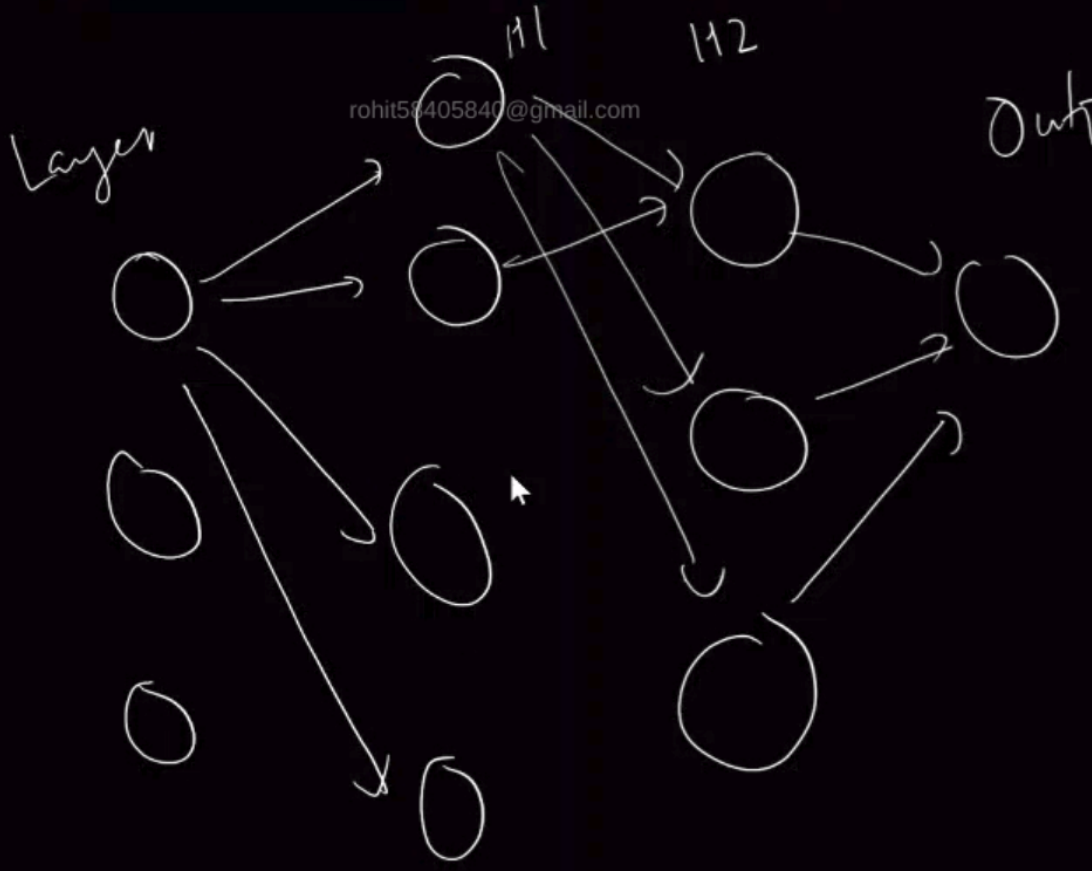
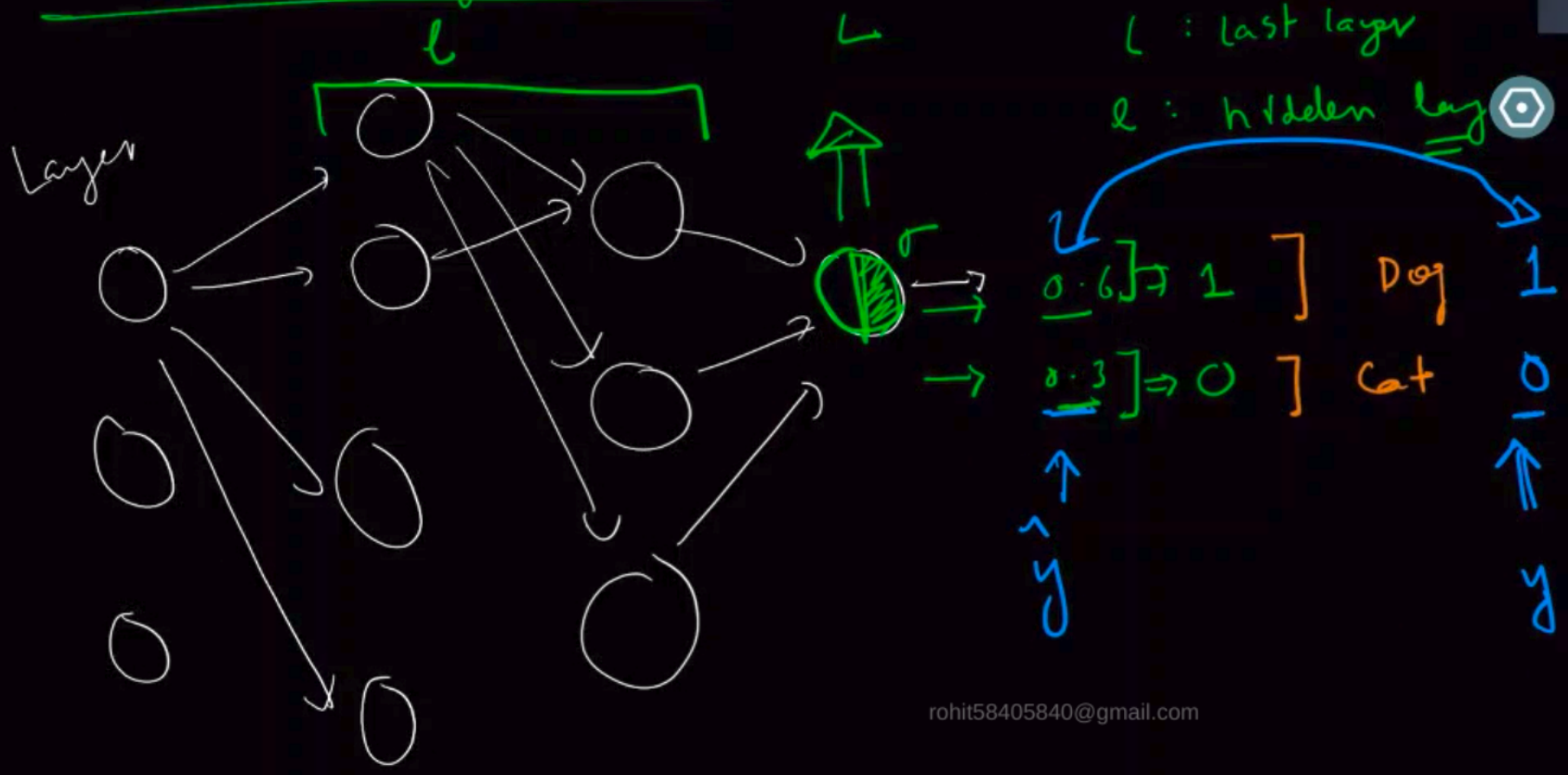
