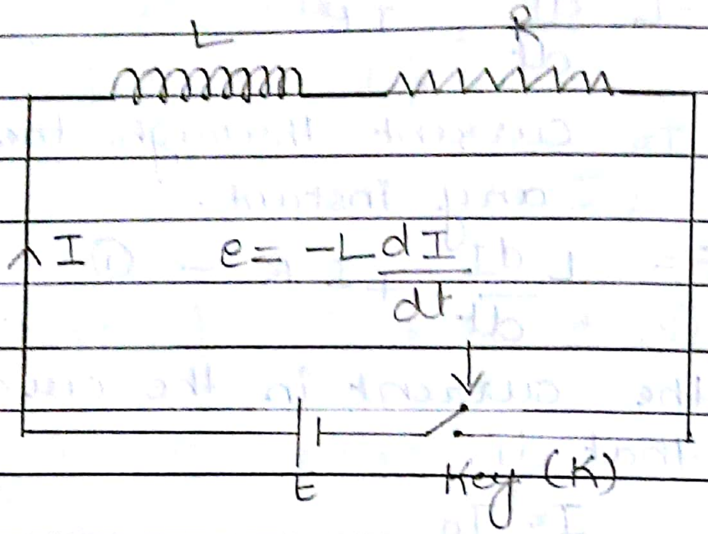
Bharti Sir (09-08-18)

current slocturial.

→ Transient.

→ Growth and ~~delta~~ decay of current in L-R circuit.

(1) Growth of current in L-R circuit.



in L-R circuit.

Hence transicent means decay in growth or decay of current / charge.

→ Let us consider a circuit consisting of inductor having inductor 'L' and register having resistance 'R' connected in series with a shell having E.M.F 'E' and a Key 'K'. When the Key is pressed the current starts growing through the circuit. Due to self inductance in the coil or inductor a back e.m.f is induced in the coil that opposses the growth of current.

In the absence of inductor,

the current reaches to max^m value (I_0). therefore

$$I_0 = \frac{E}{R}.$$

The equation for emf of current is given by.

$$E - L\frac{dI}{dt} = IR$$

where I = current through the circuit at any instant.

or $$E = L\frac{dI}{dt} + IR \quad \text{— (1)}$$

When the current in the circuit becomes max^m that is,

$$I = I_0$$

Then $$\frac{dI}{dt} = \frac{dI_0}{dt} = 0$$

Hence from eq^n (1)

$$E = I_0 R$$

$$I_0 = \frac{E}{R} \quad \text{—(2)}$$

Putting the value of E in eq^n (1), we get

$$I_0 R = L\frac{dI}{dt} + IR$$

$$(I_0 - I)R = L\frac{dI}{dt}$$

or $$\frac{dI}{(I_0 - I)} = \frac{R}{L}dt.$$

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Integrational both side

$$\int \frac{dI}{I_0 - I} = \frac{R}{L} \int dt$$

let $I_0 - I = x$

then $0 - dI = dx$

$dI = -dx$

$$\int \frac{dx}{x} = -\log x$$

$$= -\log (I_0 - I)$$

$$= -\log (I_0 - I) = \frac{R}{L} t + C \quad — (3)$$

applying initial condition

when $t = 0$ (i.e., Key is not present)

then $I = 0$.

from eqn (3)

$-\log I_0 = C$.

$-\log (I_0 - I) = t - \log I_0$

$\log (I_0 - I) = \frac{R}{L} t + \log I_0$

$\log (I_0 - I) - \log I_0 = -\frac{R}{L} t$

$$\log \left[\frac{I_0 - I}{I_0}\right] = -\frac{R}{L} t$$

or $\frac{I_0 - I}{I_0} = e^{-R/L\, t}$

$$1 - \frac{I}{I_0} = e^{-R/L\,t}$$

$$\frac{I}{I_0} = 1 - e^{-R/L\,t}$$

$$I = I_0\left[1 - e^{-R/L\,t}\right] \quad \text{-(I)}$$

This eqn shows that current through the circuit will grow exponentially to the max^m value I_0.

