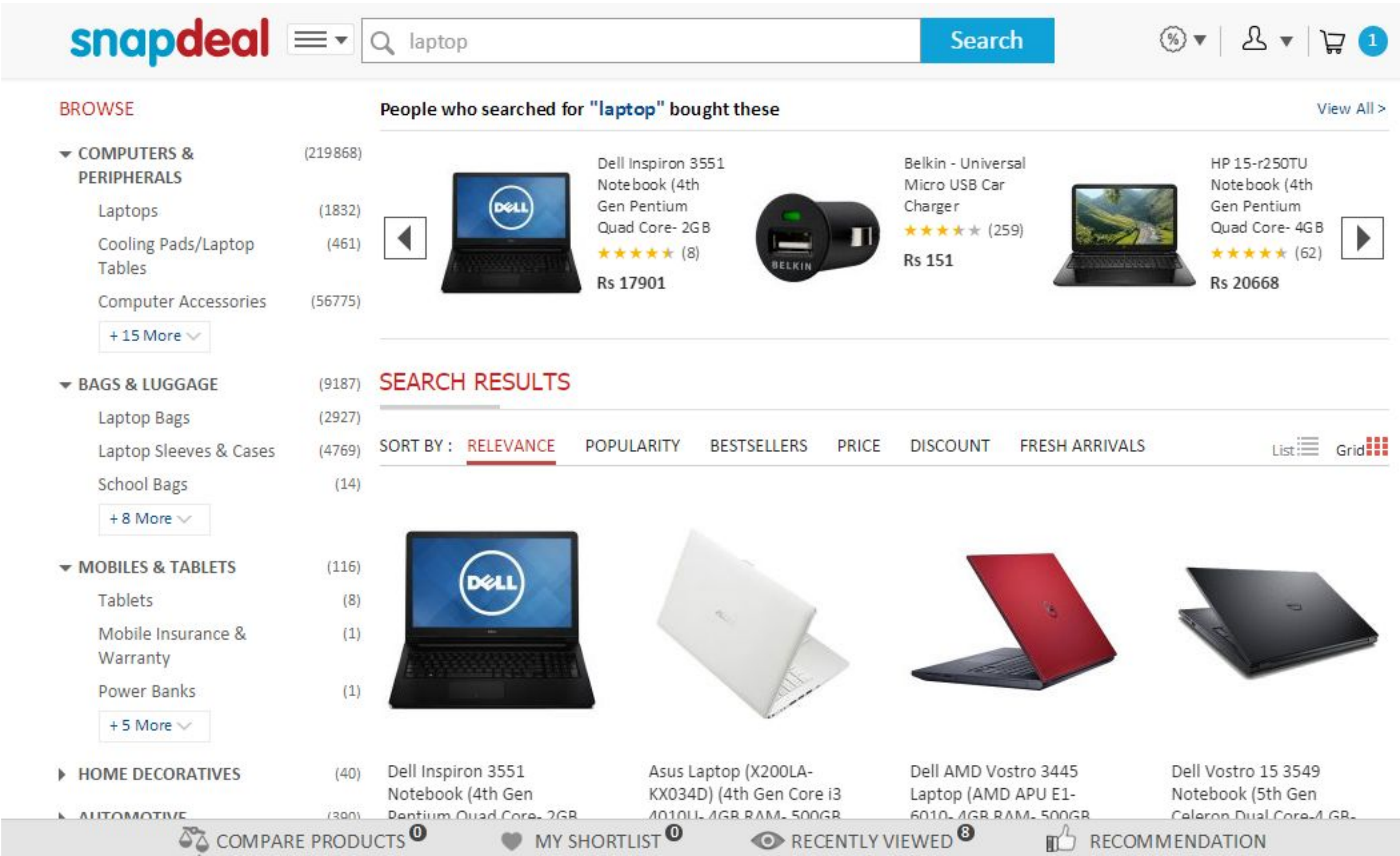
Sorting Algorithms

We use Sorting…



Sorting:Definition

Sorting: arranging elements in a list or collection in increasing or decreasing order of some property.

A = { 3 1 6 2 1 3 4 5 9 0 }

A = { 0 1 1 2 3 3 4 5 6 9 }

Applications of sorting

Sorting is useful in:

1. Searching in database.
2. Binary Search
3. Closest Pair
4. Element Uniqueness.

Different sorting algorithms

Some sorting algorithms are:-

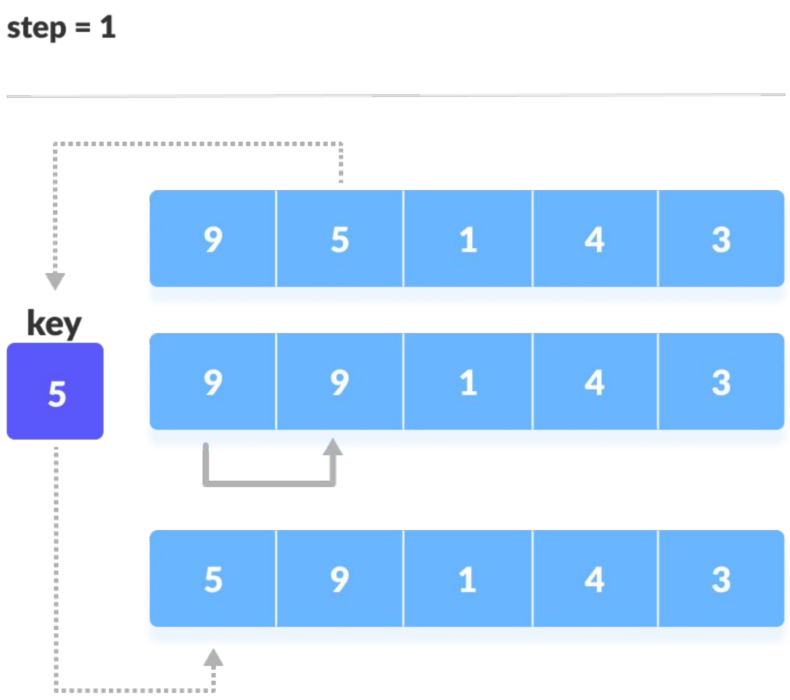
- Insertion Sort
- Bubble Sort
- Selection Sort
- Merge Sort
- Quick Sort

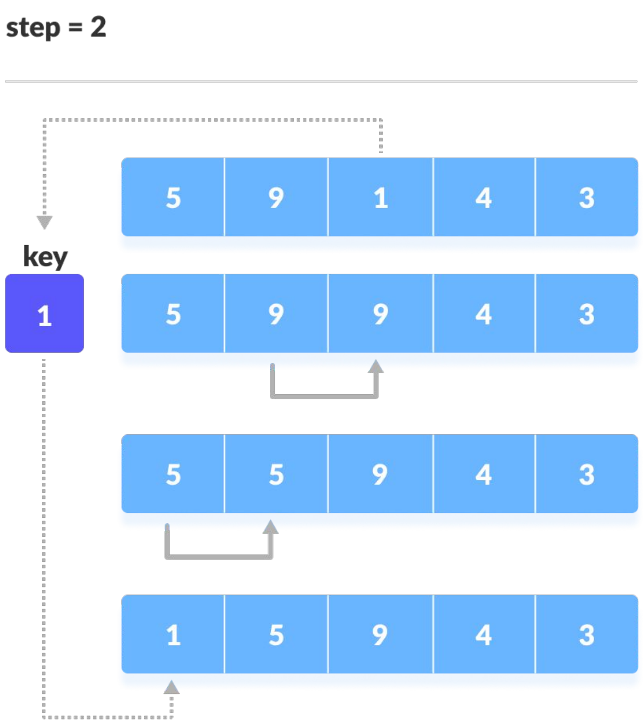
Insertion Sort Algorithm

INSERTION-SORT(A)

```
for j = 2 to A.length
    key = A[j]
    // Insert A[j] into the sorted sequence A[1 .. j - 1].
    i = j - 1
    while i > 0 and A[i] > key
        A[i + 1] = A[i]
        i = i - 1
    A[i + 1] = key
```

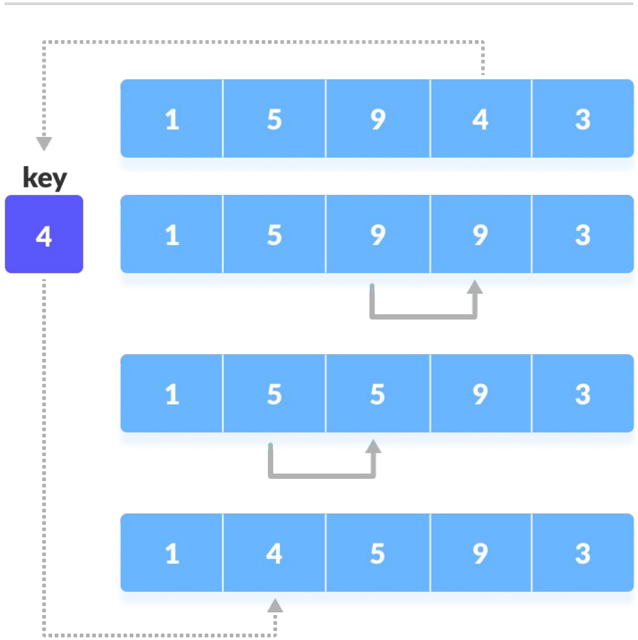
Example


step = 1
key
5
9 5 1 4 3
9 9 1 4 3
5 9 1 4 3

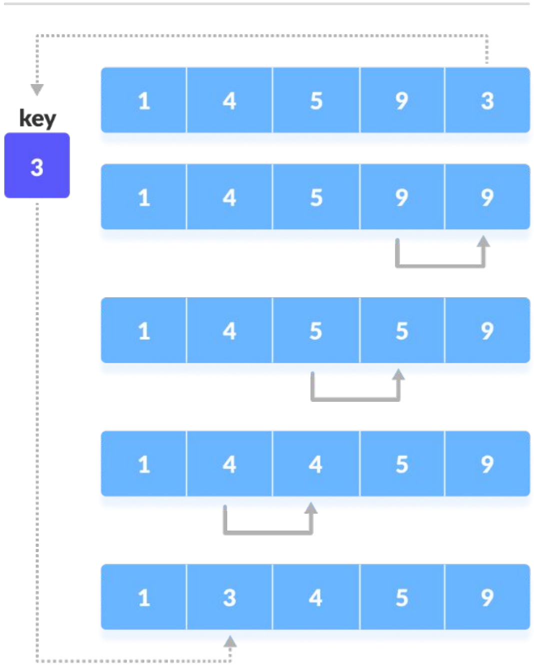

step = 2
key
1
5 9 1 4 3
5 9 9 4 3
5 5 9 4 3
1 5 9 4 3

Contd..

