GATE

GRADUATE APTITUDE TEST IN ENGINEERING

MINING ENGINEERING

B. N. BHAGAT
&
SACHIN LAL

LOVELY PRAKASHAN, DHANBAD

Scanned by CamScanner

CONTENT

●

Scanned by CamScanner

GATE : GENERAL INFORMATION

The Graduate aptitude Test in Engineering (GATE), is an All-India Examination, conducted by the five Indian Institutes of Technology and Indian Institute of Science, Bangalore, on behalf of the National Coordinating Board-GATE, Department of Education, Ministry of Human Resources Development (MHRD), Government of India.

OBJECTIVES OF GATE

1. To identify meritorious and motivated candidates for admission to Postgraduate Programmes in Engineering. Technology, Architecture, and Pharmacy at the National level.
2. To serve as benchmark for normalisation of the Undergraduate Engineering Education in the Country.

Admission to Postgraduate Courses, with MHR Scholarship/Assistantship, in Engineering/Technology/Architecture/Pharmacy at Engineering Colleges/Institutes in the Country will be open only to those who qualify through GATE.

Some Engineering Colleges/Institutes specify GATE as mandatory qualification even for admission of students to Postgraduate Programmes.

The candidates are required to find out the procedures of final selection and award of Scholarship/Assistantship from the Institutions to which they seek admission.

The following categories of candidates are eligible to appear at GATE

a) Bachelor's degree holders in Engineering/Technology/Architecture/Pharmacy and those who are in the final year of such programmes.

b) Master's degree holders in any branch of Science of M.A. in Mathematics and those who are in the final year of such programmes. However, if the degree is Master of Computer Applications, the Bachelors degree should be in Mathematics or Science with Mathematics.

c) Candidates in the second or higher year of four-year Integrated Master's degree programme (Post B.Sc) in Engineering/Technology or in the fourth or higher year of five-year Integrated Master's degree programme in Engineering/Technology.

d) Candidates with professional qualification like AMIE by examination (recognised by UPSCIAICTE as equivalent to B.E./B. Tech.) and those who have completed section A or equivalent of such professional degree course.

IMPORTANT FOR GATE EXAMINATION

Information Brochure and Application Form issued from	:	**Generally first week of October**
Date of the Examination	:	2nd Sunday of February
Announcement of Results	:	31st March

Candidates will be required to appear in a single paper of three hours duration. The examination is scheduled to 2nd Sunday of February starting at 9.30 a.m.

Before applying for GATE, candidates must assure themselves that they have chosen the right paper which qualifies them to become eligible to seek admission to the specific programme they are interested in. The criteria for postgraduate admission with Scholarship/Assistantship differ from Institution to Institution. For more details, the candidates are required to apply separately to the Institutes/Universities to which they are interested in seeking admission.

The choice of the paper is the responsibility of the candidate.

A list of examination centres is given in the Table 1. The candidate should send the completed application form to the Chairman GATE of the zone in which the centre is located.

The authority to declare a candidate qualified in GATE is vested with the GATE Committee.

The question papers will be in English and they must be answered in **English.**

Scanned by CamScanner

STRUCTURE OF GATE AND GATE RESULTS

All candidates for GATE will appear in a single paper of 3 hours duration and of 150 marks. GATE usually have the following papers :

Papers	Code	Papers	Code
Agricultural Engg.	(AG)	Mathematics	(MA)
Architecture	(AR)	Mechanical Engg.	(ME)
Civil Engg.	(CE)	Mining Engg.	(MN)
Chemical Engg.	(CH)	Metallurgical Engg.	(MT)
Computer Science & Engg.	(CS)	Physics	(PH)
Chemistry	(CY)	Pharmaceutical Sciences	(PY)
Electronics & Comm. Engg.	(EC)	Textile Engg. & Fibre Sci.	(TF)
Electrical Engg.	(EE)	Engineering Sciences	(XE)
Geology & Geophysics	(GG)	Life Sciences	(XL)
Instrumentation Engg.	(IN)		

Papers XE and XL are of general nature will comprise of the following sections :

Engineering Sciences (XE)		Life Sciences (XL)	
Engg. Mathematics (Compulsory)	(A)	Chemistry (Compulsory)	(H)
Computational Science	(B)	Biochemistry	(I)
Electrical Sciences	(C)	Botany	(J)
Fluid Mechanics	(D)	Microbiology	(K)
Material Sciences	(E)	Zoology	(L)
Solid Mechanics	(F)	Biotechnology	(M)

Candidates appearing in XE or XL papers are required to answer THREE Sections, one compulsory as mentioned above, and two others of candidate's choice out of the sections mentioned against the respective papers. **Candidates qualified in XE or XL paper are required to be interviewed by the admitting institute.**

Candidates generally receive Admit Cards by last week of January, if they do not receive it by 1st February, they should contact respective GATE Chairmen.

GATE RESULTS :

a) GATE score will cease to be valid after 2 years from the date of issue.

b) Score cards will be sent only to the qualified candidates. No information will be sent to the candidates who have not qualified.

c) The GATE score will become valid only after the candidate completes all the requirement of his/her qualifying degree.

d) Results of GATE will be declared and the score card of Qualified Candidates will give All India Rank and percentile score for that paper. The percentile score in each paper is calculated as follows : Let N be the total number of candidates appearing in that paper, and let n_c be the number of candidates who have the same all India rank c in the same paper (there can be bunching at a given all India rank). Then all the candidates, whose all India rank is 'r', will have the same percentile score p, where,

$$p = \frac{N - \sum_{c=1}^{r} n_c}{N} \times 100$$

i.e. p=[(number of candidates who have secured marks less than the candidate concerned)/N] x100

e) Requests for rechecking of answer scripts and retotalling of marks is NOT entertained.

Scanned by CamScanner

MINING ENGINEERING (Syllabus)

Mining Methods-Surface and Underground, Mine Transport and Machinery : Open c methods of mining-layout, development and design; loading and transport, mechan quarrying; continuous mining systems; underground methods of working coal by longw board and pillar, mining of thick seams, trends of new developments in coal mining; und ground mining methods for metalliferrous deposits-development and stopping, handlin broken ore, waste and materials, slope mechanisation, mine filling, Materials handlin mines-haulages, conveyors, ropeways; face and development machinery; mine hoists; m pumps.

Rock Mechanics and Ground Control, Mine Development : Physico-mechanical prop ties of rocks, rock mass classification, stress measurement techniques; theories of r failure, stress distribution around mine working; mine subsidence, ground control Inst mentation and monitoring, design of supports and support system in mine roadways a workings; pit stope stability; drilling methods and machines, explosives, blasting devic and practices, shaft sinking ordinary and special methods.

Mine Ventilation, Environment and Safety : Underground mine atmosphere, sampl and analysis, mechanics of air flow, distribution and control; natural and mechanical vent tion fans-types, selection, and operation; heat and humidity-sources, estimation a airconditioning; ventilation planning; mine fires and explosions; Inundation; illuminati noise, and mine dust-measurement, standards, and control; mine rescue; mine safety a legislation.

Mine Surveying, Mineral Economic, Mine Planning and Management : Mine Survey instruments, developments in instrumentation and techniques, correlation , undergrou surveying methods; mineral industry; concepts and measurements of resources a reserves; sampling methods and practices; national mineral policy; mining companies a mine accounts; cash flow concepts and mine valuation; mining finance royalty and taxati principles of planning, optimization of output, life, size, and other mine parameters; sh range and long range planning; preparation of feasibility reports and OPR, producti planning, and productivity in mining; project planning and control; principles of scient management, organisation and staffing; work study and operations research in mining.

Mathematics applied to Mining Engineering : Limit, continuity and differentiability functions of several variables, partial derivates and their geometrical interpretatio differentiations, derivatives of composite and implicit functions, derivate of higher orde Taylor expansion for functions of several variables, maxima and minima of functions several variables-Lagrange methods of multipliers; ordinary differential equations-first ord equation-separable, exact, homogeneous, linear and Bernoulli's form; matrices-algebra matrices, vector space-linear dependence of vectors, rank and inverse of a matrix, solutio of algebric equations-consistency conditions, eigen-values and eigen-vectors similar transformatio-reduction to a diagonal matrix; probability and statistics-definition and laws probability, probability mass function, probability density function, mathemati expectation, mean variance moment, moment generating function; correlation a regression-simple, multiple and partial; tests of significance.

Scanned by CamScanner

VENTILATION

Reynolds number

$$Re = \frac{VD\rho}{\mu} = \frac{VD}{\upsilon}$$

V = velocity of flow

D = diameter of duct

ρ = density of fluid

μ = viscosity of fluid

$\upsilon = \frac{\mu}{\rho}$, Kinetic viscosity $(1.6\times10^{-5}$ for air$)$.

If $Re \leq 2000$, flow is laminar $\rightarrow V_{max} = 2V$

if $Re \geq 2000$, flow is turbulent $\rightarrow V_{max} = V.(1.43\sqrt{f}+1)$.

Pressure drop

$$\Delta P_f = f.\frac{L}{D}.\frac{\rho V^2}{2} \qquad \text{.................. (1)}$$

L = length of airway

D = dia of airway

ρ = density

where f = resistance coefficient

$$f = \frac{64}{Re}, \text{ for } Re \leq 2000$$

$$f = \frac{0.316}{Re^{1/4}}, \text{ for } Re > 2000$$

Equivalent diameter

$$D = \frac{4A}{P},$$

A = Area

P = Perimeter

hence

$$\Delta P_f = \frac{fLP\rho V^2}{8A} = \frac{f\rho}{8}.\frac{LPV^2}{A}$$

$$= \frac{KSV^2}{A},$$

$K = \frac{f.\rho}{8}$ (coefficient of friction)

$S = LP$ (area of rubbing)

$$= \frac{KSQ^2}{A^3} P_a \qquad \text{.................. (2)}$$

Q = quantity of flow through airway m^3s^{-1}

Scanned by CamScanner

also,

$$\Delta P\rho = \frac{fLV^2}{2gD} \quad \text{.................. (3)}$$

Resistance of airway

$$R = \frac{\Delta P}{Q^2} = \frac{KS}{A^3} = \frac{KPL}{A^3} = \frac{fLP}{2DA^2}$$

$R = R_1 + R_2 + R_3 + \ldots\ldots\ldots\ldots$ for *series* roadways

$\frac{1}{\sqrt{R}} = \frac{1}{\sqrt{R_1}} + \frac{1}{\sqrt{R_2}} + \frac{1}{\sqrt{R_3}} + \ldots\ldots$ for parallel roadways

Power of ventilation

$AP = \Delta PQ = RQ^3$ watt

Equivalent orifice

$$A = \frac{1.29}{\sqrt{R}}$$

A = Area of equivalent orifice

R = Resistance of mine NS^2m^{-8}

Fans

(1) Centrifugal type (low capacity, high head)

(2) Axial flowtype (high capacity, low head)

head developed (Euler's) by fan

$$He = \frac{U^2 - U\,V_R \operatorname{Cot}\beta}{g},$$ where

U = Peripherial velocity of impeller (m/s)

V_R = Radial component of discharge velocity

β = inclination of vanes

$$U = 2\pi r_{imp} \cdot \left(\frac{r.p.m}{60}\right) \quad \text{.................. (1)}$$

$$V_R = \frac{Q}{2\pi r_{imp} \cdot W_{imp}} \quad \text{.................. (2)}$$

$$\Rightarrow 2\pi r_{imp} \cdot W_{imp} = \frac{Q}{V_R}$$ where,

r_{imp} = radius of impellar

W_{imp} = width of impellar

Scanned by CamScanner

if $\beta = \frac{\pi}{2} \rightarrow$ radial fan

$< \frac{\pi}{2} \rightarrow$ backward bladed fan

$> \frac{\pi}{2} \rightarrow$ forward bladed fan

maximum theoretical head developed by fan $= \frac{U^2}{g}$

V_R (for maximum theoretical capacity) $= U \tan\beta$

Q (maximum theoretical capacity) $= V_R \left(2\pi r_{imp} \cdot B_{imp}\right)$

Power $= \Delta P \times Q$

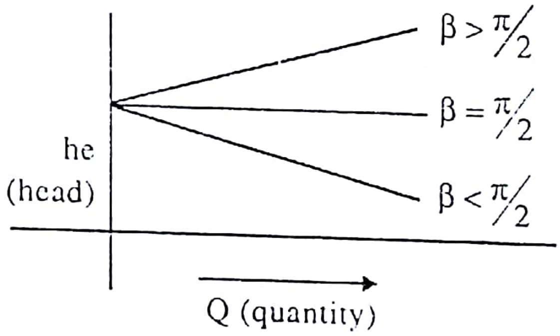
$= (h\rho g) \times Q$

Total efficiency of fan

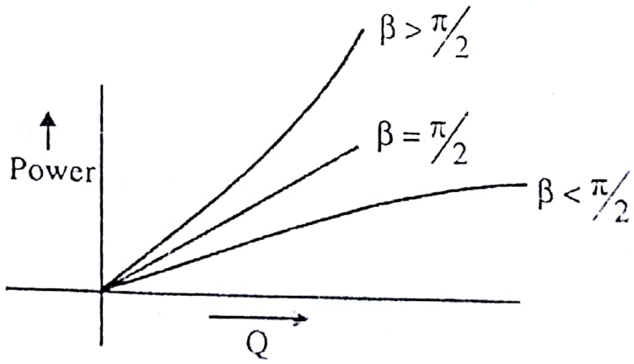
$$\eta = \frac{\text{Air power}}{\text{input power}} = \frac{QH\rho g}{\text{input power}} = \frac{Q(P)}{\text{input power}}$$

Characteristic of fan

1. Head



2. Power



Operating point of fan

head losses in fan

1. friction and diffusion losses $h_{fd} = K_3Q^2$
2. eddy and separation losses $h_s = K_4 (Q - Q_s)^2$

K_3 & K_4 are constants. Q_s is called *shockless capacity* where the impellar inlet and outlet velocities agree with the vane angles. At this capacity (Q_s) eddy and separation losses are minimum.

Scanned by CamScanner

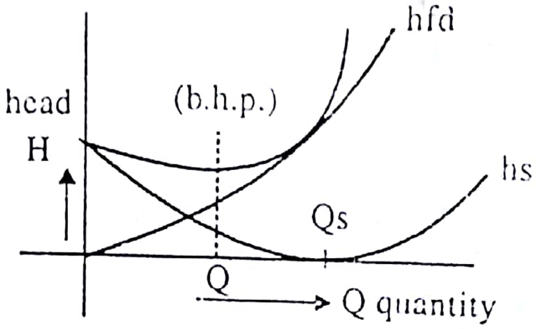
Total head losses

$$
\begin{aligned}
h &= h_{fd} + h_s \\
&= K_3Q^2 + K_4\,(Q - Q_s)^2
\end{aligned}
$$

Best efficiency point (b.e.p.)

It is that operating point at which for a given Q (quantity) total head loss is minimum

i.e. $K_3Q^2 + K^4\,(Q - Q_s)^2$ is minimum.



Head generated by fan

$$\frac{H_1}{H_2} = \frac{\omega_1^{\,2}}{\omega_2^{\,2}}$$ ω = speed (ωn)

n = no. of rev. per sec.

$$\frac{Q_1}{Q_2} = \frac{\omega_1}{\omega_2}$$ Q = quantity

$$\frac{P_1}{P_2} = \frac{\omega_1^{\,3}}{\omega_2^{\,3}}$$ P = power

Natural ventilation pressure (NVP)

$$NVP = \frac{gDB}{287.1}\left(\frac{T_U - T_D}{T_U \times T_D}\right) \times 10^3\,\text{Pa}$$

D = Depth of the shaft in (m)

B = avg Barometer reading in (KPa) usually taken 100 KPa

T_U = Temp. of upcast shaft in K

T_D = Temp. of downcast shaft in K

$g = 9.8\ \text{m/s}^2$ (K = 273 + °C)

density of air $\ell_0 = 1.2\ \text{kg/m}^2$

$$P + \rho gh + \frac{1}{2}\rho V^2 = \text{constant}$$

Regulator

Area of regulator

$$A = \frac{1.2}{\sqrt{R}},$$ where R is the resistance of regulator.

Scanned by CamScanner

Graham's Index

$$= \frac{\text{CO Produced}}{O_2 \text{ Consumed}} \times 100$$

If i it is >1%, there is active fire Composition of atmospheric air

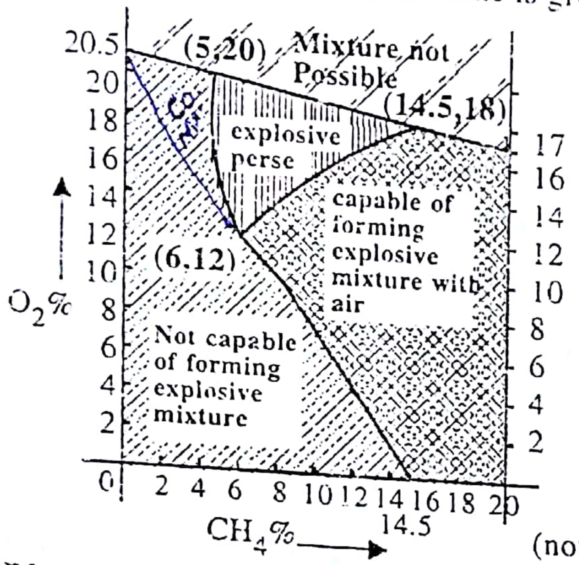
$O_2 = 20.93\%$

$N_2 = 79.04\%$

$CO_2 = 0.03\%$

Coward diagram

Nature of different composition of air and methane is given by coward diagram



(not to scale)

Dilution of methane

The distance required for complete mixing of methane in air by turbulent diffusion in a duct is given by equation with an accuracy of ± 2%.

$$L = 22 \frac{r}{\sqrt{f}}$$

L = Distance from source of gas

f = Dimension less resistance coefficient (f commonly varies between 0.12-0.012)

r = radius of the duct.

Velocities in Coal Mines

A. Maximum Velocities

Location	Maximum velocity (m/s)
1. Ventilation shafts not provided with winding equipments	15
2. Ventilation shafts without manwinding (hoisting shaft only)	12
3. Manwinding shafts & haulage roads (other than conveyor roads)	8

Scanned by CamScanner

4. Conveyor roads, loading points and transfer points 4
5. Working faces including longwall faces 4

B. Minimum Velocities

Degree of gassiness	Place where velocity of air to be measured	velocity of air (m/mm.)
I, II or III degree	1. Outbye ventilation connection from face	30
I or II degree	1. 4.5 m from any face on the intake side of brattice	30
	2. 7.5 m outbye of the discharge end of air pipe	15
	3. At the maximum span of a longwall face	60
III degree	1. 4.5 m from any face on the intake side of brattice	45
	2. 7.5 m outbye of the discharge end of air pipe	25
	3. at the maximum span of a longwell face	75

Black damp → $CO_2 + N_2$
Fire damp → $90\% \ CH_4 + 9.9\% \ N_2 + CO_2$
After damp → $CO_2 + CO + CH_4$
White damp → $CO + CO_2 +$
Stink damp → $H_2S +$

Value of Respirable Dust

$= 3 \text{ mgm}^{-3}$ for dust having $\leq 5\%$ free silica (in case of coal mines)

Otherwise,

$$\text{Threshold value} = \frac{15}{\%\text{ respirable Quartz for free Silica}} \text{ mgm}^{-3}$$

Stone dust barriers

Type	Quantity	Loading Per meter of seif	Distance between two selves	width of selves
Primary type (light barrier)	$> 110 \text{ kg/m}^2$	$< 30\text{kg/m}$	0.9m< d< 2.0m	< 35cm
Intermediate	195 kg/m^2	1/2 (P) 1/2 (S)	1/2 (P) 1/2 (S)	1/2 (P) 1/2 (S)
Secondary type (hearing type)	390 kg/m^2	<60kg/m	0.125<d<0.0270	<50cm

Dust Sampling

Method of Sampling		Instrument used
1. Filtration	→	MRE 113A Sampler

Scanned by CamScanner

2. Sedimentation →
3. Inertial Precipitation (Impaction) → Konimeter
4. Electrical Precipation → Electrical Precipitator
5. Thermal Precipitation → Thermal Precipitator
6. Optical methods based on light scattering → SIMSLIN dust sampler

General lighting Belowground (Recommended Standard)

Place		Minimum level (lumen/m^2)
a. Pit bottom	→	10
b. Main junction	→	15
c. Roadways	→	4.5
d. Haulage engine		
1. floor	→	10
2. Drum	→	25
3. Controller	→	20

Noise Level

1. 85 dBA → warning limit
2. 90 dBA → danger limit
3. 115 dBA → Ear Protection required
4. 140 dBA → Not allowed

Thickness of a Dam

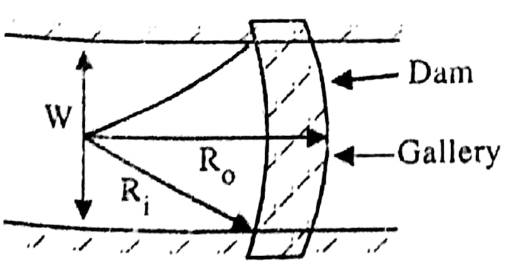
1. Rectangular or flat dam

$$T = \frac{W.H.P.}{2(W+H)S},$$ where

T = Thickness (in m)
W = entry width (m)
H = entry heigh (m)
P = water pressure (kg/m^2)
S = safe shear strength of material used (kg/cm^2) or of surrounding, strata which ever is less.

2. Arch dam (cylindrical dam)

$$T = \frac{PR_o}{f} = \frac{PR_i}{f-P}$$



P = ρgh = Vertical Pressure
f = Safe Compressive strength of construction material
R_o = Outer radius of dam
R_i = Inner radius of dam

Scanned by CamScanner

EXAMPLE

Example 1. The average air velocity as measured in a 2.0 m diameter smooth lined airway is 0.01 ms^{-1}. What is the maximum velocity of flow and where does it occur in the airway? If the average velocity is raised to 0.5 ms^{-1}, what would be the maximum velocity?

Solution :

Taking ϑ for air = 0.000016 m^2s^{-1} = 1.6×10^{-5}

$$R_e = \frac{\overline{V}D}{\vartheta} = \frac{0.01 \times 2.0}{1.6 \times 10^{-5}} = 1250 < 2000$$

So, for $\overline{V} = 0.01\ m/s$

flow is laminar.

thus maximum velocity $(V_{max}) = 2\overline{V} = 0.02\ ms^{-1}$ and it occurs at the centre of the airway.

For $\overline{V} = 0.50\ ms^{-1}$

$$Re = \frac{0.5 \times 2.0}{1.6 \times 10^{-5}} = 62{,}500 > 2000$$

hence the flow is turbulent, for which $V_{max} = \overline{V}\left(1.43\sqrt{f} + 1\right)$

$$f = \frac{0.316}{Re^{1/4}} = \frac{0.316}{(62500)^{1/4}} = 0.02$$

$$V_{max} = 0.5\left(1.43\sqrt{0.02} + 1\right) = 0.6\ ms^{-1}$$

Example 2. The impeller of a backward bladed fan has a diameter of 2500 mm and a width of 1200 mm at the outlet. Calculate the maximum theoretical head the fan will develop when rotating at a speed of 1760 rad min^{-1} (280 rpm). Assume meridianal entry into the impellar. What will be the theoretical head developed by the fan when circulating 50 m^3 of air per sec. If the outlet vane angle is 1.13 rad? What is the maximum theoretical capacity of the fan?

(Assume Dimp. = 2.5 m, Wimp = 1.2 m)

Solution :

$$U = \omega r = \frac{1760}{60} \times \frac{2.5}{2} = 36.67\ m/s$$

hence, $H_{max} = \frac{U^2}{g}$

$$= \frac{(36.67)^2}{9.8} = 137.2 \text{ m of air column}$$

Scanned by CamScanner

$$= 137.2 \times 1.2 \times 9.8 \text{ (heg)}$$

$$= 1613.5 \text{ P}_a \quad \left(\rho\text{air} = 1.2 \text{ kgm}^{-3}\right)$$

$$2\pi r_{imp} \cdot W_{imp} = \frac{Q}{V_R}$$

$$\Rightarrow V_R = \frac{Q}{\pi D_{imp} \cdot W_{imp}} = \frac{50}{3.4 \times 2.5 \times 1.2}$$

$$H_T \text{ (Theoretical head)} = \frac{U(U - V_R \cdot \cot\beta)}{g}$$

$$= \frac{36.67(36.67 - 5.308 \times \cot 1.13)}{9.8}$$

$$= 1503.3 \text{ P}_a$$

V_R' (for maximum theoretical capacity)

$$= U \tan\beta = 36\ 67 \times \tan 1.13$$

$$= 77.64 \qquad (\beta = 1.13 \text{ rad})$$

Maximum theoretical capacity

$$Q = V_R'\left(2\pi\, r_{imp} \cdot w_{imp}\right)$$

$$= 3.14\ (2.5 \times 1.2 \times 77.64)$$

$$= 731.37 \text{ m}^3\text{/sec.}$$

Example 3. Calculate the pressure developed by a backward bladed centrifugal fan having the following specification :

fan diameter = 4m, rpm = 300

velocity of flow = 4 m/s, manometeric efficiency = 70%

blade angle = 40^o, e_{air} = 1.2 kgf/m^3

(GATE-99)

Solution :

$$h_T \text{(head)} = \frac{U(U - V_R \cot\beta)}{g}$$

$$U = 2\pi\, r_{imp} \cdot \left(\frac{\text{r.p.m.}}{60}\right)$$

$$= 2 \times 3.14 \times 2 \times \frac{300}{60} = 62.8 \text{ m/s}$$

$$V_R = 4 \text{ m/s}$$

$$h_T = \frac{62.8\left(62.8 - 62.8 \times 4 \times \cot 40^o\right)}{9.8}$$

$$= \frac{3700}{9.8} = 370 \text{ m of air}$$

$$h_A \text{ (Actual)} = h_T \times \eta = 370 \times 0.7 \text{ m of air}$$

Scanned by CamScanner

$= 259$ m of air

$= 259 \times 1.2 \times 9.8\ P_a$ (heg)

$= 3.06\ kP_a$

Example 4. A longwall face is being supplied with 6000 m^3/min. of air by single gate road of 4m x 2.5 m cross section. If another gate road of 5m x 2m cross section and same length is added in parallel to the existing gate road, find the new quantity of air supplied to the longwall face assuming that pressure difference across the start and end of the gate roads remain unaltered. Assume that both the gate roads have identical coefficient of frictional resistance.

Solution :

$Q_1 = 6000\ m^3 / min. = 100\ m^3 / sec.$

$A_1 = 4 \times 2.5\ m^2,\ A_2 = 5 \times 2\ m^2$

Resistance of 1st gate road

$$R_1 = \frac{K.S}{A^3} = \frac{K.P.L}{A^3}$$

$$= \frac{K \times 2\,(4 + 2.5)\,L}{(4 \times 2.5)^3}$$

$$= 0.01\ 3\ KL$$

Resistance of 2nd gate road

$$R_2 = \frac{K.S}{A^3}$$

$$= \frac{K \times 2\,(5 + 2)L}{(5 \times 2)^2}$$

$$= 0.01\ 4\ KL$$

Pressure drop across 1st gate road

$$\Delta P = R_1 {Q_1}^2$$

$$= 0.013 \times (100)^2\ KL$$

$$= 130\ KL\ F_a$$

Quantity in 2nd gate road

$$Q_2 = \sqrt{\frac{\Delta P}{R_2}} = \sqrt{\frac{130\ KL}{0.014\ KL}}$$

$$= 96.36\ m^3/sec.$$

New Quantity $= Q_1 + Q_2$

$= 100 + 96.36 = 196.36\ m^3/sec.$

$= 11781.7\ m^3/min.$

Scanned by CamScanner

EXERCISE

1. A retreating longwall face is to produce 2000te of coal per day from a degree III gassy mine having a gas emission of $15 m^3/te$. If the percentage of gas in the return and the veloity of air in the gate roads should not exceed 0.5% and 4 m/s respectively, calculate the total minimum number of gate roads of 4m x 2.5 m cross section to be driven for the development of longwall panel. (Ans. 2) **GATE-99**
2. A fan running at 200 rpm develops a pressure of 50 mm water gause. The fan speed is increased to 300 rpm. What will be the new watergause developed. (Ans. 112.50 mm) **GATE-99**
3. The average temparature in upcast and downcast shaft in a mine are 40°C and 30°C respectively. The shafts are 300 m deep. What will be the motive coloum of air (downcast shaft air) causing natural ventilation. (Ans. 9.83 m)
4. A total flow of 85 m^3/s of air is allowed to distribute itself according to the natural splitting among the following four parallel splits each having the coefficient of friction

   $K = 0.0098\ NS^2M^{-4}$

Split	Cross section (m2)	Length (m)
A	1.5 × 6	610
B	2 × 5	450
C	2 × 4	380
D	2 × 5.5	580

   What will be the distribution of air in the four splits.

   (Ans. 17, 24, 20.24, 23) **GATE-98**
5. A fan running at 300 rad/s in mine has a total resistance of 1.1 NS^2m^8. The fan characteristics at this speed is given by the relationship

   $P = 0.1\ Q^2 - 35\ Q + 2100$

   where P is pressure in P_a and Q is quantity in m^3/s. It is desired to send an air quantity of 40 m^3/s by adjusting the speed of fan. Find the required percentage change in fan speed. (Ans. 26%) **GATE 98**
6. 10 m^3/s of air is required to be sent through each of two parallel splits × and y of resistances 2.2 NS^2m^{-8} and 5.0 NS^2m^8, respectively. The size of the regulator in split × will be what? (Ans. 0.717 m^2)
7. A standard pitot-static tube placed at the centre of a 300 mm diameter circular duct records Q velocity of 200 P_a. Assuming the air density of 1.24 kg/m^3 and the method factor of 0.84, find the quantity of air flowing through the duct in m^3/s. (Ans. 1.07)
8. For the same roadway roughness, area, length and velocity conditions the ratio of resistances of a square roadway to that of a rectangular roadway hav-

Scanned by CamScanner

ing width : height as 2 : 1 is – ? (Ans. $\frac{2\sqrt{2}}{3}$) GATE-98

9. Samples of air collected in the intake and return gate of an advancing longwall face show 0.25% and 0.75% CH_4 respectively. The panel has a production of 400 t in an 8 hour shift when an air quantity of 18 m^3/s passes through it then find out the methane emission in m^3/tone of coal mined. (Ans. 6.48 m^3/tone)

10. A single main fan in a mine delivers 40 m^3/s of air at a pressure of 560 P_a. The fan is proposed to be replaced by a new one having characteristics

$$P = 0.1\,Q^2 - 37.5Q + 2500$$

where P is pressure in P_a and Q the quantity in m^3/s. What is the operating point, once the new fan is installed? (Ans. 875, 50 m^3/s) GATE-97

11. A ventilation system provides only 40 m^3/s air at a pressure of 800 P_a. A consultant identified that the return ventilation shaft of diameter 3.0 m is consuming 60% of the mine head and suggested that widening of the shaft diameter to 4.0 m to improve the quantity. If it can be assumed that the shaft frictional characteristics and mine head remain unchanged, what will be increase in the quantity once the shaft is widen ded? (Ans. 54 m^3/sec.) GATE-97

12. In take air to a logwall panel has CO at 10 PPm and O_2 at 20.5%, whereas the return air has CO at 30 PPm and O_2 at 20.10%. Find out the Grahm's ratio (%) for the oxidation process in the panel. (Ans. 0.5)

13. A fan drift of size 4.0 m × 3.0 m, having an air flow of 60 m^3/s experiences a shock loss of 15 P_a. Find the shock factor for the drift. (Ans. 1)

14. The fan pressure of mine is 1000 P_a whereas the mine equivalent orifice is 2.0 m^2. The fan pressure is increased such that at the new conditions the fan generates pressure of 1200 P_a. If the fan efficiency is 0.7 determine the fan quantity and input power for the new speed conditions. (Ans. 99 KW) GATE-96

15. A pressure drop of 50 P_a is noticed over a square roadway of 100 m length. For a circular roadway having the same flow rate, velocity and roadway characteristics find the corresponding length to experience the same pressure drop. (Ans. 112.84)

16. 10,000 m^3 mine air per minute is supplied to two splits of equal length and surface characeteristics, having cross section of 3.0 × 3.0 m and 3.0 × 4.0 m respectively. Find out the flow of air in each split. (Ans. 5915 m^3/min) GATE-95

17. Find the Grahm's Index. Given the following results of analysis of an air sample drawn from a mine.

CH_4	2.90%
CO	0.02%
N_2	78.00%
CO_2	0.08%
O_2	19.00%
	100%

(Ans. 1.03%)

Scanned by CamScanner

18. The following is the analysis of return air samples in coal mine.

O_2	19.90%
N_2	78.65%
CH_4	0.98%
CO_2	0.45%
CO	0.02%

calculate CO/O_2 deficiency and CO_2/O_2 deficiency ratio and comment on the likelihood of active fire in the mine. The composition of atmospheric air is 20.93% O_2, 79.04% N_2 and 0.03% CO_2. (Ans. 1.9%, 40.8%)

19. A total quantity 30 m^3/sec. of air is entering a mine when the surface fan pressure is 100 mmwg. The total quantity is dividing into two splits 'A' and 'B' in parallel with a pressure drop of 40 mmwg aross them, with 'A' split getting 20 m^3/sec. and the 'B' split 10 m^3/sec. The quantity in A split is to be reduced to 10 m^3/sec. by installing a regulator. Calculate the quantity in split 'B' after the installation of the regulator and also the size of the regulator. Assume the fan pressure to remain constant.(Ans. 12 m^3/sec., 0.5) **GATE-99** .55

20. Calculate the thickness of a flat down to with stand a water head of 30 m to be constructed in an underground roadway. The details are as follows : **GATE-94**
Width of roadway = 5m, Height of roadway = 3m
Safe shearing strength of dam material = 0.15 MP_a
Safe shearing strength of surrounding strata = 0.30 MP_a
Safe compressive strength of dam construction material = 1 MP_a
Compare this with the thickness of an arched dam at the same location having an outer radius of 4m. (Ans. 1.84 m, 1.176 m)

21. The analysis of a sample of air from old workings is reported as O_2 – 15%, CO_2 – 2.8%, CH_4 – 4.3% and N_2 – 77.9%. Find the percentage of air and black damp in the sample as well as the composition of black damp assuming air to contain 20.95% O_2, 0.03% CO_2 and 79.02% N_2.
(Ans. N_2 88%, CO_2 12%)

□

Scanned by CamScanner

SOLUTION

Solution 1.

Total emission of gas

$= 2000 \times 15\ m^3/day$

$= 30{,}000\ m^3/day$

air quantity required to keep the gas level below 0.5%

$$Q = \frac{30{,}000}{\left(0.5/100\right)}\ m^3/day$$

$$= \frac{30{,}000}{\left(0.5/100\right) \times 24 \times 60 \times 60}\ m^3/min.$$

$= 69.44\ m^3/sec$

Maximum quantity of air through one roadway

$q = A \times V$

$= (4 \times 2.5) \times 4 \times 60$

$= 10 \times 4 \times 60$

$= 2400\ m^3/min$

$= 40\ m^3/sec$

no. of roadways required

$$n = \frac{Q}{q} = \frac{69.44}{40}$$

$= 1.7$

$= 2$ nos.

Solution 2.

$$\frac{h_1}{h_2} = \frac{\omega_1^2}{\omega_2^2}$$

$$\Rightarrow h_2 = \frac{\omega_2^2}{\omega_1^2} \times h_1$$

$$= \left(\frac{300}{200}\right)^2 \times 50$$

$= 112.50$ mm.

Solution 3.

$$NVP\ (\rho gh) = \frac{gDB}{2871}\left(\frac{T_U - T_D}{T_U \times T_D}\right) \times 10^3\ P_a$$

$$h = \frac{gDB}{287.1}\left(\frac{T_U - T_D}{T_U \times T_D}\right) \times 10^3 \times \frac{1}{\rho_o g}$$

Scanned by CamScanner

$$= \frac{DB}{287.1}\left(\frac{T_U - T_D}{T_U \times T_D}\right) \times 10^3 \times \frac{1}{\rho_o}$$

$$= \frac{300 \times 100}{287.1}\left(\frac{10}{313 \times 303}\right) \times 10^3 \times \frac{1}{1.2}$$

$$= \frac{11.8}{1.2}$$

$$= 9.83 \text{ m}$$

Solution 4.

$$R_A = \frac{KLP}{A^3} = \frac{KL \times 2(1.5 + 6)}{(1.5 \times 6)^3} = \frac{2 \times 7.5 \times 610}{(1.5 \times 6)^3} K = 12K$$

$$R_B = \frac{KL(2 + 5) \times 2}{(2 \times 5)^3} = \frac{K \times 14 \times 450}{10^3} = 6.3K$$

$$R_C = \frac{KL(2 + 4) \times 2}{(2 \times 4)^3} = \frac{K \times 380 \times 12}{8^3} = 9K$$

$$R_D = \frac{KL(2 + 5.5) \times 2}{(2 \times 5.5)^3} = \frac{K \times 580 \times 15}{11^3} = 6.5K$$

equivalent registrance (R)

$$\frac{1}{\sqrt{R}} = \frac{1}{\sqrt{R_A}} + \frac{1}{\sqrt{R_B}} + \frac{1}{\sqrt{R_C}} + \frac{1}{\sqrt{R_D}}$$

$$= \frac{1}{\sqrt{12K}} + \frac{1}{\sqrt{6.3K}} + \frac{1}{\sqrt{9K}} + \frac{1}{\sqrt{6.5K}}$$

$$= \frac{1}{\sqrt{k}}\left(\frac{1}{3.5} + \frac{1}{2.5} + \frac{1}{3} + \frac{1}{2.5}\right)$$

$$= \frac{1}{\sqrt{k}}(0.27 + 0.4 + 0.33 + 0.4)$$

$$= \frac{1.4}{\sqrt{K}}$$

$$\Rightarrow R = \left(\frac{\sqrt{K}}{1.4}\right)^2 = \frac{K}{2}$$

$$\Delta P_f = RQ^2 = \frac{K}{2}(85)^2$$

$$Q_A = \sqrt{\frac{\Delta P_f}{R_A}} = \sqrt{\frac{K(85)^2}{2 \times 12K}} = \frac{85}{\sqrt{24}} = 17 \text{ m}^3/\text{sec.}$$

$$Q_B = \sqrt{\frac{\Delta P_f}{R_B}} = \sqrt{\frac{K(85)^2}{2 \times 6.3K}} = \frac{85}{\sqrt{12.6}} = 24 \text{ m}^3/\text{sec.}$$

$$Q_C = \sqrt{\frac{\Delta P_f}{R_C}} = \sqrt{\frac{K(85)^2}{2 \times 9K}} = \frac{85}{\sqrt{18}} = 20.24 \text{ m}^3/\text{sec.}$$

Scanned by CamScanner

$$Q_D = \sqrt{\frac{\Delta P_f}{R_D}} = \sqrt{\frac{K(85)^2}{2 \times 6.5K}} = \frac{85}{\sqrt{15}} = 23 \text{ m}^3/\text{sec.}$$

Solution 5.

Pressure developed by fan

$$P_1 = RQ^2 = 0.1Q^2 - 35Q + 2100$$

$$\Rightarrow 1.1Q^2 = 0.1Q^2 - 35Q + 2100$$

$$\Rightarrow Q^2 + 35Q - 2100 = 0$$

$$Q = \frac{-35 \pm \sqrt{(35)^2 - 4 \times 2100}}{2}$$

$$= \frac{-35 + 98.1}{2} = 31.56 \text{ m}^3/\text{s}$$

$$\frac{\omega_2}{\omega_1} = \frac{Q_2}{Q_1}$$

$$\frac{\omega_2}{\omega_1} = \frac{40}{31.56}$$

$$\Rightarrow \frac{\omega_2}{\omega_1} - 1 = \frac{40}{31.56} - 1$$

$$\Rightarrow \frac{\omega_2 - \omega_1}{\omega_1} = \frac{40 - 31.56}{31.56} = 1.26$$

$\Rightarrow$ % change in fan speed = 26%

Solution 6.

For same quantity of air through both the splits, resistances of both the air (splits) should be same.

Hence resistance of regulator required

$P_R = 5.0 - 2.2 = 2.8$

Area of regulator

$$A = \frac{1.2}{\sqrt{P_R}} = \frac{1.2}{\sqrt{2.8}} = 0.717 \text{ m}^2$$

Solution 7.

$$P_1 + \frac{1}{2}\rho V_1^2 = \frac{1}{2}\rho V_2^2$$

$$P = \frac{1}{2}\rho V^2 \quad \left(P_{actual} = \frac{P}{0.84}\right)$$

$$V = \sqrt{\frac{2P}{\rho}} = \sqrt{\frac{2 \times 200}{1.24 \times 0.84}}$$

$$= \sqrt{400} = 10\sqrt{2}$$

Scanned by CamScanner

$$Q = V \times A = 10 \times 1.41 \times \pi r^2$$
$$= 10 \times 1.41 \times 3.14 \times (0.25)^2$$
$$= 1.27 \text{ m}^3/\text{sec}$$
$$Q_{oetual} = 1.27 \times 0.84 = 1.07 \text{ m}^3/\text{s}$$

Solution 8.

Let area of roadway = A

For square

side = $\sqrt{A}$

Perimeter $(P_S) = 2\left(\sqrt{A} + \sqrt{A}\right) = 4\sqrt{A}$

For rectangular

W = width

H (height) = $W/2$

$$W \times \frac{W}{2} = A$$
$$W = \sqrt{2}\,A$$
$$H = \frac{A}{\sqrt{2}}$$

$$\text{Perimeter } (P_R) = 2\left(\sqrt{2}\,A + \frac{A}{\sqrt{2}}\right) = 2\left(\frac{2A + A}{\sqrt{2}}\right) = 3\sqrt{2}\,A$$

$$\text{Resistance } (R) = \frac{KPL}{A^3}$$

$$\therefore \frac{R_{square}}{R_{rectangle}} = \frac{KP_SL/A^3}{KP_RL/A^3} = \frac{P_S}{P_R}$$

$$= \frac{4\sqrt{A}}{3\sqrt{2}A} = \frac{2\sqrt{2}}{3}$$

Solution 9.

$$\text{Production per hour} = \frac{400}{8} = 50\,t$$

$$\text{Methane per hour emitted} = \frac{0.5}{100} \times 18 \times 60 \times 60$$
$$= 321 \text{ m}^3$$

$$\text{methane/tone} = \frac{324}{50} = 6.48 \text{ m}^3/\text{tone}$$

Solution 10.

$$P_1 = 560\,P_a, \; Q_1 = 40 \text{ m}^3/\text{s}$$

Scanned by CamScanner

Resistance of mine

$$R = \frac{P_1}{Q_1^2} = \frac{560}{40 \times 40} = 0.35\ NS^{-2}m^{-8}$$

hence,

$$P = PQ^2$$

$$P = 0.35\ Q^2$$

also $$P = 0.1\ Q^2 - 37.5\ Q + 2500 \quad \text{.................(I)}$$

hence,

$$0.35\ Q^2 = 0.1\ Q^2 - 37.5\ Q + 2500$$

$$\Rightarrow 0.25Q^2 + 37.5Q - 2500 = 0$$

$$Q = \frac{-37.5 \pm \sqrt{(37.5)^2 + 4 \times 0.25 \times 2500}}{2 \times 0.25}$$

$$Q = \frac{-37.5 \pm \sqrt{2000}}{0.5}$$

$$= \frac{-37.5 \pm 62.5}{0.5}$$

$$= 50\ m^3 / sec$$

$$P = RQ^2$$

$$= 0.35 \times (50)^2$$

$$= 875\ P_a$$

Operating Points

$$Q = 50\ m^3 / sec$$

$$P = 875\ P_a$$

Solution 11.

Pressure drop across return ventilation shaft

$$P_1 = \frac{60}{100} \times 800 = 480\ P_a$$

Resistance of shaft

$$R_1 = \frac{P_1}{Q^2} = \frac{480}{40 \times 40} = 0.3\ NS^2m^{-8}$$

Pressure drop across rest of mine

$$P_2 = 800 - 480 = 320\ P_a$$

Resistance of rest of mine

$$R_2 = \frac{P_2}{Q^2} = \frac{320}{40 \times 40} = \frac{2}{10} = 0.2\ NS^2m^{-8}$$

$$R_1 = \frac{KPL}{A^3} = \frac{K(\pi \times 3)L}{\left(\frac{\pi \times 3^2}{4}\right)^3}$$

Scanned by CamScanner

When diameter changed to 4.0 m

$$R_1' = \frac{K(\pi \times 4)L}{\left(\frac{\pi \times 4^2}{4}\right)^3}$$

$$\frac{R_1'}{R_1} = \frac{\frac{K(\pi \times 4)L}{\left(\frac{\pi \times 16}{4}\right)^3}}{\frac{K(\pi \times 3)L}{\left(\frac{\pi \times 9}{4}\right)^3}}$$

$$= \frac{K(\pi \times 4)L}{\left(\frac{\pi \times 16}{4}\right)^3} \times \frac{\left(\frac{\pi \times 9}{4}\right)^3}{K(\pi \times 3)L}$$

$$= \frac{4 \times 9^3}{16^3 \times 3} = 0.237$$

$\therefore \quad R_1' = R_1 \times 0.237 = 0.3 \times 0.237 = 0.071$

Total registance of mine

$$= R_1' + R_2 = 0.071 + 0.2 = 0.271\ NS^2m^{-8}$$

hence,

$$Q\,(\text{new quantity}) = \sqrt{\frac{P}{R}}$$

$$= \sqrt{\frac{800}{0.271}}$$

$$= 54\ m^3/sec$$

Solution 12.

$$\text{Graham's ratio} = \frac{\text{Increase in CO content}}{\text{Decrease in } O_2 \text{ content}} \times 100$$

$$= \frac{(30-10)\,PPM}{20.5\% - 20.1\%} \times 100$$

$$= \frac{\frac{20}{100}}{\frac{0.4}{100}} \times 100$$

$$= \frac{20 \times 100 \times 100}{0.4 \times 10^6}$$

$$= 0.5$$

Scanned by CamScanner

Solution 13.

Shock factor = f_s

$$P_r = f_s \times \frac{\rho V^2}{2}$$

$$f_s = \frac{2P_r}{\rho V^2}$$

$$= \frac{2 \times 15}{1.2\left(\frac{60}{12}\right)^2}$$

$$= \frac{2 \times 15}{1.2 \times 25}$$

$$= 1$$

Solution 14.

Resistance of mine

$$\because \quad A = \frac{1.2}{\sqrt{R}}$$

$$R = \left(\frac{1.2}{A}\right)^2$$

$$= \left(\frac{1.2}{2}\right)^2$$

$$= 0.36 \text{ NS}^2\text{m}^{-8}$$

when $P = 1200\ P_a$

$$Q = \sqrt{\frac{P}{R}} \quad \left(\because P = RQ^2\right)$$

$$= \sqrt{\frac{1200}{0.36}} = 57.7 \text{ m}^3/\text{sec}$$

$$\text{Fan Power} = Q\,(h.\rho.g)$$

$$= Q \times P$$

$$\text{Input Power} = \frac{Q \times P}{\text{efficiency}}$$

$$= \frac{1200 \times 57.7}{0.7} = 99 \text{ KW}$$

Solution 15.

$$P = \frac{KPL}{A^3} = \frac{K.4a.L_s}{A^3} \text{ (square of side 'a')}$$

$$= \frac{K.2\pi r.L_c}{A^3} \text{ (circular road)}$$

$$\therefore \quad \frac{K.4a.L_s}{A^3} = \frac{K.2\pi r.L_c}{A^3}$$

Scanned by CamScanner

$$\Rightarrow \quad L_c = \frac{4a}{2\pi r} L_s \qquad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Now $\quad \pi r^2 = a^2$

$$\therefore \quad a = r\sqrt{\pi} \qquad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Putting value of 'a' in eqn. (1)

$$L_c = \frac{4r.\sqrt{\pi}}{2\pi r} L_s = \frac{2}{\sqrt{3.14}} \times 100$$

$$= 112.84 \text{ m}$$

Solution 16.

$$R_1 = \frac{KP_1L_1}{A_1{}^3} = \frac{K.2(3+3)L}{9^3} = \frac{12\text{ KL}}{9^3} = \frac{4\text{KL}}{243}$$

$$R_2 = \frac{KP_2L_2}{A_2{}^3} = \frac{K.2(3+4)L}{(12)^3} = \frac{14\text{ KL}}{12^3} = \frac{7\text{ KL}}{864}$$

Equivalent resistance 'R'

$$\frac{1}{\sqrt{R}} = \frac{1}{\sqrt{R_1}} + \frac{1}{\sqrt{R_2}}$$

$$= \frac{1}{\sqrt{\frac{4\text{ KL}}{243}}} + \frac{1}{\sqrt{\frac{7\text{KL}}{864}}}$$

$$= \frac{1}{\sqrt{K^2}} = \left(\sqrt{\frac{243}{4}} + \sqrt{\frac{864}{7}}\right)$$

$$= \frac{1}{\sqrt{K^2}}\left(\sqrt{61} + \sqrt{122}\right)$$

$$= \frac{19}{\sqrt{KL}}$$

$$\Rightarrow \quad R = \left(\frac{\sqrt{KL}}{19}\right)^2 = \frac{KL}{19^2}$$

$$\Delta P = RQ^2$$

$$= \frac{KL}{19^2} \times \left(\frac{10{,}000}{60}\right)^2$$

Quantity in 1st split

$$Q_1 = \sqrt{\frac{\Delta P}{R_1}} = \sqrt{\frac{\frac{KL}{19^2}\left(\frac{10{,}000}{60}\right)^2}{\frac{4\text{KL}}{243}}}$$

Scanned by CamScanner

$$= \sqrt{\frac{243}{4}} \times \frac{10,000}{19 \times 60} \text{ m}^3/\text{sec}$$

$$= 4119 \text{ m}^3/\text{min}$$

Quantity in 2nd split

$$Q_2 = \sqrt{\frac{\Delta P}{R_2}} = \sqrt{\frac{\frac{KL}{19^2}\left(\frac{10,000}{60}\right)^2}{\frac{7KL}{864}}}$$

$$= \frac{11}{9} \times \left(\frac{10,000}{60}\right) \text{ m}^3/\text{sec}$$

$$= 5915 \text{ m}^3/\text{sec}$$

Solution 17.

Compositon of atmospheric air

N_2 = 79.04%

O_2 = 20.93%

CO_2 = 0.03%

$$\text{Grahm's ration} = \frac{\text{CO Produced}}{O_2 \text{ Consumed}} \times 100$$

$$= \frac{\left(\frac{0.02}{100} - 0\right)}{\left(\frac{20.93}{100} - \frac{19}{100}\right)} \times 100$$

$$= \frac{0.02}{1.93} \times 100 = 1.03\%$$

Solution 18.

$$\frac{CO}{O_2} \text{ deficiency} = \frac{\text{CO Produced}}{O_2 \text{ Consumed}}$$

$$= \frac{0.02\%}{20.93\% - 19.90\%}$$

$$= \frac{0.02}{1.03} = \frac{2}{103}$$

$$= 0.019$$

$$= 1.9\%$$

$$\frac{CO_2}{O_2} \text{ deficiency} = \frac{CO_2 \text{ Produced}}{O_2 \text{ Consumed}}$$

$$= \frac{0.45\% - 0.03\%}{\left(\frac{20.93 - 19.90}{100}\right)}$$

Scanned by CamScanner

$$= \frac{\frac{0.42}{100}}{\frac{1.03}{100}}$$

$$= \frac{42}{103}$$

$$= 0.408 = 40.8\%$$

As CO/O_2 deficiency is 1.9% (>1%) and CO_2/O_2 deficiency is 40.8%, there is an active fire in the panel.

Solution 19.

Pressure drop across both splits = 40 mm Wg

$$= 40 \times 10^{-3} \times 10^3 \times 9.8$$

$$= 392\ P_a$$

Pressure drop across rest of mine = 60 mm Wg

$$= 60 \times 9.8$$

$$= 588\ P_a$$

Equivalent resistance of splits

$$R_1 = \frac{P_1}{Q^2} = \frac{392}{(30)^2} = \frac{392}{900}$$

$$= 0.44\ NS^2m^{-8}$$

Resistance of rest of mine

$$R_2 = \frac{P_2}{Q^2} = \frac{588}{900} = 0.65\ NS^2m^{-8}$$

Resistance of splits A & B

$$R_A = \frac{392}{20^2} = \frac{392}{400} = 0.98\ NS^2m^{-8}$$

$$R_B = \frac{392}{100} = 3.92\ NS^2m^{-8}$$

Let Q' be the new quantity

Pressure drop across rest of mine

$$= 0.65 \times Q'^2 \qquad \text{.............. (1)}$$

Pressure drop across splits = $980 - 0.65\ Q'^2$ .... (2)

Pressure drop across split B

$$= 3.92(Q' - 10)^2 \qquad \text{..... .... (3)}$$

Hence from eqn. (2) & (3)

$$3.92\ (Q' - 10)^2 = 980 - 0.65\ Q'^2$$

$$\Rightarrow \quad 3.92\ Q'^2 - 20 \times 3.92Q' + 392 = 980 - 0.65\ Q'^2$$

$$\Rightarrow \quad 4.57Q'^2 - 78Q' + 392 - 980 = 0$$

$$\Rightarrow \quad 4.57Q'^2 - 78Q' - 588 = 0$$

Scanned by CamScanner

$$Q' = 78 \pm \sqrt{\frac{(78)^2 + 4 \times 4.57 \times 588}{2 \times 4.57}} = \frac{78 \pm 112}{9.5}$$

$$= \frac{210}{9.5} = 22\,m^3/sec$$

Quantity of air in split 'B' = 22 − 10 = 12 m^3/sec

New resistance of split $A = \frac{\Delta P}{Q_A{}^2} = \frac{980 - 0.65(22)^2}{10^2}$

$= 6.6$

Size of regulator (Area)

$$A = \frac{1.2}{\sqrt{6.6 - 0.98}} = \frac{1.2}{2.4}$$

$$= 0.5\ m^2$$

Solution 20.

$P = 30 \times 1 \times 10^3 \times 9.8\ P_a$

$W = 5m$

S = less of 1 & 2 = 0.15 P_a

$H = 3m$

Flat Dam

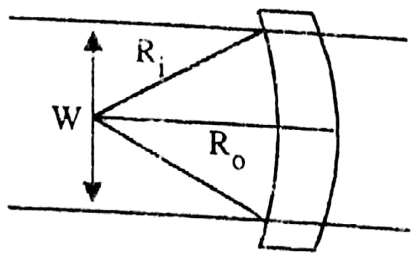
$$T = \frac{W.H.P.}{2(W + H)s}$$

$$= \frac{5 \times 3 \times 30 \times 10^3 \times 9.8}{2(5+3) \times 0.15 \times 10^6}$$

$$= \frac{14.7}{8} = 1.84\ m$$

Arched Dam

$$T = \frac{P \times R_o}{f} \quad \text{or} \quad T = \frac{P \times R_i}{f - P}$$



f = safe compressive strength of dam constn. material

R_o = Outer radius

R_i = Inner radius

$$T = \frac{(30 \times 10^3 \times 9.8) \times 4}{1 \times 10^6} = \frac{117.6}{100}$$

$$= 1.176\ m$$

Scanned by CamScanner

Solution 21.

$$\% \text{ air} = \frac{15}{20.95} \times 100 = 72\% \text{ air}$$

$$\frac{72}{100} \times 79.02 = 56\% \; N_2 \text{ in air}$$

$$\frac{72}{100} \times 0.03 = 0.02\% \; CO_2 \text{ in air}$$

excess of N_2 = 77.90% – 56% = 21.9%

excess of CO2 = 2.8% – 0.02% = 2.78%

Black damp (mixture of excess of N_2 and CO_2)

= 21.9 + 2.78 = 24.68%

Composition of blackdamp

$$N_2 = \frac{21.9}{24.68}\% = 88\%$$

$$CO_2 = \frac{2.78}{24.68}\% = 12\%$$

Scanned by CamScanner

PUMP

Pump

1. **Rotary Pumps**

   These pumps are directly coupled to electric motors (two types)

   (i) **Turbine Pump**
   (Multi Stage Pump)
   – Many impellars (< 10)
   – Priming of water required

   – High head, Low capacity

   (ii) **Centrifugal Pump**

   – Low Head
   – Large capaity (upto 400,000 l/min)
   – Single impeller
   – Priming of water required

2. **Reciprocating Pumps**
   – Bulky
   – Slow speed
   – Gearing arrangment required

3. **Roto Pumps**

   – Self Priming
   – Suitable for head of 8 m.

Axial Thrust

In both types of Pumps (Turbine & Centrifugal) a considerable end thrust is veloped which acts towards the suction end of the Pump. It occurs because water un Pressure leaks into clearance spaces on both sides of each impellar.

Axial thrust can be counteracted by use of following means :

1. Balancer disc → Most commenly used
2. Thrust Bearing →
3. In turbine Pumps (only) → By placing impellar back to back.
4. In centrifugal Pumps (only) → By drilling the eye of the single impell

Heads required to ovecome by a Pump

1. Suction head → (H_s)
   Difference of vertical height between Pump and suction end.
2. Delivery head → (H_d)
   Difference of vertical height between Pump and delivery end.
3. Frictional head → (H_f)
   Head required to overcome frictional losses.

Scanned by CamScanner

Frictional Loss (D'arcy's formula)

Frictional losses due to a flowing fluid in a pipe is given by formula

$$H_f = \frac{4f.\ell.v^2}{2g.d}$$

where,

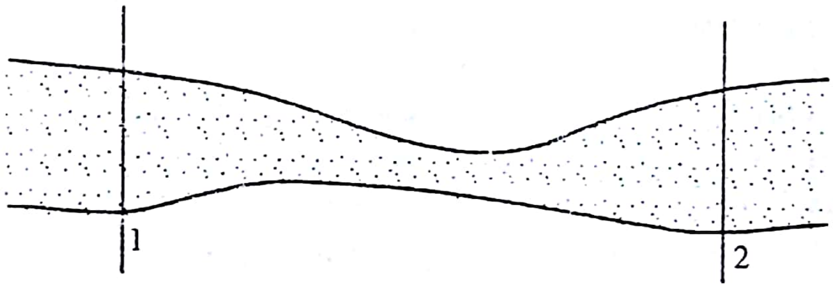
f = coefficient of friction (for water 0.01)
ℓ = length of pipe in mtr. (m)
v = velocity of fluid in (m/s)
$g = 9.8\ m/s^2$
d = diameter of the pipe (m)

Conservation of energy in fluid flow

During the entire flow of fluid in a system total energy is always conserved.

$$P + \rho.gh + \frac{1}{2}\rho V^2 = \text{constant}$$

$$P_1 + \rho_1 g h_1 + \frac{1}{2}\rho_1 V_1^2 = P_2 + \rho_2 . g . h_2 + \frac{1}{2}\rho_2 V_2^2$$



where

P_1, P_2	=	Pressure at point (1) & (2)
$\rho_1 \rho_2$	=	Density of fluid
h_1, h_2	=	Height of points above datum level.
V_1, V_2	=	velocities

Head Developed by a Pump

theoretical head per stage

$$H_T = \frac{U(U - V_R \cot\theta)}{g}$$

θ = angle of blade tip
U = tangential speed of blade
V_R = radial speed of discharge

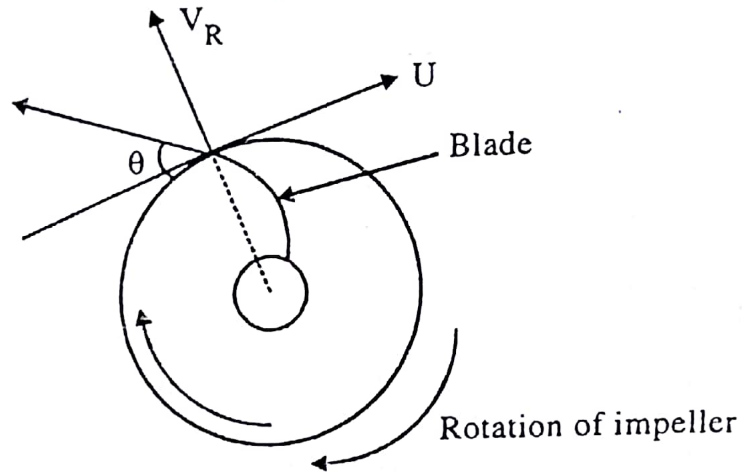
$$U = 2\pi r_{imp}\left(\frac{r.p.m.}{60}\right) \quad m/s$$

$$V_R = \frac{Q}{2\pi r_{imp.} w_{imp}} \quad m/s$$

$r_{imp.}$ = radius of impellar (m)

Scanned by CamScanner

r.p.m. = no. of revolution of impeller per minute.
Q = quantity of flow (m^3/sec.)
W_{imp} = width of impeller (m)



For the purpose of solving problems following typical values of different quantities can be used (only if they are not specified in the problem given)

Inlet velocity (V.)	= 1.7 cm
Inlet dia.	= 18.2 cm
Outlet dia	= 15.7 cm
Impeller dia.	= 35 cm
Head per stage	= 40m– 45m.
No. of impellers	= 6 – 8
Radial velocity	= 2 m/sec.
width of impellar	= 2 cm.

