Chapter_12_Implementing_a_Multilayer_Artificial_Neural_Network_from_

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0.1 Modeling complex functions with Artificial Neural Networks

0.1.1 Classifying handwritten digits

[1]:
```
import os
import struct
import numpy as np
```

[6]:
```
def load_mnist(path, kind='train'):
    """Load MNIST data from `path`"""
    labels_path = os.path.join(path,'%s-labels.idx1-ubyte' % kind)
    images_path = os.path.join(path,'%s-images.idx3-ubyte' % kind)
    with open(labels_path, 'rb') as lbpath:
        magic, n = struct.unpack('>II',lbpath.read(8))
        labels = np.fromfile(lbpath,dtype=np.uint8)
    with open(images_path, 'rb') as imgpath:
        magic, num, rows, cols = struct.unpack(">IIII",imgpath.read(16))
        images = np.fromfile(imgpath,dtype=np.uint8).reshape(len(labels), 784)
        images = ((images / 255.) - .5) * 2
    return images, labels
```

[7]:
```
X_train, y_train = load_mnist('', kind='train')
```

[8]:
```
print('Rows: %d, columns: %d' % (X_train.shape[0], X_train.shape[1]))
```

```
Rows: 60000, columns: 784
```

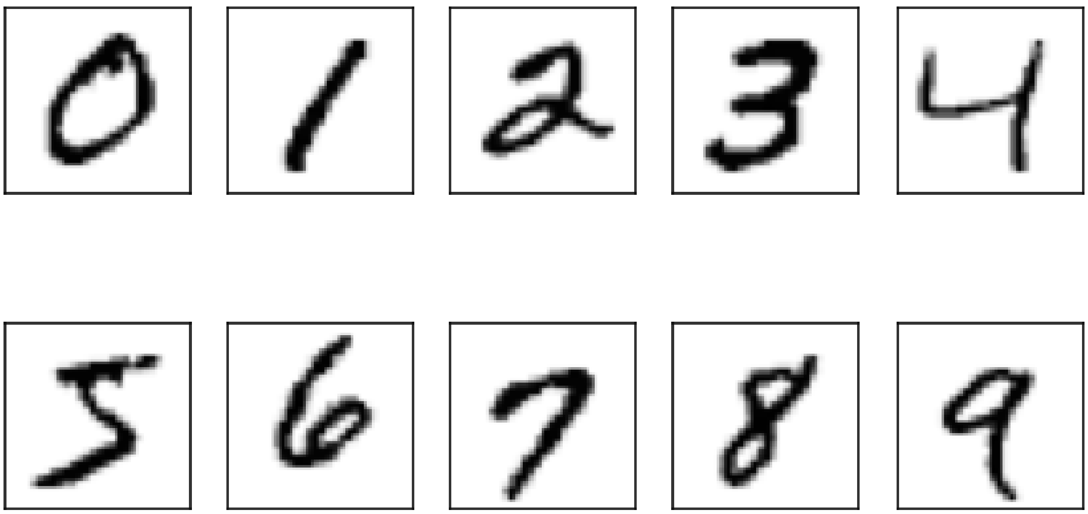
[9]:
```
X_test, y_test = load_mnist('', kind='t10k')
```

[10]:
```
print('Rows: %d, columns: %d' % (X_test.shape[0], X_test.shape[1]))
```