step = 3

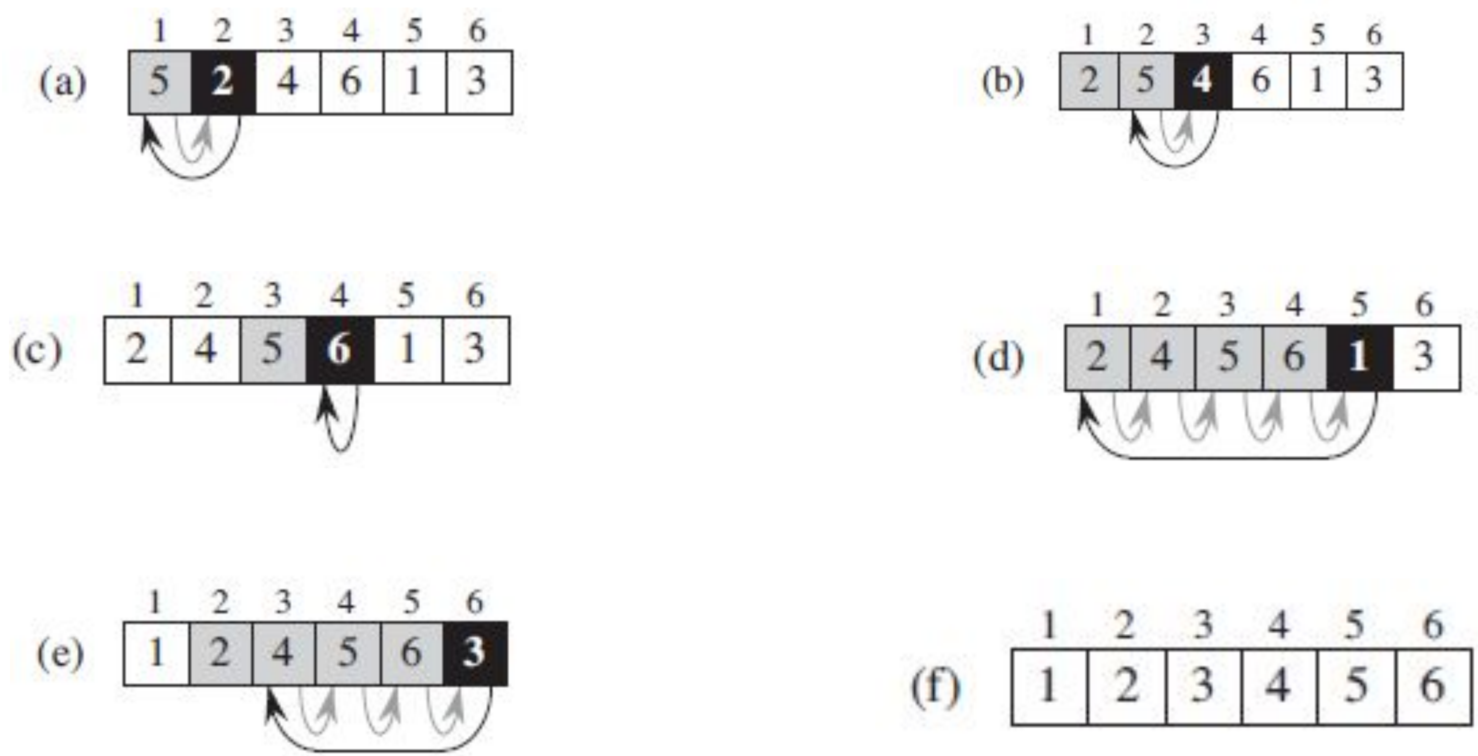


step = 4



Example 2

- Show how this algorithm works for A =[5, 2, 4, 6, 1,3]



Bubble Sort Algorithm

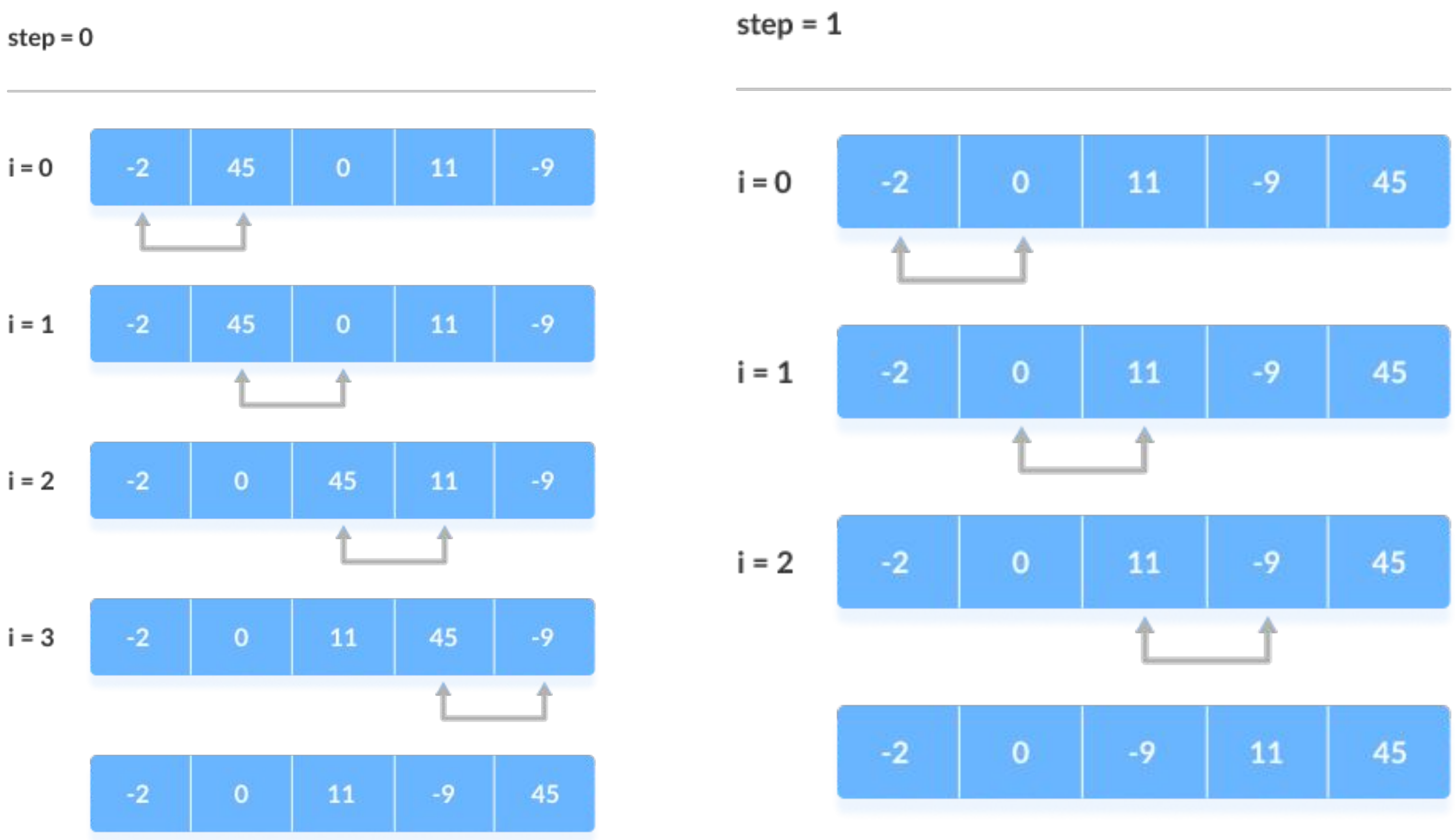
1. Start at index zero, compare the element with the next one (a[0] & a[1] (a is the name of the array)), and swap if a[0] > a[1]. Now compare a[1] & a[2] and swap if a[1] > a[2]. Repeat this process until the end of the array. After doing this, the largest element is present at the end. This whole thing is known as a pass. In the first pass, we process array elements from [0,n-1].

2. Repeat step one but process array elements [0, n-2] because the last one, i.e., a[n-1], is present at its correct position. After this step, the largest two elements are present at the end.
3. Repeat this process n-1 times.

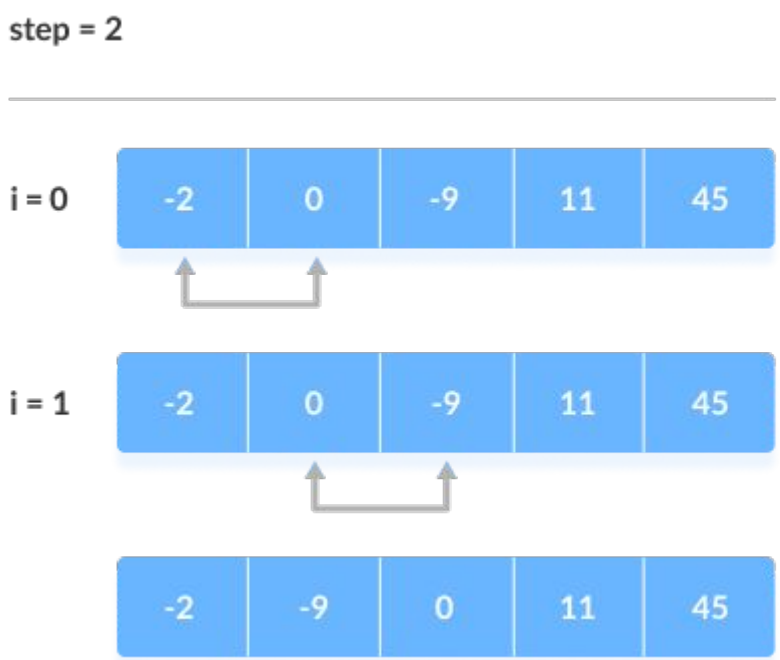
Code

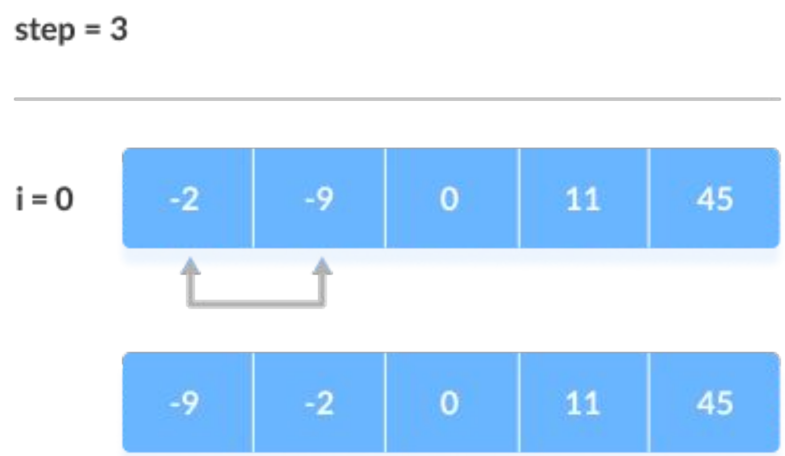
```
for (c = 0 ; c < n - 1; c++)
   {
     for (d = 0 ; d < n - c - 1; d++)
     {
       if (array[d] > array[d+1])          {
         swap        = array[d];
         array[d]    = array[d+1];
         array[d+1]  = swap;
       }
     }
   }
```

Example

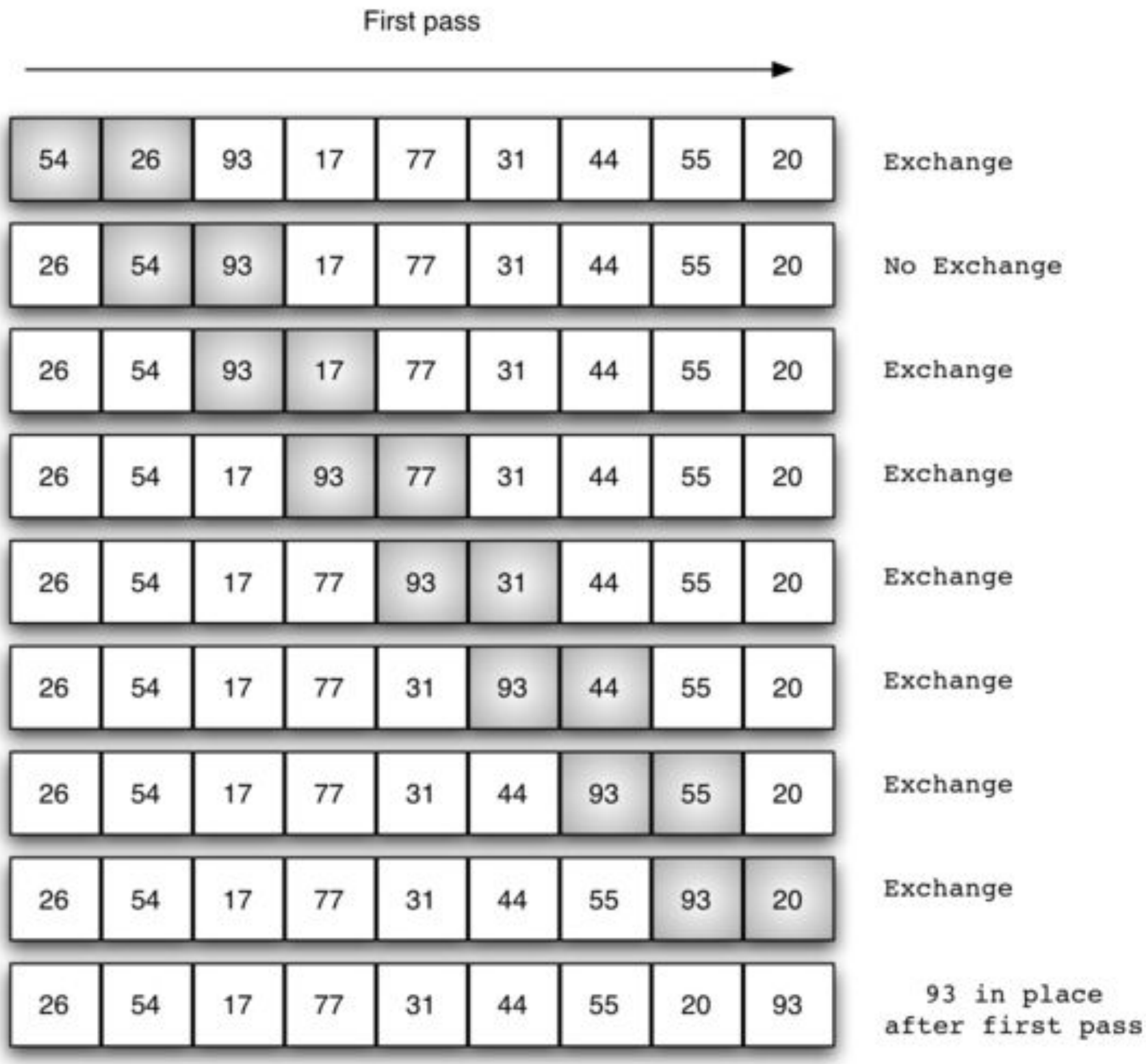

step = 0
i = 0
-2 45 0 11 -9
i = 1
-2 45 0 11 -9
i = 2
-2 0 45 11 -9
i = 3
-2 0 11 45 -9
-2 0 11 -9 45
step = 1
i = 0
-2 0 11 -9 45
i = 1
-2 0 11 -9 45
i = 2
-2 0 11 -9 45
-2 0 -9 11 45