EXAMPLES

Example 1.

A pumping installation has following specifications
Suspension pipe : 200 cm ϕ (dia.), 10 cm length.
Delivery pipe : 16 cm ϕ, 1000 m length.
Quantity of water pumped = 2.4 m^3/min.
Vertical height through which water is raised : 250 m.

Calculate 1. Total head against which pump is working.
2. Required power of the driving motor assuming pump to be directly driven by the motor.

(GATE -9

Take coefficient of friction as 0.01.

Solution :

Frictional loss (head of water lost by friction)

$$H_f = \frac{4f\ell v^2}{2g.d}$$

in suction pipe

$$H_{fs} = \frac{4 \times .01 \times 10 \times (1.27)^2}{2 \times 10 \times .2}$$

$$\left(V_s = \frac{Q}{A} = \frac{2.4/60}{(.1)^2} = 1.27 \text{ m/s} \right)$$

$= 0.165$ m

in delivery pipe

$$H_{fd} = \frac{24 \times .01 \times 1000 \times (2)^2}{2 \times 10 \times .16} \qquad \left(V_d = \frac{2.4/60}{(.8)^2} = 2 \text{ m/s} \right)$$

$= 51$ m

Total Head $= H + H_{fs} + H_{fd}$

$= 250 + 0.165 + 51 = 301.16$ m

Power required by the motor

= work done in pumping water per sec

$= mg\,H_T$

$= \left(\frac{2.4}{60} \times 1000 \right) \times 9.8 \times 301.16$

$= 120$ kw

Example 2.

A six stage turbine pump having six impellars is directly driven by an electric motor running at 1440 rpm. The pump is used to raise 3.3 m^3/min. of water against a total head of 240 m. If the radial velocity of water is 1.8 m/s, the mano metric efficiency is 74%, the curveture of impellar ranes is 30° and the velocity of water at impellar inlet is 1.7 m/s, find the diameter and width of the impellars. **(GATE-98)**

Solution :

Total head = 240 m.

head per stage $= \frac{240}{6}$ (no. of impellars = 6)

H_T (Theoretical head per stage) $= \frac{40}{\eta} = \frac{40}{0.74}$

$= 57$ m, $(\eta = 74\%)$

But $\quad H_T = \frac{U(U - V\cot\theta)}{g}$

$$57 = \frac{U^2 - UV\cot\theta}{g}$$

$\Rightarrow \quad 57 \times 9.8 = U^2 - 1.8 \times U \times \cot 30°$

$\Rightarrow \quad U^2 - 3.1\,U = 560$

$\Rightarrow \quad U - 3.1\,U - 560 = 0$

$$U = \frac{3.1 \pm \sqrt{(3.1)^2 + 4 \times 560}}{2}$$

Scanned by CamScanner

Taking positive root

$$U \text{ (tangential velocity)} = 25 \text{ m/s}$$

$$U = 2\pi r\left(\frac{\text{r.p.m}}{60}\right)$$

$$\Rightarrow \quad 25 = \pi D\left(\frac{1440}{60}\right)$$

$$\Rightarrow \quad D = \frac{25 \times 60}{1440 \times \pi} = 32.4 \text{ cm (diameter of impellar)}$$

Area of impellar outlet

$$A = \frac{\text{volume of water}}{\text{radial velocity}}$$

$$= \frac{3.3/60}{18} = 0.03 \text{ m}^2$$

Now, $A = 2\pi r_{imp} \times W_{imp}$

$$W_{imp} = \frac{A}{2\pi r_{imp}} = \frac{0.03}{\pi \times 0.32}$$

$$= 0.03 \text{ m} = 3 \text{ cm (width of impellar)}$$

Example 3.

A backward bladed centrifugal pump produces a theoretical head of 60 m of water at a radial velocity of 2.0 m/s. Given the curvature of the impellar vane is 30^0 and the impellar diameter 0.04 m, determine the tangential speed for the tip of the impellar blade, what is the impellar r.p.m. **(GATE-97)**

Solution :

Theoretical head

$$h = \frac{U(U - V\cot\theta)}{g}$$

$$60\text{m} = \frac{V^2 - U \times 20 \times \cot 30°}{9.8}$$

$$\Rightarrow \quad U^2 - 3.46\,V - 588 = 0$$

$$U = \frac{3.46 \pm \sqrt{(3.46)^2 + 4 \times 588}}{2}$$

$$= 26 \text{ m/s}$$

Now tangential speed

$$U = \pi\, D_{imp} \times \left(\frac{\text{r.p.m.}}{60}\right)$$

$$\Rightarrow \quad \text{r.p.m.} = \frac{U \times 60}{\pi\, D_{imp}}$$

$$= \frac{26.00 \times 60}{3.14 \times 0.4} = 1245$$

□

Scanned by CamScanner

EXERCISE

1. A turbine pump has 5 impellars, 30 cm. in diameter running at 1440 rpm. The delivery branch is 15 cm. in bore and suction branch 20 cm. For what rate of delivery water and the head the pump is suitable? Assume inlet velocity as 1.5 m/sec. and manometic efficiency 0.6.
   What will be the width of impellar outlet around its periphery, assuming a radial velocity of 2.4 m/s.
2. A turnine pump is required to work under the following conditions : (GATE-95)
   R.P.M. 1440, capacity 2700 litres per min, total head from all causes 300 m, angle of curvature of the impellar blades $\theta = 30°$ (backward curved) manometric efficiency 0.7, radial velocity of water 2 m/s. Find the no. of impellars, their diameter and width of the impellar.
3. A turbine pump has 6 impellars, 30 cm. diameter running at 1440 rpm. The delivery branch is 18 cm. bore and suction branch is 20 cm. bore. For what rate of delivery of water and head should the pump be suitable? Also find the width of impeller outlet around its periphery
   Given, radial velocity = 2.4 m/s
   suction velocity = 1.68 m/s
4. Calculate the diameter, width and no. of impellars of a multistage pump, which is suitable for raising 2 m^3/min. of water against a total head of 240 m. Also determine the diameters of suction and delivery branches.
   What should be the power of a motor for driving the pump coupled directly. Ignore the blade curvature.
5. In a mine the waterlogged workings have to be dewatered. The distance from the pit bottom sump to the dipmost face which dewatering will have to be done is 1200 m. The vertical level difference between the bottom most point and the pit bottom sump is 150 m. The total quantity of water in the lodgement to be pumped out is 1200 kilo leters and percolation from strata is 2000 kilo leters in 24 hours. It had been decided to pump out the water in 30 days. Net hour of run of the pump is 20 hours a day. Diameter of pipe is 150 mm. Find head, capacity and power of pump needed to install.
   Assume coefficient of friction 0.015 and efficiency of pumping set 75%.
6. Determine the diameter & width of a impellar of a turbine pump for raising 2000 ℓ/min. of water at 1440 rpm. against a head of 308 m. Also find the no. of impellars required.
   Assume radial velocity 1.8 m/sec., manometric efficiency 0.07 and currature of impellar vanes 30° backward.

Scanned by CamScanner

SOLUTIONS (EXERCISE-1)

Solution 1.

$V = 2.4$ m/s

U = tangential speed of tip of impellar blade

$$U = 2\pi r_{imp} \times \left(\frac{\text{r.p.m.}}{60}\right)$$

$$= \pi \times .3 \times \left(\frac{1440}{60}\right) = 22.5 \text{ m/s}$$

$$h_T \text{ (Theoretical head per stage)} = \frac{U(U - V\cot\theta)}{g}$$

$$h_T = \frac{U^2}{g} \text{ (neglecting cot } \theta \text{ term as } \theta \text{ is not given).}$$

$$= \frac{(22.5)^2}{9.8} = 51.5 \text{m}$$

Actual head for 5 stage pump

$$H_A = 51.5 \times 0.6 \times 5$$

$$= 125 \text{ m.}$$

Rate of delivery of water

$$Q = V_d \times A_d$$

$$V_{inet} = 1.5 \text{ m/s}$$

$$A_{inlet} = \pi\left(\frac{0.20}{2}\right)^2$$

$$\text{Adischarge} = \pi\left(\frac{0.15}{2}\right)^2$$

$$V_d \times A_d = V_{inlet} \times A_{inlet}$$

$$V_d = \frac{1.5 \times \pi(0.1)^2}{\pi\left(\frac{0.15}{2}\right)^2}$$

$$= 2.66 \text{ m/s}$$

$$Q = V_{inlet} \times A_{inlet}$$

$$= 1.5 \times \pi(0.1)^2$$

$$= 0.0407 \text{ m}^3\text{/sec}$$

Width of impellar

$$Q = \left(2\pi r_{imp} \times W_{imp}\right) \times V_R$$

$$W_{imp} = \frac{Q}{V_R \times \pi D_{imp}}$$

Scanned by CamScanner

$$= \frac{0.0407}{2.4 \times \pi \times 0.3} \text{ m}$$

$= 2.5$ cm.

Solution 2.

Assume an inlet velocity of 1.7 m/s

Area of suction inlet $= \frac{2700}{1000} \times \frac{1}{1.7} \times \frac{1}{60} = 0.026 \text{ m}^2$

$$A = \frac{\pi (D_{inlet})^2}{4}$$

$$D_i = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{4 \times 0.026}{\pi}}$$

$= 0.182$ m

$= 18.2$ cm.

Diameter of delivery outlet

$D_o = 18.2 - 2.5$ (usually 2.5 cm less then D_i)

$= 15.7$ cm.

($D_{imp.}$) Assume impellar diameter = 35 cm.

$$U = 2\pi r_{imp} \left(\frac{\text{r.p.m.}}{60}\right)$$

$$= \pi \times 0.7 \times \left(\frac{1440}{60}\right)$$

$= 26.4$ m/s

Head per stage

$$h_t \text{ (theoretical)} = \frac{U(U - V \cot\theta)}{g}$$

$$h_A \text{ (Actual)} = \frac{U(U - V\cot\theta)}{g} \times 2$$

$$h_A/\text{stage} = \frac{26.4(26.4 - 2 \times 1.73)}{9.81} \times 0.7$$

$= 43.21$ m

Number of impellars required

$$n = \frac{300}{43.21} = 7 \text{ nos.}$$

Area of impellar outlet

$$= \frac{\text{vol. of water}}{V_R}$$

$$= \frac{2.7}{60} \times \frac{1}{2} = 0.0225 \text{ m}^2$$

$$W_{imp} = \frac{A_{imp}}{2\pi r_{imp}}$$

Scanned by CamScanner

$$= \frac{0.0225}{1.10} = 0.0204 \text{ m}$$
$$= 2.04 \text{ cm}$$

Solution 3.

$$A_s \text{ (suction)} = \frac{\pi}{4}(0.20)^2 = 0.031 \text{ m}^2$$
$$Q = V_s \times A_s$$
$$= 0.031 \times 1.68 \times 60$$
$$= 3.12 \text{ m}^3/\text{min}$$
$$A_d \text{ (delivery)} = \frac{\pi}{4} \times (0.20)^2 = 0.025 \text{ m}^2$$
$$V_d = \frac{Q}{A_d} = \frac{3.12/60}{0.025}$$
$$= 2.08 \text{ m/s}$$

Head

$$h_a/\text{stage} = \frac{U^2}{g} \times \eta$$
$$U = 2\pi r_{imp}\left(\frac{\text{r.p.m.}}{60}\right)$$
$$= \pi \times 0.30 \times \left(\frac{1440}{60}\right)$$
$$= 22.62 \text{ m/s}$$
$$h_a = \frac{U^2}{g} \times \eta$$
$$= \frac{22.62}{9.81} \times 0.60 \quad (\eta = 60\% \text{ – manometric efficiency})$$
$$= 31.3 \text{ m}$$
$$h_a = h_a \times n \text{ (no. of impellars)}$$
$$= 31.3 \times 6 = 187.8 \text{ m.}$$

Width of impellar

$$2\pi r_{imp} \times W_{imp} = \frac{Q}{\sqrt{R}}$$
$$W_{imp} = \frac{Q}{\pi D_{imp} \times V_R}$$
$$= \frac{3.12/60}{3.14 \times 0.3 \times 2.4}$$
$$= 2.32 \text{ cm.}$$

Solution 4.

$$Q = 2 \text{ m}^3/\text{min.}$$
$$H_T = 240 \text{ m}$$

Scanned by CamScanner

Assuming an inlet velocity of 1.7 m/s

$$A_s \text{ (suction)} = \frac{2/60}{1.7} = .0196 \text{ m}^2$$
$$= 196 \text{ cm}^2$$

$$D_S = \sqrt{\frac{196 \times 4}{\pi}} = 15.8 \text{ cm}$$

Let r.p.m. = 1440 & η (efficiency) = 60%

Also assume head per stage = 30 m

$$\text{no. of impellars (n)} = \frac{240}{30} = 8$$

Impellar Dia ($D_{imp.}$)

$$H_A \text{ (Actual head per stage)} = \frac{U^2}{g} \times 0.6 = 30$$

$$\Rightarrow \quad U = \sqrt{\frac{30 \times 9.81}{0.6}}$$
$$= 22.15 \text{ m/s}$$

$$U = 2\pi r_{imp}\left(\frac{\text{r.p.m.}}{60}\right)$$

$$D_{imp} = \frac{U \times 60}{\pi \times \text{r.p.m.}} = \frac{22.15 \times 60}{\pi \times 1440}$$
$$= .294 \text{ m} = 29.40 \text{ cm.}$$

Power required

$$\text{Power} = \frac{2 \times 1000 \times 9.81}{60} \times 240$$
$$= 78.5 \text{ KW}$$

Power of motor (assuming 70% pump efficiency)

$$\frac{78.5}{0.70} = 112.14 \text{ KW}$$

Solution 5.

L = 1200 m, f = 0.015

d (dia. of pipe) = 0.15 m

Head = 150 m

Total water to be dewatered in 30 days

$$= (12000 + 2000 \times 30) \text{ K litres}$$
$$= (1200 + 60000) \text{ K litres}$$
$$= 72{,}000 \text{ m}^3$$

Q (quantity of water to be pumped out per min.)

$$= \frac{72{,}000}{20 \times 30 \times 60} = 2 \text{m}^3/\text{min.}$$

Scanned by CamScanner

Frictional loss

$$h_f = \frac{4f\ell v^2}{2g.d}$$

now, $$v = \frac{Q}{A} = \frac{2/60}{\frac{\pi(0.15)^2}{4}}$$

$$= 1.886 \text{ m/s}$$

$$h_f = \frac{4 \times 0.015 \times 1200 \times (1.886)^2}{2 \times 9.81 \times 0.15}$$

$$= 87.13 \text{ m}$$

Total head (H_T) $= h_v + h_f$

$= 150 + 87.13$

$= 237.13$ m

h.p. of pump needed

$$\text{Power} = \frac{\left(\frac{2 \times 10^3}{60}\right) \times 9.8 \times 237.13}{\cancel{60 \times} 0.75}$$

$$= 103.28 \text{ KW}$$

$$\text{h.p.} = \frac{103.28}{0.746} = 138.44 \text{ hp}$$

Solution 6.

Total head $(H_T) = 308$ m

Let head per impellar be 44 m

no. of impellars $= \frac{308}{44} = 7$ nos.

h_T (Theoretical head per impellar) $= \frac{44}{0.7} = 63$ m.

$$h_T = \frac{U(U - V \cot\theta)}{g}$$

$$63 = \frac{U^2 - U \times 1.8 \times \cot 30^\circ}{9.81}$$

$$\Rightarrow \quad U^2 - U \times 1.8 \times 1.73 = 63 \times 9.81$$

$$\Rightarrow \quad U^2 - 3.1\,U - 616 = 0$$

$$U = \frac{3.1 \pm \sqrt{(3.1)^2 + 4 \times 616}}{2} \quad \left(\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\right)$$

Scanned by CamScanner

$$= \frac{3.1 \pm 50}{2} = \frac{3.1 + 50}{2}$$

$$= 26.6 \text{ m/s}$$

Diameter of impellar

$$U = 2\pi r_{imp} \left(\frac{\text{r.p.m.}}{60} \right)$$

$$D_{imp} = \frac{U \times 60}{\pi \times \text{r.p.m.}}$$

$$= \frac{26.6 \times 60}{3.14 \times 1440}$$

$$= 0.345 \text{ m.}$$

Width of Impellar

$$2\pi r_{imp.} \times W_{imp.} = \frac{Q}{V_R}$$

$$W_{imp.} = \frac{Q}{\pi \times D_{imp.} \times V_R}$$

$$= \frac{2/60}{3.14 \times 0.345 \times 1.8} \quad (Q = 2 m^3 / min)$$

$$= 1.7 \times 10^{-2} \text{ m}$$

$$= 1.7 \text{ cm}$$

Scanned by CamScanner

HAULAGES, WINDERS & CONVEYORS

Important formulii

1. Mass of rope per meter

   $m = kd^2$ kg

   where $k = 0.35$

   d = dia. of rope in cm.

2. Strength of rope

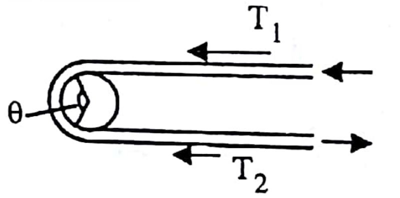
   $B = sd^2$ KN

   $S = 50$ KN/cm^2

   d = dia. of rope in cm.

3. If T_1 & T_2 are tensions in upper rope (conv.) and bottom rope (conv.) respectively then,

   $$\frac{T_1}{T_2} = e^{\mu\theta}, \quad \theta \text{ in radians}$$



μ has a value between 0.35 to 0.5 condition for rope to slip

$$\frac{T_1}{T_2} > 1.5$$

when rope is moving

$$\frac{T_1}{T_2} > 1,$$

if it exceeds 1.5 rope will start slipping

Friction winding (Koepe winding)

A belt or hoist rope passing over a wheel driven by a prime mover, has different tensions at points where the belt or rope enters the wheel and where it leaves the wheel.

The relationship between these tensions is given by

$$\frac{T_1}{T_2} = e^{\mu\theta}$$

T_1 = Tension of the rope entering the wheel.

T_2 = Tension of the rope leaving the whee.

μ = 0.35 to 0.5

θ = angle of wrap of rope around friction wheel in radians

Scanned by CamScanner

EXAMPLES

Example 1.

Calculate the power of the motor of an endless haulage running at a speed of 3.6 km/hr. The haulage is pulling coal up a gradient of 1 in 10 over a distance of 1000 m. The number of mine cars in the loaded as well as enpty set is 10. The payload capacity of each mine car is 2 te and its tare is 1 te. Take the weight of the haulage rope as 2 kgf/m, frictional resistance of mine cars as 1/1000, frictional resistance of rope as 1/10 and gearing efficiency as 80%. **(GATE-99)**

Solution :

m_c (mass of loaded tub) $= 2 \times 10^3$ kg

m_e (mass of empty tub) $= 1 \times 10^3$ kg

$\sin\theta = 1/10$

$\mu_t = 1/100$

$\mu_r = 1/10$

$\cos\theta = 1$ (approax.)

$V = 3.6 \text{ km/h}$

$$= \frac{3.6 \times 1000}{60 \times 60} = 1 \text{ m/s}$$

n = no. of tub = 10.

Forces due to empty tub and rope will be balanced.

Force due to load (coal) (mg sin θ)

$$F_1 = 10\, m_c \cdot g \times \sin\theta$$

$$= 10 \times 10^3 \times 9.8 \times \frac{1}{10} \times 2$$

$$= 2000 \times g$$

Friction forces (μ mg cosθ)

due to load tub & emplty tub ($m_c g \cos\theta \cdot \mu_t + m_t g \cos\theta\, \mu_t$)

$$F_2 = \left\{(2 \times 10) \times 10^3 g \times 1 \times \frac{1}{100}\right\} + \left\{1 \times 10 \times 10^3 g \times \frac{1}{100}\right\}$$

= 200 g + 100 g

= 300 g

due to rope on both sides ($m_r \cdot g \cdot \cos\theta \cdot \mu_r$)

$$F_3 = 2 \times \left(1000 \times 2 \times g \times 1 \times \frac{1}{10}\right)$$

= 400 g

Total force required

$$F = F_1 + F_2 + F_3$$

Scanned by CamScanner

$= 2000\ g + 300\ g + 400\ g$

$= 2700\ g$

Power required

$$P = F \times V$$
$$= 2700\ g \times 1$$
$$= 2700\ g$$

as gearing efficiency is 80%

actual power required is

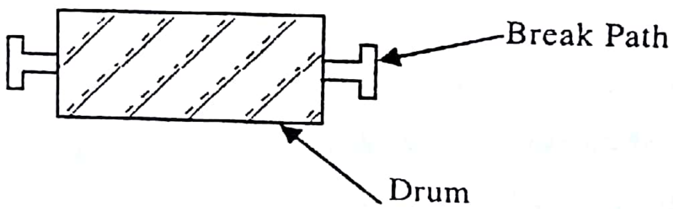
$$\frac{2700\ g}{0.80}$$

~~=20.4 KW~~

Example 2.

A train of mine cars of total 50 tef is attached to a *direct rope haulage* and travelling down a gradient of 1 in 10 (avg.). The length of haulage plane being 100 m. The coefficient of friction of mine cars is 1/50 and that of the rope is 1/20. Th rope weighs 2.73 kgf/m. The radius of the drum is 1 m and that of the brake path i 666 mm. Width of the breakpath is 150 mm. There are two brake blocks, each hav ing an angle of contact of 90° on the brake path. The brake is lined with bonde asbestoes with a coefficient of friction of 0.3. Find out the radial pressure to be ex erted on the brake path.

Solution :



(a) Force on rope due to desending tain

$$F_1 = \frac{50 \times 1000 \times 9.81}{10}$$
$$= 49050\ N$$

(b) Force in rope due to the own weight

$$F_2 = \frac{26.8 \times 1000}{10}\ N \quad (2.73 \times 9.8 = 26.8)$$

(c) Breaking force due to friction of trains

$$F_3 = \frac{50 \times 1000 \times 9.81}{50}$$
$$= 1000 \times 9.81$$
$$= 9810\ N$$

(d) Breaking force due to friction of rope

$$F_4 = \frac{26.8 \times 1000}{20}$$
$$= 1340\ N$$

Scanned by CamScanner

The net force at the drum shaft is

$$F = (F_1 + F_2) - (F4_3 + F_4)$$

$$= (49050 + 2680) - (9810 + 1340)$$

$$= 40580 \text{ N}$$

Thus pull on rope is

$$F = 40580$$

$$\text{Torque } \tau = F \times r(\text{radius of Pull})$$

$$= 40580 \times 1\text{m}$$

$$= 40580 \text{ Nm}$$

Torque due to the breaking force should be equal to the tarque due to force in rope.

If BF is the breaking force, then

$$BF \times 0.66\text{m} = 40580$$

$$BF = \frac{40580}{0.66} = 61485 \text{ N}$$

Frictional circumferential pull on brake is 61485 N.

Total break in contact with drum is

= (circumference of breakpath) × (width of breakpath) × (arc of contact)

$$= 2\pi \times 0.66 \times \frac{150}{1000} \times \frac{90 \times 2}{360}$$

$$= 0.311\text{m}^2$$

Radial pressure per m^2 of break surface is

$$P \times 0.311 \times 0.3 = 61485 = 0.311\text{m}^2$$

$$P = \frac{61485}{0.311 \times 0.3}$$

$$= 659003 \text{ Nm}^{-2}$$

$$= 659 \text{ KNm}^{-2}$$

Example 3.

A direct rope haulage pulls 10 loaded tubs at a time up an incline dipping at 1 in 8. An empty tub weighs 400 kgf and has a capacity of 900 kgf of mineral. Length of the roadway covered by the haulage is 500 m. The rope diameter is 25 mm. Rope speed is 12 km/hr. coefficient of friction for the tubs and for rope is 1/20. Find the power required by the rope and by the engine.

Solution :

Rope dia. is 25 mm

∴ mass in kg/m length of rope = kd^2,

where d is dia. in cm. and K is 0.35 for a fibre core rope made from steel having breaking strength of 1570 MN/m^2 (i.e. 160 kgf/mm^2)

Scanned by CamScanner

$\therefore$ mass in kg/m $= 0.35 \times (2.5)^2 = 2.18\ kg/m.$

wt. of rope/m $= 2.18 \times 9.81\ N$

$= 21.39\ N$

Rope speed is 12 km/h = 3.33 m/s

Total force to overcome—

= (Component of the wt. of loaded tubs) + (Component of full length of rope) + (friction of load) + (friction of rope)

$$= 1300 \times 9.81 \times 10 \times \frac{1}{8} + \frac{2.18 \times 9.81 \times 500}{8}$$

$$+ 1300 \times 9.81 \times 10 \times \frac{1}{20} + \frac{2.18 \times 9.81 \times 5000}{20}$$

= 15941 + 1336.6 + 6376.6 + 534.6

= 24.189 KN

Power = Force × Velocity

= 24189 × 3.33 = 80549 W

= 80.55 KW.

This would be the power required by the rope when rope moves at full speed and it would decrease a little as the rope coils on the drum. It would be necessary to provide a motor of atleast 50% more power for the following reasons :

(a) Additional force required to accelerate the journey from rest.

(b) Allowance for power lost due to gearing efficiency between motor and haulage drum.

(c) A reserve of power is always necessary in case the tubs derail and they have to be pulled up when the wheels are on the floor and the coefficient of friction is high.

Scanned by CamScanner

EXERCISE

1. What is the least gradient for *self acting incline*, 500 m long, assuming 8 tubs per train and each tub weighing 500 kgf and carring 1000 kgf of coal? The rope weighs 1 kgf/m.

   Assume $\mu_t = \frac{1}{50}$ for tubs and

   $\mu_r = \frac{1}{10}$ for the rope.

   (Ans. 1 in 18)

2. An endless haulage operates on a roadway 800 m long dipping at 1 in 15 and draws 600 tonnes of coal a shift (1 shift = 7.5 hrs. effective time). The loads weigh 1500 kgf each and the empties 500 kgf. The rope weighs 3 kgf/m, tub friction is 1/50 and rope friction is 1/10, speed of the rope is 4 Km/hr. Estimate the power required at the surge wheel. The loads are pulled up the gradient.

   (Ans. 30 KW)

3. Calculate the motor power of an electric endless rope haulage to haul 350 tonnes in 7 hrs. effective hauling time, up an incline 1200 m long having a gradient of 1 in 12. An empty tub weighs 500 kgf and its carrying capacity is 1000 kgf of coal.

   (Ans. 31.57 KW)

4. A locomotive weighing 15 tef travels round a curve of 80 m radius at a speed of 30 kmh. If the gause is 1 mtr. What should be the super elevation of outer rail over the inner rail, so that there is no thrust between the flanges of the outer wheels and the outer rails.

   (Ans. 88.5 mm)

5. If the r.m.s. torque for a winder is 109 KNm, the winding drum (cylindrical) dia. is 4.4 m and the maximum rope speed is 70 m/s, calculate the power of the motor, assume 95% efficiency.

   (Ans. 364 KW)

6. A winder installation has the following details :

   h = depth of the shaft = 300 m.

   h' = height of the headgear pulle = 20 m

   W_{cs} = weight of the cage and suspension gear = 5000 kgf.

   W_ℓ = weight of the loaded mine cars in the cage = 5000 kgf.

   d = rope diameter = 40 mm.

   If the weight of the rope is 0.36 kgf/m and the breaking strength is 52 d^2KN, d being the rope diameter in cms, calculate the static factor of safety of the winding rope.

   (Ans. 7) **GATE-99**

7. A tower mounted friction winder hoists mineral from a depth of 294 m, with a cycle time of 58 s. The constant acceleration and deceleration values are 0.6 m/s^2 and 0.75 m/s^2 respectively. Assuming a standard 3-period speed time diagram determine the maximum hoisting speed and the distances travelled during the accelaration and deceleration periods.

   (Ans. 13.46, 10.77 m) **GATE-98**

8. Calculate the torques at different stages for a tower mounted friction winder with the following data.

Loaded skip wt.	= 8 tef
Empty skip wt.	= 4.5 tef

Scanned by CamScanner

Rope wt.	= 5.78 kgf/m length
Friction drum dia.	= 2 m
Acceleration time	= 16 sec.
Constant speed time	= 30 sec.
Retardation speed time	= 10 sec.
Decking time	= 15 sec.
Maximum rope speed	= 8.15 m/s
Shaft depth	= 350 m.
Tower height	= 30 m.
Bottom rope loop	= 10 m.

M.I. (of the friction pulley) and the motor is 24 te m^2.

Static torque due to friction 0.08 times the torque due to loaded skip plus empty skip.

9. An incline mine has to be equipped to produce 600 te/day, the average gradient of the seam is 1 in 20, the length of the haulage plane is 2000 metres. Given rope friction is 1/20 and tub friction is 1/50, tub capacity is 1 tone and the tare is 1/2 of the coal it carries. Speed of the rope is 4 km/hour. Net working time of the haulage is 6 hours/shift. Static factor of safety of the rope is 6. Ultimate tensile strength of the rope is 160 kgf/min. Find out the h.p. of the haulage.
10. Calculate the size of the rope and the dia. meter of the drum for a direct haulage to haul 500 te in 6 hrs. effective hauling time up an incline 1000 m long having a gradient of 1 in 6. Assume each tub to weigh 0.5 tonne and to carry 1.0 tone of coal. Speed of haulage 12 km/h.
11. Determine h.p. of a motor to a operate a direct haulage, length of plane is 1000 m dipping 1 in 5. no. of coal tubs hauled up in sets of 10, each tub has a tare wt. of 500 kgf and its capacity is 1000 kgf. Diameter of rope is 18 mm. Also find out the weight, round stranded with fibre core and the diameter of the haulage drum. Speed of the rope is 12 km/h.
12. A winding engine hoists per wind 3 tef of pay load of copper ore in 2 mine cars up a vertical shaft, 600 m deep. As the loaded cage comes up, the empty cage with 2 mine cars goes down. Cage used has 2 deeks, each deck accomodating 1 mine car of tare 0.75 tef. The duty cycle consists of acceleration 10 sec. constant speed 30 sec, decelaration 10 sec, deeking period 10 sec. The length of the rope is 36 m from top cage to drum when decking also there is dead rope of 40 m always on each side of the drum. Weight of the winding rope is 5.59 kgf/m length. The diameter of headgear pulleys are 4.2 m and each weighs 2 tef. Calculate the torques at different stages of winding for cylinderical drum

(1) without tail rope (2) with tail rope.

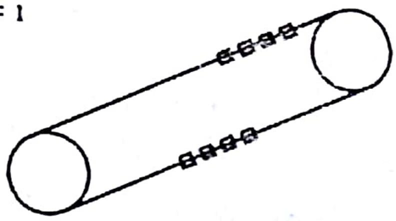
Scanned by CamScanner

SOLUTION

Solution 1.

Let the least gradient = i

$= \frac{1}{n}$



g = gradient resistance of empty tub.

$(F + f_1 + f)$ = frictional force of loaded tubs, empty tubs and the rope.

g_1 = Gradient resistance of rope at the time of start of the rope drawing empty tub.

G = force due to weight of the coal.

Considering number of loaded tubs = bumber of empty tubs

In order that tubs move by itself the condition shown below must be satisfied.

$$G = g + g_i + F + f_1 + f$$

$$\Rightarrow \quad G - g - g_1 = F + f_1 + f$$

$$\Rightarrow \quad (\omega_{ct}\omega_t).i - \omega_t .i - \omega_r .i = F + f_1 + f$$

$$\Rightarrow \quad i = \frac{F + f_1 + f}{(\omega_{ct} + \omega_t)} = \frac{F + f_1 + f}{\omega_c - \omega_r}$$

$$= \frac{\left(8 \times 1500 \times 9.81 \times \frac{1}{50}\right) + \left(8 \times 500 \times 9.81 \times \frac{1}{50}\right) + (500 \times 2) \times i \times 9.81 \times \frac{1}{10}}{8 \times 1000 \times 9.81 - 500 \times 1 \times 9.81}$$

$$= \frac{2354.4 + 784.8 + 981}{78480 - 4905}$$

$$= 0.056 = 1 \text{ in } 18$$

In actual condition a steeper gradient will be required to ensure efficient running.

Solution 2.

In 7.5 hours, amount of coal drawn is 600 t

$\therefore$ In 1 hour, amount of coal drawn is 80 t.

80 t of coal = 80 tub.

Now, in 1 hour distance travelled is 4 km. i.e.

over 4000 mtrs. no. of tubs distributed = 80

over 800 mtrs. no. of tubs distributed

$$= \frac{80 \times 800}{4000} = 16 \text{ tubs.}$$

Scanned by CamScanner

The forces which balance each other are :

(i) The wt. of the tubs on either ropoes.

(ii) The wt. of rope on loaded side and on empty side.

The forces to overcome are as follows :

(i) Component in direction of incline due to forces of gravity on coal content

$$= \frac{16 \times 1000 \times 9.81}{15}$$

$$= 10464 \text{ N} \quad \text{...............(i)}$$

(ii) Frictional forces of loads & empties

$$= \frac{(16 \times 9810 \times 1.5) + (16 \times 9810 \times 0.5)}{50}$$

$$= \frac{9810 \times 16 \times 2}{50}$$

$$= 6278.4 \text{ N} \quad \text{.....................(ii)}$$

(iii) Friction of rope of both sides

$$= \frac{3 \times 800 \times 9.81 \times 2}{10}$$

$$= 4708.8 \text{ N} \quad \text{........................(iii)}$$

Total force = (i) + (ii) + (iii)

$$F = 21451.2 \text{ N}$$

Power required

$$P = \text{Force} \times \text{Velocity}$$

$$21451.2 \times \frac{10}{9} = 23.835 \text{ KW}$$

Let us add 25% for the power required during acceleration.

∴ Power required at the surge weel

$$= 1.25 \times 23.835$$

$$= 29.79 \text{ KW}$$

$$= 30 \text{ KW.}$$

Solution 3.

To find the motor power we must know

(a) the tractive force to be exerted by the rope

(b) the speed of the rope.

Let speed of the haulage be 4 km/hr.

$$V = \frac{10}{9} = 1.11 \text{ m/s}$$

Let tub friction $\mu_t = \frac{1}{50}$

and rope friction $\mu_r = \frac{1}{10}$

Coal on rope per hour $= \frac{350}{7}$

$$= 50 \text{ te}$$

Scanned by CamScanner

This is delivered by 4 km of rope

$$\therefore \quad \text{Coal on rope of 1200 m} = \frac{50 \times 1200}{4000}$$

$$= 15 \text{ te}$$

No. of loaded tubs (15) = no. of empty tubs (15)

Size of rope

Maximum tension in the rope is due to forces acting on loaded side, namely :-

(a) (Gradient resistance of the full tubs) + (frictional resistance of full tub) = G + F

(b) (Gradient resistance of rope on load side) + (frictional resistance of ascending part of rope) = $g_1 + f_1$

As part (b) cannot be calculated readily, we therefore neglect this part momentarilly, later on a suitable allowance will be added.

Para (a)

Tension in full rope

$$T = \frac{15 \times 1.5 \times 9810}{12} + \frac{15 \times 1.5 \times 9810}{50}$$

$$= 18393.75 + 4414.5$$

$$= 22808.25 \text{ N}$$

$$= 22.81 \text{ KN}$$

adding 20% extra for part (b)

maximum tension in rope

$$T_{max.} = T \times 1.2$$

$$= 27.372 \text{ KN}$$

Assuming a factor of safety as 7

Breaking strength $= 7 \times 27.372$

$= 191.604$ KN

For a round stranded rope with fibre core

$$B = sd^2 \text{KN}$$

$$191.604 = 50 \text{ KN/cm}^2 \times d^2 \quad (d = \text{dia. in cm.})$$

or $$d = \sqrt{\frac{191.604}{50}}$$

$$= 1.96$$

$$= 2 \text{ cm.}$$

Mass of the rope

$$m(\text{kg/m}) = kd^2$$

$$= 0.35 \times (1.96)^2$$

$$= 1.344 \text{ kg/m.}$$

wt. per meter length $= 1.344 \times 9.81$ N

Scanned by CamScanner

Power of motor

In endless rope haulage the gravity effect of the rope and tubs on the load side and empty side is balanced and thus the tractive force to be exerted by the motor comprises of :

(1) Gradient resistance of coal alone = G_1

(2) Friction of both full and empty tub = F + f

(3) Friction of the whole of the rope = $2f_1$

Tractive force required = $G + F + f + 2f_1$

$$= \frac{15 \times 1.0 \times 9810}{12} + \frac{1.5 \times 1.5 \times 9810}{50} + \frac{15 \times 0.5 \times 9810}{50} + \frac{2 \times 1200 \times 1.344 \times 9.81}{10}$$

= 12262.5 + 441.5 + 1471.5 + 3164.31 N

= 21.313 KN

Power required by the rope = Power of the surge wheel

= Force × Velocity

$$= 21.313 \times \frac{10}{9} \text{ KW}$$

= 23.68 KW.

Considering 75% transmission efficiency

Motor Power = 31.573 KW.

Solution 4.

$$V = 30 \text{ kmph} = \frac{30 \times 1000}{3600} = 8.33 \text{ m/s}$$

$$\text{Superelevation} = \frac{AV^2}{gr}$$

$$= \frac{1 \times 8.33 \times 8.33}{9.8 \times 80}$$

= 0.0885 m

= 88.5 mm

Solution 5.

Power of motor (in KW) = F × V

$$= \left(\frac{109 \text{ KNM}}{2.2}\right) \times 7 \text{ m/s}$$

= 346.8 KW

add 5% extra

Total motor power = 1.05 × 346.8

= 364 KW.

Solution 6.

Total force on rope (maximum)

F_{max} = (wt. of cage and suspension gear)

Scanned by CamScanner

+ (wt. of loaded mine cars)

+ wt. of rope (maximum length)

$$= 5000 \times 9.8 + 5000 \times 9.8 + \frac{0.36 \times 4 \times 4 \times 9.8 \times 320}{100}$$

$$= (10,000 + 1843) \times 9.8$$

$$= 11843 \times 9.8 \text{ N}$$

Breaking strength $= 52 \times (4)^2 \times 10^3$

factor of safety $= \dfrac{52 \times 4 \times 4 \times 1000}{11843 \times 9.8}$

≈ 7

Solution 7.

$h = 294$

$T = t_1 + t_2 + t_3 = 58$ sec.

t_1 = time of acceleration

t_2 = time of constant speed

t_3 = time of deceleration.

$U = at_1 = 0.6\ t_1$

$0.6t_1 = 0.75\ t_3$

$\Rightarrow \quad t_1 = 1.25\ t_3$

$\& \quad t_2 = 58 - (t_3 + 1.25\ t_3)$

$= 58 - 2.25\ t_3$

Distances travelled

in t_1 sec.

$$h_1 = ut_1 + \frac{1}{2}a_1t_1^2$$

$$= \frac{1}{2} \times 0.6 \times (1.25\ t_3)^2$$

$$= 0.469\ t_3^2$$

in t_2 sec.

$$h_2 = ut_2 + \frac{1}{2}a_2t_2^2$$

$$= 0.6t_1 \times t_2 \qquad (\text{as } a_2 = 0)$$

$$= 0.6 \times 1.25\ t_3\ (58 - 2.25\ t_3)$$

$$= 43.5\ t_3 - 1.687\ t_3^2$$

in t_3 sec.

$$h_3 = \frac{1}{2}a_3t_3^2$$

$$= \frac{1}{2} \times 0.75\ t_3^2$$

Scanned by CamScanner

$$= 0.375\, t_3^2$$

$$h = h_1 + h_2 + h_3$$

$$294 = \left(0.469\, t_3^2\right) + \left(43.5\, t_3 - 1.687\, t_3^2\right) + \left(0.375\, t_3^2\right)$$

$$\Rightarrow \quad 294 = -0.843\, t_3^2 + 43.5\, t_3$$

$$\Rightarrow \quad 0.843\, t_3^2 - 43.5\, t_3 + 294 = 0$$

$$\Rightarrow \quad t_3^2 - 51.6\, t_3 + 247.84 = 0$$

$$t_3 = \frac{51.6 \pm \sqrt{(51.6)^2 - 4 \times 247.84}}{2}$$

$$t_3 = \frac{51.6 \pm 40.88}{2}$$

$$t_3 = 5.36 \text{ sec.}$$

$$t_1 = 1.25\, t_3$$

$$= 6.7 \text{ sec.}$$

$$t_2 = t - (t_1 + t_3)$$

$$= 45.94$$

Maximum speed

$$V = at1$$

$$= 0.6 \times 6.7$$

$$= 4.02 \text{ m/sec.}$$

Distance travelled

during acceleration $= \frac{1}{2} a_1 t_1^2$

$$= \frac{1}{2} \times 0.6 \times (6.7)^2$$

$$= 13.46 \text{ m}$$

during deceleration $= \frac{1}{2} a_3 t_3^2$

$$= \frac{1}{2} \times 0.75 \times (5.36)^2$$

$$= 10.77 \text{ m}$$

Solution 6.

The total torque consists of

(A) static torque and

(B) dynamic torque

(A) Static torque is due to :

(i) unbalanced load

Scanned by CamScanner

(ii) due to friction

(i) static torque due to unbalanced load

$= (8 - 4.5) \times 1000 \times 9.81 \times 1m$

$= 34.3$ KNm

(ii) static torque due to friction

$= 0.08 (8 + 4.5) \times 1000 \times 9.81 \times 1m$

$= 9.81$ KNm.

total static torque = 34.3 + 9.8 = 44.11 KN−m.

Since the length of th erope on the empty side and load side is equal there is no unbalanced rope and no rope torque.

(B) dynamic torque

The total equivalent inertia is due to following masses :

loaded skip $= 8 \times 1000$

$= 8000$ kg

empty skip $= 4.5 \times 1000$

$= 4500$ kg

total rope length $= 390 \text{ m} \times 2$

$= 780$ m

mass $= 780 \times 5.78$

$= 4508$ kg

mass of the friction wheel, rotating parts of motor, gearing etc. is

$$= \frac{24 \text{ te m}^2}{1 \text{ te m}^2} = 24000 \text{ kg}$$

Total masses for dynamic torque

$= 8000 + 4500 + 4508 + 24000$

$= 41008$ kg.

T_a, dynamic torque during acceleration

$$= 41008 \times \frac{8.15}{15} \times 1$$

$= 20.88$ KNm

T_r, dynamic torque during retardation

$$= 41008 \times \frac{8.15}{10} \times 1$$

$= 33.42$ KNm

The torque in KNm can now be tabulated as follows :

	T_a	T_c	T_r
static torque →	44.11	44.11	44.11
dynamic torque →	20.88	0	−33.42
Total →	64.99	44.11	10.69

Scanned by CamScanner

Solutiom 12.

Torque diagram for cylindrical drum without tail rope

$$5.59 \text{ kgf/m} = kd^2$$

$$\Rightarrow \quad d(\text{rope dia.}) = \sqrt{\frac{5.59 \times 9.81}{k}} \quad (K = 0.35)$$

$$= 38 \text{ mm}$$

Drum dia $= 120 \times$ rope dia.

$= 120 \times 0.038$

$= 4.4$ m

Let's assume weight = 30 tef

(i) If γ is the constant speed, then distance travelled by cage during constant speed = 30 γm.

Considering acceleration and deceleration as uniform, during acceleration period distance travelled is $\frac{10\gamma}{2}$m and during deceleration period it is $\frac{10\gamma}{2}$m.

$$\text{So,} \quad \frac{10\gamma}{2} + 30\gamma + \frac{10\gamma}{2} = 600$$

$$\Rightarrow \quad \gamma = 15 \text{ m/s}$$

(ii) Torque due to cages, mine cars and mineral is constant through out the wind. It is unbalanced load × drum radius

So, $3 \times 1000 \times 9.81 \times 2.2 = 64.746$ KNm.

(iii) Torque due to unbalanced rope varies throughout the wind. At the start of the wind it is

$$5.59 \times 9.81 \times 600 \times 2.2 = 72.38 \text{ KNm}$$

(iv) During acceleration travel of rope is 75 m. Loaded cage rope of equivalent length is unwond on the empty side. The unbalanced rope at the end of 10 sec. is therefore 600 – 150 = 450 m and the corresponding torque is

$= 450 \times 5.59 \times 9.81 \times 2.2$

$= 54.29$ KNm

(v) At mid point the ropes will balance each other and the torque will be zero. After that, length of ascending rope is smaller than the descending rope and there will be a negative torque gradually increasing in value. At lthe beginning of the retardation period the torque will be :

$450 \times 5.59 \times 9.81 \times 2.2$

$= -54.290$ KNm

and at the end of the wind it will be

$5.59 \times 9.81 \times 600 \times 2.2$

$= -72.386$ KNm

(vi) Friction torque, based on experience is taken to be equal to that exerted by a force which is $\frac{1}{16}$th the weight of the 2 cages and their contents, acting at the drum radius. This will therefore be

$$\frac{(5+5+1.5+1.5+3)1000 \times 9.81}{16} \times 2.2\,\text{Nm}$$

$= 21582$ Nm

$= 21.582$ Nm

(vii) The force required to accelerate the moving masses can be divided into 2 parts.

(a) force required to accelerate masses movng into linear direction, viz. the 2 caes and their contents and the 2 ropes

(b) force required to accelerate the rotating masses, viz. the winding drum and the head gear pulleys.

$$\text{Linear acceleration} = \frac{15\text{m}}{10} = 1.5\ \text{m}/\text{s}^2$$

$F = ma$

$= [(10+3+3)1000 + 2(672 \times 5.59 \times 5.59)] \times 1.5$ N

$= (16000 + 7513) \times 1.5$

$= 35370$ N

$\tau = F \times r$

$= 35270 \times 2.2$

$= 77.594$ KNm

(viii) A similar torque will required for (77.594 KNm) retardation but will be negative in direction.

(ix) Torque necessary to accelerate the rotating masses

= moment of inertia × angular acceleration

= mass × K^2 × d (K = radius of gyration = 0.8 × r normally taken)

$$\alpha = \frac{\omega}{t} = \frac{v/r}{t}$$

$$= \frac{15/2.2}{10}$$

$= 0.682\ \text{rad}/\text{s}^2$

$\tau = 30 \times 1000\,(0.8 \times 2.2)^2 \times 0.682$

$= 63.385$ Knm (neglecting wt. of dead rope)

Torque required to accelerate the two head gear puley is

$(2+2) \times 1000 \times (0.8 \times 2.1)^2 \times 0.682$

$= 7698.4$ Nm

$= 7.698$ KNm

Total dynamic torque $= 63.385 + 7.698$

$= 71.083$ KNm

Having calculated the torque (in KNm) at different stages in the winding cycle of the winder, they are now tabulated

Scanned by CamScanner

Time(s) from start of wind	0	10	10	40	40	50
stage	acceleration		const. speed		retardation	
Load torque	64.75	64.75	64.75	64.75	64.75	64.75
Rope torque	72.38	54.29	54.29	–54.29	–54.29	–72.38
Friction torque	21.58	21.58	21.58	21.58	21.58	21.58
Travelling torque	77.594	77.59	0	0	–77.59	–77.59
Rotating torque	71.08	71.08	0	0	–71.08	–71.08
Net torque	307.39	289.28	140.62	32.04	–116.63	–134.73

(B) For cylindrical drum with tair rope

If a tail rope having the same diameter as the main hoisting rope is used, the conditions and the data remain the same as before with a value of 64.75 KNm. throughout the wind but the torque required to accelerate the travelling masses in linear direction will be increased by that required to accelerate the extra mass of tail rope. The wt. of the tail rope will be 600 × 5.59 = 3354 kgf = 32902 N and the extra force required to accelerate this extra travelling mass of tail rope = 33.54 × 1.5 = 5031 N. The torque on the drum shaft due to extra force is 5031 N × 2.2 m = 11068 Nm. The torque for the rotating masses remains unchanged and the friction torque is assumed to remain unchanged.

So the table becomes

Time(s) from start of wind	0	10	10	40	40	50
stage	acceleration		const. speed		retardation	
Load torque	64.75	64.75	64.75	64.75	64.75	64.75
Rope torque	0	0	0	0	0	0
Friction torque	21.58	21.58	21.58	21.58	21.58	21.58
Travelling torque	88.81	88.81	0	0	–88.81	–88.81
Rotating torque	71.08	71.08	0	0	–71.08	–71.08
Net torque	246.22	246.22	86.33	86.33	–73.56	–73.56

Important : It shows that the effect of tail rope is to reduce the starting torque by 20%.

Scanned by CamScanner

SURVEYING
(Definition, formulii & Concepts)

Surveying : In order to establish relative positions of different points or objects it is necessary to make linear and directional measurements and this process is known as surveying.

Levelling : The art of determining the relative heights or elevations of different points on the surface of the earth with respect to a datum is called levelling. It is the process of determining the positions of points in a vertical plane.

In winder sense surveying and levelling are one operations.

Survey

1. Plane Survey
Effect of the curvature
of earth is not considered

2. Geodatic Survery
effect of the curvature
of earth is considered.

Survey

1. Traingulation Survey
2. Traverse Survey

1. Traingulation Survey : The system of surveying in which all the survey stations selected in the area are connected with survey lines in such a way that they form network of traingles is called traingulation survey.

2. Traverse Survey : The system of surveying in which whole of the area to be surveyed is divided into various traverse without any figure formed by four or more lines is called traverse survey.

Survey

- Chain Survey
- Compass Survey
- Plane table Survey
- Theadolite Survey
- Tacheo-metric Survey
- Photo-graphic Survey
- Arial Survey

In Chain Surveying : In chain surveying the entire area to be surveyed is divided into network of traingles and hence it can be used when the area is comparitively small and is fairly flat. When the area is large and full of details, methods of chain surveying alone are not sufficient and convenient. In such cases it becomes essential to use some sort of instrument which enables angles or directions of the survey lines to be observed. The following are the instruments for such measurements :

Scanned by CamScanner

(a) Instruments for the direct measurements of directions :

(1) Prismatic compass

(2) Surveyor's compass or diai.

(b) Instruments for measurements of angles :

(1) Sextornt

(2) Theodolite

Traversing : There are two types of traversing :

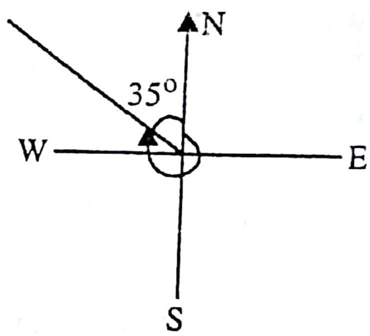
1. **Close traverse :** In close traversing the lines either return to the starting poi thus forming a closed polygon or they finish upon another station having positioned accuracy equal to the starting point.
2. **Open traverse :** It consists of a series of lines which are connected but don return to the starting point or close upon a point of equal or greater occurac

Angles, Bearings and Azimuths

Bearing : The bearings are angles expressed in two different ways :

1. Quadarant bearing (or reduced bearing)

   e.g. bearing showing fig. = N 35° W.



2. Whome circle bearing (or Azimuth)

   Angle taken clockwise from North.

   e.g. Azimuth shown in above fig.

   $= 360^\circ - 35^\circ$

   $= 325^\circ$

More Examples :

Reduced bearing		Azimuth
1. N 35° E	=	35°
2. S 30° E	=	$180^\circ - 30^\circ = 150^\circ$
3. S 40° W	=	$180 + 40^\circ = 220^\circ$
4. N 50° W	=	$360^\circ - 50^\circ = 310^\circ$

Levelling

Levelling is the process of finding the difference in vertical height between tw or more points on earth's surface, or the relative attitudes of a number of points wit respect to a given reference line, termed datum line.

Levelling instruments : (1) A level, (2) A staff

Types of staff : (1) Self reading staff, (2) Target staff

Types of level	Types of levelling
1. Dumpy level	1. Simple levelling.
2. Why level	2. Differential levelling.
3. Cook's reversible level	3. Flying levelling.
4. Cushing's level	4. Check levelling.
5. Tillting level	5. Reciprocal levelling.
6. Autoset level	6. Longitudinal levelling.

Scanned by CamScanner

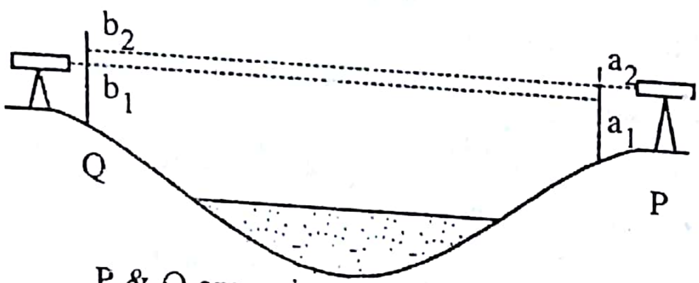
7. Cross levelling.
8. Precise levelling.

The angle of inclination between the lines of sight in radians :

$$\boxed{\alpha = \frac{s}{D} = \frac{n\ell}{R}}$$

α = angle in radians
s = difference between two staff readings
D = distance from the instrument to the staff
n = no. of divisions through which centre of bubble is moved.
ℓ = length of division (one) on the bubble tube (normally 2 mm).
R = radius of curvature of bubble tube.

Reciprocal levelling



P & Q are point where staff is held.

Level difference between P & Q

$$= \frac{(b_1 - a_1) + (a_2 - b_2)}{2}$$

Simson rule

If there are 'n' offsets taken at equal intervals on an irregular boundry line, then the area enclosed by the first and last offset, survey line and the irregular boudry is given by simpson rule :

$$\text{Area} = \frac{W}{3}\{\text{first ordinate} + \text{Last ordinate}$$
$$+ 2 \times (\text{sum of all other odd ordinates})$$
$$+ 4 \times (\text{sum of all even ordinates})$$

Where, W = width of strip (interval)

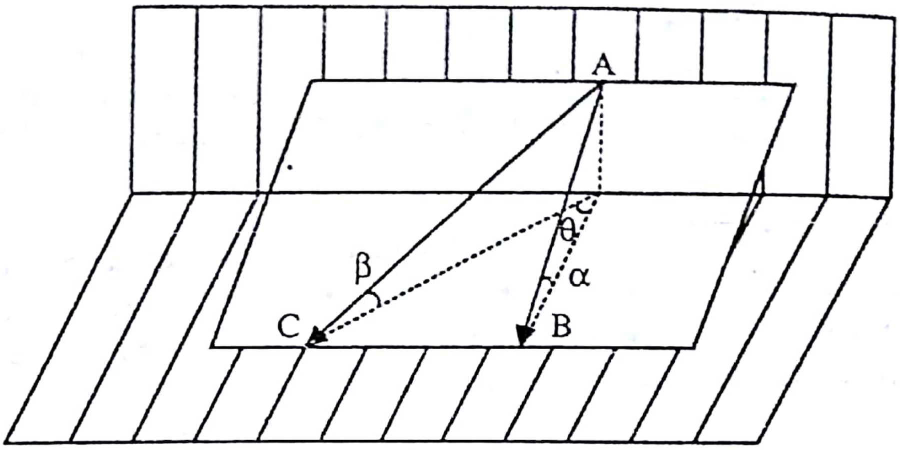
Dip & Fault Problems

The strike line is an inclined seam or bed, is a level line, all points in it having the same altitute. The strike line and direction of full dip are always at right angle to each other.

The full dip also known as true dip, is the maximum inclination of a bed or seam from the horizontal and its direction is always at right angles to the line of strike.

The apparent dip is a dip observed in any direction and it is always less than the true dip.

Scanned by CamScanner

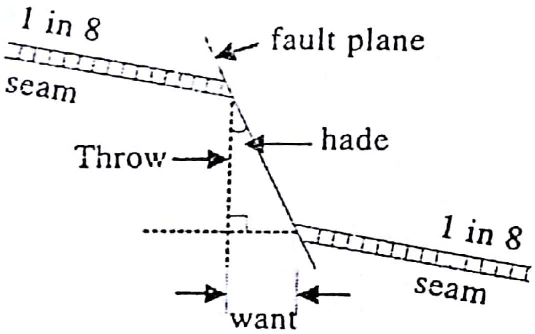


$\boxed{\cot\beta = \cot\alpha \times \sec\theta}$ or $\boxed{\tan\beta = \tan\alpha \times \cos\theta}$

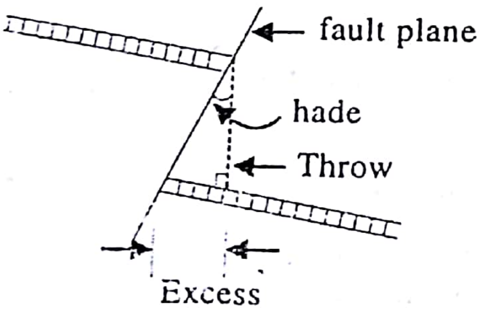
Cot (apparant dip) = Cot (full dip) × Sec (angle between angle of apparent dip and full dips)

Types of faults

1. Normal faults



2. Reverse faults



3. Transcurrent faults

Some definitions

(a) **Fault plane :** The fracture against which the rocks are relatively displayed is called fault plane.

(b) **Direction :** The direction of the strike along the fault plane is known as direction of a fault.

(c) ***Hade :*** The angle between the vertical and the fault plane is known as hade.

(d) ***Throw :*** The amount of vertical displacement of bed is known as the throw.

(e) ***Excess (want) :*** The horizontal displacement of beds is known as want. It is also called shift or heave.

Scanned by CamScanner

Theodolite Surveying

Theodolite is the most precise instrument designed for the measurement of horizontal as well as vertical angles with considerable accuracy.

Types of theodolites

(1) **Transit theodolite** is one in which its telescope can be revolved through complete revolution about its horizontal axis in the vertical plane.

(2) **Non-transit theodolite** is one in which telescope cannot be revolved.

Closing error (e)

If a closed traverse is plotted according to the field measurements the end point of the traverse will not coincide exactly with the starting point, owing to the errors in the field melasurements of angles and distances. Such error is known as closing error.

Balancing the traverse

by Bowditch's method

$$\text{Cor.} = \frac{e}{L} \times S$$

Cor. = Correction to a latitude (or departure)
L = Total length of survey
S = Length of particular draft
e = Total error in sum of lats. (Deps.) with sign changed.

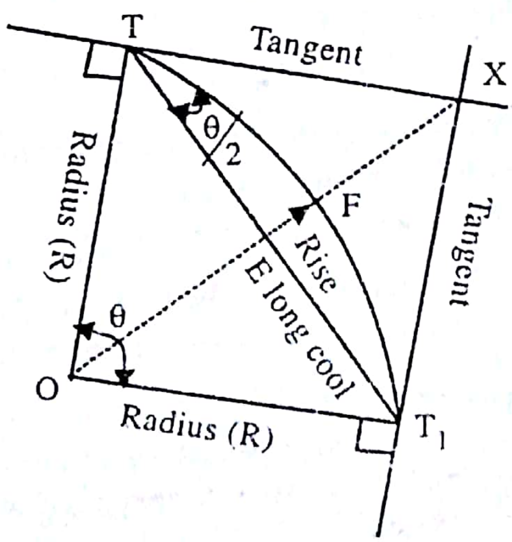
Permissible error

(a) Angular error : $(20 + X)\sqrt{n}$ seconds

n = no. of observing sations
X = 10 seconds or smallest reading in seconds of varnier (least count)

(b) Error of closure : The ratio of closing error and sum of horizontal length of draft should not exceed 1 : 1500

Setting of curves



Scanned by CamScanner

Curve TFT_1

Lenglth of chord (TT_1)

$$\frac{ET}{OT} = \frac{\frac{1}{2}\text{ chort}}{R} = \sin\left(\theta/2\right)$$

$$\Rightarrow \text{chord} = 2R\sin\frac{\theta}{2}$$

Length of Tangent Distance (TX or T_1X)

$$\frac{TX}{OT} = \tan\frac{\theta}{2}$$

$$TX = R\tan\frac{\theta}{2}$$

Rise of the curve (EF)

$$\begin{aligned} EF &= OF - OE \\ &= R - R\cos\frac{\theta}{2} \\ &= R\left(1-\cos\frac{\theta}{2}\right) \end{aligned}$$

Apex distance (XF)

$$\begin{aligned} XF &= OX - OF \\ &= R\sec\frac{\theta}{2} - R \\ &= R\left(\sec\frac{\theta}{2} - 1\right) \end{aligned}$$

Length of curve

$$\frac{\text{Length of curve}}{2\pi R} = \frac{\theta}{360^\circ}$$

$$\text{Length curve} = 2\pi R\,\frac{\theta}{360^\circ}$$

Traingulation, Trilateration & Traversing

Traingulation

Large areas are covered by network of interlacing traingles. Given the three angles and the length of one side of a traingle, the length of the other two sides can be calculated by the application of sine rule.