When time $t = \infty$

then from eqn (I)

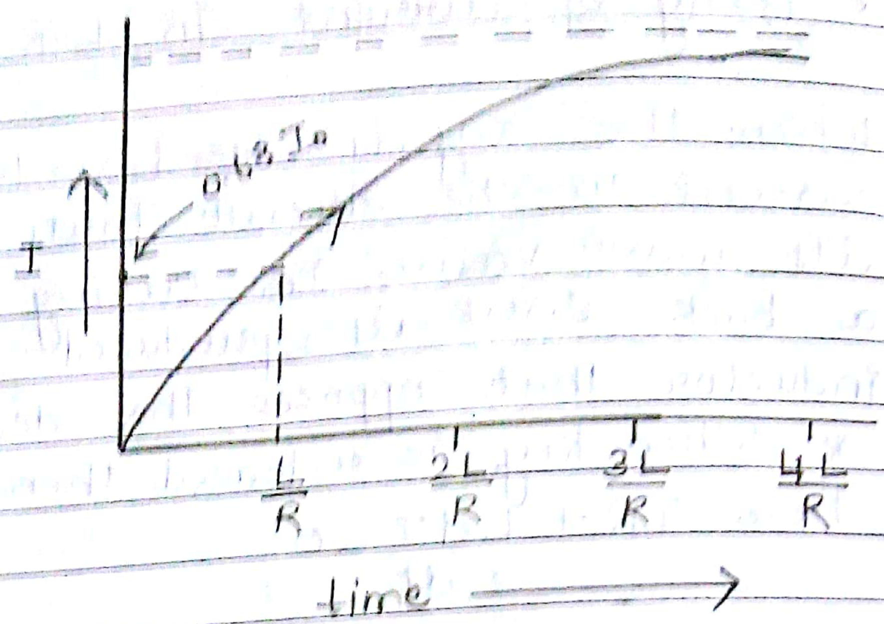
$$I = I_0\left[1 - e^{-\infty}\right]$$

$$= I_0\left[1 - \frac{1}{e^{\infty}}\right]$$

$$= I_0\,(1 - 0)$$

$$\boxed{I = I_0}$$

This shows that the current will take infinite time to reach the max^m value I_0. But L/R is small and hence the current attains a value near the maximum value in a short time.



Time constant :

when $t = \frac{L}{R}$

from eqn (9), we have

$$I = I_0 \left[1 - e^{-R/L \cdot L/R}\right]$$

$$= I_0 (1 - e^{-1}) = I_0 [1 - 1/e]$$

$$= I_0 \left[1 - \frac{1}{2.72}\right] = I_0 (1 - 0.37)$$

$$= 0.63\, I_0$$

or 63% of I_0

Hence time constant of the circuit may be defined as time taken by the current to grow to 0.63 point or 63% of its maximum value I_0.

* Decay of current in L-R circuit.

When the key is ~~closed~~ released then the current in the circuit starts decaying from its maxm value I_0. During decay, and a back E.M.F is produced across the inductor that opposes the decay of current.

* When key is released then,

here $IR + L\frac{dI}{dt} = 0$

$$L \cdot \frac{dI}{dt} = -IR$$

$$= -\frac{dI}{I} = \frac{R}{L} dt$$

or $\int \frac{dI}{I} = \frac{-R}{L}\int dt$

$\log I = -\frac{R}{L} \cdot t + c'$ — (5)

When $t=0$, $I = I_0$ (maximum)

from eqn (5)

$\log I_0 = c'$

Putting the value of c' in eqn (5), we get.

