

`In[ ]:= Clear["`*"]`

`In[ ]:= r = Sqrt[Lr^2 + Lp^2/4 Sin[α[t]]^2]`

$$\text{Out[ ]}= \sqrt{Lr^2 + \frac{1}{4} Lp^2 \sin[\alpha[t]]^2}$$

`In[ ]:= th = θ[t] - ArcTan[Lp Sin[α[t]]/(2 Lr)]`

$$\text{Out[ ]}= -\text{ArcTan}\left[\frac{Lp \sin[\alpha[t]]}{2 Lr}\right] + \theta[t]$$

`In[ ]:= z = Lp Cos[α[t]]/2`

$$\text{Out[ ]}= \frac{1}{2} Lp \cos[\alpha[t]]$$

`In[ ]:= vsq = D[r, t]^2 + r^2 D[th, t]^2 + D[z, t]^2`

$$\begin{aligned} \text{Out[ ]}= & \frac{1}{4} Lp^2 \sin[\alpha[t]]^2 \alpha'[t]^2 + \frac{Lp^4 \cos[\alpha[t]]^2 \sin[\alpha[t]]^2 \alpha'[t]^2}{16 \left( Lr^2 + \frac{1}{4} Lp^2 \sin[\alpha[t]]^2 \right)} + \\ & \left( Lr^2 + \frac{1}{4} Lp^2 \sin[\alpha[t]]^2 \right) \left( -\frac{Lp \cos[\alpha[t]] \alpha'[t]}{2 Lr \left( 1 + \frac{Lp^2 \sin[\alpha[t]]^2}{4 Lr^2} \right)} + \theta'[t] \right)^2 \end{aligned}$$

`In[ ]:= vsqs = FullSimplify[vsq, Trig → False]`

$$\text{Out[ ]}= \frac{1}{4} \left( Lp^2 \alpha'[t]^2 - 4 Lp Lr \cos[\alpha[t]] \alpha'[t] \theta'[t] + (4 Lr^2 + Lp^2 \sin[\alpha[t]]^2) \theta'[t]^2 \right)$$

`In[ ]:= Collect[vsqs, {θ'[t], θ'[t], α'[t], α'[t], α'[t] × θ'[t]}]`

$$\text{Out[ ]}= \frac{1}{4} Lp^2 \alpha'[t]^2 - Lp Lr \cos[\alpha[t]] \alpha'[t] \theta'[t] + \frac{1}{4} (4 Lr^2 + Lp^2 \sin[\alpha[t]]^2) \theta'[t]^2$$

`In[ ]:= T = 1/2 Jr D[θ[t], t]^2 + 1/2 Jp D[α[t], t]^2 + 1/2 mp vsqs`

$$\begin{aligned} \text{Out[ ]}= & \frac{1}{2} Jp \alpha'[t]^2 + \frac{1}{2} Jr \theta'[t]^2 + \\ & \frac{1}{8} mp \left( Lp^2 \alpha'[t]^2 - 4 Lp Lr \cos[\alpha[t]] \alpha'[t] \theta'[t] + (4 Lr^2 + Lp^2 \sin[\alpha[t]]^2) \theta'[t]^2 \right) \end{aligned}$$

`In[ ]:= V = mp g Lp/2 Cos[α[t]]`

$$\text{Out[ ]}= \frac{1}{2} g Lp mp \cos[\alpha[t]]$$

`In[ ]:= L = T - V`

$$\begin{aligned} \text{Out[ ]}= & -\frac{1}{2} g Lp mp \cos[\alpha[t]] + \frac{1}{2} Jp \alpha'[t]^2 + \frac{1}{2} Jr \theta'[t]^2 + \\ & \frac{1}{8} mp \left( Lp^2 \alpha'[t]^2 - 4 Lp Lr \cos[\alpha[t]] \alpha'[t] \theta'[t] + (4 Lr^2 + Lp^2 \sin[\alpha[t]]^2) \theta'[t]^2 \right) \end{aligned}$$

In[\*]:= **eq1** = D[D[L,  $\theta'$ [t]], t] - D[L,  $\theta$ [t]]

Out[\*]= 
$$\text{Jr } \theta''[t] + \frac{1}{16} \text{mp} \left( 8 \text{Lp Lr Sin}[\alpha[t]] \alpha'[t]^2 + 4 \text{Lp}^2 \text{Sin}[2 \alpha[t]] \alpha'[t] \theta'[t] - \right. \\ \left. 8 \text{Lp Lr Cos}[\alpha[t]] \alpha''[t] + 2 \left( \text{Lp}^2 + 8 \text{Lr}^2 - \text{Lp}^2 \text{Cos}[2 \alpha[t]] \right) \theta''[t] \right)$$

In[\*]:= **Collect**[eq1, { $\theta''$ [t],  $\theta'$ [t],  $\alpha''$ [t],  $\alpha'$ [t],  $\alpha'[t] \times \theta'[t]$ }]

Out[\*]= 
$$\frac{1}{2} \text{Lp Lr mp Sin}[\alpha[t]] \alpha'[t]^2 + \frac{1}{4} \text{Lp}^2 \text{mp Sin}[2 \alpha[t]] \alpha'[t] \theta'[t] - \\ \frac{1}{2} \text{Lp Lr mp Cos}[\alpha[t]] \alpha''[t] + \left( \text{Jr} + \frac{1}{8} \text{mp} \left( \text{Lp}^2 + 8 \text{Lr}^2 - \text{Lp}^2 \text{Cos}[2 \alpha[t]] \right) \right) \theta''[t]$$

