Chapter_3_Linear_Regression

April 2, 2024

0.1 Importing packages

We import our standard libraries at this top level.

[1]:
```
import numpy as np
import pandas as pd
from matplotlib.pyplot import subplots
```

0.2 New imports

Throughout this lab we will introduce new functions and libraries. However, we will import them here to emphasize these are the new code objects in this lab. Keeping imports near the top of a notebook makes the code more readable, since scanning the frst few lines tells us what libraries are used.

[2]:
```
import statsmodels.api as sm
```

Besides importing whole modules, it is also possible to import only a few items from a given module. This will help keep the namespace clean. namespace We will use a few specifc objects from the statsmodels package which we statsmodels import here.

[3]:
```
from statsmodels.stats.outliers_influence import variance_inflation_factor as␣
  ↪VIF
from statsmodels.stats.anova import anova_lm
```

We will also use some functions written for the labs in this book in the ISLP package.

[4]:
```
from ISLP import load_data
from ISLP.models import (ModelSpec as MS,summarize,poly)
```

0.3 Inspecting Objects and Namespaces

The function dir() provides a list of objects in a namespace.

[5]:
```
dir()
```

[5]:
```
['In',
 'MS',
 'Out',
 'VIF',
```

```
 '_',
 '__',
 '___',
 '__builtin__',
 '__builtins__',
 '__doc__',
 '__loader__',
 '__name__',
 '__package__',
 '__session__',
 '__spec__',
 '_dh',
 '_i',
 '_i1',
 '_i2',
 '_i3',
 '_i4',
 '_i5',
 '_ih',
 '_ii',
 '_iii',
 '_oh',
 'anova_lm',
 'exit',
 'get_ipython',
 'load_data',
 'np',
 'open',
 'pd',
 'poly',
 'quit',
 'sm',
 'subplots',
 'summarize']
```

This shows you everything that Python can fnd at the top level. There are certain objects like **builtins** that contain references to built-in functions like print().

Every python object has its own notion of namespace, also accessible with dir(). This will include both the attributes of the object as well as any methods associated with it. For instance, we see 'sum' in the listing for an array

[6]:
```
A = np.array([3,5,11])
dir(A)
```

[6]:
```
['T',
 '__abs__',
 '__add__',
```

```
'__and__',
'__array__',
'__array_finalize__',
'__array_function__',
'__array_interface__',
'__array_prepare__',
'__array_priority__',
'__array_struct__',
'__array_ufunc__',
'__array_wrap__',
'__bool__',
'__class__',
'__class_getitem__',
'__complex__',
'__contains__',
'__copy__',
'__deepcopy__',
'__delattr__',
'__delitem__',
'__dir__',
'__divmod__',
'__dlpack__',
'__dlpack_device__',
'__doc__',
'__eq__',
'__float__',
'__floordiv__',
'__format__',
'__ge__',
'__getattribute__',
'__getitem__',
'__gt__',
'__hash__',
'__iadd__',
'__iand__',
'__ifloordiv__',
'__ilshift__',
'__imatmul__',
'__imod__',
'__imul__',
'__index__',
'__init__',
'__init_subclass__',
'__int__',
'__invert__',
'__ior__',
'__ipow__',
```

```
'__irshift__',
'__isub__',
'__iter__',
'__itruediv__',
'__ixor__',
'__le__',
'__len__',
'__lshift__',
'__lt__',
'__matmul__',
'__mod__',
'__mul__',
'__ne__',
'__neg__',
'__new__',
'__or__',
'__pos__',
'__pow__',
'__radd__',
'__rand__',
'__rdivmod__',
'__reduce__',
'__reduce_ex__',
'__repr__',
'__rfloordiv__',
'__rlshift__',
'__rmatmul__',
'__rmod__',
'__rmul__',
'__ror__',
'__rpow__',
'__rrshift__',
'__rshift__',
'__rsub__',
'__rtruediv__',
'__rxor__',
'__setattr__',
'__setitem__',
'__setstate__',
'__sizeof__',
'__str__',
'__sub__',
'__subclasshook__',
'__truediv__',
'__xor__',
'all',
'any',
```

```
'argmax',
'argmin',
'argpartition',
'argsort',
'astype',
'base',
'byteswap',
'choose',
'clip',
'compress',
'conj',
'conjugate',
'copy',
'ctypes',
'cumprod',
'cumsum',
'data',
'diagonal',
'dot',
'dtype',
'dump',
'dumps',
'fill',
'flags',
'flat',
'flatten',
'getfield',
'imag',
'item',
'itemset',
'itemsize',
'max',
'mean',
'min',
'nbytes',
'ndim',
'newbyteorder',
'nonzero',
'partition',
'prod',
'ptp',
'put',
'ravel',
'real',
'repeat',
'reshape',
'resize',
```

```
 'round',
 'searchsorted',
 'setfield',
 'setflags',
 'shape',
 'size',
 'sort',
 'squeeze',
 'std',
 'strides',
 'sum',
 'swapaxes',
 'take',
 'tobytes',
 'tofile',
 'tolist',
 'tostring',
 'trace',
 'transpose',
 'var',
 'view']
```

This indicates that the object A.sum exists. In this case it is a method that can be used to compute the sum of the array A as can be seen by typing A.sum?

[7]:
```
A.sum?
```

```
Docstring:
a.sum(axis=None, dtype=None, out=None, keepdims=False, initial=0, where=True)

Return the sum of the array elements over the given axis.

Refer to `numpy.sum` for full documentation.

See Also
--------
numpy.sum : equivalent function
Type:      builtin_function_or_method
```

[8]:
```
A.sum()
```

[8]:
```
19
```

0.4 Simple Linear Regression

In this section we will construct model matrices (also called design matrices) using the ModelSpec() transform from ISLP.models.