Trilateration

It is based on the trigonometric proposition that if the three sides of a traingle are known, the three angles can be calculated.

Traversing

Traversing is that type of survey in which the frame work consists of a series of connected lines, the lengths and directions of which are measured with a chain or tape and with one angular instrument respectively.

Traverse is classified as :

1. Closed traverse
2. Open traverse.

Scanned by CamScanner

Correlation

The purpose of correlation survey is to determine with a high degree of exactitude the relative positions of the underground workings and the surface features, so that the underground roadways and faces may be laid down accurately on the working plane.

Mine correlation involves determination of the bearing or azimuth of a line belowground with respect to a true north and also of the co-ordinate of one of the end stations of the line called Reference base.

Tacheometric Surveying

Tacheometry is a branch of angular surveying in which the horizontal and vertical distances of points are obtained by instrumental observation, chaining being thus complelly eliminated.

The instruments usually employed in tacheometry are

1. tacheometer
2. levelling or stadia rod.

A tacheometer in a general sense is a transit theodolite having a stadia telescope i.e. a telscope equipped with two horizontal hairs called stadia hairs.

The instrument has two constants :

1. Multiplying constant $\left(\frac{f}{2}\right)$ usually 100
2. Additive constant (f + d) usually (8 cm. to 60 cm.)

Horizontal distance

$$D = \frac{f}{i} s.\cos^2\theta + (f+d)\cos\theta$$

Vertical distance

$$V = \frac{f}{i} s.\frac{\sin 2\theta}{2}(f+d)\sin\theta$$

For instruments fitted with axallatic lens

$$(f+d) = 0 \ \& \ \frac{f}{i} = 100$$

When readings taken on an inclined staff held perpendicular to line of sight

Horizontal distance

$$D = \frac{f}{i} s.\cos\theta + (f+d)\cos\theta + h\sin\theta$$

Vertical distance

$$V = \frac{f}{i} s\sin\theta + (f+d)\sin\theta$$

Where, h = middle reading of staff.

Scanned by CamScanner

EXAMPLES

Example 1.

Find the angles between lines AB and AC if their respective bearings are

(a) (1) 35^{0} 40') and 142^{0} 45'

(2) 130^{0} 20' and 252^{0} 45'

(b) bearing of line AB = N 85^{0} E

AC = S 45^{0} E

(c) Angle between AO & OB bearing AO = N 60^{0} 30'E bearing OB = S 45^{0} 20' E

Solution :

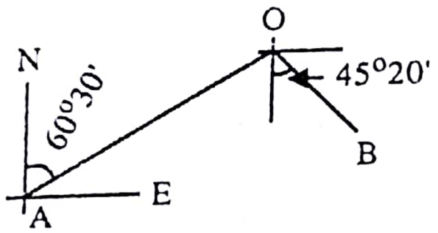
(a) (1) $\angle BAC = 142^{\circ}20' - 35^{\circ}40' = 106^{\circ}40'$

(2) $\angle BAC = 252^{\circ}45' - 130^{\circ}20' = 122^{\circ}25'$

(b) AB = S 45^{0} E = $180^{0} - 45^{0} = 135^{0}$

AC = S 45^{0} E = $180^{0} - 45^{0} = 135^{0}$

$\angle BAC = 135^{\circ} - 85^{\circ} = 50^{\circ}$



(c) Angle between AO & OB

bearing AO = N 60^{0} 30' E

bearing OB = S 45^{0} 20' E

R.B (Reduced bearing) of AO = N 60^{0} 30' E

B.B (back bearing) of AO = S 60^{0} 30' W

or F.B (forward bearing) of OA = S 60^{0} 30' W

F. B of OB = S 45^{0} 20' W

$\therefore \quad \angle AOB = 60^{\circ}\ 30' + 45^{\circ}\ 20'$

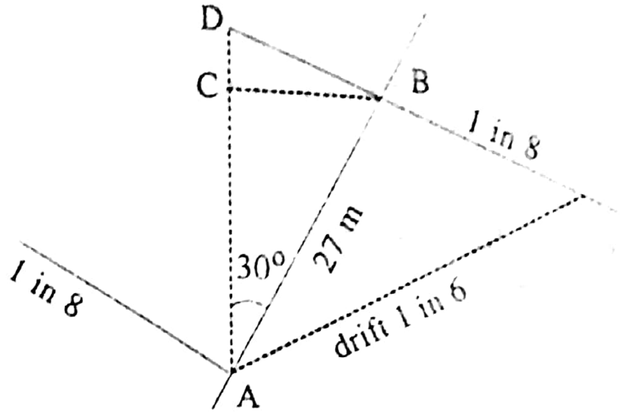
$= 105^{\circ}\ 50'$

Example 2.

A roadway dipping at a gradient of 1 in 8 in the direction of full dip of a seam, strikes an upthrow fault bearing at right angles thereto. The fault is subsequently found to have thrown up the seam a distance of 27 mtrs, measured from floor to floor on the hade of the fault. The head of the fault is 30^{0} from the vertical. Calculate the length of the cross measures drift to win the seam, commencing at the lower side of the fault and rising at 1 in 6 the same direction of the roadway.

Scanned by CamScanner

Solution :



Vertical displacement of the seam by fault

$= AC = AB \cos 30^\circ = 27 \times \cos 30^\circ = 23.382$ m

vertical distance AD $= AC + AD$

$$= 23.382 + \frac{27 \times \sin 30^\circ}{8}$$

$= 23.382 + 1.688$

$= 25.07$ m

Effective grade of seam and drift

$$= \frac{1}{8} + \frac{1}{6} = \frac{7}{24}$$

Plane length of drift

$$= 25.07 \times \frac{24}{7}$$

$= 85.95$ m

Inelined length of drift

$$= 85.95 \times \frac{\sqrt{37}}{6}$$

$= 87.14$ m

Example 3.

A tacheometer was set up at a station P and the following readings were obtained on a vertically held staff :

Station	Staff station	Vertical angle	Hair Readings	Remarks
P	B.M	$-4°22'$	1.050, 1.103, 1.156	R.L of B.M. = 1958.300
	Q	$+10°0'$	0.952, 1.055, 1.158	

The constants of instrument were 100 and 0.1. Find the horizontal distance from P to Q and the reduced level of Q.

Solution :

The horizontal distance to the staff station and the vertical distance of the axial reading above or below the instrument axis may be obtained by :

Scanned by CamScanner

$$D = \frac{f}{i} . s . \cos^2 \theta + (f + d) \cos \theta$$

$$V = \frac{f}{i} . s \sin \frac{2\theta}{2} (f + d) \sin \theta$$

$$\frac{f}{i} = 100 \text{ and } (f + d) = 0.1$$

First observation

$$s_1 = 1.156 - 1.050$$
$$= 0.106$$

$$\theta_1 = -4°22'$$

$$\therefore \quad V_1 = 100 \times 0.106 \frac{\sin(4°22' \times 2)}{2} + 0.1 \times \sin 4°22'$$
$$= 0.812 \text{ m}$$

Second observation

$$s_2 = 1.158 - 0.952$$
$$= 0.206$$

$$Q_2 = 10°$$

$$V_2 = 100 \times 0.206 \frac{\sin 20°}{2} + 0.1 \sin 10°$$
$$= 3.54 \text{ m}$$

$$D_2 = 100 \times 0.206 \cos^2 10° + 0.1 \cos 10°$$
$$= 20.078 \text{ m}$$

Now, R.L of inst. axis = R.L of B.M + Back sight + V_1
= 1958.300 + 1.103 + 0.812 = 1960.215 m
R.L of Q = 1960.215 + 3.54 − 1.055 = 1962.682 m
Distance PQ = 20.78 m.

Example 4.

From a survey line perpendicular offsets are taken at 9.0 m intervals to an irregular boundary line. The offset values recorded in metres are 10.8, 13.9, 18.6, 16.9 20 8, 22.9, 25.5, 23.2 and 19.9. Based on simpson's rule compute the area enclosed by the first and the last offsets, the surveyline and the irregular boundary. **GATE-97**

Solution :

Width of strip = 9 m

Applying simpson rule

Area = $\frac{W}{3}$ (first ordinate + last ordinate + 2 × sum of all other odd ordinates + 4 × sum of all even ordinates)

$$= \frac{9}{3}[10.8 + 19.9 + 2(18.6 + 20.8 + 25.5) + 4(13.9 + 16.9 + 22.9 + 23.2)]$$
$$= 3[30.7 + 2 \times 64.9 + 4 \times 76.9]$$
$$= 3\ [30.7 + 129.8 + 307.6]$$
$$= 3 \times 468.1$$
$$= 1404.3 \text{ m}^2$$

Scanned by CamScanner

EXERCISE

1. The two underground roads AB and BC intersect at B and the deflection angle is 105°. These roads are to be connected by a circular curve of 50 m radius.

   Calculate : (a) Tangent distance (b) main chord
   (c) Length of curve (d) Rise of curve

   (Ans. 195.48, 238, 274.88, 58.68 m) **GATE-99**

2. The full dip of a coal seam is 1 in 6 in direction of S 40° E. A cross measure drift rising at 1 in 7 and bearing S 20° W is to be driven to intersect another parallel coal seam 40 m vertically above. Calculate the length of the drift from floor to floor of the seams. (Ans. 178.63 m) **GATE-98**

3. With the bubble in the centre the reading taken on a staff point 160 m away is 2.30. The bubble is then moved out of centre by two divisions. The new staff reading is 2.24. If the length of one division is 2mm the radius of curvature of the tube is. (Ans. 10.66 m)

4. Find the permissible closing error of an underground traverse having 16 stations measured with 10" theodolite. (Ans. 120 sec.) **GATE-99**

5. If the bubble of a spirit level moves 5 mm for a change of inelination of 25 sec, then the radius of curvature of the spirit level will be how much.

   (Ans. 43.2 m)

6. Two observations were taken to a vertical staff by means of theodolite. In the first observation the angle of elevation is 5° 40' and the staff reading is 0.50. The reduced level (RL) of trunion axis of the theodalite is 150.75. Calculate the RL of the staff station and distance from instrument.

   (Ans. 165.25 m, 200 m) **GATE-96**

7. An instrument is set up at station A and the staff is held vertically at point B. The angle of depression is 8° and the readings on the staff at B are 2.57, 1.73, 0.90. Calculate the RL of B and the distance AB. The height of the instrument is 1.40 m and the reduced level of R = 1000.05 m. The multiplying factor is 100 and (f + d) = 0.15. (Ans. 976.7, 160.5) **GATE-99**

8. Readings taken on a vertical staff held at 'A' were 2.45, 3.10 and 3.75 m. The angle of eleration was 4°. Readings taken on an inclined staff held at B perpendicular to the line of sight were 3.50 m, 3.00 and 2.50 m. The angle of elevation was 5° 30'. If the height of the instrument was 1.70 m and the R.L. of ground was 1285.40, determine the R.L. of A and B. (The instrument multiplying constant is 100). (Ans. 1293.04, 1293.64) **GATE-98**

9. A tacheometer has a multiplying factor of 100 and an additive constant 0.3 m. During one measurement with the telescope being horizontal, the readings of the three diaphram webs on a vertical staff are 1.15 m, 2.30 m and

Scanned by CamScanner

3.45 m. Find the distance from the instrument to the staff and the reduced (RL) of the staff station. Given the RL of the instrument axis is 75.75 m.

(Ans. 73.45 m)

10. The RL of the mouth of shaft A, 200 m deep is 273.5 m and that of shaft B, 150 m deep is 178.5 m. The latitude and departure of the centre of shaft A are N 731.8 and W 259.48 respectively and those of shaft B are N 119.7 and E 273.85. Calculate the gradient of x-drift which connects the bottom of shaft A with that of shaft B.

(Ans. 1:18) **GATE-94**

11. A fault heading at 30° has thrown a cool seam dipping at 1 in 7, up by 20 m. Calculate the length of a drift, rising at 1 in 5, driven to meet the local seam on the other side of the fault. Coal seams have identical conditions on both sides of the fault.

12. The following bearings were taken in running a traverse ABCD :

Line		**Reduced Bearing**
AB	—	N 50° 20' E
BC	—	S 51° 40' E
CD	—	S 20° 30' E
DA	—	N 59° 20' W

Calculate the interior angles and apply the usual cheeks.

(Ans. 70° 20', 102°, 107° 50', 79° 50')

13. In levelling between two point P and Q on the opposite banks of a seam, the level was set up close to P an the staff reading on P and Q were 1.425 and 2.045 respectively. The level was then moved and set up near Q. The readings on the staff at P and Q were 0.74 m and 1.320 m. Find the true difference of level between P and Q. Also determine the RL of peg Q if that of P was 200.050 m.

(Ans. 0.60, 199.450)

14. Two seams of coal are 50 m apart vertically and dip 1 vertical in 5 horizontal. Calculate in meters the length of a cross-measure drift driven to connect them if it dips 1 in 3 in the same directions as the seams. What would be its length if it rises in 1 in 10 towards the dip of the seams.

(Ans. 395.285, 167.49)

15. The full dip of a seam is 1 in 6 in direction S 40° E. A cross measure drift rising at 1 in 7 bearing S 20° W is to be driven to intersect another seam which lies 40 mtrs. vertically above. Calculate the length of the drift from floor to floor of the seams.

(Ans. 178.63)

16. A roadway dipping at gradient of 1 in 8 in the direction of full dip of a seam strikes an upthrow fault bearing at right angles there to. The fault is subsequently found to have thrown up the seam a distance of 27 mtrs. measured from floor to floor on the hade of the fault. The hade of the fault is 30° from the vertical. Calculate the length of the cross-measures drift to win the seam commencing at the lower side of the fault and rising at 1 in 6 in the same direction of the roadways.

(Ans. 87.14)

Scanned by CamScanner

17. A cool seam dips at 1 in 8. A roadway driven to the full dip meets a 15 mtrs. down throw normal fault heading at 30° to the vertical. The roadway is continued on the same bearing and on a slope of 1 in 5 what will be the length of the roadway driven to meet the seam on the other side of the fault.

(Ans. 189.24 m)

18. In a weisbach traingle, the azimuth of a plumb-plane marked by two wires A and B is 115° 23' 49" and C is a theodolite station on the south side of the estern prolongation of AB. Given the following data, calculate the azimuth of the line CD. Illustrate your answer by sketch.

AB = 3.481 m	∠ACD = 179° 14' 33"
BC = 2.674 m	∠BCD = 179° 10' 17"
CA = 6.155 m	

(Ans. 114° 41' 39")

19. The following notes refer to a line levelled tacheometrically with an anallatic tacheometer, the multiplying constant being 100.

Inst. station	Height of axis	Staff station	Vertical Angle	Hair readings	Remarks
P	1.50	B.M	−6°12'	0.963,1.515,2.067	RL of BM
P	1.50	Q	+7°5'	0.819,1.341,1.863	= 460.650m
Q	1.50	R	+12°27'	1.860,2.445,3.030	staff being held vertically

Complete the reduced level of P, Q & R and the horizontal distances PQ and PR.

(Ans. 472.548, 485.484, 509.175, 102.84, 111.54)

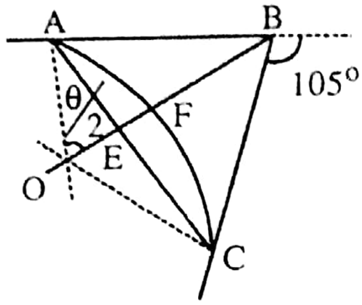
❑

Scanned by CamScanner

SOLUTIONS

Solution 1.

Tangent Distance



$$AB = R \tan\frac{\theta}{2}$$

$$= 150 \times \tan\frac{105^\circ}{2}$$

$$= 150 \times 1.30$$

$$= 195.48 \text{ m}$$

Main chord

$$AC = 2R \sin\frac{\theta}{2}$$

$$= 2 \times 150 \times \sin\left(\frac{105^\circ}{2}\right)$$

$$= 300 \times 0.793$$

$$= 238 \text{ m}$$

Length of curve

$$\overset{\frown}{AC} = 2\pi R \cdot \frac{\theta}{360}$$

$$= 2\pi \times 150 \times \frac{105}{360}$$

$$= 274.88 \text{ m}$$

Rise of the curve

$$EF = R\left(1 - \cos\frac{\theta}{2}\right)$$

$$= 150\left(1 - \cos\frac{105}{2}\right)$$

$$= 58.68 \text{ m}$$

Solution 2.

The rate and direction of dip of both seams are considered to be same.

angle between S 40° E & S 20° W = 60°

Now, apparent dip of the top seam towards the direction of drift :

Cot (apparent dip) = cot (full dip) × sec. (angle between)

= 6 × sec 60° = 12

tan (apparent dip) = 1/12

hence apparent dip of seam in the given direction (S 20° W) is 1 in 12.

Parting between two seams = 40 m.

Since the drift at 1 in 7 and the seam at 1 in 12 (apparent dip) are approaching each other with dips in opposite directions—

Rate of approach $= \frac{1}{7} + \frac{1}{12} = \frac{19}{84}$

Horizontal length of drift

$$= 40 \times \frac{84}{19} = 176.84 \text{ m}$$

Slope length of drift $= 176.84 \times \frac{\sqrt{50}}{7}$

$= 178.63$ m

$\therefore$ The length of drift is 178.63 m.

Solution 3.

$\alpha = \frac{S}{D} = \frac{n\ell}{R}$, α in radian

$S = 2.30 - 2.24$

$= 0.06$ m

$D = 160$ m

$n = 2$

$\ell = 2 \times 10^{-3}$

$$R = \frac{n\ell D}{S} = \frac{2 \times 2 \times 10^{-3} \times 160}{0.06}$$

$$= \frac{2 \times 2 \times 16}{6} = \frac{32}{3}$$

$= 10.66$ m

Solution 4.

Permissible closing error

$= (20 + X)\sqrt{n}$

$= (20 + 10)\sqrt{16}$

$= 30\sqrt{16}$

$= 30 \times 4$

$= 120$ sec.

Solution 5.

$$\alpha = \frac{S}{D} = \frac{n\ell}{R}$$

$$R = \frac{n\ell}{\alpha}$$

$\alpha = 25''$

$$= \frac{25}{60 \times 60} \times \frac{2\pi}{360}$$

$n\ell = 5 \times 10^{-3}$

$$R = \frac{5 \times 10^{-3}}{\left(\frac{2\pi \times 25}{60 \times 60 \times 360}\right)}$$

Scanned by CamScanner

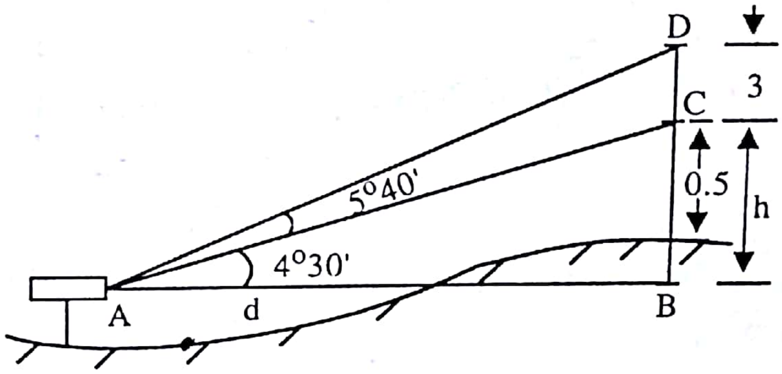
$$= \frac{5 \times 10^{-3} \times 60 \times 60 \times 360}{25 \times 2 \times 3.14}$$

$$= 6 \times 3.6 \times 2$$

$$= 21.6 \times 2$$

$$= 43.2 \text{ m}$$

Solution 6.



ΔABC

$$\frac{h}{d} = \tan\theta \approx \theta$$

$$\frac{h}{d} = \frac{4.5 \times (2 \times 3.14)}{360} \qquad \text{.............(1)}$$

ΔABD

$$\frac{h+3}{d} = \frac{5.66 \times (2 \times 3.14)}{360} \qquad \text{.............(2)}$$

$$\frac{\text{equ. (1)}}{\text{equ. (2)}} = \frac{h}{h+3} = \frac{4.55}{5.66}$$

$$\Rightarrow \quad \frac{h+3}{h} = \frac{5.66}{4.55} = 1.26$$

$$\Rightarrow \quad h + 3 = 1.26\,h$$

$$\Rightarrow \quad h = \frac{3}{.26} = 11.53 \text{ m}$$

from eqn. (1)

$$d = \frac{h \times 360}{4.5 \times (2 \times 3.14)}$$

$$= \frac{11.53 \times 360}{4.5 \times 2 \times 3.14}$$

$$= 146 \text{ m}$$

RL of staff station

= RL of theodolite + (15 – 0.5)

= 150.75 + 14.5

= 165.25 m

Solution 7.

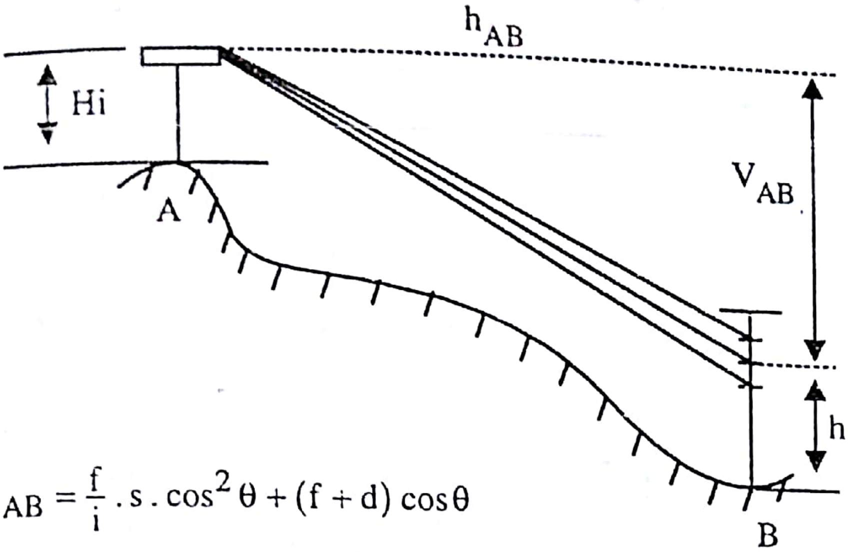
$$\frac{f}{i} = 100, \ (f + d) = 0.15$$

$h = 1.73$ $\qquad$ $S = 2.57 - 0.90$

Scanned by CamScanner

$\theta = -8^{o}$ $= 1.67$

The horizontal distance between A & B (AB)



$$h_{AB} = \frac{f}{i}.s.\cos^2\theta + (f+d)\cos\theta$$

$$= 100 \times 1.67 \times \cos^2(.8^\circ) + 0.15\cos(.8^\circ)$$

$$= (100 \times 1.67 \times 0.95) + (0.15 \times \cos 8^\circ)$$

$$= 160 + 0.15 = 160.15 \text{ m}$$

$$V_{AB} = \frac{f}{i}.s.\frac{\sin 2\theta}{2} + (f+d)\sin\theta$$

$$= 100 \times 1.67 \times \frac{\sin 16^\circ}{2} + 0.1\sin 8^\circ$$

$$= 23.02 \text{ m}$$

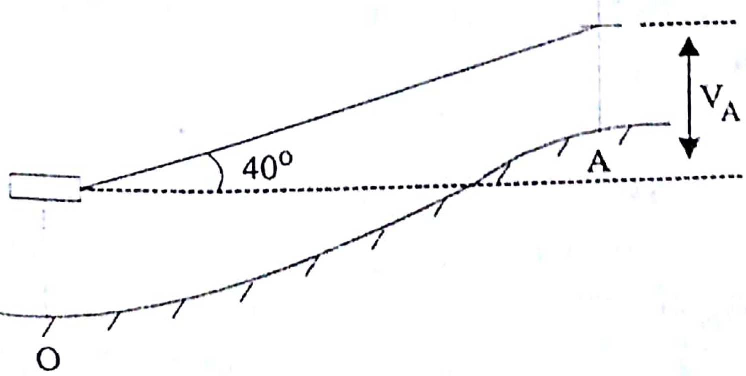
R.L of B

$= \text{RL of A} + 1.40 - V_{AB} - h$
$= 1000.05 + 1.40 - 23.02 - 1.73$
$= 1001.45 - 13.42$
$= 976.7$ m

Distance AB

$$= \sqrt{(160.15)^2 + (11.69)^2}$$
$= 160.5$ m

Solution 8.



RL of A

$$V_A = \frac{f}{i}\, s\, \frac{\sin 2\theta}{2}$$

$$= 100 \times (3.75 - 2.45) \times \frac{\sin 8^\circ}{2}$$

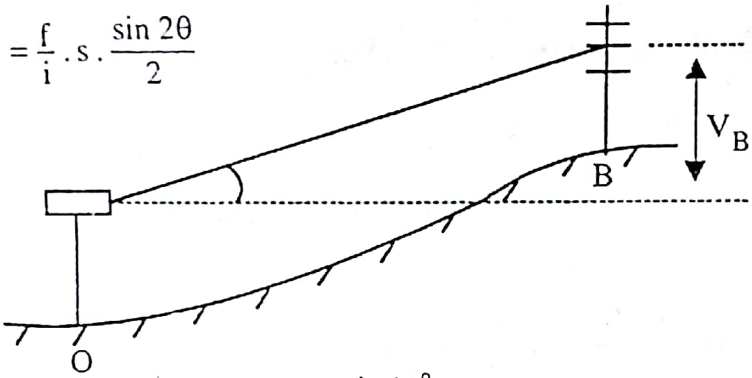
$$= 100 \times 1.3 \times 0.0695$$
$$= 9.046 \text{ m}$$

RL of A $= (1285.40) + (\text{ht. of instrument}) + (V_A) - (\text{axial reading})$

$$= 1285.40 + 1.70 + 9.046 - 3.10$$
$$= 1293.04 \text{ m}$$

RL of B

$$V_B = \frac{f}{i} . s . \frac{\sin 2\theta}{2}$$



$$= 100 \times (3.50 - 2.50) \times \frac{\sin 11^\circ}{2}$$
$$= 9.54 \text{ m}$$

RL of B $= (1285.40) + (\text{ht. of instrument}) + V_B - (\text{axial reading})$

$$= 1285.40 + 1.70 + 9.54 - 3$$
$$= 1293.64 \text{ m}$$

Solution 9.

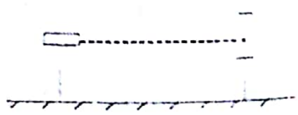
$S = 3.45 - 1.15 = 2.30$

$\frac{f}{i} = 100$

$(f + d) = 0.3$

$\theta = 0^\circ$ (horizontal)

D (distance between instrument & staff)



$$= \frac{f}{i} . S . \cos^2\theta + (f + d)\cos\theta$$
$$= 100 \times 2.3 \times 1 + 0.3$$
$$= 230.3 \text{ m}$$

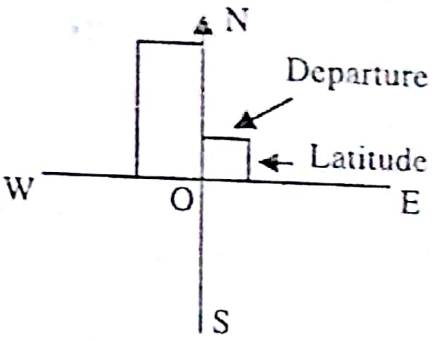
V = 0 (as the instrument telescope is horizontal)

RL of staff station

$$= \text{RL of inst. axis} - \text{axial reading}$$
$$= 75.75 - 2.30$$
$$= 73.45 \text{ m}$$

Scanned by CamScanner

Solution 10.



Distance between

A & B (horizontal)

$$h = \sqrt{(L_A - L_B)^2 + (D_A - D_B)^2}$$

$$= \sqrt{(731.8 - 119.7)^2 + (-259.48 - 273.85)^2}$$

$$= \sqrt{(612.1)^2 + (533.33)^2} = 821$$

RL of A (bottom) $= 273.5 - 200$

$= 73.5$ m

RL of B (bottom) $= 178.5 - 150$

$= 28.50$ m

Level difference between A & B

$V_{AB} = 73.5 - 28.5$

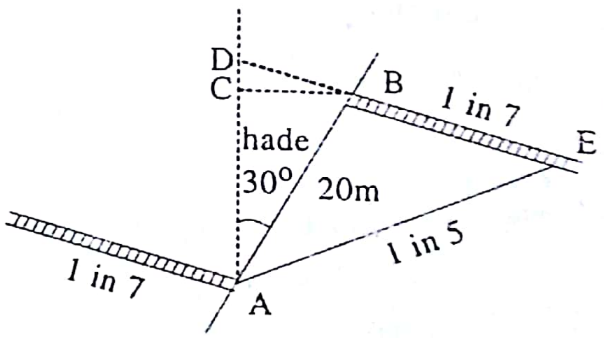
$= 45$

Gradient of X-drift

$$= \frac{V_{AB}}{h} = \frac{45}{821}$$

$= 1 : 18$

Solution 11.



$AC = AB \cos 30^\circ$

$$= 20 \times \frac{\sqrt{3}}{2}$$

$= 17.3$ m

$$CD = \frac{CB}{7} = \frac{AB \sin 30^\circ}{7}$$

$$= \frac{20}{7} \times \frac{1}{2}$$

$= 1.43$ m

Vertical distance

$AD = AC + CD$

$= 17.3 + 1.43$

$= 18.73$ m

Effective grade of seam and drift

$$= \frac{1}{7} + \frac{1}{5}$$

$$= \frac{12}{35}$$

Plane length of drift

$$= 18.73 \times \frac{35}{12} \approx 56.5 \text{ m}$$

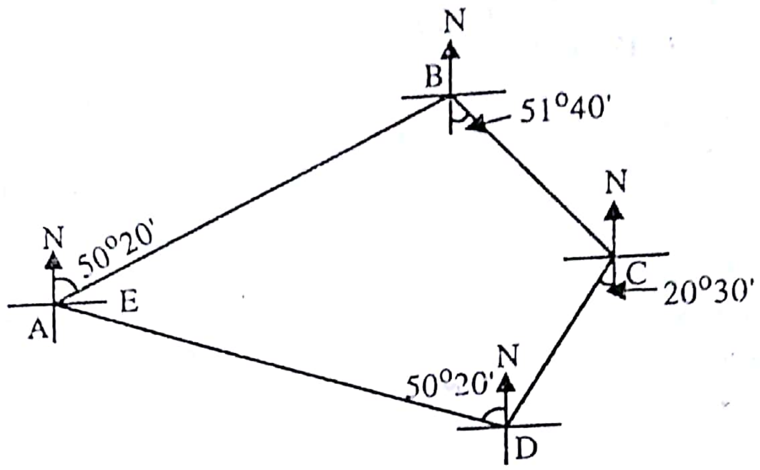
Scanned by CamScanner

Inclined length of drift

$$= 56.5 \times \frac{\sqrt{26}}{5}$$

$$\approx 57.5 \text{ m}$$

Solution 12.



$$\angle A = (90^\circ - 50^\circ + 20') + (90^\circ - 59^\circ 20')$$

$$= 70^\circ 20'$$

$$\angle B = 50^\circ 20' + 51^\circ 40'$$

$$= 102^\circ$$

$$\angle C = (90^\circ - 51^\circ 40') + (90^\circ - 20^\circ 30')$$

$$= 107^\circ 50'$$

$$\angle D = 50^\circ 20' + 20^\circ 30'$$

$$= 79^\circ 50'$$

Check

The sum of all interior angles

$$= (n-2) \times 180$$

$$= 360^\circ \quad \text{.........(1)}$$

$$\angle A + \angle B + \angle C + \angle D = 360^\circ \quad \text{.........(2)}$$

as (1) & (2) both are equal

checked.

Solution 13.

Level at P :

Apparant difference of level between P & Q

$= 2.045 - 1.425 = 0.620$ m (fall from P)

Level at Q :

Apparant difference of level between P and Q

$= 1.320 - 0.740$

$= 0.580$ (fall from P)

Scanned by CamScanner

True difference of level between P and Q.

$$= \frac{0.620 + 0.580}{2}$$
$$= 0.600 \text{ m}$$

The RL of P = 200.050 m

RL of Q = 200.050 − 0.600

= 199.450 m

Solution 14.

Drift dipping 1 in 3 : In this case the drift and the seam dip in the same direction and

$$\text{Rate of approach} = \frac{1}{3} - \frac{1}{5}$$
$$= \frac{2}{15}$$

∴ horizontal length of the drift

$$= 50 \times \frac{15}{2}$$
$$= 375 \text{ m}$$

But the drift dip at 1 in 3

∴ Length of drift

$$= 375 \times \frac{\sqrt{10}}{3}$$
$$= 395.285 \text{ m}$$

Drift rising 1 in 10 : In this case the drift and the seam dip in opposite directions and rate of approach

$$= \frac{1}{5} + \frac{1}{10}$$
$$= \frac{3}{10}$$

∴ horizontal length of drift

$$50 \times \frac{10}{3} = \frac{500}{3} \text{ m}$$

But the drift rises at 1 in 10

∴ Length of drift

$$= \frac{500}{3} \times \frac{\sqrt{101}}{10}$$
$$= 167.48 \text{ m.}$$

Solution 15.

The rate and directions of both the seams are considered to be same.

The drift is rising at 1 in 7 in a direction S 20° W

∴ The azimuth of drift = (180° + 20°)

= 200°

The seam is dipping at 1 in 6 in direction S 40° E

∴ Azimuth of full dip of seam = 180° − 40°

= 140°

Scanned by CamScanner

Now, apprant dip of the top seam towards the direction of the drift.

Cot (apparent dip) = cot (full dip) × sec. (angle between)

$$= 6 \times \sec\left(200^\circ - 140^\circ\right)$$

$$= 6 \sec 60^\circ$$

$$= 12$$

Hence apparant dip of seam = 1 in 12

Parting between two seams = 40 m

Since the drift at 1 in 7 and the seams at 1 in 12 (appaent dip) are approaching each other with dips in opposite directions.

$$\text{Rate of approach} = \frac{1}{7} + \frac{1}{12}$$

$$= \frac{19}{84}$$

Horizontal length of drift

$$= 40 \times \frac{84}{19}$$

$$= 176.84 \text{m}$$

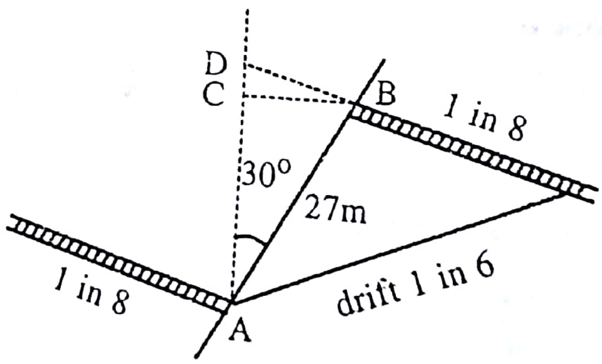
Slope length of drift

$$= 176.84 \times \frac{\sqrt{50}}{7}$$

$$= 178.63 \text{ m}$$

∴ The length of the drift = 178.63 m

Solution 16.



Vertical displaement of the seam by fault

$$AC = AB \cos 30^\circ$$

$$= 27 \times \cos 30^\circ$$

$$= 23.382 \text{ m}$$

Vertical distance AD

$$= AC + CD$$

$$= 23.382 + \frac{27 \times \sin 30^\circ}{8}$$

$$= 23.382 + 1.688$$

$$= 25.07 \text{ m.}$$

Effective grade of seam and drift

$$= \frac{1}{8} + \frac{1}{6} = \frac{7}{24}$$

Scanned by CamScanner

Plane length of drift

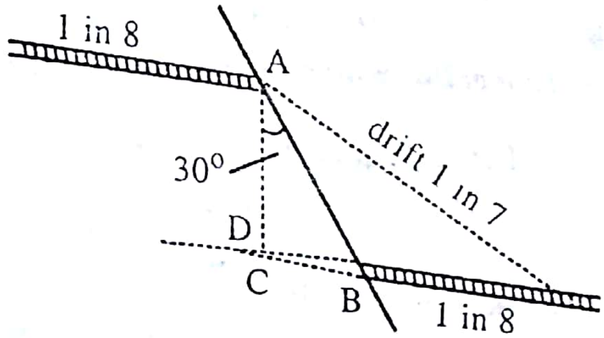
$$= 25.07 \times \frac{24}{7}$$

$$= 85.95 \text{ m}$$

Inclined length of drift

$$= 85.95 \frac{\sqrt{37}}{6} = 87.11 \text{ m}$$

Solution 17.



Width of the fault

$$BC = 15 \tan 3^o$$

$$= 8.660 \text{ m}$$

$$\therefore \quad CD = \frac{8.660}{8}$$

$$= 1.083$$

Vertical distance AD

$$= AC - CD$$

$$= 15 - 1.083$$

$$= 13.917 \text{ m}$$

Now the rate of approach of AE

$$= 13.917 \times \frac{40}{3}$$

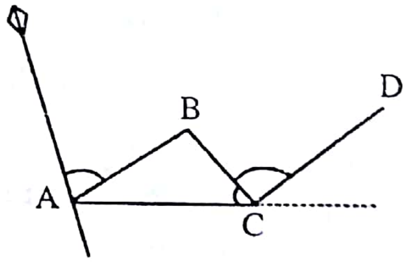
$$= 185.56$$

$\therefore$ Inclined length of drift

$$= 185.56 \times \frac{\sqrt{26}}{5}$$

$$= 189.241 \text{ m}$$

Solution 18.



In taingle ABC

Angle ACB

$$= 179^o\ 14'\ 33'' - 179^o\ 17''$$

$$= 4'\ 16''$$

As the sides of a very small angles are proportional to the angles themselves,

hence $\angle BAC = \frac{BC \times \angle ACB}{AB}$

$$= \frac{2.67 \times 4'16''}{3.481}$$

$$= 3'17''$$

Azimuth $AC = 115^o\ 23'\ 40'' + 0^o\ 3'\ 17''$

$= 115^o\ 27'\ 06''$

Azimuth $CA = 115^o\ 27'\ 06'' + 180^o\ 00'\ 00''$

$= 295^o\ 75'\ 06''$

Azimuth CD = Azimuth of CA + Angle ACD

$= 295^o\ 27'\ 06'' + 179^o\ 14'\ 33''$

Scanned by CamScanner

$= 474^\circ 41' 39'' - 360^\circ$
$= 114^\circ 41' 39''$

Solution 19.

Since the instrument is fitted with anallatic lense, the additive constant (f + c) = 0, the multiplying constant $\left(\frac{f}{2}\right) = 100$

(1) Distance to the staff station

$$D = \frac{f}{i}.s.\cos^2\theta$$

Shaft intercept (S) $= 1.863 - 0.819$
$= 1.044$

$$\theta = 7°5'$$

$$PQ = 100 \times 1.044 \cos^2 7°5'$$
$$= 102.84 \text{ m}$$

Staff intercept $= 3.03 - 1.86$
$= 1.17$

$$\theta = 12°27'$$

$$QR = 100 \times 1.17 \cos^2 12°97'$$
$$= 111.54 \text{ m}$$

(ii) Vertical distance of the axial reading above or below the inst. axis—

$$= \frac{f}{i}.s.\frac{\sin 2\theta}{2}$$

$$S_1 = (2.067 - 0.963)$$
$$= 1.104$$
$$\theta = -6°12'$$

$$V_1 = 100 \times 1.104 \frac{\sin 12°24'}{2}$$
$$= 11.853 \text{ m}$$

$$S_2 = (1.863 - 0.819)$$
$$= 1.044$$
$$\theta = 7°5'$$

$$V_2 = 100 \times 1.044 \frac{\sin 14°10'}{2}$$
$$= 12.777 \text{ m}$$

$$S_3 = 3.03 - 1.86$$
$$= 1.17$$
$$\theta = 12°27'$$

$$V_3 = 100 \times 1.17 \frac{\sin 24°54'}{2}$$
$$= 24.636 \text{ m}$$

Where RL of inst. axis at P

$= 460.650 + 1.1515 + 11.883$

Scanned by CamScanner

$= 474.048$ m

RL of P $= 474.048 - 1.500$

$= 472.548$ m

RL of Q $= 474.048 + 12.777 - 1.341$

$= 485.484$ m

RL of inst. axis a to Q

$= 485.484 + 1.500$

$= 486.984$ m

RL of R $= 486.984 + 24.636 - 2.445$

$= 509.175$ m

Scanned by CamScanner

ROCK MECHANICS

Point load test (for uniaxial compressive strength)

Point load Index

$I_S = \frac{P}{D^2}$, P = load required to break the specimen (in N)

D = diamenter of core (in m)

(Length of core ≥ 1.5 D)

Uniaxial compressive strength

$\sigma_c = (14 + 0.175\, D)\, I_S$

D = diameter in mm

I_S = point load Index

Usual values

$\sigma_c = 50$ to $900\ MP_a$

$I_S = 5$ to $50\ MP_a$

Young's Modules (E) $= 100\ GP_a$

Shear Modules (G) $= 40\ GP_a$ (Giga Pascal)

RQD (Rock Quality Designation)

RQD is defined as the percentage of core removed in intact pieces of 100 mm or more in length in the total length of a borehole.

$$RQD(\%) = 100 \times \frac{\text{Length of core inpieces} > 100\text{ mm}}{\text{Length of borehole}}$$

Schimidt Rebound Hammer Test

$$\text{Log}\,\sigma_c = 0.00874.\rho.R + 2.007\ kg/cm^2$$

where, ρ = density of rock

R = rebound no.

Tensile Strength (Brazilian Test)

$\sigma_t = \frac{2F}{2\pi.r.h}$, F = compressive load at which specimen failed.

r = radius

h = height of specimen.

Scanned by CamScanner

Influence of specimen size on strength of rock

$$\frac{\sigma_c}{\sigma_{c50}} = \left(\frac{50}{d}\right)^{0.18}$$

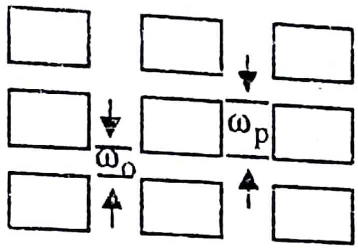
σ_c = Uniaxial compressive strength of specimen of 'd' diameter.

σ_{c50} = Uniaxial compressive strength of a specimen of 50mm diameter

d = diameter of the specimen in mm.

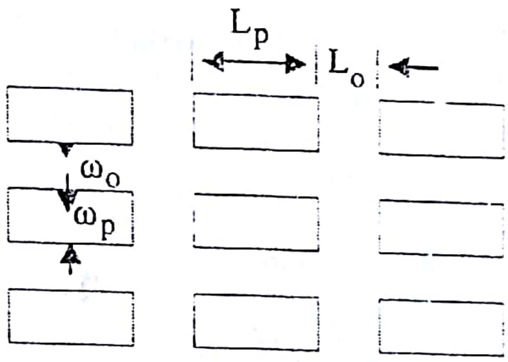
Average Pillar Stress

Square Pillars



$$\sigma_c = r\,.\,Z\,.\left(1 + \frac{\omega_o}{\omega_p}\right)^2$$

Rectangular Pillars



$$\sigma_p = r\,.\,Z\,.\left(1 + \frac{\omega_o}{\omega_p}\right)\left(1 + \frac{L_o}{L_p}\right)$$

where,

r = unit wt. of rock/m

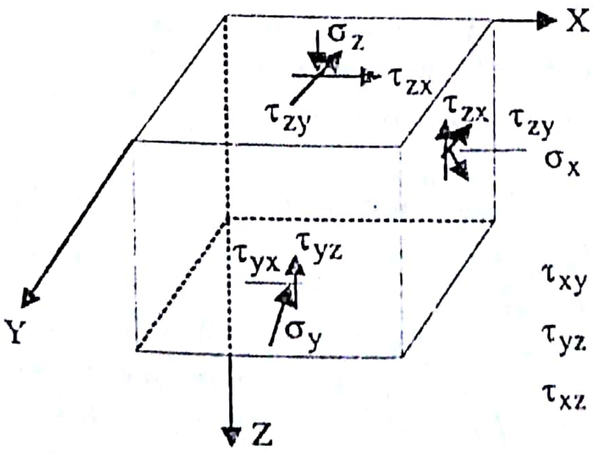
Z = depth of cover (in m)

Stress around underground axcavations

σ = normal or direct stress $\left[\frac{F(\text{normal})}{A}\right]$

τ = shear stress $\left[\frac{\text{foxe}(\tan\text{gential})}{A}\right]$

$\sigma_x, \sigma_y, \sigma_z, \tau_{xy}, \tau_{yz}$ and τ_{zx} are known as components of stress at a point. The value of 6 (six) components of stress at a point will vary with orientation



$\tau_{xy} = \tau_{yx}$,

$\tau_{yz} = \tau_{zy}$

$\tau_{xz} = \tau_{zx}$

Scanned by CamScanner

of the axes to which they are reffered. Whatever the state of stress at a point, it is always possible to find a particular orientation of the co-ordinate oxces for which all shear stress components vanish. These axes are called principal axes of stress and stresses are called principal stresses and denoted by σ_1, σ_2 & σ_3 $(\tau_{xy}, \tau_{yz}$ & $\tau_{zx} = 0)$

$\sigma_1 \geq \sigma_2 \geq \sigma_3$. σ_1 = Major principal stress

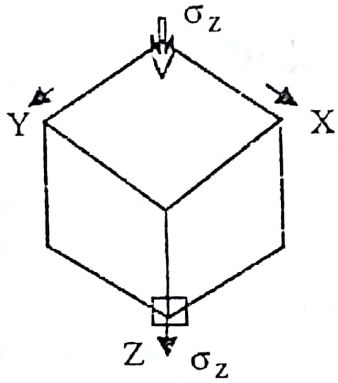
σ_2 = Intermediate principal stress

σ_3 = Minor principal stress.

Stress on a solid body (Plane stress cordition)

Consider a cube of rock loaded vertically by an average axial stress σ_z as shown $(\sigma_y = \tau_{yx} = 0)$

Lateral Strain



$$\varepsilon_x = \frac{1}{E}(\sigma_x - \upsilon\sigma_2)$$

$$\varepsilon_z = \frac{1}{E}(\sigma_z - \upsilon\sigma_x)$$

$$\varepsilon_y = \frac{\upsilon}{E}(\sigma_x + \sigma_z)$$

where, υ = poisson's ratio

Shear Strain

$$\gamma_{xz} = \frac{\tau_{xz}}{G} \qquad \gamma = \frac{\text{lateral strain}}{\text{longitudinal strain}}$$

$$\gamma_{xz} = \frac{2(1+\upsilon)}{E}\tau_{xz} \qquad \left\{\text{As } G = \frac{E}{2(1+\upsilon)}\right\}$$

Usual values

G (shear modulus) $= 40 \times 10^3$ MPa

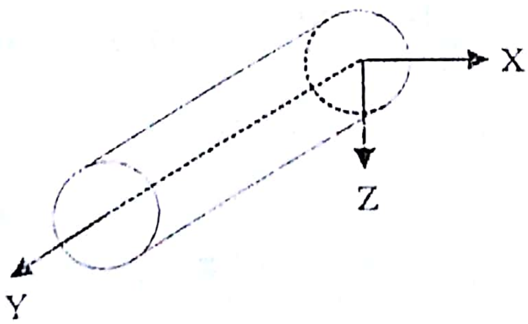
also called modulus of rigidity

E = young's modulus $= 35 \times 10^3$ to 105×10^3 MPa

υ = Poisson's ration = (0.15 to 0.30)

Stress on an excavation (Plane strain condition)

Let a tunnel of arbitrary but constant cross-section be driven parallel to y-axis



Stress and strain induced by creation of an excavation

Scanned by CamScanner

$$\varepsilon_x = \frac{1}{E'}(\sigma_x - \upsilon'\sigma_z)$$

$$\varepsilon_z = \frac{1}{E'}(\sigma_z - \upsilon'\sigma_x)$$

$$\varepsilon_y = 0$$

$$\gamma_{xz} = \frac{\tau_{xz}}{G}$$

$$= \frac{2(1+\upsilon')}{E'} Z_{xz}$$

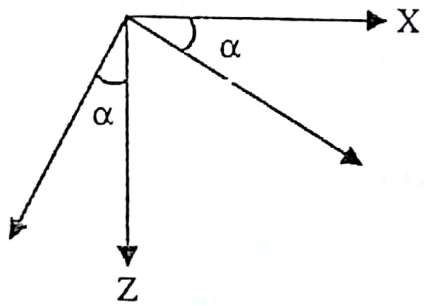
$$E' = \frac{E}{1-\upsilon^2}$$

$$\boxed{\upsilon' = \frac{\upsilon}{1-\upsilon}}$$

Two dimentional stress transformation

Let σ_x, σ_z and τ_{zx} are stress components associated with axes X and Z.

Let us introduce 'n' and 'm' axes which are inclined to x and z axes and associated stress components σ_n, σ_m and τ_{nm}. If α is the angle between 'n' and 'X' axes then



$$\sigma_n = \frac{1}{2}(\sigma_x + \sigma_z) + \frac{1}{2}(\sigma_x + \sigma_z)\cos 2\alpha + \tau_{zx}\sin 2\alpha$$

$$\sigma_m = \frac{1}{2}(\sigma_x + \sigma_z) - \frac{1}{2}(\sigma_x - \sigma_z)\cos 2\alpha - \tau_{zx}\sin 2\alpha$$

$$\tau_{nm} = \tau_{zx}\cos 2\alpha - \frac{1}{2}(\sigma_x - \sigma_z)\sin 2\alpha$$

The magnitude of *corresponding principal stresses* are found by determining value of α at which $\tau_{nm} = 0$ and σ_n and σ_m take maximum and minimum values :

$$\sigma_1 = \frac{1}{2}(\sigma_x + \sigma_z) + \sqrt{\frac{1}{4}(\sigma_x - \sigma_z)^2 + \tau_{zx}^2}$$

$$\sigma_2 = \frac{1}{2}(\sigma_x + \sigma_z) - \sqrt{\frac{1}{4}(\sigma_x - \sigma_z)^2 + \tau_{zx}^2}$$

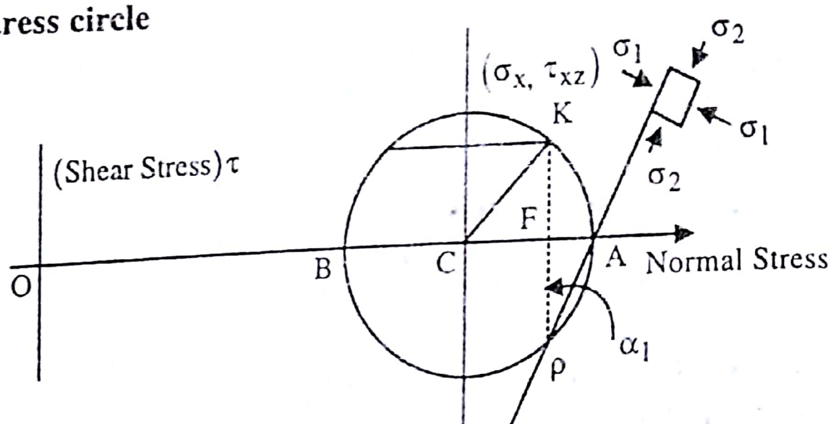
and their directions are

$$\tan 2\alpha_1 = \frac{2\tau_{zx}}{(\sigma_1 - \sigma_z)}$$

$$\alpha_2 = \alpha_1 + 90°$$

Scanned by CamScanner

Mohr's stress circle



$OF = \sigma_x$

$FK = \tau_{xz}$

$OC = \frac{1}{2}(\sigma_x + \sigma_z) \rightarrow$ centre of circle

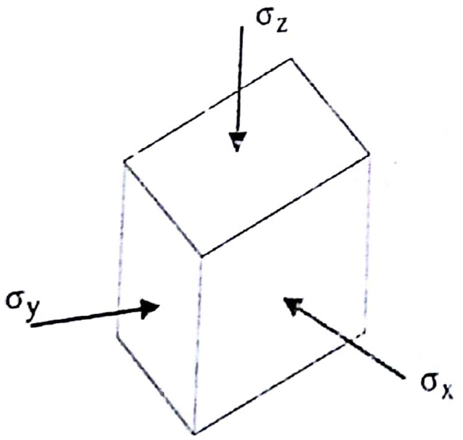
Principal Stress

$$OA = \sigma_1 = \frac{1}{2}(\sigma_x + \sigma_z) + \left[\frac{1}{4}(\sigma_x - \sigma_z)^2 + \tau_{zx}{}^2\right]^{\frac{1}{2}}$$

$$OB = \sigma_2 = \frac{1}{2}(\sigma_x + \sigma_z) - \left[\frac{1}{4}(\sigma_x - \sigma_z)^2 + \tau_{zx}{}^2\right]^{\frac{1}{2}}$$

$$\text{Tan } \alpha_1 = \left(\frac{\sigma_x - \sigma_z}{\tau_{zx}}\right)$$

In situ state of stress (Strain)



Let vertical stress σ_2 and lateral stress σ_x and σ_y then *strain* associated with these stresses are :

$$\varepsilon_z = \frac{1}{E}\left[\sigma_z - \upsilon(\sigma_x + \sigma_y)\right]$$

$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \upsilon(\sigma_y + \sigma_z)\right]$$

$$\varepsilon_y = \frac{1}{E}\left[\sigma_y - \upsilon(\sigma_x + \sigma_z)\right]$$

Vertical stress

$\sigma_z = \gamma . z$ where γ = unit wt. of rock usually 20 to 30 KN/m^3

$= f. g. z$

K

$$\sigma_x = \sigma_y = \frac{\upsilon}{1-\upsilon}\sigma_z \qquad \text{.............(1)}$$

$$\Rightarrow \quad \sigma_x = \frac{\upsilon}{1-\upsilon}\sigma_z$$

Scanned by CamScanner

$$\Rightarrow \quad \frac{\sigma_x}{\sigma_z} = \frac{\upsilon}{1-\upsilon}$$

$$\Rightarrow \quad K = \frac{\sigma_x}{\sigma_z} = \frac{\upsilon}{1-\upsilon}$$

$$\Rightarrow \quad K = \frac{\text{Avg. horizontal stress}}{\text{vertical stress}} = \frac{\upsilon}{1-\upsilon} \qquad \text{..............(2)}$$

There are four possible cases for the stress field existing at a point within the earth :

Case 1. At very shallow depths, close to faults

No confining pressure or restraint

So $\quad \sigma_x = 0$

hence $\quad K = \frac{\sigma_x}{\sigma_z} \qquad\qquad K = 0$

this situation also occurs at a place adjoing a bench face.

Case 2. At moderate depths in elastic rocks

In this case $\upsilon = 0.25$ and no lateral deformation or strain takes place hence

$$K = \frac{\upsilon}{1-\upsilon} = \frac{0.25}{1-0.25}$$

$$K = \frac{1}{3}$$

Case 3. At great depth, or in wet, squeezing and running ground

here, $\quad \sigma_x = \sigma_z$

so, $\quad K = \frac{\sigma_x}{\sigma_z}$

$\quad K = 1$

Case 4. At high lateral pressure due to tectonic forces

here, $\quad \sigma_x > \sigma_z$

$$\Rightarrow \quad \frac{\sigma_x}{\sigma_z} > 1$$

$$\Rightarrow \quad K = \frac{\sigma_x}{\sigma_z} > 1$$

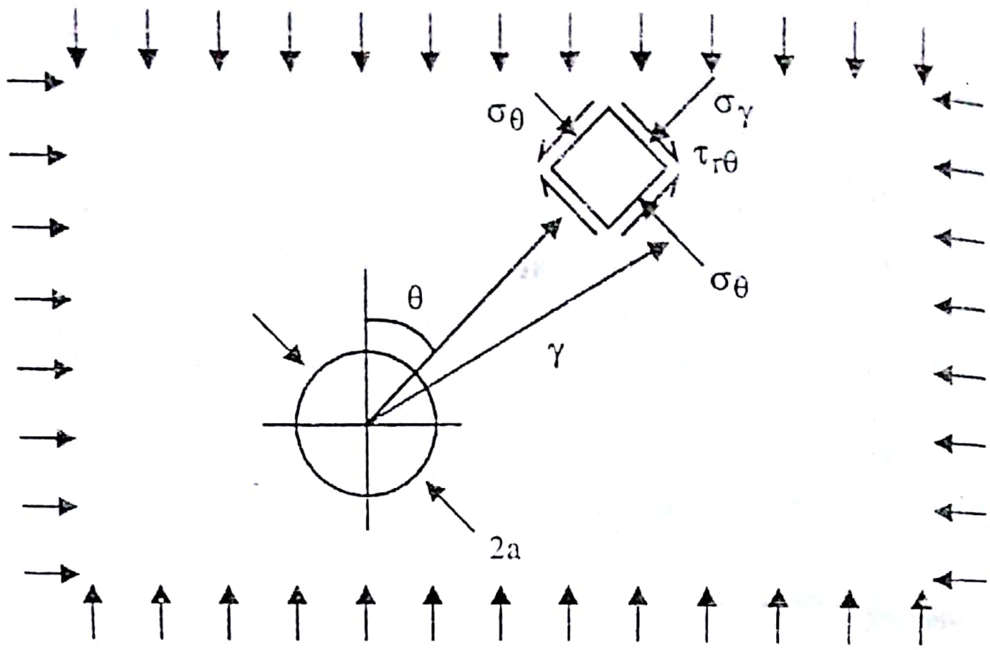
$$\Rightarrow \quad K > 1$$

Stresses at the excavation boundry

Let $\quad P_z$ = vertical stress applied stress compononents at point (r, θ)

radial

$$\sigma_r = \frac{1}{2}P_z\left\{(1+K)\left(1-\frac{a^2}{r^2}\right)+(1-K)\left(\frac{1-4a^2}{r^2+\frac{3a^4}{r^4}}\right)\cos 2\theta\right\}$$



Tangential

$$\sigma_\theta = \frac{1}{2}P_z\left\{(1+k)\left(1+\frac{a^2}{r^r}\right)-(1-k)\left(1+\frac{3a^4}{r^4}\right)\cos 2\theta\right\}$$

$$\tau_{r\theta} = \frac{1}{2}P_z\left\{-(1-k)\left(1+\frac{2a^2}{r^4}-\frac{3a^4}{r^4}\right)\sin 2\theta\right\}$$

(A) Principal Stresses

at point (r, θ)

$$\sigma_1\,(\text{max.}) = \frac{1}{2}(\sigma_r+\sigma_\theta)+\left\{\frac{1}{4}(\sigma_r-\sigma_\theta)^2+\tau_{r\theta}{}^2\right\}^{\frac{1}{2}}$$

$$\sigma_2\,(\text{min.}) = \frac{1}{2}(\sigma_r+\sigma_\theta)-\left\{\frac{1}{4}(\sigma_r-\sigma_\theta)^2+\tau_{r\theta}{}^2\right\}^{\frac{1}{2}}$$

$$\text{Tan}\,2\alpha = \frac{2\tau_{r\theta}}{\sigma_\theta-\sigma_r}$$

(B) Stresses at the boundry of opening

at boundary $r = a$

hence $\sigma_r = 0$

$\tau_{r\theta} = 0$ and

$\sigma_\theta = P_z\{(1+k)+2(1-k)\cos 2\theta\}$

Therefore,

Case 1.

Stress in roof & floor

here, $\theta = 0°$ — roof

$\theta = 180°$ — floor

hence,

$\sigma_\theta = P_z\,(3k-1)$

Scanned by CamScanner

Case 2.

