

An Extension of the Swarmalator Model

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Synchronization occurs at many natural and technological systems. Such an emergent properties is observed in cardiac pacemaker cells, Japanese tree frogs, colloidal suspensions of magnetic particles, and other biological and technological systems in which synchronization interact. We consider the system where oscillators can sync and swarm. A detailed analysis was proposed by Kevin P. O’Keeffe in the paper “Oscillators the sync and swarm”. We studied an extended model of the Swarmalator model proposed in the paper. Understanding the dynamics of this model could possibly give insight to the generalized model where the phase coupling function could be a fourier series.

1 Introduction

2 The Model

We consider swarmalators free to move in the plane. The governing equations are

$$\dot{\mathbf{x}}_i = \mathbf{v}_i + \frac{1}{N} \sum_{j=1}^N [\mathbf{I}_{att}(\mathbf{x}_j - \mathbf{x}_i) F(\theta_j - \theta_i) - \mathbf{I}_{rep}(\mathbf{x}_j - \mathbf{x}_i)] \quad (1)$$

$$\dot{\theta}_i = \omega_i + \frac{K}{N} \sum_{j=1}^N H_{att}(\theta_j - \theta_i) G(\mathbf{x}_j - \mathbf{x}_i) \quad (2)$$

for $i = 1, \dots, N$, where N is the number of swarmalators, \mathbf{x}_i is the position of the i -th swarmalator, and θ_i, ω_i , and \mathbf{v}_i are it’s phase, natural frequency and background velocity. The functions \mathbf{I}_{att} and \mathbf{I}_{rep} represent spatial attraction and repulsion between the swarmalators where as phase interaction is governed by H_{att} . We considered the following model:

$$\dot{\mathbf{x}}_i = \mathbf{v}_i + \frac{1}{N} \left[\sum_{j \neq i}^N \frac{\mathbf{x}_j - \mathbf{x}_i}{|\mathbf{x}_j - \mathbf{x}_i|} \left(1 + J \cos(\theta_j - \theta_i) \right) - \frac{\mathbf{x}_j - \mathbf{x}_i}{|\mathbf{x}_j - \mathbf{x}_i|^2} \right] \quad (3)$$

$$\dot{\theta}_i = \omega_i + \frac{K}{N} \sum_{j \neq i}^N \frac{\gamma_1 \sin(\theta_j - \theta_i) + \gamma_2 \sin(2(\theta_j - \theta_i))}{|\mathbf{x}_j - \mathbf{x}_i|} \quad (4)$$

We considered identical swarmalators so that $\omega_i = \omega$ and $\mathbf{v}_i = \mathbf{v}$. Using this assumption, using a suitable choice of reference frame we can set $\omega = 0$, and $\mathbf{v} = \mathbf{0}$. The system has four parameters $(J, K, \gamma_1, \gamma_2)$.

The parameter J measures the extend to which phase similarity enhances spatial attraction. For $J > 0$, swarmalators prefer to be near other swarmalators with similar phase. When $J < 0$, the opposite behavior is observed: swarmalators attract those with opposite phase. When $J = 0$, they show no phase based spatial behavior, i.e, their spatial attraction is independent of phase. To maintain $\mathbf{I}_{att} > 0$, we constrain J to $-1 \leq J \leq 1$. The parameter K is the phase coupling strength which scales γ_1 , and γ_2 . The relative strengths of γ_1 and γ_2 determine the stability of one or two clusters.

Before stating the dynamics of the system, we pause to state the features of this model. This model’s purpose is to study the interplay between swarming and synchronization. Our model accounts for aggregation, but not alignment. There are no alignment terms. We chose to neglect orientation because it adds another layer of complexity; it makes each swarmalator have four state variables. For rest of the report we will refer to our model as ‘Dual phase coupled model’ because of the presence of double angle sin function in Eq.4. In a similar fashion, the original model proposed by Kevin P. O’Keeffe will be referred as ‘Single phase coupled model’.

3 Optimization of the ode solver

The simulations were run using MATLAB’s ODE integrator ‘ode45’. Absolute and Relative tolerance for the integrator was set to 10^{-6} . Before large computations were performed, optimisation of the existing code was necessary. As this project involves computations of large matrices using ode45, optimising the code for speed was essential for any extensive calculation. The initial code was profiled using MATLAB’s performance profiler, and bottlenecks were identified. Calculating the pairwise inverse distance was the most

computationally expensive part in the ode function. The original code took 70 seconds for a simulation with $N = 100$ and $T = 10$ -time units. This speed was too slow for any extended time simulations. After removing unwanted function calls, changing the algorithm, and using functions which supports vectorization, the new code took 0.509 sec for the same task. The performance was good enough for an interactive user simulation where the user can change the parameters of the system and can see the results almost instantly.