In[\*]:= **Coefficient**[eq1,  $\alpha'[t]^2$ ]

Out[\*]= 
$$\frac{1}{2} \text{Lp Lr mp Sin}[\alpha[t]]$$

In[\*]:= **eq2** = D[D[L,  $\alpha'$ [t]], t] - D[L,  $\alpha$ [t]] // Expand

Out[\*]= 
$$-\frac{1}{2} g \text{Lp mp Sin}[\alpha[t]] - \frac{1}{8} \text{Lp}^2 \text{mp Sin}[2 \alpha[t]] \theta'[t]^2 + \\ \text{Jp } \alpha''[t] + \frac{1}{4} \text{Lp}^2 \text{mp } \alpha''[t] - \frac{1}{2} \text{Lp Lr mp Cos}[\alpha[t]] \theta''[t]$$

In[\*]:= **Collect**[eq2, { $\theta''$ [t],  $\theta'$ [t],  $\alpha''$ [t],  $\alpha'$ [t],  $\alpha'[t] \times \theta'[t]$ }]

Out[\*]= 
$$-\frac{1}{2} g \text{Lp mp Sin}[\alpha[t]] - \frac{1}{8} \text{Lp}^2 \text{mp Sin}[2 \alpha[t]] \theta'[t]^2 + \\ \left( \text{Jp} + \frac{\text{Lp}^2 \text{mp}}{4} \right) \alpha''[t] - \frac{1}{2} \text{Lp Lr mp Cos}[\alpha[t]] \theta''[t]$$

In[\*]:=

$$\mathbf{H} = \left\{ \left\{ \left( \text{Jr} + \frac{1}{4} \text{mp Lp}^2 \text{Sin}[\alpha[t]]^2 + \text{mp Lr}^2 \right), -\frac{1}{2} \text{Lp Lr mp Cos}[\alpha[t]] \right\}, \right. \\ \left. \left\{ -\frac{1}{2} \text{Lp Lr mp Cos}[\alpha[t]], \left( \text{Jp} + \frac{\text{Lp}^2 \text{mp}}{4} \right) \right\} \right\}$$

Out[\*]= 
$$\left\{ \left\{ \text{Jr} + \text{Lr}^2 \text{mp} + \frac{1}{4} \text{Lp}^2 \text{mp Sin}[\alpha[t]]^2, -\frac{1}{2} \text{Lp Lr mp Cos}[\alpha[t]] \right\}, \right. \\ \left. \left\{ -\frac{1}{2} \text{Lp Lr mp Cos}[\alpha[t]], \text{Jp} + \frac{\text{Lp}^2 \text{mp}}{4} \right\} \right\}$$

In[\*]:= **C1** =  $\left\{ \left\{ \frac{1}{4} \text{Lp}^2 \text{mp Sin}[2 \alpha[t]] \alpha'[t] + \text{Br} + \text{n Kg}^2 \text{Kt km} / \text{Rm}, \frac{1}{2} \text{Lp Lr mp Sin}[\alpha[t]] \right\}, \right. \\ \left. \left\{ -\frac{1}{8} \text{Lp}^2 \text{mp Sin}[2 \alpha[t]] \theta'[t], \text{Bp} \right\} \right\}$

Out[\*]= 
$$\left\{ \left\{ \text{Br} + \frac{\text{Kg}^2 \text{km Kt n}}{\text{Rm}} + \frac{1}{4} \text{Lp}^2 \text{mp Sin}[2 \alpha[t]] \alpha'[t], \frac{1}{2} \text{Lp Lr mp Sin}[\alpha[t]] \right\}, \right. \\ \left. \left\{ -\frac{1}{8} \text{Lp}^2 \text{mp Sin}[2 \alpha[t]] \theta'[t], \text{Bp} \right\} \right\}$$

$$\text{In[*]:= } \mathbf{G} = \left\{ 0, -\frac{1}{2} g \text{ Lp mp Sin}[\alpha[t]] \right\}$$

$$\text{Out[*]:= } \left\{ 0, -\frac{1}{2} g \text{ Lp mp Sin}[\alpha[t]] \right\}$$

$$\text{In[*]:= } \mathbf{B} = \left\{ n \text{ Kg Kt u}[t] / \text{Rm}, 0 \right\}$$

$$\text{Out[*]:= } \left\{ \frac{\text{Kg Kt n u}[t]}{\text{Rm}}, 0 \right\}$$

$$\text{In[*]:= } \mathbf{q} = \{\theta[t], \alpha[t]\}$$

$$\text{Out[*]:= } \{\theta[t], \alpha[t]\}$$

$$\text{In[*]:= } \text{vecqdd} = \text{Inverse}[\mathbf{H}] \cdot (-\mathbf{C1.D}[\mathbf{q}, t] - \mathbf{G} + \mathbf{B}) \quad // \quad \text{Simplify}$$

$$\begin{aligned} \text{Out[*]:= } & \left\{ \left( 2 g \text{ Lp}^2 \text{ Lr mp}^2 \text{ Rm Sin}[2 \alpha[t]] + 4 \text{ Kg Kt} (4 \text{ Jp} + \text{Lp}^2 \text{ mp}) n u[t] - \right. \right. \\ & 16 \text{ Jp Kg}^2 \text{ km Kt n } \theta'[t] - 4 \text{ Kg}^2 \text{ km Kt Lp}^2 \text{ mp n } \theta'[t] - 16 \text{ Br Jp Rm } \theta'[t] - \\ & 4 \text{ Br Lp}^2 \text{ mp Rm } \theta'[t] + \text{Lp}^3 \text{ Lr mp}^2 \text{ Rm Cos}[\alpha[t]] \text{ Sin}[2 \alpha[t]] \theta'[t]^2 - \text{Lp mp Rm } \alpha'[t] \\ & \left. (8 \text{ Bp Lr Cos}[\alpha[t]] + 2 \text{ Lr} (4 \text{ Jp} + \text{Lp}^2 \text{ mp}) \text{ Sin}[\alpha[t]] + \text{Lp} (4 \text{ Jp} + \text{Lp}^2 \text{ mp}) \text{ Sin}[2 \alpha[t]] \theta'[t]) \right) / \\ & \left( \text{Rm} (-4 \text{ Lp}^2 \text{ Lr}^2 \text{ mp}^2 \text{ Cos}[\alpha[t]]^2 + (4 \text{ Jp} + \text{Lp}^2 \text{ mp}) (4 (\text{Jr} + \text{Lr}^2 \text{ mp}) + \text{Lp}^2 \text{ mp Sin}[\alpha[t]]^2)) \right), \\ & \left( 8 \text{ Kg Kt Lp Lr mp n Cos}[\alpha[t]] u[t] - 2 \text{ Rm } \alpha'[t] (8 \text{ Bp Jr} + 8 \text{ Bp Lr}^2 \text{ mp} + 2 \text{ Bp Lp}^2 \text{ mp Sin}[\alpha[t]]^2 + \right. \\ & \left. \text{Lp}^2 \text{ Lr}^2 \text{ mp}^2 \text{ Sin}[2 \alpha[t]] + 2 \text{ Lp}^3 \text{ Lr mp}^2 \text{ Cos}[\alpha[t]]^2 \text{ Sin}[\alpha[t]] \theta'[t]) + \text{Lp mp} \right. \\ & \left. (2 g \text{ Rm Sin}[\alpha[t]] (4 (\text{Jr} + \text{Lr}^2 \text{ mp}) + \text{Lp}^2 \text{ mp Sin}[\alpha[t]]^2) - 8 \text{ Lr} (\text{Kg}^2 \text{ km Kt n} + \text{Br Rm}) \text{ Cos}[\alpha[t]] \right. \\ & \left. \theta'[t] + \text{Lp Rm} (\text{Lp}^2 \text{ mp Cos}[\alpha[t]] \text{ Sin}[\alpha[t]]^3 + 2 (\text{Jr} + \text{Lr}^2 \text{ mp}) \text{ Sin}[2 \alpha[t]] \theta'[t]^2)) \right) / \\ & \left. \left( \text{Rm} (-4 \text{ Lp}^2 \text{ Lr}^2 \text{ mp}^2 \text{ Cos}[\alpha[t]]^2 + (4 \text{ Jp} + \text{Lp}^2 \text{ mp}) (4 (\text{Jr} + \text{Lr}^2 \text{ mp}) + \text{Lp}^2 \text{ mp Sin}[\alpha[t]]^2)) \right) \right\} \end{aligned}$$

$$\begin{aligned} \text{In[*]:= } \mathbf{fstate} &= \{\theta'[t], \alpha'[t], \text{vecqdd}[1], \text{vecqdd}[2]\} /. \\ &\{\theta[t] \rightarrow x_1, \alpha[t] \rightarrow x_2, \theta'[t] \rightarrow x_3, \alpha'[t] \rightarrow x_4\} \end{aligned}$$

$$\begin{aligned} \text{Out[*]:= } & \left\{ x_3, x_4, \left( 2 g \text{ Lp}^2 \text{ Lr mp}^2 \text{ Rm Sin}[2 x_2] - 16 \text{ Jp Kg}^2 \text{ km Kt n } x_3 - 4 \text{ Kg}^2 \text{ km Kt Lp}^2 \text{ mp n } x_3 - \right. \right. \\ & 16 \text{ Br Jp Rm } x_3 - 4 \text{ Br Lp}^2 \text{ mp Rm } x_3 + \text{Lp}^3 \text{ Lr mp}^2 \text{ Rm Cos}[x_2] \text{ Sin}[2 x_2] x_3^2 - \\ & \text{Lp mp Rm} (8 \text{ Bp Lr Cos}[x_2] + 2 \text{ Lr} (4 \text{ Jp} + \text{Lp}^2 \text{ mp}) \text{ Sin}[x_2] + \text{Lp} (4 \text{ Jp} + \text{Lp}^2 \text{ mp}) \text{ Sin}[2 x_2] x_3) x_4 + \\ & 4 \text{ Kg Kt} (4 \text{ Jp} + \text{Lp}^2 \text{ mp}) n u[t] \left. \right) / \\ & \left( \text{Rm} (-4 \text{ Lp}^2 \text{ Lr}^2 \text{ mp}^2 \text{ Cos}[x_2]^2 + (4 \text{ Jp} + \text{Lp}^2 \text{ mp}) (4 (\text{Jr} + \text{Lr}^2 \text{ mp}) + \text{Lp}^2 \text{ mp Sin}[x_2]^2)) \right), \\ & \left( \text{Lp mp} (2 g \text{ Rm Sin}[x_2] (4 (\text{Jr} + \text{Lr}^2 \text{ mp}) + \text{Lp}^2 \text{ mp Sin}[x_2]^2) - 8 \text{ Lr} (\text{Kg}^2 \text{ km Kt n} + \text{Br Rm}) \text{ Cos}[x_2] x_3 + \right. \\ & \left. \text{Lp Rm} (\text{Lp}^2 \text{ mp Cos}[x_2] \text{ Sin}[x_2]^3 + 2 (\text{Jr} + \text{Lr}^2 \text{ mp}) \text{ Sin}[2 x_2] x_3^2) - \right. \\ & 2 \text{ Rm} (8 \text{ Bp Jr} + 8 \text{ Bp Lr}^2 \text{ mp} + 2 \text{ Bp Lp}^2 \text{ mp Sin}[x_2]^2 + \text{Lp}^2 \text{ Lr}^2 \text{ mp}^2 \text{ Sin}[2 x_2] + \\ & 2 \text{ Lp}^3 \text{ Lr mp}^2 \text{ Cos}[x_2]^2 \text{ Sin}[x_2] x_3) x_4 + 8 \text{ Kg Kt Lp Lr mp n Cos}[x_2] u[t] \left. \right) / \\ & \left. \left( \text{Rm} (-4 \text{ Lp}^2 \text{ Lr}^2 \text{ mp}^2 \text{ Cos}[x_2]^2 + (4 \text{ Jp} + \text{Lp}^2 \text{ mp}) (4 (\text{Jr} + \text{Lr}^2 \text{ mp}) + \text{Lp}^2 \text{ mp Sin}[x_2]^2)) \right) \right\} \end{aligned}$$

$$\text{In[*]:= } \mathbf{xvec} = \{x_1, x_2, x_3, x_4\}$$

$$\text{Out[*]:= } \{x_1, x_2, x_3, x_4\}$$

```
In[ ]:= D[fstate, {xvec}] /. {x1 -> 0, x2 -> 0, x3 -> 0, x4 -> 0, u -> 0} // FullSimplify // MatrixForm
```

```
Out[ ]//MatrixForm=
```

$$\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{g L p^2 L r m p^2}{J r L p^2 m p + 4 J p (J r + L r^2 m p)} & -\frac{(4 J p + L p^2 m p) (K g^2 k m K t n + B r R m)}{(J r L p^2 m p + 4 J p (J r + L r^2 m p)) R m} & -\frac{2 B p L p L r m p}{J r L p^2 m p + 4 J p (J r + L r^2 m p)} \\ 0 & \frac{2 g L p m p (J r + L r^2 m p)}{J r L p^2 m p + 4 J p (J r + L r^2 m p)} & -\frac{2 L p L r m p (K g^2 k m K t n + B r R m)}{(J r L p^2 m p + 4 J p (J r + L r^2 m p)) R m} & -\frac{4 B p (J r + L r^2 m p)}{J r L p^2 m p + 4 J p (J r + L r^2 m p)} \end{pmatrix}$$

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In[ ]:=
```

**B2 =**

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D[fstate, u[t]] /. {x1 -> 0, x2 -> 0, x3 -> 0, x4 -> 0, u -> 0} // FullSimplify // MatrixForm
```

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Out[ ]//MatrixForm=
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$$\begin{pmatrix} 0 \\ 0 \\ \frac{K g K t (4 J p + L p^2 m p) n}{(J r L p^2 m p + 4 J p (J r + L r^2 m p)) R m} \\ \frac{2 K g K t L p L r m p n}{4 J p J r R m + J r L p^2 m p R m + 4 J p L r^2 m p R m} \end{pmatrix}$$

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In[9]:= A = D[fstate, {xvec}] // FullSimplify
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$$Out[\bullet] = \{ \{0, 0, 1, 0\}, \{0, 0, 0, 1\},$$

$$\begin{aligned} & \left\{ \emptyset, \left( \text{Lp mp} \left( \frac{1}{2} \text{Rm} \left( -4 \text{Lp}^2 \text{Lr}^2 \text{mp}^2 \text{Cos} [x_2]^2 + (4 \text{Jp} + \text{Lp}^2 \text{mp}) \left( 4 (\text{Jr} + \text{Lr}^2 \text{mp}) + \text{Lp}^2 \text{mp Sin} [x_2]^2 \right) \right) \right. \right. \right. \\ & \quad \left( \text{Lp Lr mp} \left( 8 \text{g Cos} [2 x_2] + \text{Lp} \left( \text{Cos} [x_2] + 3 \text{Cos} [3 x_2] \right) x_3^2 \right) + \right. \\ & \quad 4 \left( -\text{Lr} \left( 4 \text{Jp} + \text{Lp}^2 \text{mp} \right) \text{Cos} [x_2] + 4 \text{Bp Lr Sin} [x_2] - \text{Lp} \left( 4 \text{Jp} + \text{Lp}^2 \text{mp} \right) \text{Cos} [2 x_2] x_3 \right) x_4 - \\ & \quad \text{Lp} \left( 4 \text{Jp} + (\text{Lp}^2 + 4 \text{Lr}^2) \text{mp} \right) \text{Sin} [2 x_2] \left( \text{Lp}^3 \text{Lr mp}^2 \text{Rm Cos} [x_2] \text{Sin} [2 x_2] x_3^2 - 2 \text{Lp Lr} \right. \\ & \quad \left. \text{mp Rm} \left( -\text{g Lp mp Sin} [2 x_2] + (4 \text{Bp Cos} [x_2] + (4 \text{Jp} + \text{Lp}^2 \text{mp}) \text{Sin} [x_2] \right) x_4 \right) - (4 \text{Jp} + \text{Lp}^2 \text{mp}) \\ & \quad \left. x_3 \left( 4 \text{Kg}^2 \text{km Kt n} + 4 \text{Br Rm} + \text{Lp}^2 \text{mp Rm Sin} [2 x_2] x_4 \right) + 4 \text{Kg Kt} \left( 4 \text{Jp} + \text{Lp}^2 \text{mp} \right) \text{n u} [\text{t}] \right) \left. \right) \left. \right) / \\ & \quad \left( \text{Rm} \left( -4 \text{Lp}^2 \text{Lr}^2 \text{mp}^2 \text{Cos} [x_2]^2 + (4 \text{Jp} + \text{Lp}^2 \text{mp}) \left( 4 (\text{Jr} + \text{Lr}^2 \text{mp}) + \text{Lp}^2 \text{mp Sin} [x_2]^2 \right) \right)^2 \right), \\ & \quad \frac{4 \text{Lp}^3 \text{Lr mp}^2 \text{Rm Cos} [x_2]^2 \text{Sin} [x_2] x_3 - (4 \text{Jp} + \text{Lp}^2 \text{mp}) \left( 4 \text{Kg}^2 \text{km Kt n} + 4 \text{Br Rm} + \text{Lp}^2 \text{mp Rm Sin} [2 x_2] x_4 \right)}{\text{Rm} \left( -4 \text{Lp}^2 \text{Lr}^2 \text{mp}^2 \text{Cos} [x_2]^2 + (4 \text{Jp} + \text{Lp}^2 \text{mp}) \left( 4 (\text{Jr} + \text{Lr}^2 \text{mp}) + \text{Lp}^2 \text{mp Sin} [x_2]^2 \right) \right)} \\ & \quad \left. , - \frac{\text{Lp mp} \left( 8 \text{Bp Lr Cos} [x_2] + 2 \text{Lr} \left( 4 \text{Jp} + \text{Lp}^2 \text{mp} \right) \text{Sin} [x_2] + \text{Lp} \left( 4 \text{Jp} + \text{Lp}^2 \text{mp} \right) \text{Sin} [2 x_2] x_3 \right)}{-4 \text{Lp}^2 \text{Lr}^2 \text{mp}^2 \text{Cos} [x_2]^2 + (4 \text{Jp} + \text{Lp}^2 \text{mp}) \left( 4 (\text{Jr} + \text{Lr}^2 \text{mp}) + \text{Lp}^2 \text{mp Sin} [x_2]^2 \right)} \right\}, \\ & \left\{ \emptyset, \left( \text{Lp mp} \left( \left( -4 \text{Lp}^2 \text{Lr}^2 \text{mp}^2 \text{Cos} [x_2]^2 + (4 \text{Jp} + \text{Lp}^2 \text{mp}) \left( 4 (\text{Jr} + \text{Lr}^2 \text{mp}) + \text{Lp}^2 \text{mp Sin} [x_2]^2 \right) \right) \right. \right. \right. \\ & \quad \left( 2 \text{g Rm Cos} [x_2] \left( 4 (\text{Jr} + \text{Lr}^2 \text{mp}) + 3 \text{Lp}^2 \text{mp