Computational Interpretations (CI) in L fragment are essentially Reducibility Sets, as all terms in CI reduce to a value. So, we can augment CIs as defined now with a well defined map, mapping types to their corresponding reducibility sets as shown :

$$RED_T[\underline{R}/\underline{X}] =_\beta \mathscr{C}[\![T]\!]$$

where β is the mapping $\forall i, X_i \rightarrow {}_\beta\mathscr{C}[\![X_i]\!]$.
We will need β in the interpretations if there is any type variables. If not, then we can get rid of β. e.g.

$$X \rightarrow \mathscr{C}[\![X]\!]$$

if X has no type vars in it.

Now we can define the value interpretation for polymorphic types as shown:

$${}_\beta\mathscr{V}[\![\forall\alpha.A]\!]_k^\theta = \{\wedge\alpha.e|.\vdash \wedge\forall\alpha.e : \forall\alpha.A \wedge \forall T : type, e[T/\alpha] \in_{\beta[\alpha \rightarrow {}_\beta\mathscr{C}[\![T]\!]]} \mathscr{C}[\![[T/\alpha]A]\!]\}$$

And so, the proof to Univ-T-app will go according to the definition.
Hence the soundness theorem remains unchanged, as defined in the paper.

I am not sure if the interpretation should have θ in it or just L.
Can we leave the P fragment definition with $j < k$, or would it be better to use this same definition in P fragment too?