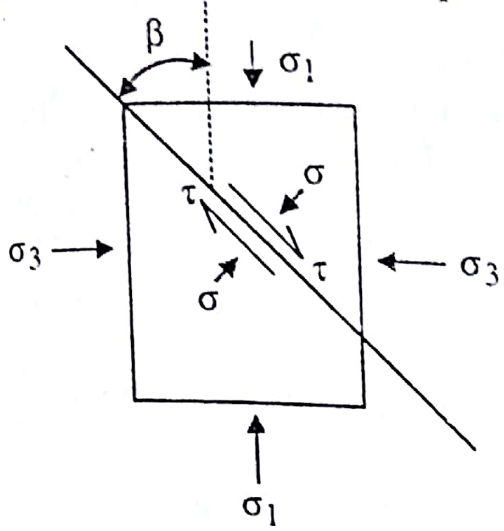
Stress in side walls

here $\theta = 90°$

and $\theta = 270°$

hence $\sigma_\theta = P_z(3-k)$

Relationship between principal stresses at failure



σ_1, σ_2 = principal stresses

σ = normal stress of failure

τ = shear stress of failure

$$\sigma = \frac{1}{2}(\sigma_1 + \sigma_3) - \frac{1}{2}(\sigma_1 - \sigma_3)\cos 2\beta$$

$$\tau = \frac{1}{2}(\sigma_1 - \sigma_3)\sin 2\beta$$

β is angle between the failure surface and the direction of the maximum principal stress σ_1.

Important points about stresses at the excavation boundry.

1. The maximum stresses occur at the boundary of the opening and act tangentially to it; likewise the radial stress at the boundary is zero, and the shear stress is equal to one-half the tangential stress.
2. The stresses are independent of the size of opening (but not of the shape or the side ratio).
3. The stresses are independent of the elastic moduli of the material.
4. The peak values of the maximum tangential stress acting on a mine opening occur at the mid points of the top and sides or at the corners of the opening (if any).
5. The stress redistribution about an opening is negligible at a distance of one diameter from the edge of the hole.

→ We also adopt two conventions. The first is that tension is represented by the (+) sign and compression by (–) sign.

Triaxial compression tensile test

Tensile strength is also determined by this test, which is given by equn.

$$\sigma_3 = \frac{-P\left(de^2 - dc^2\right)}{dc^2}$$

where ρ = hydraulic pressure given

dc = diameter of specimen at center

de = diameter of specimen at end

Scanned by CamScanner

EXAMPLES

Example 1.

A cylindrical specimen of sand stone of 50 mm diameter and 125 mm length, tested in uniaxial compression gave the following results :

Load of failure = 100 KN

Longitudinal deformation upto 1.5 mm

Assuming load deformation curve to be straight line right line upto failure, find uniaxial compressive strength and the modules of elasticity of the sand stone.

GATE-99

Solution :

$$\text{Point load Index } (I_s) = \frac{P}{d^2}$$

$$= \frac{100 \times 10^3}{\left(50 \times 10^{-3}\right)^2} = 40 \times 10^6 \text{ P}_a$$

Uniaxial compressive strength

$$\sigma = (14 + 0.175\ D) \times I_s, \quad \text{where D is in mm}$$

$$= (14 + 0.175D) \times 40 \times 10^6$$

$$= (14 + 0.175 \times 50) \times 40 \times 10^6$$

$$= 910 \times 10^6\ N/m^2$$

$$= 910\ MP_a$$

$$E \text{ (Modules of elastisity)} = \frac{\sigma}{c}$$

$$E = \frac{910\ MP_a}{1.5/125}$$

$$= 75.83\ GP_a$$

Example 2.

A borehole is drilled and instrumented yielding the following values of the stress components in the plane perpendicular to the bore hole :

$$\sigma_x = 1.724\ MP_a$$

$$\sigma_y = 2.76\ MP_a$$

$$\sigma_{xy} = -0.689\ MP_a$$

Find the magnitudes and directions of the major and minor principal stresses on this plane.

GATE-98

Solution :

Major Principal Stress

$$\sigma_1 = \frac{1}{2}\left(\sigma_x + \sigma_y\right) + \left\{\frac{1}{4}\left(\sigma_x - \sigma_y\right)^2 + \tau_{xy}{}^2\right\}^{\frac{1}{2}}$$

Scanned by CamScanner

$$= \frac{1}{2}(1.724 + 2.760) + \left\{\frac{1}{4}(1.724 - 2.76)^2 + (-0.689)^2\right\}^{\frac{1}{2}}$$

$$= \frac{1}{2}(4.484) + \frac{1}{4}\left[(-1.036)^2 + (0.689)^2\right]^{\frac{1}{2}}$$

$$= 2.242 + \left[\frac{1}{4} \times 1 + 0.48\right]^{\frac{1}{2}}$$

$$= 2.242 + [0.25 + 0.48]^{\frac{1}{2}}$$

$$= 2.242 + (0.93)^{\frac{1}{2}}$$

$$= 2.242 + 0.96 = 3.202 \text{ MP}_a$$

Minor Principal Stress

$$\sigma_2 = \frac{1}{2}(\sigma_x + \sigma_y) - \left[\frac{1}{4}(\sigma_x - \sigma_y)^2 + \tau_{xy}{}^2\right]^{\frac{1}{2}}$$

$$= \frac{1}{2}(1.724 + 2.760) + \left\{\frac{1}{4}(1.724 - 2.76)^2 + (0.689)^2\right\}^{\frac{1}{2}}$$

$$= 2.242 - 0.96 = 1.282 \text{ MP}_a$$

xample 3.

The shear stress on a plane on which the major principal stress acts is.

lution :

'Zero'

xample 4.

Following is the state of stress in a biaxial stress field :

$$\sigma_{xx} = 100 \text{ MP}_a$$
$$\sigma_{yy} = 30 \text{ MP}_a$$
$$\sigma_{xy} = 15 \text{ MP}_a$$

Calculate the major principal stress.

olution :

Major principal stress

$$\sigma_1 = \frac{1}{2}(\sigma_{xx} + \sigma_{yy}) + \left[\frac{1}{4}(\sigma_{xx} - \sigma_{yy})^2 + \sigma_{xy}\right]^{\frac{1}{2}}$$

$$= \frac{1}{2}(100 + 30) + \left[\frac{1}{4}(100 - 30)^2 + 15^2\right]^{\frac{1}{2}}$$

$$= 65 + \left[\frac{1}{4} \times 4900 + 225\right]^{\frac{1}{2}}$$

$$= 65 + (1450)^{\frac{1}{2}}$$

$$= 65 + 38$$

$$= 103 \text{ MP}_a$$

Scanned by CamScanner

PROBLEMS

1. The specimen obtained from the overlaying strata in a coal mine were tested in uniaxial compression and the following results were obtained at 50% of average compressive strength

   Longitudinal strain $= 6 \times 10^{-2}$

   Lateral strain $= 1.2 \times 10^{-2}$

   Find the ratio of vertical to horizontal stress in the mine.

   (Ans. 4) **GATE-99**

2. For a circular opening in rock under hydrostatic condition at great depths where will the maximum tangential tensile stress occur ?(Ans. 0^o & 90^o with vertical)

3. A typical hard rock with $E = 100 \times 10^3\ MP_a$ and poisson's ratio, $\upsilon = 0.25$ what will be the modulus of rigidity ? (Ans. 40 GP_a) **GATE-98**

4. The length of consecutive cores obtained from 2m section of a borehole ($N \times$ size) are 89, 32, 18, 172, 80, 255, 280, 176, 53, 302, 102, 153, 40, 147 and 101 mm. Find RQD.

   (Ans. 84.40%)

5. Calculate the radial (σ_r), tangential (σ_θ) and shear $(\gamma_{r\theta})$ stresses on the circumference of a circular opening of 3m diameter at $\theta = 0°$ and $\theta = 90°$. The unit wt. of rock is 0.028 MN/m^2 poisson's ratio is 0.3 and the depth of the escavation is 300 m. (Ans. $\sigma_r = 0, \tau_{r\theta} = 0$

   $$\sigma_\theta(0°) = 2.5 \times 10^6$$

   $$\sigma_\theta(90°) = 21.58 \times 10^6)$$

6. A cool seam 3m thick lying at a depth of 100m has been developed by board and pillar with square pillars of 28 m centre to centre and gallery of 4 m width. The average density of overlaying strata is 24 KN/m^2. If the pillar strength is 4900 KN/m^2 factor of safety of the pillar will be how much?

   GATE-9

   (Ans. 1.5)

Scanned by CamScanner

SOULUTIONS

Solutiuon 1.

$$\upsilon(\text{Poisson's ratio}) = \frac{\varepsilon_{lateral}}{\varepsilon_{logitudinal}} = \frac{1.2}{6}$$

$$= 0.2$$

$$\frac{\sigma_n}{\sigma_\gamma} = \frac{\upsilon}{1-\upsilon}$$

$$= \frac{0.2}{1-0.2}$$

$$= \frac{0.2}{0.8}$$

$$= \frac{1}{4}$$

$$\therefore \quad \frac{\sigma_\gamma}{\sigma_n} = 4$$

Solution 2.

At great depth

$$K = 1$$

As we know that at boundary radial stress and shear stress is zero and tangential stress is given by :

$$\sigma_\theta = P_z\{(1+k) - 2(1-k)\cos 2\theta\}$$

differentiating above equn. w.r.t. θ

$$\frac{d(\sigma_\theta)}{d\theta} = \frac{d}{d\theta}\left[P_z(1+k) - 2P_z(1-k)\cos 2\theta\right]$$

$$\frac{d(\sigma_\theta)}{d\theta} = 0 - 2P_z(1-k)\times(-\sin 2\theta)\times 2$$

$$\frac{d(\sigma_\theta)}{d\theta} = 4P_z(1-k)\sin 2\theta \qquad \text{..............(1)}$$

for maxima or minima

$$\frac{d(\sigma_\theta)}{d\theta} = 0$$

hence $4\rho_z(1-k)\sin 2\theta = 0$

$\Rightarrow$ $\sin 2\theta = 0$

$\Rightarrow$ $2\theta = 0^\circ$ and 180°

$\Rightarrow$ $\theta = 0^\circ$ and 90°

For both the values of 'θ' $\frac{d\sigma_\theta}{d\theta}$ is not +ve. Hence maximum tantential tensile stress will occur at :

(1) 0° with vertical

(2) 90° with vertical on both sides.

Solution 3.

Modules of rigidity (G)

$$G = \frac{E}{2(1+\upsilon)}$$

$$= \frac{100 \times 10^3 \, MP_a}{2(1+0.25)}$$

$$= \frac{100}{2.5} \, GP_a$$

$$= 40 \, GP_a$$

Solution 4.

$$RQD = \frac{100 \times \text{Length of pieces} > 100 \text{ mm}}{\text{Length of bore hole}}$$

$$= \frac{100(172+255+280+176+302+102+153+147+101)}{2000}$$

$$= \frac{1688}{2000} \times 100$$

$$= 84.4\%$$

Solution 5.

For circular opening

$$\sigma_r = \frac{1}{2} P_z \left\{ (1+k)\left(1 - \frac{a^2}{r^2}\right) + (1-k)\left(1 - \frac{4a^2}{r^2} + \frac{3a^4}{r^4} \cos 2\theta\right) \right\}$$

$$\sigma_\theta = \frac{1}{2} P_z \left\{ (1+k)\left(1 - \frac{a^2}{r^2}\right) + (1-k)\left(1 + \frac{3a^4}{r^4}\right) \cos 2\theta \right\}$$

$$\tau_{r\theta} = \frac{1}{2} P_z \left\{ -(1+k)\left(1 + \frac{2a^2}{r^r} - \frac{3a^4}{r^4}\right) \sin 2\theta \right\}$$

when $r = a$

$$\sigma_r = \frac{1}{2} P_z \{(1+k) \times 0 + (1-k) \times 0\} = 0$$

$\sigma_r = 0$ → radial stress

$\tau_{r\theta} = 0$ → shear stress

Tangential stress

$$\sigma_\theta = \frac{1}{2} P_z \{(1+k)\, 2 - (1-k) \times 4 \times \cos 2\theta\}$$

Scanned by CamScanner

$$\sigma_\theta = P_z\left[(1+k)-2(1-k)\cos 2\theta\right]$$

$$= 0.028 \times 10^6 \times 300\left[(1+0.43)-2(1-0.43)\cos 2\theta\right]$$

$$= 8.4 \times 10^6\left[1.43 - 2 \times 0.57 \times \cos 2\theta\right]$$

$$= 8.4 \times 10^6\left[1.43 - 1.14\cos 2\theta\right]$$

when $\theta = 0^\circ$

$$\sigma_0{}^\circ = 8.4 \times 10^6\left[1.43 - 1.14\right] = 8.4 \times 10^6 \times 0.29$$

$$= 2.5 \times 10^6 \ MP_a$$

when $\theta = 90^\circ$

$$\sigma_{90}{}^\circ = 8.4 \times 10^6 (1.43 + 1.14)$$

$$= 8.4 \times 10^6 (2.57)$$

$$= 21.58 \ MN/m^2$$

Solution 6.

For square pillar

Avg. pillar stress

$$\sigma = P_z\left(1 + \frac{\omega_o}{\omega_p}\right)^2$$

$$= r \cdot z\left(1 + \frac{\omega_o}{\omega_p}\right)^2$$

where ω_o (width of opening) = 4m

ω_p (width of pillar) = 28 m

$$\sigma = 24 \times 10^3 \times 100\left(1 + \frac{4}{28}\right)^2$$

$$= 24 \times 10^5 \times 1.3 \ NM^2$$

$$\text{factor of safety} = \frac{\sigma_{strength}}{\sigma_{stress}}$$

$$= \frac{4900 \times 10^3}{2400 \times 10^3 \times 1.3}$$

$$= 1.5 \text{ (approx)}$$

●

Scanned by CamScanner

CASH FLOW CONCEPT

Compound interest

(a) If yearly compounded

$$S = P(1+i)^n$$

(b) If quarterly compounded

$$S = P\left(1+\frac{i}{4}\right)^{4n}$$

where, P = Prinicipal amount

i = rate of interest

n = no. of years

Uniform yearly payment R from a single present investment

$$R = P(1+i)^n \left[\frac{i}{(1+i)^n - 1}\right]$$

or

$$n = \frac{\log\left(\frac{R}{R - P_i}\right)}{\log(1+i)}$$

Net Present value (NPV)

$$NPV = \frac{R_1}{(1+i)} + \frac{R_2}{(1+i)^2} + \ldots\ldots \frac{R_n}{(1+i)^n} + \frac{S}{(1+i)^n}$$

$$NPV = \sum_{t=1}^{n} \frac{R_t + S}{(1+i)^n}$$

where, NPV = present value

$R_1, R_2 \ldots\ldots R_n$ = cash flow a fler taxes

n = life of asset

S = salvage value

IRR (Internal Rate of Return)

The IRR is the rate at which the present value of investment (project) is zero

Present value of cash in flow

= Present value of cash outflow

Net Present Value Index (NPVI)

$$NPVI = \frac{\text{Total present value of cash flow}}{\text{Initial investment}}$$

Profitability Index

$$= \frac{\text{Present values of cash inflow}}{\text{Initial cost outflow}}$$

$$= \sum_{t=1}^{t=n} \frac{R_i}{(1+i)^n}$$

PROBLEMS

1. Find the uniform end of year payment R which can be realised forn years from a single present investment from a single present investment, P, at i rate of interest

$$\left\{ \text{Ans. } R = P(1+i)^n \left[\frac{i}{(1+i)^n - 1} \right] \right\}$$ **GATE-99**

2. A company takes a loan of Rs. 25000 at 8% interest for the extension of a mine. To repay the loan, an additional saving of Rs. 2500 per annum is necessary. The period taken for repayment will be how much? (Ans. 21 years)
3. What will be the effective rate of interest if a borrower offers 16% nominal rate of interest with quarterly compun-ding? (Ans. 17%) **GATE-98**
4. A mining company borrows Rs. 50,000 at 6% interest in order to modernize its plant. A direct saving of Rs. 6,000 per year will result and the plan is to use this sum annually against the loan. How much time it will take to repay the borrowed money? (Ans. 12 years) **GATE-98**
5. What will be the present value of a 4 year annuity of Rs. 10,000 discounted at 10%. (Ans. 31700)
6. Given below is the cash flow of a small project. Calculate NPV (Net Present Value) of the project at a discounted rate of 8% and also find its internal rate of return.

Year	Project cash flow (Rs.)
0	−1,05,000
1	60,000
2	45,000
3	30,000
4	15,000

(Ans. 23,900)

7. For a small mining project an investment of 28.0 million is made and the net cash flow are shown as :

Year	0	1	2	3	4	5	6	7	8
Cash flow	-28.0	6.5	9.5	9.0	8.5	8.0	7.5	7.0	6.5

Find the net present value of project for a discounting rate of 20%.
(Ans. 2.49 mill) **GATE-96**

Scanned by CamScanner

SOLUTIONS

Solution 1.

Value of P after n years

$$S = P(1+i)^n \qquad \ldots\ldots\ldots\ldots(1)$$

Value of R after (n – 1) years

$$R_1 = R(1+i)^{n-1}$$

Similarly,

$$R_2 = R(1+i)^{n-2}$$

....

....

$$R_n = R$$

Hence,

$$S = R_1 + R_2 + R_3 + R_4 + \ldots\ldots\ldots\ldots\ldots\ldots\ldots R_n$$

$$= R(1+i)^{n-1} + R(1+i)^{n-2} + \ldots\ldots\ldots\ldots R$$

$$= R + R(1+i) + R(1+i)^2 + \ldots\ldots\ldots\ldots R(i+i)^{n-1}$$

$$= R\left[1 + (1+i) + (1+i)^2 + \ldots\ldots\ldots\ldots + (1+i)^{n-1}\right]$$

$$= R\left[\frac{1\left\{(1+i)^{n-1+1} - 1\right\}}{(1+i)-1}\right]$$

$$= R\left[\frac{(1+i)^n - 1}{i}\right] \qquad \ldots\ldots\ldots\ldots(2)$$

From equn. (1) and equn. (2)

$$P(1+i)^n = R\left[\frac{(1+i)^n - 1}{i}\right]$$

$$\Rightarrow \qquad R = P(1+i)^n\left[\frac{i}{(1+i)^n - 1}\right]$$

Solution 2.

$$R = P(1+i)^n\left[\frac{i}{(1+i)^n - 1}\right]$$

Scanned by CamScanner

$$\Rightarrow \quad n = \frac{\log\left(\frac{R}{R - P_i}\right)}{\log(1+i)}$$

$$= \frac{\log\left(\frac{2500}{2500 - 2500 \times \frac{8}{100}}\right)}{\log(1+0.08)}$$

$$= \frac{\log 5}{\log(1.08)} = 21 \text{ years}$$

Solution 3.

$$I = \left(1 + \frac{i}{4}\right)^{4n} - 1$$

$$= \left(1 + \frac{0.16}{4}\right)^4 - 1$$

$$= 1.17 - 1$$

$$= 0.17$$

$$= 17\%$$

Solution 4.

$$n = \frac{\log\left(\frac{R}{R - P_i}\right)}{\log(1+i)}$$

$$= \frac{\log\left(\frac{6,000}{6000 - 50000 \times 0.06}\right)}{\log(1+0.06)}$$

$$= \frac{\log 2}{\log 1.06} = 12 \text{ years}$$

Solution 5.

$$NPV = \left(\frac{10,000}{1+0.1}\right) + \frac{10,000}{(1+0.1)^2} + \frac{10,000}{(1+0.1)^3} + \frac{10,000}{(1+0.1)^4}$$

$$= \frac{10,000}{1.1} + \frac{10,000}{1.21} + \frac{10,000}{1.33} + \frac{10,000}{1.46}$$

$$= 9091 + 826.4 + 7518.7 + 6849$$

$$= 31700 \text{ Rs.}$$

Solution 6.

$$V = \frac{60,000}{(1+0.08)} + \frac{40,000}{(1+0.08)^2} + \frac{30,000}{(1+0.08)^3} + \frac{15,000}{(1+0.08)^4}$$

Scanned by CamScanner

$$= 1000\left[\frac{60}{1.08} + \frac{45}{(1.08)^2} + \frac{30}{(1.08)^3} + \frac{15}{(1.08)^4}\right]$$

$$= 1000\left[55.5 + 38.6 + 23.8 + 11.0\right]$$

$$= 12,8,900$$

$$\text{NPV} = -1,05,000 + V$$

$$= -1,05,000 + 1,28,900 = 23,900 \text{ Rs.}$$

Solution 7.

$$\text{NPV} = -28 + \sum_{t=1}^{t=n} \frac{R_t + S}{(1+i)^n} \qquad (S = 0)$$

$$= -28 + \left[\frac{6.5}{1.2} + \frac{9.5}{(1.2)^2} + \frac{9}{(1.2)^3} + \frac{8.5}{(1.2)^4} + \frac{8}{(1.2)^5} + \frac{7.5}{(1.2)^6} + \frac{7.0}{(1.2)^7} + \frac{6.5}{(1.2)^8}\right]$$

$$= -28 + [5.41 + 6.6 + 5.2 + 4.10 + 3.21 + 2.51 + 1.95 + 1.51]$$

$$= -28 + 30.49 = 2.49 \text{ millions.}$$

SAMPLING

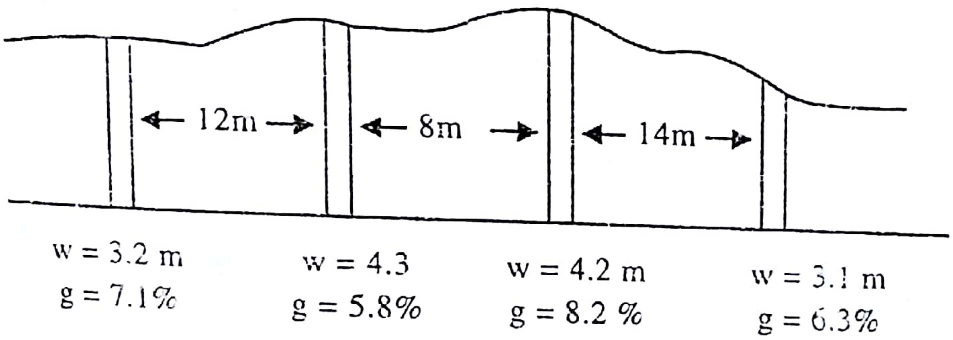
PROBLEMS

1. In an irregularly spaced chip sampling of a vein the following datas are obtained. Calculates mean width of the vein and mean assay value.

Positon of sample (m)	→	2	8	10	15	20	30	32
Width (cm)	→	75	80	70	105	92	85	50
Value (g/t)	→	15	17	8	11	20	28	12

(Ans. 0.84, 17.5) **GATE-98**

2. In a sampling programme for mettalic deposit the position of samples and their respective width and grade are shown in fig. Find the average width and grade.



(Ans. 3.6, 6.87%) **GATE-96**

3. Given below are data obtained from an irregularly spaced sampling in a drift. Calculate mean width and grade.

Position of sample	Width (cm)	Value (g/t)
2 m	83	15
8 m	76	17
10 m	125	8
15 m	160	11
20 m	125	20
30 m	70	43
32 m	50	90

(Ans. 30 cm, 32.2 g/t)

4. Find mean width and mean assay value

Width (m)	Assay value in dwts/ton
3.0	4.0
2.5	5.0
4.0	3.5
3.5	6.0
3.0	5.0

(Ans. 3.2 m, 4.66 dwt/ton)

Solution 1.

Position of sample (1)	width (m) (2)	value g/t (3)	distance of in fl. (4)	$5 = 2 \times 4$ Area of infl. (5)	$6 = 3 \times 5$ Area AssayProdt. (6)
2	0.75	15	5	3.75	56.25
8	0.80	17	4	3.20	54.40
10	0.70	8	3.5	2.45	19.60
15	1.05	11	5	5.25	57.75
20	0.92	20	7.5	6.90	138
30	0.85	28	6	5.10	142.80
32	0.50	12	2	1.00	12.0
Sum→			33.0	27.65	481.60

$$\text{mean width} = \frac{\sum \text{Area of Influence}}{\sum \text{Distance of Influence}}$$

$$= \frac{27.65}{33}$$

$$= 0.84 \text{ m}$$

mean assay value

$$= \frac{\sum \text{Area assay product}}{\sum \text{Area of Influence}}$$

$$= \frac{481.60}{27.65}$$

$$= 17.5 \text{ g/t}$$

Scanned by CamScanner

Solution 2.

Width (m) (1)	grade (%) (2)	Distance of influence (3)	Area of Influence (4)	Assay Area Product (5)
3.2	7.1	12	38.4	412.64
4.3	5.8	10	43.0	249.40
4.2	8.2	11	46.2	378.84
3.1	6.3	14	43.4	134.54
		47	171.0	1175.42

$$\text{Avg. width} = \frac{\sum \text{Area of Influence}}{\sum \text{Distance of Influence}}$$

$$= \frac{171}{47} = 3.6 \text{ m}$$

Mean assay value

$$= \frac{\sum \text{Assay Area Product}}{\sum \text{Area of influence}}$$

$$= \frac{1175.42}{171}$$

$$= 6.87\%$$

Solution 3.

Position (1)	width (m) (2)	value (g/t) (3)	distance of infl. (4)	2 × 4 Area of infl. (5)	3 × 5 Area AssayProdt. (6)
2	0.83	15	5	0.415	6.22
8	0.76	17	4	3.04	51.68
10	0.125	8	3.5	0.437	3.50
15	0.160	11	10	1.60	17.60
20	0.125	20	7.5	0.937	18.75
30	0.70	43	6	4.20	180.60
32	0.50	90	2	1.00	90.00
Sum→			38.0	11.429	368.35

$$\text{Mean width} = \frac{\sum \text{Area of Influence}}{\sum \text{Distance of Influence}}$$

$$= \frac{11.429}{38.0} = 0.30 \text{ m}$$

$$= 30 \text{ cm.}$$

Scanned by CamScanner

Mean Assay Value

$$= \frac{\sum \text{Area Assay product}}{\sum \text{Area of influence}}$$

$$= \frac{368.35}{11.429}$$

$$= 32.2 \text{ (g/t)}$$

Solution 4.

Width (m)	**Assay value (dwts/t)**	**Assay metre Product**
3.0	4.0	12
2.5	5.0	12.5
4.0	3.5	14.0
3.5	6.0	21.0
3.0	5.0	15.0
16.0		74.5

$$\text{Avg. width} = \frac{16}{5} = 3.2 \text{ m}$$

$$\text{Avg. assay value} = \frac{74.5}{16} = 4.66 \text{ (dwts / t)}$$

Scanned by CamScanner

LINEAR PROGRAMMING

PROBLEMS

1. Maximise

$$Z = 8X_1 + 7X_2$$

given, $X_1 \le 20{,}000$

$X_2 \le 40{,}000$

$X_1 + X_2 \le 45{,}000$

$$\frac{3X_1}{1000} + \frac{X_2}{1000} \le 65$$

& $X_1 \ge 0,\ X_2 \ge 0$ (Ans. 32,5000)

2. Maximise

$$Z = 6X_1 + 11X_2$$

given, $2X_1 + X_2 \le 104$

$X_1 + 2X_2 \le 76$

$X_1 \ge 0$

$X_2 \ge 0$ (Ans. 440)

3. Find the maximum and minimum values for the following linear programming problems using graphical solution method.
Objective function, $Z = 5X_1 + 3X_2$
Subject to the constrainst :

$X_1 + X_2 \le 6$

$2X_1 + 3X_2 \ge 4$

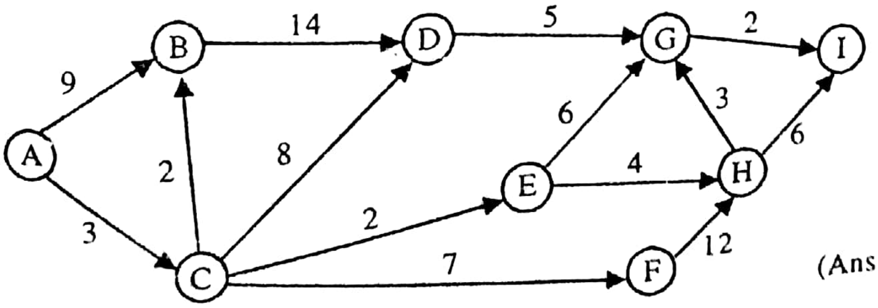
$X_1 \ge 3$

$X_2 \ge 3$

and $X_1 \ge 0,\ X_2 \ge 0$ (Ans. .24) **GATE-99**

Show the feasible region on the graph.

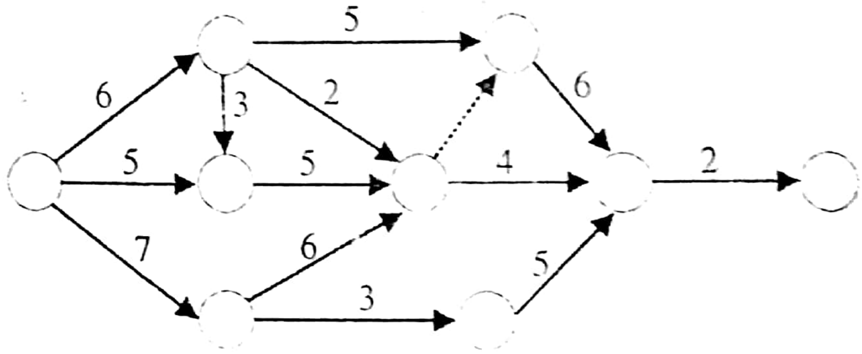
4. Find the critical path for the following network.



(Ans. ABDGI)

5. Given below is a critical path network with the duration time of each activ-
ity. The completion time of network is

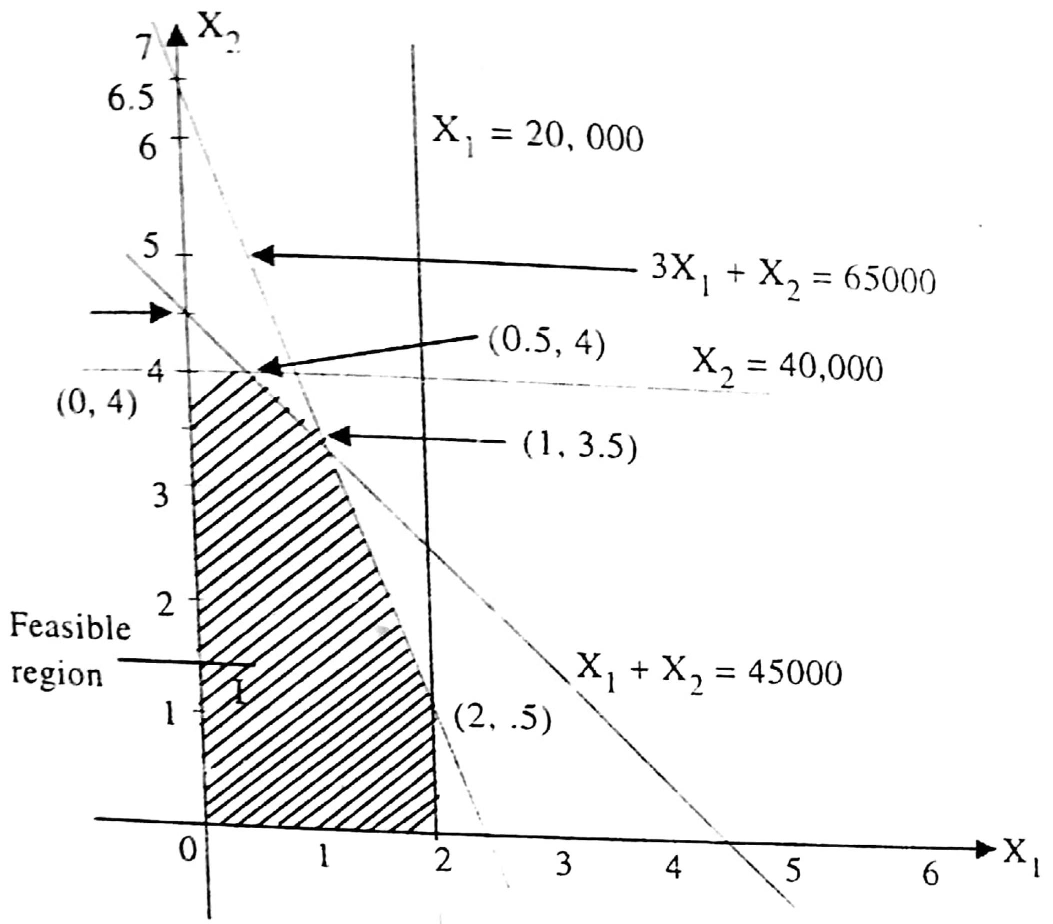
(Ans. 22 days) **GATE-**



Scanned by CamScanner

SOLUTION

Solution 1.



1 division = 10,000

Plotting all the equations we get the outer points of feasible region :

Point		Value of $Z = 8x_1 + 7X_2$	
(2, 0)	→	16,0000	
(0, 4)	→	28,0000	
(1, 3.5)	→	32,5000	→ Maximum
(0, 0)	→	00	
(0.5, 4.0)	→	32,0000	
(2, 0.5)	→	19,5000	

Hence maximum value of Z = 32,500

Solution 2.

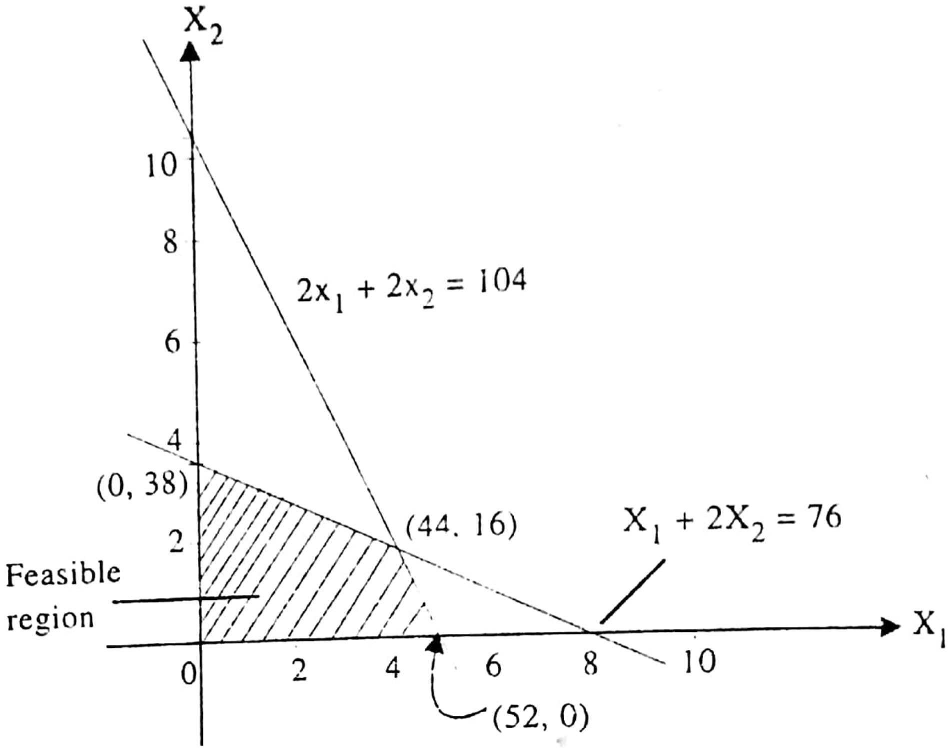
By plotting all the given conditions we get the outerpoints of the feasible region.

Point		Value of $Z = 6X_1 + 11X_2$
(0, 0)	→	0
(0, 38)	→	418
(52, 0)	→	312

Scanned by CamScanner

(44, 16) → 440 → maximum

Hence maximum value of Z = 440



(1 div. = 10)

Solution3.

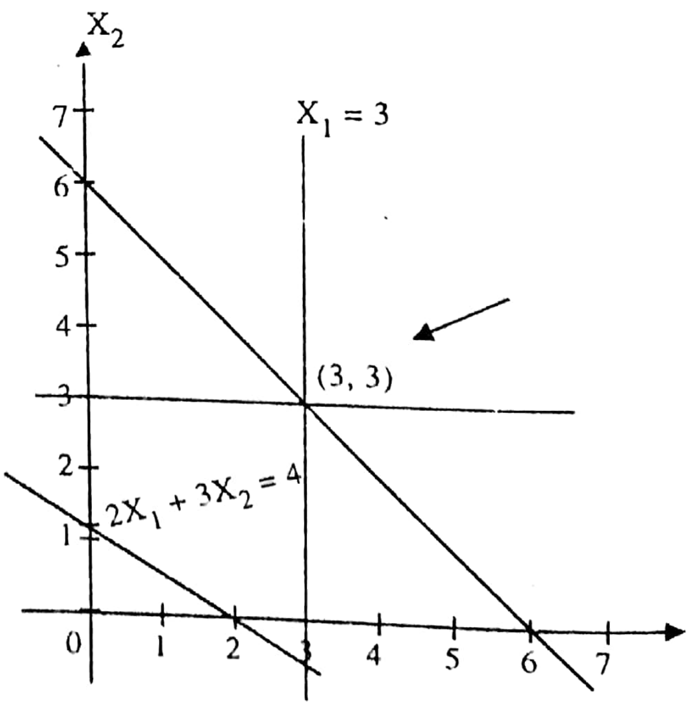
Either by solving all the conditions given or by plotting them we find that feasible region is only a point

that is $(X_1, X_2) = (3, 3)$

Hence value of Z $= 5X_1 + 3X_2$

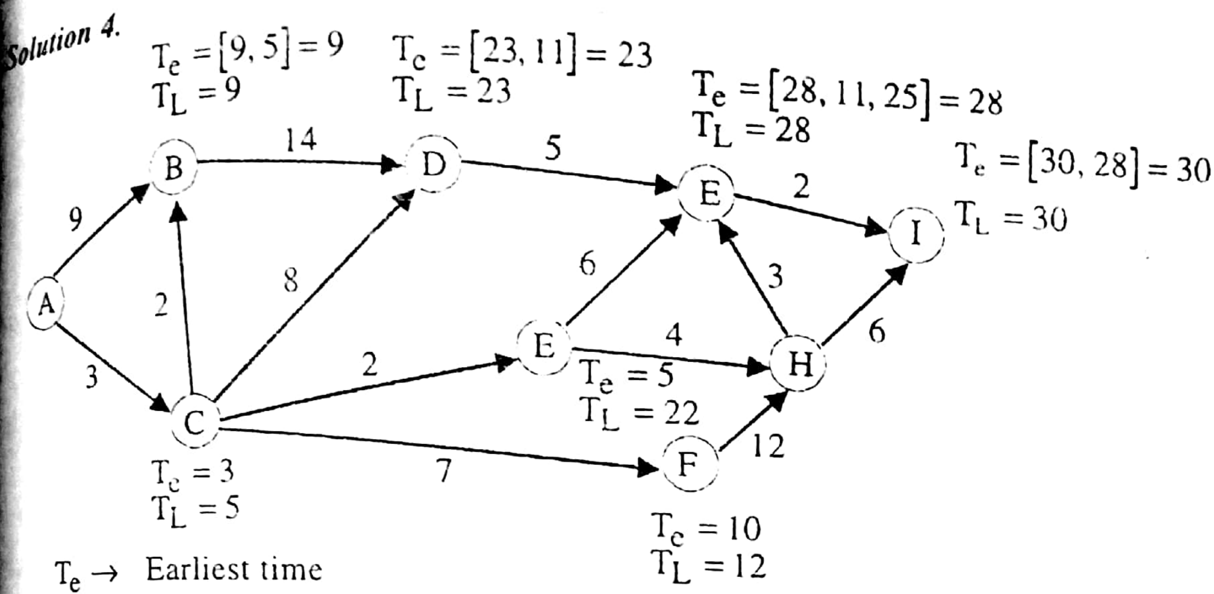
(max. & min both) $= 5 \times 3 + 3 \times 3$

$= 24$



Scanned by CamScanner

Solution 4.



$T_e \rightarrow$ Earliest time
$T_L \rightarrow$ Latest time
$a_t \rightarrow$ Activity time
Slack time for events A, B, C, ...

event	T_E	T_L	Slack
A	0	0	0
B	9	9	0
C	3	5	2
D	23	23	0
E	5	22	17
F	10	12	2
G	28	28	0
H	22	24	2
I	30	30	0

Slack time for Activity

	Activity	Slack = $[T_{L+1} - (T_e + a_t)]$
1	A – B	0
2	B – D	0
3	D – G	0
4	G – I	0
5	A – C	2
6	C – B	7
7	C – D	12
8	C – E	17
9	C – F	2
10	E – G	17
11	E – H	15
12	F – H	2
13	H – G	3
14	H – I	2

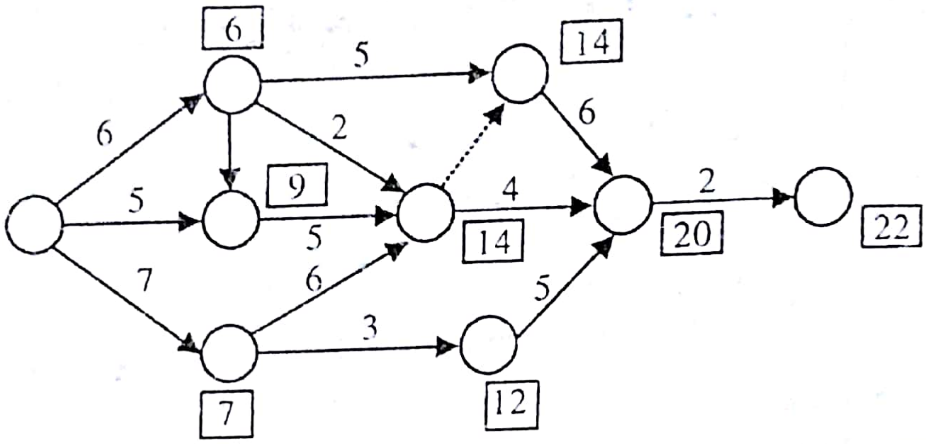
Hence critical Path = ABDGI

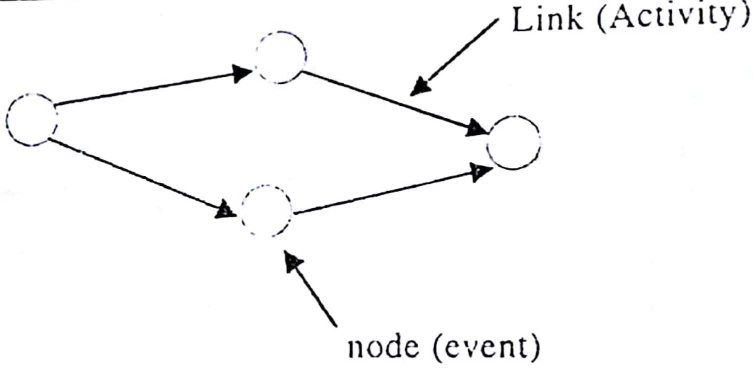
Scanned by CamScanner

Solution 5.

Writing the earliest time to reach each node :

We get to reach the last node in 22 days hence completion time of network 22 days.





CPM → Critical Path Method

PERT → Project evaluation and Review Technique.

MATHEMATICS

Partial differentiation and its application

Function : Any quantity z depends on two or more variables (x, y, t..........) is called a function of independent variables x, y, t......

for example $z = f(x, y)$

denotes that z is a function of two independent vairables x and y.

Partial Derivatives

If $z = f(x, y)$ is a function of two independent variables, then partial derivative of z with respect to x is denoted by $\frac{\delta z}{\delta x}$

where $$\frac{\delta z}{\delta x} = \underset{\delta x \to o}{\text{Lt}} \frac{f(x+\delta x, y) - f(x, y)}{\delta x}$$

similarly,

Partial derivative of z with respect to y is

$$\frac{\delta z}{\delta x} = \underset{\delta y \to 0}{\text{Lt}} \frac{f(x, y+\delta x) - f(x, y)}{\delta x}$$

Partial differentiation and its application **(function of two variables)**

Homogeneous functions

Any function f (x, y) which can be expressed in the form $x^n \phi\left(y/x\right)$ is called homogeneous function of order n in x and y.

for example : $u = x^3 \cos\left(y/x\right)$

Homogeneous function of order 3.

Euler's theorm

If u be a homogeneous function of order 'n' in x and y then.

$$x\frac{\delta u}{\delta x} + y\frac{\delta u}{\delta y} = nu$$

Total derivative

If $u = f(x, y)$

and $x = \phi(t)$

$y = \psi(t)$

then $$\frac{du}{dt} = \frac{\delta u}{\delta x} \cdot \frac{\delta x}{dt} + \frac{\delta u}{\delta y} \cdot \frac{dy}{dt}$$

for implicit functions [i.e. $f(x, y) = c$]

Scanned by CamScanner

$$\frac{dy}{dx} = \frac{-\delta f/\delta x}{\delta f/\delta y}$$

Equation of tangent to a plane at point P (x, y, z)

$$\frac{\delta F}{\delta x}(X - x) + \frac{\delta F}{\delta y}(Y - y) + \frac{\delta F}{\delta z}(Z - z) = 0$$

Equation of normal to the surface at P (x, y, z)

$$\frac{(X - x)}{\delta F/\delta x} = \frac{(Y - y)}{\delta F/\delta y} = \frac{(Z - z)}{\delta F/\delta z}$$

Tailor's theorm

$$f(x+h, y+k) = f(x,y) + \left(h\frac{\delta f}{\delta x} + k\frac{\delta t}{\delta y}\right)$$

$$+\frac{1}{2!}\left(h^2\frac{\delta^2 f}{\delta x^2} + 2hk\frac{\delta^2 f}{\delta x \delta y} + k^2\frac{\delta^2 f}{\delta^2 y^2}\right) + \ldots$$

Error and aproximation

$$\delta f = \frac{\delta f}{\delta x}.\delta x + \frac{\delta f}{\delta y}.\delta y \Rightarrow \text{approximately}$$

for percentage error, use (log)

Maxima & Minima

1. If f is a function of single variable x

$\frac{\delta f}{\delta x} = 0$ find x (?)

$\frac{\delta^2 f}{\delta x^2}$ if < 0, maxima

> 0, minimum

2. If f is a function of two variables x and y

$\frac{\delta f}{\delta x} = 0$

$\frac{\delta f}{\delta y} = 0$

solve to get pair of values like (a, b), (c, d)........

Find :

$$\left.\begin{aligned} \frac{\delta^2 f}{\delta x^2} &= r \\ \frac{\delta^2 f}{\delta x \delta y} &= s \\ \frac{\delta^2 f}{\delta y^2} &= t \end{aligned}\right\} \text{for each pair or values}$$

if $rt - s^2 > 0$, $r < 0$, pair is maxima

Scanned by CamScanner

$r > 0$, pair is minima

$rt - s^2 < 0$, its not a maxima or minima (its called a saddle point)

$rt - s^2 = 0$, it is doubtful and its needs further investigation.

Lagarange method

$$F = f(x, y, z) + \lambda\phi(x, y, z)$$

$$\frac{\delta F}{\delta x} = 0$$

$$\frac{\delta F}{\delta y} = 0$$

$$\frac{\delta F}{\delta z} = 0$$

and $\phi(x - y - z) = 0$.

solve to get values of x, y, z

Differential equations of first order

Order and Degree of equations

(i) $e^x\,dx + e^y\,dy = 0$ (first order and first degree)

(ii) $\frac{d^2x}{dt^2} + n^2x = 0$ (second order and first degree)

(iii) $y\frac{dy}{dx} = x\left(\frac{dy}{dx}\right)^2 + 1$ (first order and second degree)

Ordinary and Partial differential equations

1. Ordinary differential equation is that in which all the differential coefficient have references to a single independent variable.
2. Partial differential equation is that in which there are two or more independent variables

example : $x\frac{\delta u}{\delta x} + y\frac{\delta u}{\delta y} = 24$

Equation of first order and first degree

1. **Equations where variables are separable :**

for equations $\frac{dy}{dx} = f(ax + by + c)$

Put $ax + by + c = t$

2. **Homogeneous equations :**

$$\frac{dy}{dx} = \frac{f(x, y)}{\phi(x, y)}$$

Put $y = vx$

Scanned by CamScanner

then $\frac{dy}{dx} = v + x\frac{dv}{dx}$

separate variables v and x and integrate.

Special case :

$$\frac{dy}{dx} = \frac{ax + by + c}{a'x + b'y + c'}$$

when $\frac{a}{a'} \neq \frac{b}{b'}$,

Put $x = X + h$

$y = Y + k$

then $\frac{dy}{dx} = \frac{ax + by + (ah + bk + c)}{a'x + b'y + (a'h + b'k + c')}$

choose h, k so that

$ah + bk + c = 0$

$a'h + b'k + c' = 0$

when $\frac{a}{a'} = \frac{b}{b'}$

and $\frac{a}{a'} = \frac{b}{b'} = \frac{\ell}{m}$

3. Linear Equations :

$$\frac{dy}{dx} + Py = \theta$$

PQ are function of (x)

Soln :

$$y \cdot e^{\int Pdx} = \int Qe^{\int pdx} = C$$

$$I.F = e^{\int pdx}$$

Bernoulli's equations

$$\frac{dy}{dx} + Py = Qy^n$$

divide both side by y^n and put $y^{1-n} = z$

4. Exact differential equation :

$$M(x, y) + N(x, y) \,.\, dy = 0$$

Soln :

$$\int M dx \text{ (y const)} + \int \text{(terms of N not containing x)}\, dx\, dy = c$$

Provided $\frac{\delta M}{\delta y} \phi = \frac{\delta N}{\delta x}$

Scanned by CamScanner

EXAMPLES

Example 1.

The most probable value and the error in calculating the area of a circle whose radius in meters is 25.35 ± 0.05 will be.

(GATE-99)

Solution :

$$A = \pi r^2$$

$$dA = \frac{\delta A}{\delta r} \cdot \delta r$$

$$= 2\pi r \cdot \delta r$$

$$= 2 \times 3.14 \times 25.35 \times 0.05$$
$$= 7.96 \text{ m}^2$$

Example 2. The solution of differential equation

$$4y \cdot \frac{dy}{dx} + 9x = 0 \text{ is}$$

(GATE-99)

Solution :

$$\int 4y \cdot dy = \int -9x \, dx \qquad \text{(variables are seprable)}$$

$$\Rightarrow \quad \frac{4y^2}{2} = -9\frac{x^2}{2} + c$$

$$\Rightarrow \quad \frac{y^2}{9} + \frac{x^2}{4} = c$$

Example 3.

If the area of the circle increases at uniform rate, the rate of increase of the perimeter is proportion to ....

Solution :

(GATE-98)

$$A = \pi r^2$$

$$\frac{dA}{dt} = c$$

$$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$$

or

$$\frac{dr}{dt} = \frac{dA}{dt} \frac{1}{2\pi r}$$

$$P = 2\pi r$$

$$\frac{dP}{dt} = 2\pi \frac{dr}{dt}$$

Scanned by CamScanner

$$= 2\pi \frac{dA}{dt} \frac{1}{2\pi r}$$

$$= \frac{1}{r}\frac{dA}{dt} = \frac{C}{r} \text{ as } \frac{dA}{dt} = \text{constant}$$

Hence $\frac{dP}{dt} = \frac{C}{r}$

or $\frac{dp}{dt} \alpha \frac{1}{r}$

Example 4.

Find the general soln. of the differential equation

$$\frac{d^2y}{dx^2} = \frac{a}{y^3}$$

Solution :

$$\frac{d^2y}{dx^2} = \frac{a}{y^3}$$

$$\Rightarrow \int d^2y \,.\, y^3 = \int a\, dx^2$$

$$\Rightarrow \frac{y^4}{4} dy = ax\, dx$$

$$\Rightarrow \int \frac{y^4}{4} dy = \int ax \,.\, dx$$

$$\Rightarrow \frac{y^5}{20} = \frac{ax^2}{2} + c$$

$$\Rightarrow y^5 - 10\, ax^2 = c$$

Example 5.

If $f(x, y) = x^2 - 2xy - 4x + 2y + 2y^2$

Calculate the maxima or minima.

Solution :

$$f(x,y) = x^2 - 2xy - 4x + 2y + 2y^2$$

$$\frac{\delta f}{\delta x} = 2x - 2y - 4 = 0 \quad \text{.................. (i)}$$

$$\frac{\delta f}{\delta y} = -2x + 2 + 4y = 0 \quad \text{.................. (ii)}$$

Adding (i) & (ii), we get

$$-2y - 2 = 0$$

or $y = 1$

for $y = 1$ from equation (i), we get

$$2x - 2 - 4 = 0$$

or $$x = 3$$

Hence point P (x, y) is (3, 1)

Now

$$r = \frac{\delta f^2}{\delta x^2} = 2$$

$$s = \frac{\delta f^2}{\delta x \delta y} = -2$$

$$t = \frac{\delta^2 f^2}{\delta y^2} = 4$$

$$rt - s^2 = +8 - 4 = 4 > 0$$

& $$r = 2 > 0$$

Hence point (3, 1) is a minima.

Example 6.

For the function $Z = y^2e^x + x^2y^3 + 1$, Match **(GATE-97)**

Partial Derivative	**Solution**	
1. $\frac{\delta z}{\delta y}$	(A) $2ye^x + 6y^2$	(2)
2. $\frac{\delta^3 z}{\delta x^2 \delta y}$	(B) $y^2 e^x + 2y^3$	(3)
3. $\frac{\delta^2 z}{\delta x^2}$	(C) $2y\, e^x + 3x^2 y^2$	(1)
4. $\frac{\delta^2 z}{\delta y \delta z}$	(D) $2y\, e^x + 6\, x\, y^2$	(4)

Example 7.

For a circle whose radius is 25.35 ± 0.03 m the most probable value of its area in m^2 is **(GATE-97)**

Solution :

$$R = 25.35 \pm 0.03$$

$$A = \pi R^2$$

$$\delta A = \frac{\delta A}{\delta R} \cdot \delta R$$

$$= 2\pi R \,.\, \delta R$$

$$= 2\pi\, 25.35 \times 0.03$$

$$= \pm 4.78$$

Scanned by CamScanner

Example 8.

If z is a homogeneous function of order n in x and y show that

$$x^2\frac{\delta^2 z}{\delta x^2}+2xy\frac{\delta^2 z}{\delta x\delta y}+\frac{\delta^2 z}{\delta y^2}=n(n-1)z$$

Solution :

From Euler's theorm (if z is a homogeneous function of n order)

$$X\frac{\delta z}{\delta x}+Y\frac{\delta z}{\delta y}=nz \qquad \text{............. (A)}$$

differentating with respect to x

$$x.\frac{\delta^2 z}{\delta x^2}+\frac{\delta z}{\delta x}+y\frac{\delta^2 z}{\delta y\delta x}=n\frac{\delta z}{\delta x} \qquad \text{............. (i)}$$

differentiating (A) w.r.t. y

$$X\frac{\delta^2 z}{\delta y\delta x}+Y\frac{\delta^2 z}{\delta y^2}+\frac{\delta z}{\delta y}=n.\frac{\delta z}{\delta y} \qquad \text{............. (ii)}$$

multiplying (i) with x and (ii) y

$$x^2\frac{\delta^2 z}{\delta x^2}+x\frac{\delta z}{\delta x}+xy\frac{\delta^2 z}{\delta y\delta x}=hx\frac{\delta z}{\delta x} \qquad \text{............. (iii)}$$

$$xy\frac{\delta^2 z}{\delta x\delta y}+y^2\frac{\delta^2 z}{\delta y^2}+y\frac{\delta z}{\delta y}=ny\frac{\delta z}{\delta y} \qquad \text{............. (iv)}$$

Adding (iii) and (iv)

$$\Rightarrow \quad x^2\frac{\delta^2 z}{\delta x^2}+2xy\frac{\delta^2 z}{\delta x\delta y}+y^2\frac{\delta^2 z}{\delta y^2}$$

$$=nx\frac{\delta z}{\delta x}-x\frac{\delta z}{\delta x}+ny\frac{\delta z}{\delta y}-y\frac{\delta z}{\delta y}$$

$$\Rightarrow \quad x^2\frac{\delta^2 z}{\delta x^2}+2xy\frac{\delta^2 z}{\delta x\delta y}+y^2\frac{\delta^2 z}{\delta y^2}=(n-1)\left[x\frac{\delta z}{\delta x}+y\frac{\delta z}{\delta y}\right]$$

$$=n(n-1)z$$

□

EXERCISE

1. $e^{xy} + c^{x^2 t} \cdot \frac{dy}{dx} = 0$ is (GATE-96)
   (a) a linear differential equation
   (b) an exact differential equation
   (c) not an exact differential equation
   (d) an equation with no solution

2. If f (0) = 2 and f' (x) = $\frac{1}{2 - x}$ the f (1) lies between. (GATE-96)

3. Using the method of Lagrange's multipliers find the dimensions of a rectangle of greatest area for a given perimeter P_m (GATE-96)

4. The sides of a rectangle in meters are [100 ± 0.02] and [180 ± 0.03]. The probable erer of the area of the rectangle is....... (GATE-96)

5. If $u = x^2 \tan^{-1}\frac{y}{x} - y\tan^{-1}\frac{x}{y}$ and $xy \neq 0$

   Prove that $\frac{\delta^2 u}{\delta x \delta y} = \frac{x^2 - y^2}{x^2 + y^2}$

6. Maximise the value of R when
   $R = q^2 + 3q\,a + a^2$
   Subjected to constraint q + a = 100

7. Find $\frac{\delta z}{\delta x}$

   where $z = \tan^{-1}\left\{(x^2 + y^2)\big/(x + y)\right\}$

8. If $z = e^{ax+by} f(ax - by)$

   Prove that $b\frac{\delta z}{\delta x} + a\frac{\delta z}{\delta y} = 2abz$

9. If $u = \sin^{-1}\frac{x}{y} + \tan^{-1}\frac{y}{x}$

   Prove that $x\frac{\delta u}{\delta x} + y\frac{\delta u}{\delta y} = 0$

10. The diameter and altitude of a cane in the shape of right circular cylinder are measured as 5 cm. and 6 cm. respectively. The possible error in each measurement is 0.1 cm. Find approximately the maximum possible error in volume and lateral surface.

Scanned by CamScanner

11. Find $\frac{\delta z}{\delta x}$; $z = \tan^{-1}\left\{\left(x^2 + y^2\right)\Big/(x + y)\right\}$

12. If $z = e^{ax+by} f(ax - by)$ Prove that

$$b\frac{\delta z}{\delta x} + a\frac{\delta z}{\delta y} = 2abz$$

13. $u = \sin^{-1}\frac{x}{y} + \tan^{-1}\frac{y}{x}$

Prove that $x\frac{\delta u}{\delta x} + y\frac{\delta u}{\delta y} = 0$

Scanned by CamScanner

SOLUTIONS

Solution 1.

$$e^{xy} + e^{x^2y}\frac{dy}{dx} = 0$$

$\Rightarrow \quad ex^2y\,dy + e^{xy}\,dx = 0$

$\Rightarrow \quad e^{xy}\,dx + e^{x^2y}\,dy = 0$

$\Rightarrow \quad Mdx + Ndy = 0$

$$\frac{\delta M}{\delta y} = xe^{xy}$$

$$\frac{\delta M}{\delta x} = 2xye^{x^2y}$$

as $\quad \dfrac{\delta M}{\delta y} \neq \dfrac{\delta N}{\delta x}$

Hence it is not an exact differential equation.

Solution 2.

$$f'(x) = \frac{1}{2-x}$$

then $\quad f(x) = -\log(2-x) + c$

as $\quad f(0) = -\log(2) + c = 2$

$\quad c = 2 + \log(2)$

$\quad = 2.3$

hence $\quad f(1) = -\log(2-1) + 2.3$

$\quad = -\log 1 + 2.3$

$\quad = 0 + 2.3$

$\quad = 2.3$

lies between 2 to 2.5

Solution 3.

Area = x, y

ferimeter $= 2(x+y) = Pm$

$= 2(x+y) - Pm = 0 \quad$ .... **(A)**

Lagrange's method

$F \quad = f(x, y) + \lambda\phi(x, y)$

$= (x-y) + \lambda\left[2(x+y) - P_m\right] = 0$

$= xy + 2\lambda x + 2\lambda y - \lambda Pm = 0$

Scanned by CamScanner

$$\frac{\delta F}{\delta x} = y + 2\lambda = 0 \qquad \text{.................. (i)}$$

$$\frac{\delta F}{\delta x} = x + 2\lambda = 0 \qquad \text{.................. (ii)}$$

solving (i) and (ii), we get

$$x = y$$

Hence from equn. (A), we get

$$2(x + y) = Pm$$

$$4x = Pm$$

$$x = \frac{Pm}{4}$$

$$y = \frac{Pm}{4}$$

i.e. it should be square.