$\log I = -\frac{R}{L} t + \log I_0$

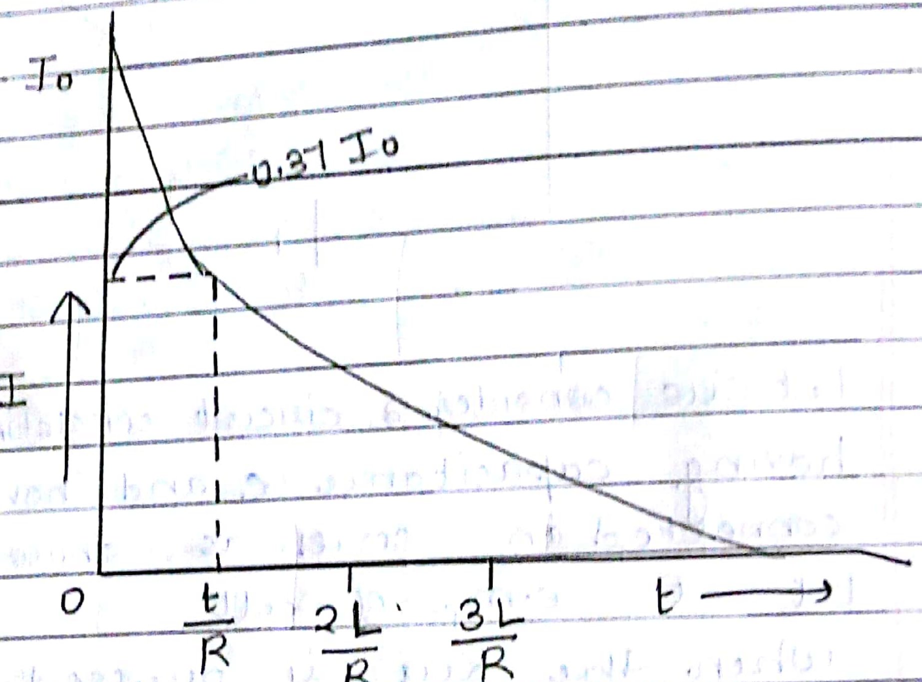
$\log I - \log I_0 = \frac{-R}{L} t$

$$\log_e \left[\frac{I}{I_0}\right] = \frac{-R}{L} t$$

$$\frac{I}{I_0} = e$$

$$\boxed{I = I_0 e^{-R/L t}} \quad \text{—— (6)}$$

This eqn shows that the current through the circuit been decay experimentally to



Time Constant

when $t = \frac{L}{R}$

from eqn (6)

$$I = I_0 e^{-L} = \frac{I_0}{e} = \frac{I_0}{2.72}$$

$$I = 0.37\, I_0$$

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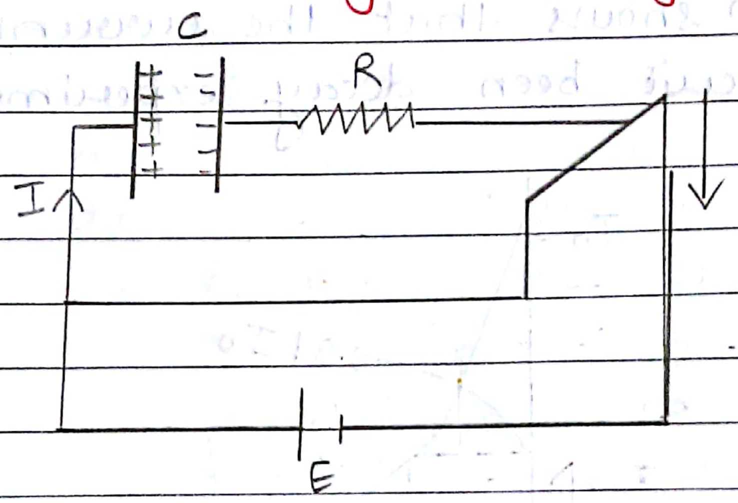
$$\boxed{I = I_0 e^{-R/L\, t}} \quad \text{--- (6)}$$

Hence time constant of this circuit may be defined as time taken by current to decay from max^m value to 0.37 times its max^m value in the circuit.

→ Growth and decay of charge in C-R circuit.