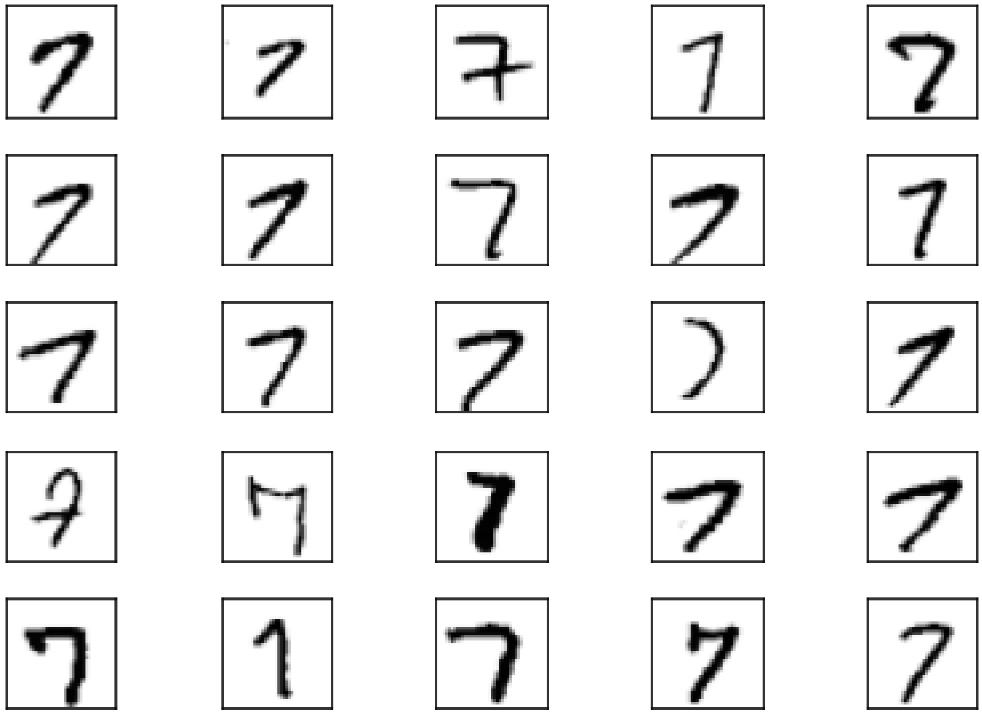
```
Rows: 10000, columns: 784
```

To get an idea of how those images in MNIST look, let's visualize examples of the digits 0-9 after reshaping the 784-pixel vectors from our feature matrix into the original 28 × 28 image that we can plot via Matplotlib's imshow function:

[11]:
```python
import matplotlib.pyplot as plt
fig, ax = plt.subplots(nrows=2, ncols=5,sharex=True, sharey=True)
ax = ax.flatten()
for i in range(10):
    img = X_train[y_train == i][0].reshape(28, 28)
    ax[i].imshow(img, cmap='Greys')
ax[0].set_xticks([])
ax[0].set_yticks([])
plt.tight_layout()
plt.show()
```



[12]:
```python
fig, ax = plt.subplots(nrows=5,ncols=5,sharex=True,sharey=True)
ax = ax.flatten()
for i in range(25):
    img = X_train[y_train == 7][i].reshape(28, 28)
    ax[i].imshow(img, cmap='Greys')
ax[0].set_xticks([])
ax[0].set_yticks([])
plt.tight_layout()
plt.show()
```



[13]:
```
import numpy as np
np.savez_compressed('mnist_scaled.
 ↪npz',X_train=X_train,y_train=y_train,X_test=X_test,y_test=y_test)
```