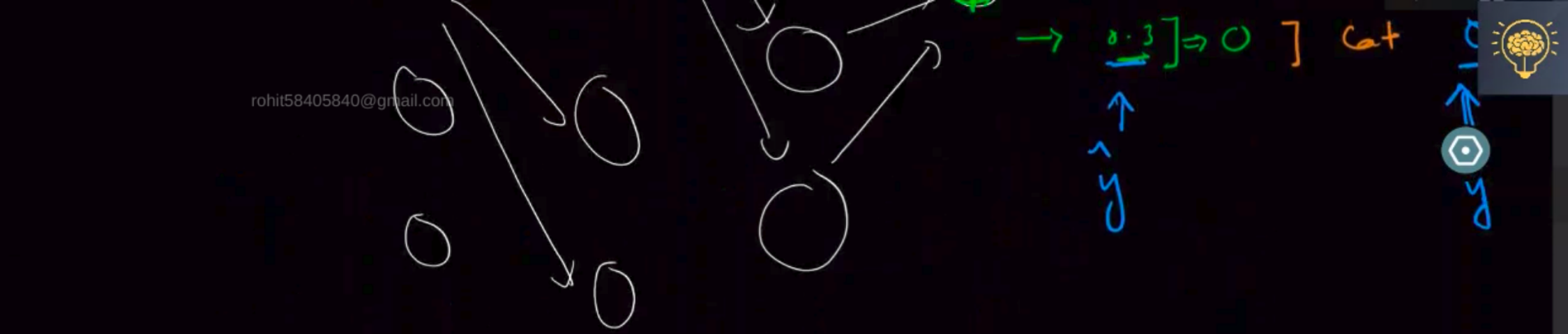
PART-3

