

Practical -1 (e,f,h)

Section : e Predator-Prey Model

1.1 Predator-Prey Model (basic Volterra model)

Example 1 :- The differential equations describing the Predator Prey Model are

$$\frac{dx}{dt} = b_1x - c_1xy$$

$$\frac{dy}{dt} = c_2xy - a_2y$$

Generate time dependent graph of the population over the time with $b_1 = 1$, $a_2 = 0.5$, $c_1 = 0.01$, $c_2 = 0.005$. Initially with $[x_0, y_0] = [200, 80]$.

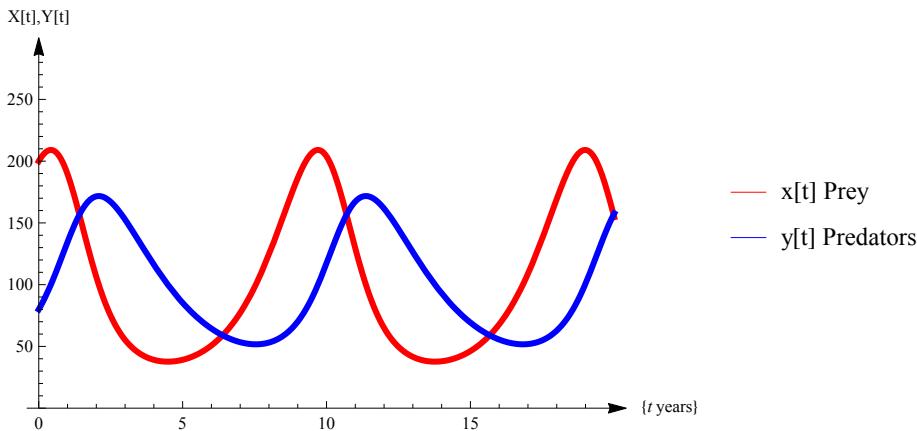
Solution :-

```

b1 = 1;
a2 = 0.5;
c1 = 0.01;
c2 = 0.005;
eqn1 = x'[t] == b1 x[t] - c1 x[t] y[t]
eqn2 = y'[t] == c2 x[t] y[t] - a2 y[t]
sol = NDSolve[{eqn1, eqn2, x[0] == 200, y[0] == 80}, {x, y}, {t, 0, 20}]

Plot[Evaluate[{x[t], y[t]} /. sol, {t, 0, 20}], PlotRange -> {0, 300},
PlotStyle -> {{Red, Thickness[0.01]}, {Blue, Thickness[0.01]}},
PlotLegends -> {"x[t] Prey", "y[t] Predators"},
AxesLabel -> {t {years}, "X[t],Y[t]"}, AxesStyle -> Arrowheads[{0, 0.03}]]
```

$x'[t] == x[t] - 0.01 x[t] y[t]$
 $y'[t] == -0.5 y[t] + 0.005 x[t] y[t]$
 $\{ \{x \rightarrow \text{InterpolatingFunction}[\{\{0., 20.\}\}, \text{<>}],$
 $y \rightarrow \text{InterpolatingFunction}[\{\{0., 20.\}\}, \text{<>}] \} \}$



1.2 Predator-Prey Model with effect of DDT

Example 2 :- The differential equations describing the Predator Prey Model with effect of DDT are

$$\frac{dx}{dt} = b_1 x - c_1 x y - p_1 x$$

$$\frac{dy}{dt} = c_2 x y - a_2 y - p_2 y$$

Generate time dependent graph of the population over time with $b_1 =$

1 , $a_2 = 0.5$, $c_1 = 0.01$ and $c_2 = 0.005$ and $p_1 = p_2 = 0.1$. Initially with $[x_0, y_0] = [200, 80]$.

