PRACTICAL 1

(i) Lake Polution Model

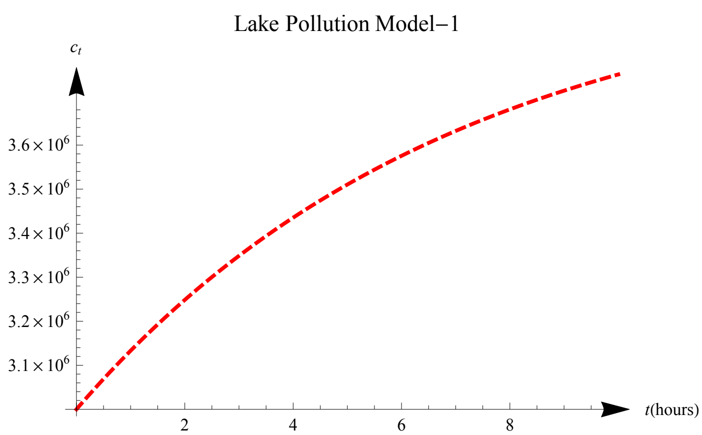
```
DE = C'[t] == F/V c_in - F/V C[t]
soln = DSolve[{DE, C[0] == c0}, C[t], t]
soln /. {c_in → 4 * 10^6, V → 28 * 10^6, F → 4 * 10^6, c0 → 3 * 10^6}
```

$$C'[t] == -\frac{F\,C[t]}{V} + \frac{F\,c_{in}}{V}$$

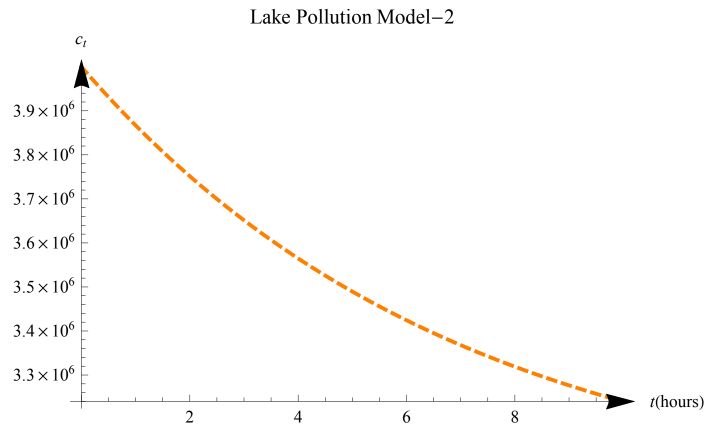
$$\left\{\left\{C[t] \to e^{-\frac{F t}{V}} \left(c0 - c_{in} + e^{\frac{F t}{V}} c_{in}\right)\right\}\right\}$$

$$\left\{\left\{C[t] \to e^{-t/7} \left(-1\,000\,000 + 4\,000\,000\, e^{t/7}\right)\right\}\right\}$$

```
Plot[
 Evaluate[C[t] /. soln /. {c_in → 4 * 10^6, V → 28 * 10^6, F → 4 * 10^6, c0 → 3 * 10^6}],
 {t, 0, 10}, AxesLabel → {t[hours], C[t]}, AxesStyle → Arrowheads[{0, 0.05}],
 PlotStyle → {Red, Thick, Dashed}, PlotLabel → "Lake Pollution Model-1"]
```



```
Plot[Evaluate[C[t] /. soln /. {cin → 3 * 10^6, V → 28 * 10^6, F → 4 * 10^6, c0 → 4 * 10^6}],
 {t, 0, 10}, AxesLabel → {t[hours], C[t]}, AxesStyle → Arrowheads[{0, 0.05}],
 PlotStyle → {Orange, Thick, Dashed}, PlotLabel → "Lake Pollution Model-2"]
```

