Chapter_4_Classifcation

April 2, 2024

0.1 4.7.1 The Stock Market Data

In this lab we will examine the Smarket data, which is part of the ISLP library. This data set consists of percentage returns for the S&P 500 stock index over 1,250 days, from the beginning of 2001 until the end of 2005. For each date, we have recorded the percentage returns for each of the fve previous trading days, Lag1 through Lag5. We have also recorded Volume (the number of shares traded on the previous day, in billions), Today (the percentage return on the date in question) and Direction (whether the market was Up or Down on this date).

[1]:
```
import numpy as np
import pandas as pd
from matplotlib.pyplot import subplots
import statsmodels.api as sm
from ISLP import load_data
from ISLP.models import (ModelSpec as MS,summarize)
```

[2]:
```
from ISLP import confusion_table
from ISLP.models import contrast
from sklearn.discriminant_analysis import (LinearDiscriminantAnalysis as␣
  ↪LDA,QuadraticDiscriminantAnalysis as QDA)
from sklearn.naive_bayes import GaussianNB
from sklearn.neighbors import KNeighborsClassifier
from sklearn.preprocessing import StandardScaler
from sklearn.model_selection import train_test_split
from sklearn.linear_model import LogisticRegression
```

[3]:
```
Smarket = load_data('Smarket')
Smarket
```

[3]:

	Year	Lag1	Lag2	Lag3	Lag4	Lag5	Volume	Today	Direction
0	2001	0.381	-0.192	-2.624	-1.055	5.010	1.19130	0.959	Up
1	2001	0.959	0.381	-0.192	-2.624	-1.055	1.29650	1.032	Up
2	2001	1.032	0.959	0.381	-0.192	-2.624	1.41120	-0.623	Down
3	2001	-0.623	1.032	0.959	0.381	-0.192	1.27600	0.614	Up
4	2001	0.614	-0.623	1.032	0.959	0.381	1.20570	0.213	Up
…	…	…	…	…	…	…	…	…	…
1245	2005	0.422	0.252	-0.024	-0.584	-0.285	1.88850	0.043	Up
1246	2005	0.043	0.422	0.252	-0.024	-0.584	1.28581	-0.955	Down

```
1247  2005 -0.955  0.043  0.422  0.252 -0.024  1.54047  0.130        Up
1248  2005  0.130 -0.955  0.043  0.422  0.252  1.42236 -0.298      Down
1249  2005 -0.298  0.130 -0.955  0.043  0.422  1.38254 -0.489      Down

[1250 rows x 9 columns]
```

[4]:
```
Smarket.columns
```

[4]:
```
Index(['Year', 'Lag1', 'Lag2', 'Lag3', 'Lag4', 'Lag5', 'Volume', 'Today',
       'Direction'],
      dtype='object')
```

We compute the correlation matrix using the corr() method for dataframes, which produces a matrix that contains all of the pairwise correlations among the variables. (We suppress the output here.) The pandas library does not report a correlation for the Direction variable because it is qualitative.

[5]:
```
Smarket.corr()
```

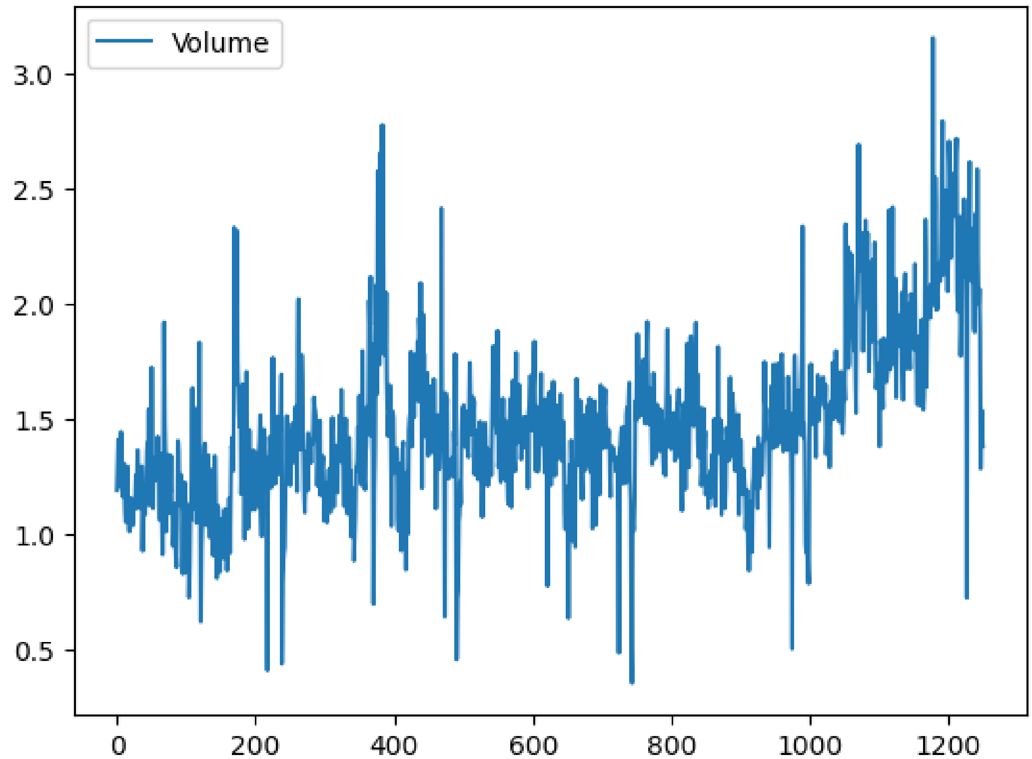
```
C:\Users\ankit19.gupta\AppData\Local\Temp\ipykernel_10472\1907124636.py:1:
FutureWarning: The default value of numeric_only in DataFrame.corr is
deprecated. In a future version, it will default to False. Select only valid
columns or specify the value of numeric_only to silence this warning.
  Smarket.corr()
```

[5]:
```
            Year      Lag1      Lag2      Lag3      Lag4      Lag5    Volume  \
Year    1.000000  0.029700  0.030596  0.033195  0.035689  0.029788  0.539006
Lag1    0.029700  1.000000 -0.026294 -0.010803 -0.002986 -0.005675  0.040910
Lag2    0.030596 -0.026294  1.000000 -0.025897 -0.010854 -0.003558 -0.043383
Lag3    0.033195 -0.010803 -0.025897  1.000000 -0.024051 -0.018808 -0.041824
Lag4    0.035689 -0.002986 -0.010854 -0.024051  1.000000 -0.027084 -0.048414
Lag5    0.029788 -0.005675 -0.003558 -0.018808 -0.027084  1.000000 -0.022002
Volume  0.539006  0.040910 -0.043383 -0.041824 -0.048414 -0.022002  1.000000
Today   0.030095 -0.026155 -0.010250 -0.002448 -0.006900 -0.034860  0.014592

           Today
Year    0.030095
Lag1   -0.026155
Lag2   -0.010250
Lag3   -0.002448
Lag4   -0.006900
Lag5   -0.034860
Volume  0.014592
Today   1.000000
```

As one would expect, the correlations between the lagged return variables and today's return are close to zero. The only substantial correlation is between Year and Volume. By plotting the data we see that Volume is increasing over time. In other words, the average number of shares traded daily increased from 2001 to 2005.

[6]:
```
Smarket.plot(y='Volume');
```



0.2 4.7.2 Logistic Regression

Next, we will ft a logistic regression model in order to predict Direction using Lag1 through Lag5 and Volume. The sm.GLM() function fts generalized linear models, a class of models that includes logistic regression. generalized Alternatively, the function linear model sm.Logit() fts a logistic regression model directly. The syntax of sm.GLM() is similar to that of sm.OLS(), except that we must pass in the argument family=sm.families.Binomial() in order to tell statsmodels to run a logistic regression rather than some other type of generalized linear model.

[7]:
```
allvars = Smarket.columns.drop(['Today', 'Direction', 'Year'])
design = MS(allvars)
X = design.fit_transform(Smarket)
y = Smarket.Direction == 'Up'
glm = sm.GLM(y,X,family=sm.families.Binomial())
results = glm.fit()
summarize(results)
```

[7]:
```
                coef  std err      z  P>|z|
intercept -0.1260    0.241 -0.523  0.601
Lag1      -0.0731    0.050 -1.457  0.145
```

```
Lag2        -0.0423    0.050 -0.845  0.398
Lag3         0.0111    0.050  0.222  0.824
Lag4         0.0094    0.050  0.187  0.851
Lag5         0.0103    0.050  0.208  0.835
Volume       0.1354    0.158  0.855  0.392
```

The smallest p-value here is associated with Lag1. The negative coefcient for this predictor suggests that if the market had a positive return yesterday, then it is less likely to go up today. However, at a value of 0.15, the p-value is still relatively large, and so there is no clear evidence of a real association between Lag1 and Direction.

We use the params attribute of results in order to access just the coefcients for this ftted model.

