

After 20 days,

Average stock = 11.9 units

Service level = 81%.

**Table 7.1 : SIMULATION OF INVENTORY PROBLEM**

| *day* | *stk* | *cstk* | *ord 1* | *ddate 1* | *ord 2* | *ddate 2* | *dmd* | *cdmd* | *short* | *cshort* |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 10 | 10 | 15 | **1 + 3 = 4** | 0 | 0 | 5 | 5 | 0 | 0 |
| 2 | 5 | 15 | 15 | 4 | 15 | **2 + 2 = 4** | 4 | 9 | 0 | 0 |
| 3 | 1 | 16 | 15 | 4 | 15 | 4 | 4 | 13 | 3 | 3 |
| 4 | 30 | 46 | 0 | 0 | 0 | 0 | 6 | 19 | 0 | 3 |
| 5 | 24 | 70 | 0 | 0 | 0 | 0 | 5 | 24 | 0 | 3 |
| 6 | 19 | 89 | 0 | 0 | 0 | 0 | 4 | 28 | 0 | 3 |
| 7 | 15 | 104 | 0 | 0 | 0 | 0 | 5 | 33 | 0 | 3 |
| 8 | 10 | 114 | 15 | **8 + 4 = 12** | 0 | 0 | 3 | 36 | 0 | 3 |
| 9 | 7 | 121 | 15 | 12 | 0 | 0 | 5 | 41 | 0 | 3 |
| 10 | 2 | 123 | 15 | 12 | 15 | **10 + 3 = 13** | 6 | 47 | 4 | 7 |
| 11 | 0 | 123 | 15 | 12 | 15 | 13 | 5 | 52 | 5 | 12 |
| 12 | 15 | 138 | 0 | 0 | 15 | 13 | 4 | 56 | 0 | 12 |
| 13 | 26 | 164 | 0 | 0 | 0 | 0 | 5 | 61 | 0 | 12 |
| 14 | 21 | 185 | 0 | 0 | 0 | 0 | 5 | 66 | 0 | 12 |
| 15 | 16 | 201 | 0 | 0 | 0 | 0 | 4 | 70 | 0 | 12 |
| 16 | 12 | 213 | 0 | 0 | 0 | 0 | 4 | 74 | 0 | 12 |
| 17 | 8 | 221 | 15 | **17 + 3 = 20** | 0 | 0 | 6 | 80 | 0 | 12 |
| 18 | 2 | 223 | 15 | 20 | 15 | **18 + 3 = 21** | 3 | 83 | 1 | 13 |
| 19 | 0 | 223 | 15 | 20 | 15 | 20 | 4 | 87 | 4 | 17 |
| 20 | 15 | 238 | 0 | 0 | 15 | 20 | 3 | 90 | 0 | 17 |

Average Stock = 238/20 = 11.9 units
Service level = (90 – 17)/90 = 81%.

### Computer Programme

In the simulation Table 7.1 the time progressed from day-to-day that is we used the fixed increment time flow mechanism. While developing a computer programme it is always helpful to draw the flow chart of the programme logic as given in Fig. 7.4. The variables used in the program have been defined as;

| | |
|---|---|
| q1 | Quantity of reorder 1 |
| q2 | Quantity of reorder 2 |
| dmd | Demand |
| cdmd | Cumulative demand |
| stk | Stock held |
| cstk | Cumulative stock |
| *avstk* | Average stock |
| ord 1 | State of Reorder 1 |
| ord 2 | State of Reorder 2 |
| dd1 | Due date of delivery of reorder 1 |
| dd2 | Due date of delivery of reorder 2 |
| short | Shortage of demand. |
| cshort | Cumulative shortage of demand |
| todays | Length of simulation run in days |

| | |
|---|---|
| lt1 | Lead-time for reorder 1 |
| lt2 | Lead-time for reorder 2 |
| rp1 | Reorder point 1 |
| rp2 | Reorder point 2 |
| sl | Service level |