Solution 4.

$$A = xy$$

$$\delta A = \frac{\delta A}{\delta x} . \delta x + \frac{\delta A}{\delta y} . \delta y$$

$$= y . \delta x + x\delta y$$

$$= 180 \times 0.02 + 100 \times 0.03$$

$$= 3.6 + 3$$

$$= \pm 6.6$$

Solution 5.

$$u = x^2 \tan^{-1}\frac{y}{x} - y^2 \tan^{-1}\frac{x}{y}$$

$$\frac{\delta u}{\delta x} = \left[x^2 \frac{1}{1+\left(\frac{y}{x}\right)^2} \times \frac{-y}{x^2} + 2x\tan^{-1}\frac{y}{x}\right] - y^2 \frac{1}{1+\left(\frac{x}{y}\right)^2} \times \frac{1}{y}$$

$$\frac{\delta u}{\delta x} = \frac{-yx^2}{x^2+y^2} + 2x\tan^{-1}\frac{y}{x} - y^2\frac{y^2}{x^2+y^2}\frac{1}{y}$$

$$\frac{\delta u}{\delta x} = 2x\tan^{-1}\frac{y}{x} - \frac{yx^2+y^3}{x^2+y^2}$$

$$\frac{\delta u}{\delta x} = 2x\tan^{-1}\frac{y}{x} - \frac{y\left(x^2+y^2\right)}{\left(x^2+y^2\right)}$$

$$\frac{\delta u}{\delta x} = 2x\tan^{-1}\frac{y}{x} - y$$

Scanned by CamScanner

$$\frac{\delta^2 u}{\delta x \delta y} = 2x \frac{1}{1-\left(\frac{y}{x}\right)^2} \times \frac{1}{x} - 1$$

$$\frac{\delta^2 u}{\delta x \delta y} = \frac{2x^2}{x^2 - y^2} - 1$$

$$\frac{\delta^2 u}{\delta x \delta y} = \frac{2x^2 - x^2 - y^2}{x^2 + y^2}$$

$$= \frac{x^2 - y^2}{x^2 - y^2}$$

Proved.

on 6.

Using Lagarang Method

$$F = q^2 + 3qa + a^2 + \lambda(q + a - 100)$$

$$= q^2 + 3qa + a^2 + \lambda q + a\lambda - 100\lambda$$

$$\frac{\delta F}{\delta q} = 2q + 3a + \lambda = 0 \quad \text{................ (i)}$$

$$\frac{\delta F}{\delta a} = 3q + 2a + \lambda = 0 \quad \text{................ (ii)}$$

multiplying (i) by 3 and (ii) 2 & subtraiting (ii) from (i), we get

$$6q + 9a + 3\lambda = 0$$

$$6q \pm 4a \pm 2\lambda = 0$$

$$5a \quad = -\lambda$$

Putting the value of λ in (i), we get

$$2q + 3a = 5a$$

or $q = a$

Putting this value in given equation

$$q + a = 100$$

$\therefore$ $2a = 100 \; (9 = a)$

$\therefore$ $q = a = 50$

Hence $R_{max} = 50^2 + 3\,(50)\,(50) + 50^2$

$= 2500 + 7500 + 2500$

$= 12500$

ion 7.

$$\frac{\delta z}{\delta x} = \frac{1}{1 + \left(\frac{x^2 + y^2}{x + y}\right)^2} \times \left\{\frac{2x}{x + y} - \frac{x^2 + y^2}{(x^2 + y2)^2}\right\}$$

$$= \frac{(x+y)^2}{(x+y)^2 + \left(x^2+y^2\right)^2} \left\{ \frac{2x^2+2xy-x^2y^2}{(x+y)^2} \right\}$$

$$= \frac{2x^2+2xy-x^2-y^2}{(x+y)^2+\left(x^2+y^2\right)^2}$$

$$= \frac{x^2+2xy-y^2}{(x+y)^2+\left(x^2+y^2\right)^2}$$

Solution 8.

$$\frac{\delta z}{\delta x} = \left[e^{ax+by} - af'(ax-by)\right] + e^{ax+by} f(ax-by)a$$

$$= ae^{ax+by}\left[f(ax-by) + f'(ax-by)\right]$$

Similarly

$$\frac{\delta z}{\delta y} = be^{ax+by}\left[(ax-by) + f(ax-by) - f'(ax-by)\right]$$

Now $\quad b\,.\frac{\delta z}{\delta x} + a\frac{\delta z}{\delta y}$

$$= abe^{ax+by}\left[f(ax-by)+f'(ax-by)\right] + abe^{ax+by}\left[f(ax-by) - f'(a\right.$$

$$= abe^{ax+by}\, 2f(ax-by)$$

$$= 2abe^{ax+by}\, 2f(ax-by)$$

$$= 2\,abz$$

Solution 9.

$$u = x\left[\sin^{-1}\frac{x}{y} + \tan^{-1}\frac{y}{x}\right]$$

here u is a homogeneous function of order $n = 0$ in x and y.

thus $\quad x\,.\frac{\delta u}{\delta x} + y\frac{\delta u}{\delta y} = n\,.\,u$

as $\quad n = 0$

$\therefore \quad x\,.\frac{\delta u}{\delta x} + y\frac{\delta u}{\delta y} = 0\,.\,u$

$$x\,.\frac{\delta u}{\delta x} + y\frac{\delta u}{\delta y} = 0$$

Solution 10.

Let x be dia and y be the height of the cane

Thus its volume

$$v = \frac{\pi}{4}x^2y$$

Scanned by CamScanner

$$\delta v = \frac{\delta v}{\delta x}.\delta x + \frac{\delta v}{\delta y}.\delta y$$

$\therefore$

$$= \frac{\pi}{4}\left(2xy\,\delta x + x^2\,\delta y\right)$$

$$\delta v = \frac{\pi}{4}\left(2\times5\times6\times0.1 + 5^2\times0.1\right)$$

$\therefore$

$$= 2.12\pi\ cm^2$$

$$= 6.66\ cm^2$$

Lateral surface $s = \pi x.y$

$\therefore$ $\delta s = \pi(y\delta x + x\,\delta y)$

or $\delta s = \pi(6\times0.1 + 5\times0.1)$

$$= 1.1\,\pi\ cm^2$$
$$= 3.46\ cm^2$$

Solution 11.

$$\frac{\delta z}{\delta x} = \frac{1}{1+\left(\frac{x^2+y^2}{x+y}\right)^2}\left\{\frac{2x}{x+y} - \frac{x^2+y^2}{(x+y)^2}\right\}$$

$$= \frac{(x+y)^2}{(x+y)^2+\left(x^2+y^2\right)^2}\left[\frac{2x^2+2xy-x^2y^2}{(x+y)^2}\right]$$

$$= \frac{2x^2+2xy-x^2-y^2}{(x+y)^2+\left(x^2+y^2\right)^2}$$

$$= \frac{x^2+2xy-y^2}{(x+y)^2+\left(x^2+y^2\right)^2}$$

Solution 12.

$$\frac{\delta z}{\delta x} = \left[e^{ax+by} - af(ax-by)\right] + e^{ax+by}.f'(ax-by)a$$

$$= ae^{ax+by}\left[f(ax-by) + f'(ax-by)\right]$$

Similarly

$$\frac{\delta z}{\delta y} = be^{ax+by}\left[f(ax-by) - f'(ax-by)\right]$$

Now, $b\frac{\delta z}{\delta x} + a\frac{\delta z}{\delta y}$

$$= abe^{ax+by}\left[f(ax-by)+f'(ax-by)\right] + abe^{ax+by}\left[f(ax-by)+f'(ax-by)\right]$$

$$= abe^{ax+by}\,2f(ax-by)$$

Scanned by CamScanner

$$= 2abe^{ax+by} f(ax - by)$$

$$= 2\ abz$$

Solution 13.

$$u = x^{\circ}\left[\sin^{-1}\frac{x}{y} + \tan^{-1}\frac{y}{x}\right]$$

here u is a homogeneous function of order $n = 0$ in x and y

thus $\quad x\frac{\delta u}{\delta x} + y\frac{\delta u}{\delta y} = nu$

$$= 0u$$

$$= 0$$

$\therefore \quad x\frac{\delta u}{\delta x} + y\frac{\delta u}{\delta y} = 0$

Scanned by CamScanner

MATRICS

Matrix of order mn has m rows and n columns.

ecial matrics

(i) Row & column matrix

$$[1, 2, 3], \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

(ii) Leading or principal diagonal

if $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$

then elements 1, 5 & 9 are called principal diagonal

(iii) Singular & non-singular matrix
A square matrix is said to be singular if its determinants is zero. Otherwise it is singular, If the rank of the $r \times r$ matrix is r it is non-singular, if $(r - 1)$ it is singular.

(iv) Diagonal matrix

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

(v) Scalar matrix

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

(vi) Unit matrix

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(vii) Null matrix

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

(viii) Symetric & skew-symetric matrices.
In a square matrix
if $a_{ij} = a_{ji}$ then it is called symetric matrix
if $a_{ij} = -a_{ji}$ then it is called skew symetric matrix

Scanned by CamScanner

(ix) Traingular matrix

$$\begin{bmatrix} a & h & g \\ 0 & b & f \\ 0 & 0 & c \end{bmatrix}$$ is called upper traingular matrix

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & 0 \\ 1 & -5 & 4 \end{bmatrix}$$ is called lower triangular matrix.

Multiplication of matrices :

Two matrices can be multiplid only when the number of columns in the firs equal to the number of rows in the second. Such matrices are said to be confo able.

Transpose of matrix

If $A = \begin{bmatrix} 1 & 2 \\ 4 & 5 \\ 7 & 8 \end{bmatrix}$

Transpose of A is $A' = \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \end{bmatrix}$ inter changing the row and column.

- The transpose of the product of two matrices is the product of their tra poses taken in reverse order.

  $(AB)' = B'A'$

- Every square matrix can be uniquely expressed as sum of a symmetric skew symmetric.

  $$A = \frac{1}{2}(A + A') + \frac{1}{2}(A - A')$$

Adjoint of a square matrix

Adjoint (Adj A) of a square matrix A is the transposed matrix of co-factors A.

If $A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c3 \end{bmatrix}$

then $\text{adj } A = \begin{bmatrix} A_1 & A_2 & A_3 \\ B1 & B_2 & B_3 \\ C_1 & C_2 & C3 \end{bmatrix}$

Reciprocal matrix and inverse matrix

$$A^{-1} = \frac{\text{adj } A}{\Delta}$$

Δ = determinant of matrix A

A^{-1} exists only when

$\Delta \neq 0$ i.e., A is not singular matrix

Scanned by CamScanner

verse of matrix

If $AB = BA = I$
then B is called Inverse of matrix A

$$A^{-1}A = AA^{-1} = I$$

hence A^{-1} is Inverse matrix
Inverse of a matrix is unique.

$$(AB)^{-1} = B^{-1}A^{-1}$$

atrix method of solution of simultaneous equations

$$AX = D$$

$$X = A^{-1}D$$

ank of matrix

A matrix is said to be of rank 'r' when

(i) it has atleast one non-zero minor of order 'r' and

(ii) every minor of order higher than 'r' partishes.

Gouss-Jordan reduction method for finding Inverse of Matrix.

$$\left[\begin{array}{ccc:ccc} 1 & 1 & 3 & 1 & 0 & 0 \\ 1 & 3 & -3 & 0 & 1 & 0 \\ -2 & -4 & -4 & 0 & 0 & 1 \end{array}\right]$$

$$\underbrace{\qquad\qquad}_{A}$$

do elementary transformations to get

$$\left[\begin{array}{ccc:ccc} 1 & 0 & 0 & 3 & 1 & 3/2 \\ 0 & 1 & 0 & -5/4 & -1/4 & -3/4 \\ 0 & 0 & 1 & -1/4 & -1/4 & -1/4 \end{array}\right]$$

$$\underbrace{\qquad\qquad}_{A^{-1}}$$

inear dependence of vectors

if $x_1, x_2, x_3, \ldots\ldots x_r$ are vectors

and $\lambda_1, \lambda_2, \lambda_3 \ldots\ldots \lambda_r$ are number not all zero.

such that

$$\lambda_1 x_1 + \lambda_2 x_2 + \lambda_3 x_3 + \ldots + \lambda_r x_r = 0$$

the vectors are said to be linearly dependent

if $\lambda_1 = \lambda_2 = \lambda_3 = \ldots\ldots = 0$

the vectors are lineraly independent.

onsistency of a system of linear equations

Rank of co-efficent matrix $A = r$

Rank of argumented matrix $K = r^1$

(i) If $r \neq r'$ equations are inconsistant, no solution

(ii) If $r = r' = n$ equations are consistant and there is a unique solution

(iii) If $r = r' < n$ consistant and infinite number of solutions.

Scanned by CamScanner

Chatracteric equations & eigen values

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} a-\lambda & b_1 & c_1 \\ a_2 & b_2-\lambda & c_2 \\ a_3 & b_3 & c_3-\lambda \end{vmatrix} = 0$$

Solve to get values of λ, which are called eigen values.

Eigen vectors

$$|A - \lambda I| = 0$$

for each eigen value find values of X which are called eigen vectors.

Cayley Hamilton theorem

$$A - \lambda I = 0 \text{ from characteristic equations}$$

$$a_1\lambda^n + a_2\lambda^{n-1} + \ldots\ldots + aI = 0$$

Put $\lambda = A$

$$aI = -a_1\lambda^n - a_2\lambda^{n-1} - \ldots\ldots\ldots$$

$$aI = -a_1A^n - a_2A^{n-1} - \ldots\ldots\ldots$$

$$A^{-1} = \frac{1}{a}\left[-aA^{n-1} - a_2A^{n-2} - \ldots\ldots\right]$$

Properties of eigen values

(i) Sum of the eigen values of a matrix is the sum of the elements of the prin pal diagonal (trace).

(ii) If x is eigen value of A then λ^{-1} is eigen value of A^{-1}.

(iii) If λ is eigen value of orthgonal matrix, then $1/\lambda$ is also its eigen value. for orthogonal Matrix A^{-1} is same as A^1 (transpose)

Scanned by CamScanner

EXAMPLES

Example 1.

Find the inverse of the Matrix A **(GATE-99)**

$$A = \begin{bmatrix} -2 & 3 & -3 \\ -2 & -2 & -3 \\ -3 & 2 & -2 \end{bmatrix} = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$$

Solution :

Let A_1, A_2, A_3....... be cofactors of elements a_1, a_2, a_3 respectively

$$A_1 = 4 + 6 = 10$$

$$A_2 = -(-6 + 6) = 0$$

$$A_3 = (-9 - 6) = -15$$

$$B_1 = -(4 - 9) = 5$$

$$B_2 = (4 - 9) = -5$$

$$B_3 = -(+6 - 6) = 0$$

$$C_1 = (-4 - 6) = -10$$

$$C_2 = (-4 + 9) = -5$$

$$C_3 = (4 + 6) = 10$$

$$\Delta = a_1A_1 + a_2A_2 + a_3A_3$$

$$= (-2 \times 10) + 0 + (.3 \times -15)$$

$$= -20 + 0 + 45$$

$$= 25$$

$$\text{Adj } A = \begin{bmatrix} A_1 & B_1 & C_1 \\ A_2 & B_2 & C_2 \\ A_3 & B_3 & C_3 \end{bmatrix}$$

$$= \begin{bmatrix} 10 & 0 & -15 \\ 5 & -5 & 0 \\ -10 & -5 & 10 \end{bmatrix}$$

$$A^{-1} = \frac{1}{\Delta} \text{adj } A$$

$$= \begin{bmatrix} 2/5 & 0 & -3/5 \\ 1/5 & -1/5 & 0 \\ -2/5 & -1/5 & 2/5 \end{bmatrix}$$

Scanned by CamScanner

Example 2. (GATE-98)

Find the eigen values of the matrix

$$\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ 1 & -2 & 0 \end{bmatrix}$$

Solution :

Characteristic equation

$$|A-\lambda I| = 0$$

$$= \begin{vmatrix} -2-\lambda & 2 & -3 \\ 2 & 1-\lambda & -6 \\ 1 & -2 & 0.\lambda \end{vmatrix}$$

$\Rightarrow \quad -(2+\lambda)[(1-\lambda)x-\lambda-12]-2[2x-\lambda-6] \; -3[-4+1(1-\lambda)]=0$

$\Rightarrow \quad -(2+\lambda)[-\lambda-\lambda^2-12]-2[-2\lambda-6]-3[-3-\lambda]=0$

$\Rightarrow \quad -[-2\lambda+2\lambda^2-24-\lambda^2+\lambda^3-12\lambda]-[-4\lambda-12]-3[-3-\lambda]=0$

$\Rightarrow \quad 2\lambda-2\lambda^2+24+\lambda^2-\lambda^3+12\lambda+4\lambda+12+9+3\lambda=0$

$\Rightarrow \quad -\lambda^3-\lambda^2+21\lambda+45=0$

$\Rightarrow \quad \lambda^3+\lambda^2-21\lambda-45=0$

$\Rightarrow \quad \lambda^3+6\lambda^2+9\lambda-5\lambda^2-30\lambda-45=0$

$\Rightarrow \quad \lambda(\lambda-3)^2-5(\lambda^2+6\lambda+9)=0$

$\Rightarrow \quad (\lambda-5)(\lambda+3)^2=0$

$\Rightarrow \quad \lambda=5 \;\&\; \lambda=-3$

Example 3.

Prove that for the matrix

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 2 & 3 & 4 \\ -1 & -1 & -2 \end{bmatrix}$$

all the given values are real and distinct.

Solution :

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 2 & 3 & 4 \\ -1 & -1 & -2 \end{bmatrix}$$

characteristic equation

$$|A-\lambda I| = 0$$

$$= \begin{vmatrix} 2-\lambda & 1 & 1 \\ 2 & 3-\lambda & 4 \\ -1 & -1 & -2-\lambda \end{vmatrix} = 0$$

Scanned by CamScanner

$$(2-\lambda)[(3-\lambda)(-2-\lambda)+4]-1[2(-2-\lambda)+4]+[-2+1(3-\lambda)]=0$$

$\Rightarrow$ $$(2-\lambda)\left[-6-3\lambda+2\lambda+\lambda^2+4\right]-1[4-2\lambda+4]+[-2+3-\lambda]=0$$

$\Rightarrow$ $$(2-\lambda)\left(\lambda^2-\lambda-2\right)+2\lambda+1-\lambda=0$$

$\Rightarrow$ $$(2-\lambda)(\lambda-2)(\lambda+1)=0$$

$\Rightarrow$ $$(\lambda+1)[(2-\lambda)(\lambda-2)+1]=0$$

$\Rightarrow$ $$(\lambda+1)\left[-\lambda^2+4\lambda-4+1\right]=0$$

$\Rightarrow$ $$(\lambda+1)\left[\lambda^2-4\lambda+3\right]=0$$

$\Rightarrow$ $$(\lambda+1)\left[\lambda^2-\lambda-3\lambda+3\right]=0$$

$\Rightarrow$ $$(\lambda+1)[\lambda(\lambda-1)-3(\lambda-1)]=0$$

$\Rightarrow$ $$(\lambda+1)(\lambda-1)(\lambda-3)=0$$

or $\lambda=-1,\ 1\ \&\ 3$ **Proved**

Example 4.

$$D_1=\begin{vmatrix}1 & 2\\ 2 & 1\end{vmatrix}\ \&\ D_2=\begin{vmatrix}2 & 1\\ 1 & 2\end{vmatrix}$$

Find $D_1 \times D_2$

Solution :

$$D=\begin{vmatrix}1 & 2\\ 2 & 1\end{vmatrix}\times\begin{vmatrix}2 & 1\\ 1 & 2\end{vmatrix}$$

$$=\begin{vmatrix}1.2+2.1 & 1.1+2.2\\ 2.2+1.1 & 2.1+1.2\end{vmatrix}$$

$$=\begin{vmatrix}4 & 4\\ 5 & 4\end{vmatrix}$$

Scanned by CamScanner

EXERCISE

1. If [A] [B] = I, then the relationship between [A] and [B] is
2. Identify the solution of the wave equation

$$\frac{\delta^2 u}{\delta t^2} = 16\frac{\delta^2 u}{\delta x^2}$$

3. If the determinant [A] of a (3 × 3) matrix [A] is zero then the rank of the matrix [A] is always ....
4. The value of λ for which following system of equations will not have a unique solution.

$$x + 2y + 3z = 1,\ \ x + 2y - 3z = 1,\ \ \lambda x + y + z = 1$$

5. Find the eigen values and corresponding eigen vectors for the following matrix.

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

6. Determine the corresponding eigen vectors for the matrix for eigen values of 1 and 3.
7. Find the co-factor matrix A^c of matrix 'A' given below

$$A = \begin{bmatrix} 4 & 2 & 1 \\ 2 & 6 & 4 \\ 1 & 4 & 6 \end{bmatrix}$$

Scanned by CamScanner

SOLUTIONS

Solution 1.

If $[A][B] = I$ then

$[B] = [A]^{-1}$ is the relationship between them.

Solution 2.

$$\frac{\delta^2 u}{\delta t^2} = 16\frac{\delta^2 u}{\delta x^2}$$

differentiating these solutions given below

$$k = x^2 + 16\,t^2$$

is found the required solution.

Solution 3.

The rank of the matrix is always less than 3.

Solution 4.

Co-efficient matrix [A] should be of

$$\begin{vmatrix} 1 & 2 & 3 \\ 1 & 2 & -3 \\ \lambda & 1 & 1 \end{vmatrix} = 0$$

$\Rightarrow \quad 1(2+3) - 1(2-3) + \lambda[-6-6] = 0$

$\Rightarrow \quad 5 + 1 - 12\lambda = 0$

$\Rightarrow \quad 12\lambda = 6$

or $\quad \lambda = \frac{1}{2}$

Solution 5.

Characteristic equations

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} -\lambda & 1 & 0 \\ 1 & -\lambda & 0 \\ 0 & 0 & 1-\lambda \end{vmatrix} = 0$$

$\Rightarrow \quad \lambda[-\lambda(1-\lambda) - 0] - 1[(1-\lambda) - 0] - 0 = 0$

$\Rightarrow \quad -\lambda(-\lambda + \lambda^2) - 1 + \lambda = 0$

$\Rightarrow \quad -\lambda^2(\lambda - 1) + (\lambda - 1) = 0$

$\Rightarrow \quad (\lambda - 1)(1 - \lambda^2) = 0$

$\Rightarrow \quad (\lambda - 1)(1 - \lambda)(1 + \lambda) = 0$

Scanned by CamScanner

or $\lambda = 1, -(1)$ eigen values.

let x, y, z be the components of eigen vectors corresponding eigen values 'λ' we have

$$\begin{bmatrix} -\lambda & 1 & 0 \\ 1 & -\lambda & 0 \\ 0 & 0 & 1-\lambda \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

for $\lambda = 1$

$-x + y = 0, \quad x - y = 0, \quad 0 . z = 0, \quad z =$ any number

$\Rightarrow \quad x = y$

so eigen vector corresponding to $\lambda = 1$ is

(1, 1, 0), (2, 2, 0), ...

for $\lambda = -1$

$x + y = 0, \quad x + y = 0, \quad 2z = 0$

$y = -x$ & $2 = 0$

so eigen vectors are

(1, –1, 0) (–1, 1, 0) etc.

Solution 6.

Let x, y, z be the components of eigen vectors, corresponding to eigen value λ, then,

$$\begin{vmatrix} 1-\lambda & 6 & 1 \\ 1 & 2-\lambda & 0 \\ 0 & 6 & 3-\lambda \end{vmatrix} \begin{vmatrix} x \\ y \\ z \end{vmatrix} = 0$$

for $\lambda = 1$

$$\begin{vmatrix} 0 & 6 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 2 \end{vmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$\Rightarrow \quad 6y + z = 0$

$x + y = 0 \qquad\qquad x, y, z = (0, 0, 0)$

$2z = 0$

for $\lambda = 3$

$$\begin{bmatrix} -2 & 6 & 1 \\ 1 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$-2x + 6y + z = 0$ .... (i)

$x - y = 0$ .... (ii)

$x = y$

from eqn. (i), we have

$z = -4x$

Hence eigen vectors can be

(1, 1, –4), (2, 2, –8), (–1, –1, 4)...... etc.

Scanned by CamScanner

Solution 7.

$$A = \begin{bmatrix} 4 & 2 & 1 \\ 2 & 6 & 4 \\ 1 & 4 & 6 \end{bmatrix} = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$$

Let A_1, A_2 & A_3 .... be cofactors of demonts a_1, a_2, a_3 ..... respectively

$A_1 = 4\,[36 - 14] = 80$

$A_2 = -2\,[12 - 4] = -16$

$A_3 = 1\,[8 - 6] = 2$

$B_1 = -2\,(12 - 4) = -16$

$B_2 = 6\,(24 - 1) = 138$

$B_1 = -4\,(16 - 27) = -56$

$C_1 = 1\,(8 - 6) = 2$

$C_2 = -4\,(16 - 2) = -56$

$C_3 = 6\,(24 - 4) = 120$

$\therefore$ cofactor matrix

$$= \begin{bmatrix} 80 & -16 & 2 \\ -16 & 138 & -56 \\ 2 & -56 & 120 \end{bmatrix}$$

$$= \begin{bmatrix} 20 & -8 & 2 \\ -8 & 23 & -14 \\ 2 & -14 & 20 \end{bmatrix}$$

❑

Scanned by CamScanner

PROBABILITY

Probability density function

If tne probability of a varlate 'x' falling in interval dx is f (x) . dx then f (x) is called probability density function.

Moment Generating function

If f (x) is the density function of a continous variate x, then the moment generating function about x = a , is given by

$$M_a(t) = \int_{-\alpha}^{\alpha} e^{t(x-a)} . f(x)dx$$

$$\mu_1 \text{ (mean)} = \frac{d}{dt} M_o(t)$$

$$\mu_2 \text{ (S.D)} = \frac{d^2}{dt^2} M_o(t)$$

Binomial distribution

It is concerned with trials of a repetitive nature in which only occurence or non-occurences, sucess or failure, yes or no of a particular event.

Probability of sucess is $n_{cr}\, p^r\, q^{n-r}$

P = probability of sucess.

q = probability of failure.

The probability of atleast r sucess in n trials

$$= n_{cr}\, p^r\, q^{n-r} + n_{cr+1}\, p^{r+1}\, q^{n-r-1} + \ldots + n_{cn}\, p^n$$

Mean $(\mu) = np$

S.D. $= \sqrt{npq}$

Poisson Distribution

It is a discrete distribution related to the probabilities of events which are extremely rare.

This distribution can be derived as a limiting case of the binomial distribution by making 'n' very large and p very small.

$$p(r) = \frac{\lambda^r}{r!} e^{-\lambda}$$

So that the probabilities of 0, 1, 2 ..... r successes in a poison distribution are given by,

$$e^{-\lambda}, \lambda e^{-\lambda}, \lambda^2 \frac{e^{-\lambda}}{2!}, \ldots\ldots \frac{\lambda^r e^{-\lambda}}{r!}$$

λ = mean

$\sqrt{\lambda}$ = S.D.

Scanned by CamScanner

rmal Distribution

Any quanity whose variation depends on random causes is distributed accord- to the normal law.

Mean – mode = 3 (mean – median)

Median— If the values a variable are aranged in ascending order of magnitude, median is the middle item if the number is odd and is the mean of the two mid- if the number is even.

Mode— The mode is defined as that value of the variable which occurs most uently i.e. the value of maximum frequency.

$$f_x(x) = \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{\frac{-(x-\mu)^2}{2\sigma^2}}$$

❑

Scanned by CamScanner

EXAMPLES

Example 1.

The probability of 4 turning up atleast once in two tosses of a fair die is

Solution :

4 in both the tosses $= \frac{1}{6} \times \frac{1}{6}$

4 in one, other in second $= \frac{1}{6} \times \frac{5}{6}$

Other in one, 4 in second $= \frac{1}{6} \times \frac{5}{6}$

$= \frac{1}{36}(1+5+5)$

$= \frac{11}{36}$

Example 2.

A machine produces bolts which are 30% defective. In a random sample of 6 bolts produced by this machines the probability that less than 2 bolts will be defective is

Solution :

$$p = \frac{30}{100}$$

$= 0.3$ (probability of defective bolt)

$q = 1 - 0.3 = 0.7$ (probability of correct bolts)

(p= probability that no bolt is defective + probability that one bolt is defective)

$$= n_{c0}\, p^0 q^n + n_{c_1}\, p^1 q^{n-1}$$

$$= (0.7)^6 + 6\,(0.3)^1\,(0.7)^5$$

$$= 0.1176 + 0.3025$$

$$= 0.42$$

Example 3.

The expected value of x^2 is .... if the density function of a random variable is given by

$$f(x) = \begin{cases} x/3 \text{ for } 0 < x < 2 \\ 0 \text{otherwise} \end{cases}$$

$$f(x) = \frac{x}{3}$$

Scanned by CamScanner

$$E\left[x^2\right]=\int_0^2 x^2 \, . f(x) \, . dx$$

$$=\int_0^2 \frac{x^3}{3} \, . dx$$

$$=\left[\frac{x^4}{12}\right]_0^2=\frac{16}{12}$$

$$=\frac{4}{3}$$

Example 4.

Concentric circles of radius 1.0 cm. and 3.0 cm. are drawn on a target of radius 5.0 cm. Depending on hitting the target in the inner most circle, in the middle annular space and in the outer annular space a persons gets 10, 5 or 3 points respectively. If the probability of hitting the target is 0.5 with equal likely most of hitting any point on the target, what is the expected number of points one would score by firing once.

Solution :

Probability of hitting inner most circle

$$=0.5\times 1\times\frac{\pi(1)^2}{\pi(5)^2}$$

$$=0.5\frac{\pi}{25\pi}$$

Point earned

$$=0.5\frac{\pi}{25\pi}\times 10$$

$$=\frac{1}{5}$$

Similarly expected point earned due to probability of hitting middle and outer space.

$$=0.5\times\frac{8}{25}\times 5 \quad \text{and} \quad 0.5\times\frac{16}{25}\times 3$$

Thus expected no. of point one should score by firing once

$\Rightarrow \quad \frac{1}{5}+\frac{8}{10}+\frac{4.8}{5}$

$\Rightarrow \quad \frac{9.8}{5}$

$\Rightarrow \quad 1.96$

Ans.

Scanned by CamScanner

EXERCISE

Exercise 1.

A miner is trapped in a underground location of a mine containing three doors. The first door leads to tunnel which takes him safely after 3 hrs. travel. The 2nd door leads to another tunnel, which returns him to the same place after one-hour travel. The 3rd door leads to yet another tunnel, which again returns him to the same place after 2 hours travel. Assuming that the miner is at all times equally likely to choose any of the doors, what is expected length of time until miner reaches safely.

Exercise 2.

In a car factory, the chances of producing defective car is 1 in 2500 in a production lot of 50 cars. Using poission distribution calculate the approximate number of lots containing no defective, one defetive and two defective cars respectively in a yearly production of 17,500 lots.

Solution 1.

The miner car come out of the underground in 5 ways.

1^{st} tunnel = 3 hrs.

2^{nd} & 1st tunnel = 1 + 3 = 4 hrs.

3^{rd} & 1^{st} tunnel = 2 + 3 = 5 hrs.

2^{nd}, 3^{rd} and 1^{st} tunnel = 1 + 2 + 3 = 6 hrs.

3^{rd}, 2^{nd} & 1^{st} tunnel = 2 + 1 + 3 = 6 hrs.

∴ expected time of coming out

$$= \frac{1}{5}(3+4+5+6+6)$$

= 4 hrs. & 48 mins.

Solution 2.

$$\lambda = np$$
$$= \frac{1}{2500} \times 50$$
$$= \frac{1}{50}$$
$$= 0.02$$

no defective car in lot

$$= \frac{\lambda^0 e^{-\lambda}}{0!}$$
$$= e^{-0.02}$$
$$= 0.98$$

number of lots of no defective cars

$$= 0.98 \times 17{,}500$$

$$= 17{,}153$$

Similarly,

number of lots of one defective cars

$$= \frac{(0.02)}{1!} e^{-0.02} \times 17{,}500$$

$$= 343$$

number of lots of two defective cars

$$= (0.02)^2 \frac{e^{-0.02}}{2!} \times 17{,}500$$

$$= 3$$

Scanned by CamScanner

QUESTION PAPER

2000

MN : Mining Engineering

Duration : Three hours Maximum Marks : 150

This question paper contains 15 pages

Read the following instructions carefully

1. Write all the answers in the ANSWER BOOK only.
2. This question paper consists of **TWO SECTIONS : A** and **B.**
3. **Section A** has **TWO** questions of the multiple choice type. Question 1 consists of **TWENTY FIVE** sub-questions of one mark each and Question 2 consists of **TWENTY FIVE** sub-questions of two marks each. Answer all the sub-questions of both these questions on the page of the answer book specially marked for this purpose.
4. Answers to **Section B** should be started on a fresh page and should not be mixed with the answers to **Section A.** Question numbers must be written legibly and correctly in the answer book.
5. **Section B** consists of TWENTY questions of five marks each. ANY FIFTEEN out of these have to be answered. If more questions are attempted, score off the answers not to be evaluated, else only the first fifteen unscored answers will be considered strictly.
6. In all 5 mark questions, clearly show the important steps in your answers. These steps will carry partial credit.
7. There will be no negative marking.
8. Use of calculator is allowed.
9. Graph paper will be supplied if needed.

Scanned by CamScanner

SECTION A

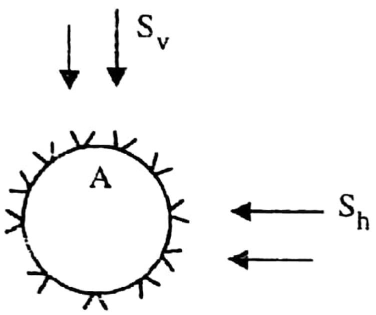
(75 marks)

1. In each of the sub-questions below four alternatives, of which one is correct. Choose the alternative and write in your answer book the alphabet A, B, C, or D against the corresponding sub-question number. The answers to the multiple choice question MUST be written only in the boxes corresponding to the question in the first page of the answer book.

(25 × 1 = 25)

1.1. The sequence of attachments between the winding rope and the cage is
(A) bull chains, triangular plate, capel, detaching hook.
(B) detaching hook, capel, detaching hook.
(C) capel, detaching hook, bull chains, triangular plate.
(D) triangular plate, detaching hook, bull chains, capel.

1.2 A circular opening in a biaxial stress field is as shown, with Sv and S_h being the vertical and horizontal compressive stresses respectively. The point A is under tensile stress when



(A) $S_h = 0$
(B) $S_h = S_v / 3$
(C) $S_h = S_v$
(D) $S_h = 3\ S_v$

1.3 The most important chemical in a self rescuer is
(A) hopcalite.
(B) lithium chloride.
(C) protosorb.
(D) calcium bromide.

1.4 The feasible region of an LP problem in two variables is defined by

$$X_1 \leq 4.5,\quad X_2 \leq 3.7,\ \text{and}\ X_1 + X_2 \leq 7.0$$

along with the nonnegativity constraints. The number of corner feasibility points are
(A) 2
(B) 3
(C) 4
(D) 5

1.5 What is the slope of the tangent drawn to the circle $X^2 + y^2 = 10$, at (1,3)?
(A) 1/3

Scanned by CamScanner

(B) 3
(C) –1/3
(D) –3

1.6 In a mine for one shovel six trucks are assigned. The shovel loading time per truck is 5 min, and truck cycle time 20 min. Calculate the match factor.
(A) 0.66
(B) 1.00
(C) 1.20
(D) 1.50

1.7 A weak sandstone has modulus of elasticity of 1000 MP a and Poisson's ratio 0.25. Its shear modulus is
(A) 4 GPa
(B) 400 MPa
(C) 500 MPa
(D) 800 MPa

1.8 Intrinsically safe apparatus is designed to operate within a maximum voltage of
(A) 25V
(B) 50V
(C) 75V
(D) 100V

1.9 The location of a vertical shaft for a horizontal coal seam depends on the shaft
(A) depth.
(B) horizontal distance from each production location.
(C) distance from surface fan.
(D) depth and horizontal distance from each production location.

1.10 Given $Y = X^2$, What is d/dx (in Y) at x = 1?
(A) in 0.5
(B) in 2.0
(C) 0.5
(D) 2.0

1.11 High production rates coupled with large scale and extensive subsidence results from the method of
(A) top slicing.
(B) block caving.
(C) sublevel caving.
(D) VCR mining.

1.12 A core of 10.0 cm height and 5.4 cm diameter is subjected to point loading, The ram of the hydraulic jack has a cross-sectional area of 12.5 cm^2 and the dial gage reading is 3.0 MPa. What is the point load strength?
(A) 1.63 MPa
(B) 30 MPa

Scanned by CamScanner

(C) 37.5 MPa

(D) 68.7 Mpa

1.13 Draeger 174 does not contain

(A) by-pass valve.

(B) breathing bag.

(C) CO_2 absorbent.

(D) coolant.

1.14 The type of organisational structure characterised by the wastage of material, manpower, and lack of specialisation is

(A) functional.

(B) scalar.

(C) line and staff.

(D) line, staff and committee.

1.15 The series 1, 1/3, 1/9, 1/27........... converges to

(A) 1.44

(B) 1.50

(C) 1.63

(D) 2.00

1.16 The process of surface highwall recovery by making openings into the seam without removing the overburden is called

(A) auger mining.

(B) contour mining.

(C) area mining.

(D) strip mining.

1.17 Select the correct order with respect to the core sizes

(A) AX > BX > EX > NX

(B) NX > BX > AX > EX

(C) AX > BX > NX > EX

(D) EX > AX > BX > NX

1.18 Underground coal mines are necessarily ventilated by exhaust system due to the hazard of

(A) cole dust.

(B) fire in the intake shaft.

(C) spontaneous heating.

(D) methane.

1.19 Ore grade for which revenue from the recoverable reserve exactly equals the cost of mining, treatment, and marketing is known as

(A) cut-off grade.

(B) average grade.

(C) break-even grade.

(D) liquidation grade.

1.20 A solid cylinder is measured to have a height of 1.0 m and radius 0.5 m. The actual radius is however 0.55 m. The resultant error in the estimation

Scanned by CamScanner

of the cylinder volume in m^3 is

(A) 0.005π

(B) $0.05\ \pi$

(C) $0.5\ \pi$

(D) 0.12

1.21 The height of the caveable roof in a depillaring of a coal mine is 6.0m. If the mining height is 2.5 m, what is the bulking factor of the roof strata?

(A) 2.0

(B) 1.6

(C) 1.4

(D) 1.2

1.22 In VCR method of mining the charge length to diameter ratio is restricted to

(A) 2 : 1

(B) 4 : 1

(C) 6 : 1

(D) 8 : 1

1.23 Velocity as measured by an anemometer is 2.12 m/s while the true velocity for the air stream is 2.17 m/s. If the error is due to yaw, what is the angle of yaw?

(A) 5.3^{o}

(B) 12.3^{o}

(C) 77.7^{o}

(D) 84.7^{o}

1.24 By the 'written down' method the depreciation charge of an equipment of Rs. 1,50,000 at the end of the 5th year at 10% depreciation rate is

(A) Rs. 15, 009

(B) Rs. 10,401

(C) Rs. 9,842

(D) Rs. 8,857

1.25 An orthogonal matrix A is such that

(A) $A^{-1} = A^{T}$

(B) $A = A^{T}$

(C) $AA^{-1} = A^{-1}A$

(D) $AA^{T} = A^{T}A$

MN2. In each of the sub-questions below four alternative are provided of which one is correct. Choose the correct alternative and write in your answer book the alphabet A, B, C or D against the corresponding sub-question number. (25 × 2 = 50)

2.1 An AFC of length 114 m runs at a speed of 0.535 m/s. The mass of chains and flights is 54 kg/m, and the coefficient of friction between the chain and trough 0.33. What is the power needed to drive the empty conveyor?

Scanned by CamScanner

(A) 6.94 kW
(B) 10.68 kW
(C) 21.36 kW
(D) 25.74 kW

2.2 At a current of 0.8 A, ignition time of detonator fuse head is 7 ms. If 192 J is the D.C. Exploder power required to ignite the blasting circuit what is the total resistance of the circuit?

(A) 38Ω
(B) 40Ω
(C) 43Ω
(D) 47Ω

2.3 Floor illumination at a point directly below a light source in an underground garage of height 4.0 m is 40 Lux. What is the floor illumination in Lux at a point 8.0m away from the light source?

(A) 2
(B) 5
(C) 10
(D) 20

2.4 The amount Rs.5000 compounded quarterly at 8% rate of interest in 10 years time becomes

(A) Rs. 10,795
(B) Rs. 10,975
(C) Rs. 11,040
(D) Rs. 11,400

2.5 The eigen vectors of the matrix $\begin{bmatrix} 3 & 2 \\ 1 & 2 \end{bmatrix}$ are

(A) 4, 2, 1
(B) 4, 2
(C) 2,1
(D) 4,1

2.6 If a pump develops 10 bars of pressure and has a flow rate of 1500 l/min, what will be the pump motor power at 90% efficiency?

(A) 25 kW
(B) 28 kW
(C) 30 kW
(D) 45 kW

2.7 A sandstone specimen of diameter 54 mm and height 60 mm has moisture content of 20%, and specific gravity 2.65. The porosity of the sandstone is

(A) 0.346
(B) 1.530
(C) 1.732
(D) 2.887

Scanned by CamScanner

2.8 The downcast and upcast shafts of depth 500 m in mine have mean air densities 1.15 and 1.12 kg/m3 respectively. If the mine resistance is 1.0 Ns^2/m^8 how much air flows through the mine when fan is not operating?

(A) 1.2 m^3/s

(B) 6.9 m^3/s

(C) 12.3 m^3/s

(D) 16.9 m^3/s

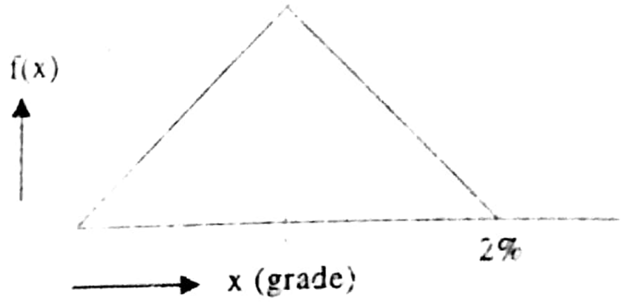
2.9 The probability density function of the grade values obtained from an exploration process is shown in the figure. If the cut off grade is 0.5%, what percentage of the mineralisation will be mined?

(A) 12.5

(B) 25.0

(C) 75.0

(D) 87.5



2.10 A force P(i + j + k) acts on a particle which is displaced from (1, -1, 2) to (2, 1, 3). The units of work done is

(A) 1

(B) 2

(C) 3

(D) 4

2.11 The r.m.s. torque of a 3.8 m diameter winder is 109 kNm. If the rope speed is 7.6 m/s, the motor power required to run the winder is

(A) 436 kW

(B) 506 kW

(C) 872 kW

(D) 1574 kW

2.12 A resin grouted rock bolt is able to provide an yield strength of 200 MPa. If the bolt diameter is 2.5 cm and the grout length 150 cm, what is the adherence (shear strength) between the bolt and the grout?

(A) 800 kg/cm^2

(B) 800 kPa

(C) 80 MN

(D) 8000 N/m

2.13 If f_1, f_2 and f_3 are the consecutive mid-band frequencies, then octave band analysis of noise means that

(A) $f_2^2 = f_1 \cdot f_3$

(B) $f_2 - f_1 = f_3 - f_2$

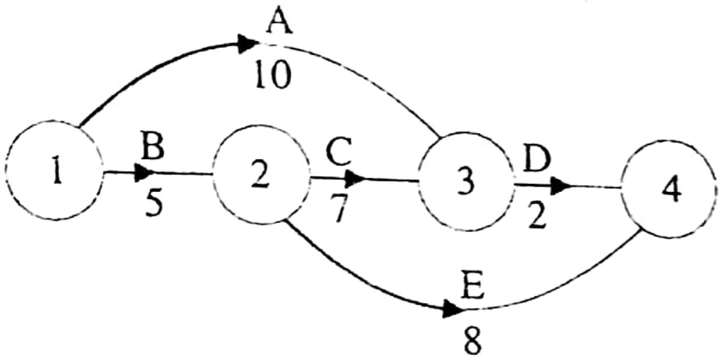
(C) $1/f_2 = (1/f_1) + (1/f_3)$

(D) $f_2/f_1 = f_3/f_2 = 8$

Scanned by CamScanner

2.14 In the project network shown activity durations are given in months. If one activity can be crashed to expedite the project, which one would you crash and by how many months?

(A) C,1
(B) C,2
(C) D,1
(D) E,1



2.15 Thickness values of a coal seam are distributed normally with mean of 2.5 m and variance 0.25 m^2. 90% of the thickness values observed lie in the range of

Hint: $z = 1.65$ for $\varnothing(z) = 0.95$

(A) 2.0875 to 2.1925 m
(B) 1.675 to 3.325 m
(C) 0 to 2.9125 m
(D) 0 to 3.325 m

2.16 A conveyor is 600m and long and coal of bulk density 0.8 t/m^3 up a gradient of 1 in 60 at the rate of 220 t/hr. If the belt width (w) is 0.75 m and cross-sectional area of the material 0.1 w^2, determine the belt speed.

(A) 0.04 m/s
(B) 0.96 m/s
(C) 1.36 m/s
(D) 1.80 m/s

2.17 A tension crack of 5.0 m depth behind a rock slope crest is completely filled with water. What is the horizontal force exerted on a unit slope length by the water in the tension crack.

(A) 122.60 kN
(B) 245.00 kN
(C) 12.26 MPa
(D) 24.50 Nm

2.18 Identify the correct sequence of numbers that match with a, b, c and d respectively.

Constituents of Coward flammability diagram	Explosibility status
a. O_2 : 21.5%, CH_4 : 5.0 %	1. Non explosive
b. O_2 : 10.0%, CH_4: 20.0%	2. Explosive per se
c. O_2 : 10.0%, CH_4: 4.0%	3. Impossible mixture
d. O_2 : 15.0%, CH_4 : 7.0%	4. Potentially explosive

Scanned by CamScanner

(A) 4, 3, 2, 1
(B) 3, 4, 1, 2
(C) 3, 1, 2, 4
(D) 2, 4, 3, 1

2.19 The cost table of an assignment problem is as shown. Optimal assignment would result in the cost units of

		Coal face 1	2	3	4
LHD	1	15	10	20	30
	2	10	30	40	15
	3	30	20	40	15
	4	20	25	10	15

(A) 40
(B) 45
(C) 60
(D) 75

2.20 A box contains 4 black and 7 white balls. Another box contains 3 black and 5 yellow balls. If one box is selected at random, and one ball is drawn, the probability for the ball being black is
(A) 0.738
(B) 0.432
(C) 0.369
(D) 0.245

2.21 A round stranded fibre core winding rope of diameter 2.54 cm has a space factor of 0.5. The tensile strength of the rope steel is 1500 MPa. At a safety factor of 10.0, what is the maximum load the rope can take?
(A) 38 kN
(B) 76 kN
(C) 3.8 MPa
(D) 7.6 kg/mm^2

2.22 During a laboratory permeability experiment on mill tailings 20 ml of water percolated in 30 min through a tailings of 5.0 cm diameter, and 20 cm length. If the tailings column is subjected to a steady hydraulic gradient of 1.2 m during the experiment, what is the hydraulic conductivity of the tailings?
(A) 94 Darcys
(B) 172 Darcys
(C) 0.94 cm/s
(D) 17.2 m^3/s

2.23 Identify the correct sequence of numbers that match with a, b, c and d respectively.

Scanned by CamScanner

Type of auxiliary ventilation	characteristic
a. Forcing	1. Inefficient flushing of face
b. Exhausting	2. Delivers fresh air to the face
c. Overlap	3. Fans in series can not be used in duct
d. Reversible	4. Expensive and hinders movement

(A) 4, 3, 1, 2
(B) 3, 2, 4, 1
(C) 2, 1, 4, 3
(D) 3, 4, 2, 1

2.24 On a certain spirit level, the bubble moves through 2 mm for a change of inclination of 20 seconds. What is the radius of curvature of the spirit level?
(A) 40.26 m
(B) 34.38 m
(C) 28.73 m
(D) 20.63 m

2.25 Given the set of observations 51.6, 48.7, 50.3, 49.5, and 48.7 identify the correct sequence of numbers that match with a, b, c and d respectively.

Parameter	Value
a. Variance	1. 49.8
b. Median	2. 1.2
c. Mode	3. 49.5
d. Mean	4. 48.7

(A) 2,4,1,3
(B) 4,1,2,3
(C) 1,4,3,2
(D) 2,3,4,1

SECTION B

(75 marks)

is section consists of TWENTY questions of FIVE marks each. ANY FIFTEEN t of these have to be answered. If more questions are attempted, score off the an-ers not to be evaluated, else only the first fifteen unscored answers will be consid-ed for marking.

MN 3. A bord and pillar of a coal mine lying at a depth of 200 m from surface is developed with square pillars of size 30 m, and galleries of width 4.6 m. The seam height 3.0 m is fully extractable and the overlying rock density is 2400 kg/m^3. Given the following equation to obtain the strength of the coal pillar

2400 kN/m^3.

$$\sigma_8 = 11.0\ w^{+0.5} h^{-0.5}\ \text{MPa}$$

determine the factor of safety of the pillar.

MN4. The immediate roof of a coal mine opening has strata in three layers of thickness 2.5 m, 1.5 m, and 2.0 m. If the roof bolting is done bind the layers together, by what percentage does the bending stress reduce compared to the unsupported roof span?

MN5. A level duct to size 1.0 m x 1.0 m and length 100.0 m has air flowing at a density of 1.2 kg/m^3 and velocity 100.0 m/s. The duct friction coefficient is 0.005 Ns^2/m^4 and fan is situated in the middle of the duct. Plot the static, total, and velocity pressure profiles over the length of the duct if inlet shock factor is 1.0.

MN 6. In an inventory model the order cost per cycle is 'a' rate of consumption 'd', order quantity per cycle 'q', and the holding cost per unit time per item 'h'. Develop the expression for inventory cost per unit time. Minimise this cost with respect to 'q' to obtain the 'Economic Order Quantity'.

Consumption rate of explosives in an opencast mine is 200 t per month, order cost Rs. 31,250 and holding cost per tonne per month Rs. 5,000. What is the EOQ?

MN 7. In a right angle triangle ABC, side b is measured to be 121.56 m long, With an absolute error such that $|\Delta b| = 0.05$m. The angle A is measured to be 25° 21'40" with an absolute error $|\Delta A| = 12''$. If the length of side a is calculated from the relationship a = b tan A, what is the absolute error in such calculation?

(Hint : $|\Delta A|$ must be considered in radians)

MN 8. A longwall mine uses a double ended ranging drum shearer with drums of web depth 0.5 m and diameter 1.24 m. The average speed of the shearer is 0.4 km/hr and the longwall face length150 m. Each shearer cycle has an average administrative time delay of 4 hours. Considering the panel is 400 m long and fully extractable, how many days of operation would be required before the panel is exhausted?

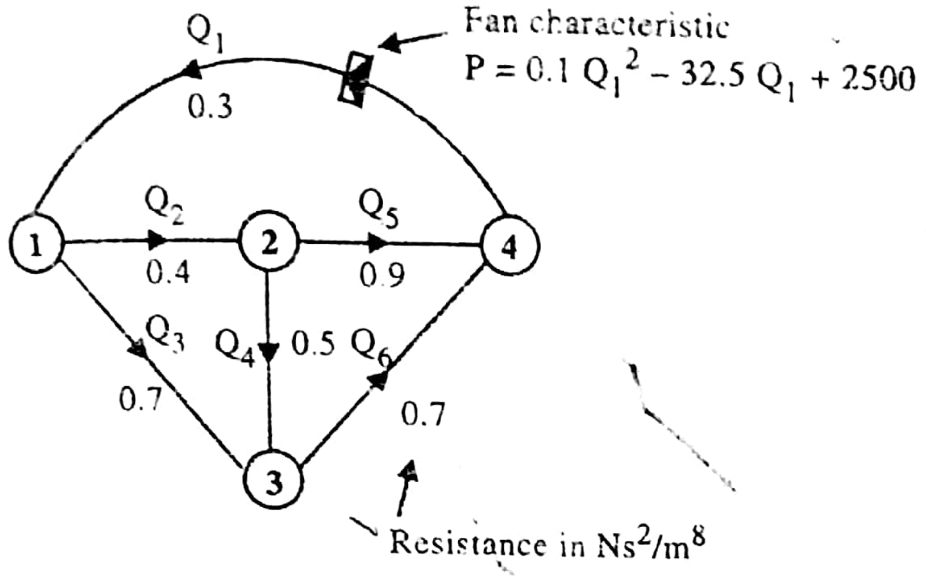
There are two production shifts each of 8 hours duration per day.

MN 9. During a triaxial loading test a core specimen failed along a plane at 50° from normal, for an axial load of 120 MPa and confining pressure 25 MPa. Using Mohr's circle derive the expressions for σ_n and τ on the failure plane in terms of principal stresses. Calculate the cohesion and the angle of internal friction from the experimental data.

MN 10. For the ventilation network problem shown develop (need not solve) the set of equations that need to be solved to obtain the flow rates Q_1 to Q_6.

MN 11. In a surface mine trucks arrive at a shovel in a Poisson process with a

Scanned by CamScanner



mean arrival rate of 9.6 trucks per hour. The shovel service time is exponentially distributed with a mean of 4.4 min, if there is no limit on the number of trucks that can wait, determine the expected

(i) number of trucks in the queue system.

(ii) number of trucks waiting for the service.

(iii) truck waiting time for service.

(iv) truck waiting time in the queue.

IN 12. In set of diamond drilling experiments on granitie the torque was measured as a function of RPM for a 55 mm diameter drill bit at 500 N thrust. Determine the best fit straight line relationship between the torque and the RPM.

RPM/100	Torque (Nm)
3.00	1.2
4.75	1.4
7.50	1.8
11.80	2.7

IN 13. A train of 200 tonnes moves up a slope of 1 in 100 against a resistance of 60 N per tonne with uniform acceleration of 2 m/s^2. The train in the process increases its speed from 7.2 to 21.6 kmph. Find the average power during acceleration.

IN 14. The effective principal stresses at a point underground where a major fault plane exists are,

$\sigma_1 = 12.2$ MPa, $\sigma_2 = 7.2$ MPa and $\sigma_3 = 5.2$ MPa If the direction angles between the principal axes and the fault plane are 24°, 71° and 104°, determine the normal and resultant shear stress on the fault plane.

IN 15. A ventilation system operates at a fan capacity of 40.0 m^3/s and pressure 600 pa. To improve the mine quantity the management is willing to consider widening and dressing the upcast shaft of depth 500 m. The diameter is projected to change from the present 3.0 to 3.5 m and the friction

Scanned by CamScanner

coefficient from 0.013 to 0.010 Ns^2/m^4. If the fan pressure remains unchanged, what will be the new mine quantity?

MN 16. The true bearing of a pole observed from station A is 350°35'20" and the magnetic bearing of the pole by the theodolite is 3°16'45". The magnetic bearing of the line AB observed from the same station is 137°27'15". What is the true bearing of the line AB?

MN 17. The probabilities for three men to hit a target 1/6, 1/4, and 1/3. If each person shoots once at the target, find the probability that exactly one of them hits the target.

MN 18. A roadheader and two shuttle cars are deployed to drive two headings interconnected with crosscuts under the following conditions :

Heading and crosscut dimensions	6.0 m × 1.8 m
Advance per cut	12.0 m
Tonnage factor	0.8 m^3/t
Shift working time	6.0 hours
Capacity of each shuttle car	5.0 t
Delay in loading and changing the shuttle car	1.0 min/trip
Delay to shift roadheader from face to fece	3.0 min/cut
Shift production target	972 t

Calculate the capacity of the roadheader in tonnes per minute to achieve the shift production goal.

MN 19. Distinguish among the sub critical, critical, and super subsidence phenomena with proper illustrations.

MN 20. Air enters an intake shaft at a dry bulb temperature of 30°C, flow rate 60 m^3/s, and density 1.15 kg/m^3. A sensible heat load transfer of 100 kW takes place from the rock to the air in the shaft. Assuming a specific heat of 1005 j/kg°C for the air, for the shaft depth of 500 m, predict the dry bulb temperature of the air at the shaft bottom, Consider that no moisture exchange takes place in the air.

MN 21. Three boreholes A,B, and C intersect a seam of coal at depths of 200m, 315m, and 270 m respectively. A is due south of B by 360m, and C is N 60°W of A by 255m. Calculate the direction and rate of full dip.

MN 22. Find du/dx if $u = x \cdot \log(xy)$, and $x^3 + y^3 + 3xy = 1$

●

Scanned by CamScanner

QUESTION PAPER

1999
MN : Mining Engineering

ration : Three hours

Maximum Marks : 150

This question paper contains 13 pages

Read the following instructions carefully

1. Write all the answers in the **ANSWER BOOK** only.
2. This question paper consists of **TWO SECTIONS : A** and **B**.
3. **Section A** has **TWO** questions of the multiple choice type. Question 1 consists of **TWENTY FIVE** sub-questions of one mark each and Question 2 consists of **TWENTY FIVE** sub-questions of two marks each. Answer all the sub-questions of both these questions on the page of the answer book specially marked for this purpose.
4. Answers to **Section B** should be started on a fresh page and should not be mixed with the answers to **Section A.** Question numbers must be written legibly and correctly in the answer book.
5. **Section B** consists of TWENTY questions of five marks each. ANY FIFTEEN out of these have to be answered. If more questions are attempted, score off the answers not to be evaluated, else only the first fifteen unscored answers will be considered strictly.
6. In all 5 mark question, clearly show the important steps in your answers. These steps will carry partial credit.
7. There will be no negative marking.
8. Use of calculator is allowed.
9. Graph paper will be supplied.
10. te = 1000 kgf = tonnes.

Scanned by CamScanner

SECTION A

(75 marks)

1. In each of the sub-questions below four alternatives are provided of which one is correct. Choose the correct alternative and write in your answer book the letter. A, B, C, or D against the corresponding sub-question number. The answers to the multiple choice question MUST be written only in the boxes corresponding to the questions in the first page of the answer book. (25 × 1 = 25)

1.1 The feasible region of a linear programming problem is
(A) concave (B) convex
(C) both concave and convex (D) none of the above.

1.2 Misfires in blast holes charged with LOX can be dealt by
(A) Pouring water in the blast hole to desensitize explosive.
(B) digging out the explosive carefully with a shovel.
(C) putting another detonator on the top the hole and blasting it.
(D) none of the above.

1.3 The formulae recommended by the Bureau of Indian Standards for calculating the peak particle velocity, in ground, due to blasting relating; peak particle velocity (U), mm/sec; constant, (K); charge per delay, (Q) kg; and distance from the blast (R), m; is
(A) $U = K (Q^2/R^2)^{1.5}$ (B) $U = K (Q^{2/3}/R)^{1.25}$
(C) $U = K (Q/R^{2/3})^{1.5}$ (D) $U = K (Q/R)^{0.5}$

1.4 The fluid used in liquid type of compensator is
(A) alcohol (B) silicon oil
(C) water (D) machine oil

1.5 The permissible closing error of an underground traverse having 16 stations measured with a 10 theodolite is
(A) 120 secs (B) 140 secs
(C) 150 secs (D) 160 secs

1.6 The net present value of Rs. 400 at the end of 5 years at 10% discount rate is
(A) Rs. 273 (B) Rs. 226
(C) Rs. 248 (D) Rs. 465

1.7 A square matrix of order 3 is non-singular if its rank is
(A) 3 (B) 2
(C) 1 (D) none of the above

1.8 A locomotive haulage train transports 50 te of coal in its outby journey. In the return journey it takes back 50 te of material inby. The Ideal gradi-

Scanned by CamScanner

ent for this locomotive haulage system is

(A) 1 in 50 in favour of coal movement

(B) 1 in 50 in favour of material movement

(C) zero

(D) none of the above.