[8]:
```
results.params
```

[8]:
```
intercept   -0.126000
Lag1        -0.073074
Lag2        -0.042301
Lag3         0.011085
Lag4         0.009359
Lag5         0.010313
Volume       0.135441
dtype: float64
```

Likewise we can use the pvalues attribute to access the p-values for the coefcients (not shown).

[9]:
```
results.pvalues
```

[9]:
```
intercept    0.600700
Lag1         0.145232
Lag2         0.398352
Lag3         0.824334
Lag4         0.851445
Lag5         0.834998
Volume       0.392404
dtype: float64
```

The predict() method of results can be used to predict the probability that the market will go up, given values of the predictors. This method returns predictions on the probability scale. If no data set is supplied to the predict() function, then the probabilities are computed for the training data that was used to ft the logistic regression model. As with linear regression, one can pass an optional exog argument consistent with a design matrix if desired. Here we have printed only the frst ten probabilities.

[10]:
```
probs = results.predict()
probs[:10]
```

[10]:
```
array([0.50708413, 0.48146788, 0.48113883, 0.51522236, 0.51078116,
       0.50695646, 0.49265087, 0.50922916, 0.51761353, 0.48883778])
```

In order to make a prediction as to whether the market will go up or down on a particular day, we must convert these predicted probabilities into class labels, Up or Down. The following two commands create a vector of class predictions based on whether the predicted probability of a market increase is greater than or less than 0.5.

[11]:
```
labels = np.array(['Down']*1250)
labels[probs >0.5] = "Up"
```

[12]:
```
labels
```

[12]:
```
array(['Up', 'Down', 'Down', …, 'Up', 'Up', 'Up'], dtype='<U4')
```

The confusion_table() function from the ISLP package summarizes these confusion_ table() predictions, showing how many observations were correctly or incorrectly classifed. Our function, which is adapted from a similar function in the module sklearn.metrics, transposes the resulting matrix and includes row and column labels. The confusion_table() function takes as frst argument the predicted labels, and second argument the true labels.

[13]:
```
confusion_table(labels, Smarket.Direction)
```

[13]:
```
Truth      Down   Up
Predicted
Down        145  141
Up          457  507
```

The diagonal elements of the confusion matrix indicate correct predictions, while the of-diagonals represent incorrect predictions. Hence our model correctly predicted that the market would go up on 507 days and that it would go down on 145 days, for a total of $507 + 145 = 652$ correct predictions. The np.mean() function can be used to compute the fraction of days for which the prediction was correct. In this case, logistic regression correctly predicted the movement of the market 52.2% of the time.

[14]:
```
(507+145)/1250, np.mean(labels == Smarket.Direction)
```

[14]:
```
(0.5216, 0.5216)
```

At frst glance, it appears that the logistic regression model is working a little better than random guessing. However, this result is misleading because we trained and tested the model on the same set of 1,250 observations. In other words, $100 - 52.2 = 47.8\%$ is the training error rate. As we have seen previously, the training error rate is often overly optimistic — it tends to underestimate the test error rate. In order to better assess the accuracy of the logistic regression model in this setting, we can ft the model using part of the data, and then examine how well it predicts the held out data. This will yield a more realistic error rate, in the sense that in practice we will be interested in our model's performance not on the data that we used to ft the model, but rather on days in the future for which the market's movements are unknown.

To implement this strategy, we frst create a Boolean vector corresponding to the observations from 2001 through 2004. We then use this vector to create a held out data set of observations from 2005.

[15]:
```
train = (Smarket.Year < 2005)
Smarket_train = Smarket.loc[train]
Smarket_test = Smarket.loc[~train]
Smarket_test.shape
```

[15]: (252, 9)

[16]:
```
Smarket_train.shape
```

[16]: (998, 9)

We now ft a logistic regression model using only the subset of the observations that correspond to dates before 2005. We then obtain predicted probabilities of the stock market going up for each of the days in our test set — that is, for the days in 2005.

[17]:
```
X_train, X_test = X.loc[train], X.loc[~train]
y_train, y_test = y.loc[train], y.loc[~train]
glm_train = sm.GLM(y_train,X_train,family=sm.families.Binomial())
results = glm_train.fit()
probs = results.predict(exog=X_test)
```

Notice that we have trained and tested our model on two completely separate data sets: training was performed using only the dates before 2005, and testing was performed using only the dates in 2005. Finally, we compare the predictions for 2005 to the actual movements of the market over that time period. We will frst store the test and training labels (recall y_test is binary).

[18]:
```
D = Smarket.Direction
L_train, L_test = D.loc[train], D.loc[~train]
```

Now we threshold the ftted probability at 50% to form our predicted labels.

[19]:
```
labels = np.array(['Down']*252)
labels[probs >0.5] = 'Up'
confusion_table(labels, L_test)
```

[19]:

Truth	Down	Up
Predicted		
Down	77	97
Up	34	44

[20]:
```
np.mean(labels == L_test), np.mean(labels != L_test)
```

[20]: (0.4801587301587302, 0.5198412698412699)

The test accuracy is about 48% while the error rate is about 52%

The results are rather disappointing: the test error rate is 52%, which is worse than random guessing! Of course this result is not all that surprising, given that one would not generally expect to be able to use previous days' returns to predict future market performance. (After all, if it were possible to do so, then the authors of this book would be out striking it rich rather than writing a statistics textbook.)

We recall that the logistic regression model had very underwhelming pvalues associated with all of the predictors, and that the smallest p-value, though not very small, corresponded to Lag1. Perhaps by removing the variables that appear not to be helpful in predicting Direction, we can obtain a more efective model. After all, using predictors that have no relationship with the response tends to cause a deterioration in the test error rate (since such predictors cause an increase in variance without a corresponding decrease in bias), and so removing such predictors may in turn yield an improvement. Below we reft the logistic regression using just Lag1 and Lag2, which seemed to have the highest predictive power in the original logistic regression model.

[21]:
```
model = MS(['Lag1', 'Lag2']).fit(Smarket)
X = model.transform(Smarket)
X_train, X_test = X.loc[train], X.loc[~train]
glm_train = sm.GLM(y_train,X_train,family=sm.families.Binomial())
results = glm_train.fit()
probs = results.predict(exog=X_test)
labels = np.array(['Down']*252)
labels[probs >0.5] = 'Up'
confusion_table(labels, L_test)
```

[21]:
```
Truth      Down   Up
Predicted
Down         35   35
Up           76  106
```

Let's evaluate the overall accuracy as well as the accuracy within the days when logistic regression predicts an increase.

[22]:
```
(35+106)/252,106/(106+76)
```

[22]:
```
(0.5595238095238095, 0.5824175824175825)
```

Now the results appear to be a little better: 56% of the daily movements have been correctly predicted. It is worth noting that in this case, a much simpler strategy of predicting that the market will increase every day will also be correct 56% of the time! Hence, in terms of overall error rate, the logistic regression method is no better than the naive approach. However, the confusion matrix shows that on days when logistic regression predicts an increase in the market, it has a 58% accuracy rate. This suggests a possible trading strategy of buying on days when the model predicts an increasing market, and avoiding trades on days when a decrease is predicted. Of course one would need to investigate more carefully whether this small improvement was real or just due to random chance.

Suppose that we want to predict the returns associated with particular values of Lag1 and Lag2. In particular, we want to predict Direction on a day when Lag1 and Lag2 equal 1.2 and 1.1, respectively, and on a day when they equal 1.5 and -0.8. We do this using the predict() function.

[23]:
```
newdata = pd.DataFrame({'Lag1':[1.2, 1.5],'Lag2':[1.1, -0.8]});
newX = model.transform(newdata)
results.predict(newX)
```

[23]:
```
0    0.479146
1    0.496094
dtype: float64
```

0.3 4.7.3 Linear Discriminant Analysis

We begin by performing LDA on the Smarket data, using the function LinearDiscriminantAnalysis(), which we have abbreviated LDA(). We ft the model using only the observations before 2005.

[24]:
```
lda = LDA(store_covariance=True)
```

[25]:
```
X_train, X_test = [M.drop(columns=['intercept']) for M in [X_train, X_test]]
lda.fit(X_train, L_train)
```

[25]:
```
LinearDiscriminantAnalysis(store_covariance=True)
```

Having ft the model, we can extract the means in the two classes with the means_ attribute. These are the average of each predictor within each class, and are used by LDA as estimates of μk. These suggest that there is a tendency for the previous 2 days' returns to be negative on days when the market increases, and a tendency for the previous days' returns to be positive on days when the market declines.

[26]:
```
lda.means_
```

[26]:
```
array([[ 0.04279022,  0.03389409],
       [-0.03954635, -0.03132544]])
```

The estimated prior probabilities are stored in the priors_ attribute. The package sklearn typically uses this trailing _ to denote a quantity estimated when using the fit() method. We can be sure of which entry corresponds to which label by looking at the classes_ attribute.