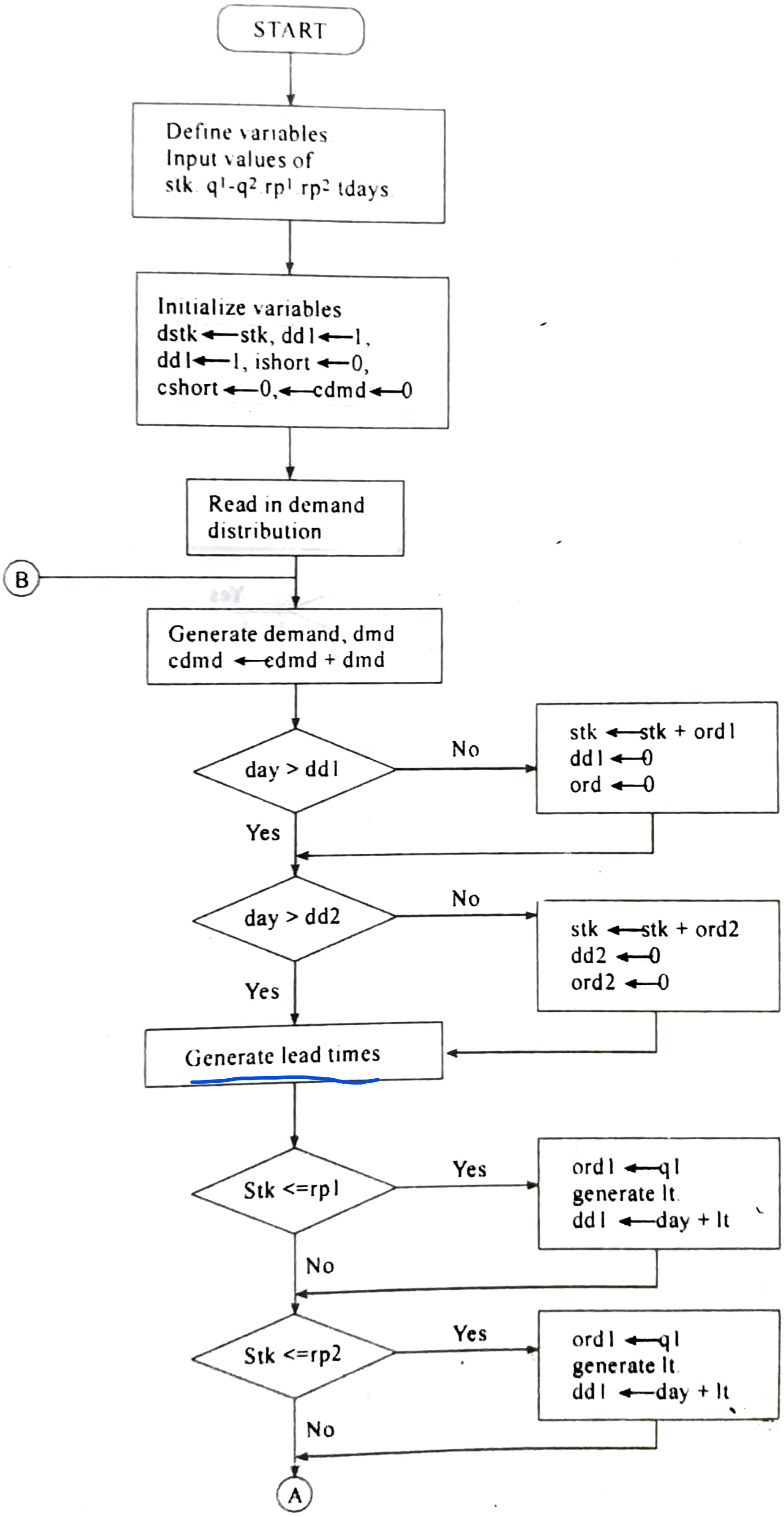


**Fig. 7.4** ***(Cont'd)***

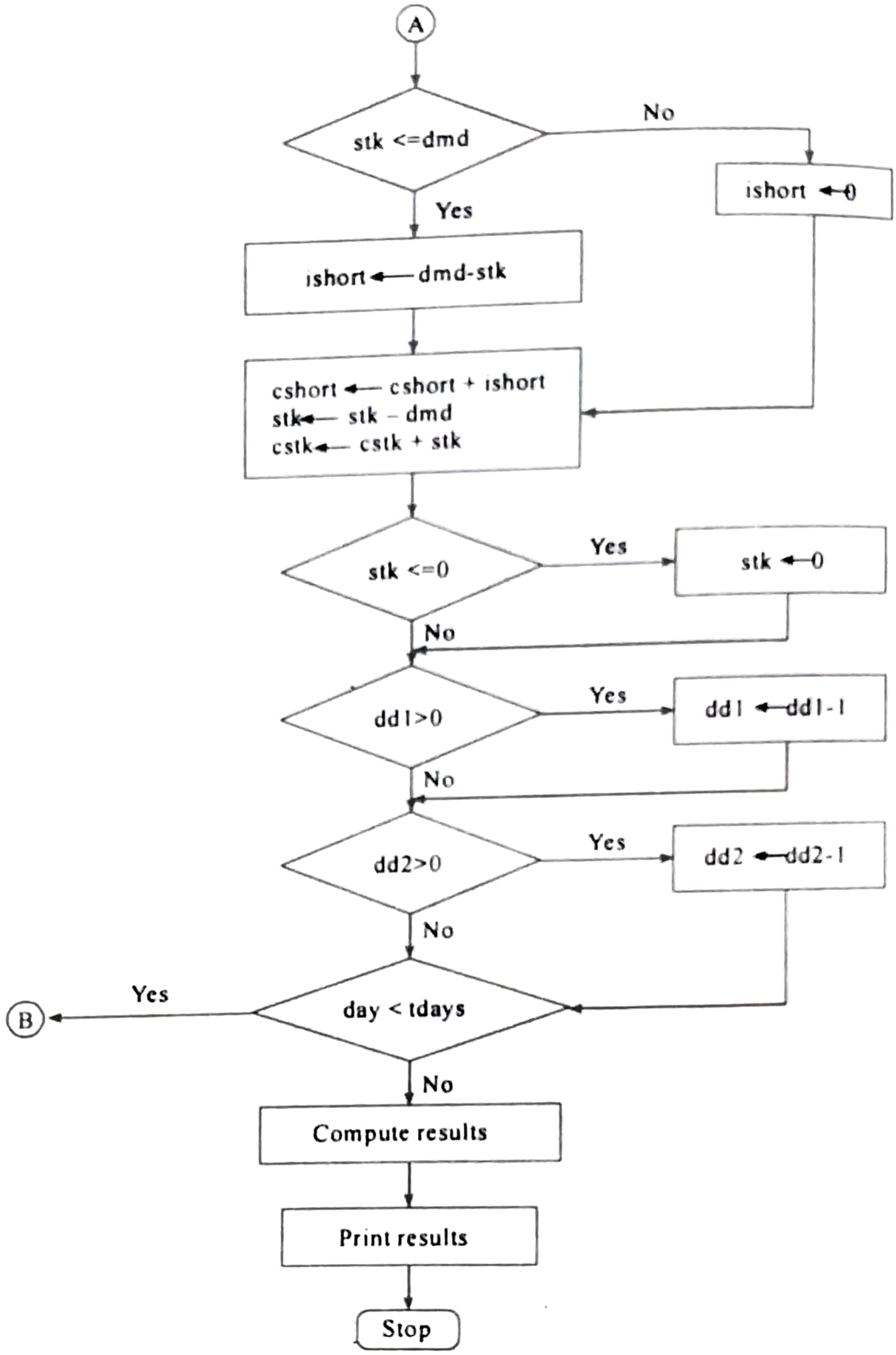


**Fig 7.4**

A computer programme for this simulation written in C language is given below.

```
#include<stdio.h>
#include<stdlib.h>
#include<math.h>
main()
    {
    /* INVENTORY PROBLEM with two orders*/
    int q1,q2,rp1,rp2,tdays,dmd,cdmd,stk,cstk,day,dd1,dd2,lt1,lt2;
    int ishort, cshort,ord1,ord2;
    float x,avstk,sl;
```

```
q1=10;q2=10;
day=1;dd1=0;dd2=0;tdays=1000;
rp1=2; rp2=2;ord1=0;ord2=0;
ishort=0;cshort=0;cdmd=0;
printf("\n Enter valueof initial stk=");
scanf("%d",&stk);
printf("\n Initial stock=%d",stk);
cstk=stk;
/* printf("\n   Day stk cstk ord1 dd1 ord2  dd2 dmd cdmd ishort
   cshort\n");*/

   /*    printf("initial stock=%d q1=%d  q2=%d",stk,q1,q2);*/
for(day=1;day<=tdays;++day)
     {
   /* Generate Demand */
   x=rand()/32768.0;
   if(x<=0.15) dmd=4;
   else if((x>0.15) && (x<=0.45)) dmd=5;
   else if((x>0.45) && (x<=0.80)) dmd=6;
   else dmd=7;
   cdmd=cdmd+dmd;
   /*    printf("\n