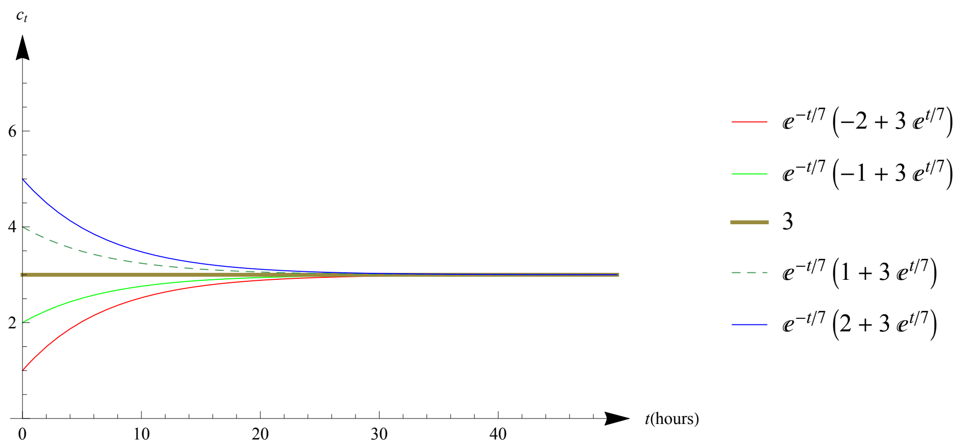


```
DE = C'[t] == F/V cin - F/V C[t]
soln = DSolve[{DE, C[0] == c0}, C[t], t]
soln /. {cin → 3, V → 28, F → 4, c0 → Range[1, 5]}
Plot[Evaluate[C[t] /. soln /. {cin → 3, V → 28, F → 4, c0 → Range[1, 5]}],
 {t, 0, 50}, AxesLabel → {t[hours], C[t]}, PlotRange → {0, 8},
 AxesStyle → Arrowheads[{0, 0.04}], PlotLegends → "Expressions",
 PlotStyle → {Red, Green, Thick, Dashed, Blue}]
```

$$C'[t] == -\frac{F\,C[t]}{V} + \frac{F\,c_{in}}{V}$$

$$\left\{\left\{C[t] \to e^{-\frac{F t}{V}} \left(c0 - c_{in} + e^{\frac{F t}{V}} c_{in}\right)\right\}\right\}$$

$$\left\{\left\{C[t] \to \left\{e^{-t/7}\left(-2+3\,e^{t/7}\right),\ e^{-t/7}\left(-1+3\,e^{t/7}\right),\ 3,\ e^{-t/7}\left(1+3\,e^{t/7}\right),\ e^{-t/7}\left(2+3\,e^{t/7}\right)\right\}\right\}\right\}$$



Interpretation of Graph :- As the time increase the concentration of pollutants in the lake will approach the cocentration of the polluted water entering the lake [C[t] = c_{in}] Here c_0 = c_{in} then the level of pollution in the lake decreases monotonically to c_{in} . If c_0< c_{in} then the level of pollution in the lake increase monotonically to c_{in}.

(ii) Case of a single cold pill and a course of cold pills

Example :- Solve drug single cold pill model and plot its graph

```
eqn1 = x'[t] == -k1 x[t];
eqn2 = y'[t] == k1 x[t] - k2 y[t];
sol = DSolve[{eqn1, eqn2, x[0] == 1, y[0] == 0}, {x[t], y[t]}, t]
solx1 = x[t] /. sol /. {k1 → 1.3860, k2 → 0.1386}
solx2 = x[t] /. sol /. {k1 → 0.6931, k2 → 0.0231}
solx3 = x[t] /. sol /. {k1 → 0.6931, k2 → 0.08}

soly1 = y[t] /. sol /. {k1 → 1.3860, k2 → 0.1386}
soly2 = y[t] /. sol /. {k1 → 0.6931, k2 → 0.0231}
soly3 = y[t] /. sol /. {k1 → 0.6931, k2 → 0.08}

Plot[{solx1, solx2, solx3}, {t, 0, 15},
 PlotStyle → {Thickness[0.02], {Red, Thickness[0.01]}, Thick},
 Epilog → Inset[Framed[GI - Tract]], PlotLegends → "Expressions"]
Plot[{soly1, soly2, soly3}, {t, 0, 15},
 PlotStyle → {Thickness[0.02], {Red, Thickness[0.01]}, Thick},
 Epilog → Inset[Framed[Bloodstream]], PlotLegends → "Expressions"]
```

$$\left\{\left\{x[t] \to e^{-k1\,t},\ y[t] \to -\frac{e^{-k1\,t-k2\,t}\left(-e^{k1\,t}+e^{k2\,t}\right)k1}{k1-k2}\right\}\right\}$$