Contd..


step = 2
i = 0
-2 0 -9 11 45
i = 1
-2 0 -9 11 45
-2 -9 0 11 45


step = 3
i = 0
-2 -9 0 11 45
-9 -2 0 11 45

Example 2


First pass
54 26 93 17 77 31 44 55 20 Exchange
26 54 93 17 77 31 44 55 20 No Exchange
26 54 93 17 77 31 44 55 20 Exchange
26 54 17 93 77 31 44 55 20 Exchange
26 54 17 77 93 31 44 55 20 Exchange
26 54 17 77 31 93 44 55 20 Exchange
26 54 17 77 31 44 93 55 20 Exchange
26 54 17 77 31 44 55 93 20 Exchange
26 54 17 77 31 44 55 20 93 93 in place after first pass

Selection sort

- The selection sort algorithm sorts an array by repeatedly finding the minimum element (considering ascending order) from unsorted part and putting it at the beginning.

The algorithm maintains two subarrays in a given array.

1) The subarray which is already sorted.
2) Remaining subarray which is unsorted.

- In every iteration of selection sort, the minimum element (considering ascending order) from the unsorted subarray is picked and moved to the sorted subarray.

Selection Sort

```
void selectionSort(int arr[], int n)
{
    int i, j, min_idx;

    // One by one move boundary of unsorted subarray
    for (i = 0; i < n-1; i++)
    {
        // Find the minimum element in unsorted array
        min_idx = i;
        for (j = i+1; j < n; j++)
          if (arr[j] < arr[min_idx])
            min_idx = j;

        // Swap the found minimum element with the first element
        swap(&arr[min_idx], &arr[i]);
    }
}
```

Selection Sort Example

Original Array	After 1st pass	After 2nd pass	After 3rd pass	After 4th pass	After 5th pass
3	1	1	1	1	1
6	6	3	3	3	3
1	3	6	4	4	4
8	8	8	8	5	5
4	4	4	6	6	6
5	5	5	5	8	8

Merge Sort

- Divide-and-conquer based algorithm:
- Divide the unsorted array into 2 halves until the sub-arrays only contain one element
- Merge the sub-problem solutions together:
  - Compare the sub-array’s first elements
  - Remove the smallest element and put it into the result array
  - Continue the process until all elements have been put into the result array

Idea

```
Mergesort(Passed an array)
  if array size > 1
    Divide array in half
    Call Mergesort on first half.
    Call Mergesort on second half.
    Merge two halves.

Merge(Passed two arrays)
  Compare leading element in each array
  Select lower and place in new array.
    (If one input array is empty then place
     remainder of other array in output array)
```

Algorithm

MERGE-SORT(A, p, r)

1 **if** $\boldsymbol{p < r}$

2 **then** $\boldsymbol{q \leftarrow (p + r)/2}$

3 MERGE-SORT(A, p, q)

4 MERGE-SORT$(A, q + 1, r)$

5 MERGE(A, p, q, r)

Merge

```
MERGE(A, p, q, r)
n1 ← q − p + 1
n2 ← r − q
create arrays L[1 .. n1 + 1] and R[1 .. n2 + 1]
for i ← 1 to n1
     do L[i] ← A[p + i − 1]
for j ← 1 to n2
     do R[j] ← A[q + j]
L[n1 + 1] ← ∞
R[n2 + 1] ← ∞
i ← 1
j ← 1
for k ← p to r
     do if L[i] ≤ R[j]
           then A[k] ← L[i]
                i ← i + 1
           else A[k] ← R[j]
                j ← j + 1
```

98	23	45	14	6	67	33	42

98	23	45	14	6	67	33	42

98	23	45	14

6	67	33	42

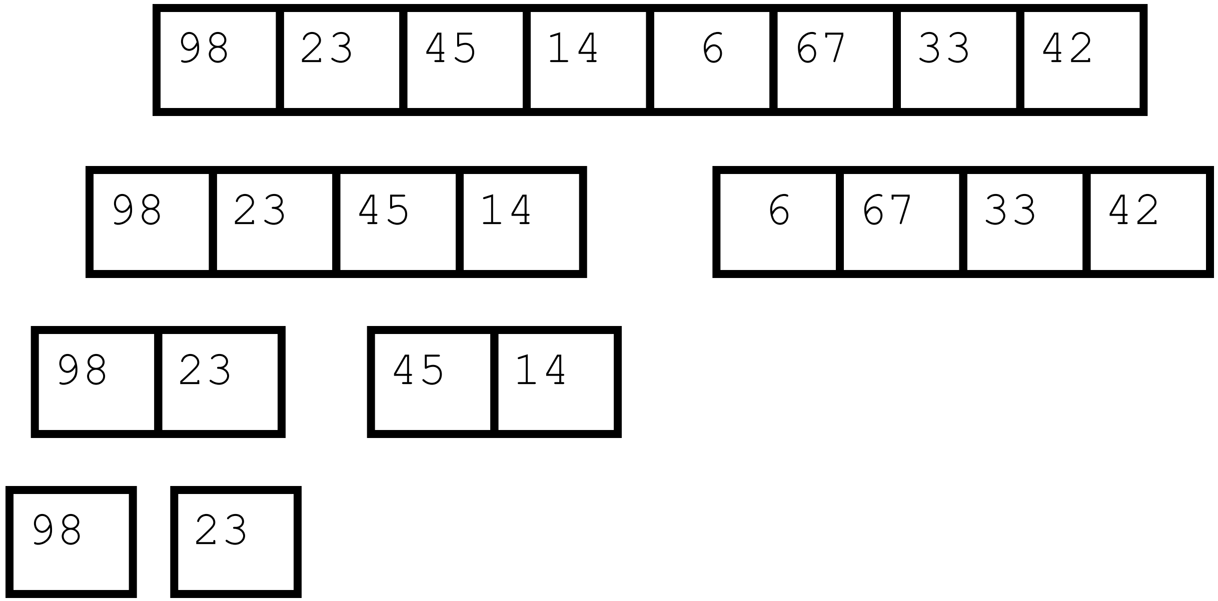
98	23	45	14	6	67	33	42

98	23	45	14

6	67	33	42

98	23

45	14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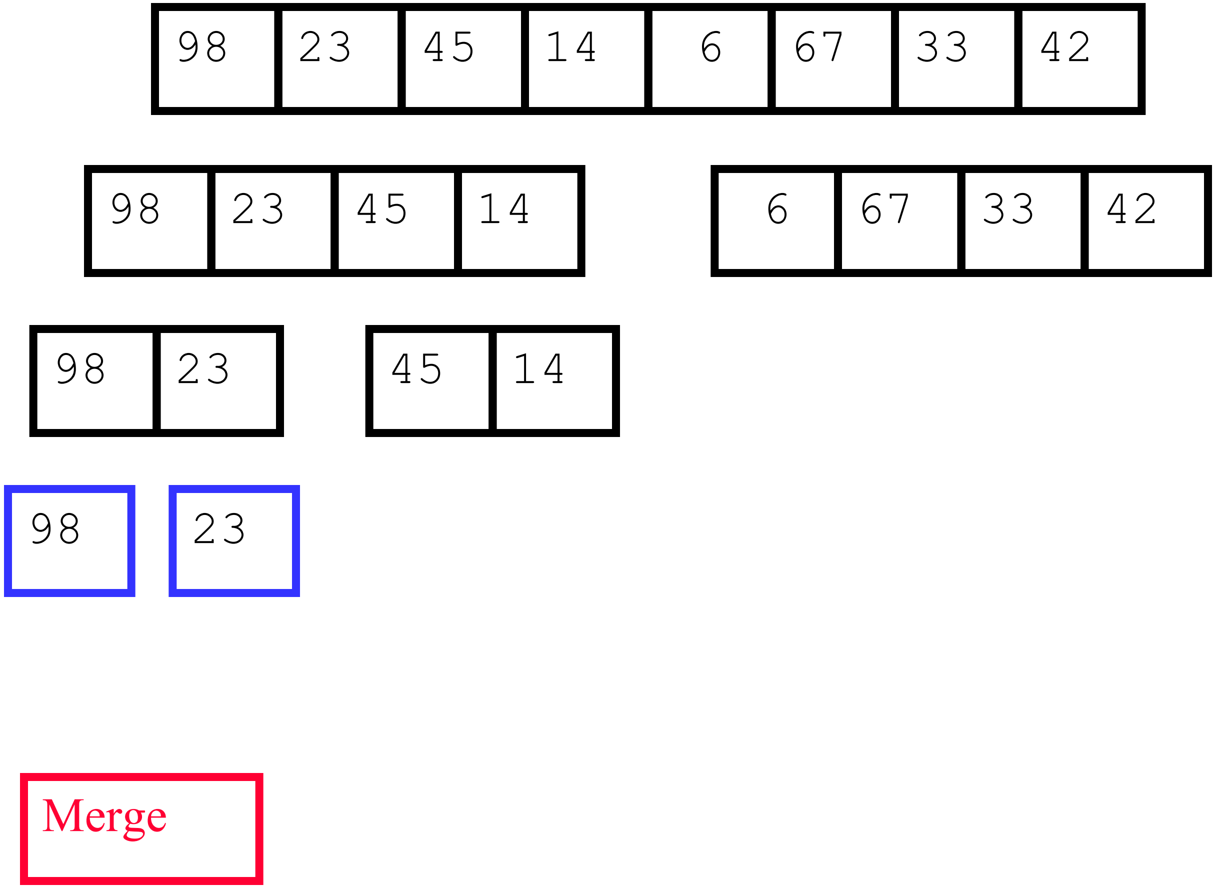