%2d %4d %4d %3d %2d %3d   %2d   %3d   %4d  %3d  %4d",
   day,stk,cstk,ord1,dd1,ord2,dd2,dmd,cdmd,ishort,cshort); */
   if(day>=dd1){
    stk=stk+ord1;
    dd1=0;
    ord1=0; }
   if(day>=dd2){
   stk=stk+ord2;
   dd2=0;
   ord2=0; }
   /*generate lead time*/
   if( stk<=rp1) {
   ord1=q1;
   x=rand()/32768.0;
   if(x<=0.2) lt1=2;
   else if((x>0.2)&&(x<=0.8)) lt1=3;
   else  lt1=4;
   dd1=day+lt1;
   }
   /*    printf("\ndd1=%d lt1=%d",dd1,lt1); */

   if(stk<=rp2) {
   ord2=q2;
   x=rand()/32768.0;
```

```
if(x<=0.2) lt2=2;
else if((x>0.2)&&(x<=0.8)) lt2=3;
else  lt2=4;
dd2=day+lt2;
}

if(dmd<=stk) ishort=0;
else ishort=dmd-stk;
cshort=cshort+ishort;
stk=stk-dmd;
cstk=cstk+stk;
if(stk<=0) stk=0;
if(dd1>0) dd1=dd1-1;
if(dd2>0) dd2=dd2-1;
/*printf("\nday= %d cstk=%d cshort%d",day,cstk,cshort); */

}
float days=tdays;
avstk=cstk/days;
sl=(cdmd-cshort)*100.0/cdmd;
printf("\n Simulation run=%d days",tdays);

printf("\n Q1=%d Q2=%d  RP1=%d  RP2=%d",q1,q2,rp1,rp2);

printf("\n Average stock=%7.2f Service level=%5.2f%",avstk,sl);

}
```

