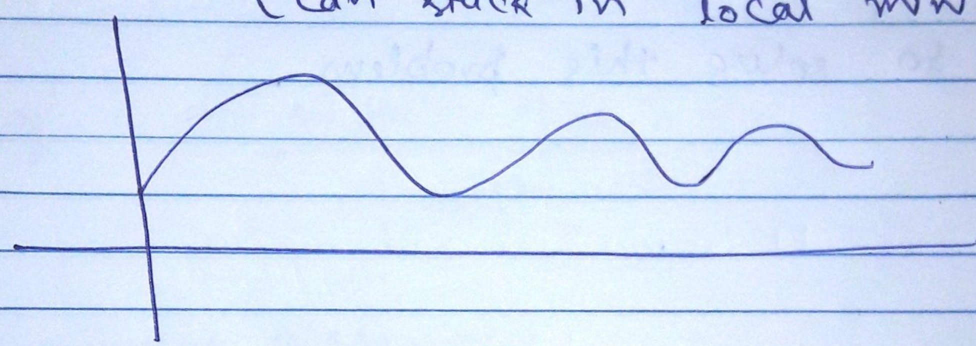
(03) SVM - Objective as Constrained Convex optimization :

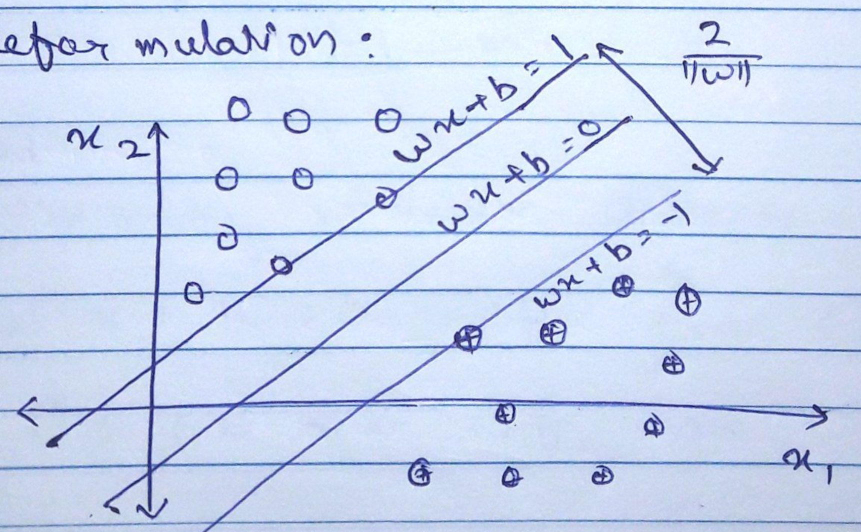
$$\gamma = \min_{i=1 \dots m} \frac{w^T x^{(i)} + b}{\|w\|}$$

This function is Nonconvex.
(can stuck in local minima)



Reform our problem :

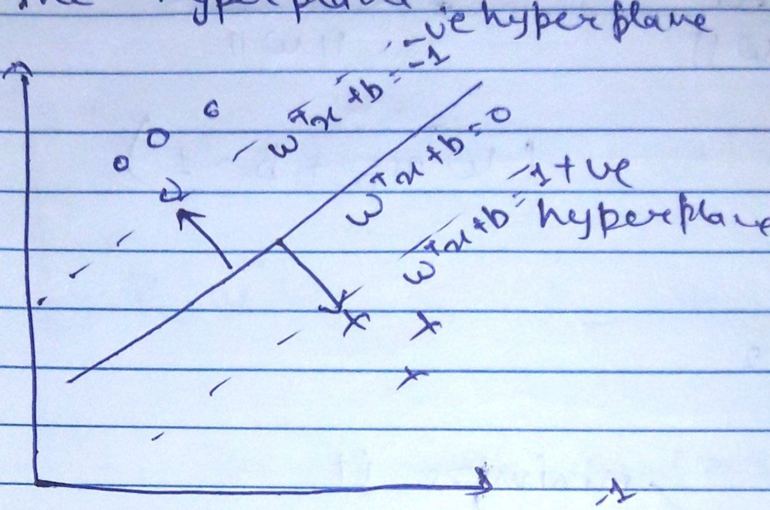
Reformulation :