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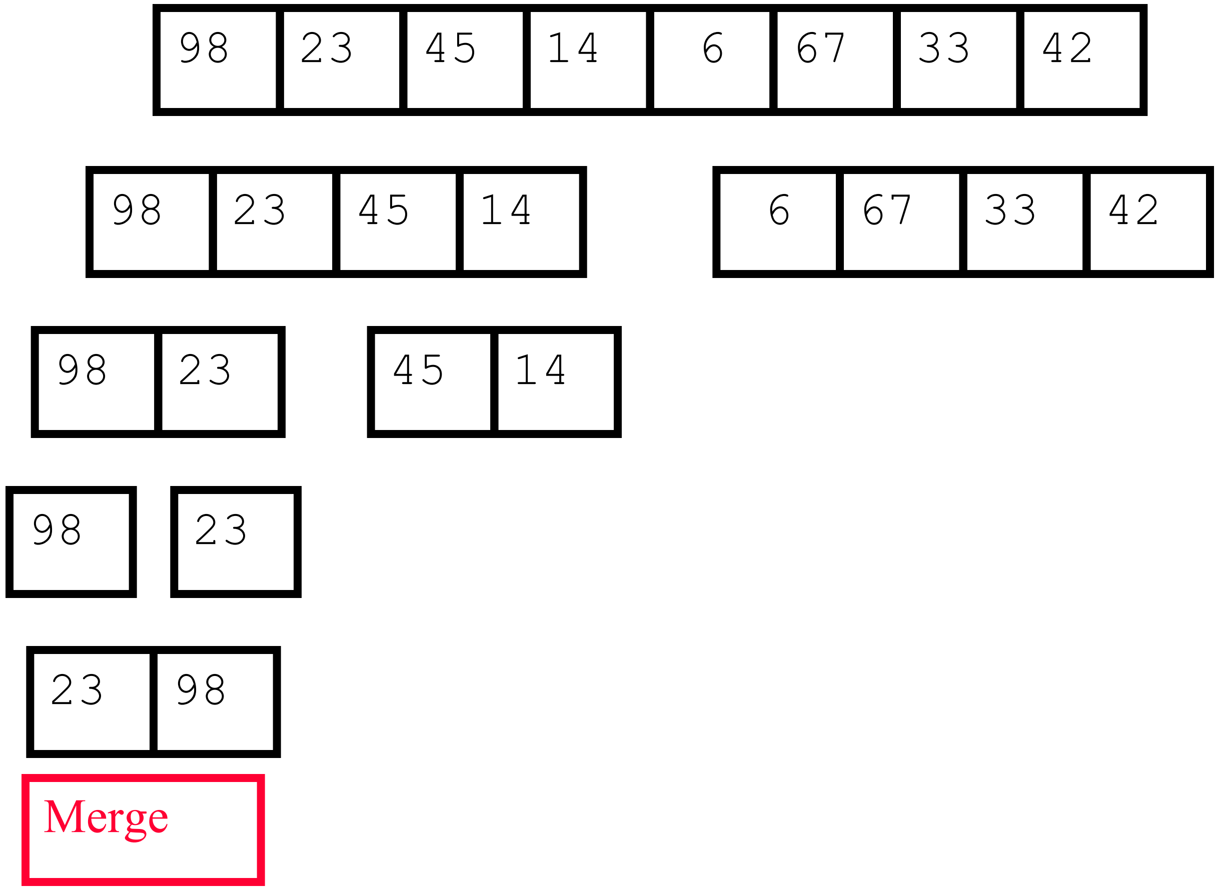
Merge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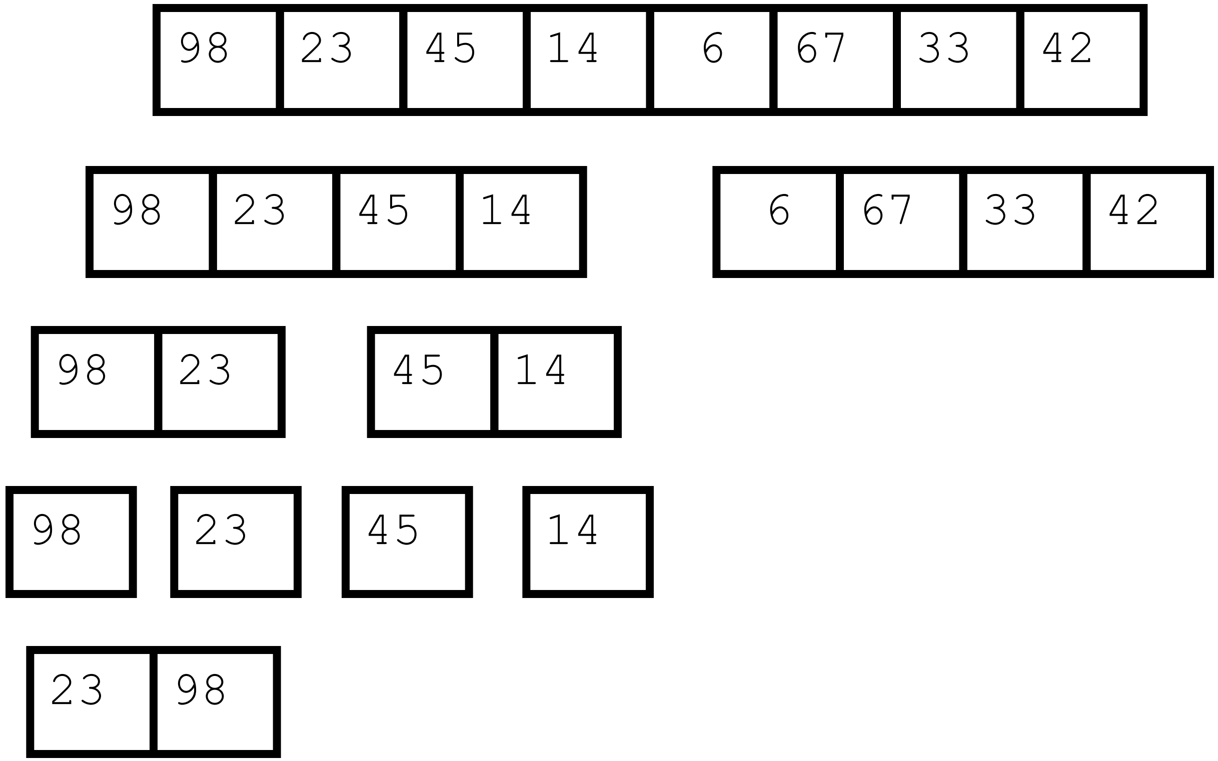
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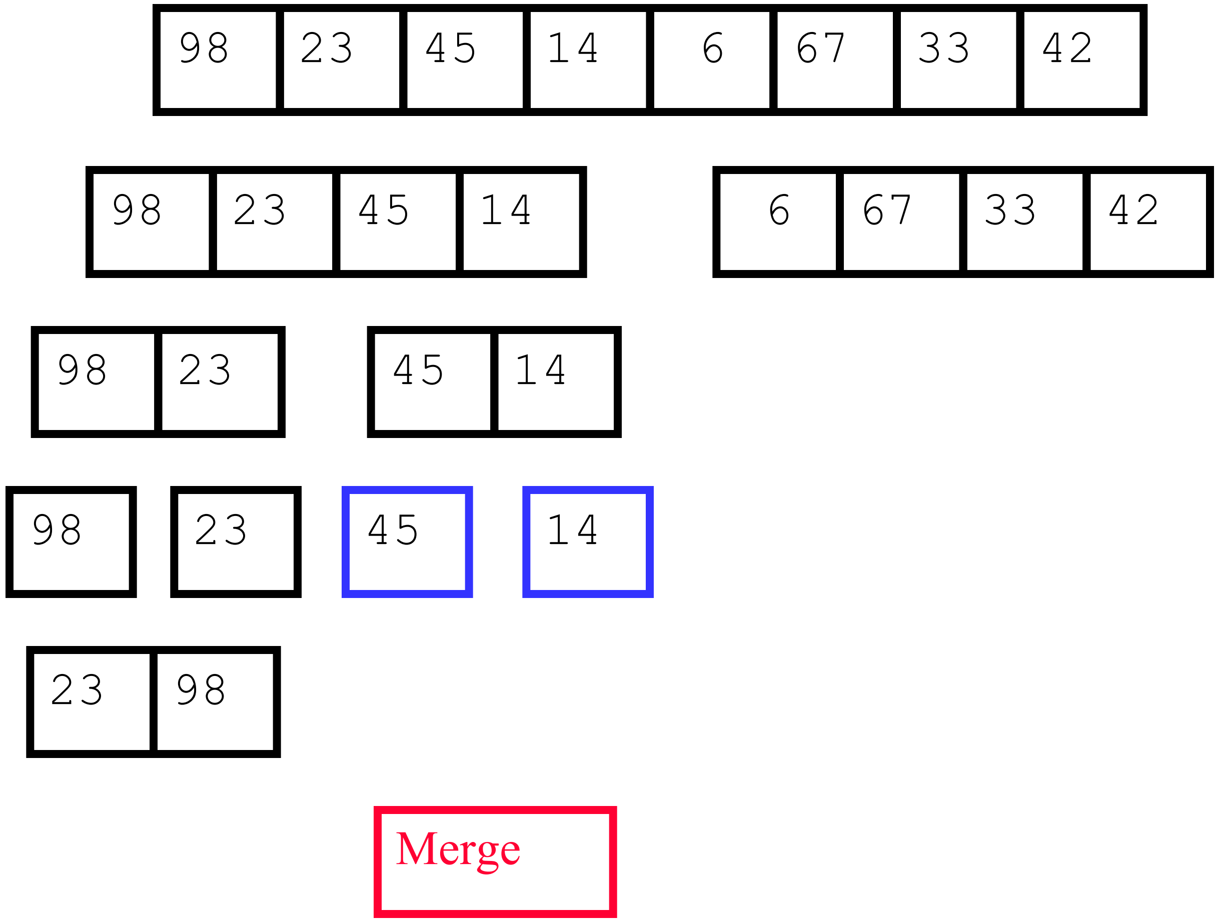