The average stock and service levels obtained for some combinations of q1, q2, rp1 and rp2 are given below.

Length of Run = 100 days

| *Q1* | *Q2* | *RP1* | *RP2* | *Initial Stk = 10* | | *Initial Stk = 5* | |
|---|---|---|---|---|---|---|---|
| | | | | *Av.Stk* | *SL* | *Av.Stk* | *SL* |
| 15 | 15 | 10 | 7 | 5.78 | 74.65 | 5.93 | 73.61 |
| 15 | 10 | 10 | 7 | 4.16 | 72.81 | 3.17 | 68.15 |
| 10 | 15 | 10 | 7 | 3.99 | 68.90 | 4.90 | 73.66 |
| 10 | 10 | 10 | 7 | 2.34 | 64.53 | 2.32 | 66.26 |
| 15 | 10 | 7 | 7 | 1.47 | 66.28 | 1.48 | 66.40 |
| 10 | 10 | 5 | 5 | 1.04 | 62.70 | 0.93 | 62.03 |
| 10 | 10 | 2 | 2 | 0.38 | 56.76 | 0.37 | 56.68 |

## 7.9 Example – Single Order Outstanding

The simulation of an inventory problem with a single order outstanding is given in Table 7.2. In this case lead-time is 4, while the reorder point and reorder quantity are 8 and 15 respectively. The manual simulation carried out for 20 days gives the service rate as 62.5% and the average stock as 5.95 units.

The same inventory situation has been simulated again in Table 7.3 by reducing the lead-time to 3 days. By reduction in lead-time, the performance of the system improves, to service level of 69.32%, but this is at increased average stock of 6.45 units.

Similarly in Table 7.4 and Table 7.5 we find that service level can be increased by increasing the reorder quantity, but this also results in increased average stock held.

**Table 7.2**

Initial stock = 10 Lead-time = 2

Demand 3.4.5.6 p (.20,.30,.35,.15)

Reorder point = 8

Reorder quantity = 15

Single order outstanding.

| *Day* | *Stock* | *ordr.due* | *date.due* | *demand* | *shortage* |
|---|---|---|---|---|---|
| 1 | 10 | 0 | 0 | 6 | 0 |
| 2 | 4 | 15 | 4 | 3 | 0 |
| 3 | 1 | 15 | 3 | 4 | 3 |
| 4 | 0 | 15 | 2 | 3 | 3 |
| 5 | 0 | 15 | 1 | 4 | 4 |
| 6 | 15 | 0 | 0 | 3 | 0 |
| 7 | 12 | 0 | 0 | 5 | 0 |
| 8 | 7 | 15 | 4 | 4 | 0 |
| 9 | 3 | 15 | 3 | 5 | 2 |
| 10 | 0 | 15 | 2 | 6 | 6 |
| 11 | 0 | 15 | 1 | 5 | 5 |
| 12 | 15 | 0 | 0 | 4 | 0 |
| 13 | 11 | 0 | 0 | 3 | 0 |
| 14 | 8 | 15 | 4 | 5 | 0 |
| 15 | 3 | 15 | 3 | 5 | 2 |
| 16 | 0 | 15 | 2 | 4 | 4 |
| 17 | 0 | 15 | 1 | 3 | 3 |
| 18 | 15 | 0 | 0 | 5 | 0 |
| 19 | 10 | 0 | 0 | 5 | 0 |
| 20 | 5 | 15 | 4 | 6 | 1 |
| | 119 | | | 88 | 33 |

Service level = (88 – 33)/88 = 62.5%.