BackPROPAgation



BackPROPAgation





For simplification —

$$MSE = L(\hat{y}, y) = \frac{1}{2} \sum_{i=1}^{m} \left(y^{(i)} - \hat{y}^{(i)} \right)^2$$

$$MSE = L(y, \hat{y}) = \frac{1}{2} \sum_{i=1}^{n} (y - \hat{y})$$

Case-1 Output layer.

$$\frac{\partial L}{\partial W^{L}_{i,j}} = ?$$

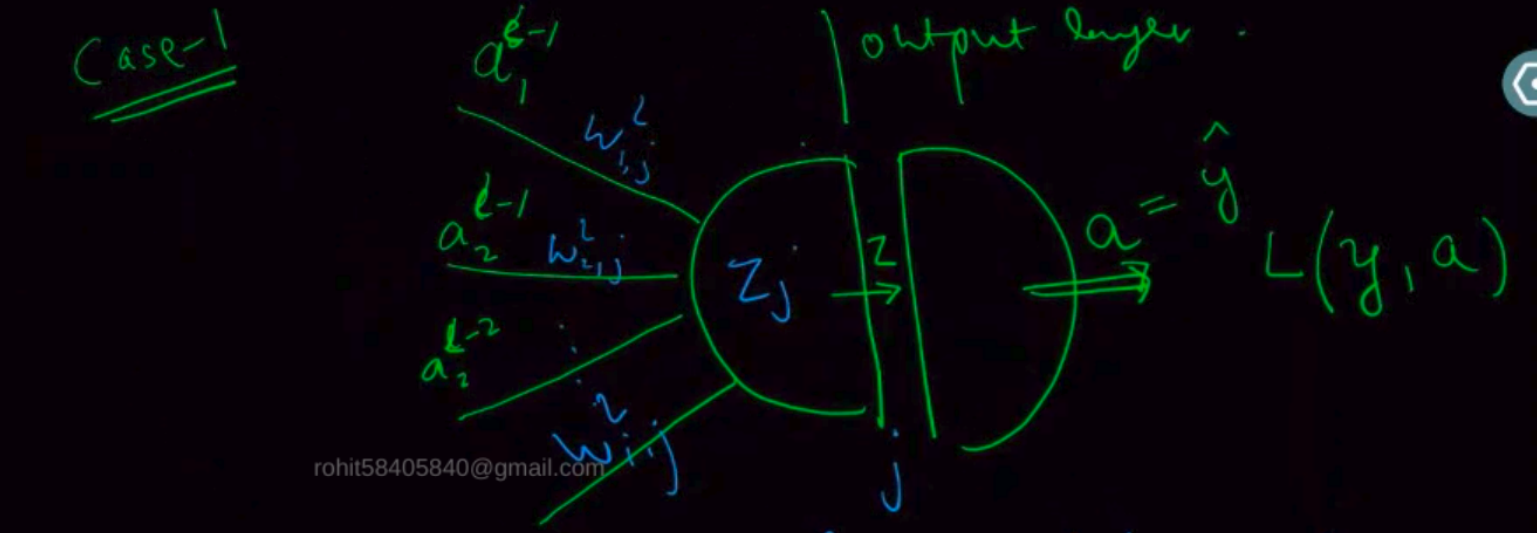
$$\frac{\partial L}{\partial b^{L}} = ?$$

Case-2 Hidden layer.

