

Let $T[\underline{X}]$ be a type where \underline{X} contains all the free variables of T . Let \underline{U} be a sequence of types of the same length; then we can define a type $T[\underline{U}/\underline{X}]$ by simultaneous substitution of types. Let $PAR\mathcal{C}[[U_i]_k^\theta]$ and $PAR\mathcal{V}[[U_i]_k^\theta]$ be the parametrised value and parametrised computational interpretations of the type U . Let \underline{R} stand for the candidates of reducibility (C.R.) for the sequence of types. Then we define the set $PAR\mathcal{V}[[T]_k^\theta[\underline{R}/\underline{X}]]$ and $PAR\mathcal{C}[[T]_k^\theta[\underline{R}/\underline{X}]]$ for terms of type $T[\underline{U}/\underline{X}]$ as follows :

- $T = X_i$, then $PAR\mathcal{C}[[T]_k^\theta[\underline{R}/\underline{X}]] = R[U_i]_k^\theta$
- $T = Unit$
 $PAR\mathcal{V}_k^\theta[\underline{R}/\underline{X}] = \{()\}$
- $T = A + B$
 $PAR\mathcal{V}[[T]_k^\theta[\underline{R}/\underline{X}]] = \{inl\ v | v \in PAR\mathcal{V}[[A]_k^\theta[\underline{R}/\underline{X}]]\} \cup \{inr\ v | v \in PAR\mathcal{V}[[B]_k^\theta[\underline{R}/\underline{X}]]\}$
- $T = A @ \theta'$
 $PAR\mathcal{V}[[T]_k^L[\underline{R}/\underline{X}]] = \{box\ v | v \in PAR\mathcal{V}[[A]_k^{\theta'}[\underline{R}/\underline{X}]]\}$
- $T = A^{\theta'} \rightarrow B$ then,
 $PAR\mathcal{V}[[T]_k^L[\underline{R}/\underline{X}]] = \{rec\ f\ x.a | \vdash^L rec\ f\ x.a : T[\underline{U}/\underline{X}] \text{ and } \forall j \leq k, \text{ if } v \in PAR\mathcal{V}[[A]_j^{\theta'}[\underline{R}/\underline{X}]] \text{ then } [v/x]a \in PAR\mathcal{C}[[B]_j^L[\underline{R}/\underline{X}]]\}$
 $PAR\mathcal{V}_k^P[[T]_k^L[\underline{R}/\underline{X}]] = \{rec\ f\ x.a | \vdash^P rec\ f\ x.a : T[\underline{U}/\underline{X}] \text{ and } \forall j < k, \text{ if } v \in PAR\mathcal{V}[[A]_j^{\theta'}[\underline{R}/\underline{X}]] \text{ then } [v/x][rec\ f\ x.a/f]a \in PAR\mathcal{C}[[B]_j^P[\underline{R}/\underline{X}]]\}$
- $T = \mu\alpha.a$
 $PAR\mathcal{V}[[T]_k^P[\underline{R}/\underline{X}]] = \{roll\ v | \vdash^P roll\ v : \mu\alpha.a[\underline{U}/\underline{X}] \text{ and } \forall j < k, v \in PAR\mathcal{V}[[\mu\alpha.a/\alpha]A]_j^P[\underline{R}/\underline{X}]]\}$
 $PAR\mathcal{V}[[T]_k^L[\underline{R}/\underline{X}]] = \phi$
- $T = \forall\alpha.A$
 $PAR\mathcal{V}[[\forall\alpha.A]_k^\theta[\underline{R}/\underline{X}]] = \{t | \forall V : \text{type s.t. } R[V]_j^\theta \text{ is C.R. of } V, tV \in PAR\mathcal{C}[[A]_k^\theta[\underline{R}/\underline{X}, R[V]_j^\theta/\alpha]]\}$

And parametrised computational interpretations are defined as :

$$PAR\mathcal{C}[[T]_k^P[\underline{R}/\underline{X}]] = \{a | \vdash^P a : T[\underline{U}/\underline{X}] \text{ and } \forall j \leq k, \text{ if } a \rightsquigarrow^j v \text{ then } v \in PAR\mathcal{V}[[T]_{k-j}^P[\underline{R}/\underline{X}]]\}$$

$$PAR\mathcal{C}[[T]_k^L[\underline{R}/\underline{X}]] = \{a | \vdash^P a : T[\underline{U}/\underline{X}] \text{ and } a \rightsquigarrow^* v \in PAR\mathcal{V}[[T]_k^L[\underline{R}/\underline{X}]]\}$$

Now, for the case $T = \forall\alpha.A$

$$PAR\mathcal{V}[[\forall\alpha.A]_k^\theta[\underline{R}/\underline{X}]] = \{t | \forall V : \text{type s.t. } R[V]_j^\theta \text{ is C.R. of } V, tV \in PAR\mathcal{C}[[A]_k^\theta[\underline{R}/\underline{X}, R[V]_j^\theta/\alpha]]\}$$

We need the candidates of reducibility to fulfill conditions :

- $R[A]_k^\theta$ needs to be Strongly Normalizing (atleast in L, what about P?)
- If $t \rightsquigarrow t'$ and $t' \in R[A]_j^\theta$, then $t \in R[A]_j^\theta$.

Now we need to define reducibility candidates for each type :

- $T=Unit : R[Unit]_k^\theta = ()$
CR1 and CR2 hold trivially.
- $T=A+B : R[A+B]_k^\theta = \{inl\ a|a \in R[A]_k^\theta\} \cup \{inr\ b|b \in R[B]_k^\theta\}$
CR1 : By I.H. both $R[A]_k^\theta$ and $R[B]_k^\theta$ are SN.
CR2 : from I.H. if $a \in R[A]_k^\theta$ then $a \in R[A]_j^\theta$ by CR2. Similarly $b \in R[B]_j^\theta$ by CR2.
Hence $\{inl\ a|a \in R[A]_k^\theta\} \cup \{inr\ b|b \in R[B]_k^\theta\} \in R[A+B]_j^\theta$.
- $T=A^{\theta'} \rightarrow B : R[A^{\theta'} \rightarrow B]_k^L = \{t|t \rightsquigarrow *v \wedge irred(v) \wedge \forall j \leq k, \text{ if } a \in R[A]_k^{\theta'} \text{ then } ta \in R[B]_j^L\}$
CR1:by def.
CR2: let $a \in R[A]_i^{\theta'}$ where $i \leq j \leq k$ then if $t \in R[A^{\theta'} \rightarrow B]_k^L$ then $ta \in R[B]_i^L$, which is true also when $t \in R[A^{\theta'} \rightarrow B]_j^L$