[14]:
```
mnist = np.load('mnist_scaled.npz')
```

[15]:
```
mnist.files
```

[15]:
```
['X_train', 'y_train', 'X_test', 'y_test']
```

[16]:
```
X_train = mnist['X_train']
```

[17]:
```
X_train, y_train, X_test, y_test = [mnist[f] for f in mnist.files]
```

0.1.2 Implementing a multilayer perceptron

[18]:
```
import numpy as np
import sys
class NeuralNetMLP(object):
    """ Feedforward neural network / Multi-layer perceptron
    classifier.
    Parameters
    ------------
```

```
    n_hidden : int (default: 30)
    Number of hidden units.
    l2 : float (default: 0.)
    Lambda value for L2-regularization.
    No regularization if l2=0. (default)
    epochs : int (default: 100)
    Number of passes over the training set.
    eta : float (default: 0.001)
    Learning rate.
    shuffle : bool (default: True)
    Shuffles training data every epoch
    if True to prevent circles.
    minibatch_size : int (default: 1)
    Number of training samples per minibatch.
    seed : int (default: None)
    Random seed for initializing weights and shuffling.
    Attributes
    -----------
    eval_ : dict
    Dictionary collecting the cost, training accuracy,
    and validation accuracy for each epoch during training.
    """
    def __init__(self, n_hidden=30,l2=0., epochs=100, eta=0.001,shuffle=True, minibatch_size=1, seed=None):
        self.random = np.random.RandomState(seed)
        self.n_hidden = n_hidden
        self.l2 = l2
        self.epochs = epochs
        self.eta = eta
        self.shuffle = shuffle
        self.minibatch_size = minibatch_size
    def _onehot(self, y, n_classes):
        """Encode labels into one-hot representation
        Parameters
        -----------
        y : array, shape = [n_samples]
        Target values.
        Returns
        -----------
        onehot : array, shape = (n_samples, n_labels)
        """
        onehot = np.zeros((n_classes, y.shape[0]))
        for idx, val in enumerate(y.astype(int)):
            onehot[val, idx] = 1.
        return onehot.T
    def _sigmoid(self, z):
        """Compute logistic function (sigmoid)"""
```

```
        return 1. / (1. + np.exp(-np.clip(z, -250, 250)))
    def _forward(self, X):
        """Compute forward propagation step"""
        # step 1: net input of hidden layer
        # [n_samples, n_features] dot [n_features, n_hidden]
        # -> [n_samples, n_hidden]
        z_h = np.dot(X, self.w_h) + self.b_h
        # step 2: activation of hidden layer
        a_h = self._sigmoid(z_h)
        # step 3: net input of output layer
        # [n_samples, n_hidden] dot [n_hidden, n_classlabels]
        # -> [n_samples, n_classlabels]
        z_out = np.dot(a_h, self.w_out) + self.b_out
        # step 4: activation output layer
        a_out = self._sigmoid(z_out)
        return z_h, a_h, z_out, a_out
    def _compute_cost(self, y_enc, output):
        """Compute cost function.
        Parameters
        ----------
        y_enc : array, shape = (n_samples, n_labels)
        one-hot encoded class labels.
        output : array, shape = [n_samples, n_output_units]
        Activation of the output layer (forward propagation)
        Returns
        ---------
        cost : float
        Regularized cost
        """
        L2_term = (self.l2 *(np.sum(self.w_h ** 2.) +np.sum(self.w_out ** 2.)))
        term1 = -y_enc * (np.log(output))
        term2 = (1. - y_enc) * np.log(1. - output)
        cost = np.sum(term1 - term2) + L2_term
        return cost
    def predict(self, X):
        """Predict class labels
        Parameters
        -----------
        X : array, shape = [n_samples, n_features]
        Input layer with original features.
        Returns:
        ----------
        y_pred : array, shape = [n_samples]
        Predicted class labels.
        """
        z_h, a_h, z_out, a_out = self._forward(X)
        y_pred = np.argmax(z_out, axis=1)
```

```
        return y_pred
    def fit(self, X_train, y_train, X_valid, y_valid):
        """ Learn weights from training data.
        Parameters
        -----------
        X_train : array, shape = [n_samples, n_features]
        ,PSOHPHQWLQJD0XOWLOD\HU$UWLÀFLDO1HXUDO1HWZRUNIURP6FUDWFK
        [ 400 ]
        Input layer with original features.
        y_train : array, shape = [n_samples]
        Target class labels.
        X_valid : array, shape = [n_samples, n_features]
        Sample features for validation during training
        y_valid : array, shape = [n_samples]
        Sample labels for validation during training
        Returns:
        ----------
        self
        """
        n_output = np.unique(y_train).shape[0] # no. of class #labels
        n_features = X_train.shape[1]
        ########################
        # Weight initialization
        ########################
        # weights for input -> hidden
        self.b_h = np.zeros(self.n_hidden)
        self.w_h = self.random.normal(loc=0.0, scale=0.1,size=(n_features,self.
↪n_hidden))
        # weights for hidden -> output
        self.b_out = np.zeros(n_output)
        self.w_out = self.random.normal(loc=0.0, scale=0.1,size=(self.
↪n_hidden,n_output))
        epoch_strlen = len(str(self.epochs)) # for progr. format.
        self.eval_ = {'cost': [], 'train_acc': [], 'valid_acc': \
        []}
        y_train_enc = self._onehot(y_train, n_output)
        # iterate over training epochs
        for i in range(self.epochs):
            # iterate over minibatches
            indices = np.arange(X_train.shape[0])
            if self.shuffle:
                self.random.shuffle(indices)
            for start_idx in range(0, indices.shape[0] -self.minibatch_size +1,␣
↪self.minibatch_size):
                batch_idx = indices[start_idx:start_idx +self.minibatch_size]
                # forward propagation
                z_h, a_h, z_out, a_out = self._forward(X_train[batch_idx])
```

```
                ##################
                # Backpropagation
                ##################
                # [n_samples, n_classlabels]
                sigma_out = a_out - y_train_enc[batch_idx]
                # [n_samples, n_hidden]
                sigmoid_derivative_h = a_h * (1. - a_h)
                # [n_samples, n_classlabels] dot [n_classlabels,
                # n_hidden]
                # -> [n_samples, n_hidden]
                sigma_h = (np.dot(sigma_out, self.w_out.T)␣
↪*sigmoid_derivative_h)
                # [n_features, n_samples] dot [n_samples,
                # n_hidden]
                # -> [n_features, n_hidden]
                grad_w_h = np.dot(X_train[batch_idx].T, sigma_h)
                grad_b_h = np.sum(sigma_h, axis=0)
                # [n_hidden, n_samples] dot [n_samples,
                # n_classlabels]
                # -> [n_hidden, n_classlabels]
                grad_w_out = np.dot(a_h.T, sigma_out)
                grad_b_out = np.sum(sigma_out, axis=0)
                # Regularization and weight updates
                delta_w_h = (grad_w_h + self.l2*self.w_h)
                delta_b_h = grad_b_h # bias is not regularized
                self.w_h -= self.eta * delta_w_h
                self.b_h -= self.eta * delta_b_h
                delta_w_out = (grad_w_out + self.l2*self.w_out)
                delta_b_out = grad_b_out # bias is not regularized
                self.w_out -= self.eta * delta_w_out
                self.b_out -= self.eta * delta_b_out
            #############
            # Evaluation
            #############
            # Evaluation after each epoch during training
            z_h, a_h, z_out, a_out = self._forward(X_train)

            cost = self._compute_cost(y_enc=y_train_enc,output=a_out)
            y_train_pred = self.predict(X_train)
            y_valid_pred = self.predict(X_valid)
            train_acc = ((np.sum(y_train ==y_train_pred)).astype(np.float) /
↪X_train.shape[0])
            valid_acc = ((np.sum(y_valid ==y_valid_pred)).astype(np.float) /
↪X_valid.shape[0])
            sys.stderr.write('\r%0*d/%d | Cost: %.2f | Train/Valid Acc.: %.2f%%/
↪%.2f%% '%(epoch_strlen, i+1, self.epochs,cost,train_acc*100, valid_acc*100))
            sys.stderr.flush()
```

```
            self.eval_['cost'].append(cost)
            self.eval_['train_acc'].append(train_acc)
            self.eval_['valid_acc'].append(valid_acc)
        return self
```