Let us consider a circuit consisting of a capacitor having capacitance C and having resistor R connected in series as shown in the figure.
Let E = e.m.f of cell
When the key is pressed then the capacitor is charged slowly.
Let I = chargeing current.
Q = Charge at any instant on the capacitor.
Then equation for e.m.f of the circuit is given by.

$$E = \frac{Q}{C} + RI$$

$$E - \frac{Q}{C} = RI$$

$$\frac{EC - Q}{C} = \frac{R\,dQ}{dt} \quad \left[\because I = \frac{dQ}{dt} \right]$$

$$EC - Q = RC \frac{dQ}{dt}$$

or $$\frac{dQ}{EC - Q} = \frac{dt}{RC}$$

Integrating both side, we get.

$$\int \frac{dQ}{EC - Q} = \int \frac{dt}{RC}$$

$$-\log_e (EC - Q) = \frac{t}{RC} + K \quad \text{—— (1)}$$

where K= Constant of integration.

Applying initial condition

That when $t=0$

$Q=0$

from eqn (1) we get.

$$-\log EC = K$$

or $K = -\log EC$

Putting value of K in eqn (1)

$$\log (EC - Q) = \frac{-t}{RC} + \log EC$$

or $$\log \left[\frac{EC - Q}{EC} \right] = \frac{-t}{RC}$$

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Or $\left[1-\frac{Q}{EC}\right] = e^{-t/RC}$

$$\frac{Q}{EC} = 1 - e^{-t/RC}$$

$$Q = EC\left[1-e^{-t/RC}\right] \quad \text{--- (2)}$$

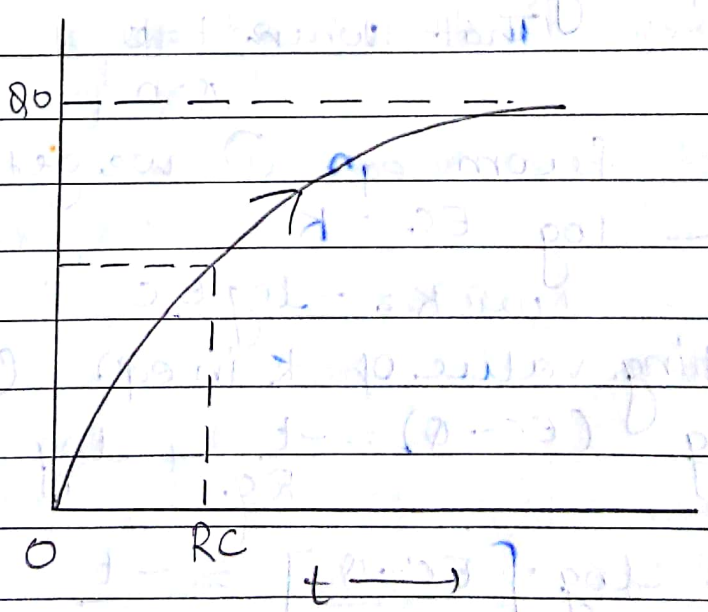
When $t = \infty$

Then $Q = EC = Q_0 =$ Maximum charge on capacitor.

Hence from eq^n (2)

$$\boxed{Q = Q_0\left[1-e^{-t/RC}\right]}$$

This eq^n shows that charge on capacitor will grow exponentially to the maximum ~~Q~~ value Q_0.