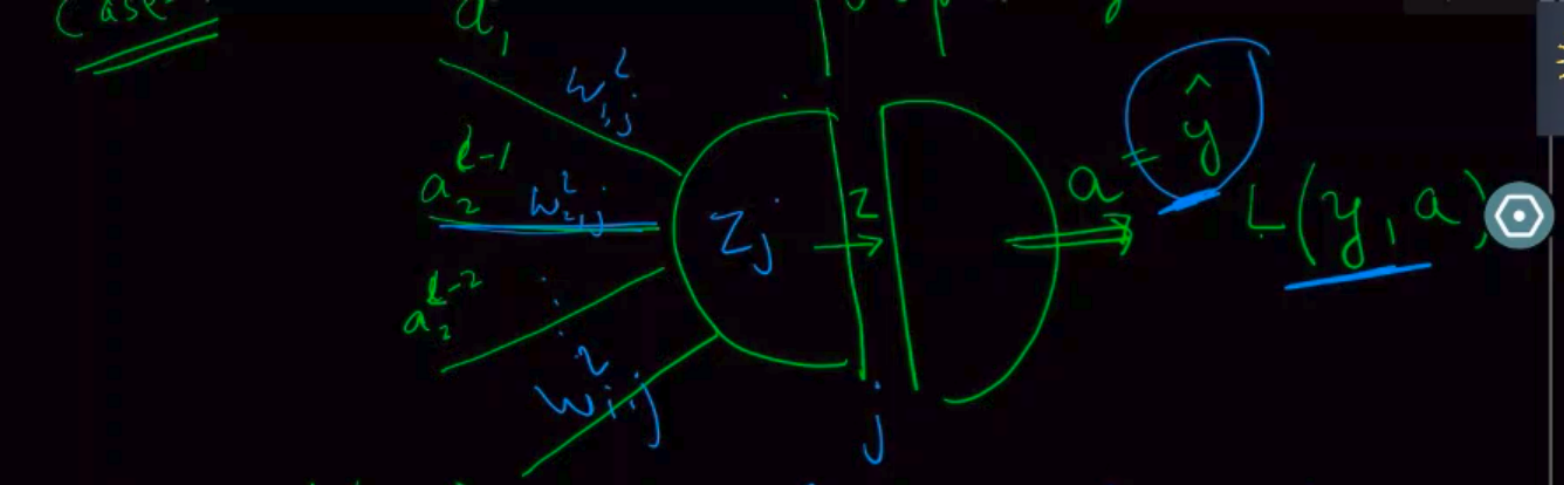
$$\frac{\partial L}{\partial W^{l}_{i,j}} = ?$$

$$\frac{\partial L}{\partial b^{l}} = ?$$

Case-1



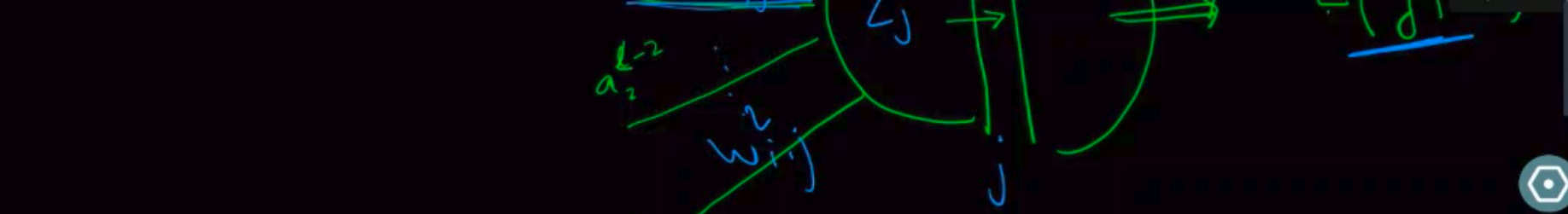
$$z_j = \sum_i^{L} w_{ij}^{L} a_i^{L-1} + b_j^{L}$$



$w_{ij} \rightarrow z_j \rightarrow a_j \rightarrow L(a, y)$

$$z_j = \sum_i^l w_{ij}^l \, a_i^{l-1} + b_j^l$$

$$\frac{\partial L}{\partial w_{ij}} = \frac{\partial L}{\partial a_j}$$



$w_{ij} \rightarrow z_j \rightarrow a_j \rightarrow L(a,y)$

$$z_j^l = \sum_i w_{ij}^l a_i^{l-1} + b_j^l$$

$$\frac{\partial L}{\partial w_{ij}} = \frac{\partial L}{\partial a_j} \cdot \frac{\partial a_j}{\partial z_j} \cdot \frac{\partial z_j}{\partial w_{ij}}$$

Chain Rule

$$\frac{\partial L}{\partial w_{i,j}} = \frac{\partial L}{\partial a_j} \cdot \frac{\partial a_j}{\partial z_j} \cdot \frac{\partial z_j}{\partial w_{i,j}}$$

(1) = (2) (3)

Chain Rule

$$\delta_j^L \Leftarrow \frac{\partial L}{\partial z_j}$$

$\frac{\partial L}{\partial w_{i,j}}$ = $\frac{\partial a_j}{\partial z_j}$ (1) × (2) $\frac{\partial z_j}{\partial w_{i,j}}$ (3)