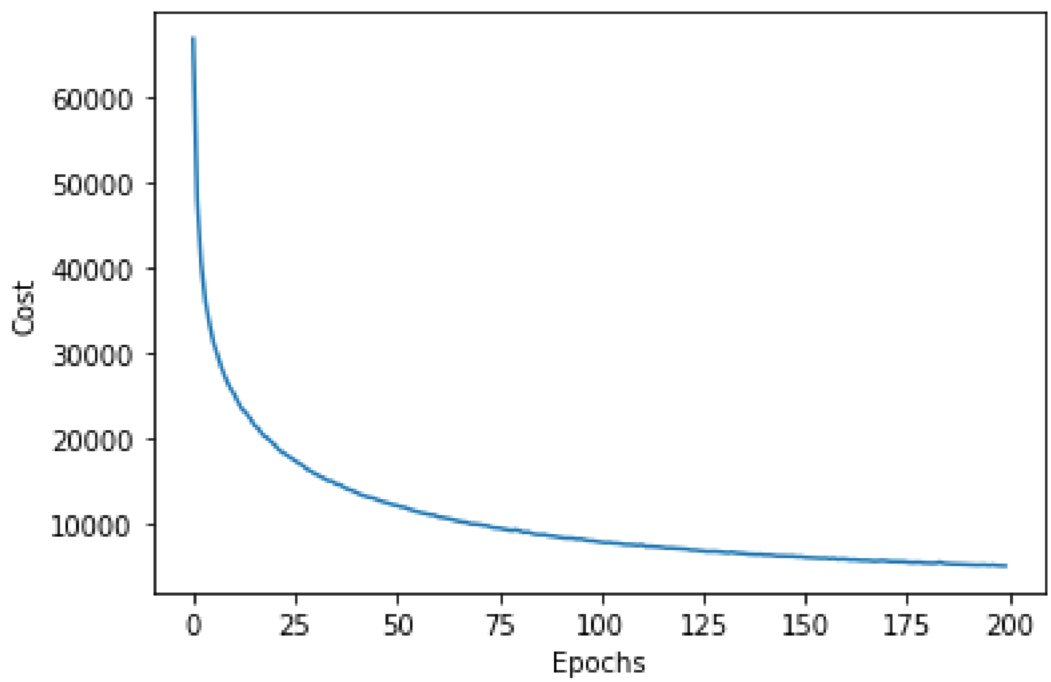
[19]:
```
nn = NeuralNetMLP(n_hidden=100,l2=0.01,epochs=200,eta=0.
 ↪0005,minibatch_size=100,shuffle=True,seed=1)
```