$$\left\{e^{-1.386\,t}\right\}$$

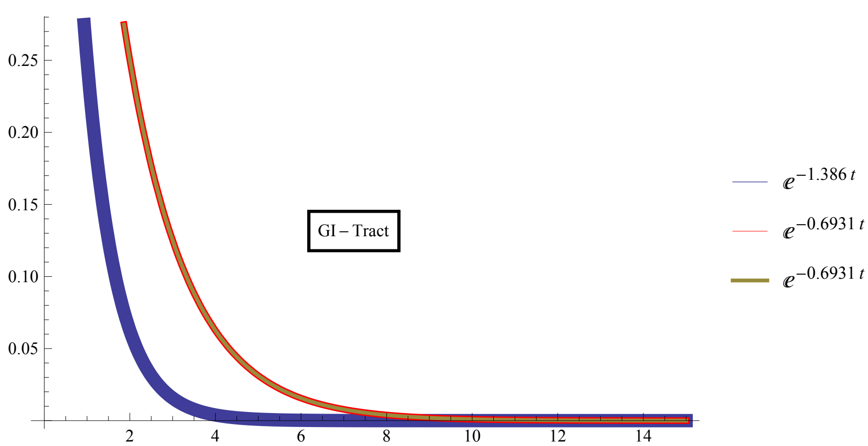
$$\left\{e^{-0.6931\,t}\right\}$$

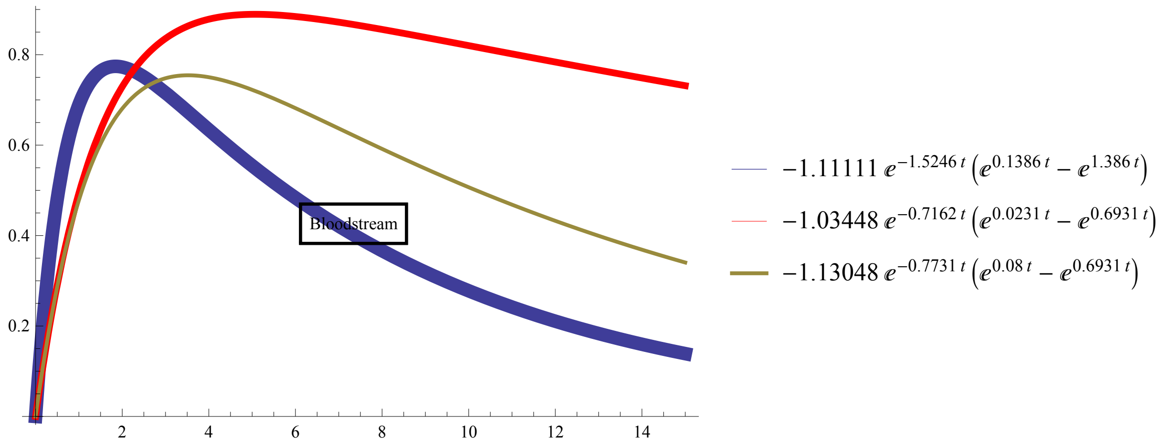
$$\left\{e^{-0.6931\,t}\right\}$$

$$\left\{-1.11111\,e^{-1.5246\,t}\left(e^{0.1386\,t}-e^{1.386\,t}\right)\right\}$$

$$\left\{-1.03448\,e^{-0.7162\,t}\left(e^{0.0231\,t}-e^{0.6931\,t}\right)\right\}$$

$$\left\{-1.13048\,e^{-0.7731\,t}\left(e^{0.08\,t}-e^{0.6931\,t}\right)\right\}$$




0.8
0.6
0.4
0.2
2
4
6
8
10
12
14
Bloodstream
$-1.11111\, e^{-1.5246\, t} \left(e^{0.1386\, t} - e^{1.386\, t}\right)$
$-1.03448\, e^{-0.7162\, t} \left(e^{0.0231\, t} - e^{0.6931\, t}\right)$
$-1.13048\, e^{-0.7731\, t} \left(e^{0.08\, t} - e^{0.6931\, t}\right)$

Example :- Solve course of cold pills model and plot its graph.