Rule

$$\delta_j^L \Leftarrow \frac{\partial L}{\partial z_j}$$

L	a_L	dt	y^L
0		0.3	0
i	a_i^L	0.5	1
2		0.2	0

(1) $\frac{\partial L}{\partial a_j} = (y_j - a_j)$

$L = \frac{1}{2} \sum_i (y_i^L - a_i^L)^2$

$\frac{\partial L}{\partial a_j} = \frac{1}{2} \times (y_j - a_j)$

$\delta_j \Leftarrow \frac{\partial L}{\partial z_j}$

$i \to a_i^L$

$j \to$

(1) $\frac{\partial L}{\partial a_j} = (y_j - a_j)$

(2) $\frac{\partial a_j}{\partial z_j} = \sigma'(z_j)$

$L = \frac{1}{2} \sum_i (y_i^L - a_i^L)^2$

$\frac{\partial L}{\partial a_j} = \frac{1}{2} \times 2 (y_j - a_j)$

$a_j = \sigma(z_j)$

$\frac{\partial a_j}{\partial z_j} = \sigma'(z_j)$

$= (1 - \sigma(z_j))\,\sigma(z_j)$

δ_j

$\frac{\partial L}{\partial z_j}$

$\delta \rightarrow$

① $\frac{\partial L}{\partial a_j} = (y_j - a_j)$

③ $\frac{\partial a_j}{\partial z_j} = \sigma'(z_j)$

③ $\frac{\partial z_j}{\partial w_{i,j}} =$

① $L = \frac{1}{2} \sum (y_j - a_j)^2$

$\frac{\partial L}{\partial a_j} = \frac{1}{2} \times 2 (y_j - a_j)$

② $a_j = \sigma(z_j)$

$\frac{\partial a_j}{\partial z_j} = \sigma'(z_j)$

$= (1 - \sigma(z_j))\,\sigma(z_j)$

$$\frac{\partial a_j}{\partial z_j} = \sigma'(z_j)$$

$$\frac{\partial z_j}{\partial w_{i,j}} = a_j^{l-1}$$

2. $a_j = \sigma(z_j)$

$$\frac{\partial a_j}{\partial z_j} = \sigma'(z_j)$$

$$= (1 - \sigma(z_j))\,\sigma(z_j)$$

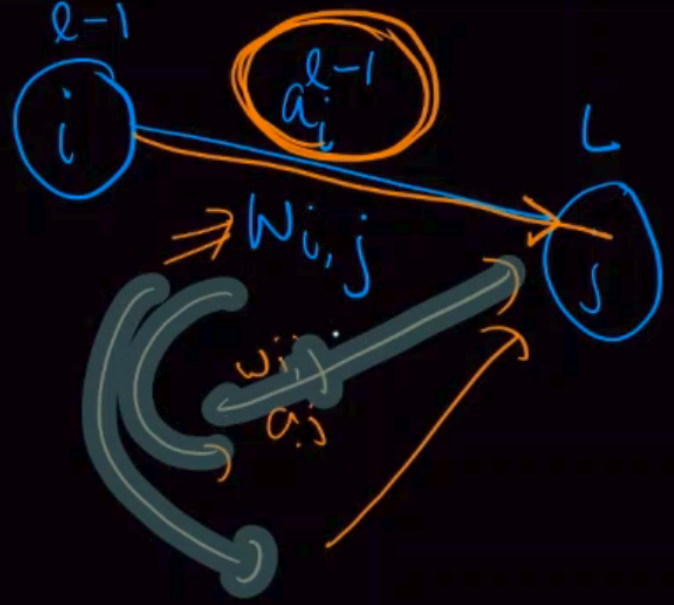
3. $z_j^l = \sum_i w_{i,j}^l a_i^{l-1} + b_j^l$