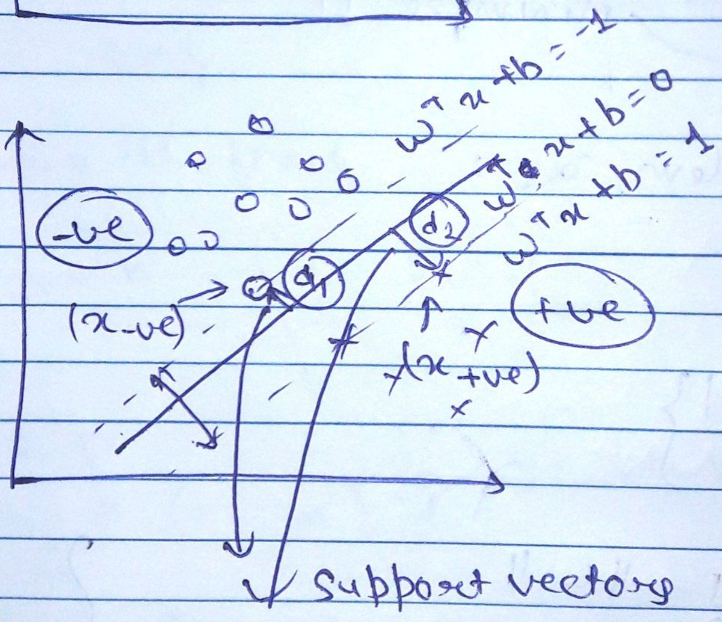


$$\left.\begin{aligned} w^T x + b &= 1 \\ w^T x + b &= -1 \end{aligned}\right\} \text{Hyperplanes}$$

Support vector : (Nearest points)

- we renormalize our dataset such that support vectors should always lie on the hyperplane





Equidistant support vectors.

Goal:

Maximize d such that

$$y_j \left(w^T x^{(i)} + b \right) \geq \gamma$$

$$d_1 = \frac{|w^T x_p + b|}{||w||} = \frac{1}{||w||}$$

positive

$$d_2 = \frac{|w^T x_{-ve} + b|}{||w||} = \frac{1}{||w||}$$

$(w^T x_p + b = 1)$

Maximize

$$d = d_1 + d_2$$

$$\boxed{d = \frac{2}{||w||}}$$

Minimize it

Rewrite our problem as:

<u>SVM Objective:</u>

$$\text{minimize} \left\{ \frac{||w||}{2} \right\}$$

$$= \min \left\{ \begin{array}{l} \frac{1}{2} ||w|| \\ \\ \frac{y_i (w^T x^{(i)} + b)}{\cancel{||w||}} \geq \frac{1}{\cancel{||w||}} \end{array} \right\}$$

<u>SVM Objective 2:</u>

Final objective:

$$\text{minimize} \quad \frac{||w||}{2} \quad (\text{L2 norm of } w)$$

such that

Such that

$\forall_i \in (1 \dots m)$

$$\{ y_i \, (w^T x^{(i)} + b) \geq 1 \}$$

Now

$$\begin{bmatrix} \|w\|_2^2 & = & w^T w \\ \nabla_w w^T w & = & w \end{bmatrix}$$