1.9 Axial thrust in a turbine pump acts

(A) from suction side to delivery side

(B) from delivery side to suction side

(C) perpendicular to pump shaft

(D) in the delivery pipe

1.10 The velocity of air in a longwall face should not exceed

(A) 2 m/s (B) 3 m/s

(C) 4 m/s (D) 5 m/s

1.11 The three time estimates of PERT network are assumed to follow

(A) exponential distribution (B) normal distribution

(C) beta distribution (D) gamma distribution

1.12 Reportable injuries are those injuries other than serious bodily injuries which involve enforced absense of persons from work for

(A) more than 24 hours (B) more than 48 hours

(C) less than 72 hours (D) more than 72 hours

1.13 The uniform end of year payment, R, which can be realised for n years from a single present investment, P, at i rate of interest is

(A) $R = P(1+i)^n \left[\dfrac{i}{(1+i)^n - i}\right]$

(B) $R = P(1+i)^n \left[\dfrac{(1+i)^n - 1}{i}\right]$

(C) $R = \dfrac{1}{p(1+i)^n} \left[\dfrac{i}{(1+i)^n - 1}\right]$

(D) $R = \dfrac{1}{p(1+i)^n} \left[\dfrac{(1+i)^n - 1}{i}\right]$

1.14 A mine producing 500 te of coal per day has a methane emission of 60000 m^3 per day. The mine can be classified for gassiness as

(A) degree I (B) degree II

(C) degree III (D) none of the above

1.15 The shear stress on a plane on which the major principal stress acts is

(A) half the major principal stress

(B) twice the major principal stress

Scanned by CamScanner

(C) shear stress is absent

(D) none of the above

1.16 In a surface mine blast holes of 120 mm dia are to be drilled. If the rock hardness is 14 on Protodiyakonov scale, than the type of the drill machine used will be

(A) down-the hole drill

(B) Jack hammer drill

(C) rotary drill with tricone roller bits having large shaped teeth.

(D) rotary drill with tricone roller bits having buttons

1.17 Hydraulic shovels can do selective mining from a bench because

(A) they are highly mobile

(B) they are located on the top of a bench

(C) their buckets have wrist action

(D) hydraulic action gives them better breakout force

1.18 "Work study" is a generic term used for conducting

(A) method study

(B) time study

(C) work measurement

(D) method study and work measurement

1.19 The inflation pressure in truck tyre as tyre as recommended by the tyre manufacturer is checked

(A) when the truck has worked for 4 hours

(B) when it is fully loaded by the shovel

(C) when it is returns to the garage at the end of the shift

(D) when it is leaves the garage at the start of the shift

1.20 A coal heading 4m wide and 2.5 high has an advance of 1m per cycle. The amount of explosive used in blasting is 6 kg. Taking specific gravity of coal as 1.5, the powder factor is

(A) 1.66 te/kg (B) 2.50 te/kg

(C) 2.99 te/kg (D) 3.32 te/kg

1.21 In an underground coal mine, 1800 workers are employed. The average production from the mine is 900 tonnes per day. The O. M. S. of the mine is

(A) 0.24 te (B) 0.60 te

(C) 0.65 te (D) none of the above

1.22 $\lim_{x \to 1} \frac{x^3 - 1}{x - 1}$ is

(A) 3 (B) 1

(C) 0 (D) ∞

1.23 The area under the function $y=\sin x$ between $x=0$ and $x = \pi$ is

(A) 1 (B) 2

(C) 3 (D) 4

Scanned by CamScanner

1.24 Resuing method is generally adopted for

(A) flat thin deposits

(B) flat thick deposits

(C) steeply dipping thick deposits

(D) steeply dipping thin deposits

1.25 Dust particles most harmful to human respiratory system are of the size of

(A) about 1 micron (B) about 0.5 micron

(C) about 5 micron (D) about 0.05 micron

2. In each of the sub-question below four alternatives are provided of which one is correct. Choose the correct alternative and write in your answer book the letter - A, B, C or D against the corresponding the sub-question number. (25 × 2 = 50)

2.1 The solution of the differential equation $4yy' + 9x = 0$ is

(A) $\frac{x^2}{81} + \frac{y^2}{16} = c$ (B) $\frac{x^2}{9} + \frac{y^2}{4} = c$

(C) $\frac{x^2}{16} + \frac{y^2}{81} = c$ (D) $\frac{x^2}{4} + \frac{y^2}{9} = c$

2.2 A mine employing 2500 persons experienced 1 fatal accident, 5 serious injuries, and 7 reportable injuries for the year 1997. The frequency rate for serious injuries per thousand persons employed for the year 1997 was

(A) 12.5 (B) 15.0

(C) 20.0 (D) 32.5

2.3 Consider the following linear programming problem.

maximize $Z = 4X_1 + 6X_1$

Subject to the constraints:

$$x_1 \le 4$$

$$2X_2 \le 12$$

$$3X_1 + 2X_2 \le 18$$

and $X_1 \ge 0$ and $X_2 \ge 0$

The corner point feasible solutions are (0, 0), (0, 6), (2, 6) and (4, 0). The optimal value of Z is

(A) 34 (B) 36

(C) 44 (D) 50

2.4 Following is the state of stress in a biaxial stress field :

$\sigma_{xx} = 100$ MPa

$\sigma_{yy} = 30$MPa

$\tau_{xy} = 15$Mpa

The major principal stress is equal to :

(A) 103.08 MPa (B) 130 MPa

(C) 145.00 MPa (D) 165.28 MPa

Scanned by CamScanner

2.5 A vertical shaft for a mine is being sunk to a depth of 300m. The finished diameter is 6 m and the thickness of concrete lining is 30 cms. The volume of concrete required for the construction of the lining is

(A) 1293 m^3 (B) 1442 m^3
(C) 1538 m^3 (D) 1780 m^3

2.6 An ore body of uniform width of 20 m and dipping vertically is being worked by surface mining method. The density of the ore is 2500 kg/m^3 and that of the surrounding waste rock is 1500 kg/m^3. the side wall slope of the mine is 45 degrees. The average stripping ratios of the mine at the depth of 60m will be

(A) 0.9 m^3/te (B) 1.2 m^3/te
(C) 2.1 m^3/te (D) 0.6 m^3/te

2.7 A block ore between two levels 30m apart is 40 m long. The ore body thickness, specific gravity, and grade are 2m, 2.8 and 2% respectively. The tonnage of copper available in the block of ore is

(A) 134.4 (B) 156.8
(C) 236.2 (D) 290.3

2.8 The thickness of a m.s. pipe of 100 mm internal diameter pumping water against a head of 1000 m, taking safe tensile strength of mild steel as 150 MN/m^2 is

(A) 1.56 mm (B) 2.43 mm
(C) 2.98 mm (D) 3.27 mm

2.9 The centre lines of winding drum and winding pulley are parallel and 0.6 m apart. When one cage is at pit bottom, its winding rope is positioned at 1.8 m from the drum centre. The plan distance between the winding drum and the winding pulley axes is 60m. The fleet angle of installation is

(A) 1.15 degree (B) 1.35 degree
(C) 1.42 degree (D) 1.82 degree

2.10 A company takes a loan of Rs. 25,000 at 8% interest for the extension of a mine. To repay the loan, an additional saving of Rs. 2500 per annum is necessary. The period taken for repayment is

(A) 25 years (B) 28 years
(C) 21 years (D) 30 years

2.11 A haul ramp of 6% grade is to be constructed connecting a bench of 15m height to the surface. If the cost of ramp construction is 1 million rupees per kilometer, the cost of this ramp construction in million rupees will be

(A) 0.25 (B) 0.10
(C) 2.50 (D) 0.50

2.12 Immediate roof of a mine roadway is supported by 1.5 m long roof bolts placed at 1.2 m centers along and across the roadway. The bolts are of 4 sq. cm cross-sectional area and are made from steel having tensile strength 300 MN/m^2. Taking the sq. Gr. Of roof rock as 2.5, the factor of safety of the roof bolt is

Scanned by CamScanner

(A) 1.86 (B) 2.26

(C) 2.98 (D) 4.34

2.13 The expansion by Taylor series for x=sinx about 0 is

(A) $x-\frac{x_2}{2}+\frac{x^3}{3}$ (B) $1-\frac{x^2}{2!}-\frac{x^4}{4!}$

(C) $x-\frac{x^3}{3}+\frac{x^5}{5}$ (D) $x-\frac{x^3}{3!}+\frac{x^5}{5!}$

2.14 The probability of a 4 turning up at least once in two tosses of a fair die is

(A) 3/36 (B) 11/36

(C) 3/12 (D) 1/6

2.15 the most probable value and the error in calculating the area of a circle whose radius in metre is 25.35 + 0.05 will be

(A) 2018, 858 + 6.04, m^2

(B) 2018.858 + 5.75, m^2

(C) 2018.858 + 7.96, m^2

(D) 2018.858 + 4.62, m^2

2.16 A machine produces bolts which are 30% defective. In a random sample of 6 bolts produced by this machine, the probability that less than 2 bolts will be defective is

(A) 0.28 (B) 0.75

(C) 0.60 (D) 0.42

2.17 A fan running at 200 rpm develops a pressure of 50mm water gauge, the fan speed is increased to 300 rpm. The water gauge developed will be

(A) 75.00mm (B) 96.80 mm

(C) 112.50 mm (D) 164.06mm

2.18 An underground panel extracted in a coal mine has a strike length of 500m. After a subsidence survey, it was found that the length of a straight line between the extreme limits of the subsided area on surface is 593.2 m along the strike direction. The depth of the extracted panel from the surface is 100m. The angle of draw for the overlying strata assuming identical subsidence on either side of the centre of the panel is

(A) 18 degrees (B) 25 degrees

(C) 28 degree (D) 35 degree

2.19 The average temperature in upcast and downcast shaft in a mine are 40°C and 30°C respectively. The shafts are 300m deep. The motive column of air (down cast. shaft air) causing natural ventilation in the mine is

(A) 9.9 m (B) 11.8 m

(C) 13.9 m (D) 15.6 m

2.20 A belt conveyor system has the following specifications :

Belt tension in loaded side belt = 10 kN

Belt tension in the return belt = 5 kN

Speed of the belt = 120m/min

Scanned by CamScanner

The power to drive the belt is

(A) 2 kW (B) 10 kW

(C) 50 kW (D) 20 kW

2.21 The density function of a random variable x is given by :

$$F(x) = \begin{cases} x/3 & \text{for } 0 < x < 2 \\ 0 & \text{otherwise} \end{cases}$$

The expected value of x^2 is

(A) 4/3 (B) 8/9

(C) 9/8 (D) 2/3

2.22 The projection of vector A = 2i − j + k on the vector B = 7i − 4j + 4k is

(A) 19/9 (B) 23/9

(C) 22/9 (D) 30/9

2.23 The rock specimens obtained from the overlying strata in a coal mine were tested in uniaxial compression and the following results were obtained at 50% of average compressive strength

Longitudinal strain = $6 * 10^{-2}$

Lateral strain = $1.2 * 10^{-2}$

The ratio of vertical to horizontal stress in the mine is

(A) 3 (B) 4

(C) 2.5 (D) 0.3

2.24 A coal heading 4m wide and 2.5m high has an advance of 1m per blasting round. The amount of explosive used in the blasting round is 6 kg. If the sp. gr. of coal is 1.5, the powder factor is

(A) 1.66 te/kg (B) 2.50 te/kg

(C) 2.99 te/kg (D) 3.32 te/kg

2.25 An underground panel in a coal mine is being developed on bord and pillar system. The pillar size is 30m centre to centre and the adjacent galleries are 4m wide. The percentage of extraction during development is

(A) 17.62 (B) 19.48

(C) 24.89 (D) 26.24

SECTION B

(75 marks)

Answer any fifteen question in these section.
All questions are of 5 marks each.

3. A miner is trapped in an underground location of a mine containing three doors. The first door leads to a tunnel, which take him to safety after three hour's travel. The second door leads to another tunnel, which returns him to the same place after one-hour's travel. The third door leads to yet another tunnel, which again returns him to the same place after two-hour's travel. Assuming that the miner is at all times equally likely to choose any

of the doors, what is the expected length of time until miner reaches safety?

4. Suppoe that the demand function for a company's product is $p = f(q) = 1200 - 3q$, where p is price (in rupees) per unit when q units are demanded per week. Find the level of production per week that maximizes the company s total revenue and determine this revenue.

5. Find the maximum and minimum values for the following linear programming problems using the graphical solution method.

Objective function $z = 5x_1 + 3x_2$

subject to the constraints:

$$x_1 + x_2 \le 6$$

$$2x_1 + 3x_2 \ge 4$$

$$x_1 \ge 3$$

$$x_2 \ge 3$$

and $x_1 \ge 0$, $x_2 \ge 0$

also show the feasible region on the graph

6. An instrument is set up at station A and the staff is held vertically at point B. The angle of depression is 8° and the readings on the staff at B are 2.57, 1.73, 0.90. Calculate the RL of B and the distance AB. The height of the instrument is 1.40 m and the RL of A = 1000.05 m. The multiplying factor is 100 and (f+d) = 0.15.

7. Caculate the area of the triangle with vertices at A (2,3,5) B (4, 6, −1) and C (3, 6, 4)

8. The two underground roads AB and BC intersect at the deflection angle is 105°. These roads are to be connected by a circular curve of 150 m radius, Calculate (a) tangent distance, (b) main chord, (c) length of the curve, (d) rise of the curve.

9. Find the inverse of the matrix A

$$A = \begin{bmatrix} -2 & +3 & -3 \\ -2 & -2 & -3 \\ -3 & +2 & -2 \end{bmatrix}$$

10. In a surface mine, a coal seam of 12m thickness located under a standstone overburden bench of 25m height is being worked by a shovel. The mine has to produce 2800 tonnes of coal per shift of eight hours. The coal is to be mined by shovel having the following operating parameters : cycle time-40 seconds; bucket fill factor-0.9; material swell factor-1.3; bulk weight of broken coal-900 kg/cu.m; shovel utilization factor during the shift-0.8. Calculate the required bucket capacity and the required bench advance per shift if width of the cut is 25m.

11. Differentiate with simple sketches between horizontal cut and fill method and post pillar method of stopping.

12. A winder installation has the following details :
depth of the shaft = 300 m; height of the headgear pulley = 20 m; weight

Scanned by CamScanner

of the cage and suspension gear = 5000 kgf; weight of the loaded mine cars in the cage = 5 000 kgf; rope diameter = 40 mm. If the weight of the rope is 0.36 d^2 kgf/m and the breaking strength is 52d^2 kN, d being the rope diameter in cms, calculate the static factor of safety of the winding rope.

13. A pumping installation gas following specifications :-
    suction pipe : 20 cm diameter, 10 m length ;
    delivery pipe : 16 cm diameter, 1000 m length;
    quantity of water pumped : 2.4 cu.m/min;
    vertical height through which water is raised : 250 m.
    Neglecting head loss at entry and exit points, calculate the total head against which the pump is working and the required power of the driving motor, assuming the pump to be directly driven by the motor. Take coefficient of friction to the flow of water in the pipe as 0.01.
14. A longwall face is being supplied with 6000 cu.m/min of air by a single gate road of 4 m × 2.5 m cross section. If another gate road of 5 m × 2 m cross-section and same length is added in parallel to the existing gate road, find the new quantity of air supplied to the long wall face assuming that pressure difference across the start and end of the gate roads remains unaltered. Assume that both gate roads have identical coefficient of frictional resistance.
15. Calculate the pressure developed by a back ward-bladed centrifugal fan having the following specifications :
    fan diameter = 4 m; rpm = 300; blade angle = 40°
    velcity of flow = 4 m/s; manometric efficiency = 70 %
    Take density of air as 1.2 kgf/cu.m.
16. A retreating long wall face is to produce 2000 te of coal per day from a degree III gassy mine having a gas emission of 15 cu.m/te. If the percentage of gas in the return and velocity of air in the gate road should not exceed 0.5% and 4 m/s respectively, calculate the total minimum number of gate roads of 4 m × 2.5 m cross section to be driven for the development of long wall panel.
17. Calculate the thickness of a flat dam to withstand a water head of 30m to be constructed in an underground roadway. The details are as follows : width of roadway = 5 m; height of roadway = 3 m; safe shearing strength of the dam material = 0.15 MPa : safe shearing strength of surrounding strata = 0.3 MPa; safe compressive strength of dam construction material = 1 MPa. Compare this with the thickness of an arched dam at the same location having an outer radius of 4m.
18. Calculate the power of the motor of an endless haulage moving at a speed of 3.6 km/hour. The haulage is pulling coal up a gradient of 1 in 10 over a distance of 1000 m The number of mine cars in the loaded as well as empty set is 10. The pay load capacity of each mine car is 2 te and its tare is 1

Scanned by CamScanner

ie. Take the weight of the haulage rope as 2 kgf/m, frictional resistance of mine cars as 1/100, frictional resistance of rope as 1/10 and gearing efficiency as 80%.

19. A bord and pillar panel in a coal mine is being developed by 5-heading development system. The average availability of working faces in the panel at any time is 9. The headings are being driven by solid blasting and have the following specifications :

width = 4.8m height = 2.4m

pull per round = 1.2m specific gravity of coal = 1.5

Assuming that every face undergoes one complete cycle per shift and the mine works three shifts per day for 300 days during a year, calculate the yearly production from the panel. What should be the minimum quantity of air to be circulated on production from the panel. What should be the minimum quantity of air to be circulated on production basis through this panel? If the total number of workers working in the three shifts in the three shifts in the panel is 500, calculate the O. M. S. of the panel.

20. The following is the analysis of return air samples in a coal mine :

oxygen 19.90%,

nitrogen 78.65%,

methane 0.98%

Co_2 o.45%

Co 0.02%

Calculate the Co/o_2 deficiency and Co_2/o_2 deficiency ratio and comment on the likelihood of active fire in the mine. The composition of the atmospheric air is 20.93% O_2, 79.04% N_2, and 0.03% CO_2.

21. A cylindrical specimen of sandstone of 50mm diameter and 125mm length, tested in uniaxial compression gave the following results :

load at failure = 100 kN

longitudinal deformation upto failue = 1.5 mm.

Assuming load-deformation curve to be a straight line right up to failure, find the uniaxial compressive strength and the modulus of elasticity of the sandstone.

22. A mechanized longwall face in a 2m thick coal seam is supported by self advancing chocks erected at 1.2m centre to centre along the face. Assuming that the maximum length of overhanging rock from the rear end of the canopy is 4.8m and the immediate roof breaks ahead of face at a distance equal to seam thickness, calculate the required support capacity of the chock to be used. Take the bulking factor and the specific gravity for immediate roof rock to be 1.5 and 2.5 respectively and factor of safety for the chock to be 2.

Scanned by CamScanner

QUESTION PAPER

1998
MN : Mining Engineering

Duration : Three hours Maximum Marks : 150

This question paper contains 16 pages

Read the following instructions carefully

1. Write all the answers in the answer book only.
2. This question paper consists of **TWO SECTIONS** : A and B.
3. **Section a** has **EIGHT** question. Answer **ALL** question of this section Answer to question 1 and 2 are to be written on the first two pages of the answer book.
4. **Section B** has **TWENTY** questions. Answer any **TEN** question from this section. Score off the answers will be evaluated.
5. Answers to **Section B** should start on a fresh page and should not be mixed with answers to **Section A.**
6. Answers to questions and answers to parts of a question should appear together in the same sequence in which they appear in question paper.
7. In all questions of 5 marks each, write clearly the important steps as they carry partial credit.
8. I here will be no negative marking.
9. Graph paper will be supplied.

Scanned by CamScanner

SECTION A

(100 marks)

1 In each of the subquestions below four alternative answers are provided of which one is correct. Choose the correct alternative and write in your answer book the letter-A, B, C or D-against the corresponding subquestion number. (15 × 1 = 15)

1.1 If the value of coefficient of determination for a particular situation is 0.49 then coefficient of correlation is

(A) 0.49

(B) 0.07

(C) 0.7

(D) cannot be determined from the given information

1.2 In a PERT network the critical path is

(A) the path having longest time

(B) the path having shortest time

(C) the path having no dummy

(D) none of the above

1.3 Which of the following properties is not desirable for a suitable stone dust to be used for stone dusting in coal mines :

(A) good dispersibility

(B) solubility to lung fluids

(C) high tendency to cake

(D) light in colour

1.4 A rock sample was subjected to diametrical point load testing. The pressure gauge reading at failure is 12.5 MPa. If the ram area is 13.36 cm^2, and the diameter of the core tested is 54 mm, the point load strength index is

(A) 5.7 MPa (B) 5.21 MPa

(C) 4.85 MPa (D) 5.46 MPa

1.5 In rock blasting the term "impedance of medium" is the product of

(A) wave pressure and Poisson's's ratio

(B) wave pressure and density of medium

(C) propagation velocity and Poisson's ratio

(D) propagation velocity and density of medium

1.6 The Burnside Safety Boring Apparatus is used for

(A) drivage of tunnels in water bearing strata

(B) tapping water from water logged working

(C) drilling small holes through fire stopping

(D) exploratory drilling work in metal mines

Scanned by CamScanner

1.7 In a waiting line situation with Posson's arrival (mean λ) and exponential service rate (mean μ) the probability that service facility is idle is

(A) $\frac{\lambda}{\mu}$ (B) $1-\frac{\lambda}{\mu}$

(C) $\frac{\lambda}{\mu-\lambda}$ (D) $\frac{1}{\mu-\lambda}$

1.8 For a circular opening in rock under hydrostatic condition at great depth, the maximum tangential tensile boundary stress will occur at

(A) 0^{o} with vertical (B) 45^{o} with vertical

(C) 90^{o} with vertical (D) no where along the boundary.

1.9 The necessary condition for use of poisson's distribution is

(A) probability of one arrival per time interval is constant

(B) number of arrivals in any one time interval is independent of arrivals in other intervals

(C) probability of two or more arrivals in the same time interval is zero

(D) all of these

1.10 Oxygen flow rate in compressed oxygen Proto breathing apparatus is

(A) 1.0 litre/min (B) 1.5 litre/min

(C) 2.0 litre/min (D) 3.0 litre/min

1.11 Sublevel stoping method belongs to

(A) heavily supported class (B) artificially supported class

(C) caving class (D) open stoping class

1.12 As per DCF analysis a project is acceptable if

(A) discounting rate > IRR

(B) discounting rate < IRR

(C) discounting rate = IRR

(D) there exists no relation between the two

1.13 Of the following types of rails which are used in mine haulage tracks, find one not connected with haulage turnouts

(A) tongue rail (B) filler rail

(C) stock rail (D) bridge rail

1.14 The pressure equalization at stoppings by pressure chamber is practised in order to

(A) prevent further explosion in the sealed-off area

(B) control leakage across stoppings

(C) pump-in fresh air inside the sealed-off area

(D) act as insulation to prevent heat transfer from fire area

1.15 Compensator device is fitted in

(A) dumpy level (B) Cooke's reversible level

(C) tilting level (D) self aligning level

2. In each of the subquestions below four alternatives are provided of which one is correct. Choose the correct alternative and write in your answer book

Scanned by CamScanner

the letter-A, B, C or D - against the corresponding subquestion number.

$(20 \times 2 = 40)$

2.1 The rank of the matrix $\begin{bmatrix} 3 & -3 & 0 \\ 1 & 4 & 5 \\ 4 & 4 & 8 \end{bmatrix}$ is

(A) 1 (B) 2

(C) 3 (D) cannot be determined

2.2 If the area of the circle increases at a uniform rate, the rate of increase of the perimeter is proportional to

(A) $1/r$ (B) r where r = radius of the circle

(C) r^2 (D) $\sqrt{r}$

2.3 $10 m^3/s$ of the air is required to be sent through each of the two parallel splits X and Y of resistances $2.2\ Ns^2\ m^8$ and $5.0\ Ns^2\ m^8$, respectively. The size of the regulator in split X is

(A) $0.072\ m^2$ (B) $0.170 m^2$

(C) $0.536\ m^2$ (D) $0.717\ m^2$

2.4 A borrower offers 16% nominal rate of interest with quarterly compounding. The effective rate of interest is

(A) 16.6% (B) 17%

(C) 17.4% (D) 18%

2.5 A coal seam 3 in thick lying at an average depth of 100m has been developed by bord and pillar with square pillars of 28 m centre to centre and gallery of 4 m width. The average density of overlying strata is 24 KN/m^3. If the pillar strength is 4900 KN/m^2 the safety factor of the pillar is

(A) 0.6 (B) 1.0

(C) 1.5 (D) 2.0

2.6 The present value of a 4 year annuity of Rs. 10,000 discounted at 10% is

(A) 35,600 (B) 33.500

(C) 32.400 (D) 31.700

2.7 The approximate head generate by a single-stage centrifugal pump of 250 mm diameter impeller running at 1440 rev/min with manometric efficiency of 0.7 is

(A) 25m (B) 36m

(C) 72 m (D) 144m

2.8 For a typical hard rock with $E = 100 \times 10^3$ MPa and Poisson's ratio ν = 0.25, the modules of rigidity in GPa is

(A) 40 (B) 66.67

(C) 80 (D) 133.33

2.9 A box contains 5 red 4 white marbles. Two marbles are drawn successively from the box without replacement and it is noted that the second one is white. The probability that the first one is also white is

(A) $\frac{3}{8}$ (B) $\frac{4}{9}$

Scanned by CamScanner

(C $\frac{4}{5}$ (D) $\frac{2}{9}$

2.10 A 8t locomotive has a coefficient of adhesion of 0.18 and a coefficient of rolling resistance of 100 N/tone mass. The maximum tractive force available to haul the trailing load in kN along a straight track is

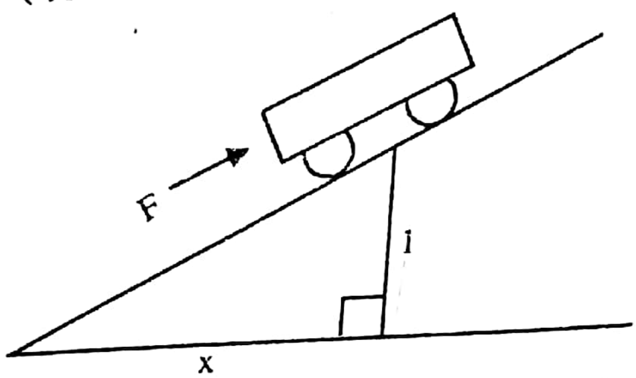
(A) 0.64 (B) 13.33

(C) 14.13 (D) 14.93

2.11 Neglecting the frictional forces, the force F required to prevent the mine car from moving down (as shown in figure) is

(A) $mg\left(\frac{1}{1+x^2}\right)$ (B) $mg\left(\frac{1}{\sqrt{1+x^2}}\right)$

(C) $mg\left(\frac{x}{\sqrt{1+x^2}}\right)$ (D) $mg\left(\frac{x^2}{1+x^2}\right)$



2.12 A standard pitot-static tube placed at the centre of a 300 mm diameter circular duct records a velocity pressure of 200 Pa. Assuming the air density of 1.24 kg/m^3 and the method factor of 0.84, the quantity of air flowing through the duct in m^3/s, is

(A) 1.07 (B) 1.27

(C) 4.27 (D) 5.07

2.13 With the bubble in the centre the reading taken on a staff point 160 m away is 2.30. The bubble is them moved out of centre by two divisions. The new staff reading is 2.24. If the length of one division is 2 mm the radius of curvature of the tube is

(A) 9.84 m (B) 16.00 m

(C) 10.66 m (D) none of these values

2.14 The length of consecutive cores obtained from 2m section of a borehole (NX size) are 89, 32, 18, 172, 80, 255, 280, 176, 53, 302, 102, 153, 40, 147 and 101 mm. Then the RQD value of rock in that section is

(A) 66.9% (B) 84.4%

(C) 95.5% (D) 100.0%

2.15 A small mining company borrows Rs. 50,000 at 6% interest in order to modernize its plant A direct saving of Rs. 6,000 per year will result and

Scanned by CamScanner

the plan is to use its sum annually against the loan. The time is will take to repay the borrowed money is

(A) 10 year (B) 12 year

(C) 13 year (D) 14 year

2.16 The corrected value (c_0) for the uni-axial compressive strength is the specimen size is not 1:1 is given by the relation where c_p is observed uni-axial compressive strength.

(A) $c_0 = \dfrac{c_p}{0.778 + 0.222 \frac{D}{L}}$

(B) $c_0 = \dfrac{c_p}{0.222 + 0.778 \frac{D}{L}}$

(C) $c_0 = \dfrac{c_p}{0.778 + 0.222 \frac{L}{D}}$

(D) $c_0 = \dfrac{c_p}{0.222 + 0.778 \frac{L}{D}}$

2.17 For the same roadway roughness, area, length and velocity condition the ratio of resistances of a square roadway to that of a rectangular roadway having width: height as 2:1 is

(A) $\frac{1}{\sqrt{3}}$ (B) $\frac{2\sqrt{2}}{3}$

(C) $\frac{3\sqrt{2}}{2}$ (D) $\frac{\sqrt{2}}{3}$

2.18 The length of a line measured by means of a 20 m chain was found to be 610.2 m. the true length of the line is known to be 612.0 m. The actual length of the chain is

(A) 20.06 m (b) 20.60 m

(C) 19.06 m (D) 20.42 m

2.19 Samples of air collected in the intake and return gate roads of an advancing longwall face show 0.25% and 0.75% CH_4, respectively. The panel has a production of 400 t in an 8 hour shift when an air quantity of 18 m^3/s passes through it. Then the methane emission in m^3 per tonne of coal mined is

(A) 0.02 (B) 0.81

(C) 2.25 (D) 6.48

2.20 For tower mounted friction hoist having a coefficient of friction (μ) between the rope and the sheave of 0.30, the wrap factor $\left(e^{\mu\theta}\right)$ is

(A) 1.60 (B) 2.57

(C) 4.11 (D) none of these value

Scanned by CamScanner

3. Match the two columns for each of the subquestion (from pairs by writing number alphabet). (10 × 2 = 20)

3.1 **Instrument** — **Parameter that can be estimated**

(1) Pilot static tube and manometer
(2) Anemometer
(3) Konimeter
(4) Asman psychrometer

(A) Dust concentration in air
(B) Relative humidity of air
(C) Velocity pressure of air
(D) Average velocity of air

3.2 **Component** — **Machine**

(1) Guide vane
(2) Sprocket drum
(3) Forepole ram
(4) Rifle bar

(A) Jack hammer drill
(B) Shield support
(C) AFC
(D) Turbine pump

3.3 **Component** — **Material handling system**

(1) Flame trap
(2) Trestle
(3) Flap gate
(4) steel cord

(A) Skip hoist
(B) Belt conveyor
(C) Bi-cable ropeway
(D) Locomotive

3.4 **Classification** — **Basis of classification**

(1) Gambie's
(2) Deere's
(3) Bieniawski's
(4) Barton's

(A) RMR
(B) Q
(C) Slake durability
(D) RQD

3.5 **drilling method** — **Principle of rock cutting**

(1) Diamond drilling
(2) Percussive drilling
(3) Rotary roller drilling
(4) Jet piercing

(A) Impact action
(B) Thermal spalling
(C) Rotary abrasive
(D) Cutting and ploughing

3.6 **Strata condition** — **Value of k in** $\sigma_x = k\,\sigma_y$

(1) Very shallow depth
(2) Moderate depth (ν=0.25)
(3) Wet, squeezing & running ground
(4) Region of recent volcanic activity

(A) $k > 1$
(B) $k = \frac{1}{3}$
(C) $0 < k < 1$
(D) $k = 1$

3.7 For the differential equation $y'' + ky' - 60\,y' = 0$

Value of K

(1) 4
(2) 7
(3) 11
(4) 17

Solutions

(A) $y_1 = e^{-12x}$, $y_2 = e^{5x}$
(B) $y_1 = e^{6x}$, $y_2 = e^{-10x}$
(C) $y_1 = e^{4x}$, $y_2 = e^{-15x}$
(D) $y_1 = e^{-20x}$, $y_2 = e^{3x}$

Scanned by CamScanner

3.8 **Ore body condition** — **Method of working**

(1) Strong, moderately thick, flatly dipping ore with strong host rock — (A) Cut and fill

(2) Strong, steeply dipping ore with strong host rock — (B) Sub-level

(3) Very weak moderately dripping ore with very weak host rock — (C) Room and pillar

(4) Strong to weak, fairly steep ore with weak host rock — (D) Square set

3.9 **Properties** — **Mine gas**

(1) colourless, odourless, extremely soluble in water, sp.gr = 1.529 — (A) SO_2

(2) Colourless, odourless, slightly soluble water highly toxic sp.gr = 0.972 — (B) CO_2

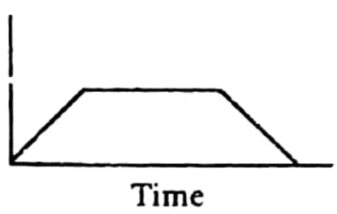
(3) Colourless, suffocating odour highly soluble in water, sp.gr = 2.264 — (C) CO

(4) Colourless, odourless, slightly soluble in water,sp.gr = 0.559 — (D) CH_4

3.10 For a standard 3-period speed-time diagram of a mine hoist with constant acceleration and deceleration,

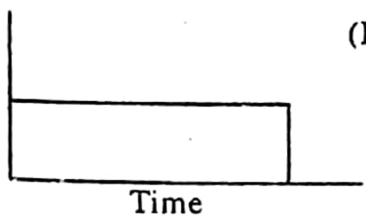
Diagram — **Y-axis parameter**

(1)



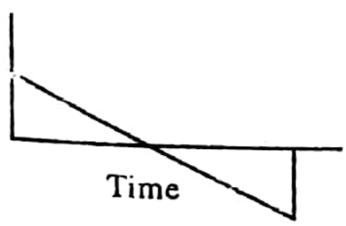
(A) Torque due to payload and friction

(2)

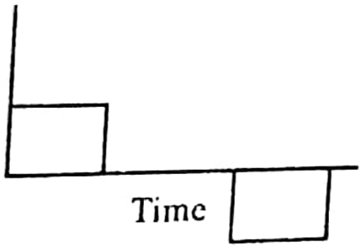


(B) Inertial torque

(3)



(C) Speed

(4) 

(D) Torque due to out-of-balance rope

4. Given below is the cashflow of a small project. Calculate Net Present Value of the project at a discount rate of 8% and also find its internal Rate of Return (5)

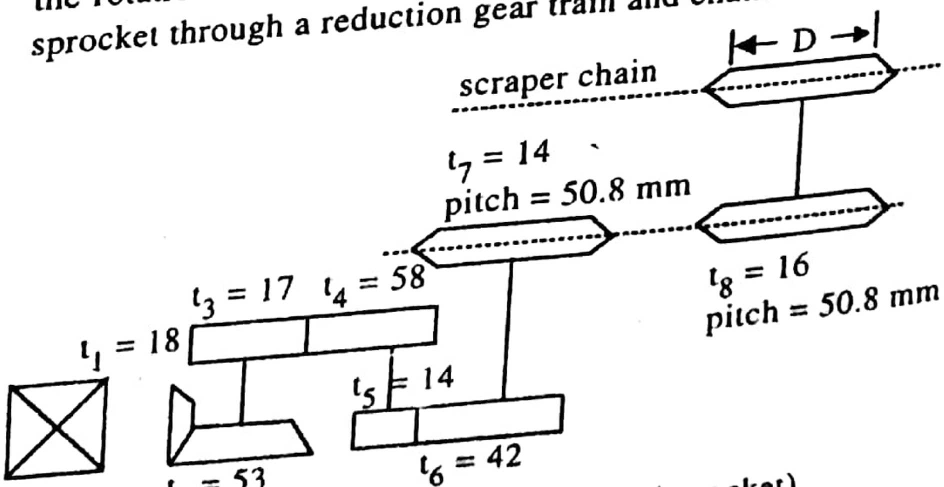
Year	Project cash-flow (Rs.)
0	-1,05,000
1	60,000
2	45,000
3	30,000
4	15,000

5. The following information is provided for overburden preparation requirement of a surface mine (5)

Spacing of drill holes	7 m
Burden of drill holes	4 m
Overburden to be rem	4,40,000 m^3
Hours of operation per shift	6
Shift per day	2
Drilling days per week	5
Drilling rate	20 m/hour

Find the total number of drills required.

6. The chain of a scraper conveyor moves at a speed of 0.57 m/s. Determine the diameter (D) of the driving sprocket as shown in the figure below, when the rotation is transmitted from the motor running at 1460 rev/min to the sprocket through a reduction gear train and chain drive.



(t_i = number of teeth in i-th gear/sprocket)

7. The analysis of a sample of air from old workings is reported as, O_2 - 15%, CO_2 - 2.8%, CH_4 - 4.3%, and N_2 - 77.9%. Find the percentage of air and

Scanned by CamScanner

blackdamp in the sample as well as the composition of blackdamp assuming air to contain 20.95% O_2, 0.03% CO_2 and 79.02% N_2. (5)

8. Calculate the radial (σ_r), tangential (σ_θ) and shear ($\gamma_{r\theta}$) stresses, on the circumference of a circular opening of 3 m diameter at $\theta = 0^0$ and $\theta = 90^0$. The unit weight of rock is 0.028 MN/m^3, Poisson's ratio is 0.3 and the depth of the excavation is 300 m. (5)

SECTION A

(~~100~~ 50 marks)

Answer any TEN question in this section.

9. Find the eigenvalues of the matrix $\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$ (5)

10. Find the general solution of the differential equation, $\dfrac{d^2y}{dx^2} = \dfrac{a}{y^3}$ (5)

11. A total flow of 85 m^3/s of air is allowed to distribute itself according to natural splitting among the following four parallel splits each having the coefficient of friction k= 0.0098 Ns^2m^{-4} (5)

Split	Cross section (m^2)	length (m)
A	1.5*6	610
B	2*5	450
C	2*4	380
D	2*5.5	580

what will be the distribution of air in the four splits?

12. A fan running at 300 rad/s in a mine gas a total resistance of 1.1 $Ns^2 m^8$ The fan characteristics at this speed is given by the relationship

$$p = 0.1Q^2 - 35Q + 2100$$

Where p is pressure in Pa and Q is quantity in m^3/s. It is desired to send an air quantity of 40 m^3/s by adjusting the speed of the fan. Find the required percentage change in fan speed. (5)

13. Readings taken on a vertical staff held at A 2.45, 3.10 and 3.75 m. The angle of elevation was 4^0. Reading taken on an inclined staff held at B perpendicular to the line of sight were 3.50, 3.00 and 2.50m. The angle of elevation was 5^0 30'. If the height of instrument was 1.70 m and the R.L.of ground was 1285.40, determine the RL's of A and B. (The instrument multiplying constant is 100) (5)

14. The full dip of a coal seam is 1 in 6 in direction of S 40^0E. A cross measure drift rising at 1 in 7 and bearing S 20° W is to be driven to intersect anther parallel coal seam which is 40 m vertically above. Calculate the length of the drift from floor to floor of the seams. (5)

Scanned by CamScanner

15. Explain briefly with sketch the principal of blasting gallery technique. State the important problems associated with the system. (5)

16. In an open pit mine a bench of 10 m height is being mined by a 6 m^3 shovel taking a face width of 10 m. The mining data are as follows:

Material swell factor	= 1.2
bulk weight of broken material	= 2.5t/m^3
bucket fill factor	= 0.8
shovel cycle time	= 40 sec

The mine works in 2 shifts / day of 8 hours duration each out of which 7 hours are used for production. The shovel is being served by 4 trucks of 50t capacity each and the round trip for a truck is 15 min. Calculate the advance of bench face in a day. (5)

17. In longwall mining with shearer. What is sumping? State the main methods of sumping with their principal of operation. (2 + 3)

18. In a depillaring district 380 m^3 of void is required to be filled up using a 280 m^3/h capacity pump to supply water. Density of sand used is 2.8 t/m^3. Assuming a hydraulic stowing factor of 0.95 calculate the volume concentration of solids in the slurry when the excavation is to be filled up in 3 hours. Finds also the density of the slurry and its velocity in a 200 mm diameter pipe. (2 + 2 + 1)

19. In a series of tri-axial compression tests on a sandstone the following represent the stresses at peak load condition.

Test	σ_3 MPa	σ_1 MPa
1	1.0	9.2
2	5.0	28.0
3	9.5	48.7
4	15.0	74.0

Determine the values of cohesion and angle of internal friction that best fit the data, and calculate the uniaxial compressive strength of the rock (3+2)

20. A borehole is drilled and instrumented, yielding the following values of the stress components in the plane perpendicular to the borehole:

$$\sigma_x = 1.724 \text{ MPa}$$
$$\sigma_y = 2.76 \text{ MPa}$$
$$\tau_{xy} = -0.689 \text{ MPa}$$

Find the magnitudes and directions of the major and minor principal stresses on this plane. (5)

21. (a) Explain the function of relays in open cast bench blasting. (3)
(B) What is presplitting and why it is used? (2)

22. In an irregularly spaced chip sampling of a vein the following data are obtained Calculate mean width of the vein and mean assay value. (5)

Position of sample (m)	2	8	10	15	20	30	32
Width (cm)	75	80	70	105	92	85	50
Value (g/t)	15	17	8	11	20	28	12

23. For a firm, total sales value is Rs. 1000,000 where total direct cost is Rs. 670.000 and total fixed cost is Rs. 230.000. Find the break even condition of the firm. (5)

24. A company owns two soft coke making plants A and B which have different production capacities for high, medium and low grade coke. The company is to supply every week, 12, 8 and 24 quintals of high, medium and low grade coke, respectively. The cost of running the plants A and B is Rs. 1000 and Rs. 800 per day. On a day the productions of high, medium and low grade cokes plants A and B are 6, 2, 4 and 2, 2, 12 quintals, respectively. Find graphically the number of days per week each plant should be operated to satisfy the supply most economically. (5)

25. (a) state the mathematical model of general transportation problem.
(b) how does it differ from assignment problem. (3+2)

26. A six stage turbine pump having six impellers is directly driven by an electric motor running at 1440 rev/min. The pump is used to raise $3.3 m^3/min$ of water against a total head of 240 m. If the radial velocity of water is 1.8 m/s, the manometric efficiency is 74%, the curvature of impeller vanes is 30^0 and the velocity of water at impeller at impeller inlet is 1.7 m/s, find the diameter and width of the impellers. (4 + 1)

27. A tower mounted friction winder hoists mineral from a depth of 294 m with a cycle time of 58 s. The constant acceleration and deceleration values are $0.6\ m/s^2$ and $0.75\ m/s^2$, respectively. Assuming a standard 3-period speed-time diagram determine the maximum hoisting speed and the distances travelled during the acceleration and deceleration periods. (3 + 2)

28. If $f(x, y) = x^2 - 2xy - 4x + 2y + 2y^2$ calculate the maxima or minima. (5)

Scanned by CamScanner

QUESTION PAPER

1997

MN : MINING ENGINEERING

SECTION A

Maximum Marks : 150

Duration : Three hours

1. Match the two columns for each of the subquestion (From pairs by writing number alphabet) (5 × 2 = 10)

1.1 For the function $z = y^2e^x + x^2y^3 + 1$

Partial derivative	Solution
(1) $\frac{\partial z}{\partial y}$	(A) $2y\lambda^x + 6y^2$
(2) $\frac{\partial^3 z}{\partial x^2 \partial y}$	(B) $y^2e^x + 2y^3$
(3) $\frac{\partial^2 z}{\partial x^2}$	(C) $2y\lambda^x + 3x^2y^2$
(4) $\frac{\partial^2 z}{\partial y \partial x}$	(D) $2y\lambda^x + 6xy^2$

1.2

Instrument	Measurement
(1) Flat jack	(A) insitu stress in rock
(2) Borchole deformation gauge	(B) Load
(3) Tape extensometer	(C) Bed separation resistance
(4) Borehole penetrometer	(D) Roof convergence

1.3

Chemical	Application
(1) Monoammonium phosphate	(A) Moisture absorption
(2) Alkaline pyrogallol	(B) Cooling agent
(3) Disodium phosphate	(C) Oxygen absorption
(4) Calcium bromide	(D) Fire extinguishing medium

1.4

Component	Machine
(1) Loading apron	(A) Road header
(2) Cowl	(B) Shield support
(3) Flight bar	(C) DFRES
(4) Lemniscate bar	(D) AFC

Scanned by CamScanner

1.5 For an emulsion explosive

Element	Constituent
(1) Dispersion redium	(A) Several nitrate salts
(2) Strengthene	(B) Microspheres of gas
(3) Oxidiser	(C) Diesel
(4) Sensitizer	(D) Aluminium powder

2. In each of the subquestions below four alternatives are provided of which one is correct. Choose the correct alternative and write the letter A, B, C or D against the corresponding subquestion number. (15 × 1 = 15)

2.1 For any moderately skewed unimodal frequency curve, the following is a valid empirical relationship

(A) Mode-Mean = 3 (Mean-Median)
(B) Median-Mode = 3 (Median-Mean)
(C) Mean-Mode = 3 (Mean-Median)
(D) Mean-Median = 3 (Median-Mode)

2.2 When a telescope is plunged, all common misadjustments of transit are compensated, accept for the misadjustment of

(A) collimation
(B) plate level
(C) telescope level
(D) horizontal axis

2.3 Activity durations in PERT and CPM are decided on the following consideration

(A) PERT-stochastic CPM-deterministic
(B) PERT-stochastic CPM-stochastic
(C) PERT-deterministic CPM-deterministic
(D) PERT-deterministic CPM-stochastic

2.4 The accident frequency rates in India are reported on the basis of

(A) one million man-hours worked
(B) one lakh man- shifts worked
(C) actual number of man-days lost
(D) one thousand persons employed

2.5 The ground in the central portion of the subsidence trough is subjected to

(A) lateral tension
(B) vertical compression
(C) lateral compression
(D) none of the above

2.6 A manometer cannot measure

(A) differential pressure
(B) velocity pressure
(C) gauge pressure
(D) absolute pressure

Scanned by CamScanner

2.7 A twin cable ropeway consists of
(A) two track ropes and one traction rope.
(B) one track rope and two traction ropes
(C) two track ropes and two traction ropes
(D) three track ropes and one traction rope

2.8 The minimum illumination standard at the coal mine pit bottom is
(A) 1.5 lux
(B) 1.5 lumens per square – feet
(C) 0.4 lumens per square – feet
(D) 2.4 lux.

2.9 Rock mass rating does not make use of
(A) compressive strength of rock
(B) drill core quality
(C) shear strength of rock
(D) groundwater condition

2.10 Terrace cut is made in an open pit mine using
(A) front – end loader
(B) bucket – wheel loader
(C) wheel tractor – scraper
(D) clamshell

2.11 One jack hammer produces a noise of 85 dBA. When four of these jack hammers work side by side and together, the resultant level is
(A) 88 dBA (B) 91 dB
(C) 91 dBA (D) 95 dB

2.12 Considering longwall mining of coal, identify the statement applicable to the advancing method.
(A) Easy maintenance of gate roads
(B) Superior ventilation
(C) prior knowledge of geological disturbances
(D) Early production from the longwall face

2.13 Internal rate of return of a project is the discount rate at which the
(A) profit after tax is zero
(B) net present value is zero
(C) written down value of the equipment is zero
(D) book value of the equipment is zero

2.14 Top slicing stoping method belongs to
(A) self supported class
(B) artificially supported class
(C) caving class
(D) in- seam mining class

2.15 The Hardy C_1 is method of ventilation network analysis makes of
(A) Taylor eries approximation
(B) Gaussian elimination

Scanned by CamScanner

(C) Cubic spline inter polation

(D) poloynomial trend surface

3. In each of the subquestions below four alternative are provided of which one is correct. Choose the correct alternative and write letter A, B, C or D against the corresponding subquestion number. (25 × 2 = 50)

3.1 A flat steel plate conveyor of width 0.72 m runs at a speed of 0.9 m/s. If the dynamic angle of repose of the material conveyed is 20^0 the conveyor theoretical capacity in m^3/hr is

(A) 152 (B) 212.3

(C) 394.6 (D) 1217

3.2 Which of the following statements is not applicable for respirable dust measurement in coal mines.

(A) Dust concentration is expressed in mg/m^3

(B) Percentage of free silica must be known

(C) Konimeter or its equivalent instrument is used

(D) Elutriator is used for dust separation

3.3 A level seam at a depth of 100 m is excavated to a width of 100 m. If the angle of draw for the subsidence over the seam is 35^0, then the subsidence phase is

(A) critical

(B) subcritical

(C) supercritical

(D) insufficient data to analyse

3.4 If A is a non-singular square matrix of order ^{n}, then det (adj A) is

(A) det (A^{-1}) (B) adj (det A)

(C) $(\det A)^n$ (D) $(\det A)^{n-1}$

3.5 In rotary drag bit drilling the primary cutting action is

(A) crushing (B) chipping

(C) spalling (D) melting

3.6 A fan drift of size 4.0 m * 3.0 m, having an air flow of 60 m^3/s experiences a shock loss of 15 pa. The shock factor for the drift is

(A) 0.1 (B) 0.5

(C) 1.1 (D) 2.3

3.7 For a circle whose radius is 25.35 ± 0.03 m, the most probable value of its area in m^2 is

(A) 2018.85 ±3.65 (B) 2018.85±4.78

(C) 2018.85±5.85 (D) 2018.85±6.12

3.8 For an open pit operation the value of metal is Rs. 210/kg, and recoverable grade is 1.2%. Production cost per tonne of ore inclusive of mining and processing but excluding stripping is Rs. 2,000. If the break even stripping ratio is $3.49m^3$/tonne, then the stripping cost is (1 tonne = 1,000kg)

(A) Rs. 149/m^3 (B Rs. 149/tonne

(C) Rs. 72/m^3 (D) Rs. 60/m^3

Scanned by CamScanner

3.9 Based on coward flammability diagram a methane air mixture containing 9.5% methane and 20% oxygen can be classified as
(A) optimum or stoichiometric mixture
(B) potentially explosive mixture
(C) explosive per se
(D) impossible mixture

3.10 During Brazilian test a rock specimen of diameter 5 cm, and height 5 cm, failed at a compression load of 19.6 kN. The indirect tensile strength of the specimen is
(A) 0.5 kPa (B) 2.5 MPa
(C) 5.0 MPa (D) 25.0 kPa

3.11 For a compressive strength teest 10 rock specimens are prepared, of which 3 are defective. If two specimens are drawn one after another at random without replacement, find the probability that none of the specimens is defective.
(A) 0.47 (B) 0.49
(C) 0.72 (D) 0.81

3.12 An increase in the fragment size from blasting can be correlated to a decrease in the cost of
(A) crushing
(B) secondary breakage
(C) primary drilling and blasting
(D) loading and hauling

3.13 Dumpers arrive at a shovel at a constant rate of 10 per hour. The service rate is constant at 15 per hour. At a time, the expected number of dumpers in the queue are
(A) 3 (B) 2
(C) 5 (D) 4

3.14 Condenser heat exchange in an underground air conditioning plant is 5.0 MW. If the plant coefficient of performance is 3.5, the evaporator heat exchange in MW is
(A) 1.43 (B) 2.43
(C) 3.89 (D) 6.43

3.15 The fraction of shines in the mill tailings used for filling cut and fill stopes should not exceed 10%. The reason being
(A) to provide smooth movement of mucking equipment
(B) to reduce solution while mucking
(C) for easy installation of chockmat
(D) to facilitate quick drainage of the fill material

$5\times10^3 \times 3600$ or 360 / $\pi\times25$ $= r$

3.16 If the bubble of a spirit level moves 5 mm for a change of inclination of 25 seconds, then the radius of curvature of the spirit level is
(A) 0.2 m (B) 41.25 m
(C) 125 m (D) 412.53 m

Scanned by CamScanner

3.17 Square pillars in a bord and pillar panel are of size 30 m centre to centre, and the galleries are of width 4.0 m. The tributary area for each pillar in m^2 is

(A) 224 (B) 256
(C) 676 (D) 884

3.18 Draw points in metalliferous mines are reinforced by

(A) chockmat
(B) bamboo mat
(C) full column grouted bolt
(D) stull and strut

3.19 Intake air to a longwall panel has CO at 10 ppm and O_2 at 20.5%. Where as the return air has CO at 30 ppm and O_2 at 20.1%. The Graham's ratio (%) for the oxidation process in the panel is

(A) 0.1 (B) 0.5
(C) 1.0 (D) 2.0

3.20 Insitu ore of 1.5% Cu got diluted with waste rock of value 0.2% Cu. If the resultant extracted ore has 1.2% Cu, what is the percentage dilution?

(A) 30 (B) 25
(C) 23 (D) 20

3.21 Given the function $n=x^2 + y^2 + z^2$, the magnitude of its gradient vector at point (1,1,1) is

(A) 2 (B) $\sqrt{6}$
(C) 6 (D) $2\sqrt{3}$

3.22 A 10,000 kg locomotive runs at a speed of 3 m/s along a level track. The coefficient of adhesion at the track is 0.23, and the rolling resistance of the locomotive is 90 N per 1000 kg. The maximum power available for hauling the train in kW is

(A) 40.7 (B) 65.0
(C) 67.1 (D) 67.7

3.23 Spiling may be necessary during drivage of underground roadways in order to

(A) protect men and machinery from falling rock
(B) dewater the face rapidly
(C) provide a stockpile room for the broken rock
(D) provide an alternative entry to the face

3.24 Failure times of a shearer are exponentially distributed. In one instance the shearer worked for one hour, and the probability of it working for the next one hour is 0.5. In the second instance the shearer worked for two hours. What is the probability that it will work for another one hour?

(A) 1.0 (B) 0.75
(C) 0.50 (D) 0.25

Scanned by CamScanner

3.25 Which of the following is an escape apparatus?

(A) Pulmotor

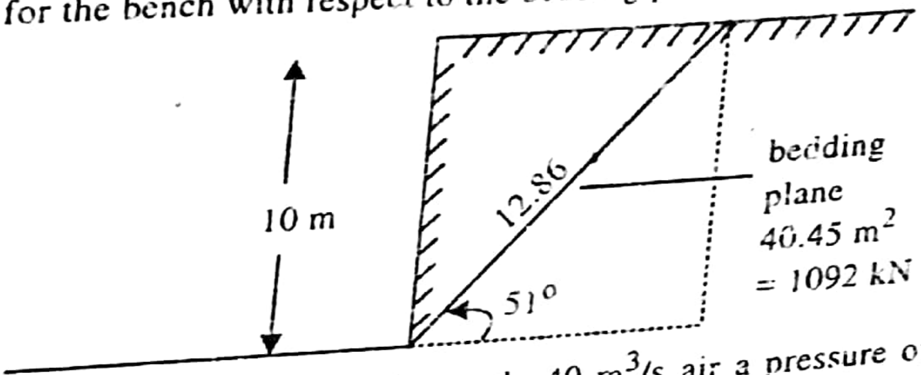
(B) Draeger BG 172

(C) Any open circuit rescue apparatus

(D) MSA self rescuer

4. Coal of bulk density 800 kg/m^3 is carried on a belt conveyor at 60 kg/s on a level roadway. The cross-sectional area of the load is 0.05 m^2, the total mass of the moving components of the conveyor 27,000 kg, and the frictional coefficient of the idler rollers 0.03. Neglecting other frictional losses, determine the component of the total power consumed by the conveyor alone. (5)

5. A 10 m high vertical bench has a bedding plane as shown in Fig. 5 which offers an angle of internal friction 12 degrees and cohesion 0.14 MPa. If the bulk weight of the rock is 27 kN/m^3, What may be the safety factor for the bench with respect to the bedding plane? (5)



6. A ventilation system provides only 40 m^3/s air a pressure of 800 Pa. A consultant identified that the return ventilation shaft of diameter 3.0 m is consuming 60% of the mine head, and suggested the widening of the shaft diameter to 4.0 m to the quantity. If it can be assumed that the shaft frictional characteristics and mine head remain unchanged, what will be the increase in the quantity once the shaft is widened? (5)

7. A tacheometer has a multiplying factor of 100, and an additive constant 0.3 m. During one measurement, with the telescope being horizontal, the readings of the three diaphragm weds on a vertical staff are 1.15 m, 2.30 m and 3.45 m. Find the distance from the instrument to the staff and the RL of the staff station, given the RL of the instrument axis is 75.75 m. (5)

8. Concentric circles of radius 1.0 cm and 3.0 cm are drawn on a target of radius 5.0 cm. Depending on hitting the target in the inner most circle, in the middle annular space, and in the outer annular space a person gets 10, 5 or 3 points respectively. If the probability of hitting the target is 0.5 with equal likelihood of hitting any point on the target, what is the expected number of points one would score by firing once. (5)

Scanned by CamScanner

SECTION B

(5 marks)

Answer any TEN question in this section

9. A face of dimensions 100 m * 2.0 m * 1.2 m requires to be filled in three hours by a hydraulic filling system. The fill material is of density 2500 kg/m^3, the hydraulic filling factor 0.9, and the ratio of the fill to the mixture (of water and fill) on volume basis 0.4 Estimate the hourly consumption of water for the filling operation. (5)

10. Design the size of regularly spaced square pillars, given the following data : (5)

depth of the opening	=	300 m
safe roof span	=	10 m
unit weight of rock	=	22 kN/m3
extraction ratio	=	0.65
safety factor	=	4.0

11. Give reasons, why (answer in 2 or 3 lines) (5 × 1 = 5)
   (A) dry kata cooling power may not be measurable in hot summer ?
   (B) evaporative cooling is not really cooling?
   (C) surface fan in a coal mine is situated 10 m away from shaft?
   (D) Ventilation door always opens towards intake side?
   (E) fan noise is continuously monitored?

12. The unit cost matrix of a transportation problem is given below

		destination 1	2	3	supply
source	1	2	3	4	10
	2	3	4	2	20
	3	4	3	2	30
demand		30	20	10	

The resource allocation is made as follows

		destination 1	2	3
source	1	10		
	2	20		
	3	0	20	10

Determine if the all allocation is feasible and optimal

(2 + 3 = 5)

Scanned by CamScanner

13. Without the use of simplex method or graph paper, determine the value of the objective function for the following linear programming problem. (5)

Maximize $W = x + y + z$

subject $0.5x + y + z \leq 18$

$x + 0.5y + z \leq 18$

$z = 8$, and $x, y \geq 0$

14. Write in brief the difference between (answer in 2 or 3 lines)
   (a) rock burst and out burst. (3)
   (b) blown out shot and misfired shot. (2)

15. For a subcritical phase of subsidence, illustrate with a neat sketch subsidence profile in the dip-rise orientation. When the coal seam is of uniform thickness but dipping moderately. Mark the following on your sketch. (2)
   (a) Angles of draw. (1)
   (b) Transition points. (1)
   (c) Maximum subsidence in the profile. (1)

16. A single, main surface fan in a mine delivers 40 m^3/s of air a pressure of 560 Pa. The fan is proposed to be replaced by a new one having the characteristics.