$$\frac{\partial z_j}{\partial w_{i,j}} =$$

$$\frac{\partial a_j}{\partial z_j} = \sigma'(z_j)$$

$$= (1 - \sigma(z_j))\,\sigma(z_j)$$

(3) $z_j^l = \sum_i w_{i,j}\, a_i^{l-1} + b_j^l$



$$\frac{\partial z_j}{\partial w_{i,j}} = a_i^{l-1}$$

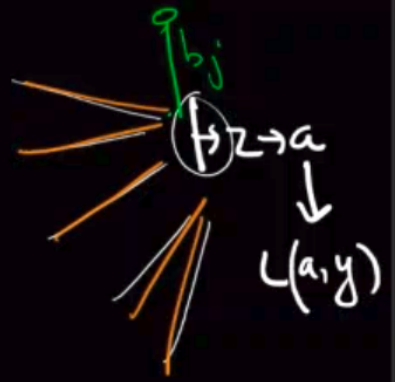
(3) $\dfrac{\partial z_j}{\partial w_{i,j}} = a_i^{l-1}$

$$\frac{\partial L}{\partial w_{ij}^{l}} = \underbrace{(y_j - a_j^l)\,\sigma'(z_j^l)}_{\delta_j^l}\, a_i^{l-1}$$

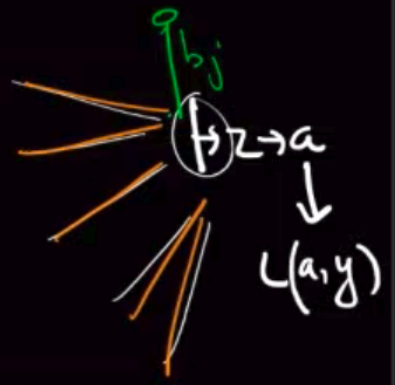
$$\Rightarrow \frac{\partial L}{\partial w_{ij}^{l}} = \delta_j^l\, a_i^{l-1}$$

$$\Rightarrow \frac{\partial L}{\partial w_{ij}} = \delta_j^l \, a_i^{l-1}$$

$$\Rightarrow \frac{\partial L}{\partial b_j} = \frac{\partial L}{\partial a_j} \cdot \frac{\partial a_j}{\partial z_j} \cdot \frac{\partial z_j}{\partial b}$$



$$\Rightarrow \frac{\partial L}{\partial b_j} = \frac{\partial L}{\partial a_j} \cdot \frac{\partial a_j}{\partial z_j} \cdot \frac{\partial z_j}{\partial b_j}$$



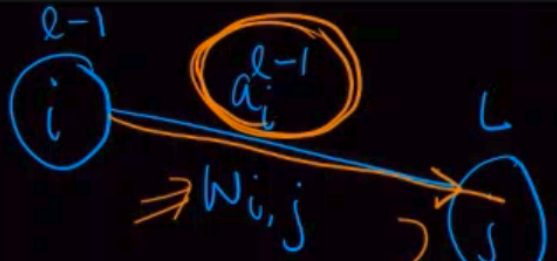
$$= \delta_j^L \cdot 1$$

$$\frac{\partial z_j}{\partial w_{i,j}} = a_i$$

$$\frac{\partial a_j}{\partial z_j} = \sigma'(z_j)$$

$$= (1-\sigma(z_j))\,\sigma(z_j)$$

$$z_j^l = \sum w_{i,j}^l a_i^{l-1} + b_j^l$$



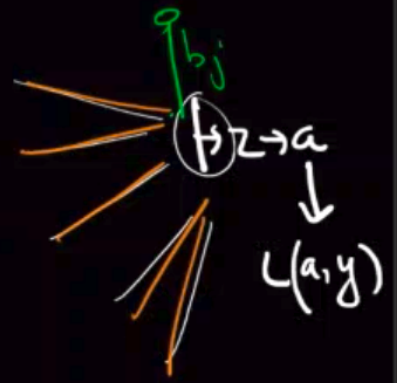
$$\frac{\partial z_j}{\partial w_{i,j}} = a_i^{l-1}$$

$$\frac{\partial z_j}{\partial b_j} = 1$$

$$\frac{\partial L}{\partial w_{i,j}} = \boxed{(y_j - a_j)\,\sigma'(z_j)}\; a_i^{l-1}$$

$$\delta_j^l$$

$$\Rightarrow \frac{\partial L}{\partial b_j} = \frac{\partial L}{\partial a_j} \cdot \frac{\partial a_j}{\partial z_j} \cdot \frac{\partial z_j}{\partial b_j}$$

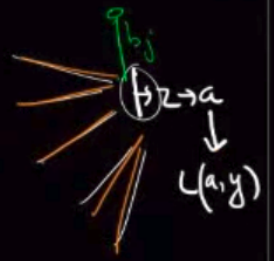


$$\frac{\partial L}{b_j} = \delta_j^L \cdot 1$$

z

$$\Rightarrow \frac{\partial L}{\partial w_{ij}} = \delta_j^L a_i^{L-1}$$

$$\Rightarrow \frac{\partial L}{\partial b_j} = \frac{\partial L}{\partial a_j} \cdot \frac{\partial a_j}{\partial z_j} \cdot \frac{\partial z_j}{\partial b_j}$$



$$\frac{\partial L}{b_j} = \delta_j^L \cdot 1$$

z

For output layer

$$\frac{\partial L}{\partial w_{ij}} =$$

For output layer

$$\frac{\partial L}{\partial w_{ij}} = \delta_j^L \cdot a_i^{L-1}$$

$$\frac{\partial L}{\partial b_j} = \delta_j^L \cdot 1$$

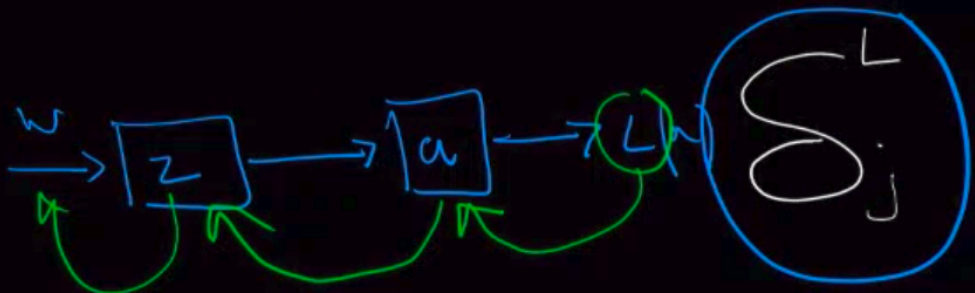
$$\frac{\partial z_j}{\partial w_{ij}} = a_i^{l-1}$$

$$\frac{\partial z_j}{\partial b_j} = 1$$

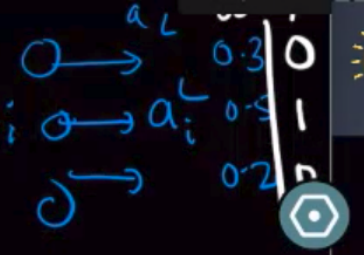
$$\frac{\partial L}{\partial w_{ij}} = \boxed{-(y_j - a_j)\sigma'(z_j)}\, a_i^{l-1}$$

$$\delta_j^l$$

$$\Rightarrow \boxed{\frac{\partial L}{\partial w_{ij}}} = \delta_j^l\, a_i^{l-1}$$



$$\frac{\partial L}{\partial z_j}$$



① $\begin{cases} \frac{\partial L}{\partial a_j} = (y_j - a_j) \end{cases}$

② $\begin{cases} \frac{\partial a_j}{\partial z_j} = \sigma'(z_j) \end{cases}$

③ $\begin{cases} \frac{\partial z_j}{\partial w_{i,j}} = a_i^{l-1} \end{cases}$

① $L = \frac{1}{2} \sum_i (y_i^L - a_i^L)^2$

$\frac{\partial L}{\partial a_j} = \frac{1}{2} \times (y_j - a_j) \times -1$

② $a_j = \sigma(z_j)$

$\frac{\partial a_j}{\partial z_j} = \sigma'(z_j)$

$= (1 - \sigma(z_j)) \sigma(z_j)$

③ $z_j^l = \sum_i w_{i,j}^l a_i^{l-1} + b_j^l$

$$\frac{\partial L}{b_j} = \delta_j^L \cdot 1$$

z

For output layer

$$\frac{\partial L}{\partial w_{ij}} = \delta_j^L \cdot a_i^{L-1}$$

$$\frac{\partial L}{\partial b_j} = \delta_j^L \cdot 1$$

$$\delta_j^L = (a_j - y_j) \odot \sigma'(z_j)$$

For output layer

$$\frac{\partial L}{\partial w_{ij}} = \delta_j^L \cdot a_i^{(L-1)}$$

$$\frac{\partial L}{\partial b_j} = \delta_j^L \cdot 1$$

$$\delta_j^L = (a_j - y_j) \odot \sigma'(z_j)$$