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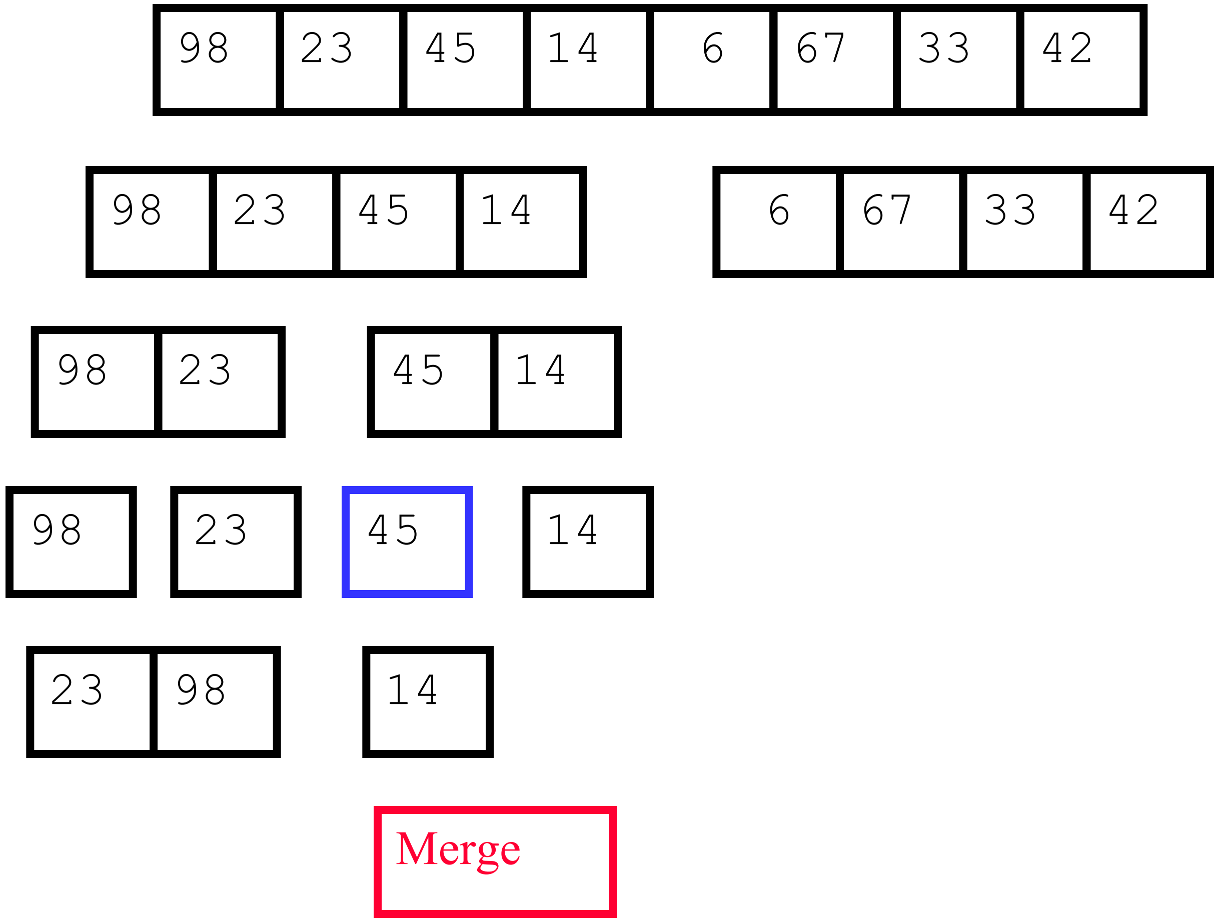
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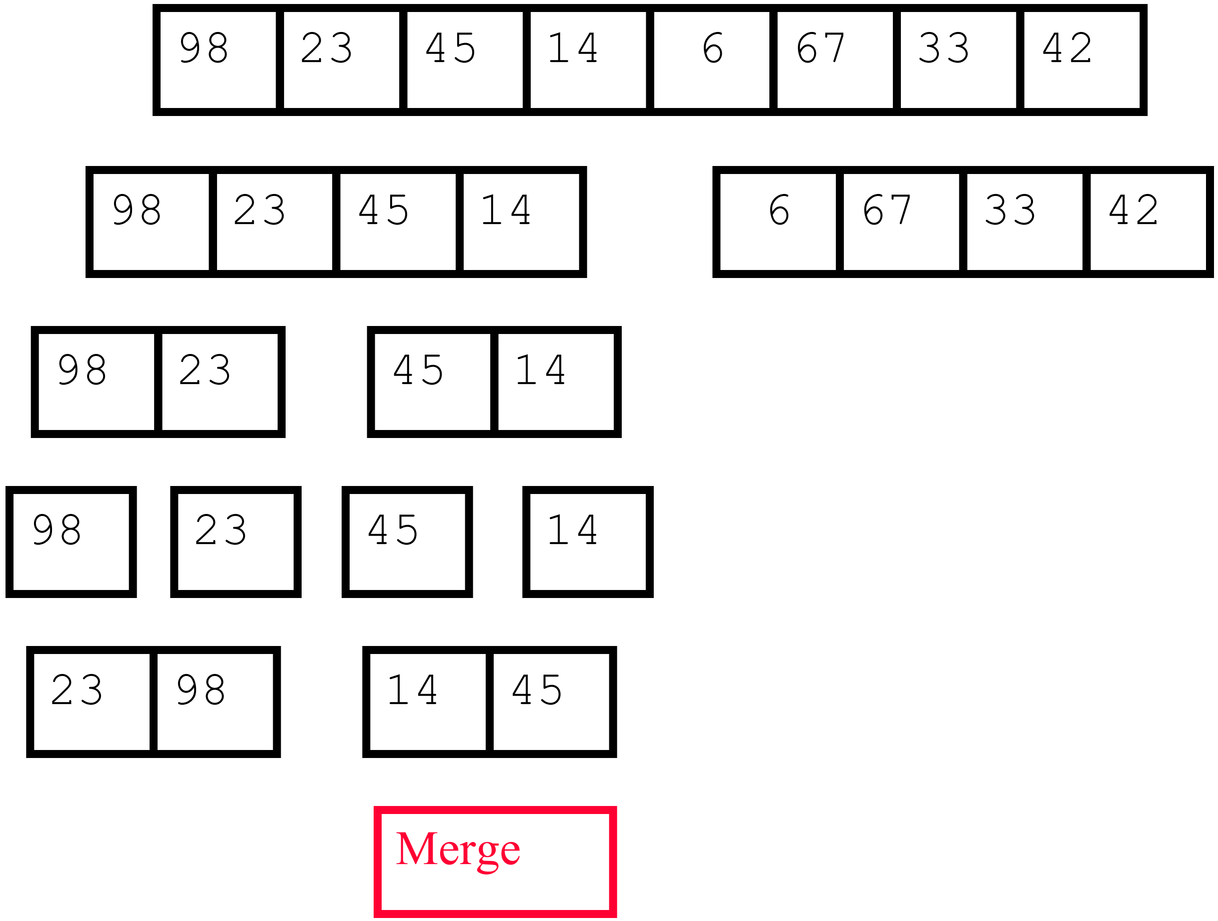
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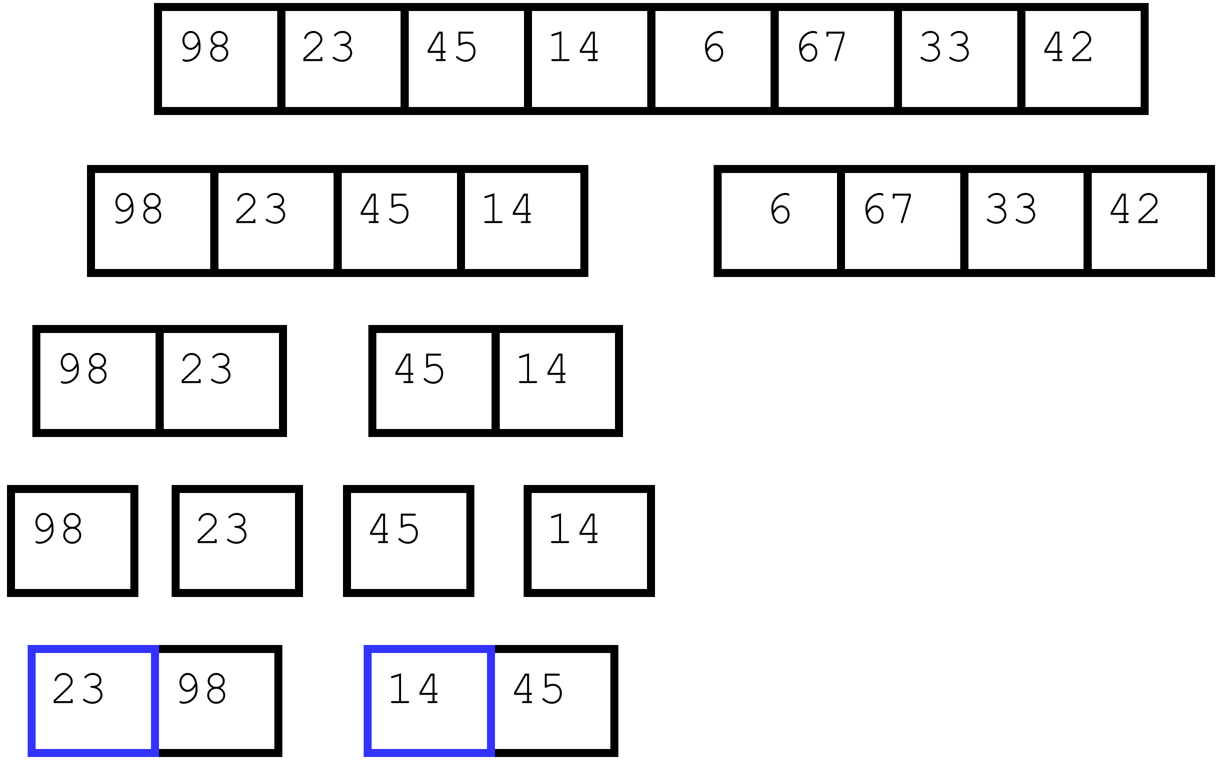
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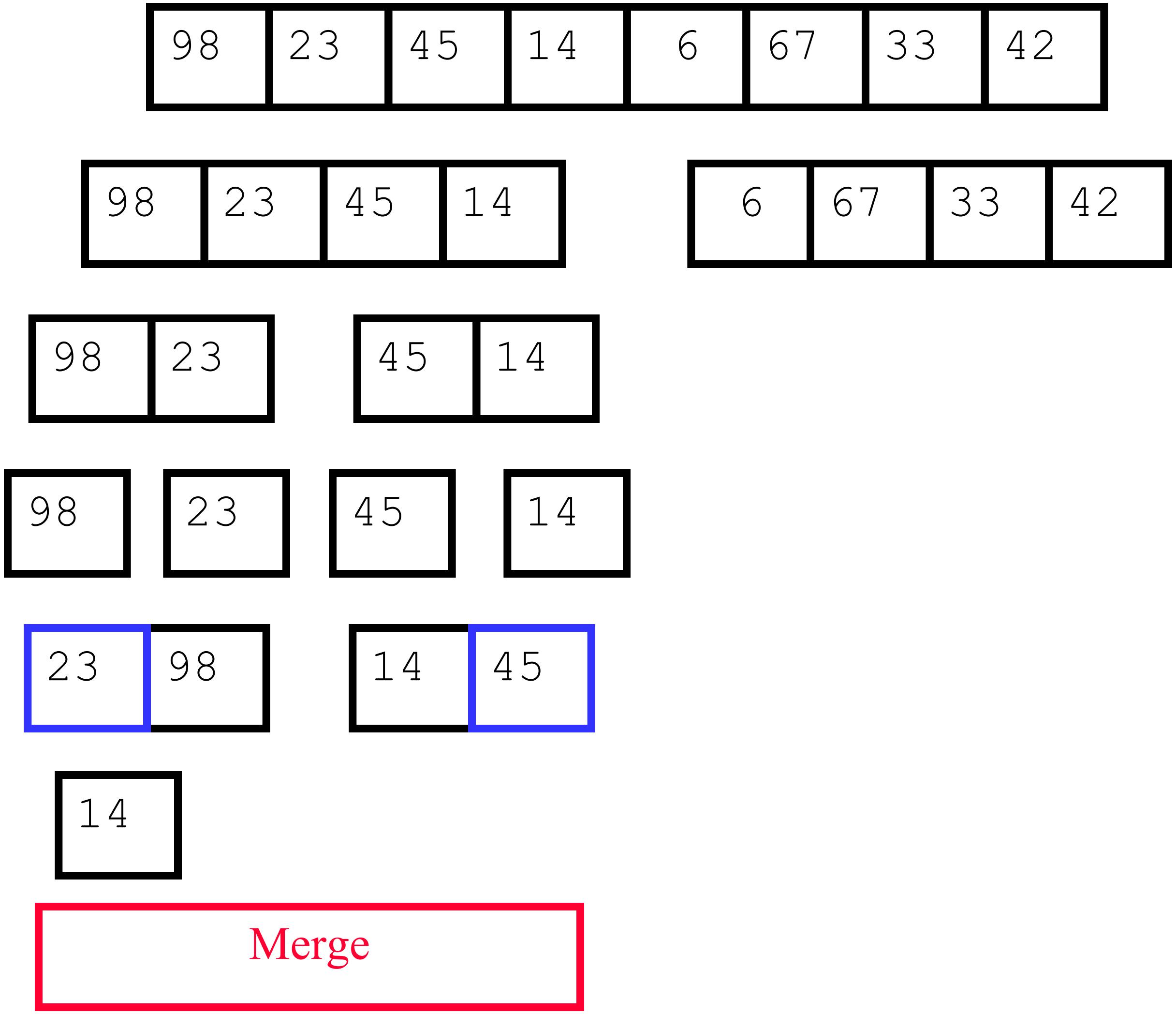