[27]:
```
lda.classes_
```

[27]:
```
array(['Down', 'Up'], dtype='<U4')
```

[28]:
```
lda.priors_
```

[28]:
```
array([0.49198397, 0.50801603])
```

The LDA output indicates that ˆDown = 0.492 and ˆUp = 0.508.

The linear discriminant vectors can be found in the scalings_ attribute:

[29]:
```
lda.scalings_
```

[29]:
```
array([[-0.64201904],
       [-0.51352928]])
```

These values provide the linear combination of Lag1 and Lag2 that are used to form the LDA decision rule. In other words, these are the multipliers of the elements of X = x in (4.24). If $-0.64 \times$ Lag1 $- 0.51 \times$ Lag2 is large, then the LDA classifer will predict a market increase, and if it is small, then the LDA classifer will predict a market decline.

[30]:
```
lda_pred = lda.predict(X_test)
```

As we observed in our comparison of classifcation methods (Section 4.5), the LDA and logistic regression predictions are almost identical.

[31]:
```
confusion_table(lda_pred, L_test)
```

[31]:
```
Truth      Down   Up
Predicted
Down         35   35
Up           76  106
```

We can also estimate the probability of each class for each point in a training set. Applying a 50% threshold to the posterior probabilities of being in class one allows us to recreate the predictions contained in lda_pred.

[32]:
```
lda_prob = lda.predict_proba(X_test)
np.all(np.where(lda_prob[:,1] >= 0.5, 'Up','Down') == lda_pred)
```

[32]:
```
True
```

Above, we used the np.where() function that creates an array with value np.where() 'Up' for indices where the second column of lda_prob (the estimated posterior probability of 'Up') is greater than 0.5. For problems with more than two classes the labels are chosen as the class whose posterior probability is highest:

[33]:
```
np.all([lda.classes_[i] for i in np.argmax(lda_prob, 1)] ==lda_pred)
```

[33]:
```
True
```

If we wanted to use a posterior probability threshold other than 50% in order to make predictions, then we could easily do so. For instance, suppose that we wish to predict a market decrease only if we are very certain that the market will indeed decrease on that day — say, if the posterior probability is at least 90%. We know that the frst column of lda_prob corresponds to the label Down after having checked the classes_ attribute, hence we use the column index 0 rather than 1 as we did above.

[34]:
```
np.sum(lda_prob[:,0] > 0.9)
```

[34]:
```
0
```

No days in 2005 meet that threshold! In fact, the greatest posterior probability of decrease in all of 2005 was 52.02%. The LDA classifer above is the frst classifer from the sklearn library.

We will use several other objects from this library. The objects follow a common structure that simplifes tasks such as cross-validation, which we will see in Chapter 5. Specifcally, the methods frst create a generic classifer without referring to any data. This classifer is then ft to data with the fit() method and predictions are always produced with the predict() method. This pattern of frst instantiating the classifer, followed by ftting it, and then producing predictions is an explicit design choice of sklearn. This uniformity makes it possible to cleanly copy the classifer so that it can be ft on diferent data; e.g. diferent training sets arising in cross-validation. This standard pattern also allows for a predictable formation of workfows.

0.4 4.7.4 Quadratic Discriminant Analysis

We will now ft a QDA model to the Smarket data. QDA is implemented via QuadraticDiscriminantAnalysis() in the sklearn package, which we ab- Quadratic Discriminant Analysis() abbreviate to QDA(). The syntax is very similar to LDA().

[35]:
```
qda = QDA(store_covariance=True)
qda.fit(X_train, L_train)
```

[35]:
```
QuadraticDiscriminantAnalysis(store_covariance=True)
```

[36]:
```
qda.means_, qda.priors_
```

[36]:
```
(array([[ 0.04279022,  0.03389409],
        [-0.03954635, -0.03132544]]),
 array([0.49198397, 0.50801603]))
```

The QDA() classifer will estimate one covariance per class. Here is the estimated covariance in the frst class:

[37]:
```
qda.covariance_[0]
```

[37]:
```
array([[ 1.50662277, -0.03924806],
       [-0.03924806,  1.53559498]])
```

The output contains the group means. But it does not contain the coeffcients of the linear discriminants, because the QDA classifer involves a quadratic, rather than a linear, function of the predictors. The predict() function works in exactly the same fashion as for LDA.

[38]:
```
qda_pred = qda.predict(X_test)
confusion_table(qda_pred, L_test)
```

[38]:
```
Truth      Down   Up
Predicted
Down         30   20
Up           81  121
```

Interestingly, the QDA predictions are accurate almost 60% of the time, even though the 2005 data was not used to ft the model.

[39]:
```
np.mean(qda_pred == L_test)
```

[39]: 0.5992063492063492

This level of accuracy is quite impressive for stock market data, which is known to be quite hard to model accurately. This suggests that the quadratic form assumed by QDA may capture the true relationship more accurately than the linear forms assumed by LDA and logistic regression. However, we recommend evaluating this method's performance on a larger test set before betting that this approach will consistently beat the market!

0.5 4.7.5 Naive Bayes

Next, we ft a naive Bayes model to the Smarket data. The syntax is similar to that of LDA() and QDA(). By default, this implementation of GaussianNB() the naive Bayes classifer models each quantitative feature using a Gaussian distribution. However, a kernel density method can also be used to estimate the distributions.

[40]:
```
NB = GaussianNB()
NB.fit(X_train, L_train)
```

[40]: GaussianNB()

[41]:
```
NB.classes_
```

[41]: array(['Down', 'Up'], dtype='<U4')

The class prior probabilities are stored in the class_prior_ attribute.

[42]:
```
NB.class_prior_
```

[42]: array([0.49198397, 0.50801603])

The parameters of the features can be found in the theta_ and var_ attributes. The number of rows is equal to the number of classes, while the number of columns is equal to the number of features. We see below that the mean for feature Lag1 in the Down class is 0.043.

[43]:
```
NB.theta_
```

[43]:
```
array([[ 0.04279022,  0.03389409],
       [-0.03954635, -0.03132544]])
```

Its variance is 1.503

[44]:
```
NB.var_
```

[44]:
```
array([[1.50355429, 1.53246749],
       [1.51401364, 1.48732877]])
```

How do we know the names of these attributes? We use NB? (or ?NB).

[45]:
```
NB?
```

```
Type:        GaussianNB
String form: GaussianNB()
```

```
File:        c:\users\ankit19.
↪gupta\desktop\self_projects\islp\myenv\lib\site-packages\sklearn\naive_bayes.py
Docstring:
Gaussian Naive Bayes (GaussianNB).

Can perform online updates to model parameters via :meth:`partial_fit`.
For details on algorithm used to update feature means and variance online,
see Stanford CS tech report STAN-CS-79-773 by Chan, Golub, and LeVeque:

    http://i.stanford.edu/pub/cstr/reports/cs/tr/79/773/CS-TR-79-773.pdf

Read more in the :ref:`User Guide <gaussian_naive_bayes>`.

Parameters
----------
priors : array-like of shape (n_classes,), default=None
    Prior probabilities of the classes. If specified, the priors are not
    adjusted according to the data.

var_smoothing : float, default=1e-9
    Portion of the largest variance of all features that is added to
    variances for calculation stability.

    .. versionadded:: 0.20

Attributes
----------
class_count_ : ndarray of shape (n_classes,)
    number of training samples observed in each class.

class_prior_ : ndarray of shape (n_classes,)
    probability of each class.

classes_ : ndarray of shape (n_classes,)
    class labels known to the classifier.

epsilon_ : float
    absolute additive value to variances.

n_features_in_ : int
    Number of features seen during :term:`fit`.

    .. versionadded:: 0.24

feature_names_in_ : ndarray of shape (`n_features_in_`,)
    Names of features seen during :term:`fit`. Defined only when `X`
    has feature names that are all strings.
```

```
    .. versionadded:: 1.0

var_ : ndarray of shape (n_classes, n_features)
    Variance of each feature per class.

    .. versionadded:: 1.0

theta_ : ndarray of shape (n_classes, n_features)
    mean of each feature per class.

See Also
--------
BernoulliNB : Naive Bayes classifier for multivariate Bernoulli models.
CategoricalNB : Naive Bayes classifier for categorical features.
ComplementNB : Complement Naive Bayes classifier.
MultinomialNB : Naive Bayes classifier for multinomial models.

Examples
--------
>>> import numpy as np
>>> X = np.array([[-1, -1], [-2, -1], [-3, -2], [1, 1], [2, 1], [3, 2]])
>>> Y = np.array([1, 1, 1, 2, 2, 2])
>>> from sklearn.naive_bayes import GaussianNB
>>> clf = GaussianNB()
>>> clf.fit(X, Y)
GaussianNB()
>>> print(clf.predict([[-0.8, -1]]))
[1]
>>> clf_pf = GaussianNB()
>>> clf_pf.partial_fit(X, Y, np.unique(Y))
GaussianNB()
>>> print(clf_pf.predict([[-0.8, -1]]))
[1]
```