Solution :-

```

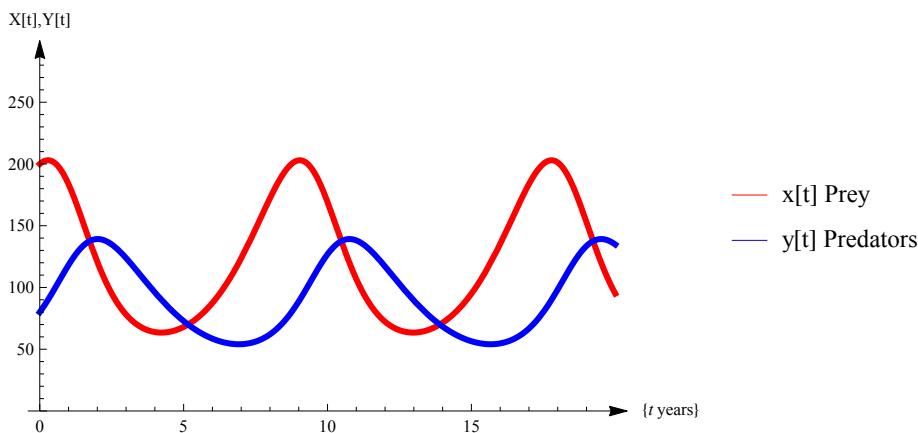
b1 = 1;
a2 = 0.5;
c1 = 0.01;
c2 = 0.005;
p1 = 0.1;
p2 = 0.1;
eqn1 = x'[t] == b1 x[t] - c1 x[t] y[t] - p1 x[t]
eqn2 = y'[t] == c2 x[t] y[t] - a2 y[t] - p2 y[t]
sol = NDSolve[{eqn1, eqn2, x[0] == 200, y[0] == 80}, {x, y}, {t, 0, 20}]

Plot[Evaluate[{x[t], y[t]} /. sol, {t, 0, 20}], PlotRange -> {0, 300},
PlotStyle -> {{Red, Thickness[0.01]}, {Blue, Thickness[0.01]}},
PlotLegends -> {"x[t] Prey", "y[t] Predators"},
AxesLabel -> {t {years}, "X[t],Y[t]"}, AxesStyle -> Arrowheads[{0, 0.03}]]
```

$$x'[t] == 0.9 x[t] - 0.01 x[t] y[t]$$

$$y'[t] == -0.6 y[t] + 0.005 x[t] y[t]$$

{ $x \rightarrow \text{InterpolatingFunction}[\{\{0., 20.\}\}, \text{<>}]$,
 $y \rightarrow \text{InterpolatingFunction}[\{\{0., 20.\}\}, \text{<>}]$ }



1.3 Predator-Prey Model with Density Dependent

Example 3 :- The differential equations describing the Predator Prey Model with density dependence are

$$\frac{dx}{dt} = b_1 x \left(1 - \frac{x}{k}\right) - c_1 xy$$

$$\frac{dy}{dt} = c_2 xy - a_2 y$$

Generate time dependent graph of the population over time with $b_1 = 1$, $a_2 = 0.5$, $c_1 = 0.01$ and $c_2 = 0.005$ and $k = 1000$. Initially with $[x_0, y_0] = [200, 80]$.

Solution :-

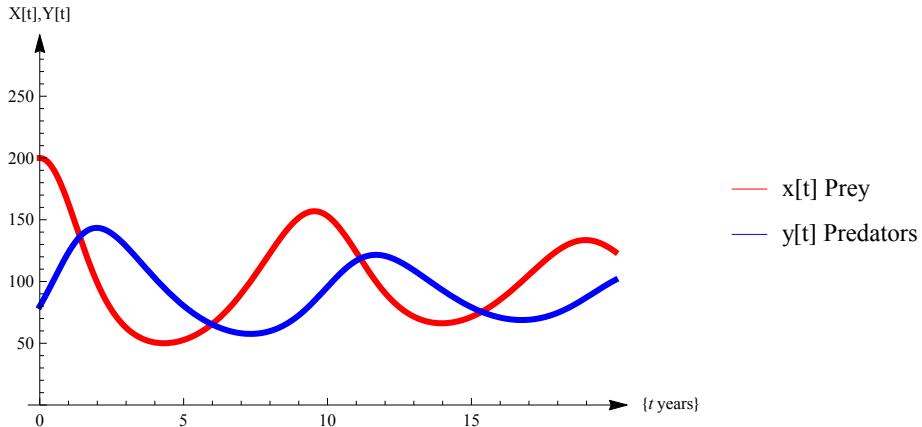
```

b1 = 1;
a2 = 0.5;
c1 = 0.01;
c2 = 0.005;
k = 1000;
eqn1 = x'[t] == b1 x[t] \left(1 - \frac{x[t]}{k}\right) - c1 x[t] y[t]
eqn2 = y'[t] == c2 x[t] y[t] - a2 y[t]
sol = NDSolve[{eqn1, eqn2, x[0] == 200, y[0] == 80}, {x, y}, {t, 0, 20}]

Plot[Evaluate[{x[t], y[t]} /. sol, {t, 0, 20}], PlotRange -> {0, 300},
PlotStyle -> {{Red, Thickness[0.01]}, {Blue, Thickness[0.01]}},
PlotLegends -> {"x[t] Prey", "y[t] Predators"},
AxesLabel -> {t {years}, "X[t],Y[t]"}, AxesStyle -> Arrowheads[{0, 0.03}]]