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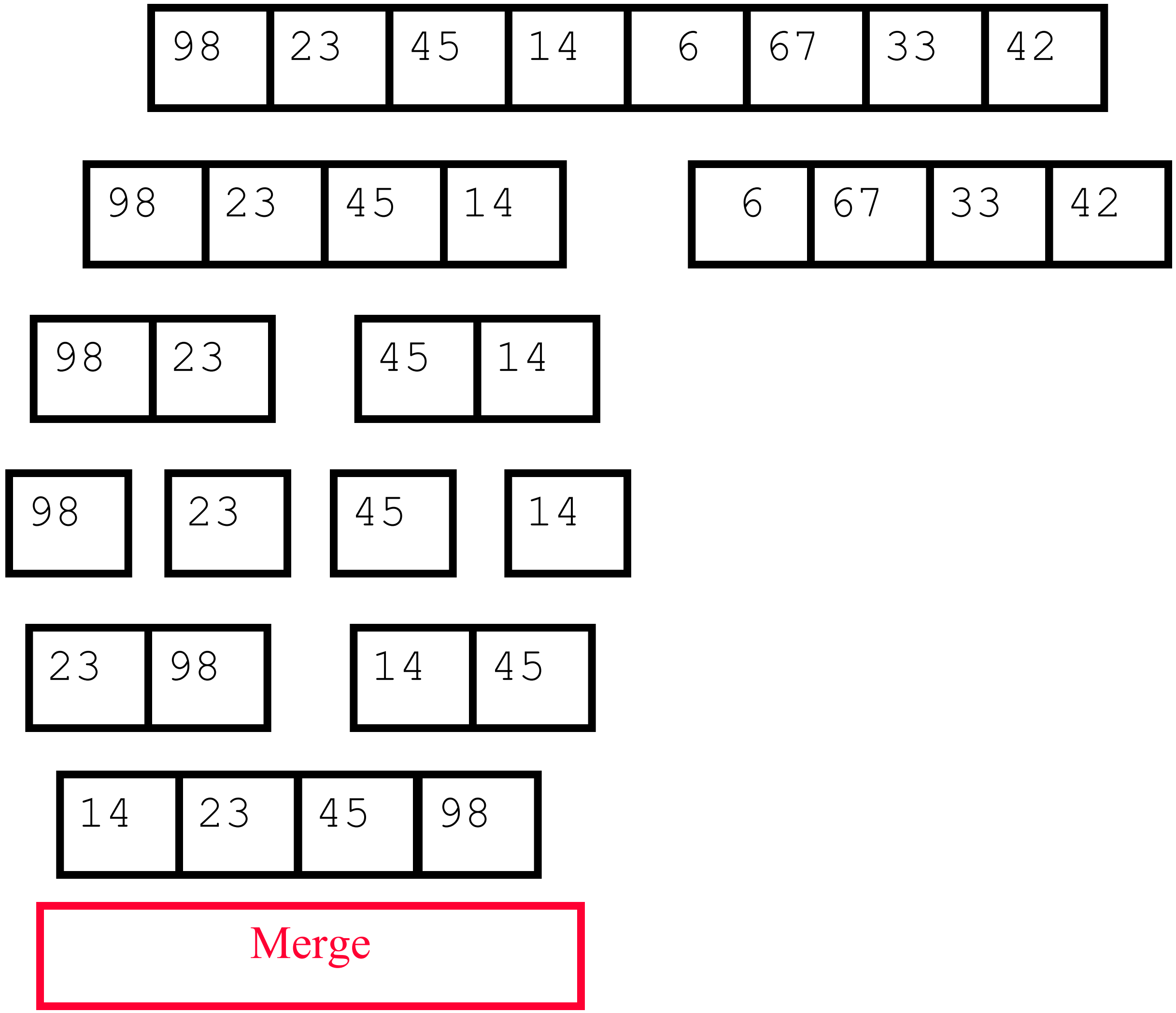
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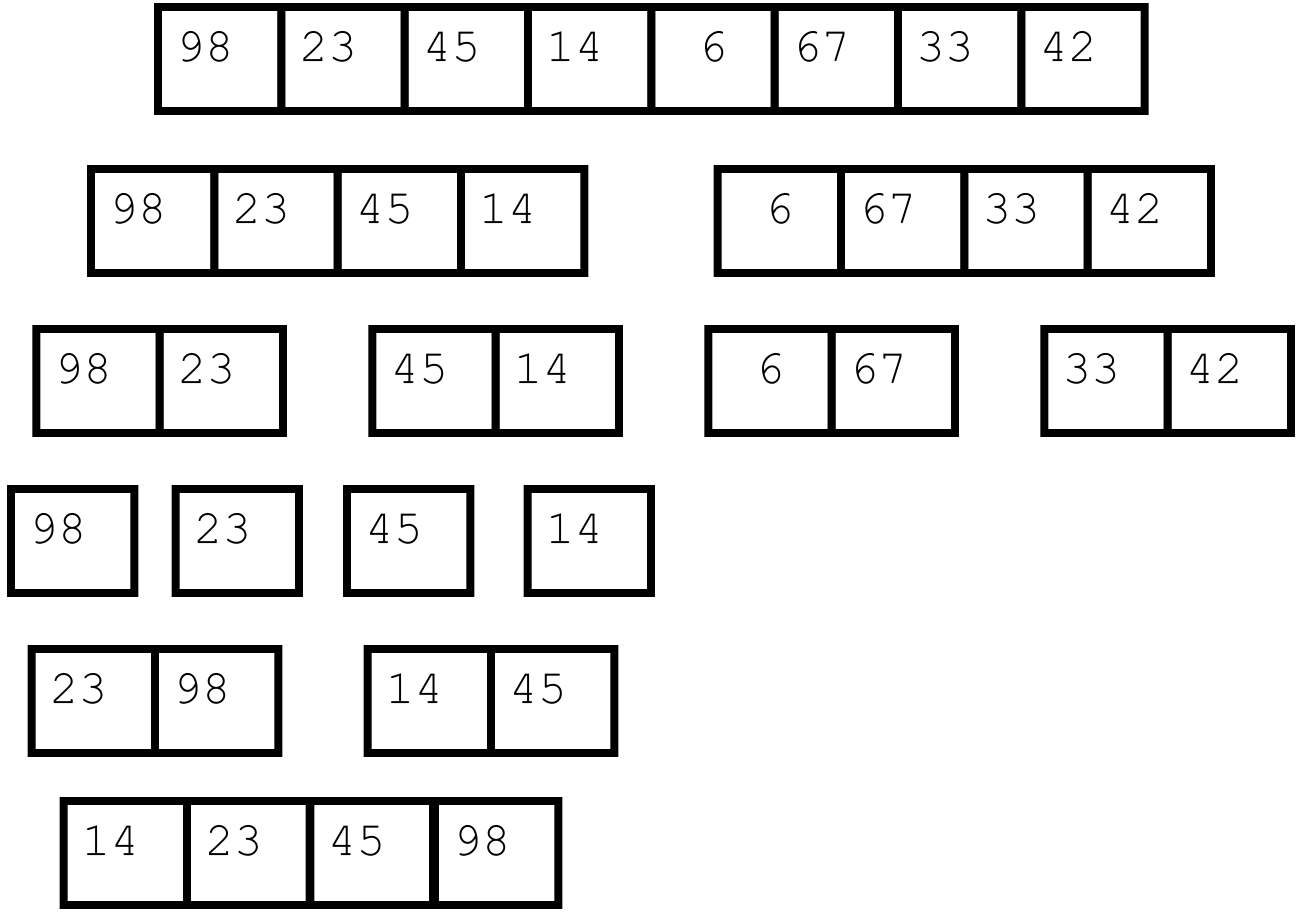
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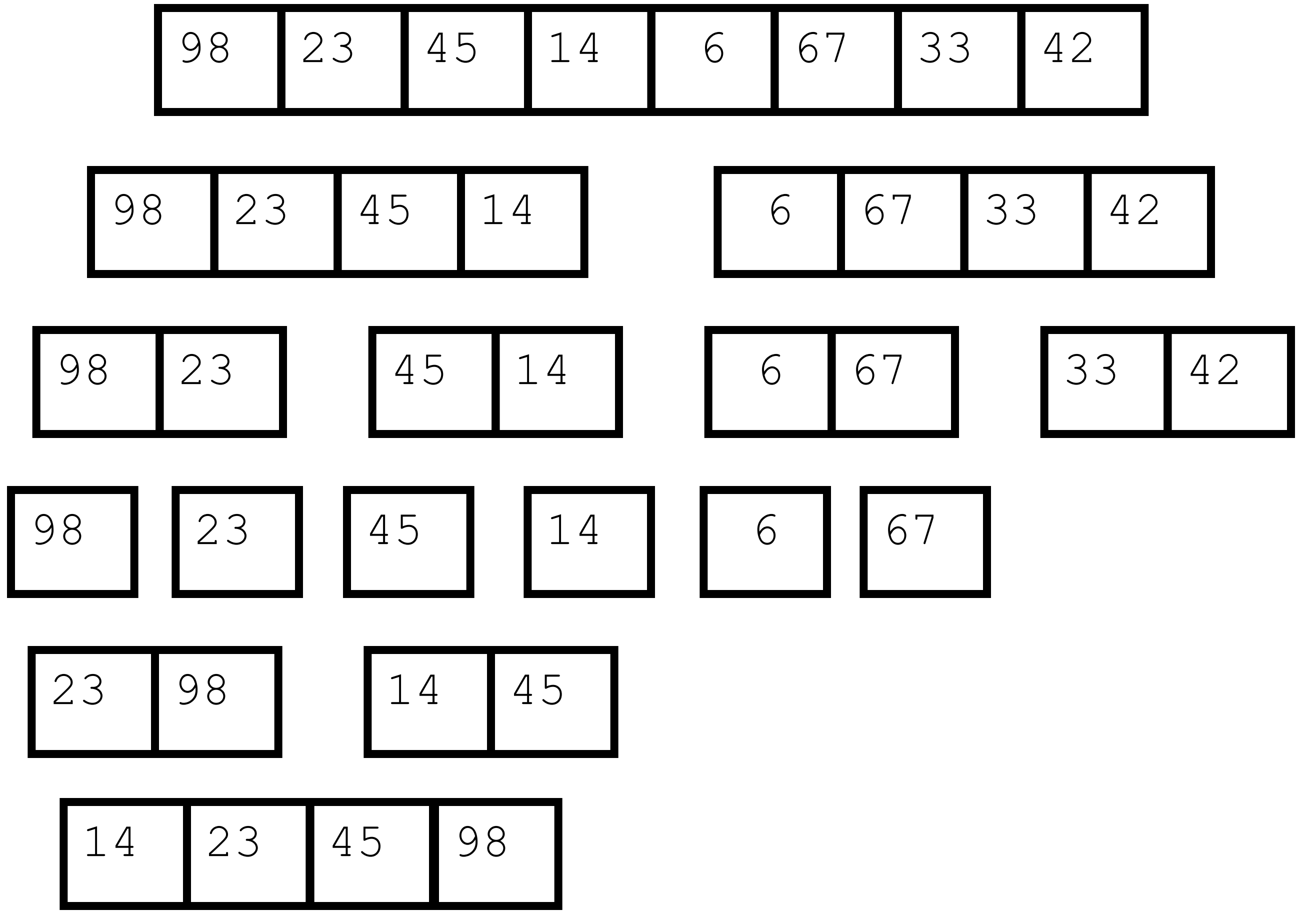
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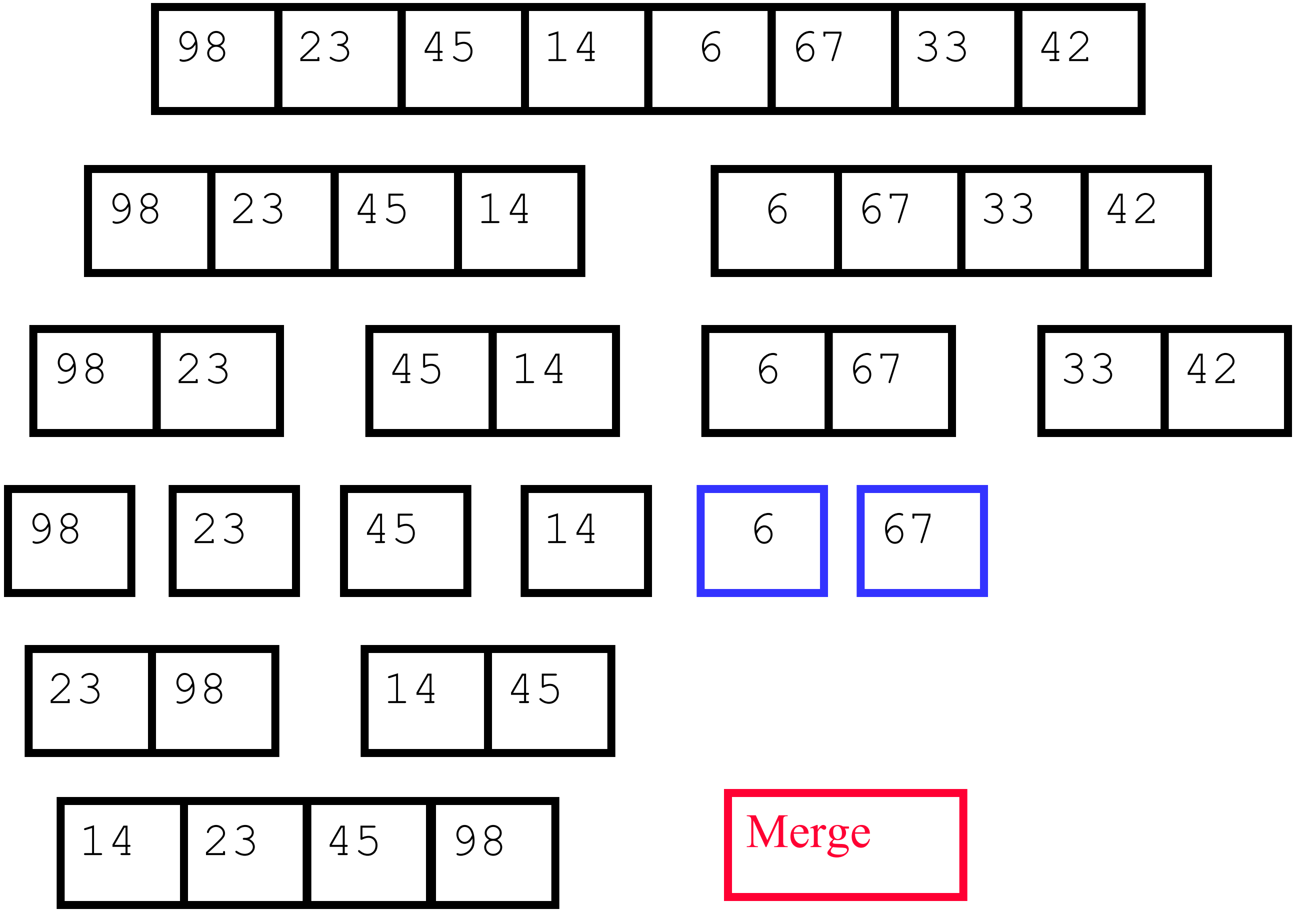