[46]:
```
?NB
```

```
Type:        GaussianNB
String form: GaussianNB()
File:        c:\users\ankit19.
↪gupta\desktop\self_projects\islp\myenv\lib\site-packages\sklearn\naive_bayes.py
Docstring:
Gaussian Naive Bayes (GaussianNB).

Can perform online updates to model parameters via :meth:`partial_fit`.
For details on algorithm used to update feature means and variance online,
see Stanford CS tech report STAN-CS-79-773 by Chan, Golub, and LeVeque:
```

```
    http://i.stanford.edu/pub/cstr/reports/cs/tr/79/773/CS-TR-79-773.pdf

Read more in the :ref:`User Guide <gaussian_naive_bayes>`.

Parameters
----------
priors : array-like of shape (n_classes,), default=None
    Prior probabilities of the classes. If specified, the priors are not
    adjusted according to the data.

var_smoothing : float, default=1e-9
    Portion of the largest variance of all features that is added to
    variances for calculation stability.

    .. versionadded:: 0.20

Attributes
----------
class_count_ : ndarray of shape (n_classes,)
    number of training samples observed in each class.

class_prior_ : ndarray of shape (n_classes,)
    probability of each class.

classes_ : ndarray of shape (n_classes,)
    class labels known to the classifier.

epsilon_ : float
    absolute additive value to variances.

n_features_in_ : int
    Number of features seen during :term:`fit`.

    .. versionadded:: 0.24

feature_names_in_ : ndarray of shape (`n_features_in_`,)
    Names of features seen during :term:`fit`. Defined only when `X`
    has feature names that are all strings.

    .. versionadded:: 1.0

var_ : ndarray of shape (n_classes, n_features)
    Variance of each feature per class.

    .. versionadded:: 1.0

theta_ : ndarray of shape (n_classes, n_features)
    mean of each feature per class.
```

```
See Also
--------
BernoulliNB : Naive Bayes classifier for multivariate Bernoulli models.
CategoricalNB : Naive Bayes classifier for categorical features.
ComplementNB : Complement Naive Bayes classifier.
MultinomialNB : Naive Bayes classifier for multinomial models.

Examples
--------
>>> import numpy as np
>>> X = np.array([[-1, -1], [-2, -1], [-3, -2], [1, 1], [2, 1], [3, 2]])
>>> Y = np.array([1, 1, 1, 2, 2, 2])
>>> from sklearn.naive_bayes import GaussianNB
>>> clf = GaussianNB()
>>> clf.fit(X, Y)
GaussianNB()
>>> print(clf.predict([[-0.8, -1]]))
[1]
>>> clf_pf = GaussianNB()
>>> clf_pf.partial_fit(X, Y, np.unique(Y))
GaussianNB()
>>> print(clf_pf.predict([[-0.8, -1]]))
[1]
```

We can easily verify the mean computation:

[47]:
```
X_train[L_train == 'Down'].mean()
```

[47]:
```
Lag1    0.042790
Lag2    0.033894
dtype: float64
```

Similarly for the variance:

[48]:
```
X_train[L_train == 'Down'].var(ddof=0)
```

[48]:
```
Lag1    1.503554
Lag2    1.532467
dtype: float64
```

The GaussianNB() function calculates variances using the 1/n formula.6 Since NB() is a classifer in the sklearn library, making predictions uses the same syntax as for LDA() and QDA() above.

[49]:
```
nb_labels = NB.predict(X_test)
confusion_table(nb_labels, L_test)
```

[49]:
```
Truth      Down   Up
Predicted
```

```
Down          29   20
Up            82  121
```

Naive Bayes performs well on these data, with accurate predictions over 59% of the time. This is slightly worse than QDA, but much better than LDA.

As for LDA, the predict_proba() method estimates the probability that each observation belongs to a particular class.

[50]:
```
NB.predict_proba(X_test)[:5]
```

[50]:
```
array([[0.4873288 , 0.5126712 ],
       [0.47623584, 0.52376416],
       [0.46529531, 0.53470469],
       [0.47484469, 0.52515531],
       [0.49020587, 0.50979413]])
```

0.6 4.7.6 K-Nearest Neighbors

We will now perform KNN using the KNeighborsClassifier() function. This function works similarly to the other model-ftting functions that we have encountered thus far.

As is the case for LDA and QDA, we ft the classifer using the fit method. New predictions are formed using the predict method of the object returned by fit().

[51]:
```
knn1 = KNeighborsClassifier(n_neighbors=1)
knn1.fit(X_train, L_train)
knn1_pred = knn1.predict(X_test)
confusion_table(knn1_pred, L_test)
```

[51]:
```
Truth      Down  Up
Predicted
Down         43  58
Up           68  83
```

The results using K = 1 are not very good, since only 50% of the observations are correctly predicted. Of course, it may be that K = 1 results in an overly-fexible ft to the data.

[52]:
```
(83+43)/252, np.mean(knn1_pred == L_test)
```

[52]:
```
(0.5, 0.5)
```

[53]:
```
knn3 = KNeighborsClassifier(n_neighbors=3)
knn3_pred = knn3.fit(X_train, L_train).predict(X_test)
np.mean(knn3_pred == L_test)
```

[53]:
```
0.5317460317460317
```

The results have improved slightly. But increasing K further provides no further improvements. It appears that for these data, and this train/test split, QDA gives the best results of the methods that we have examined so far.

KNN does not perform well on the Smarket data, but it often does provide impressive results. As an example we will apply the KNN approach to the Caravan data set, which is part of the ISLP library. This data set includes 85 predictors that measure demographic characteristics for 5,822 individuals. The response variable is Purchase, which indicates whether or not a given individual purchases a caravan insurance policy. In this data set, only 6% of people purchased caravan insurance.

[54]:
```
Caravan = load_data('Caravan')
Purchase = Caravan.Purchase
Purchase.value_counts()
```

[54]:
```
No      5474
Yes      348
Name: Purchase, dtype: int64
```

The method value_counts() takes a pd.Series or pd.DataFrame and returns a pd.Series with the corresponding counts for each unique element. In this case Purchase has only Yes and No values and returns how many values of each there are.

[55]:
```
348 / 5822
```

[55]:
```
0.05977327378907592
```

Our features will include all columns except Purchase.

[56]:
```
feature_df = Caravan.drop(columns=['Purchase'])
```

Because the KNN classifer predicts the class of a given test observation by identifying the observations that are nearest to it, the scale of the variables matters. Any variables that are on a large scale will have a much larger efect on the distance between the observations, and hence on the KNN classifer, than variables that are on a small scale. For instance, imagine a data set that contains two variables, salary and age (measured in dollars and years, respectively). As far as KNN is concerned, a diference of 1,000 USD in salary is enormous compared to a diference of 50 years in age. Consequently, salary will drive the KNN classifcation results, and age will have almost no efect. This is contrary to our intuition that a salary diference of 1,000 USD is quite small compared to an age diference of 50 years. Furthermore, the importance of scale to the KNN classifer leads to another issue: if we measured salary in Japanese yen, or if we measured age in minutes, then we'd get quite diferent classifcation results from what we get if these two variables are measured in dollars and years.

A good way to handle this problem is to standardize the data so that all standardize variables are given a mean of zero and a standard deviation of one. Then all variables will be on a comparable scale. This is accomplished using the StandardScaler() transformation.

[57]:
```
scaler = StandardScaler(with_mean=True,with_std=True,copy=True)
```

The argument with_mean indicates whether or not we should subtract the mean, while with_std indicates whether or not we should scale the columns to have standard deviation of 1 or not. Finally, the argument copy=True indicates that we will always copy data, rather than trying to do calculations in place where possible.

This transformation can be ft and then applied to arbitrary data. In the frst line below, the parameters for the scaling are computed and stored in scaler, while the second line actually constructs the standardized set of features.

```
[58]: scaler.fit(feature_df)
      X_std = scaler.transform(feature_df)
```

Now every column of feature_std below has a standard deviation of one and a mean of zero.

```
[59]: feature_std = pd.DataFrame(X_std,columns=feature_df.columns);
      feature_std.std()
```

```
[59]: MOSTYPE     1.000086
      MAANTHUI    1.000086
      MGEMOMV     1.000086
      MGEMLEEF    1.000086
      MOSHOOFD    1.000086
                    …
      AZEILPL     1.000086
      APLEZIER    1.000086
      AFIETS      1.000086
      AINBOED     1.000086
      ABYSTAND    1.000086
      Length: 85, dtype: float64
```

Notice that the standard deviations are not quite 1 here; this is again due to some procedures using the 1/n convention for variances (in this case scaler()), while others use $1/(n-1)$ (the std() method). See the footnote .std() on page 183. In this case it does not matter, as long as the variables are all on the same scale.

Using the function train_test_split() we now split the observations into a test set, containing 1000 observations, and a training set containing the remaining observations. The argument random_state=0 ensures that we get the same split each time we rerun the code.

```
[60]: (X_train,X_test,y_train,y_test) =␣
       ↪train_test_split(feature_std,Purchase,test_size=1000,random_state=0)
```

```
[61]: ?train_test_split
```

Signature:

```
train_test_split(
    *arrays,
    test_size=None,
    train_size=None,
    random_state=None,
    shuffle=True,
    stratify=None,
)
Docstring:
Split arrays or matrices into random train and test subsets.

Quick utility that wraps input validation,
``next(ShuffleSplit().split(X, y))``, and application to input data
into a single call for splitting (and optionally subsampling) data into a
one-liner.

Read more in the :ref:`User Guide <cross_validation>`.

Parameters
----------
*arrays : sequence of indexables with same length / shape[0]
    Allowed inputs are lists, numpy arrays, scipy-sparse
    matrices or pandas dataframes.

test_size : float or int, default=None
    If float, should be between 0.0 and 1.0 and represent the proportion
    of the dataset to include in the test split. If int, represents the
    absolute number of test samples. If None, the value is set to the
    complement of the train size. If ``train_size`` is also None, it will
    be set to 0.25.

train_size : float or int, default=None
    If float, should be between 0.0 and 1.0 and represent the
    proportion of the dataset to include in the train split. If
    int, represents the absolute number of train samples. If None,
    the value is automatically set to the complement of the test size.

random_state : int, RandomState instance or None, default=None
    Controls the shuffling applied to the data before applying the split.
    Pass an int for reproducible output across multiple function calls.
    See :term:`Glossary <random_state>`.

shuffle : bool, default=True
    Whether or not to shuffle the data before splitting. If shuffle=False
```

```
    then stratify must be None.

stratify : array-like, default=None
    If not None, data is split in a stratified fashion, using this as
    the class labels.
    Read more in the :ref:`User Guide <stratification>`.

Returns
-------
splitting : list, length=2 * len(arrays)
    List containing train-test split of inputs.

    .. versionadded:: 0.16
        If the input is sparse, the output will be a
        ``scipy.sparse.csr_matrix``. Else, output type is the same as the
        input type.

Examples
--------
>>> import numpy as np
>>> from sklearn.model_selection import train_test_split
>>> X, y = np.arange(10).reshape((5, 2)), range(5)
>>> X
array([[0, 1],
       [2, 3],
       [4, 5],
       [6, 7],
       [8, 9]])
>>> list(y)
[0, 1, 2, 3, 4]

>>> X_train, X_test, y_train, y_test = train_test_split(
…     X, y, test_size=0.33, random_state=42)
…
>>> X_train
array([[4, 5],
       [0, 1],
       [6, 7]])
>>> y_train
[2, 0, 3]
>>> X_test
array([[2, 3],
       [8, 9]])
>>> y_test
[1, 4]

>>> train_test_split(y, shuffle=False)
[[0, 1, 2], [3, 4]]
```

```
File:      c:\users\ankit19.
 ↪gupta\desktop\self_projects\islp\myenv\lib\site-packages\sklearn\model_selection\_split.
 ↪py
Type:      function
```

?train_test_split reveals that the non-keyword arguments can be lists, arrays, pandas dataframes etc that all have the same length (shape[0]) and hence are indexable. In this case they are the dataframe feature_std and the response variable Purchase. We ft a KNN model on the training data using K = 1, and evaluate its performance on the test data.

[62]:
```
knn1 = KNeighborsClassifier(n_neighbors=1)
knn1_pred = knn1.fit(X_train, y_train).predict(X_test)
np.mean(y_test != knn1_pred), np.mean(y_test != "No")
```

[62]:
```
(0.111, 0.067)
```

The KNN error rate on the 1,000 test observations is about 11%. At frst glance, this may appear to be fairly good. However, since just over 6% of customers purchased insurance, we could get the error rate down to almost 6% by always predicting No regardless of the values of the predictors! This is known as the null rate.

Suppose that there is some non-trivial cost to trying to sell insurance to a given individual. For instance, perhaps a salesperson must visit each potential customer. If the company tries to sell insurance to a random selection of customers, then the success rate will be only 6%, which may be far too low given the costs involved. Instead, the company would like to try to sell insurance only to customers who are likely to buy it. So the overall error rate is not of interest. Instead, the fraction of individuals that are correctly predicted to buy insurance is of interest.

[63]:
```
confusion_table(knn1_pred, y_test)
```

[63]:
```
Truth       No  Yes
Predicted
No         880   58
Yes         53    9
```

It turns out that KNN with K = 1 does far better than random guessing among the customers that are predicted to buy insurance. Among 62 such customers, 9, or 14.5%, actually do purchase insurance. This is double the rate that one would obtain from random guessing.

[64]:
```
9/(53+9)
```

[64]:
```
0.14516129032258066
```

0.6.1 Tuning Parameters

The number of neighbors in KNN is referred to as a tuning parameter, also referred to as a hyper-parameter. We do not know a priori what value to use. It is therefore of interest to see how the classifer performs on test data as we vary these parameters. This can be achieved with a for loop, described in Section 2.3.8. Here we use a for loop to look at the accuracy of our classifer in the group predicted to purchase insurance as we vary the number of neighbors from 1 to 5:

[65]:
```
for K in range(1,6):
    knn = KNeighborsClassifier(n_neighbors=K)
    knn_pred = knn.fit(X_train, y_train).predict(X_test)
    C = confusion_table(knn_pred, y_test)
    templ = ('K={0:d}: # predicted to rent: {1:>2},' +
    ' # who did rent {2:d}, accuracy {3:.1%}')
    pred = C.loc['Yes'].sum()
    did_rent = C.loc['Yes','Yes']
    print(templ.format(K,pred,did_rent,did_rent / pred))
```

```
K=1: # predicted to rent: 62, # who did rent 9, accuracy 14.5%
K=2: # predicted to rent:  6, # who did rent 1, accuracy 16.7%
K=3: # predicted to rent: 20, # who did rent 3, accuracy 15.0%
K=4: # predicted to rent:  4, # who did rent 0, accuracy 0.0%
K=5: # predicted to rent:  7, # who did rent 1, accuracy 14.3%
```

We see some variability — the numbers for K=4 are very diferent from the rest.

0.6.2 Comparison to Logistic Regression

As a comparison, we can also ft a logistic regression model to the data. This can also be done with sklearn, though by default it fts something like the ridge regression version of logistic regression, which we introduce in Chapter 6. This can be modifed by appropriately setting the argument C below. Its default value is 1 but by setting it to a very large number, the algorithm converges to the same solution as the usual (unregularized) logistic regression estimator discussed above.

Unlike the statsmodels package, sklearn focuses less on inference and more on classifcation. Hence, the summary methods seen in statsmodels and our simplifed version seen with summarize are not generally available for the classifers in sklearn

[66]:
```
logit = LogisticRegression(C=1e10, solver='liblinear')
logit.fit(X_train, y_train)
logit_pred = logit.predict_proba(X_test)
logit_labels = np.where(logit_pred[:,1] > 5, 'Yes', 'No')
confusion_table(logit_labels, y_test)
```

[66]:

Truth	No	Yes
Predicted		
No	933	67
Yes	0	0

We used the argument solver='liblinear' above to avoid a warning with the default solver which would indicate that the algorithm does not converge.

If we use 0.5 as the predicted probability cut-of for the classifer, then we have a problem: none of the test observations are predicted to purchase insurance. However, we are not required to use a cut-of of 0.5. If we instead predict a purchase any time the predicted probability of purchase exceeds 0.25, we get much better results: we predict that 29 people will purchase insurance, and we are correct for about 31% of these people. This is almost fve times better than random guessing

[67]:
```python
logit_labels = np.where(logit_pred[:,1]>0.25, 'Yes', 'No')
confusion_table(logit_labels, y_test)
```

[67]:
```
Truth       No  Yes
Predicted
No         913   58
Yes         20    9
```

[68]:
```python
9/(20+9)
```

[68]:
```
0.3103448275862069
```

0.7 4.7.7 Linear and Poisson Regression on the Bikeshare Data

Here we ft linear and Poisson regression models to the Bikeshare data, as described in Section 4.6. The response bikers measures the number of bike rentals per hour in Washington, DC in the period 2010–2012.

[69]:
```python
Bike = load_data('Bikeshare')
```

Let’s have a peek at the dimensions and names of the variables in this dataframe.

[70]:
```python
Bike.shape, Bike.columns
```

[70]:
```
((8645, 15),
 Index(['season', 'mnth', 'day', 'hr', 'holiday', 'weekday', 'workingday',
        'weathersit', 'temp', 'atemp', 'hum', 'windspeed', 'casual',
        'registered', 'bikers'],
       dtype='object'))
```

0.7.1 Linear Regression

We begin by ftting a linear regression model to the data.

[71]:
```python
X = MS(['mnth','hr','workingday','temp','weathersit']).fit_transform(Bike)
Y = Bike['bikers']
M_lm = sm.OLS(Y, X).fit()
summarize(M_lm)
```

[71]:

	coef	std err	t	P>|t|
intercept	-68.6317	5.307	-12.932	0.000
mnth[Feb]	6.8452	4.287	1.597	0.110
mnth[March]	16.5514	4.301	3.848	0.000
mnth[April]	41.4249	4.972	8.331	0.000
mnth[May]	72.5571	5.641	12.862	0.000
mnth[June]	67.8187	6.544	10.364	0.000
mnth[July]	45.3245	7.081	6.401	0.000
mnth[Aug]	53.2430	6.640	8.019	0.000
mnth[Sept]	66.6783	5.925	11.254	0.000

```
mnth[Oct]                        75.8343    4.950  15.319  0.000
mnth[Nov]                        60.3100    4.610  13.083  0.000
mnth[Dec]                        46.4577    4.271  10.878  0.000
hr[1]                           -14.5793    5.699  -2.558  0.011
hr[2]                           -21.5791    5.733  -3.764  0.000
hr[3]                           -31.1408    5.778  -5.389  0.000
hr[4]                           -36.9075    5.802  -6.361  0.000
hr[5]                           -24.1355    5.737  -4.207  0.000
hr[6]                            20.5997    5.704   3.612  0.000
hr[7]                           120.0931    5.693  21.095  0.000
hr[8]                           223.6619    5.690  39.310  0.000
hr[9]                           120.5819    5.693  21.182  0.000
hr[10]                           83.8013    5.705  14.689  0.000
hr[11]                          105.4234    5.722  18.424  0.000
hr[12]                          137.2837    5.740  23.916  0.000
hr[13]                          136.0359    5.760  23.617  0.000
hr[14]                          126.6361    5.776  21.923  0.000
hr[15]                          132.0865    5.780  22.852  0.000
hr[16]                          178.5206    5.772  30.927  0.000
hr[17]                          296.2670    5.749  51.537  0.000
hr[18]                          269.4409    5.736  46.976  0.000
hr[19]                          186.2558    5.714  32.596  0.000
hr[20]                          125.5492    5.704  22.012  0.000
hr[21]                           87.5537    5.693  15.378  0.000
hr[22]                           59.1226    5.689  10.392  0.000
hr[23]                           26.8376    5.688   4.719  0.000
workingday                        1.2696    1.784   0.711  0.477
temp                            157.2094   10.261  15.321  0.000
weathersit[cloudy/misty]        -12.8903    1.964  -6.562  0.000
weathersit[heavy rain/snow]    -109.7446   76.667  -1.431  0.152
weathersit[light rain/snow]     -66.4944    2.965 -22.425  0.000
```

There are 24 levels in hr and 40 rows in all, so we have truncated the summary. In M_lm, the frst levels hr[0] and mnth[Jan] are treated as the baseline values, and so no coefcient estimates are provided for them: implicitly, their coefcient estimates are zero, and all other levels are measured relative to these baselines. For example, the Feb coefcient of 6.845 signifes that, holding all other variables constant, there are on average about 7 more riders in February than in January. Similarly there are about 16.5 more riders in March than in January.

The results seen in Section 4.6.1 used a slightly diferent coding of the variables hr and mnth, as follows:

[72]:
```
hr_encode = contrast('hr', 'sum')
mnth_encode = contrast('mnth', 'sum')
```

Reftting again:

[73]:
```
X2 = MS([mnth_encode,hr_encode,'workingday','temp','weathersit']).
 ↪fit_transform(Bike)
M2_lm = sm.OLS(Y, X2).fit()
S2 = summarize(M2_lm)
S2
```

[73]:

	coef	std err	t	P>|t|
intercept	73.5974	5.132	14.340	0.000
mnth[Jan]	-46.0871	4.085	-11.281	0.000
mnth[Feb]	-39.2419	3.539	-11.088	0.000
mnth[March]	-29.5357	3.155	-9.361	0.000
mnth[April]	-4.6622	2.741	-1.701	0.089
mnth[May]	26.4700	2.851	9.285	0.000
mnth[June]	21.7317	3.465	6.272	0.000
mnth[July]	-0.7626	3.908	-0.195	0.845
mnth[Aug]	7.1560	3.535	2.024	0.043
mnth[Sept]	20.5912	3.046	6.761	0.000
mnth[Oct]	29.7472	2.700	11.019	0.000
mnth[Nov]	14.2229	2.860	4.972	0.000
hr[0]	-96.1420	3.955	-24.307	0.000
hr[1]	-110.7213	3.966	-27.916	0.000
hr[2]	-117.7212	4.016	-29.310	0.000
hr[3]	-127.2828	4.081	-31.191	0.000
hr[4]	-133.0495	4.117	-32.319	0.000
hr[5]	-120.2775	4.037	-29.794	0.000
hr[6]	-75.5424	3.992	-18.925	0.000
hr[7]	23.9511	3.969	6.035	0.000
hr[8]	127.5199	3.950	32.284	0.000
hr[9]	24.4399	3.936	6.209	0.000
hr[10]	-12.3407	3.936	-3.135	0.002
hr[11]	9.2814	3.945	2.353	0.019
hr[12]	41.1417	3.957	10.397	0.000
hr[13]	39.8939	3.975	10.036	0.000
hr[14]	30.4940	3.991	7.641	0.000
hr[15]	35.9445	3.995	8.998	0.000
hr[16]	82.3786	3.988	20.655	0.000
hr[17]	200.1249	3.964	50.488	0.000
hr[18]	173.2989	3.956	43.806	0.000
hr[19]	90.1138	3.940	22.872	0.000
hr[20]	29.4071	3.936	7.471	0.000
hr[21]	-8.5883	3.933	-2.184	0.029
hr[22]	-37.0194	3.934	-9.409	0.000
workingday	1.2696	1.784	0.711	0.477
temp	157.2094	10.261	15.321	0.000
weathersit[cloudy/misty]	-12.8903	1.964	-6.562	0.000
weathersit[heavy rain/snow]	-109.7446	76.667	-1.431	0.152
weathersit[light rain/snow]	-66.4944	2.965	-22.425	0.000

What is the diference between the two codings? In M2_lm, a coefcient estimate is reported for all but level 23 of hr and level Dec of mnth. Importantly, in M2_lm, the (unreported) coefcient estimate for the last level of mnth is not zero: instead, it equals the negative of the sum of the coefcient estimates for all of the other levels. Similarly, in M2_lm, the coefcient estimate for the last level of hr is the negative of the sum of the coefcient estimates for all of the other levels. This means that the coefcients of hr and mnth in M2_lm will always sum to zero, and can be interpreted as the diference from the mean level. For example, the coefcient for January of -46.087 indicates that, holding all other variables constant, there are typically 46 fewer riders in January relative to the yearly average.

It is important to realize that the choice of coding really does not matter, provided that we interpret the model output correctly in light of the coding used. For example, we see that the predictions from the linear model are the same regardless of coding:

[74]:
```
np.sum((M_lm.fittedvalues - M2_lm.fittedvalues)**2)
```

[74]:
```
5.2568472069513736e-20
```

The sum of squared diferences is zero. We can also see this using the np.allclose() function:

[75]:
```
np.allclose(M_lm.fittedvalues , M2_lm.fittedvalues)
```

[75]:
```
True
```

To reproduce the left-hand side of Figure 4.13 we must frst obtain the coefcient estimates associated with mnth. The coefcients for January through November can be obtained directly from the M2_lm object. The coefcient for December must be explicitly computed as the negative sum of all the other months. We frst extract all the coefcients for month from the coefcients of M2_lm.

[76]:
```
coef_month = S2[S2.index.str.contains('mnth')]['coef']
coef_month
```

[76]:
```
mnth[Jan]     -46.0871
mnth[Feb]     -39.2419
mnth[March]   -29.5357
mnth[April]    -4.6622
mnth[May]      26.4700
mnth[June]     21.7317
mnth[July]     -0.7626
mnth[Aug]       7.1560
mnth[Sept]     20.5912
mnth[Oct]      29.7472
mnth[Nov]      14.2229
Name: coef, dtype: float64
```

Next, we append Dec as the negative of the sum of all other months.

[77]:
```
months = Bike['mnth'].dtype.categories
coef_month = pd.concat([
coef_month,
```

```
pd.Series([-coef_month.sum()],index=['mnth[Dec]'])])
coef_month
```

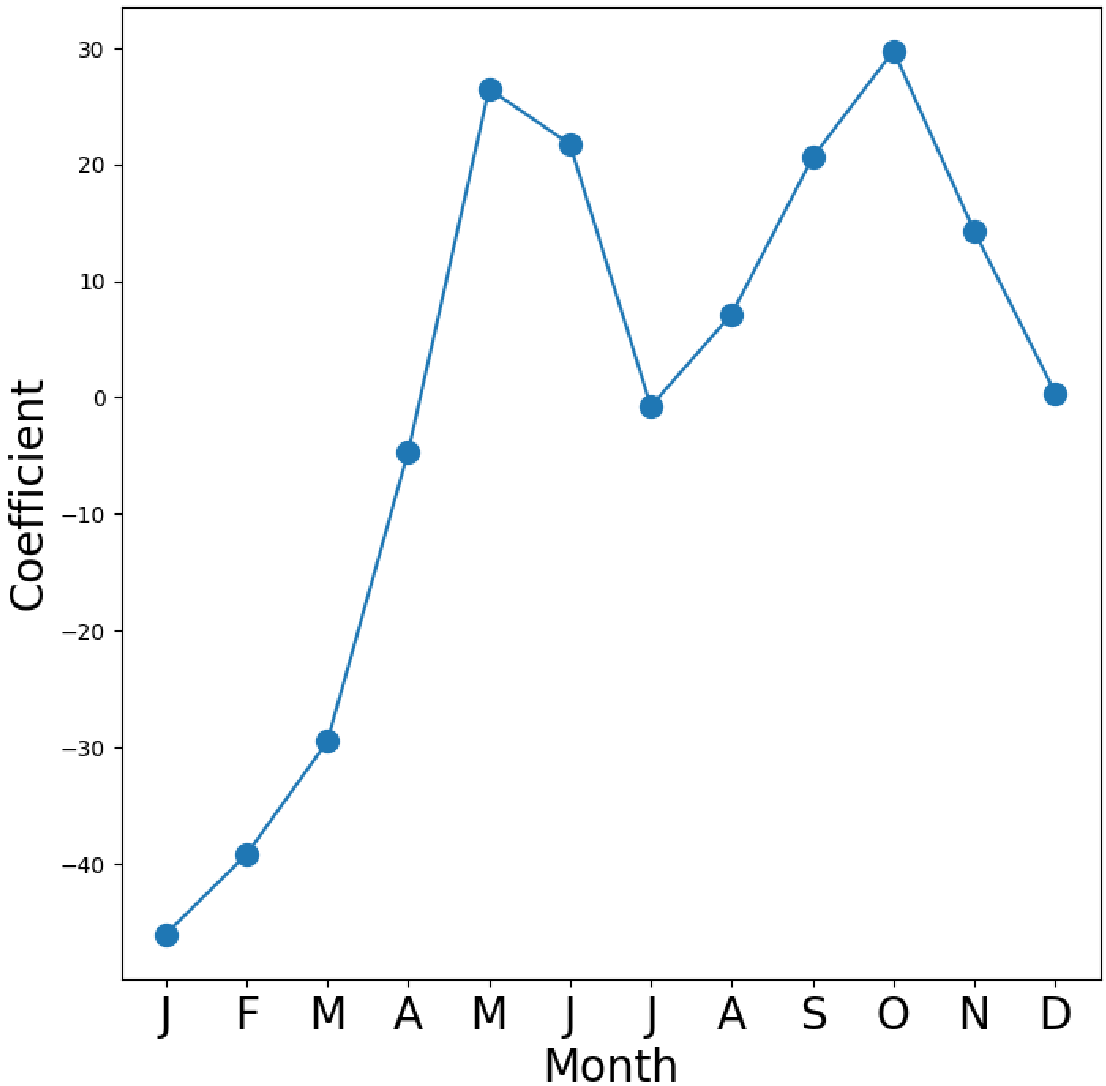
[77]:

```
mnth[Jan]      -46.0871
mnth[Feb]      -39.2419
mnth[March]    -29.5357
mnth[April]     -4.6622
mnth[May]       26.4700
mnth[June]      21.7317
mnth[July]      -0.7626
mnth[Aug]        7.1560
mnth[Sept]      20.5912
mnth[Oct]       29.7472
mnth[Nov]       14.2229
mnth[Dec]        0.3705
dtype: float64
```

Finally, to make the plot neater, we'll just use the frst letter of each month, which is the 6th entry of each of the labels in the index.

[78]:

```
fig_month, ax_month = subplots(figsize=(8,8))
x_month = np.arange(coef_month.shape[0])
ax_month.plot(x_month, coef_month, marker='o', ms=10)
ax_month.set_xticks(x_month)
ax_month.set_xticklabels([l[5] for l in coef_month.index], fontsize
=20)
ax_month.set_xlabel('Month', fontsize=20)
ax_month.set_ylabel('Coefficient', fontsize=20);
```



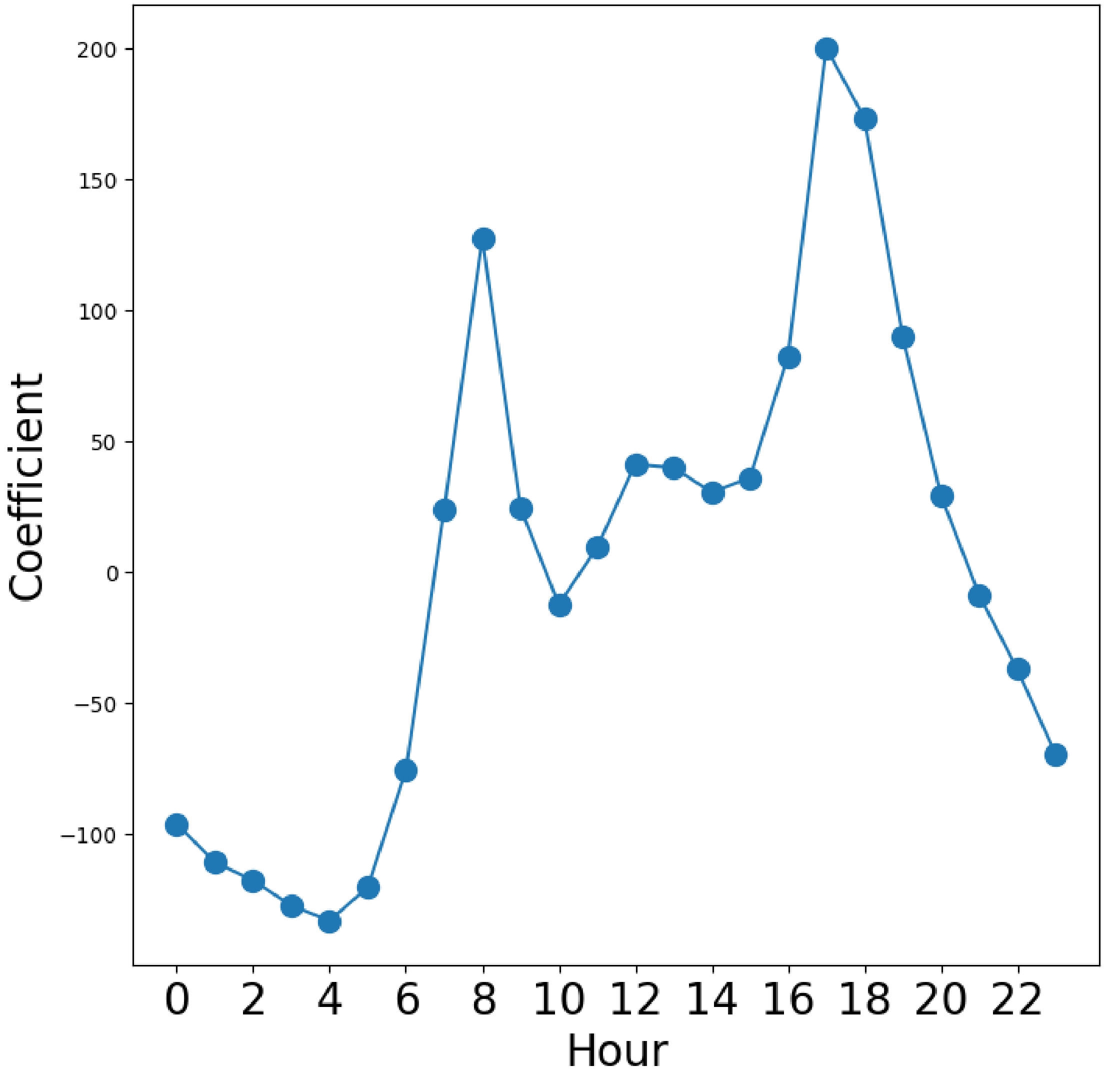
Reproducing the right-hand plot in Figure 4.13 follows a similar process.

[80]:
```
coef_hr = S2[S2.index.str.contains('hr')]['coef']
coef_hr = coef_hr.reindex(['hr[{0}]'.format(h) for h in range(23)])
coef_hr = pd.concat([coef_hr,pd.Series([-coef_hr.sum()], index=['hr[23]'])])
```

We now make the hour plot.

[81]:
```
fig_hr, ax_hr = subplots(figsize=(8,8))
x_hr = np.arange(coef_hr.shape[0])
ax_hr.plot(x_hr, coef_hr, marker='o', ms=10)
ax_hr.set_xticks(x_hr[::2])
ax_hr.set_xticklabels(range(24)[::2], fontsize=20)
ax_hr.set_xlabel('Hour', fontsize=20)
```

```python
ax_hr.set_ylabel('Coefficient', fontsize=20);
```



0.7.2 Poisson Regression

Now we ft instead a Poisson regression model to the Bikeshare data. Very little changes, except that we now use the function sm.GLM() with the Poisson family specifed:

[82]:
```python
M_pois = sm.GLM(Y, X2, family=sm.families.Poisson()).fit()
```

We can plot the coefcients associated with mnth and hr, in order to reproduce Figure 4.15. We frst complete these coefcients as before.

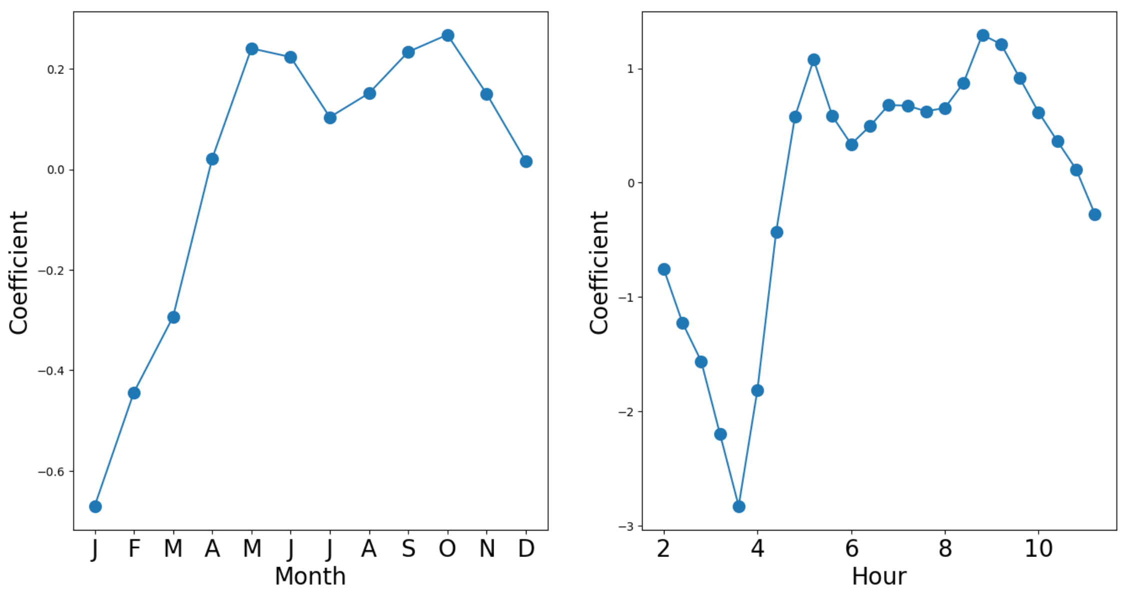
[83]:
```python
S_pois = summarize(M_pois)
coef_month = S_pois[S_pois.index.str.contains('mnth')]['coef']
```

```
coef_month = pd.concat([coef_month,pd.Series([-coef_month.
↪sum()],index=['mnth[Dec]'])])
coef_hr = S_pois[S_pois.index.str.contains('hr')]['coef']
coef_hr = pd.concat([coef_hr,pd.Series([-coef_hr.sum()],index=['hr[23]'])])
```

The plotting is as before.

[84]:
```
fig_pois, (ax_month, ax_hr) = subplots(1, 2, figsize=(16,8))
ax_month.plot(x_month, coef_month, marker='o', ms=10)
ax_month.set_xticks(x_month)
ax_month.set_xticklabels([l[5] for l in coef_month.index], fontsize=20)
ax_month.set_xlabel('Month', fontsize=20)
ax_month.set_ylabel('Coefficient', fontsize=20)
ax_hr.plot(x_hr, coef_hr, marker='o', ms=10)
ax_hr.set_xticklabels(range(24)[::2], fontsize=20)
ax_hr.set_xlabel('Hour', fontsize=20)
ax_hr.set_ylabel('Coefficient', fontsize=20);
```

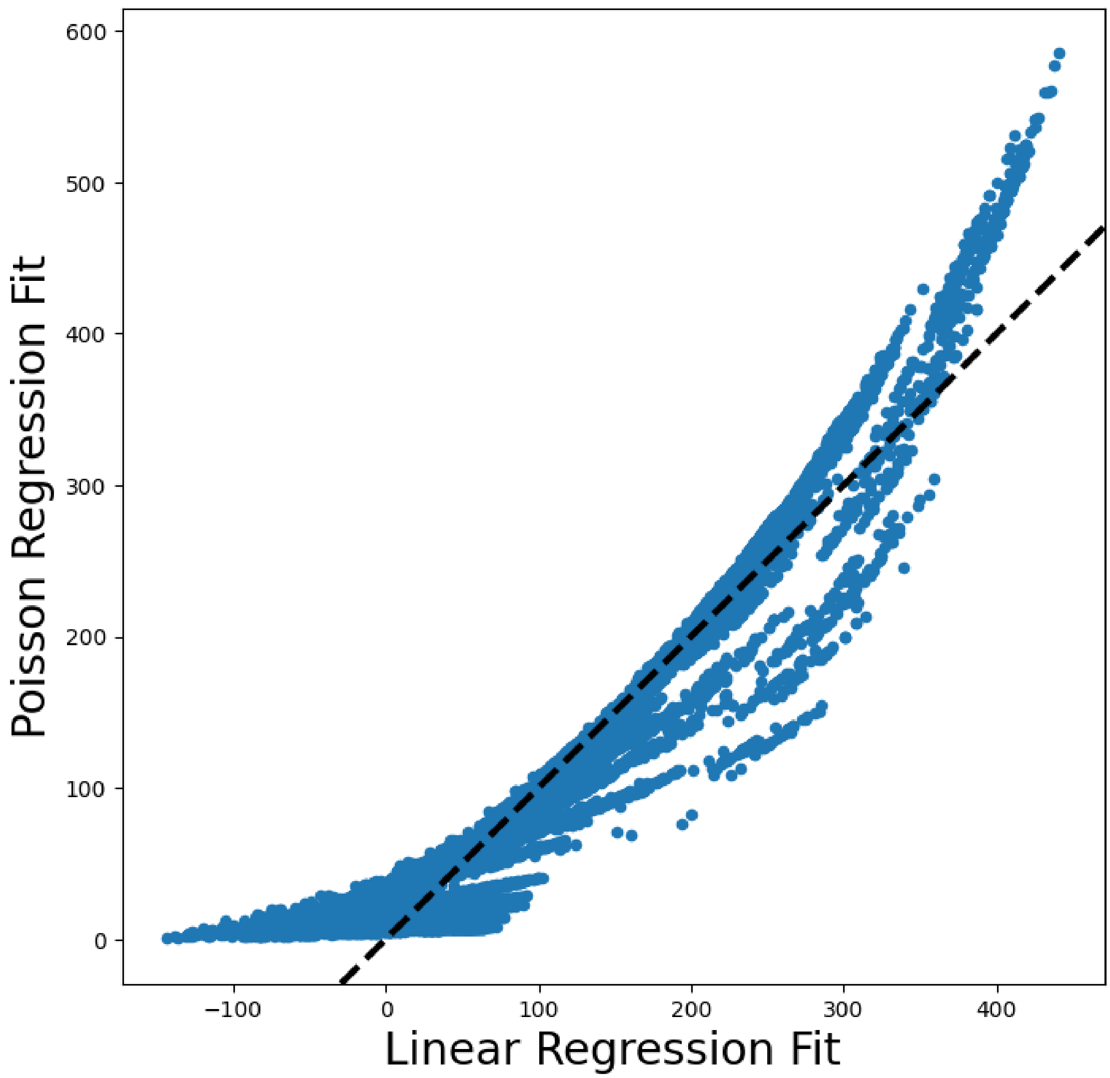
```
C:\Users\ankit19.gupta\AppData\Local\Temp\ipykernel_10472\3779510511.py:8:
UserWarning: set_ticklabels() should only be used with a fixed number of ticks,
i.e. after set_ticks() or using a FixedLocator.
  ax_hr.set_xticklabels(range(24)[::2], fontsize=20)
```



We compare the ftted values of the two models. The ftted values are stored in the fittedvalues attribute returned by the fit() method for both the linear regression and the Poisson fts. The linear predictors are stored as the attribute lin_pred.

[85]:

```
fig, ax = subplots(figsize=(8, 8))
ax.scatter(M2_lm.fittedvalues ,M_pois.fittedvalues,s=20)
ax.set_xlabel('Linear Regression Fit', fontsize=20)
ax.set_ylabel('Poisson Regression Fit', fontsize=20)
ax.axline([0,0], c='black', linewidth=3,linestyle='--', slope=1);
```



The predictions from the Poisson regression model are correlated with those from the linear model; however, the former are non-negative. As a result the Poisson regression predictions tend to be larger than those from the linear model for either very low or very high levels of ridership. In this section, we ft Poisson regression models using the sm.GLM() function with the argument family=sm.families.Poisson(). Earlier in this lab we used the sm.GLM() function with family=sm.families.Binomial() to perform logistic regression. Other choices for the family argument can be used to ft other types of GLMs. For instance, family=sm.families.Gamma() fts a Gamma regression model.

```
[ ]:
```