```

$$\begin{aligned}
x'[t] &= \left(1 - \frac{x[t]}{1000}\right) x[t] - 0.01 x[t] y[t] \\
y'[t] &= -0.5 y[t] + 0.005 x[t] y[t] \\
\{x \rightarrow \text{InterpolatingFunction}[\{\{0., 20.\}\}, \text{<>}], \\
y \rightarrow \text{InterpolatingFunction}[\{\{0., 20.\}\}, \text{<>}]\}
\end{aligned}$$



1.4 Two Prey One Predator Model

Example 4 :- The differential equations describing the Predator Prey Model two prey one predator are

$$\frac{dx}{dt} = b_1 x - c_1 xz$$

$$\frac{dy}{dt} = b_2 y - c_2 yz$$

$$\frac{dz}{dt} = d_1 xz + d_2 yz - b_3 z$$

Generate time dependent graph of the population over time with $b_1 =$

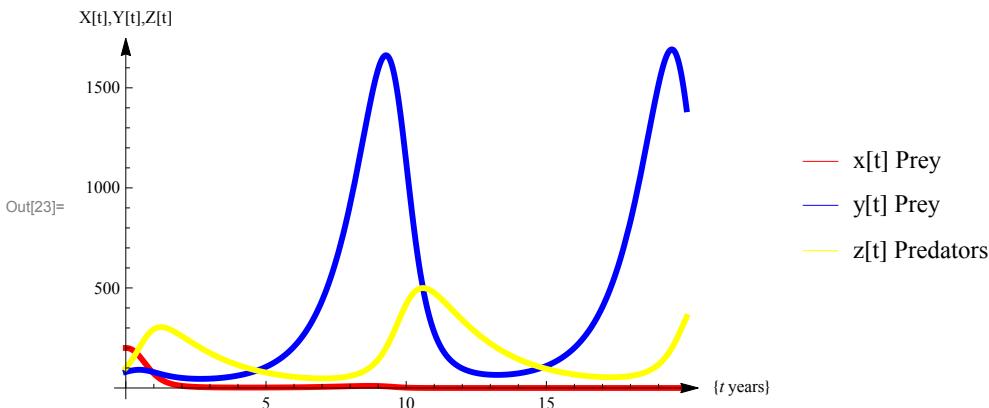
$b_2 = 1, b_3 = 0.5, c_1 = 0.01, c_2 = 0.005$, and
 $d_1 = 0.01, d_2 = 0.001$. Initially with $[x_0, y_0, z_0] = [200, 80, 100]$

Solution :-

```
In[12]:= b1 = 1;
b2 = 1;
b3 = 0.5;
c1 = 0.01;
c2 = 0.005;
d1 = 0.01;
d2 = 0.001;
eqn1 = x'[t] == b1 x[t] - c1 x[t] z[t]
eqn2 = y'[t] == b2 y[t] - c2 y[t] z[t]
eqn3 = z'[t] == d1 x[t] z[t] + d2 y[t] z[t] - b3 z[t]
sol =
NDSolve[{eqn1, eqn2, eqn3, x[0] == 200, y[0] == 80, z[0] == 100}, {x, y, z}, {t, 0, 20}]

Plot[Evaluate[{x[t], y[t], z[t]} /. sol, {t, 0, 20}], PlotStyle ->
{{Red, Thickness[0.01]}, {Blue, Thickness[0.01]}, {Yellow, Thickness[0.01]}},
PlotLegends -> {"x[t] Prey", "y[t] Prey", "z[t] Predators"},
AxesLabel -> {t {years}, X[t], Y[t], Z[t]},
PlotRange -> All, AxesStyle -> Arrowheads[{0, 0.03}]]

Out[19]= x'[t] == x[t] - 0.01 x[t] z[t]
Out[20]= y'[t] == y[t] - 0.005 y[t] z[t]
Out[21]= z'[t] == -0.5 z[t] + 0.01 x[t] z[t] + 0.001 y[t] z[t]
Out[22]= {x -> InterpolatingFunction[{{0., 20.}}, <>],
y -> InterpolatingFunction[{{0., 20.}}, <>],
z -> InterpolatingFunction[{{0., 20.}}, <>]}}
```