$P = 0.1\ Q^2 - 37.5\ Q + 2500$

Where, P is the pressure in Pa and Q the quantity in m^3/s. What is the operating point, once the new fan is installed? (5)

17. A mining equipment costing Rs. 12 million can be salvaged at a value of Rs. 3 million after 5 years of use. Determine the annual cost of depreciation and book value on the 'sum of the years digits' method? (5)

(5)

1.8 Find the angle between the vectors

$A = 2i + j + k$, and $B = i - 3j + 5k$

19. In one year 2.4 million m^3 of overburden is proposed to be removed from a dragline operated bench. Estimate the numbers of drills required for this purpose in the absence of any provision for standby machines. The other parameters being, (5)

expected rate of drilling	= 5 m/hr
bench height	= 14 m
burden	= 8 m
spacing	= 5m
subgrade drilling	= nil
availability of drill	= 5000hr/year
utilization	= 65 % of availability

20. At a point P in a body $\sigma_x = 100$MPa , $\sigma_y = \sigma_z = -50$ MPa; and $\tau_{xy} = \tau_{yz} = \tau_{zx} = 100$ MPa. Determine the normal and shearing stress on a plane that is equally inclined to all the three axes. (5)

21. In an arch shaped roadway as shown in Fig. 21 a 6.0 m long flat dam is constructed. The dam offers a shear strength of 200 kPa. At a safety of

Scanned by CamScanner

2.0 what is the water head the dam could withstand? (5)

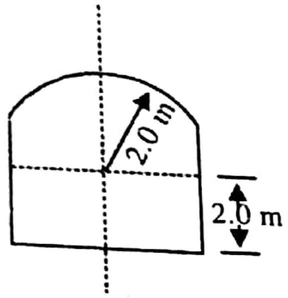


Fig. 21

22. From a surveyline perpendicular offsets are taken at 9.0 m intervals to an irregular boundary line. The offset values recorded in metres are 10.8, 13.9, 18.6, 16.9, 20.8, 22.9, 25.5, 23.2 and 19.9 Based on Simpson's rule compute the area enclosed by the first and the last offsets, the surveyline, and the irregular boundary. (5)

23. Prove that, for the matrix

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 2 & 3 & 4 \\ -1 & -1 & -2 \end{bmatrix}$$

all the eigen values use real and distinct. (5)

24. A backward bladed centrifugal pump produces a theoretical head of 60 m of water at a radial velocity of 2.0 m/s. Given th curvature of the impeller vane is 30 degrees and the impeller diameter 0.4 m. Determine the tangential speed for the impeller blade. What is the impeller RPM? (5)

25. Derive the expression for the future value (S_n) of an annuity (R)over a duration of n years at an interest rate of i per year. (3)
How much should an organization save annually so that over a period of 10 years, at an interest of 12%, the sum becomes 20 million rupees. (2)

26. Design a secondary or heavy type of stone dust barrier for use in a roadway of width 4.0 m, and height 3.0 m. (5)

27. Four samples surround a mineral block whose centre coincides with 0.0 m. Easting and 0.0 m Northing. The sample coordinates and grades are given below. Considering a radius of influence of 200 m, estimate the block value by inverse distance square method. (5)

Sample	Easting (m)	Northing	Grade (% Cu)
1	80	60	1.6
2	-90	120	1.4
3	-160	-160	1.0
4	30	-40	2.1

28. If Z is a homogeneous function of order n in x and y, show that

$$X^2 \frac{\partial^2 Z}{\partial X^2} + 2XY \frac{\partial^2 Z}{\partial X \partial Y} + Y^2 \frac{\partial^2 Z}{\partial Y^2} = n(n-1)Z$$

Scanned by CamScanner

QUESTION PAPER

1995

MINING ENGINEERING
SECTION A

Duration : Three hours

Maximum marks : 150

1. Match the two columns
(Write number = alphabet to form pairs)

1.1 Given below are some differential equation and names of their standard forms to which they belong (2)

(1) $x\,dy + (x + y)\,dx = 0$ (A) separable

(2) $\left(x^2 + y^2\right)\frac{dy}{dx} = xy$ (B) Exact

(3) $\tan\phi\, r\frac{d0}{dr}$ (C) Homogeneous

(4) $\frac{dy}{dx} = y \sec = \tan x$ (D) linear

1.2 Given below are the type of subsidence and the conditions for their occurrence (2)

(1) Subsidene plug (A) Succession of failure over small area

(2) Chimneying (B) Sudden lowering of strata

(3) Caving (C) Succession of failuresover large area in relatively

(4) Trough subsidence (D) succession of failures over large areas

1.3 Give below are the names of measuring instruments and the purpose for which they are used (2)

(1) Hot wire anemomenter (A) Cooling power

(2) Whirling hygroneter (B) Spontaneous heating

(3) DTA (C) Relative humidity

(4) Kata Thermometer (D) Velocity of air

1.4 Given below are the commercial names and their chemical compositions of the materials used in the preparation of explosives (2)

(1) Gun-Cotton (A) $C_2H_4(NO_3)_2$

(2) T.N.T. (B) $C_3H_5(NO_3)_3$

(3) Nitro-glycerine (C) $C_6H_2(NO_2)_3 \cdot CH_3$

(4) Ethylene glycol dinitrate (D) $C_{12}H_4O_4(NO_3)_6$

Scanned by CamScanner

1.5 Given below are the names of the fragmenting agents and the chemicals used for the same

(1) LOX	(A) Ammonium chloride
(2) ANFO	(B) Oxygen
(3) Dionating fuse	(C) Diesel oil
(4) Hydrox	(D) PETN

2. Write the correct alphabet against question numbers. (15 × 1 = 15)

2.1 As per the indian standard, the maximum distance from any physical evidence for the existance of coal deposit to be categorised as a proved reserve is,

(A) 100 m
(B) 200 m
(C) 300 m
(D) 400 m

2.2 The accuracy of measurement for determining the depth of a vertical shaft should be

(A) 1 in 500
(B) 1 in 1000
(C) 1 in 1500
(D) 1 in 2000

2.3 the ratio of the theoretical power input (p_1/p_2) of two centrifugal fans having the same specific speed and operating at same speed is given by

(A) $(D_1/D_2)^2$
(B) $(D_1/D_2)^3$
(C) $(D_1/D_2)^4$
(D) $(D_1/D_2)^5$

2.4 the threshold limit of respirable air borne dust containing less than 5 percent free silica at any work place to which a worker can be exposed during a shift is

(A) 3.5 milligram per cubic meter
(B) 3.0 milligram per cubic meter
(C) 2.5 milligram per cubic meter
(D) 2.0 milligram per cubic meter

2.5 The maximum wet-bulb temperature at working faces as required under mines regulations is

(A) 36°C
(B) 33°C
(C) 30°C
(D) 27°C

2.6 For a bulking factor of 1.5, the height of the immediate roof for an extraction thickness of H is

Scanned by CamScanner

(A) H

(B) 1.5 H

(C) 2.0 H

(D) 2.5 H

2.7 If the critical path of a project is 20 months long with a standard deviation of 4 months, the probabillity that the project will be completed within 20 months is

(A) 0.84

(B) 0.60

(C) 0.16

(D) 0.50

2.8 Reportable injuries are those which require the workers to be absent for a period of

(A) 7 days

(B) 14 days

(C) 21 days

(D) 28 days

2.9 Geothermal gradient in the indian coal fields is about

(A) 1^{o} C/20 m

(B) 1^{o} C/92 m

(C) 1^{o} C/38 m

(D) 1^{o} C/72 m

2.10 A drum shearer is mounted on.

(A) its own skid

(B) Separate rail

(C) A. F. C.

(D) Suport frame

2.11 Dust generation may be reduced by drilling with

(A) Low RPM

(B) Sharp bit

(C) High RPM

(D) Low thrust

2.12 SIMSLIN operters on the principal

(A) Absorption

(B) optimal interference

(C) Gravimetry

(D) Light scattring

2.13 Maximum damage to a haulage rope is caused by

(A) Fatigue

(B) wear and tear

(C) Corrosion

(D) Mine climate

Scanned by CamScanner

2.14 In koepe winding the overwind is prevented by

(A) Safety hook

(B) Breakage of rope

(C) Coveragnce of guide

(D) Thickening of guides

2.15 Fig-1shows a diagrammatic illustration of variogram, which is

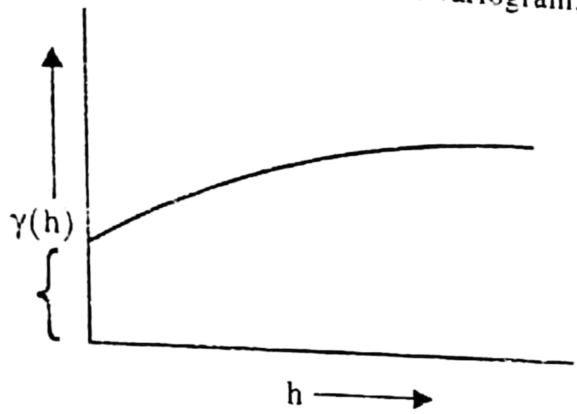


fig. - 1

(A) Linear type

(B) Nugget type

(C) Continuous type

(D) Transitive type

3. Write the correct alphabet against each question number :

(25 × 2 = 50)

3.1 For a successful VCR stoping, which one of the following is appropriate?

(A) Large diameter blast holes

(B) Spherical explosive charges

(C) Vertical downward blasthole

(D) All the above

3.2 For all practical purposes coal dust explosions in underground mine are not affcted by.

(A) Percentage of mosture in coal

(B) Relative humidity of mine air

(C) Presence of fire damp in mine air

(D) All the above

3.3 Isolation stopings are provided in underground working of coal mines to prevent

(A) Spontaneous heating

(B) Inadvertant entry of persons in to disused working

(C) Occurrence of coal dust explosion

(D) migration of methane from disused working

3.4 Which one of the following will be significantly affected by the abnormal changes in the barometric pressure?

Scanned by CamScanner

(A) Air quamtity circulated by the mine fan
(B) Rate of emission of noxious gases from sealed-off areas
(C) Rate of ecmission of gases from coal seam in an underground mine
(D) The performance of the diesel onging operating underground

3.5 Which one of the following statements is more appropriate?
(A) Only at the crown that too on the surface of the opening
(B) only at the crown but upto a certain distance in to the rock mass
(C) Only around the crown existing on both sides of it
(D) Both around the crown and the base of the opening, existing on both sides of them

3.6 Fig-2 shows the duty cycle of a hoist systam. The system is
(A) Cylindrical drum with tail rope
(B) Bicylindro conical drum
(C) Ground mounted friction sheave
(D) Cylindrical drum without tail rope

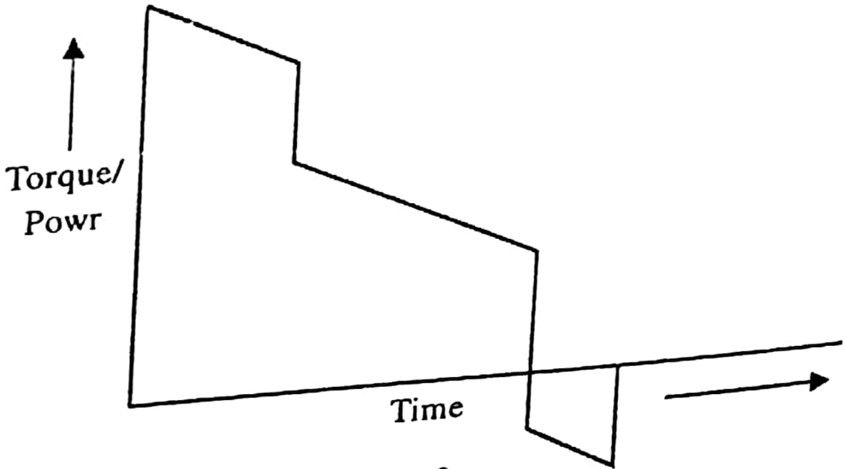
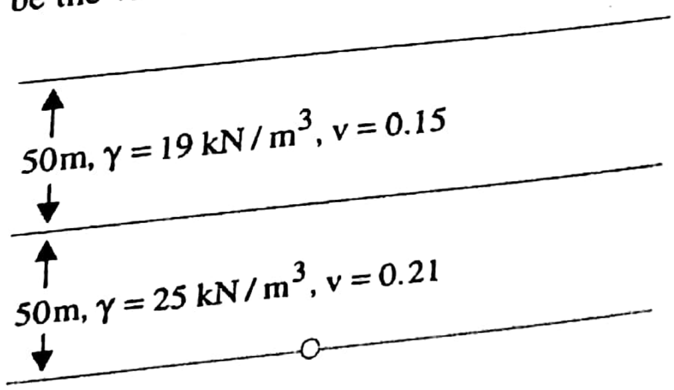


Fig. - 2

3.7 What will be the value of the horizontal stress at P in the deposit as shown in Fig-3



Average

$\gamma = (19 + 25)/2 = 22$ kN/m^3
$V = (0.15 + 0.21)/2 = 0.18$

Fig. - 3

Scanned by CamScanner

(A) 100 * 22 * 0.18/(1 - 0.18)
(B) 100 * 22 * 0.21/(1 - 0.21)
(C) 50 (19 + 25) * 0.21/(1 - o.21)
(D) 50 (19 + 25) * 0.18/! - 0.18)

3.8 While crossing an existing gallery by a power supported longwall face the support beams in the gallery should be
(A) Parallel to the face line
(B) Perpendicular to the face line
(C) Perpendicular to the gallery axis
(D) Parallel to the gallery axis

•3.9 A base lifting sevice for some of the shield supports is required in a longwall face, when
(A) The caving is intensive
(B) The AFC capacity is not adequate
(C) The shield base not adequate
(D) The floor is soft

3.10 Which of the following starement is correct?
Static gauge pressure at the inlet of any duct is,
(A) Positive for forcing and negative for exhausting ventilation system.
(B) Always negative and equal to that of velocity pressure
(C) Depends on the barometic pressure.
(D) Always equal to the total pressure at the inlet.

3.11 Evaporative cooling of mine air results in
(A) Decreased dry and wet bulb temperature of air
(B) Decreased total heat content of air
(C) Increased dry bulb temeperature but decreased wet-bulb temperture
(D) Increased wet-bulb temperature and incresed heat content of air

3.12 In an exhaustive ventilation system an evasee is used to
(A) Reduce static head of the fan
(B) Reduce shock loss at the fan inlet
(C) Recover veloctу head lost to at mosphere
(D) Improve air power of the ventilation system

3.13 For armoured face conveyor open-bottom pans are preferable to closed-bottom pans because
(A) These do not sink into soft floor being of lighter weight
(B) The return chain can not get jammed due to accumulation of coal particles
(C) These can be easily inspected and maintained
(D) These make less noise

3.14 Transporatation of sand in a pipeline flow is
(A) Homogeneous in nature
(B) Heterogeneous in nature
(C) By saltaition
(D) Due to moving bed along the floor of the pipe

Scanned by CamScanner

3.15 Explosive performance depends only on
(A) The shock energy of the explosive
(B) The bubble energy of the explosive
(C) The velocity of detonation and detonation pressure
(D) The total energy

3.16 A shock tube initiating system, such as Nonel, (wrong one).
(A) Does not need detonators for initiation
(B) It can not be used in under-water condition
(C) It is not affected by static electricity or stong currents
(D) It creates a lot of noise

3.17 Working in an underground coal mine are sectionalised into different districts for the purpose of
(A) Quick isolation in the event of a spontaneous heating or fire
(B) For better management and production control
(C) For effective strata control
(D) To contain methane emission in disued workings

3.18 It is necessary to square the differences from the mean when computing the population variance
(A) So that extreme values will be not unduly affect the calculation
(B) Because it is possible that number could be very small
(C) Some of the differences will be positive and some will be negative
(D) None of these

3.19 Which of the following is a necessary condition for use of a poisson distribution
(A) Probability of one arrival per second is constant
(B) Number of arrivals in any one second-interval is independent of arrivals in other intervals
(C) The probability of two or more arrivals in the same second is zero
(D) All of these

3.20 Flame safety temperature is not a reliable indicator of firedamp in thin layers near the roof of an underground roadway in a coal mine as
(A) it is difficult to properly observe the flame in the safety lamp at the roof level
(B) the presence of the lighted safety lamp dilutes the concentration of methane at the roof level due to the local convection air current
(C) gas keeps on migrating
(D) presence of the observer disturbs methane layering

3.21 The following are the concentration of an ore body

Ore strength	- moderate to strong
Rock strength	- weak to fairly weak
Deposit shape	- irregular
Deposit dip	- fairly steep
Ore grade	- fairly high

Scanned by CamScanner

Depth - moderate to deep

Suitable method of exploitation of the deposit is

(A) Shrinkage stoping (B) Sub level stoping

(C) Top slicing (D) Cut and fill stoping

3.22 The faster and cheaper method of under cutting a mechanized sub-level open stope, being worked by long hole drilling, is acieved by

(A) Troughing

(B) Finger raising and funneling

(C) Scram level driving and mill holing

(D) Undercut level driving, box holing and chute raising

3.23 Shrinkage stoping is generally unsuitable for pyritic ore bodies because of

(A) Poor fragmentation of ore

(B) Chances of spontaneous heating

(C) Weakness of ore leading to pressure manifestation

(D) Poor ore drawing characteristics

3.24 Zero circle condition is obtained while

(A) Correlating the surface station with the corresponding underground station

(B) Laying an underground curve with the help of a theodolite

(C) Computing the areas under subsidence with the help of a planimeter

(D) Conducting close traverse

4. If $u = x^2 \tan^{-1}\frac{y}{x} - y^2 \tan^{-1}\frac{x}{y}$ and xy # 0

Proved that $\frac{\partial^2 u}{\partial x\, \partial y} = \frac{x^2 - y^2}{x^2 + y^2}$ (5)

5. A wedge-cut blasting round with eight-cut-hoels is drilled to obtain a pull of 1.8 m in a 2m x 3m face, having a load factot of 0.0381 m^3/kN. ANFO with a specific gravity of 0.8 is to be used. Average drill hole diameter is 76 mm. Total consumption of explosives is 670 N, out of which 160 N is used in eight-cut-holes. Length of each hole to be drilled should be equal to the pull plus half the charge length of cut-hole. Holes other than the cut-holes are charged to 40 percent of the drilled length. Calculate

(a) Length of charge in cut-holes (2)

(b) total number of holes in the face (2)

(c) powder factor (1)

6. A Bord and pillar panel lying at an average depth of 100 m has been developed by galleries of 4 m width to from square pillars of 28 m centree to centre. The average density of overlying strata is 24 kN/m^3. What will be the safety factor of these pillars, if the pillar strength is 490 N/cm^2? (5)

7. Find the height of the tie-beam above the floor level from the following data taken on meter staff by levelling

R.L. of floor level = 100.00

Staff reading on the floor = 1.052

Scanned by CamScanner

Reading on the staff held inverted, the bottom touching the underside of the tie beam = 3.558 (5)

8. Draw the sketch of a simple sidecast diagram of a dragline pit for the following situation, Indicate all dimensions on the sketch wherever possible. (5)

Thickness of coal seam	= 10 m
Thickness of over burden bench	= 20 m
Dip of seam	= Horizontal
Side slope of over burden bench	= 70°
Side slope of coal bench	= 70°
Side slope of dump	= 30°
Distance of overburden bench toe and crest of coal bench	= 5 m
Distance of toe of coal bench and the toe of dump slope	= 5 m
Swell factor	= 1.3
Cut width	= 30 m

SECTION B
(100 marks)

Answer ANY TEN question in this section.

9. Determine the amount of reduction in the height of the roadway at its centre for the roof condition shown in Fig-4. Assume that the layers are clamped. (5)

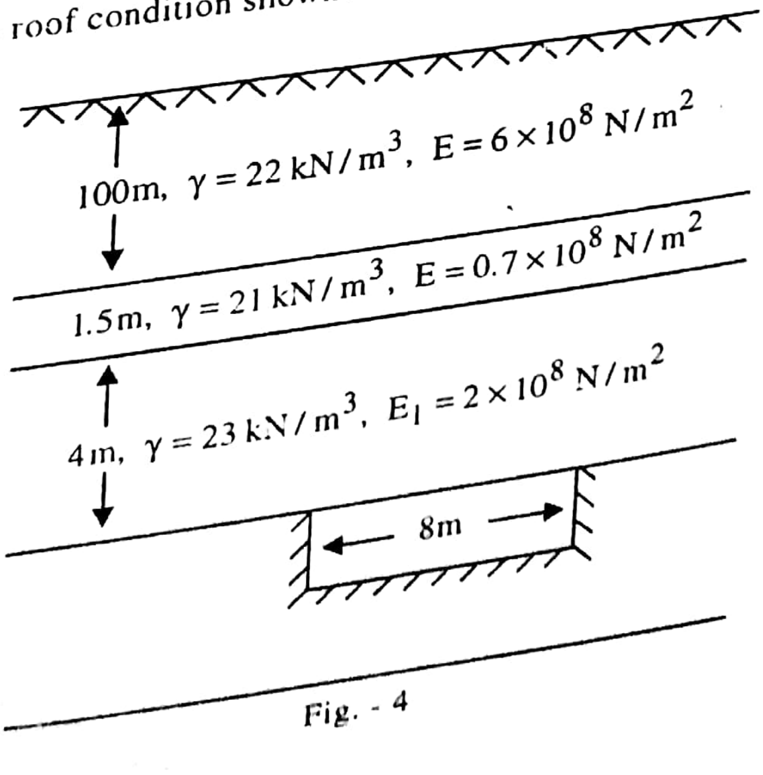


Fig. - 4

10. Explain with the help of a neat sketh how to determine absolute in-situstress in rock mass by the use of flat jack. (Answer should give only the steps without any elaboration) (2) + (3)

11. A nickel laterite deposit has been sampled by pits. An area of 1250 m^2 is allocated to each pit. Two different types of ores with different densities were encountered in the pits (Fig-5). The laterite has an in-situ density of 1.25 t/m^3 and decomposed serpentinite (zs) has an in-situ. density of 1.0 t/m^3. Calculate the nickel grade of the total tonnage. (5)

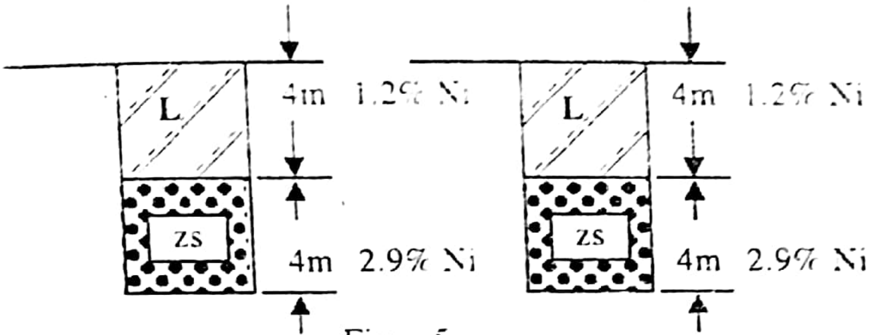


Fig. - 5

12. Maximise the value of R, when

$$R = q^2 + 3qa + a^2$$

subject to the constraint. q + a = 100 (5)

13. The average before tax cash flows associated with the discovery of an economic mineral deposit have been estimated as follws

Exploration:	Expenditure of Rs. 25 million per year 8 years
Development:	Investment of Rs. 20 million per year for 3 years
Production:	Revenues of Rs. 25 million per year and operating cost of Rs. 7 million per year for 16 years

the cost of capital is 10 per cent. Find the net present value (2) + (3)

14. Determine the coresponding eigen vectors of the matrix for eigh values of −1 and 3 (5)

$$A = \begin{pmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

15. For a production unit, the selling price $p = 21000\ n^{-1/2}$, where n is the number of units produced. The variable cost. V is Rs. 1000.00 per unit and the fixed cost. F is Rs. 100,000.00. Find the break-even number to be produced and also the number for maximum profit. (3 + 2)

16. Draw a rspresentative profile of the subsidence trough over a level coal seam showing :

(a) Critical width of extraction (1)

(b) Strain profile indicating the compressive and tensile zones (3)

(c) Angle of draw (1)

17. Show the profile of failure planes for a planar and a circular failure for a single bench section of an openpit. List the necessary conditions for such a failure to occur in each case. (3) + (2)

Scanned by CamScanner

18. 10,000 m^3 mine air per minute is supplied to two splits of equal length and surface character, having cross-sections of 3.0 × 3.0 m and 3.0 × 4.0 m respectively. Find out the flow of air in each split. (5)

19. In a shovel-dumper openpit, the following information on dumper movement are available

Loading time	= 3.50 min
Spotting time	= 0.50 min
Load and empty travel distance	= 3.00 km each
Load travel speed of dumper	= 20 KMPH
Dumping time	= 1.00 min

6 How many dumpers will be required for optimum utilisation of shoveldumper system? (5)

20. Plot the hydraulic profile of a stowing pipeline whose geometric profile is given in Fig-6. Find out if the hydraulic profile is correct. What is the hydraulic gradient?

AB : Vertical Shaft Range
BC : Pipeline along Level
CD : Pipeline along slope of 45°

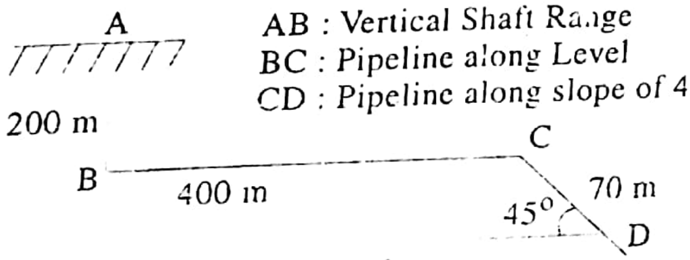


Fig. - 6

21. Why tensioning of a belt conveyor system is necessary? Show with the help of a simple sketch a gravity take-up device. (1 + 4)

22. What is an emulsified ANFO (or Heavy ANFO)? How is its explosive property achieved? How is it superior to ANFO? (2 + 1 + 2)

23. Following information are available for a small project :

Activity	Predecessor	Duration (Hrs)	Cost (Rs)	Crash Duration (Hrs)	Crash Cost (Rs.)
A	–	8	80	6	100
B	A	7	40	4	94
C	A	12	100	5	184
D	A	9	70	5	102
E	B, C, D	6	50	6	50

Assuming linear cost relationship between cost and duration and overhead cost at Rs. 25.00 per hour find the optimum duration and the corresponding cost of the project. (5)

24. Draw neat sketches of the following pit top car circuits :
(a) Shunt-back circuit (3)
(b) Turnable circuit (2)

Scanned by CamScanner

25. A mine installs a new hoisting plant at a cost of Rs. 200,000.00. It is estimated that the equipment will become obsolete in 10 years. What sinking fund must be set aside anually at 4% compound interest rate to replace the hoist investment at the end of the period. (5)

26. A mine placed order for a large number of sodium vapour lamps which is claimed by the manufacturer to have a life of 14,500 hours with a known standard deviation of 2,100 hours. From a sample of 25 lamps, the mine management finds a sample mean of 13,000 hours. At a 0.01 significance level, should the mine concude that the average life of the lamps is less than 14,500 hours? (5)

27. A turbine has 5 impellers, 30 cm in diameter, running at 1440 R.P.M. The delivery branch is 15 cm in bore and suction branch 20 cm. For what rate of delivery of water and the head the pump is suitable? Assume inlet velocity as 1.5 m/sec and manometeric coefficient 0.6. (5)

28. Explain the following terms in relation to wire ropes

(A) Fill factor (2)

(B) Efficiency of construction (2)

(C) Nominal factor of safety of winding rope (1)

2000

SECTION A

1.1	C	2.1	B
1.2	C	2.2	C
1.3	A	2.3	C
1.4	B	2.4	C
1.5	C	2.5	D
1.6	D	2.6	B
1.7	B	2.7	C
1.8	A	2.8	C
1.9	D	2.9	D
1.10	D	2.10	D
1.11	B	2.11	A
1.12	C	2.12	C
1.13	D	2.13	D
1.14	B	2.14	A
1.15	B	2.15	B
1.16	A	2.16	C
1.17	B	2.17	A
1.18	A	2.18	B
1.19	C	2.19	B
1.20	B	2.20	C
1.21	C	2.21	B
1.22	C	2.22	A
1.23	B	2.23	C
1.24	C	2.24	D
1.25	A	2.25	D

Scanned by CamScanner

SECTION B

$w_p = 30$ m

wo = 4.6 m,

z = 200 m   h = 3.0

$$\sigma_{strength} = 11.\ w^{0.5} h^{-0.5} \text{ MPa}$$

$$= 11.0 \times 4.6^{0.5} 3.0^{-0.5} \text{ MPa}$$

$$= 13.62 \text{ MPa}$$

$$\sigma_{stress} = \gamma \cdot z \left(1 + \frac{wo}{wp}\right)^2$$

$$= 2400 \times 200\left(1 + \frac{4.6}{30}\right)^2$$

$$= 6.385 \times 10^5 \text{ Pa}$$

$$\text{F.O.S.} = \frac{\sigma \text{ strength}}{\sigma \text{ stress}}$$

$$= \frac{13.62 \times 10^6}{6.385 \times 10^5}$$

$$= 21.3$$

$$\approx 21$$

$$\text{Bending stress } \alpha \frac{1}{(\text{thickness})^2}$$ 4

$$\frac{\text{New bending stress}}{\text{Old bending stress}} = \frac{(2.5)^2 + (1.5)^2 + (2)^2}{(2.5 + 1.5 + 2)^2}$$

$$\% \text{ reduction} = \left(1 - \frac{2.5^2 + 1.5^2 + 2^2}{(2.5 + 1.5 + 2)^2}\right) \times 100$$

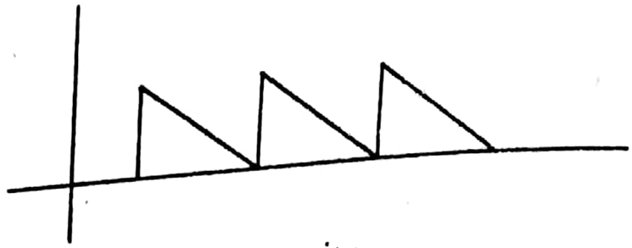
$$= \left[\frac{36 - (6.25 + 2.25 + 4)}{36}\right] \times 100$$

$$= \frac{36 - 12.5}{36} \times 100$$

$$= \frac{23.5}{36} \times 100 = 65\% \quad \text{Ans.}$$

Scanned by CamScanner

Q. 6



Let q be the economic order quantity

$$\text{Total cost (T)} = \frac{d}{q} \times a + \frac{q}{2} \times h$$

For EOQ, min. cost

$$\frac{dT}{dq} = 0 = \frac{-da}{q^2} + \frac{h}{2}$$

$$\Rightarrow \quad q = \sqrt{\frac{2ad}{h}}$$

given d = 200, a = 31, 250, h = 5000

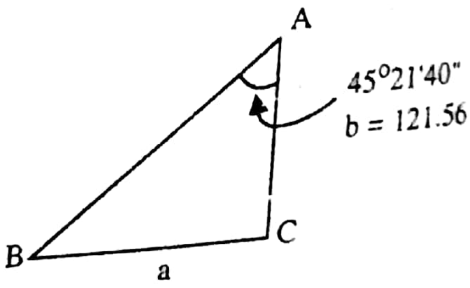
$$q = \sqrt{\frac{2 \times 200 \times 31{,}250}{5000}}$$

$$= 2 \times 25$$

$$= 50$$

Ans.

Q.7



$$\Delta b = 0.05 \text{ m}$$

$$\Delta A = 12''$$

$$= \frac{12}{60 \times 60} \times \frac{\pi}{180}$$

$$= 5.82 \times 10^{-5}$$

$$a = \tan A$$

Error in side a

$$\delta a = \frac{\delta a}{\delta b} \times \delta b + \frac{\delta a}{\delta A} \times \delta A$$

$$= \tan A \times \delta b + b \sec^2 A \times \delta A$$

$$\delta a = \left(\tan 45°21'40'' \times 0.05\right)$$

$$+ \left[121.56 \times \left(\sec^2 45°21'40''\right) \times 5.82 \times 10^{-5}\right]$$

$$= \left[\tan (45.36)° \times 0.05\right] + 121.56 \times \sec (45.36)^2 \times 5.82 \times 10^{-5}$$

$$= 0.050 + 0.0143$$

$$= 0.0643 \text{ m}$$

$\therefore$ error in side a:

$$\delta a = 0.0443 \text{ m}$$

Scanned by CamScanner

Web of drum $(\omega_d) = 0.5$ m

$d = 1.24$ m

$$v = 0.4 \text{ km/hr} = \frac{0.4 \times 10^3}{60 \times 60} = 0.11 \text{ m/s}$$

(Width of face) $\lambda = 150$ m

Lenght of panel

$L = 400$

Let height of the seam be

$h = 2.48$ m

Time taken to make one cut

$$t = 2 \times \frac{0.150}{0.4}$$
$$= 0.75 \text{ m.}$$

For one cut (total time taken)

$$t = 4.0 + t$$
$$= 4.0 + 0.75$$
$$= 4.75 \text{ m}$$

No. of cuts in 16 hrs (perday)

$$= \frac{16}{4.75}$$
$$= 3.36 \text{ cuts/day}$$

advance of face/day

$$= 3.36 \times 0.5$$
$$= 1.68 \text{ m}$$

No. of days it will need to exhaust the panel

$$N = \frac{400}{1.68}$$
$$= 238 \text{ days}$$

Q. 12.

	RPM/100 (x)	x^2	Torque (Nm) y	xy
1.	3.00	9.00	1.2	3.6
2.	4.75	22.56	1.4	6.65
3.	7.50	56.25	1.8	13.50
4.	11.80	139.24	2.7	31.86
$n = 4$	$\sum x = 27.05$	$\sum x^2 = 218.05$	$\sum y = 7.1$	$\sum xy = 55.61$

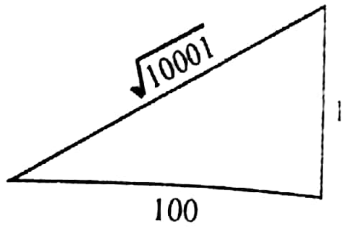
$$\sum y = na + b\sum x$$

$$a = \frac{\sum y}{n} - \frac{b\sum x}{n} \qquad \text{.......... (1)}$$

$$b = \frac{\left(\sum xy - \frac{\sum x \sum y}{n}\right)}{\left(\sum x^2 - \frac{\sum x \sum x}{n}\right)}$$

Putting the respective values we get a & b.

Q.13



$m_t = 200$

$v_1 = 7.2 \text{ kmp} = \frac{7.2 \times 10^3}{60 \times 60}$

$= 2 \text{ m/s}$

$v_2 = 21.6 \text{ kmp} = 6 \text{ m/s}$

Total force required during acceleration

F = (downward force) + (frictional force)+ (force for acceleration)

$= mg\sin\theta + (60 \times 200) + m\,.\,a$

$= \left(200 \times 10^3 \frac{1 \times 9.8}{\sqrt{10001}}\right) + (60 \times 200) + \left(200 \times 10^3 \times 0.2\right)$

$= 19.6 \times 10^3 + 12 \times 10^3 + 40 \times 10^3$

$= 71.6$ KN

Power required when speed is 7.2 kmp

$P_1 = F \times V_1 = 71.6 \times 2$

$= 143.2$ k Watt

power required when speed is 21.6 kmp

$P_2 = F \times V_2 = 71.6 \times 6$

$= 429.6$ k watt

Avg. power during a ceeleration

$P = \frac{P_1 + P_2}{2}$

$= \frac{143.2 + 429.6}{2}$

$= 286.4$ K watt

Q. 15

$Q = 40.0 \text{ m}^3/\text{s}$ $\quad f_1 = 0.013 \text{ N s}^2\text{m}^{-8}$

$P = 600$ Pa $\quad f_2 = 0.010 \text{ N s}^2 \text{ m}_{-8}$

$L = 500$ m

$d_1 = 3.0$ m

$d_2 = 3.5$ m

Initial resistance of shaft

$$R_1 = \frac{f_1 . LP_1}{A_1^3}$$

$$= \frac{0.13 \times 500 \times (\pi \times 3.0)}{\left[\pi (1.5)^2\right]^3}$$

$$= \frac{61.26}{353.1}$$

$$= 0.1735$$

Final resistance of shaft

$$R_2 = \frac{f_2 . L . P_2}{A_2^3}$$

$$= \frac{0.010 \times 500 \times (\pi \times 3.5)}{\left[\pi (1.75)^2\right]^3}$$

$$= \frac{55}{890.6}$$

$$= 0.0617$$

Resistance of total mine

$$R = \frac{P}{Q^2} = \frac{600}{(40)^2} = 0.375$$

Resistance of rest of mine (other than shaft)

$$R_3 = R - R_1 = 0.375 - 0.1735$$
$$= 0.2015$$

As resistance of rest of mine remains unchanged

New resistance of mine

$$R' = R_2 + R_3$$
$$= 0.0617 + 0.2015$$
$$= 0.2632$$

New quantity (Q')

$$Q' = \sqrt{\frac{P}{R'}}$$

$$= \sqrt{\frac{600}{0.2632}}$$

$$= 47.74 \text{ m}^3/\text{sec.}$$

Scanned by CamScanner

Q. 17.

$$P(A) = \frac{1}{6}$$

$$P(B) = \frac{1}{4}$$

$$P(C) = \frac{1}{3}$$

Probability that only 'A' hits

$$= \frac{1}{6} \times \left(1 - \frac{1}{4}\right) \times \left(1 - \frac{1}{3}\right)$$

$$= \frac{1}{6} \times \frac{3}{4} \times \frac{2}{3}$$

$$= \cancel{\frac{1}{6}}\ \frac{1}{12}$$

Probability that only 'B' hits

$$= \left(1 - \frac{1}{6}\right) \times \frac{1}{4} \times \left(1 - \frac{1}{3}\right)$$

$$= \frac{5}{6} \times \frac{1}{4} \times \frac{2}{3}$$

$$= \frac{5}{36}$$

Probability that only 'C' hits

$$= \left(1 - \frac{1}{6}\right) \times \left(1 - \frac{1}{4}\right) \times \left(\frac{1}{3}\right)$$

$$= \frac{5}{6} \times \frac{3}{4} \times \frac{1}{3}$$

$$= \frac{5}{24}$$

Probability of hitting exactly one

$$= \frac{1}{\cancel{6}\,12} + \frac{5}{36} + \frac{5}{24}$$

$$= \frac{\cancel{12}\,6 + 10 + 15}{72}$$

$$= \frac{3\cancel{7}1}{72}$$

Q. 18.

$$P = 972 \text{ t/shift}$$

tonnage/cut

$$P = 6.0 \times 1.8 \times 12 \times \frac{1}{0.8}$$

$$= 162 \text{ t/cut}$$

no. of cuts required to meet shift production target

$$n = \frac{972}{162} = 6$$

Scanned by CamScanner

Time available for 1 cut = $\frac{6}{6} = 1$ hr. = 60 min.

Let R is the capacity of roadheater

then time taken to clear one cut

t = (time taken by roadheader for one cut) + (time to load and change suttle car) + (delay to shift road header fromone face to other)

$$t = \frac{162}{R} + \left(\frac{162}{5} \times 1\right) + 3.0$$

$$60 \text{ min} = \frac{162}{R} + 33 + 3.0$$

$$\Rightarrow \quad 60 = \frac{162}{R} + 33 + 3.0$$

$$\Rightarrow \quad \frac{162}{R} = 60 - 33 = 27$$

$$\Rightarrow \quad R = \frac{162}{27} = 6 \text{ t/min}$$

Capacity of Road header

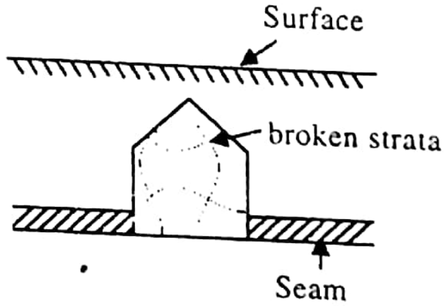
$R = 6$ t/min.

$= 6 \times 60$ t/hr.

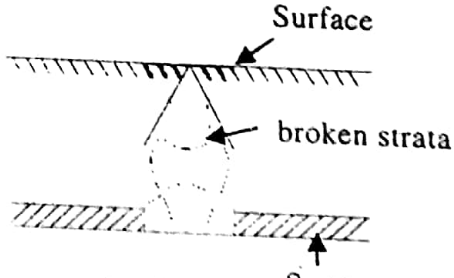
$= 360$ t/hr. or $288 \text{ m}^3/\text{hr}$.

Q. 19.

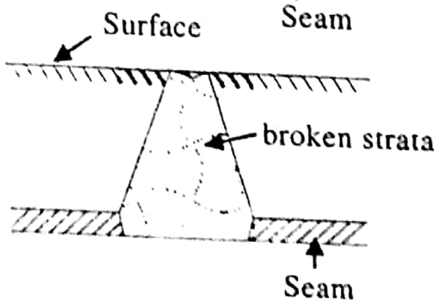
Sub-Critical Subsidence



Critical Subsidence



Super Critical Subsidence



Scanned by CamScanner

Q. 20.

Amount of air flowing throught shaft

$$Q = 60 \text{ m}^3/\text{s}$$

$$= 60 \times 1.15 \text{ kg/s}$$

$$= 69 \text{ kg/s}$$

Amount of heat transferred/sec. by the time air reaches to shaft bottom.

$$H = 100 \text{ KJ (per sec.)}$$

Increase in temparature of air

$$H = Q.S. (\Delta t)$$

$$\Delta.t = \frac{H}{Q.S}$$

$$= \frac{100 \times 10^3}{69 \times 1005}$$

$$= \left(1.44^{\circ} C\right)$$

Thus dry bulb temperature of air when it reaches shaft bottom

$$T_2 = T_1 + \Delta t$$

$$= 30^{\circ} + 1.44^{\circ}$$

$$= 31.44^{\circ} \text{ C.}$$

Q. 22.

1. $U = X . \log (xy)$ & 2. $x^3 + y^3 + 3xy = 1$

from equation (2), we get

$$x^3 + y^3 + 3xy - 1 = 0$$

$$y^3 + x^3 + 3xy - 1 = 0$$

differentiating w.r.t. x

$$3y^2 . \frac{dy}{dx} + 3x^2 + 3y + 3x . \frac{dy}{dx} = 0$$

$$\Rightarrow \quad \frac{dy}{dx}\left(3y^2 + 3x\right) = -3x^2 - y$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{-\left(x^2 + y\right)}{y^2 + x}$$

$$U = x . \log (xy) \qquad \text{.................... (1)}$$

$$\frac{dU}{dx} = \log(xy) + x . \frac{d}{dx}[\log (xy)]$$

$$= \log (xy) + x\left[\frac{y}{xy} + \frac{1}{xy} . x . \frac{dy}{dx}\right]$$

$$= \log(xy) + x\left[\frac{1}{x} + \frac{1}{y} \cdot \frac{dy}{dx}\right]$$

$$= \log(xy) + x\left[\frac{1}{x} + \frac{1}{y} \cdot \frac{-(x^2 + y)}{(y^2 + x)}\right]$$

– (putting value of $\frac{dy}{dx}$)

$$= \log(xy) + x\left[\frac{y(y^2 + x) - x(x^2 + y)}{xy(y^2 + x)}\right]$$

$$= \log xy + \left[\frac{y^3 + xy - x^3 - xy}{y(y^2 + x)}\right]$$

$$= \log xy + \left[\frac{y^3 - x^3}{y(y^2 + x)}\right]$$

hence

$$\frac{du}{dx} = (\log xy) + \left[\frac{(y^3 - x^3)}{y(y^2 + x)}\right]$$

Scanned by CamScanner

1999

SECTION A

1.1	D
1.2	D
1.3	B
1.4	A
1.5	A
1.6	C
1.7	A
1.8	C
1.9	B
1.10	C
1.11	~~A~~ C
1.12	D
1.13	A
1.14	C
1.15	C
1.16	D
1.17	A
1.18	D
1.19	B
1.20	B
1.21	~~B~~ D
1.22	A
1.23	B
1.24	D
1.25	A

2.1	D
2.2	C
2.3	C
2.4	A
2.5	D
2.6	D
2.7	A
2.8	D
2.9	A
2.10	C
2.11	A
2.12	B
2.13	D
2.14	B
2.15	C
2.16	D
2.17	C
2.18	C
2.19	A
2.20	B
2.21	B
2.22	C
2.23	B
2.24	B
2.25	C

(12) — $FOS = \frac{Strength}{Stress} = \frac{300 \times 10^6 \, N/m^2}{\frac{1000 \times (2.4)^2 \times 1.5 \times 205 \times 9.81}{(4 \times 4 \times 10^{-4})}}$

$= \underline{2.66}$

Scanned by CamScanner

SECTION B

3.

The miner can come out of the underground in 5 ways

1st tunnel	$\rightarrow$ 3 hrs
2nd & 1st tunnel	$\rightarrow$ 1 + = 4 hrs.
3rd & 1st tunnel	$\rightarrow$ 2 + 3 = 5hrs.
2nd, 3rd & 1st tunnel	$\rightarrow$ 1 + 2 + 3 = 6hrs.
3rd, 2nd & 1st tunnel	$\rightarrow$ 2 + 1 + 3 = 6 hrs

Honce expceted time of coming out

$$= \frac{1}{5}(3+4+5+6+6)$$

= 4 hrs & 48 mins.

4.

$$P = 1200 - 3q$$

Total price (Revernu) at a production level of q

$$R = P \times q = (1200 - 3q).q$$

$$\therefore \quad R = 1200q - 3q^2$$

$$\frac{dR}{dq} = 1200 - 6q \qquad \text{.........................(1)}$$

$$\& \quad \frac{d^2R}{dq^2} = -6 < 0 \qquad \text{.........................(2)}$$

For maximum or minimum R

$$\frac{dR}{dq} = 0$$

$$\Rightarrow \quad 1200 - 6q = 0$$

$$\Rightarrow \quad q = \frac{1200}{6}$$

$$= 200 \text{ units}$$

as $\frac{d^2R}{dq} < 0$, it is a max ima

So,

Maximum revenue

$$R = (12{,}00 - 200 \times 3) \times 200$$

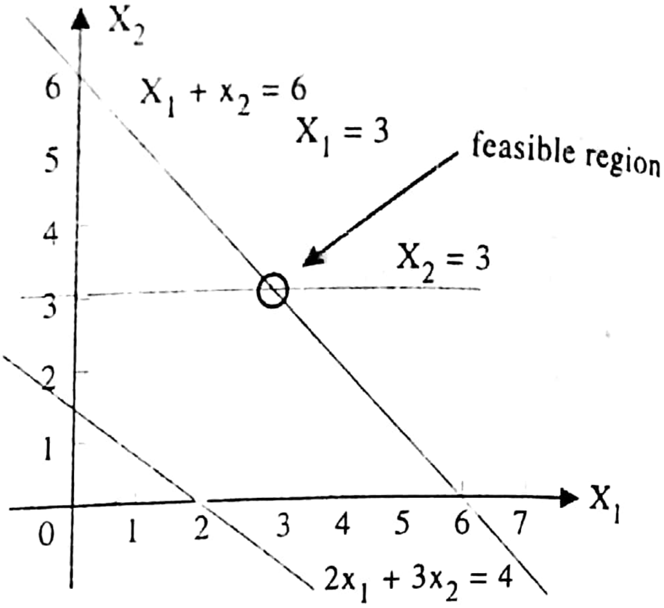
$$= (1200 - 600) \times 200$$

$$= 600 \times 200$$

$$= 12{,}0000 \text{ Rs.}$$

Scanned by CamScanner

Q. 5.



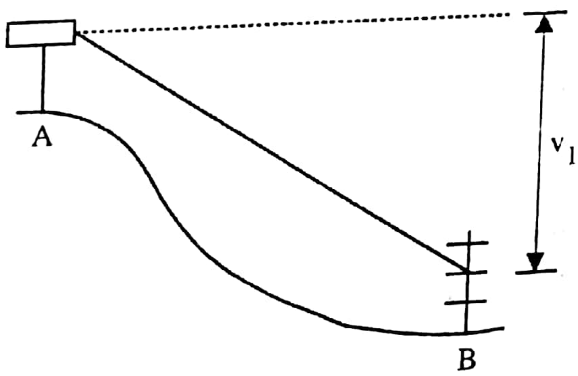
Plotting all the conditions we find that feasible region is only a point which i (3, 3)

Hence,

Maximum & minimum value of funition $z = 5x_1 + 3x_2$

$$= 5 \times 3 + 3 \times 3$$
$$= 15 + 9$$
$$= 24$$

Q. 6.



Horizontal Distance

$$AB = \frac{f}{i} . s. \cos^2 \theta + (f + d) \cos\theta$$

$$= \left[100 \times (2.57 - 0.90) \cos^2 8^0\right] + 0.15 \times \cos 8^0$$

$$= (100 \times 1.67 \times 0.98) + 0.148$$

$$= 163.76 + 0.148$$

$$= 163.9 \text{ m}$$

$$v_1 = \frac{f}{i} . s. \frac{\sin 2\theta}{2} + (f + d) \sin\theta$$

$$= (2.57 - 0.90) \frac{\sin 16^0}{2} + 0.15 \times \sin 8^0$$

Scanned by CamScanner

$= 100 \times 1.67 \times 0.137 + 0.021$

$= 2301 + 0.021$

$= 23.03$ in

RL of B

= RL of A + Inst. ht. $- v_1$ − Axial reading

$= 1000.05 + 1.40 - 23.03 - 1.73$

$= 976.06$ m

Distance AB

$$= \sqrt{(163.9)^2 + (1000.05 - 976.69)^2}$$

$= 165.55$ m

Q. 7.

Vertices of the traingle

A = (2, 3, 5)

B = (4, 6, −1)

C = (3, 6, 4)

Length of sides

$$a = AB\sqrt{(2-4)^2 + (3-6)^2 + (5+1)^2} = 7$$

$$b = BC\sqrt{(4-3)^2 + (6-6)^2 + (-1-4)^2} = 5.1$$

$$c = BC\sqrt{(2-3)^2 + (3-6)^2 + (5-4)^2} = 3.32$$

$$S = \frac{a+b+c}{2} = \frac{7+5.1+3.32}{2} = 7.71$$

Area of traingle

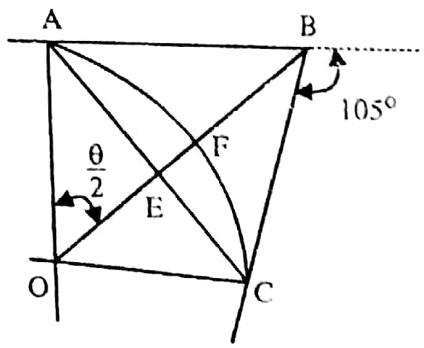
$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{7.71(7.71-7)(7.71-5.1)(7.71-3.30)}$$

$$= \sqrt{7.71 \times 0.71 \times 2.61 \times 4.39}$$

$= 7.92$ sq. units

Q. 8.



Tangent distance

$$AB = R\tan\frac{\theta}{2}$$

$$= 150 . \tan\frac{105^0}{2}$$

$= 150 \times 1.30$

$= 195.48$ m

Main chord

$$AC = 2R\sin\frac{\theta}{2}$$

$$= 2 \times 150 \times \sin\left(\frac{105^0}{2}\right)$$

$$= 300 \times 0.793$$

$$= 238\text{m}$$

Length of curue

$$\widehat{AC} = 2\pi R \cdot \frac{\theta}{360^0}$$

$$= 2 \times 3.14 \times 150 \times \frac{105^0}{360^0}$$

$$= 274.88 \text{ m}$$

Rise of curve

$$EF = R\left(1 - \cos\frac{\theta}{2}\right)$$

$$= 150\left(1 - \cos\frac{105^0}{2}\right)$$

$$= 58.68 \text{ m}$$

Q.9

Inverse of matrix A

$$A = \begin{bmatrix} -2 & +3 & -3 \\ -2 & -2 & -3 \\ -3 & +2 & -2 \end{bmatrix} = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$$

Let A_1, A_2, A_3......................be cofactors of a_1, a_2, a_3 respectively.
Then,

$A_1 = (4 + 6) = 10$ $\qquad \Delta = a_1A_1 + a_2A_2 + a_3A_3$

$A_2 = -(-6 + 6) = 0$ $\qquad = (-2 \times 10) + (-2 \times b) + (-3 \times 15)$

$A_3 = (-9 - 6) = -15$ $\qquad = -20 + 45 = 25$

$B_1 = -(4 - 9) = 5$

$B_2 = (4 - 9) = -5$

$B_3 = -(6 - 6) = 0$

$C_1 = (-4 - 6) = -10$

$C2 = -(4 + 9) = -5$

$C_3 = (4 + 6) = 10$

$$\text{adj } A = \begin{bmatrix} A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \\ C_1 & C_2 & C_3 \end{bmatrix} = \begin{bmatrix} 10 & 0 & -15 \\ 5 & -5 & 0 \\ -10 & -5 & 10 \end{bmatrix}$$

$$A^{-1} = \frac{1}{\Delta} \text{adj} . A$$

Scanned by CamScanner

$$A^{-1} = \frac{1}{25}\begin{bmatrix} 10 & 0 & -15 \\ 5 & -5 & 0 \\ -10 & -5 & 10 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} \frac{2}{5} & 0 & -\frac{3}{5} \\ \frac{1}{5} & -\frac{1}{5} & 0 \\ -\frac{2}{5} & -\frac{1}{5} & \frac{2}{5} \end{bmatrix}$$

Q. 12.

Weight on rope

= (wt. of suspension gear) + (wt. of load mine cars) + (wt. of rope)

$= (5000 \times 9.8) + (5000 \times 9.8) + 0.36 . (4)^2 . (320) \times 9.8$

$= 98 \times 10^3 + 18.06 \times 10^3$

$= 166.06 \times 10^3$ N

Strength of rope

$= 52d^2$

$= 52 . (4)^2 \times 10^3$

$= 832$ kN.

Static factor of safety

$$= \frac{\text{strength of rope}}{\text{Wt. on rope}}$$

$$= \frac{832 \times 10^3}{166.06 \times 10^3}$$

$= 7.168$

≈ 7

Q. 13.

Head loss in suction pipe

$$h_1 = \frac{4f\ell . v^2}{2gd}$$

$$\Rightarrow \quad h_1 = \frac{4 \times 0.01 \times 10\,(1.27)^2}{2 \times 9.8\,(0.2)}$$

$= 0.165$ m

$$v = \frac{Q}{\pi r^2} = \frac{\frac{2.4}{60}}{\pi\,(0.1)^2}$$

$= 1.27$ m/sec.

Head loss in delivery pipe

$$h_2 = \frac{4f\ell . v^2}{2gd}$$

$$= \frac{4 \times 0.01 \times 1000 \times 2^2}{2 \times 9.8 \times 0.016}$$

$$v = \frac{\frac{2.4}{60}}{\pi\,(0.08)^2}$$

$= 2$ m/s

Scanned by CamScanner

$$\Rightarrow \quad h_2 = 51$$

Total head against which pump is working

$$= h + h_1 + h_2$$
$$= 250 + 0.16 + 51$$
$$= 301.16 \text{ m}$$

Q. 14.

$$Q = \frac{6000 m^3}{min} = \frac{1000 m^3}{sec}$$

$$A = 4 \times 2.5$$

Restance of I'st gate road

$$R_1 = \frac{K\,S}{A^3} = \frac{K.P.L}{A^3}$$
$$= \frac{K.2\,(4-2.5)\,L.}{(4 \times 2.5)^3}$$
$$= 0.013 \text{ KL}$$

Resistance of 2nd gate road

$$R_2 = \frac{K.PL}{A^3}$$
$$= \frac{K.2\,(5+2)\,L}{(5 \times 2)^3}$$
$$= 0.014 \text{ KL}$$

Pressure drop across 1st gate road

$$\Delta p = R_1 Q_1^2$$
$$= 0.013 \text{ KL} \times (100)^2$$
$$= 130 \text{ KL Pa}$$

Quantity in 2nd gate road

$$Q_2 = \sqrt{\frac{\Delta P}{R_2}}$$
$$= \sqrt{\frac{130 \text{ KL}}{0.014 \text{ KL}}}$$
$$= \frac{96.36 m^3}{sec}$$

New total quantity

$$Q = Q_1 + Q_2$$
$$= 100 + 96.36 = 196.36 \text{ m}^3/\text{sec.}$$
$$= \frac{11781.7 m^3}{min}$$

Scanned by CamScanner

5.

Pressure dereloped by fan

$$h = \frac{U^2 - U \cdot V_R \cdot \cot\beta}{g}$$

$V_R = 4$ m/s

$$U = 2\pi \text{ rimp}\left(\frac{\text{r.p.m}}{60}\right)$$

$$= \pi \times 4 \times \frac{300}{60}$$

$= 62.83$ m/s

Theoretical Head

$$h = \frac{(62.83)^2 - (62.83 \times 4 \times \cot 40^0)}{9.8}$$

$$= \frac{3648}{9.8}$$

$= 372.25$ m

$= (372.25 \times 1.2 \times 9.8)$ Pa $(h\rho g)$

Actual head

$$= (372.25 \times 1.2 \times 9.8) \times \frac{70}{100} \quad (\eta = 70\%)$$

$= 3.06$ KPa

16.

Total amount of gas emitted

$= 15 \times 2000$ per day

$$= \frac{15 \times 2000}{24 \times 60 \times 60} \text{ per sec}$$

$$= \frac{0.347 \text{m}^3}{\text{sec}}$$

Amount of air required if the percentage is not to exceed 0.5%

$$Q \text{ air} = \frac{0.347 \times 100}{0.5} = 69.4 \text{ m}^3/\text{sec.}$$

air thought one gate road (at 4m/s)

$q = \text{Area} \times v$

$= (4 \times 2.5) \times 4$

$= 40$ m^3/sec.

no. of gate roads required for sending 69.4 m^3/sec. of air

$$n = \frac{Q}{q}$$

$$= \frac{69.4}{40}$$

$$= 1.735$$

$$\approx 2 \text{ no}$$

Q. 17.

Thickness of flat dam

$$T = \frac{W.H.P}{2(W+H)\sigma s}$$

$$= \frac{5 \times 3 \times (30 \times 10^3 \times 9.8)}{2(5+3) \times 0.15 \times 10^6}$$

$$= 1.83 \text{ m}$$

Thickness of arched dam

$$T^1 = \frac{\rho\gamma_o}{\sigma c} \quad (\gamma_o = \text{outer radius})$$

$$= \frac{(30 \times 10^3 \times 9.8) \times 4}{1 \times 10^6}$$

$$= 1.176 \text{ m}$$

so thickness of an arched dam is less than the flat dam.

Q. 18.

$$m_c = 2 \times 10^3 \text{ kg} \qquad v = 3.6 \text{ km/h}$$

$$m_t = 1 \times 10^3 \text{ kg} \qquad = 1 \text{ m/s}$$

$$\sin\theta = \frac{1}{10}$$

$$\mu_t = \frac{1}{100}$$

$$\mu_l = \frac{1}{10}$$

no. of tub (n) = 10

Force due to empty tub and load with be balanced.

Force due to load (coal)

$$= 10 \text{ mcg} \times \sin\theta$$

$$= 10 \times 10^3 \times 9.8 \times \frac{1}{10}$$

$$= 1000g.$$

Friction force due to load tub & empty tub

$$= \left(2 \times 10 \times 10^3 g \times \frac{1}{100}\right) \times \left(1 \times 10 \times 10^3 \times gx \frac{1}{100}\right)$$

$$= 200g + 100g$$

$$= 300g$$

Scanned by CamScanner

Friction force due to rope on both sides

$$= 2 \times \left(1000 \times 2 \times g \times 1 \times \frac{1}{10}\right)$$

$$= 400g$$

Total force $= 1000\ g + 300\ g + 400\ g$

$F = 1700\ g$

Power required

(at 80% efficiency)

$$= F \times V \times \eta$$

$$= 1700g \times 1 \times 1.2$$

$$= 20.4 \text{ KW}$$

Q. 19.

Production/Shift

$$= 5 \times (4.8 \times 2.4 \times 1.2 \times 1.5)$$

$$= 103.68 \text{ t}$$

Production/Year

$$= (3 \times 103.68) \times 300$$

$$= 93312 \text{ t}$$

Minimum quantity of air

$$= (103.68 \times 3) \times 2.5$$

$$= \frac{777 m^3}{sec}$$

O.M.S. of panel

$$= \frac{3 \times 103.68}{500}$$

$$= 0.622$$

Q. 20.

$$\frac{CO}{O_2} \text{ deficiency} = \frac{CO \text{ Produced}}{O_2 \text{ Consumed}}$$

$$= \frac{0.02 / 100}{\frac{20.93}{100} - \frac{19.90}{100}}$$

$$= \frac{0.02}{1.03} = 0.0194$$

$$= 1.94\%$$

$$\frac{CO_2}{O_2} \text{ deficiency} = \frac{CO_2 \text{ Produced}}{O_2 \text{ Consumed}}$$

Scanned by CamScanner

Friction force due to rope on both sides

$$= 2 \times \left(1000 \times 2 \times g \times 1 \times \frac{1}{10} \right)$$

$$= 400g$$

Total force $= 1000\ g + 300\ g + 400\ g$

$F = 1700\ g$

Power required

(at 80% efficiency)

$$= F \times V \times \eta$$

$$= 1700g \times 1 \times 1.2$$

$$= 20.4\ KW$$

Q. 19.

Production/Shift

$$= 5 \times (4.8 \times 2.4 \times 1.2 \times 1.5)$$

$$= 103.68\ t$$

Production/Year

$$= (3 \times 103.68) \times 300$$

$$= 93312\ t$$

Minimum quantity of air

$$= (103.68 \times 3) \times 2.5$$

$$= \frac{777 m^3}{sec}$$

O.M.S. of panel

$$= \frac{3 \times 103.68}{500}$$

$$= 0.622$$

Q. 20.

$$\frac{CO}{O_2} \text{ deficiency} = \frac{CO \text{ Produced}}{O_2 \text{ Consumed}}$$

$$= \frac{0.02/100}{\frac{20.93}{100} - \frac{19.90}{100}}$$

$$= \frac{0.02}{1.03} = 0.0194$$

$$= 1.94\%$$

$$\frac{CO_2}{O_2} \text{ deficiency} = \frac{CO_2 \text{ Produced}}{O_2 \text{ Consumed}}$$

Scanned by CamScanner

$$= \frac{0.45\% - 0.03\%}{20.93\% - 19.90\%}$$

$$= \frac{0.42}{1.03} = 0.407$$

$$= 40.7\%$$

as $\frac{CO}{O_2}$ deficiency is 1.94% (>1%)

& $\frac{CO_2}{O_2}$ deficency is 40.7%

There is a likelyhood of fire existing in panel.