[20]:
```
nn.fit(X_train=X_train[:55000],y_train=y_train[:55000],X_valid=X_train[55000:
 ↪],y_valid=y_train[55000:])
```

```
200/200 | Cost: 5065.78 | Train/Valid Acc.: 99.28%/97.98%
```

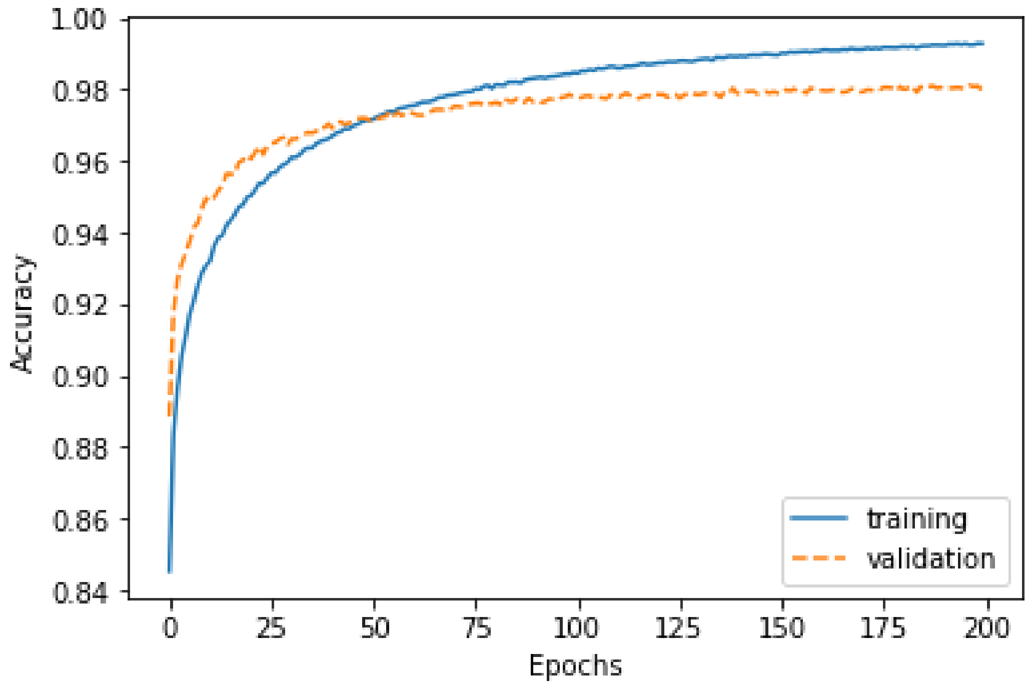
[20]:
```
<__main__.NeuralNetMLP at 0x1e701dcff60>
```

[21]:
```
import matplotlib.pyplot as plt
plt.plot(range(nn.epochs), nn.eval_['cost'])
plt.ylabel('Cost')
plt.xlabel('Epochs')
plt.show()
```



[22]:
```
plt.plot(range(nn.epochs), nn.eval_['train_acc'],label='training')
plt.plot(range(nn.epochs), nn.eval_['valid_acc'],label='validation',␣
 ↪linestyle='--')
plt.ylabel('Accuracy')
```