Sin} [x_2]^2 \right) + 8 \text{Lr} \left( \text{Kg}^2 \text{km Kt n} + \text{Br Rm} \right) \text{Sin} [x_2] x_3 + \right. \\ & \quad \frac{1}{2} \text{Lp Rm} \left( (8 \text{Jr} + (\text{Lp}^2 + 8 \text{Lr}^2) \text{mp}) \text{Cos} [2 x_2] - \text{Lp}^2 \text{mp Cos} [4 x_2] \right) x_3^2 - \\ & \quad \text{Lp Rm} \left( 4 \left( \text{Lr}^2 \text{mp Cos} [2 x_2] + \text{Bp Sin} [2 x_2] \right) + \text{Lp Lr mp} \left( \text{Cos} [x_2] + 3 \text{Cos} [3 x_2] \right) x_3 \right) x_4 - \\ & \quad \left. 8 \text{Kg Kt Lr n Sin} [x_2] \text{u} [\text{t}] \right) - \text{Lp} \left( 4 \text{Jp} + (\text{Lp}^2 + 4 \text{Lr}^2) \text{mp} \right) \text{Sin} [2 x_2] \\ & \quad \left( \text{Lp}^2 \text{mp Rm} \left( \text{Lp}^2 \text{mp Cos} [x_2] \text{Sin} [x_2]^3 + 2 (\text{Jr} + \text{Lr}^2 \text{mp}) \text{Sin} [2 x_2] \right) x_3^2 - \right. \\ & \quad 4 \text{Rm} \left( 4 \text{Bp} \left( \text{Jr} + \text{Lr}^2 \text{mp} \right) + \text{Lp}^2 \text{mp Sin} [x_2] \left( \text{Lr}^2 \text{mp Cos} [x_2] + \text{Bp Sin} [x_2] \right) \right) x_4 - \\ & \quad 2 \text{Lp Lr mp Cos} [x_2] x_3 \left( 4 \text{Kg}^2 \text{km Kt n} + 4 \text{Br Rm} + \text{Lp}^2 \text{mp Rm Sin} [2 x_2] x_4 \right) + \\ & \quad \left. 2 \text{Lp mp} \left( \text{g Rm Sin} [x_2] \left( 4 \text{Jr} + 4 \text{Lr}^2 \text{mp} + \text{Lp}^2 \text{mp Sin} [x_2]^2 \right) + 4 \text{Kg Kt Lr n Cos} [x_2] \text{u} [\text{t}] \right) \right) \left. \right) \left. \right) / \\ & \quad \left( \text{Rm} \left( -4 \text{Lp}^2 \text{Lr}^2 \text{mp}^2 \text{Cos} [x_2]^2 + (4 \text{Jp} + \text{Lp}^2 \text{mp}) \left( 4 (\text{Jr} + \text{Lr}^2 \text{mp}) + \text{Lp}^2 \text{mp Sin} [x_2]^2 \right) \right)^2 \right), \\ & \quad \left( \text{Lp mp Cos} [x_2] \left( \text{Lp Rm} \left( 8 \text{Jr} + \text{Lp}^2 \text{mp} + 8 \text{Lr}^2 \text{mp} - \text{Lp}^2 \text{mp Cos} [2 x_2] \right) \text{Sin} [x_2] x_3 - \right. \right. \\ & \quad \left. 2 \text{Lr} \left( 4 \text{Kg}^2 \text{km Kt n} + 4 \text{Br Rm} + \text{Lp}^2 \text{mp Rm Sin} [2 x_2] x_4 \right) \right) \left. \right) / \\ & \quad \left( \text{Rm} \left( -4 \text{Lp}^2 \text{Lr}^2 \text{mp}^2 \text{Cos} [x_2]^2 + (4 \text{Jp} + \text{Lp}^2 \text{mp}) \left( 4 (\text{Jr} + \text{Lr}^2 \text{mp}) + \text{Lp}^2 \text{mp Sin} [x_2]^2 \right) \right) \right), \\ & \quad - \left( \left( 4 \left( 4 \text{Bp} \left( \text{Jr} + \text{Lr}^2 \text{mp} \right) + \text{Lp}^2 \text{mp Sin} [x_2] \left( \text{Lr}^2 \text{mp Cos} [x_2] + \text{Bp Sin} [x_2] \right) \right) + \right. \right. \\ & \quad \left. \left. \text{Lp}^3 \text{Lr mp}^2 \text{Cos} [x_2]^2 \text{Sin} [x_2] x_3 \right) \right) / \\ & \quad \left( -4 \text{Lp}^2 \text{Lr}^2 \text{mp}^2 \text{Cos} [x_2]^2 + (4 \text{Jp} + \text{Lp}^2 \text{mp}) \left( 4 (\text{Jr} + \text{Lr}^2 \text{mp}) + \text{Lp}^2 \text{mp Sin} [x_2]^2 \right) \right) \left. \right) \left. \right\} \end{aligned}$$