Q. 21.

Unixial compressive strength

$$\sigma_c = (14 + 0.175D)\, I_s$$

$$I_s = \frac{P}{d_2} = \frac{100 \times 10^3}{(50 \times 10^{-3})^2} = 40 \times 10^6 N$$

$$\sigma_c = (14 + 0.175 \times 50) \times 40 \times 10^6$$

$$= 910 \times 10^6\ N/m^2 = 910\ MPa$$

$$E = \frac{\sigma_c}{\epsilon_c} = \frac{Stress}{Strain}$$

$$= \frac{910\ MPa}{\left(\frac{1.5}{125}\right)}$$

$$= 75.83 \times 10^3\ MPa$$

Scanned by CamScanner

1998

SECTION A

1.1	C
1.2	A
1.3	C
1.4	A
1.5	D
1.6	B
1.7	B
1.8	C
1.9	D
1.10	C
1.11	D
1.12	B
1.13	D
1.14	B
1.15	D

2.1	B
2.2	A
2.3	D
2.4	B
2.5	C
2.6	D
2.7	A
2.8	A
2.9	A
2.10	B
2.11	B
2.12	A
2.13	C
2.14	B
2.15	B
2.16	C
2.17	B
2.18	A
2.19	D
2.20	B

3.1

1.	C
2.	D
3.	A
4.	B

3.2

1.	D
2.	C
3.	B
4.	A

3.3

1.	D
2.	C
3.	A
4.	B

3.4

1.	C
2.	D
3.	A
4.	B

3.5

1.	C
2.	A
3.	D
4.	B

3.6

1.	A
2.	B
3.	D
4.	D

3.7

1.	B
2.	A
3.	C
4.	D

3.8

1.	C
2.	B
3.	D
4.	A

3.9

1.	B
2.	C
3.	A
4.	D

3.10

1.	C
2.	B
3.	D
4.	A

Scanned by CamScanner

Q.4

NVP → Net Present value of the Project

$$NVP = -1,05,000 + \frac{60,000}{1.08} + \frac{45,000}{(1.08)^2} + \frac{30,000}{(1.08)^3} + \frac{15,000}{(1.08)^4}$$

$= -1,05,000 + 55,555.5 + 38,580.27 + 23,814.96 + 11,976.22$

$= 23,976.22$ Rs.

IRR (Internal rate of return) is the rate at which the Net Present value of the project becomes zero.

Q.5

Length of drill hole required per month to be drilled

$$= \frac{4,40,000}{7 \times 4}$$

$= 15,714.28$ mt.

Total length which can be drilled in one month.

$$= (20 \times 6 \times 2 \times 5) \times \frac{30}{7}$$

$= 5142.85$ mt./drill/month

no. of drills required

$$= \frac{15,714.28}{5142.85}$$

$= 3.05$

$= 3$

around 3 drill machines in operation.

Q.7

Sample air	**Normal air**
$O_2 - 15\%$	$O_2 \rightarrow 20.95\%$
$CO_2 - 2.8\%$	$CO_2 \rightarrow 0.03\%$
$CH_4 - 4.3\%$	$N_2 \rightarrow 79.02\%$
$N_2 - 77.9\%$	

Precentage of air in the sample air

$$= \frac{15\%}{20.95\%}$$

$= 71.60\%$

Percentage of black damp

$= 1 - 71.6\% = 28.4\%$

Composition of black damp

$CO_2 + N_2$

$$CO_2 \text{ of Sample} = \left(2.8 - .03 \times \frac{71.6}{100}\right)\%$$

$= 2.78\%$

$$N_2 \text{ of sample} = \left(77.9\% - 79.02 \times \frac{71.60}{100}\right)\%$$
$$= 21.32\%$$

$$\% \text{ of } CO_2 \text{ in Sample} = \frac{2.78}{2.780 + 21.32} = 12\%$$

$$\% \text{ of } N_2 \text{ in Sample} = \frac{21.32}{2.780 + 21.32} = 88\%$$

$$CO_2 \rightarrow 12\%$$
$$N_2 \rightarrow 88\%$$

$$\upsilon = 0.3$$
$$\gamma = \frac{0.028 \times 10^6 \text{ N}}{m^3}$$
$$h = 300m$$
$$P_z = \gamma.h = 0.028 \times 300$$
$$= 8.4 \text{ MN/m}^2$$
$$K = \frac{\upsilon}{1-\upsilon} = \frac{0.3}{1-0.3} = \frac{3}{7}$$

$$\sigma_r = \frac{P_z}{2}\left[(1+k)\left(1-\frac{a^2}{r^2}\right) + (1-k)\left(1-\frac{4a^2}{r^2}+\frac{3a^4}{r^4}\right)\cos 2\theta\right]$$

$$\sigma_\theta = \frac{P_z}{2}\left[(1+k)\left(1+\frac{a^2}{r^2}\right) - (1-k)\left(1+\frac{3a^4}{r^4}\right)\cos 2\theta\right]$$

$$z_{r\theta} = \frac{P_z}{2}\left[-(1-k)\left(1+\frac{2a^2}{r^2}-\frac{3a^4}{r^4}\right)\sin 2\theta\right]$$

at boundry

$$r = a = 3m$$
$$\sigma_r = 0$$
$$\tau_{r\theta} = 0$$
$$\sigma_\theta = P_z\left[(1+k) - 2(1-k)\cos 2\theta\right]$$

at $\theta = 0°$

$$\sigma_{0°} = P_z\left[3k - 1\right]$$
$$= 8.4\left[3 \times \frac{3}{7} - 1\right]$$

at $\theta = 90°$

$$\sigma_{0°} = 2.4 \text{ MPa}$$
$$\sigma_{90°} = P_z\left[3 - k\right]$$
$$= 8.4\left(3 - \frac{3}{7}\right)$$
$$= 21.6\text{MPa}$$

Scanned by CamScanner

SECTION B

Q. 9.

$$A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$$

Characteristic equation

$$|A - \lambda I| = \begin{vmatrix} -(2+\lambda) & 2 & -3 \\ 2 & (1-\lambda) & -6 \\ -1 & -2 & -\lambda \end{vmatrix}$$

$\Rightarrow \quad -(2+\lambda)\left[(1-\lambda)-\lambda-12\right]-2\left[-2\lambda-6\right]-3\left[-4+(1-\lambda)\right]=0$

$\Rightarrow \quad (2+\lambda)\left[-\lambda+\lambda^2-12\right]+2(-2\lambda-6)+3(-4+1-\lambda)=0$

$\Rightarrow \quad (2+\lambda)\left(-\lambda+\lambda^2-12\right)-4\lambda-12-9-3\lambda=0$

$\Rightarrow \quad (2+\lambda)\left(\lambda^2-\lambda-12\right)-7\lambda-21=0$

$\Rightarrow \quad (2+\lambda)\left(\lambda^2-4\lambda+3\lambda-12\right)-7(\lambda+3)=0$

$\Rightarrow \quad (2+\lambda)(\lambda-4)\;(\lambda+3)-7(\lambda+3)=0$

$\Rightarrow \quad (\lambda+3)\left[(\lambda+2)(\lambda-4)-7\right]=0$

$\Rightarrow \quad (\lambda+3)\left(\lambda^2-2\lambda-8-7\right)=0$

$\Rightarrow \quad (\lambda+3)\left(\lambda^2-2\lambda-15\right)=0$

$\Rightarrow \quad (\lambda+3)(\lambda+3)(\lambda-5)=0$

hence $\lambda = -3 \,\&\, 5$

Q. 10.

Find General Solution

$$\frac{d^2y}{dx^2} = \frac{a}{y^3}$$

$\Rightarrow \quad d^2y \,.\, y^3 = adx^2$

$\Rightarrow \quad \int y^3 \,.\, d^2y = \int adx^2$

$\Rightarrow \quad \int \frac{y^4}{4} \,.\, dy = \int a \,.\, x \,.\, dx + c''$

$\Rightarrow \quad \frac{y^5}{20} = \frac{ax^2}{2} + c'$

Scanned by CamScanner

$$\Rightarrow \quad y^5 = 10\,ax^2 + 10\,c'$$

$$\Rightarrow \quad y^5 - 10\,ax^2 + c = 0$$

11.

Resistances

$$R_A = \frac{KPL}{A^3} = \frac{0.0098 \times 2\,(1.5 + 6) \times 610}{(1.5 \times 6)^3} = 0.123$$

$$R_B = \frac{KPL}{A^3} = \frac{0.0098 \times 2\,(2 + 5) \times 450}{(2 \times 5)^3} = 0.0617$$

$$R_C = \frac{KPL}{A^3} = \frac{0.0098 \times 2\,(2 + 4) \times 380}{(2 \times 4)^3} = 0.0873$$

$$R_D = \frac{KPL}{A^3} = \frac{0.0098 \times 2(2 + 5.5) \times 580}{(2 \times 5.5)^3} = 0.064$$

equivalent resistance

$$\frac{1}{\sqrt{R}} = \frac{1}{\sqrt{R_A}} + \frac{1}{\sqrt{R_B}} + \frac{1}{\sqrt{R_C}} + \frac{1}{\sqrt{R_D}}$$

$$= \frac{1}{\sqrt{0.123}} + \frac{1}{\sqrt{0.617}} + \frac{1}{\sqrt{0.873}} + \frac{1}{\sqrt{0.064}}$$

$$= 14.21$$

$$\Rightarrow \quad R = \left(\frac{1}{14.21}\right)^2 = 4.95 \times 10^{-3}\, NS^2 m^{-8}$$

$$\Delta P = RQ^2 = 4.95 \times 10^{-3} \times (85)^2 = 35.76$$

$$Q_A = \sqrt{\frac{\Delta P}{R_A}} = \sqrt{\frac{35.76}{0.123}} = 17\ m^3/sec.$$

$$Q_B = \sqrt{\frac{\Delta P}{R_B}} = \sqrt{\frac{35.76}{0.617}} = 24\ m^3/sec.$$

$$Q_C = \sqrt{\frac{\Delta P}{R_C}} = \sqrt{\frac{35.76}{0.873}} = 20.24\ m^3/sec.$$

$$Q_D = \sqrt{\frac{\Delta P}{R_D}} = \sqrt{\frac{35.76}{0.064}} = 23\ m^3/sec.$$

Q. *12.*

$$W_1 = \frac{300\ rad}{sec}$$

Let Q be the initial quantity

Scanned by CamScanner

$$\Delta P = RQ^2 = 0.1Q^2 - 35Q + 2100$$

$$\Rightarrow \quad 1.1Q^2 = 0.1Q^2 - 35Q + 2100$$

$$\Rightarrow \quad Q^2 + 35Q - 2100 = 0$$

$$\Rightarrow \quad Q = \frac{-35 \pm \sqrt{(35)^2 + 4 \times 2100}}{2}$$

$$= \frac{35 + 98.1}{2}$$

$$= 31.56 \text{ m}^3/\text{sec.}$$

New quantity

$$Q' = 40 \text{ m}^3/\text{sec.}$$

Change in speed

$$\frac{Q'}{Q} = \frac{W'}{W}$$

$$\therefore \quad \frac{\omega'}{\omega} = \frac{Q'}{Q}$$

$$\Rightarrow \quad \frac{\omega' - \omega}{\omega} = \frac{Q' - Q}{Q}$$

$$\Rightarrow \quad \frac{\omega' - \omega}{\omega} = \frac{40 - 31.56}{40}$$

$$\Rightarrow \quad \frac{\omega' - \omega}{\omega} = 0.211$$

$\therefore$ % Change in speed $= 0.211 \times 100$

$= 21.1\%$

Q. 14.

Length of the drift

angle between drift and seam

$= 40^\circ + 20^\circ$

$= 60^\circ$

apparent dip of coal seam in the direction of drift

$$\cot\beta = \cot\alpha \cdot \cot\theta$$

$$\cot\beta = \cot\alpha \cdot \sec 60^\circ$$

$$= 6 \times 2 = .2$$

$$\tan\beta = \frac{1}{12} \text{ (1 in 12)}$$

rate of approach of drift

$$= \frac{1}{12} + \frac{1}{7} = \frac{7 + 12}{84} = \frac{19}{84}$$

Scanned by CamScanner

horizontal length of drift

$$= \frac{40}{\frac{19}{84}}$$

$$= 176.84 \text{ m}$$

Actual (inelined) length of drift

$$= 176.84\left(\sqrt{\frac{50}{7}}\right)$$

$$= 178.63 \text{ m}$$

Q. 16.

Capacity of shovel

Per 40 sec $= \frac{6 \times .8}{1.2} = 4 \text{ m}^3$

Per hour $= \frac{4}{40} \times 60 \times 60$

Per day $= \frac{4}{40} \times 60 \times 60\,(2 \times 7)$

$= 5040 \text{ m}^3/\text{day}$

Capacity of trucks

Per truck $= 50 \text{ t}/15 \text{ min.}$

$= \frac{50}{2.5} \text{ m}^3/15 \text{ min}$

$= 20 \text{ m}^3/15 \text{ min.}$

Capacity/hr./truck $= \frac{20}{15} \times 60$

/day/truck $= \frac{20}{15} \times 60\,(2 \times 7)$

Capacity/4 truck/day $= 4 \times 1120$

$= 4480 \text{ m}^3$ ....(2)

so net capacity is less of (1) and (2) which is $4480 \text{ m}^3/\text{day}$

advance / day $= \frac{4480 \text{ m}^3/\text{day}}{\omega \times h}$

$= \frac{4480}{10 \times 10}$

$= 44.80 \text{ m / day}$

Q. 13.

$$\frac{f}{i} = 100 \;\&\; (f + d) = 0$$

Scanned by CamScanner

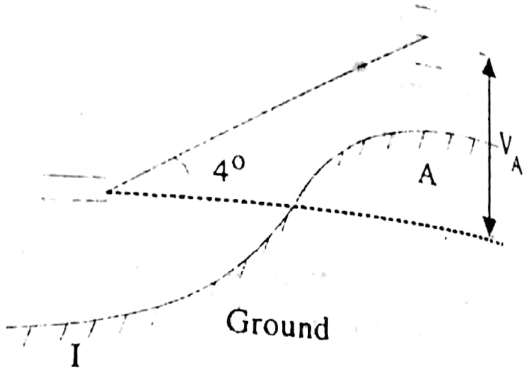
R_L of A

$$V_A = \frac{f}{i} \cdot \frac{\sin 2\theta}{2} \cdot s$$

$$= 100 \cdot \frac{\sin 8^\circ}{2}(3.75 - 2.45)$$

$$= 100 \times 0.0695 \times 1.3$$

$$= 9.046 \text{ m}$$

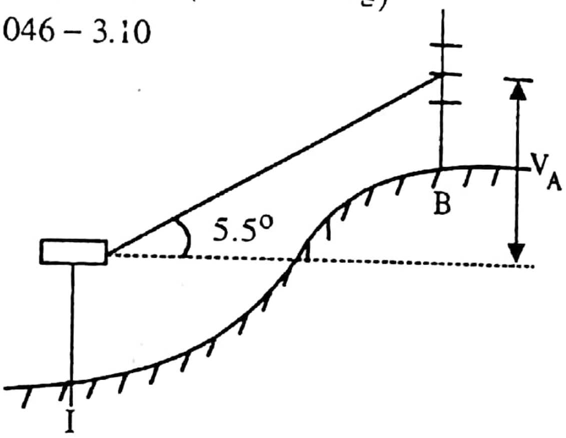


R_L of A

$$V_A = (1285.40) + (\text{ht. of inst}) + v_A - (\text{axial reading})$$
$$= 1285.40 + 1.70 + 9.046 - 3.10$$
$$= 1293.04 \text{ m}$$

R_L of B

$$V_B = \frac{f}{i} \cdot s \cdot \frac{\sin\theta}{2}$$



$\Rightarrow$ $$V_B = 100 \times (3.50 - 2.50) \times \frac{\sin 11^\circ}{2}$$
$$= 9.54 \text{ m}$$

R_L B $= (1285.40) + (\text{Inst. ht.}) + VB - (\text{axial reading})$
$$= 1285.40 + 1.70 + 9.54 - 3$$
$$= 1293.64 \text{ m}$$

Q.18.

sand required to fill the void

$$= \frac{380 \times 0.95}{3} = 120.3\,\text{m}^3/\text{hr.}$$

Vol. Conc. of sand in fluid

$$= \frac{120.3}{12.3 + 280} = 30\%$$

wt. of sand $= 120.3 \times 2.8 = 337\ t/hr.$

wt. of water $= 280 \times 1.0 = 280\ t/hr.$

density of slurry $= \dfrac{337 + 280}{120.3 + 280}$

$$= \frac{617}{400.3} = 1.54\ t/m^3$$

Scanned by CamScanner

$$Q = 400 m^3/hr. = 0.11\ m^3/sec.$$

Velocity

$$V = \frac{Q}{A} = \frac{0.11}{\pi(.1)^2} = 3.54\ m/s$$

Q. 20.

Find major & minor Principal Stresses

Major Principal Stress

$$\sigma_1 = \frac{1}{2}(\sigma_x + \sigma_y) + \sqrt{\frac{1}{4}(\sigma_x - \sigma_y)^2 - \tau_{xy}^2}$$

$$= \frac{1}{2}(1.721 + 2.76) + \sqrt{\frac{1}{4}(1.724 - 2.76)^2 + (-.689)^2}$$

$$= 2.24 + 1.24$$

$$= 3.48\ \text{MPa}$$

Minor Principal Stress

$$\sigma_2 = \frac{1}{2}(\sigma_x + \sigma_y) - \sqrt{\frac{1}{4}(\sigma_x - \sigma_y)^2 + \tau_{xy}^2}$$

$$= \frac{1}{2}(1.72 + 2.76) - \sqrt{\frac{1}{4}(1.724 - 2.76)^2 + (-0.689)^2}$$

$$= 2.24 - 1.24$$

$$= 1\ \text{MPa}$$

$$\tan\alpha_1 = \frac{\sigma_x - \sigma_y}{\tau_{xy}} = \frac{1.724 - 2.76}{-0.689}$$

$$= 1.5$$

$$\alpha_1 = \tan^{-1} 1.5$$

$$= (56.37)^o$$

$$\alpha_2 = 90^o + \alpha_1 = 146.37^o$$

Q. 22.

Position	Width (m)	Value (g/t)	Distance of Infl.	Area of Infl.	Area Assay Prod.
2	0.75	15	5	3.75	56.25
8	0.80	17	4	3.2	54.40
10	0.70	8	3.5	2.45	19.60
15	1.05	11	5	5.25	57.75
20	0.92	20	7.5	6.9	138.0
30	0.85	28	6	5.1	142.0
32	0.50	12	2	1.0	12.0
Σ			33.0	27.65	481.60

Scanned by CamScanner

$$\text{Mean width} = \frac{\sum \text{Area of Influence}}{\sum \text{Dist.}^{\text{n}} \text{of Influence}} \quad \text{(inm)}$$

$$= \frac{27.65}{33} = 0.84 \text{ m}$$

$$\text{Mean assay value} = \frac{\sum \text{Area Assay Product}}{\sum \text{Area of Influence}}$$

$$= \frac{481.60}{27.65} = 17.5 \text{ (g/t)}$$

Q. 26.

$n = 6$

$rpm = 1440/min = 24/sec.$

$Q = 3.3 \ m^3/min.$

$H = 240$

$V_R = 1.8 \ m/s$

$\eta_m = 74\%$

$\theta = 30^\circ$

$V_i = 1.7 \ m/s$

$$\text{head / stage} = \frac{240}{6} = 40\text{m}$$

$$\text{theoretical head / stage } h_t = \frac{40}{0.74}$$

$$h_t = \frac{U^2 - UV_R \cot\theta}{g}$$

$$\Rightarrow \quad 54 = \frac{U^2 - U \times 1.8 \times \cot 30^\circ}{9.8}$$

$$\Rightarrow \quad 529.2 = U^2 - 3.118U$$

$$\Rightarrow \quad U^2 - 3.118U - 529.2 = 0$$

$$\Rightarrow \quad U = \frac{3.118 \pm \sqrt{(3.118)^2 + 4 \times 529.2}}{2}$$

$$= \frac{3.118 + 46.114}{2}$$

$$= 24.6 \ \text{m/sec.}$$

$$U = 2\pi r_{imp} \cdot \left(\frac{\text{r.p.m.}}{60}\right)$$

$$\Rightarrow \quad D = \frac{24.60}{\pi \times 24} = 0.326 \text{ m}$$

width of impeller

$$2\pi r_{imp.} \times \omega_{imp} = \frac{Q}{V_R}$$

$$\Rightarrow \qquad \omega\ \text{imp} = \frac{33/60}{1.8\pi \times 0.326}$$

$$= 0.0298\ \text{m}$$

$$= 3\ \text{cm}$$

Q. 27.

$h = 294$ m

$T = t_1 + t_2 + t_3 = 58$ sec.

$u = at_1 = 0.6\ t_1$

$0.6t_1 = 0.75t_3$

$\Rightarrow \quad t_1 = 1.25t_3$

& $\quad t_2 = 58 - (t_3 + 1.25t_3)$ sa

$= 58 - 2.25\ t_3$

Distances travelled in (t_1)

$$h_1 = ut_1 + \frac{1}{2} a_1 t_1^2$$

$$= \frac{1}{2} \times 0.6 \times (1.25\ t_3)^2$$

$$= 0.469 t_3^2$$

in (t_2)

$$h_2 = ut_2 + \frac{1}{2} a_2 t_2^2$$

$$= 0.6t_1 \times t_2$$

$$= 0.6t_1\ (58 - 2.25t_3)$$

$$= 43.5t_3 - 1.687t_3^2$$

in (t_3)

$$h_3 = \frac{1}{2} a_3 t_3^2$$

$$= \frac{1}{2} \times 0.75t_3^2$$

$$= 0.375t_3^2$$

$$h = h_1 + h_2 + h_3$$

$$\Rightarrow \qquad 294 = 0.469\ t_3^2 + 43.5t_3 - 1.687t_3^2 - 3.75t_3^2$$

$$294 = -0.843t_3^2 + 43.5t_3$$

$$0.84t_3^2 - 43.5t_3 + 294 = 0$$

$$t_3^2 - 51.6t_3 + 247.84 = 0$$

Solving we get

$t_3 = 5.36$ sec

$\therefore \quad t_1 = 1.25t_3 = 6.7$ sec

& $\quad t_2 = 45.94$

Scanned by CamScanner

Maximum speed

$$V = at_1 = 0.6 \times 6.7$$
$$= 4.02 \text{ m / sec.}$$

Distance travelled during acceleration

$$= \frac{1}{2} a_1 t_1^2$$
$$= \frac{1}{2} \times 0.6 \times (6.7)^2$$
$$= 13.46 \text{ m}$$

Distance during decederation

$$= \frac{1}{2} a_3 t_3^2 = \frac{1}{2} \times 0.75 (5.36)^2 = 10.77 \text{ m}$$

Q. 28.

$$f(x, y) = x^2 - 2xy - 4x + 2y + 2y^2$$
$$\frac{\delta f}{\delta x} = 2x - 2y - 4 = 0 \quad \text{............(1)}$$
$$\frac{\delta f}{\delta x} = 4y + 2 - 2x = 0 \quad \text{............(2)}$$

adding the two equations

$$2y - 2 = 0$$
$$\Rightarrow \quad y = 1$$

from equn. (1) we get

$$2x - 2 - 4 = 0$$
$$\Rightarrow \quad x = 3$$

So point is (x, y), i.e. (3, 1)

$$r = \frac{\delta^2 f}{\delta x^2} = 2$$

$$t = \frac{\delta^2 f}{\delta y^2} = 4$$

$$s = \frac{\delta^2 f}{\delta x . sy} = -2$$

Now $\quad rt - s^2 = 2 \times 4 - (-2)^2 = 12$

So $\quad rt - s^2 > 0$

& $\quad r > 0$

hence point (3, 1) is a minima.

1997

SECTION A

1.1

(1) C
(2) A
(3) B
(4) D

1.2

(1) B
(2) A
(3) D
(4) C

1.3

(1) D
(2) C
(3) B
(4) A

1.4

(1) A
(2) C
(3) D
(4) B

1.5

(1) C
(2) D
(3) A
(4) B

Q. 2.

2.1 C
2.2 B
2.3 A
2.4 D
2.5 B
2.6 D
2.7 A
2.8 B
2.9 C
2.10 B
2.11 C
2.12 D
2.13 B
2.14 D
2.15 A

Q. 3

3.1 A
3.2 D
3.3 B
3.4 A
3.5 B
3.6 C
3.7 B
3.8 A
3.9 D
C 3.10 ~~B~~
3.11 ~~A~~ B
3.12 A
3.13 ~~A~~ B
3.14 D
3.15 D
3.16 B
3.17 A
3.18 C
3.19 B
3.20 B
3.21 D
3.22 B
3.23 A
3.24 D
3.25 D

Scanned by CamScanner

Q. 4.

Celculate the power consumed by conveyor

$$\rho = 800 \text{ kg/m}^3$$

vol. of cal / sec.

$$= \frac{60}{800} = 0.075 \text{ m}^3/\text{sec.}$$

speed of belt

$$V = \frac{0.075}{0.05} = 1.5 \text{ m/sec.}$$

Frictional force

$$= \mu \text{ mg}$$

$$= 27{,}000 \times 0.03 \times g$$

$$= 7.938 \times 10^3 \text{ N}$$

Power

$$P = F.V$$

$$= 7.938 \times 10^3 \times 1.5$$

$$= 11.9 \times 10^3 \text{ watt}$$

Q. 6.

$$Q_1 = 40 \text{ m}^3/\text{sec.}$$

$$\Delta\rho = 800\ \rho_a$$

$$\Delta P = RQ^2$$

initial registrance of the mine

$$R = \frac{\Delta P}{Q^2} = \frac{800}{(90)^2} = 0.5 \text{ NS}^2\text{m}^{-8}$$

Pressure drop acrosss shaft

$$\Delta P_1 = \frac{60}{100} \times 800$$

$$= 480\ p_a$$

Resistance of the shaft

$$R_1 = \frac{480}{(40)^2} = 0.3 \text{NS}^2\text{m}^{-8}$$

Resistance of rest of mine

$$R_2 = 0.5 - 0.3 = 0.2 \text{ Ns}^2\text{m}^{-8}$$

Now, $R = \frac{K.S}{A^3} = \frac{K.P.L}{A^3}$

So for shaft

$$R_2 = \frac{K.P.L}{A^3}$$

Scanned by CamScanner

$$\Rightarrow \quad 0.3 = \frac{K(2\pi r)L}{(\pi r^2)^3} = \frac{KL(\pi \times 3)}{\left[\pi(1.5)^2\right]^3} = 0.0267\ KL$$

$$\Rightarrow \quad 0.3 = 0.0267\ KL$$

$$\Rightarrow \quad KL = \frac{0.3}{0.0267}\ 11.24$$

New resistance of the shaft

$$R'_1 = \frac{KL.\Delta P'}{A^3} = \frac{(2\pi \times 2)\ KL}{(\pi . 2^2)^3}$$

$$= \frac{11.24\ (4\pi)}{(4\pi)^3}$$

$$= 0.0711$$

New resistance of mine

$$R_n = R_1' + R_2$$

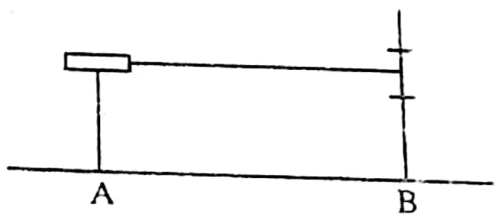
$$= 0.0711 + 0.2$$

$$= 0.2711$$

New Quantity

$$Q' = \sqrt{\frac{\Delta P}{R}} = \sqrt{\frac{800}{0.2711}} = 54.3\ m^3/sec.$$

Q. 7.



$$\frac{f}{i} = 100,\ (f + d) = 0.3$$

$$\theta = 0°$$

Distance

$$AB = \frac{f}{i}.s.\cos^2\theta + (f + d)\cos\theta$$

$$= 100 \times (3.45 - 1.15) \times (\cos 0°)^2 + 0.3 \times \cos 0°$$

$$= 100 \times 2.3 \times 1 + 0.3$$

$$= 230 + 0.3$$

$$= 230.3\ m$$

RL of staff station

$$R_{LB} = R_{LA} - \text{ht. of instrument}$$

$$= 75.75 - 2.30$$

$$= 73.45\ m$$

Scanned by CamScanner

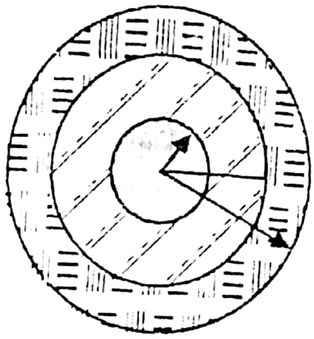
Q. 8.

Area of inner Circle $= \pi r^2 = \pi 1^2 = \pi\ cm^2$

Area of middle space $= \pi\,(3)^2 - \pi \times 1^2 = 8\,\pi\ cm^2$

Area of outer space $= \pi\,(5)^2 - \pi\,(3)^2 = 16\,\pi\ cm^2$

Total area $= \pi(5)^2 = 25\pi$



Probability of hitting inner Circle

$$P_1 = \frac{\pi}{25\,\pi} \times 0.5 = \frac{0.5}{25} = 0.02$$

Point earned $T_1 = 0.02 \times 10 = 2$

Probability of hitting middle area

$$P_2 = \frac{8\pi}{25\pi} \times 0.5 = \frac{0.5}{25} = 0.02$$

Point eaearned $T_2 = 0.16 \times 5 = 0.8$

Probability of hitting outer space

$$P_3 = \frac{16\pi}{25\pi} \times 0.5 = 0.32$$

Point earned $T_3 = 0.96$

Total points earned

$= T_1 + T_2 + T_3$

$= 2 + 0.8 + 0.96$

$= 3.76$

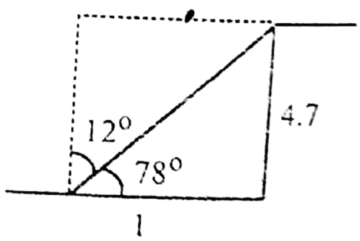
Q. 5.

Total force which the material can withstand

$= mg \sin\theta$

$= 0.14 \times 10^6 \left(\frac{1}{2} \times 4.7\right) \times 0.978$

$= 0.32 \times 10^6$



Actual load on bedding plane

$= mg \sin\theta$

$= 27 \times 10^3 \left(\frac{1}{2} \times 1.23\right) \times \sin 51°$

$= 12.9 \times 10^3\ N/\text{min.}$

Factor of safety

$$= \frac{0.32 \times 10^6}{12.9 \times 10^3}$$

$= 24.8$

≈ 25

Scanned by CamScanner

SECTION B

Q. 9.

Amount of sand required to fill the void

$$V_s = (100 \times 2.0 \times 1.2) \times 0.9$$
$$= 216 \text{ m}^3$$

hourly consumption of sand

$$= \frac{216}{3} = 72 \text{ m}^3/\text{hr}.$$

Amount of slurry required / hour

$$= \frac{72}{0.4} = 180 \text{ m}^3/\text{hr}.$$

hourly consumption of water

$$= 180 - 72 = 108 \text{ m}^3/\text{hr}.$$

Q. 13.

Maximise

$$w = x + y + z$$

subject to $0.5x + y + z \leq 18$ ....(1)

$x + 0.5y + z \leq 18$ ....(2)

$z = 8$ & $x, y \geq 0$

For $z = 8$ equations (1) & (2) become

$$0.5x + y \leq 10$$
$$x + 0.5y \leq 10$$

or $0.5x + y = 10$ ....(3)

$x + 0.5y = 10$ ....(4)

multiply (3) by 2 and (4) by 1

$$x + 2y = 20$$
$$\underline{-\ x + 0.5y = 10}$$
$$1.5y = 10$$

$\Rightarrow$ $y = \dfrac{10}{1.5} = 6.67$

$x = 3.33$

so point is $(x, y, z) = (3.33, 6.67, 8) = p_1$

in equn. (3) for $x = 0$, $y = 0$ pointis $= (0, 10, 8)\ p_2$

in equn. (4) for $y = 0$, $x = 10$, point is $= (10, 0, 8) = p_3$

for $x = 0$, $y = 0$, point is $= (0, 0, 8) = P_4$

out of above four points points "P_1" has maximum value of z.

Scanned by CamScanner

Hence

$$Z_{max} = 3.33 + 6.67 + 8 = 18$$

Q. 16.

$$Q = 40\ m^3/s, \quad \Delta P = 560\ P_a$$

Resistance of mine

$$\Delta P = RQ^2$$

$$R = \frac{\Delta P}{Q^2} = \frac{560}{(40)^2} = 0.35\ Ns^2m^{-8}$$

For new fan

new quantity = Q'

$$\Delta P = RQ'^2 = 0.1Q'^2 - 37.5\ Q' + 2500$$

$$\Rightarrow \quad 0.35 \times Q'^2 = 0.1Q'^2 - 37.5\ Q' + 2500$$

$$\Rightarrow \quad 0.25 \times Q'^2 + 37.5\ Q' - 2500 = 0$$

Solving we get

$$Q' = \frac{-37.5 \pm \sqrt{(37.5)^2 - 4 \times 0.25 \times 2500}}{2 \times 0.25}$$

$$= \frac{-37.5 + 62.5}{2 \times 0.25}$$

$$= 50\ m^3/sec$$

$$P = RQ'^2$$

$$= 0.35 \times (50)^2$$

$$= 875\ P_a$$

Hence operating point (new)

$Q = 50\ m^3/sec.$

$P = 875\ p_a$

Q.18

Find qugles between vectors

$$\vec{A} . \vec{B} = |A| . |B| . \cos\theta$$

$$\cos\theta = \frac{\vec{A} . \vec{B}}{|A| . |B|}$$

$$= \frac{(2i + j + k).(i - 3j + 5k)}{\left(\sqrt{2^2 + 1^2 + 1^2}\right).\left(\sqrt{1^2 + (-3)^2 + 5^2}\right)}$$

Scanned by CamScanner

$$= \frac{2 - 3 + 5}{\sqrt{6} \times \sqrt{35}}$$

$$= \frac{4}{\sqrt{6} \times \sqrt{35}}$$

$$= 0.276$$

$$\theta = \cos^{-1}(0.276)$$

$$\theta = 74^{\circ}$$

Q. 19.

Drilling required / year

$$= \frac{2.4 \times 10^6}{(8 \times 5)} = 60 \times 10^3 \text{ m/yr.}$$

Drilling length/yr. by a drill m/c

$$= 15 \times 5000 \times 0.65$$

$$= 16.25 \times 10^3 \text{ m/yr.}$$

no. of drill m/c required

$$n = \frac{60 \times 10^3}{16.25 \times 10^3}$$

$$= 3.69$$

$$\approx 4 \text{ nos.}$$

Q. 20.

on plane which is equally inclined to all the there axes $\alpha = 45^{\circ}$

normal stress

$$\sigma_{x'} = \frac{1}{2}(\sigma_x - \sigma_y) + \frac{1}{2}(\sigma_x - \sigma_y)\cos 2\alpha - \tau_{xy}\sin 2\alpha$$

$$= \frac{1}{2}(100 - 50) + \frac{1}{2}(100 + 50)\cos 90^{\circ} - 100 \times \sin 90^{\circ}$$

$$= 50 - 100 = -50 \text{ MPa}$$

$$\sigma_{y'} = \frac{1}{2}(\sigma_x - \sigma_y) + \frac{1}{2}(\sigma_x - \sigma_y) - \tau_{xy}\sin 2\alpha$$

$$= \frac{1}{2}(100 - 50) - 0 - 100 \times \sin 90^{\circ}$$

$$= -50 \text{ MPa}$$

Shearing stress

$$\tau_{x'y'} = \tau_{xy}\cos 2\alpha - \frac{1}{2}(\sigma_x - \sigma_y)\sin 2\alpha$$

$$= 100 \times \cos 90^{\circ} - \frac{1}{2}(100 - 50)\sin 90$$

$$= -75 \text{ MPa}$$

Scanned by CamScanner

Q. 22

$\omega = 9$ m

simpson rule

$$\text{Area} = \frac{\omega}{3n}[\text{first ordinate + Last ordinate + 2 (all the other odd ordinates)} + 4 \text{(all the other even ordinates)}]$$

$$= \frac{9}{3}[10.8 + 19.9 + 2(18.6 + 20.8 + 25.5) + 4(13.9 + 16.9 + 22.9 + 23.2)]$$

9

$$= \frac{9}{3}[30.7 + 2(64.9) + 4(76.9)]$$

$$= \frac{1}{3}[468.1]$$

~~= 152.7 m²~~ 156.03 m²

Q. 23.

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 2 & 3 & 4 \\ -1 & -1 & -2 \end{bmatrix}$$

Characteristic equation

$$|A - \lambda I| = 0$$

$$\Rightarrow \begin{vmatrix} 2-\lambda & 1 & 1 \\ 2 & 3-\lambda & 4 \\ -1 & -1 & -2-\lambda \end{vmatrix} = 0$$

$$\Rightarrow (2-\lambda)[(3-\lambda)(-2-\lambda)+4] - 1[2(-2-\lambda)+4] + 1[-2+1(3-\lambda)] = 0$$

$$\Rightarrow (2-\lambda)[-6-3\lambda+2\lambda+\lambda^2+4] - [-4-2\lambda+4] + [-2\lambda-3-\lambda] = 0$$

$$\Rightarrow (2-\lambda)(\lambda^2-\lambda^{-2}) + 2\lambda + 1 - \lambda = 0$$

$$\Rightarrow (2-\lambda)(\lambda-2)(\lambda+1) + (\lambda+1) = 0$$

$$\Rightarrow (\lambda+1)[(2-\lambda)(\lambda-2)+1] = 0$$

$$\Rightarrow (\lambda+1)[\lambda^2 + 4\lambda - 4x + 1] = 0$$

$$\Rightarrow (\lambda+1)[\lambda^2 - 4\lambda + 3] = 0$$

$$\Rightarrow (\lambda+1)[\lambda(\lambda-1) - 3(\lambda-1)] = 0$$

$$\Rightarrow (\lambda+1)(\lambda-1)(\lambda-3) = 0$$

$\Rightarrow$ $\lambda = 1, -1$ & 3 which are real & distinct.

Scanned by CamScanner

24.

$H_T = 60$ m

$V_R = 2.0$ m / s

$\theta = 30^{o}$

$d_{imp} = 0.4$ m

$$H_T = \frac{U^2 - UV_R \cot B}{g}$$

$$60 = \frac{U^2 - U \times 2 \times \cot 30^{o}}{9.8}$$

$\Rightarrow \quad 588 = U^2 - 3.46\,U$

$\Rightarrow \quad U^2 - 3.46U - 588 = 0$

$$\Rightarrow \quad U = \frac{3.46 \pm \sqrt{(3.46)^2 + 4 \times 588}}{2}$$

$$= \frac{3.46 + 48.62}{2} = \frac{52.1}{2} \text{ m/s}$$

$= .26$ m/s

$\therefore$ Tangential speed = 26 m/s

$$2\pi\, r_{imp.} \left(\frac{rpm}{60}\right) = U$$

$$\Rightarrow \quad rpm = \frac{U \times 60}{\pi \times (0.4)}$$

$$= \frac{26 \times 60}{\pi_x (0.4)}$$

$= 1241$ / minute

25.

Future value S_n

$$S_n = R(1 + i')^n + R(1 + i')^{n-1} + R(1 + i)^{n-2} \ldots\ldots\ldots + R(1 + i)$$

$$= R(1 + i) + R(1 + i)^2 + \ldots\ldots\ldots\ldots n \text{ terms}$$

$$= R(1 + i)\left[1 + (1 + i) + (1 + i)^2 + \ldots\ldots\ldots\ldots n \text{ terms}\right]$$

$$= R(1 + i)\left[\frac{1(1 + i)^n - 1}{(1 + i) - 1}\right]$$

$$= R(1 + i)\left[\frac{(1 + i)^n - 1}{i}\right]$$

Scanned by CamScanner

$$S_n = R(1+i)\left[\frac{(1+i)^n - 1}{i}\right] \quad \text{..........(A)}$$

saving per year for n = 10

$i = 12\%$

& $S_n = 20 \times 10^6$ Rs.

From equation (A)

$$R = \frac{S_n}{(1+i)^n - 1}\left(\frac{i}{1+i}\right)$$

$$= \frac{20 \times 10^6}{\left[(1+0.12)^{10} - 1\right]}\left[\frac{0.12}{1.12}\right]$$

$$= \frac{20 \times 10^6}{2.1} \times \frac{0.12}{1.12}$$

= 1.017 million/yr.

Q. 26.

Area = $4 \times 3 = 12 m^2$

Heavy type stone dust barrier

Quantity of stone dust required

$= 12 \times 390$

= 4680 kg

no. of self (@ 60 kg/m)

$$= \frac{4680}{60 \times 4}$$

= 19.5

= 20 nos.

self width = 50 cm

Distance between shelves $0.125 \text{ m} < d \leq 0.27$

Q. 28.

From euler's theorm for homogeneous equn.

$$x \cdot \frac{\delta u}{\delta x} + y \frac{\delta u}{\delta y} = nu \quad \text{............(1)}$$

differentiating w.r.t. x

$$x \cdot \frac{\delta^2 u}{\delta x^2} + \frac{\delta u}{\delta x} + y \frac{\delta^2 u}{\delta y \cdot \delta x} = n \cdot \frac{\delta u}{\delta x} \quad \text{............(2)}$$

differentiating equation (1) ω.r.t.y

$$x \cdot \frac{\delta u}{\delta x \cdot \delta y} + \frac{\delta u}{\delta y} + y \frac{\delta^2 u}{\delta y^2} = n \cdot \frac{\delta u}{\delta y}$$

Scanned by CamScanner

multiplying equation (2) with x and (3) with y and adding both

$$x^2 \cdot \frac{\delta^2 u}{\delta x^2} + x \cdot \frac{\delta u}{\delta y} + xy \frac{\delta^2 u}{\delta x \cdot \delta y} = n \cdot x \frac{\delta u}{\delta x} \qquad \text{.............(4)}$$

$$x \cdot y \frac{\delta u}{\delta x \cdot \delta y} + y \frac{\delta u}{\delta y} + y^2 \frac{\delta^2 u}{\delta y^2} = n \cdot y \cdot \frac{\delta u}{\delta y} \qquad \text{...............(5)}$$

Adding (4) & (5)

$$\left(x^2 \frac{\delta^2 u}{\delta x^2} + 2\,xy \frac{\delta^2 u}{\delta x \cdot \delta y} + y^2 \frac{\delta^2 u}{\delta y^2} \right) + \left(x \frac{\delta u}{\delta x} + x \frac{\delta u}{\delta y} \right) = n \left(x \frac{\delta u}{\delta x} + y \frac{\delta u}{\delta y} \right)$$

$$\Rightarrow \quad \left(x^2 \frac{\delta^2 u}{\delta x^2} + 2\,xy \frac{\delta^2 u}{\delta x \cdot \delta y} + y^2 \frac{\delta^2 u}{\delta y^2} \right) = (n-1) \left(x \cdot \frac{\delta u}{\delta y} + y \cdot \frac{\delta u}{\delta y} \right)$$

$$\Rightarrow \quad x^2 \frac{\delta^2 u}{\delta x^2} + 2\,xy \frac{\delta^2 u}{\delta x \cdot \delta y} + y^2 \frac{\delta^2 u}{\delta y^2} = n\,(n-1)\,u$$

Proved

Scanned by CamScanner

1995
SECTION A

1.1

(1)	B
(2)	C
(3)	D
(4)	A

1.2

(1)	B
(2)	B
(3)	D
(4)	C

1.3

(1)	D
(2)	C
(3)	B
(4)	A

1.4

(1)	D
(2)	C
(3)	B
(4)	A

1.5

(1)	B
(2)	C
(3)	D
(4)	A

Q. 2.

2.1	B
2.2	C
2.3	B
2.4	B
2.5	B
2.6	B
2.7	D
2.8	B
2.9	C
2.10	C
2.11	B
2.12	D
2.13	A
2.14	C
2.15	B

Q. 3

3.1	C
3.2	B
3.3	A
3.4	B
3.5	D
3.6	D
3.7	C
3.8	B
3.9	C
3.10	A
3.11	B
3.12	C
3.13	B
3.14	D
3.15	C
3.16	~~C~~ D
3.17	A
3.18	C
3.19	~~C~~ D
3.20	B
3.21	D
3.22	A
3.23	B
3.24	D
3.25	B

Scanned by CamScanner

Q. 4.

$$u = x^2 \tan^{-1}\frac{y}{x} - y^2 \tan^{-1}\frac{x}{y} \text{ and } xy \neq 0$$

$$\frac{\delta u}{\delta x} = 2x \cdot \tan^{-1}\frac{y}{x} + x^2 \frac{1}{1+\left(\frac{y}{x}\right)^2} \times \frac{-y}{x^2} - y^2 \frac{1}{1+\left(\frac{x}{y}\right)^2} \times \frac{1}{y}$$

$$= 2x \cdot \tan^{-1}\frac{y}{x} + \frac{-x^2 y}{x^2+y^2} - \frac{y^3}{x^2+y^2}$$

$$\therefore \quad \frac{\delta u}{\delta x} = 2x \cdot \tan^{-1}\frac{y}{x} + \frac{-x^2 y - y^2}{x^2+y^2}$$

$$\frac{\delta u}{\delta x} = 2x \cdot \tan^{-1}\frac{y}{x} + \frac{-y\left(x^2+y^2\right)}{x^2+y^2}$$

$$\frac{\delta u}{\delta x} = 2x \cdot \tan^{-1}\frac{y}{x} - y$$

$$\frac{\delta u}{\delta x \cdot \delta y} = 2x \cdot \frac{1}{1+\left(\frac{y}{x}\right)^2} \times \frac{1}{x} \times 1 - 1$$

$$= \frac{2x^3}{\left(x^2+y^2\right)} \times \frac{1}{x} - 1$$

$$= \frac{2x^2}{\left(x^2+y^2\right)} - 1$$

$$= \frac{2x^2 - x^2 + y^2}{x^2+y^2}$$

$$= \frac{x^2-y^2}{x^2+y^2}$$

$$\frac{\delta^2 u}{\delta x \cdot \delta y} = \frac{x^2-y^2}{x^2+y^2}$$

Proved

Q. 5.

$n = 8$ holes

Pull = 1.8 m

Face = 2×3

$\rho = 0.8$

$d(o) = 76$ mm $= 0.76$ m

$Q = 670$ N

Scanned by CamScanner

Load factor = $0.381\ m^3 / KN$ Incut holes $\rightarrow$ 160 N

Total length of charge

$$Vol = \frac{\pi}{4}(0.076)^2 \times h \times 0.8 \times 10^3 \times 9.8 = 670$$

$$\Rightarrow \quad h = \frac{670 \times 4}{\pi (0.076)^2 \times 0.8 \times 10^3 \times 9.8}$$

$$= 4.71 \text{ meter.} \times 4 = 18.81$$

In cut holes length of charge

$$L = \frac{160}{670} \times 4.71$$

$$= 1.125 \text{ m.}$$

In length other them cut holes

$$= 4.71 - 1.25 = 3.585 \text{ m.}$$

as 40% of these holes are charged

$$\text{Length of holes} = \frac{3.385}{0.6} = 5.97 \text{ m.}$$

Length of holes (each) (other than cut holes)

$$= \left(1.8 + \frac{1.125}{8 \times 2}\right)$$

$$= 1.8 + 0\ .\ 22 = 2.02$$

no. of holes other than cut holes

$$= \frac{5.97}{2.02} = 3 \text{ no.}$$

Powder factor

$$\text{Mineral} = \frac{1.8 \times 2 \times 3}{0.0381 \times 9.8} t$$

$$= 29\ t$$

$$\text{Powder factor} = \frac{29}{\frac{670}{9.8}}$$

$$= 0.42\ t/kg.$$

Q. 6.

depth = 100 m

$\omega_0 = 4\,m$

$\omega_p = 28 - 4 = 24\,m$

$\gamma = 24\ N/m^3$

strength $= 490\ N/cm^2$

$= 490 \times 10^4\ N/m^2$

Scanned by CamScanner

stress on pillars

$$\sigma_{stress} = \gamma \cdot z \cdot \left(1 + \frac{\omega o}{\omega p}\right)^2$$

$$= 24 \times 10^3 \times 100 \left(1 + \frac{4}{24}\right)^2$$

$$= 3266 \times 10^3 \ n/m^2$$

safety factor

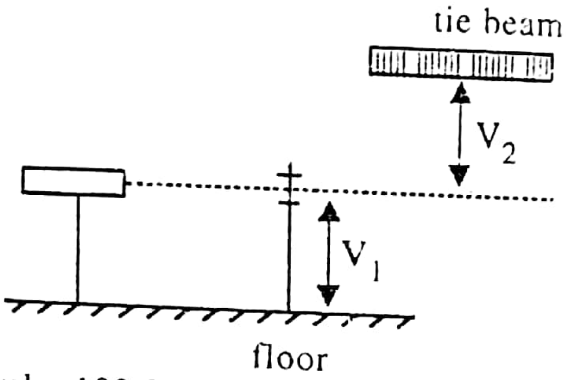
$$= \frac{\text{strength}}{\text{stress}}$$

$$= \frac{490 \times 10^4}{3266 \times 10^3}$$

$= \cancel{1.35}$

safety factor = ~~1.35~~ 1.5

Q. 7.



R_L of floor level = 100.00

staff reading on floor V_1 = 1.052

Reading on staff held inverted on the bottom touhing the underside of the tie beam = 3.558

R_L of tie beam = 100.00 + 1.052 +3.558 = 104.61

Ht. of tie beam = 1.052 + 3.558 = 4.61

H. $\frac{1.2 \times 1.25 + 1 \times 12.9}{200} =$ .022 te.

Scanned by CamScanner

SECTION B

Q. 12.

Maximise

$$R = q^2 + 3qa + a^2$$

subjeet to

$$q + a = 100$$

$\Rightarrow$ $q + a - 100 = 0$ ....(1)

Using Lagarange's method

$$F = q^2 + 3qa + a^2 + \lambda (a + a - 100) = 0$$

$$\frac{\delta F}{\delta q} = 2q + 3a + \lambda = 0 \qquad \text{............(2)}$$

$$\frac{\delta F}{\delta a} = 3q + 2a + \lambda = 0 \qquad \text{............(3)}$$

substracting (2) from (3)

$$q - a = 0$$

$\Rightarrow$ $q = a$ ....(4)

from equation (1) and (4) we get $2q = 100$

$\Rightarrow$ $q = 50$

& $a = 50$

so maximum R is

$$R = (50)^2 + 3 \times 50 \times 50 + (50)^2$$

$$= 5 \times (50)^2$$

$$= 12500$$

Q. 14

eigen values = − 1 & 3

$$A = \begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

Characteristic equation

$$|A - \lambda I| = 0$$

$$\Rightarrow \begin{vmatrix} (1-\lambda) & 6 & 1 \\ 1 & 2-\lambda & 0 \\ 0 & 0 & 3-\lambda \end{vmatrix} = 0$$

Let, x, y, & z be the components of eigen veetors then

Scanned by CamScanner

$$\begin{bmatrix} 1-\lambda & 6 & 1 \\ 1 & 2-\lambda & 0 \\ 0 & 0 & 3-\lambda \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

for $\lambda = -1$

$$\begin{bmatrix} 2 & 6 & 1 \\ 1 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$2x + 6y + z = 0$

$x + 3y + 0 = 0$

$4z = 0$

$\Rightarrow \quad z = 0$

$x + 3y = 0$

$\Rightarrow \quad x = -3y$

eigen vectors corresponding to $\lambda = -1$

$(1, -3, 0), (-1, 3, 0), (2, -6, 0)$ ....etc.

For $\lambda = 3$

$$\begin{bmatrix} 2 & 6 & 1 \\ 1 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$-2x + 6y + z = 0$

$x - y = 0$

$\Rightarrow \quad x = y$

hence $z = -4y$

So, eigen vectors corresponding to $\lambda = 3$

$(1, 1, -4), (-1, -1, 4), (2, 2, -8)$ ....etc.

Selling price

$$P = 21000\, n^{-\frac{1}{2}}$$

Cost

$$C = 100{,}000 + 1000\, n$$

For break even Point

Cost = Price (Selling)

$$(21{,}000)n^{-\frac{1}{2}} \times n = 100{,}000 + 1000\, n$$

$$\Rightarrow \quad (21{,}000)n^{\frac{1}{2}} \times n = 100{,}000 + n \times 1000$$

$$\Rightarrow \quad 21\sqrt{n} = 100 + n$$

$$\Rightarrow \quad n - 21\sqrt{n} + 100 = 0$$

$$\Rightarrow \quad n = \frac{21\sqrt{(21)^2 - 400}}{2}$$

Scanned by CamScanner

$$= \frac{21 - 6.4}{2}$$

$$= 7.3$$

$\Rightarrow$ $n = (7.3)^2$

$= 53$ nos

For maximum profit

Profit $R = P - C$

$$= 21,000\, n^{-\frac{1}{2}} \times n - (100,000 + 1000 . n)$$

$$= 21,000\, n^{\frac{1}{2}} - 100,000 - 1000\, n$$

$$\frac{dR}{dn} = \frac{1}{2} \times 21,000\, n^{-\frac{1}{2}} - 1000 = 0$$

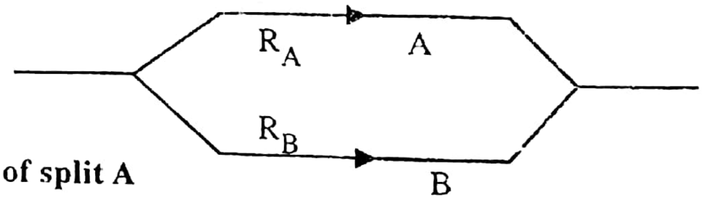
$\Rightarrow$ $21 n^{-\frac{1}{2}} - 2 = 0$

$\Rightarrow$ $n^{-\frac{1}{2}} = \frac{2}{21}$

$\Rightarrow$ $n = \left(\frac{21}{2}\right)^2 = 110$ nos

Q. 18.

Find the air flow in both the splits



Resistance of split A

$$R_A = \frac{Ks}{A^3} = \frac{K.P.L}{A^3}$$

$$= \frac{K.L \times 2(3+3)}{(3 \times 3)^3}$$

$= 0.0165$ KL

Resistance of split B

$$R_B = \frac{K.S'}{(A')^3} = \frac{K.P'.L'}{A'^3}$$

$$= \frac{K.L \times 2(3+4)}{(3 \times 4)^3}$$

$= 0.0081$ KL

Total Resistrence

$$\frac{1}{\sqrt{R}} = \frac{1}{\sqrt{R_A}} + \frac{1}{\sqrt{R_B}}$$

Scanned by CamScanner

$$= \frac{1}{\sqrt{0.0165\ KL}} + \frac{1}{\sqrt{0.0081\ KL}}$$

$$= (7.785 + 11.111)\frac{1}{\sqrt{KL}}$$

$$= \frac{18.90}{\sqrt{KL}}$$

$$\Rightarrow \quad R = \frac{KL}{(18.9)^2}$$

$$= 0.00280\ \ KL$$

$$\Delta P = R\,Q^2 = 0.0028\ KL\left(\frac{10{,}000}{60}\right)^2$$

$$= 77.76 Pa$$

Quantity in A

$$Q_A = \sqrt{\frac{\Delta P}{R}} = \sqrt{\frac{77.76\ KL}{0.0165\ KL}}$$

$$= 68.65\ m^3 / sec.$$

$$= 4119\ m^3 / min.$$

Quantity in B

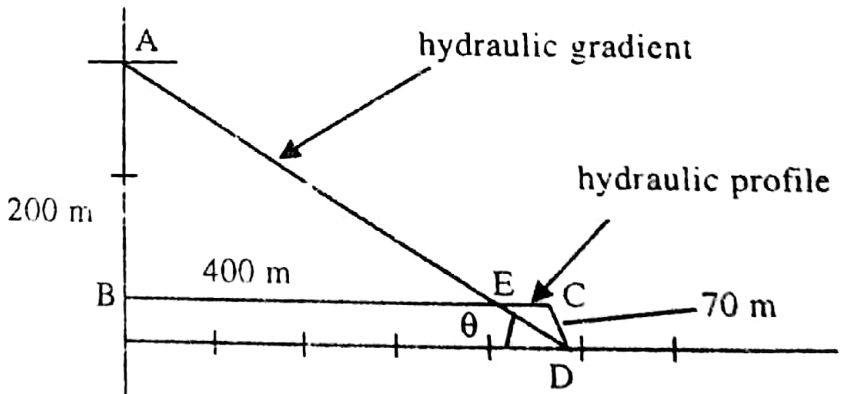
$$Q_B = \sqrt{\frac{77.76\ KL}{0.008\ KL}}$$

$$= 98.59\ m^3 / sec.$$

$$= 5915\ m^3 / min.$$

Q. 20.

Hydraulic profile



hydraulic gradient

$$= \frac{249.5}{449.5}$$

$$= 0.55 = 1 \text{ in } 1.8$$

$$\tan\theta = 0.55$$

$$\theta = \tan^{-1}(0.55)$$

$$= 29^{\circ}$$

hydraulic Profile is not correct as it cuts gradient line at Point E.

Q. 25.

$P = 200{,}000$
$i = 4\%$
$n = 10$ years

Scanned by CamScanner

Sinking fund required to be accumulated per year = R

$$R = p\,(1+i)^n \left[\frac{i}{(1+i)^n - 1}\right]$$

$$= 200{,}000\,(1+0.04)^{10}\left[\frac{0.04}{(1+0.04)^{10} - 1}\right]$$

$$= \text{Rs. } 24{,}658.18 \text{ per year}$$

Q. 27

$n = 5$

$d_{imp} = 0.3$ m $\qquad V_i = 1.5$ m/sec.

r.p.m. = 1440 $\qquad \eta = 0.6$

$d_d = 0.15$ m

$d_i = 0.20$ m

Quantity

$$Q = (\text{inlet area}) \times (\text{inlet velocity})$$

$$= \pi r_i^2 \times V_i$$

$$= \pi\,(.1)^2 \times (1.5)^2$$

$$= 4.24 \text{ m}^3/\text{min.}$$

Trangenital velocity (U)

$$U = \pi r_{imp}\left(\frac{rpm}{60}\right)$$

$$= \pi\,(0.3) \times \frac{1440}{60}$$

$$= 22.62 \text{ m / sec.}$$

Theoretical head / stage

$$h_t = \frac{U^2}{g} = \frac{(22.62)^2}{9.8}$$

$$= 52.21 \text{ mt.}$$

Actual head / stage

$$= 52.51 \times \eta$$

$$= 52.51 \times 0.6$$

$$= 31.3 \text{ mt.}$$

Total head

$$= 5 \times 31.3$$

$$= 156.63 \text{ m}$$

Quantity

$$= 4.24 \text{ m}^3 / \text{sec.}$$

Scanned by CamScanner

Reference

Deshmukh, R.T. *Mineral and Mine Economics*, Myra Publication, Nagpur

Deshmukh, D.J. *Elements of Mining Technology, Vol. I*, Vidyasewa Prakashan, Nagpur

Deshmukh, D.J. *Elements of Mining Technology, Vol. II*, Vidyasewa Prakashan, Nagpur

Deshmukh, D.J. *Elements of Mining Technology, Vol. III*, Vidyasewa Prakashan, Nagpur

Ghatak, S. *Mine Surveying & Levelling Mathematics, Vol. III*, Coalfield Publisher, Asansol, 1994

Grewal, Dr. B.S., *Higher Engineering Mathematics*, Khanna Publishers, Delhi

Hartman, L. Howard, *Introductory Mining Engineering*, Wiley-Interscience Publication

E. Hock and E.T. Brown, *Underground Excavation in Rock*, Institution ~~of Mining~~ & Metallurgy, 1994

Obert Leonard and Duvall I. Wilbur, *Rock Mechanics and the Design of Structures in Rock*, Wiley-Interscience Publication

Mishra, G.B., *Mine Environment & Ventilation*, Oxford University Press, 1993

Kanetkar, T.P. & Karni, S.N., *Surveying and Levelling*, Pune Vidyarthi Gupta Prakashan, Pune

Obert Leonard & Duvall. Wilbur, *Rock Mechanics and the Design of Structures in Rock*, Wiley-Interscience Publication

[illegible]mlu, M.A. *Mine Disasters and Mine Rescue*, Oxford and IBH Publishing Co. Pvt. Ltd., Delhi, 1991

Scanned by CamScanner