Expression for charging current.

charging current (I)

$$I = \frac{dQ}{dt}$$

$$= \frac{d}{dt}\left[Q_0\left(1 - e^{-t/RC}\right)\right]$$

$$= 0 - Q_0 e^{-t/RC} \times \left[-\frac{1}{RC}\right]$$

$$= \frac{Q_0}{RC} e^{-t/RC}$$

$$= \frac{EC}{RC} e^{-t/RC}$$

$$\therefore \boxed{I = \frac{E}{R} e^{-t/RC}} \quad \text{--- (4)}$$

Time Constant :-

When $t = RC$

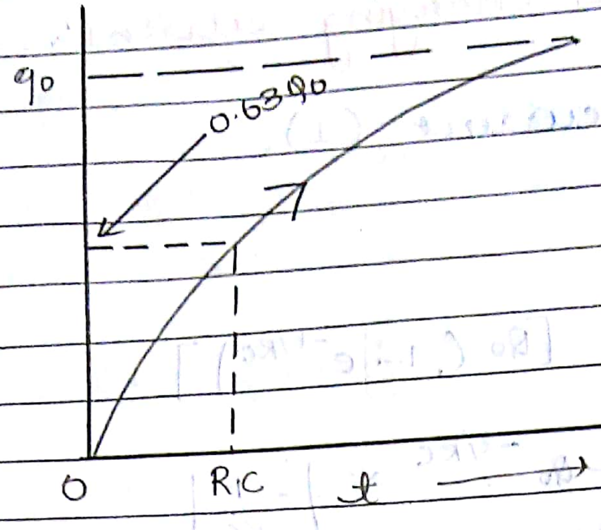
from eq^n (4)

$$Q = Q_0 (1 - e)$$

$$= Q_0\left(1 - \frac{1}{e}\right) = Q_0\left[1 - \frac{1}{2.72}\right]$$

$$= Q_0 (1 - 0.37)$$

$$\boxed{Q = 0.63\, Q_0}$$



Hence time constant of this circuit may be defined as time in which the charge on capacitor grows to 0.63 times the max^m value on capacitors.

* Decay of charge in C-R circuits.

When the Key is raised up or released, the charge on capacitor starts decaying from its max^m value q_0.

the eq^n for e.m.f of the circuit is given by

$$\frac{Q}{C} = RI$$

here $I = -\frac{dQ}{dt}$

then $\frac{Q}{C} = -R\frac{dQ}{dt}$

or $\left(\frac{dQ}{Q} = -\frac{1}{RC}\int dt\right)$

$$\log_e QC = -\frac{t}{RC} + K' \quad \text{--- (5)}$$

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where K' is const of integration.
when $t=0$, $Q = Q_0$ (maximum)

$$\log_e Q_0 = K'$$

Putting value of K' in eqn (5)
we get

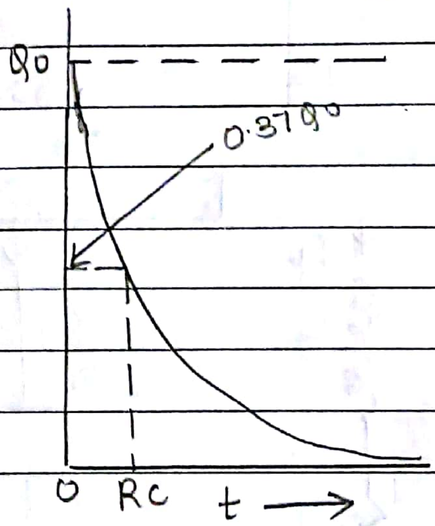
$$\log_e Q = \frac{1}{RC} + \log Q_0$$

$$\log Q - \log Q_0 = \frac{1}{RC}$$

$$\log \frac{Q}{Q_0} = \frac{-t}{RC}$$

$$\frac{Q}{Q_0} = e^{-t/RC}$$

$$Q - Q_0 = e^{-t/RC} \quad \text{— (6)}$$



This eqn shows that the charge on capacitor will decay exponentially from maxm value q_0.

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time const.

When $t = RC$

$$Q = Q_0 e^{-1} \quad [\text{from eq}^n (5)]$$

$$Q = \frac{Q_0}{2.72}$$

$$\boxed{Q = 0.37 Q_0}$$

Its the time constant of the circuit may be defined as time in which the charge of capacitor decreases to 0.37 time the max^m on capacitor.