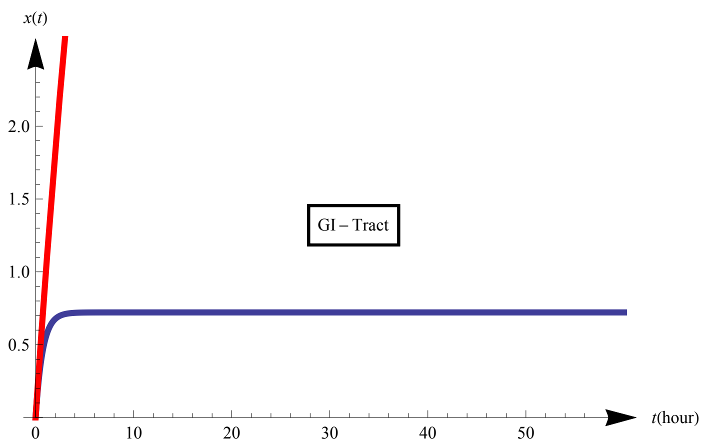
```
Clear[eqn1, eqn2, sol]
eqn1 = x'[t] == I - k1 x[t]
eqn2 = y'[t] == k1 x[t] - k2 y[t]
sol = DSolve[{eqn1, eqn2, x[0] == 0, y[0] == 0}, {x[t], y[t]}, t]
sol /. {k1 → {1.3860, 0.6931}, k2 → {0.1386, 0.0231}, I → 1}

Plot[Evaluate[x[t] /. sol /. {k1 → {1.3860, 0.0931}, I → 1}], {t, 0, 60},
 Epilog → Inset[Framed[GI - Tract]], AxesLabel → {t[hour], x[t]},
 PlotStyle → {Thickness[0.01], {Red, Thickness[0.01]}, Blue},
 AxesStyle → Arrowheads[{0, 0.05}], PlotRange → {0, 2.6}]
```

$x'[t] == i - k_1\, x[t]$

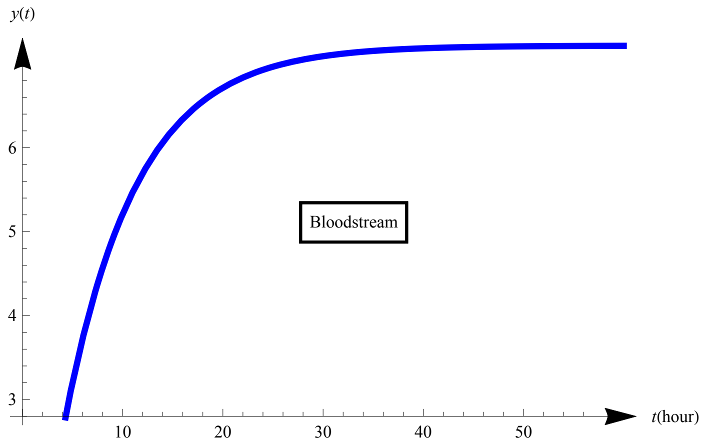
$y'[t] == k_1\, x[t] - k_2\, y[t]$

$$\left\{\left\{x[t] \to \frac{i\, e^{-t k_1} \left(-1 + e^{t k_1}\right)}{k_1},\ y[t] \to \frac{i\, e^{-t k_1 - t k_2} \left(e^{t k_1} k_1 - e^{t k_1 + t k_2} k_1 - e^{t k_2} k_2 + e^{t k_1 + t k_2} k_2\right)}{k_2 \left(-k_1 + k_2\right)}\right\}\right\}$$

$$\{\{x[t] \to \{0.721501\, e^{-1.386 t} \left(-1 + e^{1.386 t}\right),\ 1.44279\, e^{-0.6931 t} \left(-1 + e^{0.6931 t}\right)\},$$
$$y[t] \to \{-5.78404\, e^{-1.5246 t} \left(-0.1386\, e^{0.1386 t} + 1.386\, e^{1.386 t} - 1.2474\, e^{1.5246 t}\right),$$
$$-64.612\, e^{-0.7162 t} \left(-0.0231\, e^{0.0231 t} + 0.6931\, e^{0.6931 t} - 0.67\, e^{0.7162 t}\right)\}\}\}$$



```
Plot[Evaluate[y[t] /. sol /. {k1 → 1.3860, k2 → 0.1386, I → 1}], {t, 0, 60},
 Epilog → Inset[Framed[Bloodstream]], AxesLabel → {t[hour], y[t]},
 PlotStyle → {Thickness[0.01], {Red, Thickness[0.01]}, Blue},
 AxesStyle → Arrowheads[{0, 0.05}]]
```