Section : h

Battle Model (Basic battle model, Jungle warfare, Long range weapons)

2.1 Battle Model (Basic Battle Model)

Note :- In basic battle model, Red & Blue both armies uses aimed firing

Example 1 :- Consider the simple Battle Model Given by the differential equations

$$\frac{dR}{dt} = -a_1 B$$

$$\frac{dB}{dt} = -a_2 R,$$

where $a_1 = 0.0544$, $a_2 = 0.0106$. The initial values of R & B are $R_0 = 66$ and $B_0 = 18$.

Obtain the solution of the differential equations given above and plot their graphs.

Solution :-

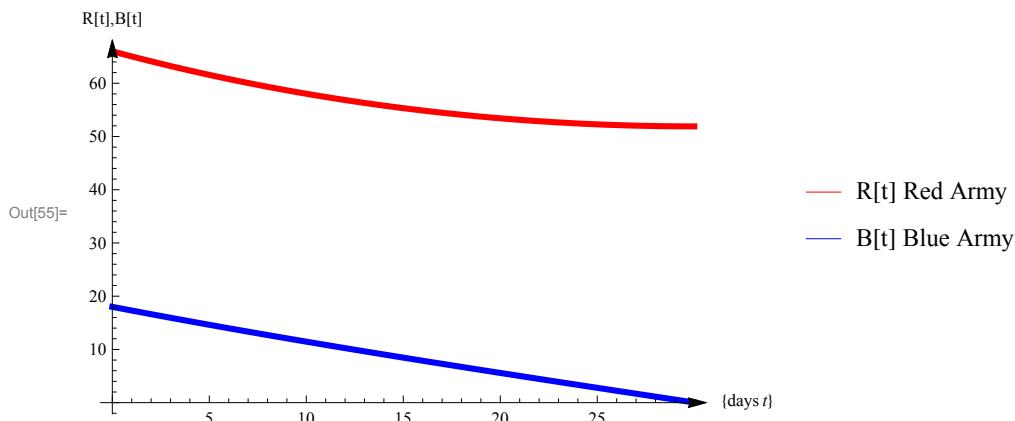
```
In[50]:= a1 = 0.0544;
a2 = 0.0106;
eqn1 = R'[t] == -a1 B[t]
eqn2 = B'[t] == -a2 R[t]
sol = NDSolve[{eqn1, eqn2, R[0] == 66, B[0] == 18}, {R, B}, {t, 0, 30}]

Plot[Evaluate[{R[t], B[t]} /. sol, {t, 0, 30}], PlotRange -> All,
PlotStyle -> {{Red, Thickness[0.01]}, {Blue, Thickness[0.01]}},
PlotLegends -> {"R[t] Red Army", "B[t] Blue Army"},
AxesLabel -> {t {days}, "R[t],B[t]"}, AxesStyle -> Arrowheads[{0, 0.03}]]
```

Out[52]= $R'[t] == -0.0544 B[t]$

Out[53]= $B'[t] == -0.0106 R[t]$

Out[54]= $\{R \rightarrow \text{InterpolatingFunction}[\{\{0., 30.\}\}, \text{<>}],$
 $B \rightarrow \text{InterpolatingFunction}[\{\{0., 30.\}\}, \text{<>}]\}$



Conclusion :- Red army wins as the number of soldiers of red army left after 30 days but blue army has no soldiers after 30 days.

2.2 Battle Model (Jungle Warfare)

Note :- In jungle warfare, one army use aimed firing and other use random firing

Example 2 :- Consider the jungle warfare Battle Model Given by the differential equations

$$\frac{dR}{dt} = -c_1 RB$$

$$\frac{dB}{dt} = -a_2 R,$$

where $c_1 = 0.0001, a_2 = 0.1$. The initial values of R & B are $R_0 = 1000$ and $B_0 = 1000$.