$$\ln[\bullet] := \mathbf{A} \text{ // ToMatlab}$$
$$\begin{aligned} \text{Out}[4] = & [0,0,1,0;0,0,0,1;0,\text{Lp}.*\text{mp}.*\text{Rm}.^{(-1)}.*( (-4).*\text{Lp}.^2.*\text{Lr}.^2.*\text{mp}.^2.* \dots \\ & \cos(\text{Subscript}(x,2)).^2+(4.*\text{Jp}+\text{Lp}.^2.*\text{mp}).*(4.*(\text{Jr}+\text{Lr}.^2.*\text{mp})+ \dots \\ & \text{Lp}.^2.*\text{mp}.*\sin(\text{Subscript}(x,2)).^2)).^{(-2)}.*((1/2).*\text{Rm}.*((-4).* \dots \\ & \text{Lp}.^2.*\text{Lr}.^2.*\text{mp}.^2.*\cos(\text{Subscript}(x,2)).^2+(4.*\text{Jp}+\text{Lp}.^2.*\text{mp}).*( \dots \\ & 4.*(\text{Jr}+\text{Lr}.^2.*\text{mp})+\text{Lp}.^2.*\text{mp}.*\sin(\text{Subscript}(x,2)).^2)).*(\text{Lp}.*\text{Lr}.* \dots \\ & \text{mp}.*(8.*\text{g}.*\cos(2.*\text{Subscript}(x,2))+\text{Lp}.*( \cos(\text{Subscript}(x,2))+3.*\cos( \dots \\ & 3.*\text{Subscript}(x,2)))).*\text{Subscript}(x,3).^2+4.*((-1).*\text{Lr}.*(4.*\text{Jp}+ \dots \\ & \text{Lp}.^2.*\text{mp})).*\cos(\text{Subscript}(x,2))+4.*\text{Bp}.*\text{Lr}.*\sin(\text{Subscript}(x,2))+(\dots \end{aligned}$$

```

-1) * Lp * (4 * Jp + Lp.^2 * mp) * cos(2 * Subscript(x, 2)) * Subscript(x, 3) ...
) * Subscript(x, 4) + (-1) * Lp * (4 * Jp + (Lp.^2 + 4 * Lr.^2) * mp) * sin(2 * ...
Subscript(x, 2)) * (Lp.^3 * Lr * mp.^2 * Rm * cos(Subscript(x, 2)) * sin( ...
2 * Subscript(x, 2)) * Subscript(x, 3).^2 + (-2) * Lp * Lr * mp * Rm * ((-1) ...
* g * Lp * mp * sin(2 * Subscript(x, 2)) + (4 * Bp * cos(Subscript(x, 2)) + ( ...
4 * Jp + Lp.^2 * mp) * sin(Subscript(x, 2))) * Subscript(x, 4) + (-1) * (4 * ...
Jp + Lp.^2 * mp) * Subscript(x, 3) * (4 * Kg.^2 * km * Kt * n + 4 * Br * Rm + ...
Lp.^2 * mp * Rm * sin(2 * Subscript(x, 2)) * Subscript(x, 4)) + 4 * Kg * Kt * ...
(4 * Jp + Lp.^2 * mp) * n * u(t)), Rm.^(-1) * ((-4) * Lp.^2 * Lr.^2 * ...
mp.^2 * cos(Subscript(x, 2)) .^2 + (4 * Jp + Lp.^2 * mp) * (4 * (Jr + Lr.^2 * ...
mp) + Lp.^2 * mp * sin(Subscript(x, 2)) .^2)) .^(-1) * (4 * Lp.^3 * Lr * ...
mp.^2 * Rm * cos(Subscript(x, 2)) .^2 * sin(Subscript(x, 2)) * Subscript( ...
x, 3) + (-1) * (4 * Jp + Lp.^2 * mp) * (4 * Kg.^2 * km * Kt * n + 4 * Br * Rm + ...
Lp.^2 * mp * Rm * sin(2 * Subscript(x, 2)) * Subscript(x, 4))), (-1) * Lp * ...
mp * ((-4) * Lp.^2 * Lr.^2 * mp.^2 * cos(Subscript(x, 2)) .^2 + (4 * Jp + ...
Lp.^2 * mp) * (4 * (Jr + Lr.^2 * mp) + Lp.^2 * mp * sin(Subscript(x, 2)) .^2)) ...
.^(-1) * (8 * Bp * Lr * cos(Subscript(x, 2)) + 2 * Lr * (4 * Jp + Lp.^2 * mp) * ...
sin(Subscript(x, 2)) + Lp * (4 * Jp + Lp.^2 * mp) * sin(2 * Subscript(x, 2)) ...
* Subscript(x, 3)); 0, Lp * mp * Rm.^(-1) * ((-4) * Lp.^2 * Lr.^2 * mp.^2 * ...
cos(Subscript(x, 2)) .^2 + (4 * Jp + Lp.^2 * mp) * (4 * (Jr + Lr.^2 * mp) + ...
Lp.^2 * mp * sin(Subscript(x, 2)) .^2)) .^(-2) * (((-4) * Lp.^2 * Lr.^2 * ...
mp.^2 * cos(Subscript(x, 2)) .^2 + (4 * Jp + Lp.^2 * mp) * (4 * (Jr + Lr.^2 * ...
mp) + Lp.^2 * mp * sin(Subscript(x, 2)) .^2)) * (2 * g * Rm * cos(Subscript( ...
x, 2)) * (4 * (Jr + Lr.^2 * mp) + 3 * Lp.^2 * mp * sin(Subscript(x, 2)) .^2) + ...
8 * Lr * (Kg.^2 * km * Kt * n + Br * Rm) * sin(Subscript(x, 2)) * Subscript( ...
x, 3) + (1/2) * Lp * Rm * ((8 * Jr + (Lp.^2 + 8 * Lr.^2) * mp) * cos(2 * ...
Subscript(x, 2)) + (-1) * Lp.^2 * mp * cos(4 * Subscript(x, 2))) * ...
Subscript(x, 3) .^2 + (-1) * Lp * Rm * (4 * (Lr.^2 * mp * cos(2 * Subscript( ...
x, 2)) + Bp * sin(2 * Subscript(x, 2))) + Lp * Lr * mp * (cos(Subscript(x, 2)) ...
+ 3 * cos(3 * Subscript(x, 2))) * Subscript(x, 3) * Subscript(x, 4) + (-8) ...
* Kg * Kt * Lr * n * sin(Subscript(x, 2)) * u(t)) + (-1) * Lp * (4 * Jp + ( ...
Lp.^2 + 4 * Lr.^2) * mp) * sin(2 * Subscript(x, 2)) * (Lp.^2 * mp * Rm * ( ...
Lp.^2 * mp * cos(Subscript(x, 2)) * sin(Subscript(x, 2)) .^3 + 2 * (Jr + ...
Lr.^2 * mp) * sin(2 * Subscript(x, 2))) * Subscript(x, 3) .^2 + (-4) * Rm * ( ...
4 * Bp * (Jr + Lr.^2 * mp) + Lp.^2 * mp * sin(Subscript(x, 2)) * (Lr.^2 * mp * ...
cos(Subscript(x, 2)) + Bp * sin(Subscript(x, 2)))) * Subscript(x, 4) + (-2) ...
* Lp * Lr * mp * cos(Subscript(x, 2)) * Subscript(x, 3) * (4 * Kg.^2 * km * ...
Kt * n + 4 * Br * Rm + Lp.^2 * mp * Rm * sin(2 * Subscript(x, 2)) * Subscript( ...
x, 4)) + 2 * Lp * mp * (g * Rm * sin(Subscript(x, 2)) * (4 * Jr + 4 * Lr.^2 * mp + ...
Lp.^2 * mp * sin(Subscript(x, 2)) .^2) + 4 * Kg * Kt * Lr * n * cos( ...
Subscript(x, 2)) * u(t))), Lp * mp * Rm.^(-1) * cos(Subscript(x, 2)) * (( ...
-4) * Lp.^2 * Lr.^2 * mp.^2 * cos(Subscript(x, 2)) .^2 + (4 * Jp + Lp.^2 * mp) ...
* (4 * (Jr + Lr.^2 * mp) + Lp.^2 * mp * sin(Subscript(x, 2)) .^2)) .^(-1) * ( ...
Lp * Rm * (8 * Jr + Lp.^2 * mp + 8 * Lr.^2 * mp + (-1) * Lp.^2 * mp * cos(2 * ...
Subscript(x, 2))) * sin(Subscript(x, 2)) * Subscript(x, 3) + (-2) * Lr * ( ...
4 * Kg.^2 * km * Kt * n + 4 * Br * Rm + Lp.^2 * mp * Rm * sin(2 * Subscript(x, 2) ...
) * Subscript(x, 4))), (-4) * ((-4) * Lp.^2 * Lr.^2 * mp.^2 * cos( ...
Subscript(x, 2)) .^2 + (4 * Jp + Lp.^2 * mp) * (4 * (Jr + Lr.^2 * mp) + Lp.^2 * ...
mp * sin(Subscript(x, 2)) .^2)) .^(-1) * (4 * Bp * (Jr + Lr.^2 * mp) + Lp.^2 * ...
mp * sin(Subscript(x, 2)) * (Lr.^2 * mp * cos(Subscript(x, 2)) + Bp * sin( ...
Subscript(x, 2))) + Lp.^3 * Lr * mp.^2 * cos(Subscript(x, 2)) .^2 * sin( ...