$\therefore$ for making derivative calculation easy

we will find

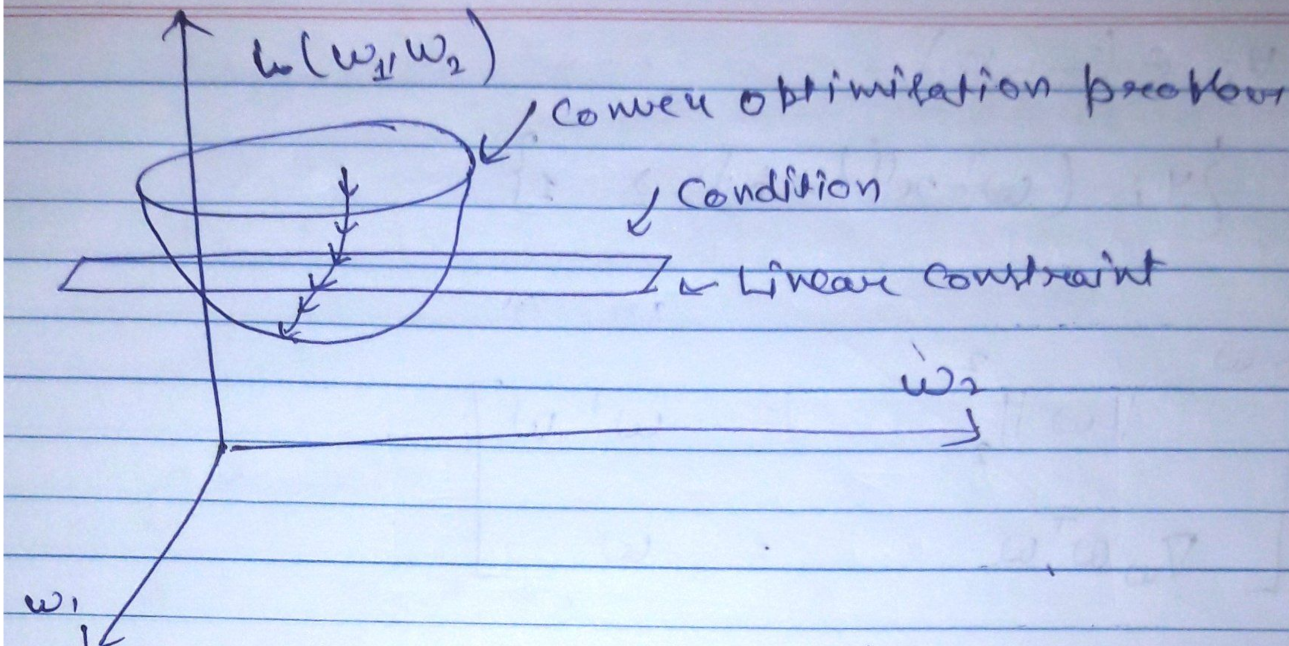
$$\begin{bmatrix} \min & \frac{1}{2} \|w\|_2^2 & \\ \text{s.T} & & y_i \, (w^T x^{(i)} + b) \geq 1 \\ \forall_i \in (1 \dots m) & & \end{bmatrix}$$

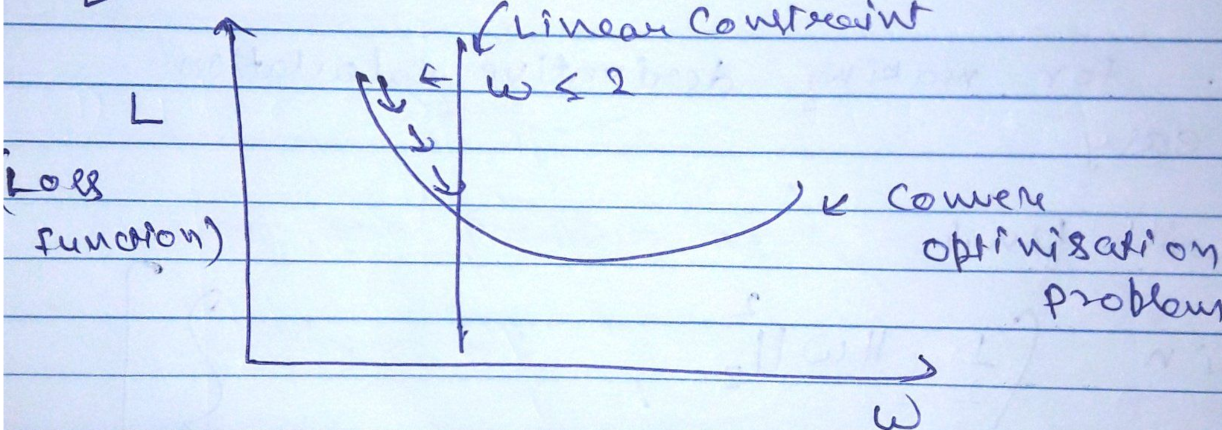
Final SVM Objective (convex optimization problem)

This is convex with linear constraint.
can be solved using:

(a) Quadratic solver

(b) Lagrangian duality





Reference:

CS 229 : Andrew NG

Machine Learning

we will solve it using a modern approach

- Pegsos

and also

one method

- Convex Optimisation without constraint

or unconstrained convex optimisation problem (then gradient descent)