Obtain the solution of the differential equations given above and plot their graphs.

Solution :-

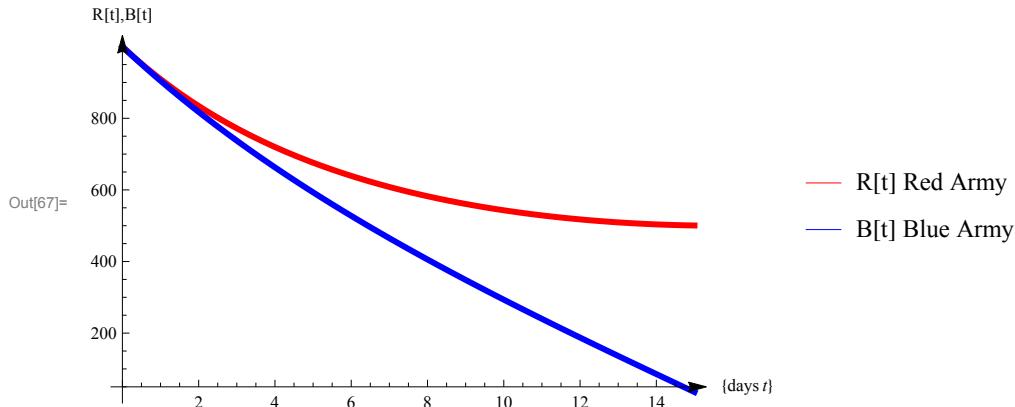
```
In[62]:= c1 = 0.0001;
a2 = 0.1;
eqn1 = R'[t] == -c1 B[t] R[t]
eqn2 = B'[t] == -a2 R[t]
sol = NDSolve[{eqn1, eqn2, R[0] == 1000, B[0] == 1000}, {R, B}, {t, 0, 15}]

Plot[Evaluate[{R[t], B[t]} /. sol, {t, 0, 15}], PlotRange -> All,
PlotStyle -> {{Red, Thickness[0.01]}, {Blue, Thickness[0.01]}},
PlotLegends -> {"R[t] Red Army", "B[t] Blue Army"},
AxesLabel -> {t {days}, "R[t],B[t]"}, AxesStyle -> Arrowheads[{0, 0.03}]]

Out[64]= R'[t] == -0.0001 B[t] R[t]

Out[65]= B'[t] == -0.1 R[t]

Out[66]= {R -> InterpolatingFunction[{{0., 15.}}, <>],
B -> InterpolatingFunction[{{0., 15.}}, <>]}]
```



Conclusion :- Red Army wins as number of soldiers left after 15 days but blue army has no soldiers. Here red army used aimed fire that is red army hidden and blue army used random fire.

2.3 Battle Model (Long range weapons or Guerilla warfare)

Note :- In Long Range weapons or Guerilla warfare, Red and Blue both armies uses random fire.

Example 3 :- Consider the Long Range Weapon Battle Model Given by the differential equations

$$\frac{dR}{dt} = -c_1 RB$$

$$\frac{dB}{dt} = -c_2 RB,$$

where $c_1 = 0.001, c_2 = 0.0001$. The initial values of R & B are $R_0 = 100$ and $B_0 = 100$. Obtain the solution of the differential equations given above and plot their graphs.

Solution :-

```
In[80]:= c1 = 0.001;
c2 = 0.0001;
eqn1 = R'[t] == -c1 B[t] R[t]
eqn2 = B'[t] == -c2 B[t] R[t]
sol = NDSolve[{eqn1, eqn2, R[0] == 100, B[0] == 100}, {R, B}, {t, 0, 30}]

Plot[Evaluate[{R[t], B[t]} /. sol, {t, 0, 30}], PlotRange -> All,
PlotStyle -> {{Red, Thickness[0.01]}, {Blue, Thickness[0.01]}},
PlotLegends -> {"R[t] Red Army", "B[t] Blue Army"},
AxesLabel -> {t {years}, "R[t],B[t]"}, AxesStyle -> Arrowheads[{0, 0.03}]]

Out[82]= R'[t] == -0.001 B[t] R[t]

Out[83]= B'[t] == -0.0001 B[t] R[t]

Out[84]= {R -> InterpolatingFunction[{{0., 30.}}, <>],
B -> InterpolatingFunction[{{0., 30.}}, <>]}
```