Average stock = 119/20 = 5.95

**Table 7.3**

Initial stock = 10 Lead-time = 3

Demand 3.4.5.6 p (.20, .30, .35, .20)

Reorder point = 8

Reorder quantity = 15

Single order outstanding.

| Day | Stock | ordr.due | date.due | demand | shortage |
|---|---|---|---|---|---|
| 1 | 10 | – | – | 6 | – |
| 2 | 4 | 15 | 3 | 3 | – |
| 3 | 1 | 15 | 2 | 4 | 3 |
| 4 | 0 | 15 | 1 | 3 | 3 |
| 5 | 15 | – | – | 4 | – |
| 6 | 11 | – | – | 3 | – |
| 7 | 8 | 15 | 3 | 5 | – |
| 8 | 3 | 15 | 2 | 4 | 1 |
| 9 | 0 | 15 | 1 | 5 | 5 |
| 10 | 15 | – | – | 6 | – |
| 11 | 9 | – | – | 5 | – |
| 12 | 4 | 15 | 3 | 4 | – |
| 13 | 0 | 15 | 2 | 3 | 3 |
| 14 | 0 | 15 | 1 | 5 | 5 |
| 15 | 15 | – | – | 5 | – |
| 16 | 10 | – | – | 4 | – |
| 17 | 6 | 15 | 3 | 3 | – |
| 18 | 3 | 15 | 2 | 5 | 2 |
| 19 | 0 | 15 | 1 | 5 | 5 |
| 20 | 15 | – | – | 6 | 0 |
| | 129 | | | 88 | 27 |

Service level = (88 – 27)/88 = 69.32%.

Average stock = 129/20 = 6.45

**Table 7.4**

Initial stock = 10 Lead-time = 3

Demand 3.4.5.6 p (.20, .30, .35, .15)

Reorder point = 8

Reorder quantity = 20

Single order outstanding.

| Day | Stock | ordr.due | date.due | demand | shortage |
|---|---|---|---|---|---|
| 1 | 10 | 0 | 0 | 6 | 0 |
| 2 | 4 | 20 | 3 | 3 | – |
| 3 | 1 | 20 | 2 | 4 | 3 |
| 4 | 0 | 20 | 1 | 3 | 3 |
| 5 | 20 | 0 | 0 | 4 | 0 |
| 6 | 16 | 0 | 0 | 3 | 0 |
| 7 | 13 | 0 | 0 | 5 | 0 |
| 8 | 8 | 20 | 3 | 4 | 0 |
| 9 | 4 | 20 | 2 | 5 | 1 |
| 10 | 0 | 20 | 1 | 6 | 6 |




| | | | | | |
|---|---|---|---|---|---|
| 11 | 20 | 0 | 0 | 5 | 0 |
| 12 | 15 | 0 | 0 | 4 | 0 |
| 13 | 11 | 0 | 0 | 3 | 0 |
| 14 | 8 | 20 | 3 | 5 | 0 |
| 15 | 3 | 20 | 2 | 5 | 2 |
| 16 | 0 | 20 | 1 | 4 | 4 |
| 17 | 20 | 0 | 0 | 3 | 0 |
| 18 | 17 | 0 | 0 | 5 | 0 |
| 19 | 12 | 0 | 0 | 5 | 0 |
| 20 | 7 | 20 | 3 | 6 | 0 |
| | 189 | | | 88 | 19 |

Service level = (88 – 19)/88 = 78.4%.

Average stock = 189/20 = 9.45 units.