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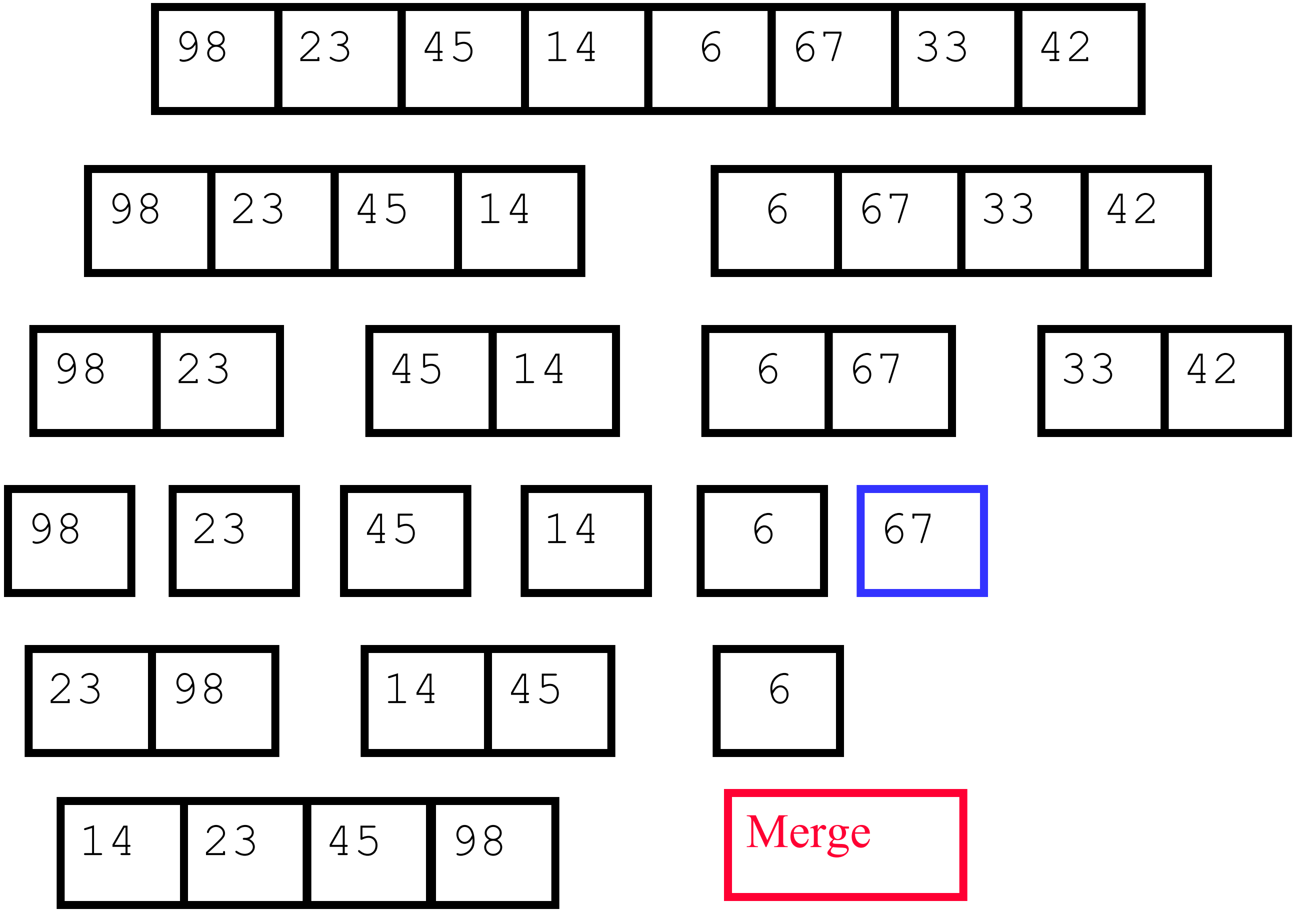
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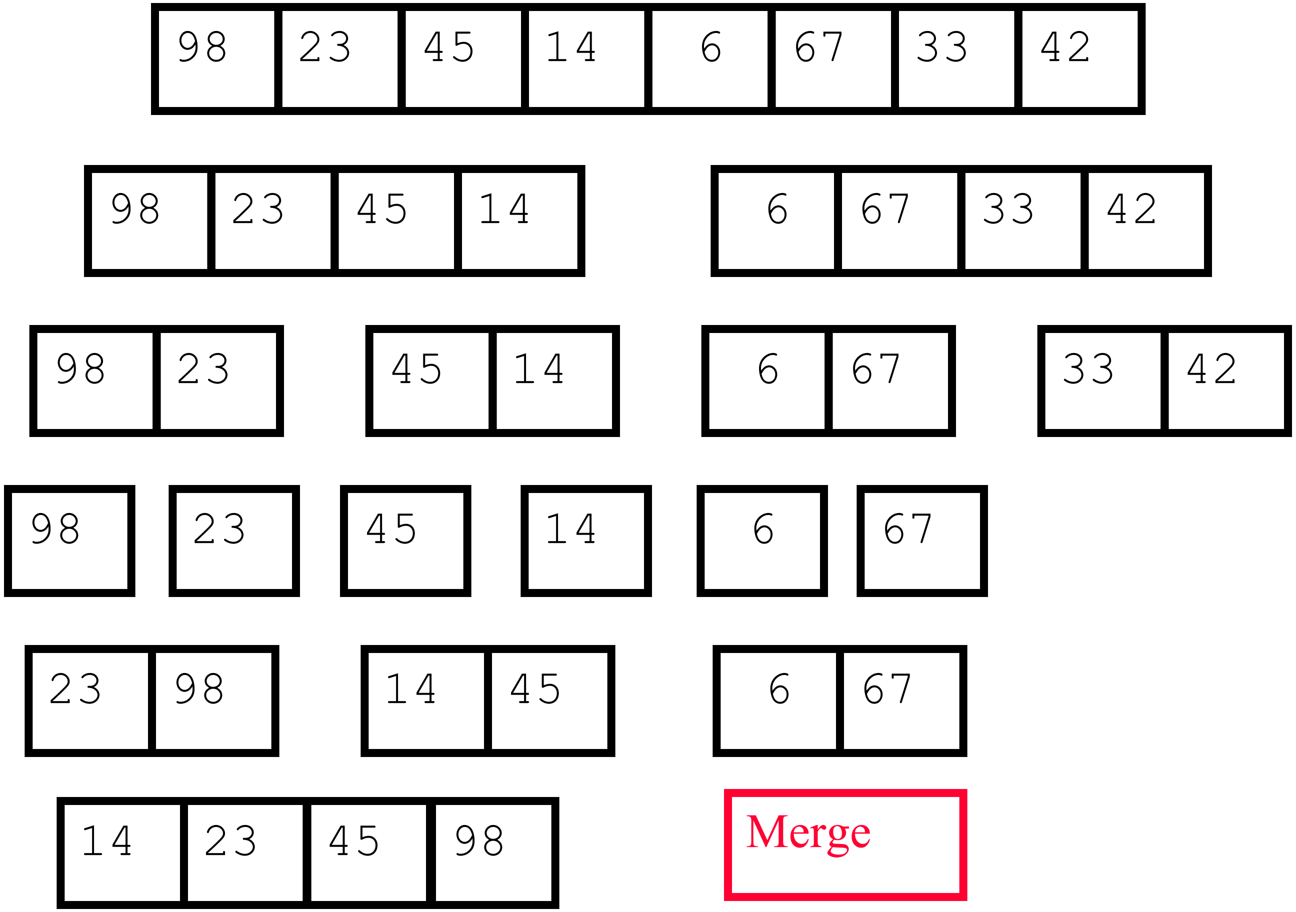
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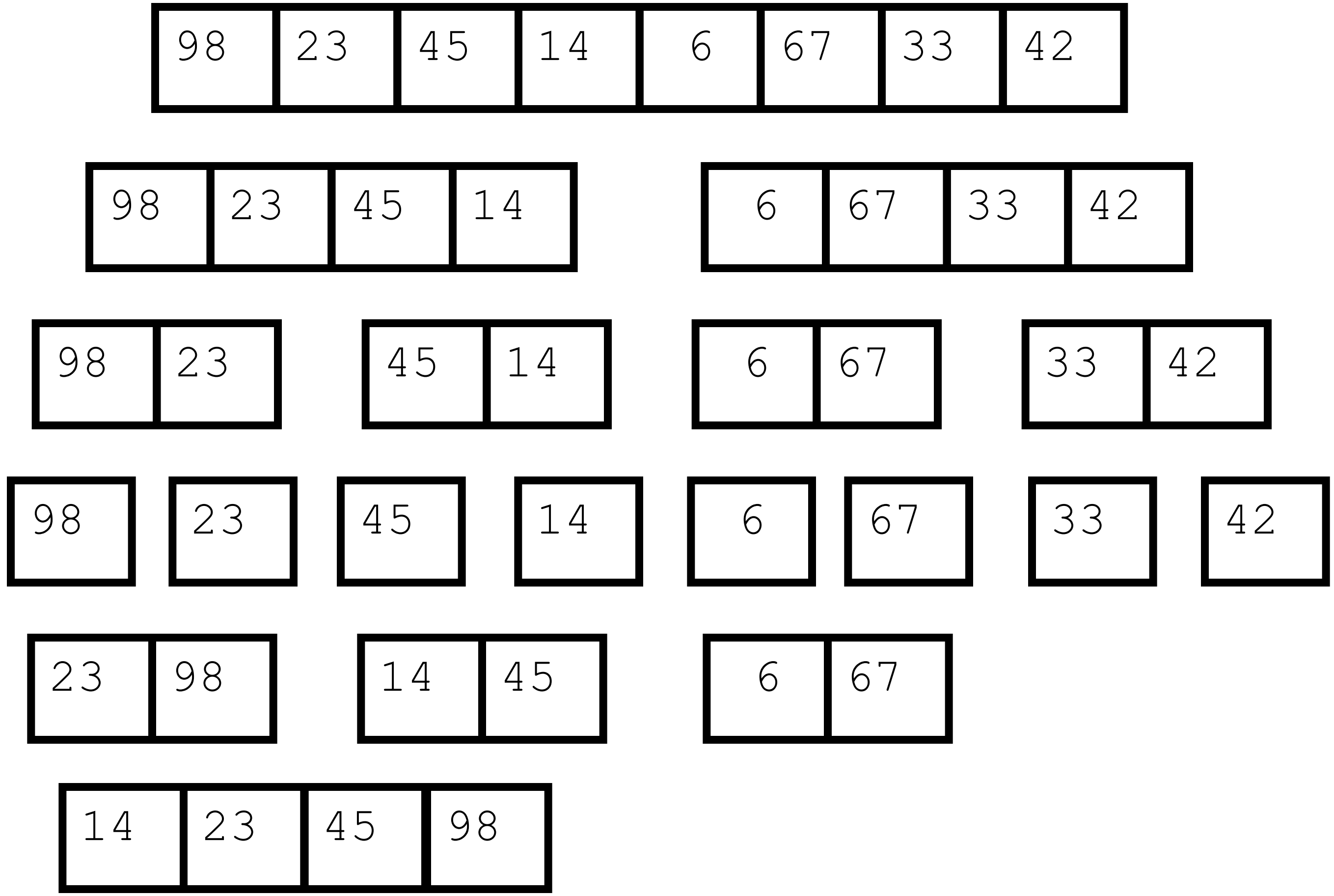
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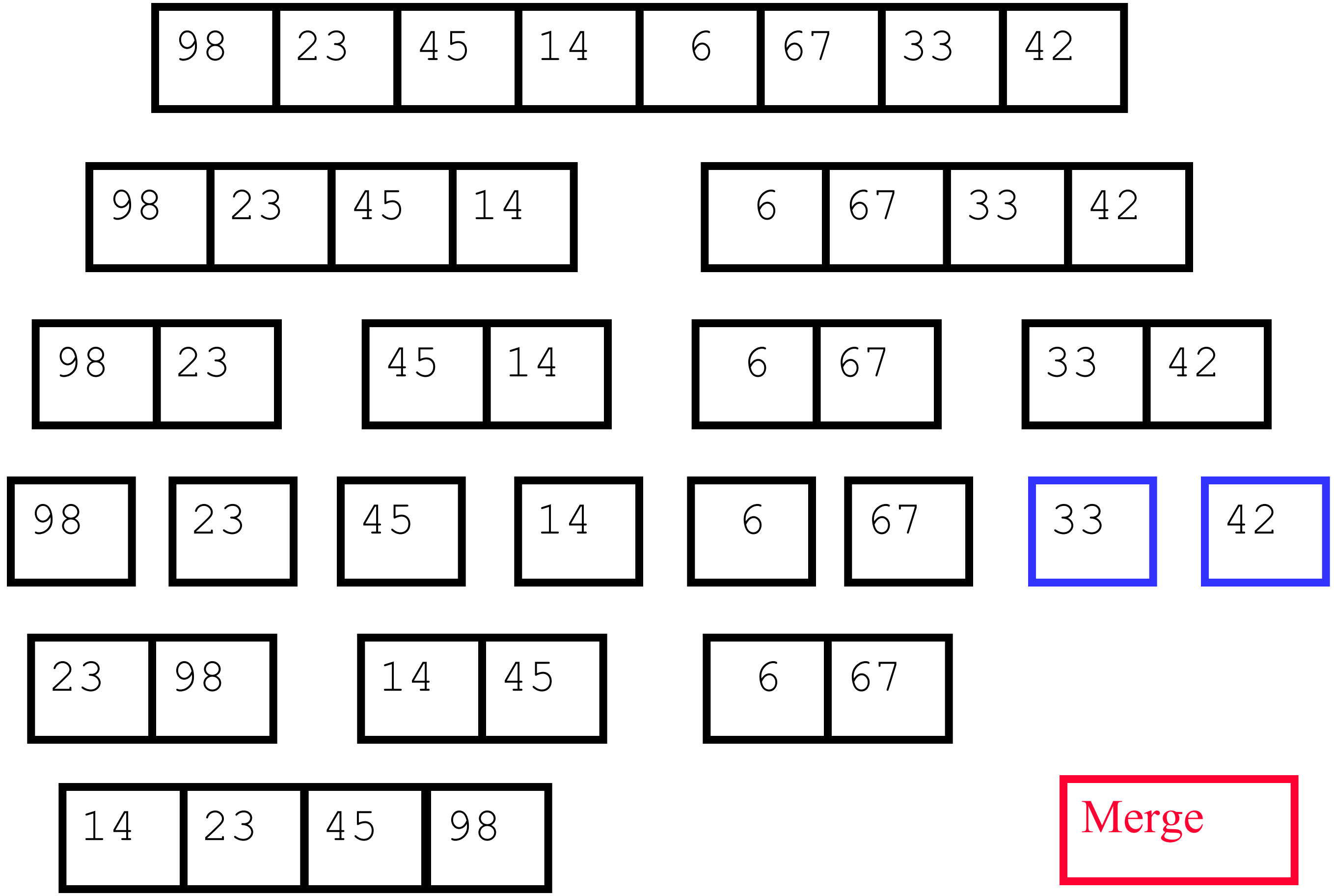