4 Cluster formation and stability

The dynamics of dual phase coupled model had some striking differences with the original single phase model. The system not only settled in all the five states which were discussed in detail by Kevin P. O’Keeffe in single phase model, but also showed many additional stable and metastable states. Understanding the behavior of the modified phase coupling term and how it affects the formation of states could give us deep insights about the generalization of the model. The additional cluster states made by the model can be broadly classified as follows

- (a) Static two cluster state
- (b) Two cluster state with rogues
- (c) Active 4 cluster state
- (d) Active single cluster state

To understand system behavior under the change of γ_1 , and γ_2 , we chose $J = 0.8$ and $K = -0.5$ which represent Active phase wave in single phase coupled model. This set of state variables were chosen because it showed the most active spacial behavior in single phase coupled model and the dynamics of the system would be more depended on the values of γ_1 and γ_2 . Unless otherwise stated, the simulations are run with $J = 0.8$ and $K = -0.5$.

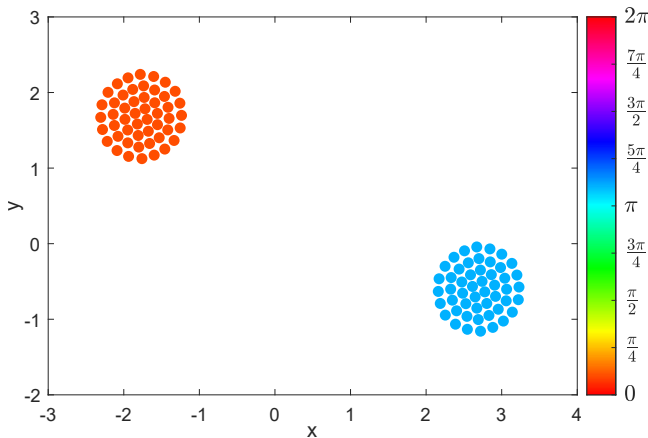


Fig. 1. Static two Cluster State

4.1 Static Two cluster state

4.2 Inter-cluster distance for Static two cluster state

By making a few assumptions, we can estimate the inter-

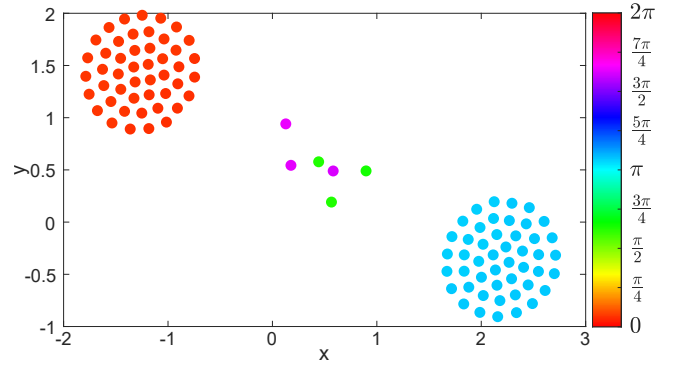


Fig. 2. Two Clusters with Rogues

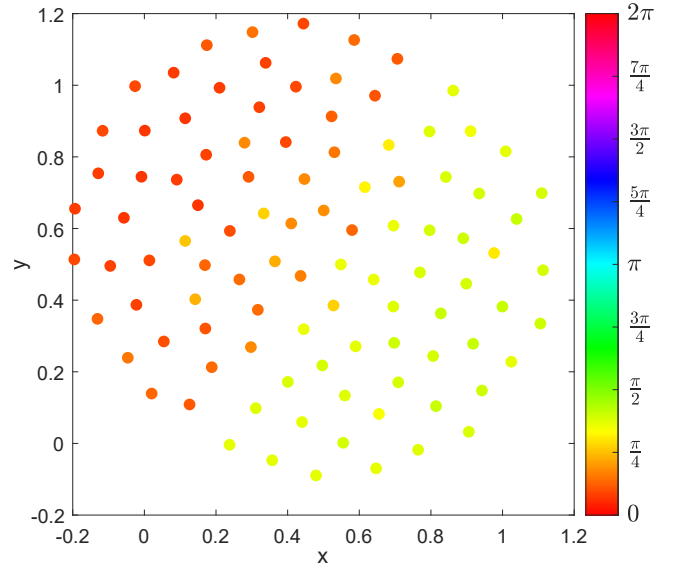


Fig. 3. Active Single cluster state

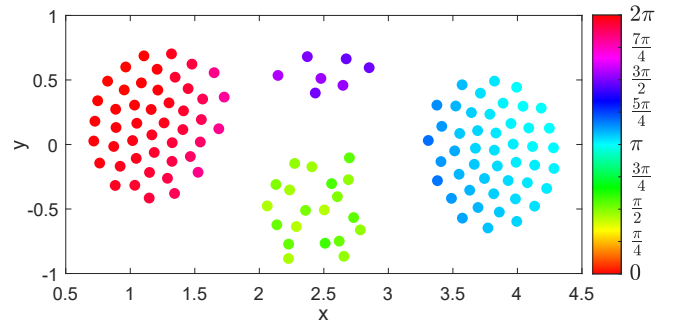


Fig. 4. Active 4 cluster state

cluster distance for static two cluster state.

4.3 Stability of Static two cluster state

5 Periodic motion of Rogues

6 Circular ring state

7 Phase Variation within the clusters

- 8 Simplified model**
- 9 Front end application**
- 10 Conclusion**
- 11 References**