**Table 7.5**

Initial stock = 10 Lead-time = 3

Demand 3.4.5.6 p (.20, .30, .35, .15)

Reorder point = 15

Reorder quantity = 20

Single order outstanding.

| Day | Stock | ordr.due | date.due | demand | shortage |
|---|---|---|---|---|---|
| 1 | 10 | 20 | 3 | 6 | 0 |
| 2 | 4 | 20 | 2 | 3 | 0 |
| 3 | 1 | 20 | 1 | 4 | 3 |
| 4 | 20 | 0 | 0 | 3 | 0 |
| 5 | 17 | 0 | 0 | 4 | 0 |
| 6 | 13 | 20 | 3 | 3 | 0 |
| 7 | 10 | 20 | 2 | 5 | 0 |
| 8 | 5 | 20 | 1 | 4 | 0 |
| 9 | 21 | 0 | 0 | 5 | 0 |
| 10 | 16 | 0 | 0 | 6 | 0 |
| 11 | 10 | 20 | 3 | 5 | 0 |
| 12 | 5 | 20 | 2 | 4 | 0 |
| 13 | 1 | 20 | 1 | 3 | 2 |
| 14 | 20 | 0 | 0 | 5 | 0 |
| 15 | 15 | 20 | 3 | 5 | 0 |
| 16 | 10 | 20 | 2 | 4 | 0 |
| 17 | 6 | 20 | 1 | 3 | 0 |
| 18 | 23 | 0 | 0 | 5 | 0 |
| 19 | 18 | 0 | 0 | 5 | 0 |
| 20 | 13 | 20 | 3 | 6 | 0 |
| | 248 | | | 88 | 5 |

Service level = (88 – 5)/88 = 94.32%.

Average stock = 248/20 = 12.4

A computer program for the above inventory system, having fixed order quantity, fixed reorder point, fixed lead-time and random demand with empirical distribution, is given below.

```
#include<stdio.h>
#include<stdlib.h>
#include<math.h>

main()

  /*Simulation of Inventory Problem- fixed reorder point, fixed order
    quantity, fixed lead time and variable demand:3,4,5,6 having
    p=(.20,.30,.35,.15).
    Terms used:
    stk - stock held
    cstk - cumulative stock held
    ordd - order due
    ddue - date on which order due
    dmd - demand
    cdmd - cumulative demand
    shortage - unfulfilled demand
    cshortage - cumulative shortage
    rp - reorder point
    avstk - average stock
    sl - service level
    tdays - total days that is length of simulation run.     */

 {
  int day,stk,cstk,ordd,ddue,dmd,cdmd,shortage,cshort,rp,q,lt;
  float x,y,avstk,sl;
  int tdays;

  stk=10;lt=3,rp=10;q=15;day=1;cstk=0;
  ordd=0;ddue=0;shortage=0;dmd=0;cdmd=0;cshort=0;

  printf("\n Length of simulation run (tdays)=");
  scanf("%d",&tdays);
  printf("\n Initial stock=%d",stk);
  printf("\n Lead time =%d",lt);
  printf("\n Reorder point=%d",rp);
  printf("\n reorder quantity=%d",q);
    /*printf("\n Day  Stock  Cstk   Ordd   Ddue  Demand  Cdmd Shortage
Cshort");*/
  for(day=1;day<=tdays;++day) {
     /*printf("\n %2d %5d %6d %6d %6d %6d %6d %6d
%6d",day,stk,cstk,ordd,
   ddue,dmd,cdmd,shortage,cshort);*/
  stk=stk-dmd;
```



```
  if(stk<=0) stk=0;
  if(ddue==0){
  stk=stk+ordd; ordd=0; }

  cstk=cstk+stk;
  if(ordd==0)   {
  if(stk<=rp){ ordd=q; ddue=lt;}
           }

  /*Generate demand*/
  x=rand()/32768.0;
  if(x<=0.20) dmd=3;
  else if((x>0.2)&&(x<=0.50)) dmd=4;
  else if((x>0.5)&&(x<=0.85)) dmd=5;
  else dmd=6;
  cdmd=cdmd+dmd;
  if(dmd>stk) shortage=dmd-stk;
  else shortage=0;
  cshort=cshort+shortage;
    /*printf("\n %2d %5d %6d %6d %6d %6d %6d %6d
%6d",day,stk,cstk,ordd,
    ddue,dmd,cdmd,shortage,cshort);*/
  if(ddue>0)  ddue=ddue-1;
  }
  sl=(cdmd-cshort)*100.0/cdmd;
  float days=tdays;
  avstk=cstk/days;
  printf("\nAverage stock= %6.2f units   Service Level=%6.2f
%",avstk,sl);
 }
```

The program written in C language can be employed to experiment with various combinations of lead-time, reorder quantity and reorder point to study their influence on the system performance. To obtain reasonably reliable results, length of simulation run can be increased. The determination of simulation run and other issues related to the design of simulation experiment are discussed in Chapter 9.

Some results obtained from the above program are given below in Table 7.6.

**Table 7.6**

| *Lead-time* | *Reorder point = 8* | | *Reorder point =10* | |
|---|---|---|---|---|
| | *Ave. stock* | *SL(%)* | *Ave. stock* | *SL(%)* |
| 1 | 11.70 | 98.97 | 13.58 | 99.96 |
| 2 | 8.23 | 82.22 | 9.48 | 91.42 |
| 3 | 6.66 | 66.80 | 7.48 | 74.84 |
| 4 | 5.54 | 55.67 | 6.07 | 60.77 |
| 5 | 4.77 | 47.86 | 5.16 | 51.88 |

**Example 7.1.** A bakery keeps stock of a popular brand of cake. Daily demand based on past experience is given below :

| Daily Demand | 0 | 15 | 25 | 35 | 45 | 50 |
|---|---|---|---|---|---|---|
| Probability | .01 | .15 | .20 | .50 | .12 | .02 |

Using the following sequence of random numbers, simulate the demand for the next 10 days.

48, 78, 09, 87, 99, 77, 15, 14, 68 and 89

Find out the stock situation, if the owner of the bakery decides to make 35 cakes every day. The unmet demand on any day is lost. [P.T.U., B.Tech. (Prod.), Dec. 2005]

**Solution :** This is a very simple inventory problem; Thirty five cakes are made every day. The demand is random and given by a discrete probability distribution. The demand for 10 days can be generated using the probability distribution and the given sequence of random numbers. The daily demand of cakes, probability and cumulative probability are given in Table 7.7. The last column of the table gives the range of two-digit random numbers, corresponding to the probability of occurrence of a particular daily demand.

**Table 7.7**

| Daily Demand | Probability | Commu. Prob. | Random Numbers |
|---|---|---|---|
| 0 | .01 | .01 | 00 |
| 15 | .15 | .16 | 01 – 15 |
| 25 | .20 | .36 | 16 – 35 |
| 35 | .50 | .86 | 36 – 85 |
| 45 | .12 | .98 | 86 – 97 |
| 50 | .02 | 1.00 | 98 – 99 |

Now the daily demand of cakes for 10 days can be generated by using the given sequence of random numbers. Random No. 48 corresponds to day 1. It lies in the random number range 37 – 86, which corresponds to daily demand of 35 cakes. The demand for 10 day is simulated as given below, in Table 7.8.

**Table 7.8**

| Day | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| Random No. | 48 | 78 | 09 | 87 | 99 | 77 | 15 | 14 | 68 | 89 |
| Demand | 35 | 35 | 15 | 45 | 50 | 35 | 15 | 15 | 35 | 45 |

The production rate of cakes is 35 per day. It can be assumed that the left over stock of cakes will be carried over to next day, and any shortage in meeting the demand will amount to sales lost. The simulation of stock is carried out in Table 7.9.

On day 1, demand is 35 cakes, production or supply is also 35, hence no stock is left at the end of the day. Similarly on day 2 demand and supply are equal. But on day 3 demand is only 15 and hence 20 cakes are left in stock at the end of the day. One day 4 while demand is 45, the supply is 20 cakes of previous day and 35 made on 4th day, amounting to 55. Thus 55 – 45 = 10 cakes are left in stock on 4th day. On 5th day demand is 50, while supply is 10 + 35 = 45 cakes, resulting in a shortage of 5 cakes. This shortage cannot be carried to next day, and hence is lost sale. The process of simulation continues as in Table 7.9. At the end of 10th day, the cakes left in stock are 30.



**Table 7.9**

| Day | Demand | Supply | Stock left |
|---|---|---|---|
| 1 | 35 | 35 | 0 |
| 2 | 35 | 35 | 0 |
| 3 | 15 | 35 | 20 |
| 4 | 45 | 20 + 35 | 10 |
| 5 | 50 | 10 + 35 | – 5 |
| 6 | 35 | 35 | 0 |
| 7 | 15 | 35 | 20 |
| 8 | 15 | 20 + 35 | 40 |
| 9 | 35 | 40 + 35 | 40 |
| 10 | 45 | 40 + 35 | 30 |