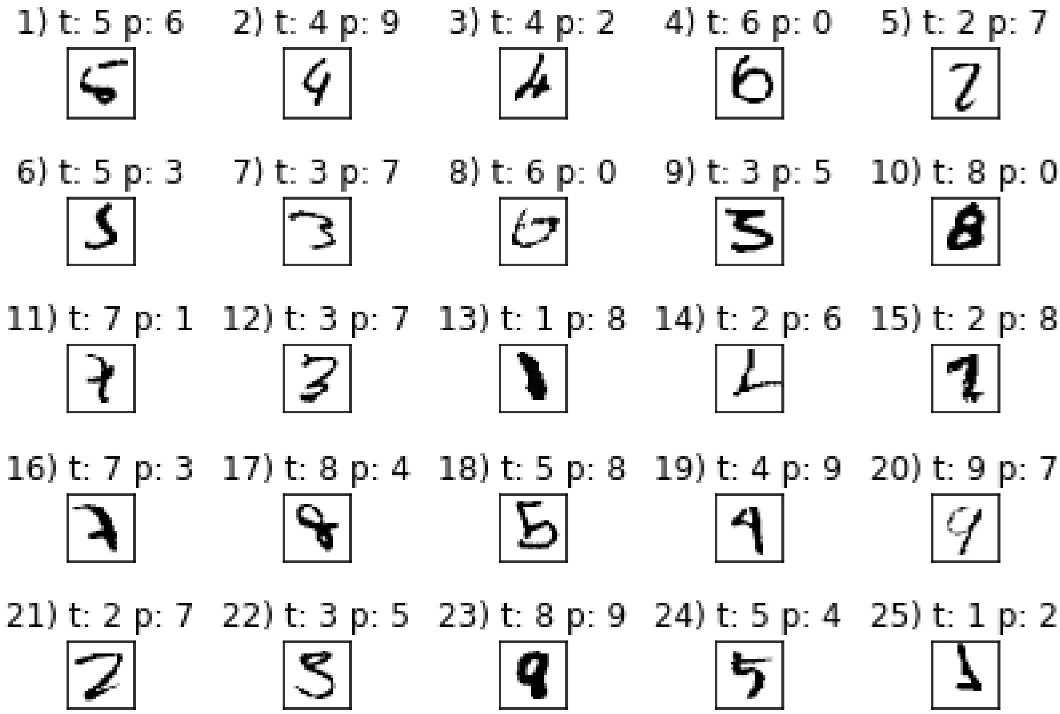
```
plt.xlabel('Epochs')
plt.legend()
plt.show()
```



[23]:
```
y_test_pred = nn.predict(X_test)
acc = (np.sum(y_test == y_test_pred).astype(np.float) / X_test.shape[0])
print('Training accuracy: %.2f%%' % (acc * 100))
```

```
Training accuracy: 97.54%
```

[29]:
```
miscl_img = X_test[y_test != y_test_pred][:25]
correct_lab = y_test[y_test != y_test_pred][:25]
miscl_lab= y_test_pred[y_test != y_test_pred][:25]
fig, ax = plt.subplots(nrows=5,ncols=5,sharex=True,sharey=True,)
ax = ax.flatten()
for i in range(25):
    img = miscl_img[i].reshape(28, 28)
    ax[i].imshow(img,cmap='Greys',interpolation='nearest')
    ax[i].set_title('%d) t: %d p: %d'% (i+1, correct_lab[i], miscl_lab[i]))
ax[0].set_xticks([])
ax[0].set_yticks([])
plt.tight_layout()
plt.show()
```


1) t: 5 p: 6
2) t: 4 p: 9
3) t: 4 p: 2
4) t: 6 p: 0
5) t: 2 p: 7
6) t: 5 p: 3
7) t: 3 p: 7
8) t: 6 p: 0
9) t: 3 p: 5
10) t: 8 p: 0
11) t: 7 p: 1
12) t: 3 p: 7
13) t: 1 p: 8
14) t: 2 p: 6
15) t: 2 p: 8
16) t: 7 p: 3
17) t: 8 p: 4
18) t: 5 p: 8
19) t: 4 p: 9
20) t: 9 p: 7
21) t: 2 p: 7
22) t: 3 p: 5
23) t: 8 p: 9
24) t: 5 p: 4
25) t: 1 p: 2

[ ]: