④ Bayes Examples : Disease or not ?

(+ve) $$P(A|B) = \frac{P(B|A)\,P(A)}{P(B)}$$

$P(+ \mid \text{disease}) = 0.98$ $\quad P(- \mid \text{disease}) = 1 - 0.98 = 0.02$

$P(- \mid \neg \text{disease}) = 0.97$ $\quad P(+ \mid \neg \text{disease}) = 1 - 0.97 = 0.03$

$P(\text{disease}) = 0.008$ $\quad P(\neg \text{disease}) = 1 - 0.008 = 0.992$

$$P(\text{disease} \mid +ve) = \frac{P(+ve \mid \text{disease}) \cdot P(\text{disease})}{P(+ve)}$$

$$= \frac{0.98 \times 0.008}{?}$$

$$P(+ve) = P(+ve \mid \text{disease})\,P(\text{disease}) + P(+ \mid \neg \text{disease})\,P(\neg \text{disease})$$

$$= 0.98 \times 0.008 + 0.03 \times 0.992$$

$$P(\text{disease} \mid +ve) = \frac{0.98 \times 0.008}{0.98 \times 0.008 + 0.03 \times 0.992}$$

$$= \frac{1}{1 + \frac{0.03 \times 0.992}{0.98 \times 0.008}}$$

$$= 0.2085 \quad = 20.85\%$$

- $P(\neg disease \mid +ve) = \frac{P(+ve \mid \neg disease)\, P(\neg disease)}{P(+ve)}$

$$= \frac{0.03 \times 0.992}{0.98 \times 0.008 + 0.03 \times 0.992}$$

$$= 0.7915$$

$$= 79.15\ \%.$$

$P(disease \mid +ve) = 1 - P(disease \mid +ve)$