We will use the Boston housing data set, which is contained in the ISLP package. The Boston dataset records medv (median house value) for 506 neighborhoods around Boston. We will build a regression model to predict medv using 13 predictors such as rmvar (average number of rooms per house), age (proportion of owner-occupied units built prior to 1940), and lstat (percent of households with low socioeconomic status). We will use statsmodels for this task, a Python package that implements several commonly used regression methods.

We have included a simple loading function load_data() in the ISLP package

[10]:
```
Boston = load_data("Boston")
Boston.columns
```

[10]:
```
Index(['crim', 'zn', 'indus', 'chas', 'nox', 'rm', 'age', 'dis', 'rad', 'tax',
       'ptratio', 'lstat', 'medv'],
      dtype='object')
```

[11]:
```
Boston?
```

```
Type:        DataFrame
String form:
crim    zn  indus  chas   nox    rm   age    dis  rad  tax  \
             0    0.00632  18.0   2.3 <…> 0   5.64  23.9
             504     21.0   6.48  22.0
             505     21.0   7.88  11.9

             [506 rows x 13 columns]
Length:      506
File:        c:\users\ankit19.
 ↪gupta\desktop\self_projects\islp\myenv\lib\site-packages\pandas\core\frame.py
Docstring:
Two-dimensional, size-mutable, potentially heterogeneous tabular data.

Data structure also contains labeled axes (rows and columns).
Arithmetic operations align on both row and column labels. Can be
thought of as a dict-like container for Series objects. The primary
pandas data structure.

Parameters
----------
data : ndarray (structured or homogeneous), Iterable, dict, or DataFrame
    Dict can contain Series, arrays, constants, dataclass or list-like objects.␣
 ↪If
    data is a dict, column order follows insertion-order. If a dict contains␣
 ↪Series
    which have an index defined, it is aligned by its index.

    .. versionchanged:: 0.25.0
       If data is a list of dicts, column order follows insertion-order.

index : Index or array-like
```

```
    Index to use for resulting frame. Will default to RangeIndex if
    no indexing information part of input data and no index provided.
columns : Index or array-like
    Column labels to use for resulting frame when data does not have them,
    defaulting to RangeIndex(0, 1, 2, …, n). If data contains column labels,
    will perform column selection instead.
dtype : dtype, default None
    Data type to force. Only a single dtype is allowed. If None, infer.
copy : bool or None, default None
    Copy data from inputs.
    For dict data, the default of None behaves like ``copy=True``.  For DataFrame
    or 2d ndarray input, the default of None behaves like ``copy=False``.
    If data is a dict containing one or more Series (possibly of different␣
 ↪dtypes),
    ``copy=False`` will ensure that these inputs are not copied.

    .. versionchanged:: 1.3.0

See Also
--------
DataFrame.from_records : Constructor from tuples, also record arrays.
DataFrame.from_dict : From dicts of Series, arrays, or dicts.
read_csv : Read a comma-separated values (csv) file into DataFrame.
read_table : Read general delimited file into DataFrame.
read_clipboard : Read text from clipboard into DataFrame.

Notes
-----
Please reference the :ref:`User Guide <basics.dataframe>` for more information.

Examples
--------
Constructing DataFrame from a dictionary.

>>> d = {'col1': [1, 2], 'col2': [3, 4]}
>>> df = pd.DataFrame(data=d)
>>> df
   col1  col2
0     1     3
1     2     4

Notice that the inferred dtype is int64.

>>> df.dtypes
col1    int64
col2    int64
dtype: object
```

To enforce a single dtype:

```
>>> df = pd.DataFrame(data=d, dtype=np.int8)
>>> df.dtypes
col1    int8
col2    int8
dtype: object
```

Constructing DataFrame from a dictionary including Series:

```
>>> d = {'col1': [0, 1, 2, 3], 'col2': pd.Series([2, 3], index=[2, 3])}
>>> pd.DataFrame(data=d, index=[0, 1, 2, 3])
   col1  col2
0     0   NaN
1     1   NaN
2     2   2.0
3     3   3.0
```

Constructing DataFrame from numpy ndarray:

```
>>> df2 = pd.DataFrame(np.array([[1, 2, 3], [4, 5, 6], [7, 8, 9]]),
…                    columns=['a', 'b', 'c'])
>>> df2
   a  b  c
0  1  2  3
1  4  5  6
2  7  8  9
```

Constructing DataFrame from a numpy ndarray that has labeled columns:

```
>>> data = np.array([(1, 2, 3), (4, 5, 6), (7, 8, 9)],
…                 dtype=[("a", "i4"), ("b", "i4"), ("c", "i4")])
>>> df3 = pd.DataFrame(data, columns=['c', 'a'])
…
>>> df3
   c  a
0  3  1
1  6  4
2  9  7
```

Constructing DataFrame from dataclass:

```
>>> from dataclasses import make_dataclass
>>> Point = make_dataclass("Point", [("x", int), ("y", int)])
>>> pd.DataFrame([Point(0, 0), Point(0, 3), Point(2, 3)])
   x  y
0  0  0
```

```
1  0  3
2  2  3
```

We start by using the sm.OLS() function to ft a simple linear regression model. Our response will be `medv` and `lstat` will be the single predictor. For this model, we can create the model matrix by hand.

[12]:
```
X = pd.DataFrame({'intercept': np.ones(Boston.shape[0]),'lstat':␣
 ↪Boston['lstat']})
X[:4]
```

[12]:

	intercept	lstat
0	1.0	4.98
1	1.0	9.14
2	1.0	4.03
3	1.0	2.94

We extract the response, and ft the model.

[13]:
```
y = Boston['medv']
model = sm.OLS(y, X)
results = model.fit()
```

Note that `sm.OLS()` does not ft the model; it specifes the model, and then `model.fit()` does the actual ftting.

Our `ISLP` function `summarize()` produces a simple table of the parame- summarize() ter estimates, their standard errors, t-statistics and p-values. The function takes a single argument, such as the object results returned here by the fit method, and returns such a summary.

[14]:
```
summarize(results)
```

[14]:

	coef	std err	t	P>|t|
intercept	34.5538	0.563	61.415	0.0
lstat	-0.9500	0.039	-24.528	0.0

Before we describe other methods for working with ftted models, we outline a more useful and general framework for constructing a model matrix X.

0.4.1 Using Transformations: Fit and Transform

Our model above has a single predictor, and constructing X was straightforward. In practice we often fit models with more than one predictor, typically selected from an array or data frame. We may wish to introduce transformations to the variables before fitting the model, specify interactions between variables, and expand some particular variables into sets of variables (e.g. polynomials). The `sklearn` package has a particular notion sklearn for this type of task: a transform. A transform is an object that is createdwith some parameters as arguments. The object has two main methods: `fit()` and `transform()`.

We provide a general approach for specifying models and constructing the model matrix through the transform `ModelSpec()` in the ISLP library. `ModelSpec()` (renamed MS() in the preamble) creates a transform object, and then a pair of methods `transform()` and `fit()` are used to construct a corresponding model matrix.

We frst describe this process for our simple regression model using a single predictor lstat in the Boston data frame, but will use it repeatedly in more complex tasks in this and other labs in this book. In our case the transform is created by the expression `design = MS(['lstat'])`.

The `fit()` method takes the original array and may do some initial computations on it, as specifed in the transform object. For example, it may compute means and standard deviations for centering and scaling. The `transform()` method applies the ftted transformation to the array of data, and produces the model matrix.

[15]:
```
design = MS(['lstat'])
design = design.fit(Boston)
X = design.transform(Boston)
X[:4]
```

[15]:

	intercept	lstat
0	1.0	4.98
1	1.0	9.14
2	1.0	4.03
3	1.0	2.94

In this simple case, the `fit()` method does very little; it simply checks that the variable `'lstat'` specifed in `design` exists in `Boston`. Then `transform()` constructs the model matrix with two columns: an intercept and the variable lstat.

These two operations can be combined with the `fit_transform()` method.

[16]:
```
design = MS(['lstat'])
X = design.fit_transform(Boston)
X[:4]
```

[16]:

	intercept	lstat
0	1.0	4.98
1	1.0	9.14
2	1.0	4.03
3	1.0	2.94

Note that, as in the previous code chunk when the two steps were done separately, the design object is changed as a result of the fit() operation. The power of this pipeline will become clearer when we ft more complex models that involve interactions and transformations.

Let's return to our ftted regression model. The object results has several methods that can be used for inference. We already presented a function summarize() for showing the essentials of the ft. For a full and somewhat exhaustive summary of the ft, we can use the summary() method (output not shown).

[17]:
```
results.summary()
```

[17]:

Dep. Variable:	medv	**R-squared:**	0.544
Model:	OLS	**Adj. R-squared:**	0.543
Method:	Least Squares	**F-statistic:**	601.6
Date:	Mon, 01 Jan 2024	**Prob (F-statistic):**	5.08e-88
Time:	18:32:47	**Log-Likelihood:**	-1641.5
No. Observations:	506	**AIC:**	3287.
Df Residuals:	504	**BIC:**	3295.
Df Model:	1		
Covariance Type:	nonrobust		

	coef	**std err**	**t**	**P> |t|**	**[0.025**	**0.975]**
intercept	34.5538	0.563	61.415	0.000	33.448	35.659
lstat	-0.9500	0.039	-24.528	0.000	-1.026	-0.874

Omnibus:	137.043	**Durbin-Watson:**	0.892
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	291.373
Skew:	1.453	**Prob(JB):**	5.36e-64
Kurtosis:	5.319	**Cond. No.**	29.7

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

The ftted coefcients can also be retrieved as the params attribute of results.

[18]:
```
results.params
```

[18]:
```
intercept    34.553841
lstat        -0.950049
dtype: float64
```

The `get_prediction()` method can be used to obtain predictions, and produce confidence intervals and prediction intervals for the prediction of `medv` for given values of `lstat`

We frst create a new data frame, in this case containing only the variable `lstat`, with the values for this variable at which we wish to make predictions. We then use the `transform()` method of `design` to create the corresponding model matrix.

[19]:
```
new_df = pd.DataFrame({'lstat':[5, 10, 15]})
newX = design.transform(new_df)
newX
```

[19]:
```
   intercept  lstat
0        1.0      5
1        1.0     10
2        1.0     15
```

Next we compute the predictions at `newX`, and view them by extracting the `predicted_mean` attribute

[20]:
```
new_predictions = results.get_prediction(newX);
new_predictions.predicted_mean
```

[20]: array([29.80359411, 25.05334734, 20.30310057])

We can produce confdence intervals for the predicted values.

[21]:
```
new_predictions.conf_int(alpha=0.05)
```

[21]:
```
array([[29.00741194, 30.59977628],
       [24.47413202, 25.63256267],
       [19.73158815, 20.87461299]])
```

Prediction intervals are computing by setting `obs=True`:

[22]:
```
new_predictions.conf_int(obs=True, alpha=0.05)
```

[22]:
```
array([[17.56567478, 42.04151344],
       [12.82762635, 37.27906833],
       [ 8.0777421 , 32.52845905]])
```

For instance, the 95% confdence interval associated with an lstat value of 10 is (24.47, 25.63), and the 95% prediction interval is (12.82, 37.28). As expected, the confdence and prediction intervals are centered around the same point (a predicted value of 25.05 for medv when lstat equals 10), but the latter are substantially wider.

Next we will plot `medv` and `lstat` using DataFrame.plot.scatter(), and wish to add the regression line to the resulting plot.

0.5 Defining Functions

While there is a function within the ISLP package that adds a line to an existing plot, we take this opportunity to defne our frst function to do so.

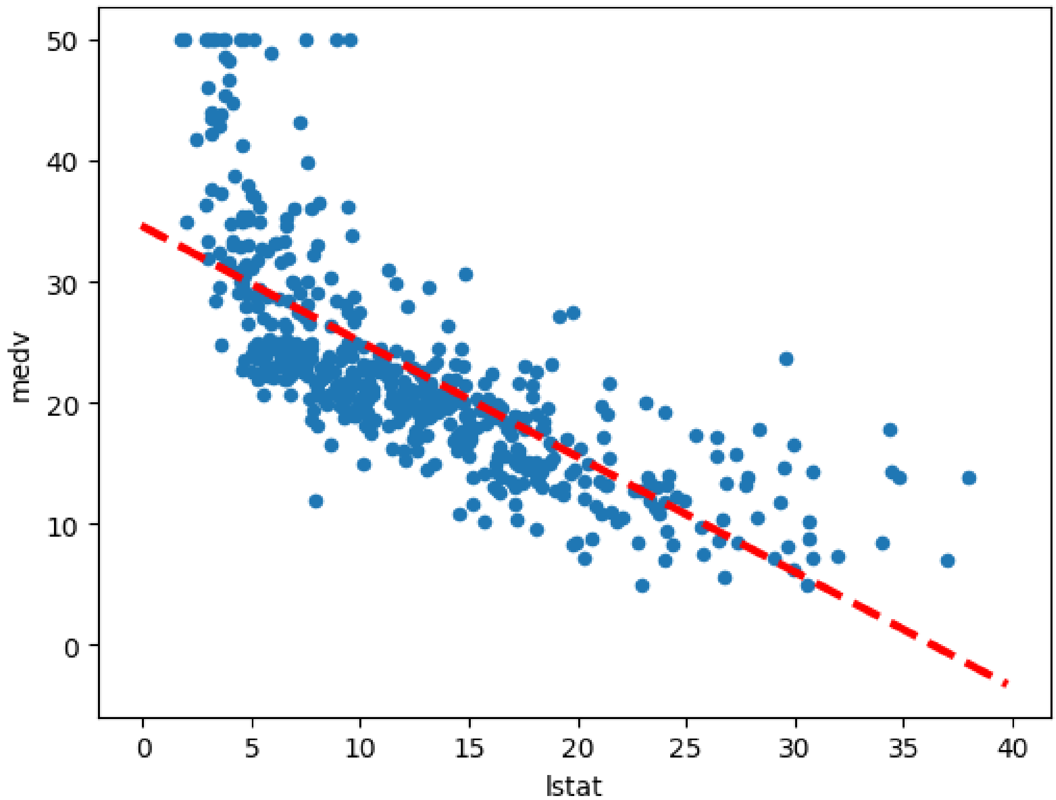
[23]:
```
def abline(ax, b, m):
    "Add a line with slope m and intercept b to ax"
    xlim = ax.get_xlim()
    ylim = [m * xlim[0] + b, m * xlim[1] + b]
    ax.plot(xlim, ylim)
```

A few things are illustrated above. First we see the syntax for defning a function: `def funcname(...)`. The function has arguments `ax, b, m` where `ax` is an axis object for an exisiting plot, `b` is the intercept and `m` is the slope of the desired line. Other plotting options can be passed on to `ax.plot` by including additional optional arguments as follows:

[24]:
```
def abline(ax, b, m, *args, **kwargs):
    "Add a line with slope m and intercept b to ax"
    xlim = ax.get_xlim()
    ylim = [m * xlim[0] + b, m * xlim[1] + b]
    ax.plot(xlim, ylim, *args, **kwargs)
```

Let’s use our new function to add this regression line to a plot of medv vs. lstat

[26]:
```
ax = Boston.plot.scatter('lstat', 'medv')
abline(ax,results.params[0],results.params[1],'r--',linewidth=3)
```



[27]:
```
results.params[0]
```

[27]: 34.55384087938308

[28]:
```
results.params[1]
```

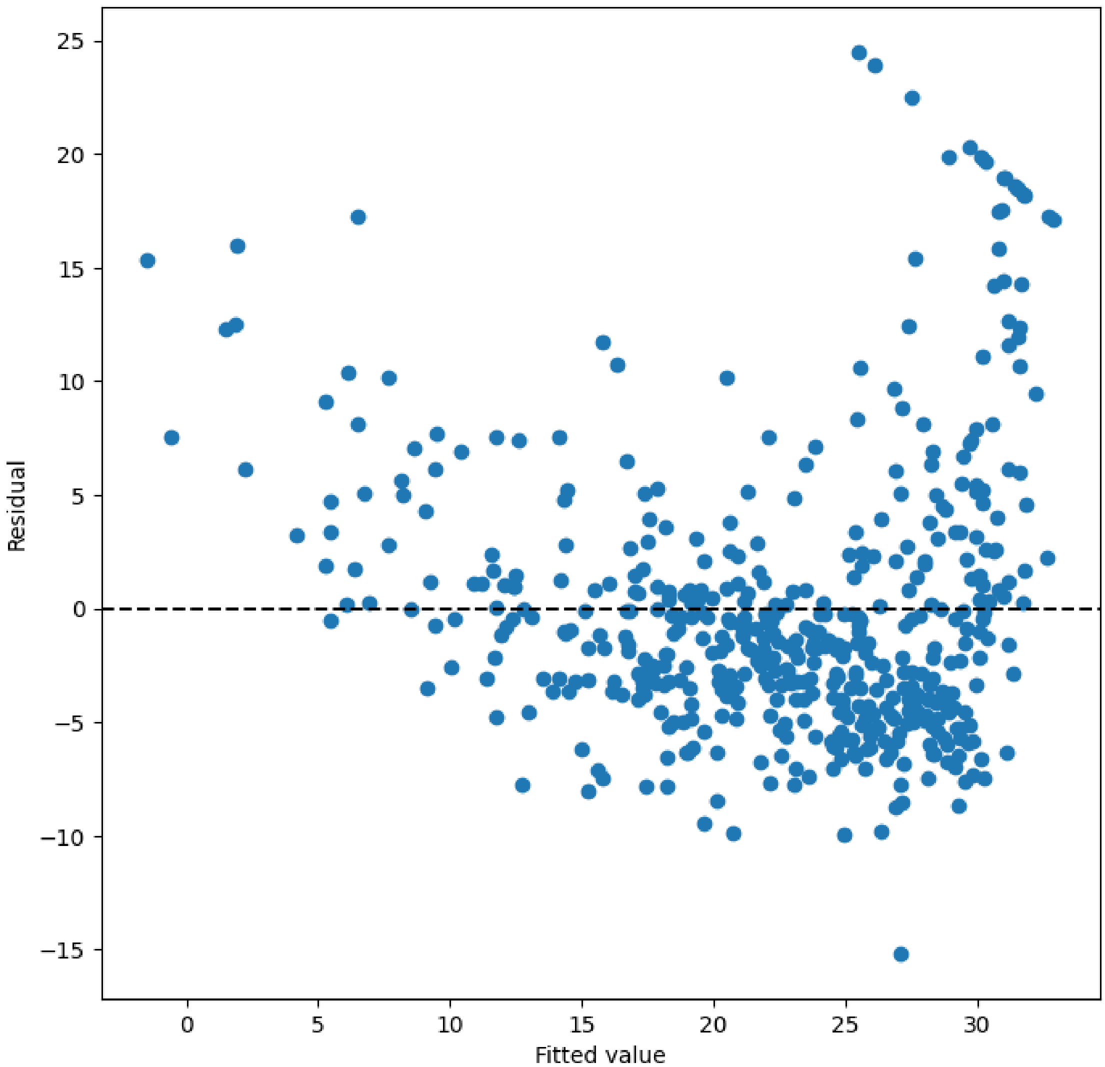
[28]: -0.9500493537579922

Thus, the final call to `ax.plot()` is `ax.plot(xlim, ylim, 'r--', linewidth=3)`. We have used the argument `'r--'` to produce a red dashed line, and added an argument to make it of width 3. There is some evidence for non-linearity in the relationship between lstat and medv. We will explore this issue later in this lab.

As mentioned above, there is an existing function to add a line to a plot — ax.axline() — but knowing how to write such functions empowers us to create more expressive displays.

Next we examine some diagnostic plots, several of which were discussed in Section 3.3.3. We can fnd the fitted values and residuals of the fit as attributes of the results object. Various infuence measures describing the regression model are computed with the `get_influence()` method. As we will not use the fig component returned as the frst value from subplots(), we simply capture the second returned value in ax below.

[29]:
```
ax = subplots(figsize=(8,8))[1]
ax.scatter(results.fittedvalues , results.resid)
ax.set_xlabel('Fitted value')
ax.set_ylabel('Residual')
ax.axhline(0, c='k', ls='--');
```



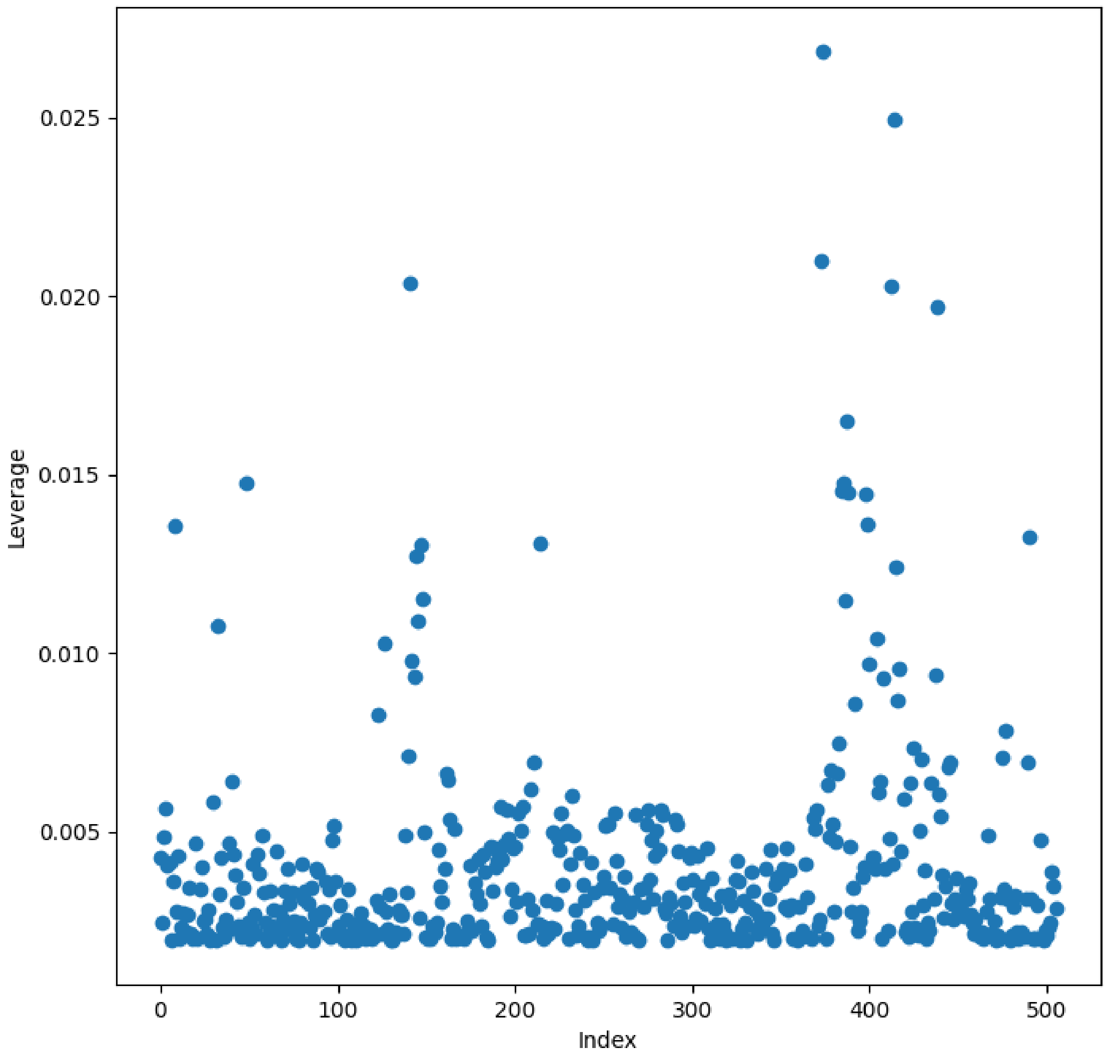
We add a horizontal line at 0 for reference using the ax.axhline() method, indicating it should be black (c='k') and have a dashed linestyle (ls='–').

On the basis of the residual plot (not shown), there is some evidence of non-linearity. Leverage statistics can be computed for any number of predictors using the `hat_matrix_diag` attribute of the value returned by the `get_influence()` method.

[31]:

```python
infl = results.get_influence()
ax = subplots(figsize=(8,8))[1]
ax.scatter(np.arange(X.shape[0]), infl.hat_matrix_diag)
ax.set_xlabel('Index')
ax.set_ylabel('Leverage')
np.argmax(infl.hat_matrix_diag)
```

[31]: 374



The np.argmax() function identifes the index of the largest element of an array, optionally computed over an axis of the array. In this case, we maximized over the entire array to determine which observation has the largest leverage statistic.

0.6 Multiple Linear Regression

In order to ft a multiple linear regression model using least squares, we again use the `ModelSpec()` transform to construct the required model matrix and response. The arguments to `ModelSpec()` can be quite general, but in this case a list of column names suffice. We consider a ft here with the two variables lstat and age.

[32]:
```
X = MS(['lstat', 'age']).fit_transform(Boston)
model1 = sm.OLS(y, X)
results1 = model1.fit()
summarize(results1)
```

[32]:

	coef	std err	t	P>|t|
intercept	33.2228	0.731	45.458	0.000
lstat	-1.0321	0.048	-21.416	0.000
age	0.0345	0.012	2.826	0.005

The Boston data set contains 12 variables, and so it would be cumbersome to have to type all of these in order to perform a regression using all of the predictors. Instead, we can use the following short-hand:

[33]:
```
terms = Boston.columns.drop('medv')
terms
```

[33]:
```
Index(['crim', 'zn', 'indus', 'chas', 'nox', 'rm', 'age', 'dis', 'rad', 'tax',
       'ptratio', 'lstat'],
      dtype='object')
```

We can now ft the model with all the variables in terms using the same model matrix builder.

[34]:
```
X = MS(terms).fit_transform(Boston)
model = sm.OLS(y, X)
results = model.fit()
summarize(results)
```

[34]:

	coef	std err	t	P>|t|
intercept	41.6173	4.936	8.431	0.000
crim	-0.1214	0.033	-3.678	0.000
zn	0.0470	0.014	3.384	0.001
indus	0.0135	0.062	0.217	0.829
chas	2.8400	0.870	3.264	0.001
nox	-18.7580	3.851	-4.870	0.000
rm	3.6581	0.420	8.705	0.000
age	0.0036	0.013	0.271	0.787
dis	-1.4908	0.202	-7.394	0.000
rad	0.2894	0.067	4.325	0.000
tax	-0.0127	0.004	-3.337	0.001
ptratio	-0.9375	0.132	-7.091	0.000
lstat	-0.5520	0.051	-10.897	0.000

What if we would like to perform a regression using all of the variables but one? For example, in the above regression output, `age` has a high p-value. So we may wish to run a regression excluding this predictor. The following syntax results in a regression using all predictors except `age` (output not shown).

[35]:
```
minus_age = Boston.columns.drop(['medv', 'age'])
Xma = MS(minus_age).fit_transform(Boston)
model1 = sm.OLS(y, Xma)
summarize(model1.fit())
```

[35]:

	coef	std err	t	P>|t|
intercept	41.5251	4.920	8.441	0.000
crim	-0.1214	0.033	-3.683	0.000
zn	0.0465	0.014	3.379	0.001
indus	0.0135	0.062	0.217	0.829
chas	2.8528	0.868	3.287	0.001
nox	-18.4851	3.714	-4.978	0.000
rm	3.6811	0.411	8.951	0.000
dis	-1.5068	0.193	-7.825	0.000
rad	0.2879	0.067	4.322	0.000
tax	-0.0127	0.004	-3.333	0.001
ptratio	-0.9346	0.132	-7.099	0.000
lstat	-0.5474	0.048	-11.483	0.000

0.7 Multivariate Goodness of Fit

We can access the individual components of results by name `(dir(results)` shows us what is available). Hence results.rsquared gives us the `R2`, and `np.sqrt(results.scale)` gives us the `RSE`.

`Variance infation factors` (section 3.3.3) are sometimes useful to assess the efect of collinearity in the model matrix of a regression model. We will compute the VIFs in our multiple regression ft, and use the opportunity to introduce the idea of list comprehension.

Often we encounter a sequence of objects which we would like to transform for some other task. Below, we compute the VIF for each feature in our X matrix and produce a data frame whose index agrees with the columns of X. The notion of list comprehension can often make such a task easier.

[36]:
```
vals = [VIF(X, i)
for i in range(1, X.shape[1])]
vif = pd.DataFrame({'vif':vals},
index=X.columns[1:])
vif
```

[36]:

	vif
crim	1.767486
zn	2.298459
indus	3.987181
chas	1.071168
nox	4.369093

```
rm         1.912532
age        3.088232
dis        3.954037
rad        7.445301
tax        9.002158
ptratio    1.797060
lstat      2.870777
```

The function `VIF()` takes two arguments: a dataframe or array, and a variable column index. In the code above we call `VIF()` on the fy for all columns in X. We have excluded column 0 above (the intercept), which is not of interest. In this case the VIFs are not that exciting.

[37]:
```
vals = []
for i in range(1, X.values.shape[1]):
    vals.append(VIF(X.values, i))
```

[38]:
```
vals
```

[38]:
```
[1.7674859154310125,
 2.2984589077358097,
 3.987180630757096,
 1.0711677737584042,
 4.369092622844795,
 1.9125324374368873,
 3.0882320397311966,
 3.954036641628298,
 7.445300760069838,
 9.002157663471797,
 1.797059593129779,
 2.8707765008417514]
```

List comprehension allows us to perform such repetitive operations in a more straightforward way.

0.8 Interaction Terms

It is easy to include interaction terms in a linear model using `ModelSpec()`. Including a tuple `("lstat","age")` tells the model matrix builder to include an interaction term between `lstat` and `age`.

[39]:
```
X = MS(['lstat','age',('lstat', 'age')]).fit_transform(Boston)
model2 = sm.OLS(y, X)
summarize(model2.fit())
```

[39]:
```
              coef  std err       t  P>|t|
intercept  36.0885    1.470  24.553  0.000
lstat      -1.3921    0.167  -8.313  0.000
age        -0.0007    0.020  -0.036  0.971
lstat:age   0.0042    0.002   2.244  0.025
```

0.9 Non-linear Transformations of the Predictors

The model matrix builder can include terms beyond just column names and interactions. For instance, the `poly()` function supplied in ISLP specifes that columns representing polynomial functions of its frst argument are added to the model matrix

[40]:
```
X = MS([poly('lstat', degree=2), 'age']).fit_transform(Boston)
model3 = sm.OLS(y, X)
results3 = model3.fit()
summarize(results3)
```

[40]:

	coef	std err	t	P>|t|
intercept	17.7151	0.781	22.681	0.0
poly(lstat, degree=2)[0]	-179.2279	6.733	-26.620	0.0
poly(lstat, degree=2)[1]	72.9908	5.482	13.315	0.0
age	0.0703	0.011	6.471	0.0

The efectively zero p-value associated with the quadratic term (i.e. the third row above) suggests that it leads to an improved model.

By default, `poly()` creates a basis matrix for inclusion in the model matrix whose columns are orthogonal polynomials, which are designed for stable least squares computations. Alternatively, had we included an argument `raw=True` in the above call to `poly()`, the basis matrix would consist simply of `lstat` and `lstat**2`. Since either of these bases represent quadratic polynomials, the fitted values would not change in this case, just the polynomial coefficients. Also by default, the columns created by `poly()` do not include an intercept column as that is automatically added by MS().

Actually, poly() is a wrapper for the workhorse and standalone function Poly() that does the work in building the model matrix.

We use the `anova_lm()` function to further quantify the extent to which `anova_lm()` the quadratic fit is superior to the linear fit.

[41]:
```
anova_lm(results1, results3)
```

[41]:

	df_resid	ssr	df_diff	ss_diff	F	Pr(>F)
0	503.0	19168.128609	0.0	NaN	NaN	NaN
1	502.0	14165.613251	1.0	5002.515357	177.278785	7.468491e-35

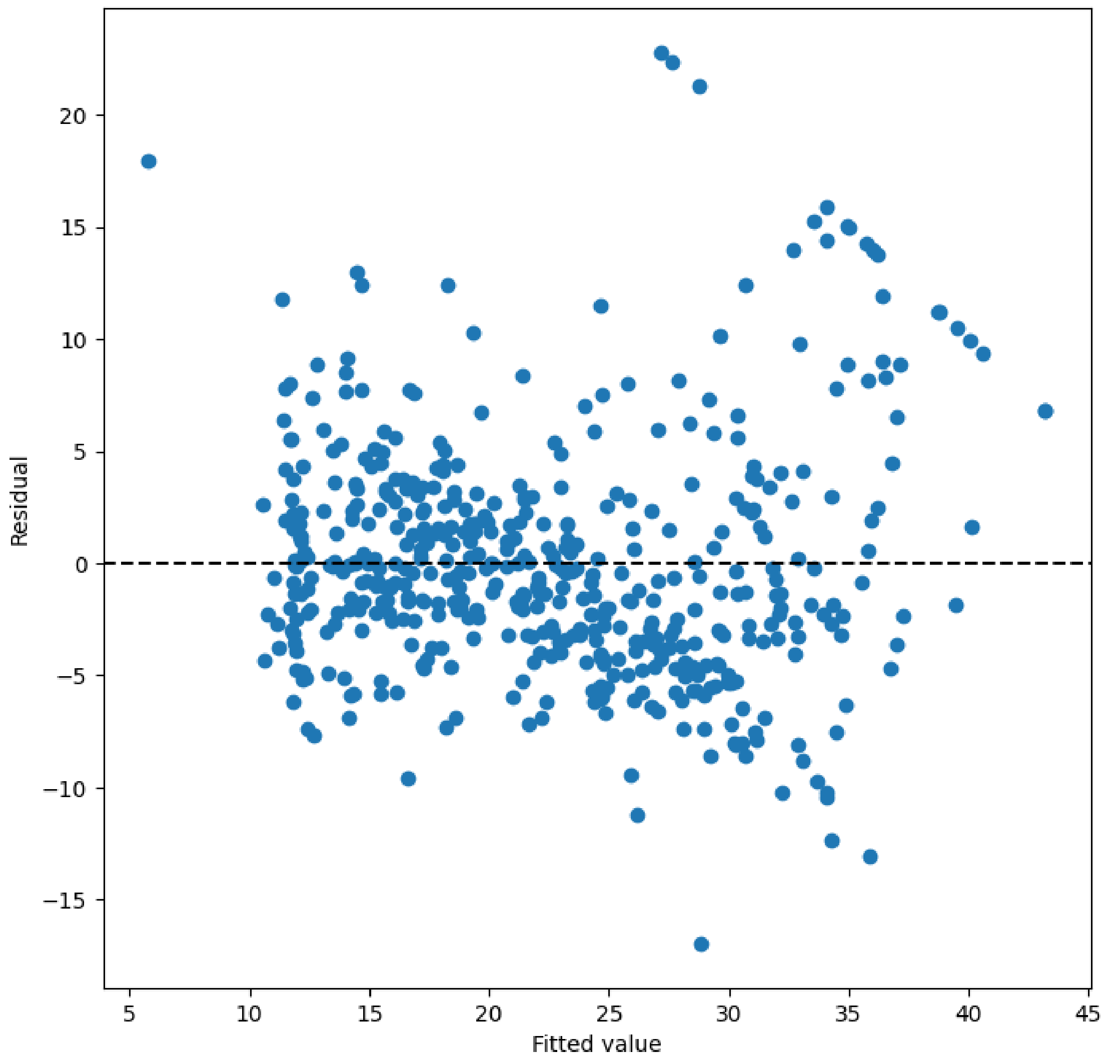
Here `results1` represents the linear submodel containing predictors `lstat` and `age`, while `results3` corresponds to the larger model above with a quadratic term in `lstat`. The `anova_lm()` function performs a hypothesis test comparing the two models. The null hypothesis is that the quadratic term in the bigger model is not needed, and the alternative hypothesis is that the bigger model is superior. Here the `F-statistic` is 177.28 and the associated p-value is zero. In this case the F-statistic is the square of the t-statistic for the quadratic term in the linear model summary for `results3`

a consequence of the fact that these nested models difer by one degree of freedom. This provides very clear evidence that the quadratic polynomial in`lstat` improves the linear model. This is not surprising, since earlier we saw evidence for non-linearity in the relationship between `medv` and

lsta
t. The function anova_lm() can take more than two nested models as input, in which case it compares every successive pair of models. That also explains why their are NaNs in ithe frst row above, since there is no previous model with which to comparei the frst.

[42]:
```
ax = subplots(figsize=(8,8))[1]
ax.scatter(results3.fittedvalues , results3.resid)
ax.set_xlabel('Fitted value')
ax.set_ylabel('Residual')
ax.axhline(0, c='k', ls='--')
```

[42]: <matplotlib.lines.Line2D at 0x1c8a2481640>



We see that when the quadratic term is included in the model, there is little discernible pattern in the residuals. In order to create a cubic or higher-degree polynomial ft, we can simply change the degree argument to poly()

0.10 Qualitative Predictors

Here we use the Carseats data, which is included in the ISLP package. We will attempt to predict Sales (child car seat sales) in 400 locations based on a number of predictors.

[44]:
```
Carseats = load_data('Carseats')
Carseats.columns
```

[44]:
```
Index(['Sales', 'CompPrice', 'Income', 'Advertising', 'Population', 'Price',
       'ShelveLoc', 'Age', 'Education', 'Urban', 'US'],
      dtype='object')
```

The Carseats data includes qualitative predictors such as ShelveLoc, an indicator of the quality of the shelving location — that is, the space within a store in which the car seat is displayed. The predictor ShelveLoc takes on three possible values, Bad, Medium, and Good. Given a qualitative variable such as ShelveLoc, ModelSpec() generates dummy variables automatically.

These variables are often referred to as a one-hot encoding of the categorical one-hot encoding feature. Their columns sum to one, so to avoid collinearity with an intercept, the first column is dropped. Below we see the column ShelveLoc[Bad] has been dropped, since Bad is the first level of ShelveLoc. Below we fit a multiple regression model that includes some interaction terms.

[45]:
```
allvars = list(Carseats.columns.drop('Sales'))
y = Carseats['Sales']
final = allvars + [('Income', 'Advertising'),
('Price', 'Age')]
X = MS(final).fit_transform(Carseats)
model = sm.OLS(y, X)
summarize(model.fit())
```

[45]:

	coef	std err	t	P>|t|
intercept	6.5756	1.009	6.519	0.000
CompPrice	0.0929	0.004	22.567	0.000
Income	0.0109	0.003	4.183	0.000
Advertising	0.0702	0.023	3.107	0.002
Population	0.0002	0.000	0.433	0.665
Price	-0.1008	0.007	-13.549	0.000
ShelveLoc[Good]	4.8487	0.153	31.724	0.000
ShelveLoc[Medium]	1.9533	0.126	15.531	0.000
Age	-0.0579	0.016	-3.633	0.000
Education	-0.0209	0.020	-1.063	0.288
Urban[Yes]	0.1402	0.112	1.247	0.213
US[Yes]	-0.1576	0.149	-1.058	0.291
Income:Advertising	0.0008	0.000	2.698	0.007
Price:Age	0.0001	0.000	0.801	0.424

In the first line above, we made allvars a list, so that we could add the interaction terms two lines down. Our model-matrix builder has created a ShelveLoc[Good] dummy variable that takes on a value of 1 if the shelving location is good, and 0 otherwise. It has also created a `ShelveLoc[Medium]` dummy variable that equals 1 if the shelving location is medium, and 0 otherwise. A bad shelving location corresponds to a zero for each of the two dummy variables.

The fact that the coefcient for `ShelveLoc[Good]` in the regression output is positive indicates that a good shelving location is associated with high sales (relative to a bad location). And `ShelveLoc[Medium]` has a smaller positive coefcient, indicating that a medium shelving location leads to higher sales than a bad shelving location, but lower sales than a good shelving location.

[ ]:
```

```