## 7.10 Exercises

1. An automobile tyre dealer operates his inventory on a single order model. As soon as the stock goes down to $P$ units he places an order of $Q$ units with the wholesaler. The order is placed at the end of the day and is received after three days that is at the beginning of 4th day. Only one order can be outstanding at a time. Any unfulfilled demand is lost forever. The demand on any day can be any number between 0 and 20, each equally probable. Write a simulation program to determine the service level for various combinations of P and Q. The levels of P can be taken as 15, 20, 25 and of Q as 20, 30, 40. Initial stock may be taken as 10 units. For each combination run the simulation for 200 days.
2. In the above problem, the lead-time is fixed at 3 days. Consider the more realistic case when the lead-time can vary slightly and can be any of 2, 3 and 4 days with probabilities of 0.15, 0.75 and 0.10 respectively. Modify the simulation model and run it for the given number of combinations of P and Q for 200 days.
3. In Exercise 1 it was assumed that the unfulfilled demand on any day is lost forever. But in many cases the orders can be held for some small durations if the customer agrees. Let us assume that 50% of the customers agree to hold unfulfilled part of their orders for one day. Modify the simulation model and run it for the said combinations of P and Q to determine the effect on service level.
4. In a single order inventory model the demand per day is distributed normally with mean of 50 units and standard deviation of 10. The lead-time follows Erlang distribution with mean of 3 days and Erlang parameter $k = 2$. The inventory carrying cost is Re. 1.00 per unit per day. Unfulfilled demand is demand lost and costs Rs. 20 per unit. Placement of each order costs Rs. 100. Taking the initial stock on hand as 100 units and no order outstanding, select the order quantity and reorder point so that the total cost is minimum. Reorder points between 115 and 200 and order quantity between 140 and 300 may be tested in some suitable steps.
5. A single order inventory model is given in Exercise 4. Develop a search procedure to determine the minimum cost inventory policy.
6. Simulate the inventory system described in Section 7.8 for two different starting conditions say with initial stock of zero in one, and with initial stock of 20 in second. Plot the variation in Average stock and service level for the first 50 days taking observations after every 10 days and give your comments on the curves obtained.
7. Develop a simulation model for a periodic review inventory system in which the highest inventory level is fixed say as Z units. The stock is checked after a fixed interval of time say $T$ days, and order is placed for varying amount which is equal to the difference between the maximum stock level Z and the stock in hand at the time of review. The inventory carrying cost is Rs. c1 per unit per day, the cost of lost sales is Rs. c2 per unit and the ordering cost is Rs. c3 per order. The demand is stochastic and follows

a normal distribution with mean 'mus' and standard deviation 'sigma'. The lead-time follows exponential distribution with mean 'lt'. The simulation has to evaluate the various (I, Z) policies for a specified simulation run.

8. In Exercise 7 the order is placed for a varying order quantity after every fixed period of time. Now consider the situation in which the order is placed after the fixed interval of time only if the stock level is at or below a specified level P. If it is above P nothing is done till the next review time. Run the simulation for 500 days for the following data.

| | | | | |
|---|---|---|---|---|
| Z = 100, | P = 60, | Demand mean = 10 and sigma = 3. | | |
| Lead-time mean, | lt = 3, | c1 = 1.25, | c2 = 20.0, | c3 = 125.0. |

What will be the effect on total cost if the value of P is made 50, 40 or 70?

9. A baker is trying to determine how many dozens of bread to bake each day. The probability distribution of the number of customers is as follows:

| | | | | |
|---|---|---|---|---|
| Number of customers per day | 10 | 12 | 14 | 16 |
| Probability | .35 | .30 | .20 | .15 |

Customers order 1, 2, 3, or 4 dozen breads according to the following distribution.

| | | | | |
|---|---|---|---|---|
| No. of Dozen ordered/customer | 1 | 2 | 3 | 4 |
| Probability | .4 | .3 | .2 | .1 |

Bread sells for Rs. 60.0 per dozen and cost Rs. 42 per dozen to make. The breads not sold at the end of the day are disposed off at half price. Determine, by simulating the system for 500 days, the dozens of bread, which should be baked each day.

10. A newsstand can buy a daily newspaper for 50 paise each and sell it for 85 paise. The unsold copies, if any, can be disposed of as waste paper at 20 paise each. The estimated daily demand distribution is as follows:

| *Demand (No. of copies)* | *Probability* |
|---|---|
| 100 | .03 |
| 110 | .07 |
| 120 | .19 |
| 130 | .28 |
| 140 | .20 |
| 150 | .10 |
| 160 | .05 |
| 170 | .05 |
| 180 | .03 |

Develop a computer simulation model of the system to determine the optimal number of newspaper copies, which should be procured, so that the expected profit is maximum.