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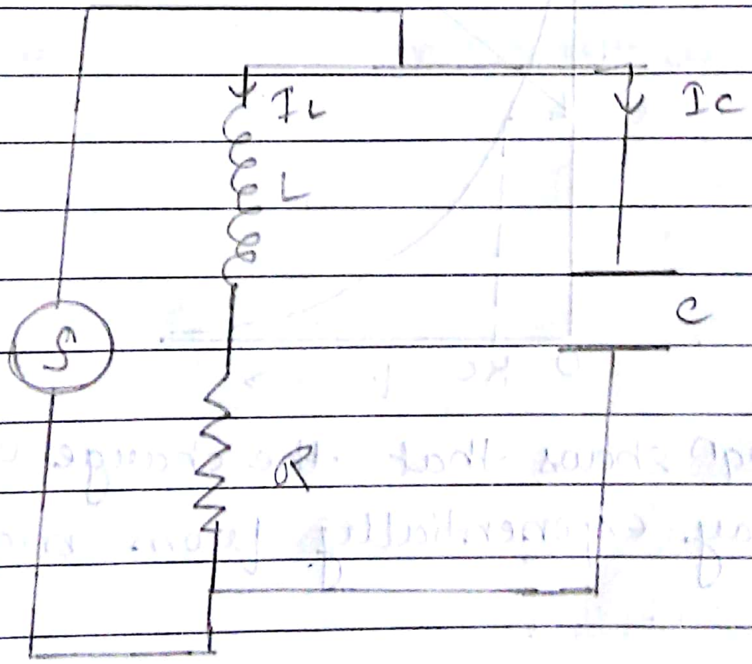
Parallel L-C-R Circuit :-



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In parallel LCR circuit, a capacitor having capacitance C is connected in parallel to the series combination of inductance(L) and resistance (R). This parallel combination is connected across the AC source.

let I = main current

$$I = I_L + I_C \qquad \left\{ I = \frac{V}{R} \right.$$

$$\frac{E}{Z} = \frac{E}{Z_1} + \frac{E}{Z_2}$$

$$\frac{1}{Z} = \frac{1}{Z_1} + \frac{1}{Z_2} \quad \text{——— (1)}$$

Z = impedence of LCR circuit.

$$Z_1 = R + j\omega L \qquad \left\{ j = \sqrt{-1} \right.$$

$$Z_2 = \frac{1}{j\omega C}$$

$$\frac{1}{Z} = \frac{1}{R + j\omega L} + \frac{1}{\frac{1}{(j\omega C)}}$$

$$\frac{1}{Z} = \frac{1}{R + j\omega L} + j\omega C$$

$$= \frac{1}{R + j\omega L} \times \frac{R - j\omega L}{R - j\omega L} + j\omega C$$

$$= \frac{R - j\omega L}{R^2 - (j\omega L)^2} + j\omega C$$

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$$= \frac{R - j\omega L}{R^2 - j^2\omega^2 L^2} + j\omega C$$

$$\frac{1}{Z} = \frac{R - j\omega L}{R^2 + \omega^2 L^2} + j\omega C \qquad \{\because j^2 = -1\}$$

$$= \frac{(R - j\omega L) + j\omega C\,(R^2 + \omega^2 L^2)}{(R^2 + \omega^2 L^2)}$$

$$= \frac{R + j\,(\omega C R^2 + \omega^3 C L^2 - \omega L)}{R^2 + \omega^2 L^2} \quad \text{——— (2)}$$

$\frac{1}{Z}$ is called "admittance" of the circuit and it is denoted by Y.

$$Y = \frac{1}{Z}$$

$$Y = \frac{R + j\,(\omega C R^2 + \omega^3 C L^2 - \omega L)}{R^2 + \omega^2 L^2}$$

$$Y = \left(\frac{R}{R^2 + \omega^2 L^2}\right) + j\left\{\frac{\omega C\,(R^2 + \omega^2 L^2) - \omega L}{R^2 + \omega^2 L^2}\right\}$$

$$= \left(\frac{R}{R^2 + \omega^2 L^2}\right) + j\left[\omega C - \frac{\omega L}{R^2 + \omega^2 L^2}\right] \quad \text{——— (3)}$$