(iii) Limited Growth of population (with and without Harvesting)

Example :- Solve Limited growth model without harvesting (Logistic Equation) and plot its graph.
Solution.

```
Clear[DE, soln, r, k, X0]
DE = X'[t] == r X[t] - r/k X[t]^2
soln = DSolve[{DE, X[0] == X0}, X[t], t]
soln /. {r → 1, k → 1000, X0 → 100}

Plot[X[t] /. soln /. {r → 1, k → 1000, X0 → 100}, {t, 0, 8}, AxesLabel → {t, x[t]},
 PlotStyle → {Red, Thick, Dashed}, AxesStyle → Arrowheads[{0, 0.05}]]
```

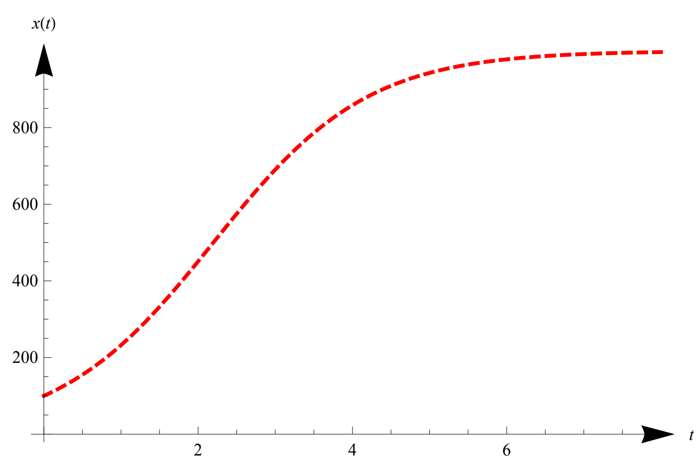
$$X'[t] == r\,X[t] - \frac{r\,X[t]^2}{k}$$

Solve::ifun :

Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information. ≫

$$\left\{\left\{X[t] \to \frac{e^{r\,t}\,k\,X0}{k - X0 + e^{r\,t}\,X0}\right\}\right\}$$

$$\left\{\left\{X[t] \to \frac{100\,000\,e^{t}}{900 + 100\,e^{t}}\right\}\right\}$$



Example :- Solve Limited growth model with harvesting and plot its graph.
Solution:-

```
Clear[X, k, r, X0, DE, soln]
DE = X'[t] == r X[t] - r/k X[t]^2 - h
soln = DSolve[{DE, X[0] == X0}, X[t], t]
soln /. {r → 1, k → 1000, X0 → 100, h → 9/10} // Simplify
Plot[X[t] /. soln /. {r → 1, k → 1000, X0 → 100, h → 9/10},
 {t, 0, 8}, AxesLabel → {t, x[t]}, PlotStyle → {Red, Thick, Dashed},
 AxesStyle → Arrowheads[{0, 0.05}]]
```

$$X'[t] == -h + r\,X[t] - \frac{r\,X[t]^2}{k}$$

Solve::ifun :

Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information. ≫

$$\left\{\left\{X[t] \to \frac{1}{2\sqrt{r}}\left(k\sqrt{r} - \sqrt{k}\sqrt{4h - k r}\,\mathrm{Tan}\left[\frac{1}{2}\left(\frac{\sqrt{r}\sqrt{4h - k r}\,t}{\sqrt{k}} - 2\,\mathrm{ArcTan}\left[\frac{-k^{3/2}\sqrt{r}\sqrt{4h - k r} + 2\sqrt{k}\sqrt{r}\sqrt{4h - k r}\,X0}{4 h k - k^2 r}\right]\right)\right]\right)\right\}\right\}$$

$$\left\{\left\{X[t] \to 10\left(50 + \sqrt{2491}\,\mathrm{Tanh}\left[\frac{\sqrt{2491}\,t}{100} - \mathrm{ArcTanh}\left[\frac{40}{\sqrt{2491}}\right]\right]\right)\right\}\right\}$$