```

`Subscript(x,2)).*Subscript(x,3))];`

`In[ ]:= B3 = D[fstate, u[t]] // FullSimplify`

$$\text{Out[ ]} = \left\{ 0, 0, \frac{4 \text{ Kg Kt} (4 \text{ Jp} + \text{Lp}^2 \text{ mp}) n}{\text{Rm} (-4 \text{ Lp}^2 \text{ Lr}^2 \text{ mp}^2 \cos[x_2]^2 + (4 \text{ Jp} + \text{Lp}^2 \text{ mp}) (4 (\text{Jr} + \text{Lr}^2 \text{ mp}) + \text{Lp}^2 \text{ mp} \sin[x_2]^2))}, \frac{8 \text{ Kg Kt Lp Lr mp} n \cos[x_2]}{\text{Rm} (-4 \text{ Lp}^2 \text{ Lr}^2 \text{ mp}^2 \cos[x_2]^2 + (4 \text{ Jp} + \text{Lp}^2 \text{ mp}) (4 (\text{Jr} + \text{Lr}^2 \text{ mp}) + \text{Lp}^2 \text{ mp} \sin[x_2]^2))} \right\}$$

`In[ ]:= B3 // ToMatlab`

`Out[ ]:= [0,0,4.*Kg.*Kt.*(4.*Jp+Lp.^2.*mp).*n.*Rm.^(-1).*((-4).*Lp.^2.* ...  
Lr.^2.*mp.^2.*cos(Subscript(x,2)).^2+(4.*Jp+Lp.^2.*mp).*(4.*(Jr+ ...  
Lr.^2.*mp)+Lp.^2.*mp.*sin(Subscript(x,2)).^2)).^(-1),8.*Kg.*Kt.* ...  
Lp.*Lr.*mp.*n.*Rm.^(-1).*cos(Subscript(x,2)).*((-4).*Lp.^2.* ...  
Lr.^2.*mp.^2.*cos(Subscript(x,2)).^2+(4.*Jp+Lp.^2.*mp).*(4.*(Jr+ ...  
Lr.^2.*mp)+Lp.^2.*mp.*sin(Subscript(x,2)).^2)).^(-1)];`

`In[ ]:= Collect[Simplify[D[L, {q}], Trig -> False],  
{θ'[t], θ'[t], α'[t], α'[t], α'[t] × θ'[t]}] // MatrixForm`

$$\text{Out[ ]} // \text{MatrixForm} = \begin{pmatrix} 0 \\ \frac{1}{2} g \text{ Lp mp} \sin[\alpha[t]] + \frac{1}{2} \text{ Lp Lr mp} \sin[\alpha[t]] \alpha'[t] \theta'[t] + \frac{1}{4} \text{ Lp}^2 \text{ mp} \cos[\alpha[t]] \sin[\alpha[t]] \theta'[t]^2 \end{pmatrix}$$