The current through the circuit is;

$$I = \frac{E}{Z} = EY$$

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$$= E\left[\frac{R}{R^2+\omega^2L^2} + j\left\{\omega C - \frac{\omega L}{R^2+\omega^2L^2}\right\}\right] \quad \text{--- (4)}$$

Then magnitude of current is;

$$I = \left[E\sqrt{\left(\frac{R}{R^2+\omega^2L^2}\right)^2 + \left[\omega C - \frac{\omega L}{R^2+\omega^2L^2}\right]^2}\right] \quad \text{--- (5)}$$

The current I in the circuit will be minimum when the second term is zero.

i.e;

$$\omega C - \frac{\omega L}{R^2+\omega^2L^2} = 0$$

$$\omega C = \frac{\omega L}{R^2+\omega^2L^2}$$

$$R^2+\omega^2L^2 \times \omega C = \omega L$$

$$R^2+\omega^2L^2 = \frac{\omega L}{\omega C}$$

or $$\boxed{R^2+\omega^2L^2 = \frac{L}{C}} \quad \text{--- (6)}$$

or $$\omega^2L^2 = \frac{L}{C} - R^2$$

or $$\omega^2 = \frac{1}{LC} - \frac{R^2}{L^2}$$

or $$\omega = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

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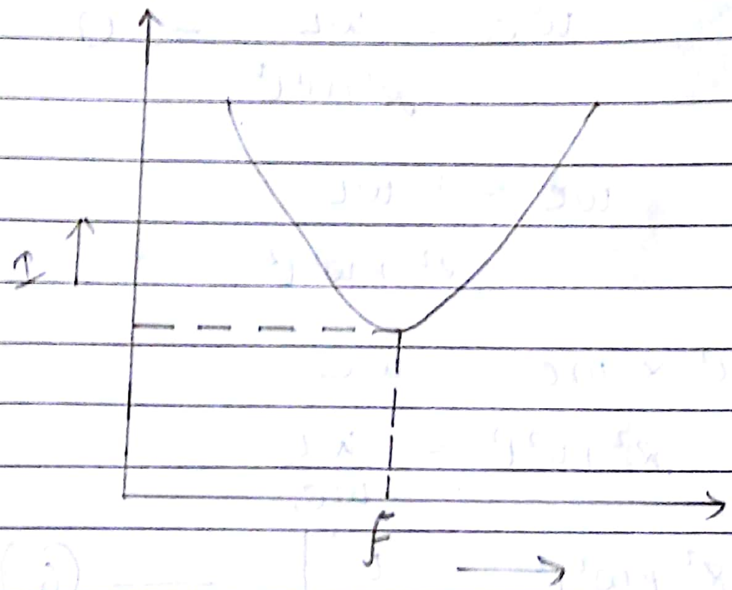
$$2\pi f = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

$$\left[f = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}} \right] \quad \text{——— (7)}$$

This is expression for frequency and the resonance of the circuit,

At resonance the current in the circuit is minimum and the admittance (Y) is also minimum but the impedence of the circuit is maximum.



* Expression for impedence at resonance :-

from eqⁿ (5) magnitude of current,

$$I = E\left[\frac{R}{R^2+\omega^2L^2}\right] = EY$$

$$\because \quad Y = \frac{1}{Z}$$

$$Z = \frac{1}{Y}$$

$$\boxed{Z = \left[\frac{R^2+\omega^2L^2}{R}\right]} \quad \text{—— (8)}$$

This is expression for impedence of the circuit at resonance.

from eqⁿ (6) :-

$$R^2+\omega^2L^2 = \frac{L}{C}$$

$$\therefore \quad Z = \frac{\frac{L}{C}}{R}$$

$$\boxed{Z = \frac{L}{CR}} \quad \text{—— (9)}$$

END.

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