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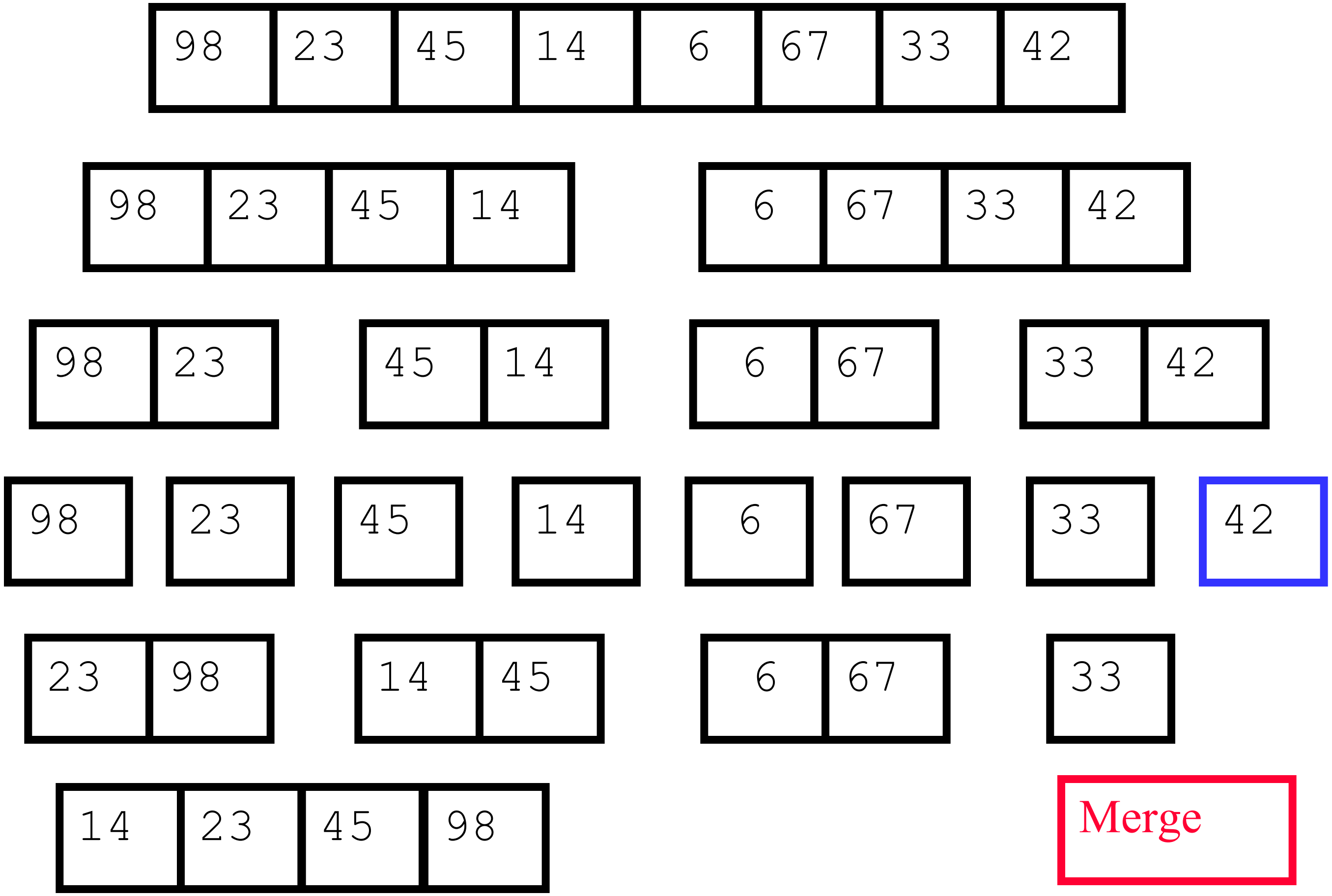
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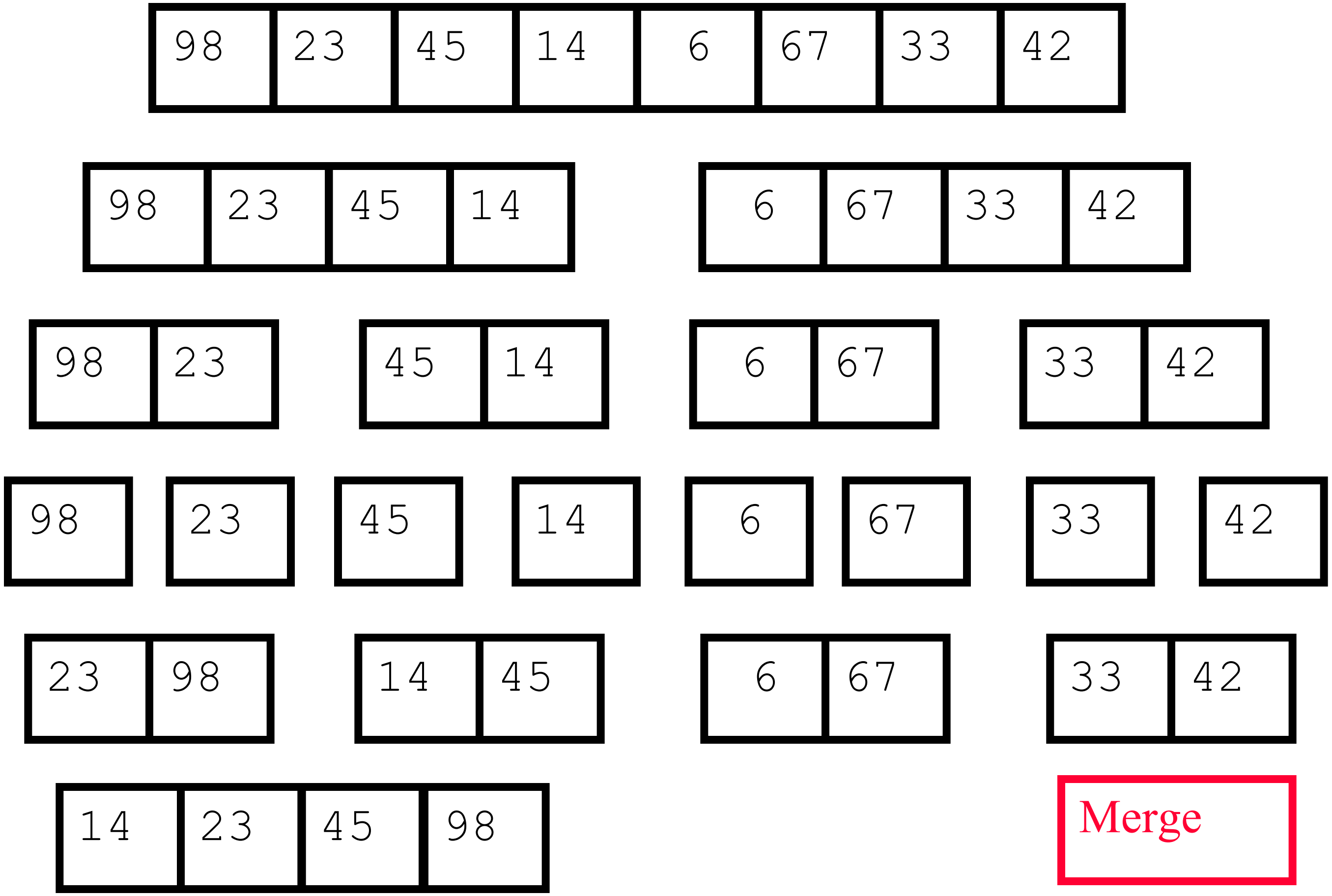

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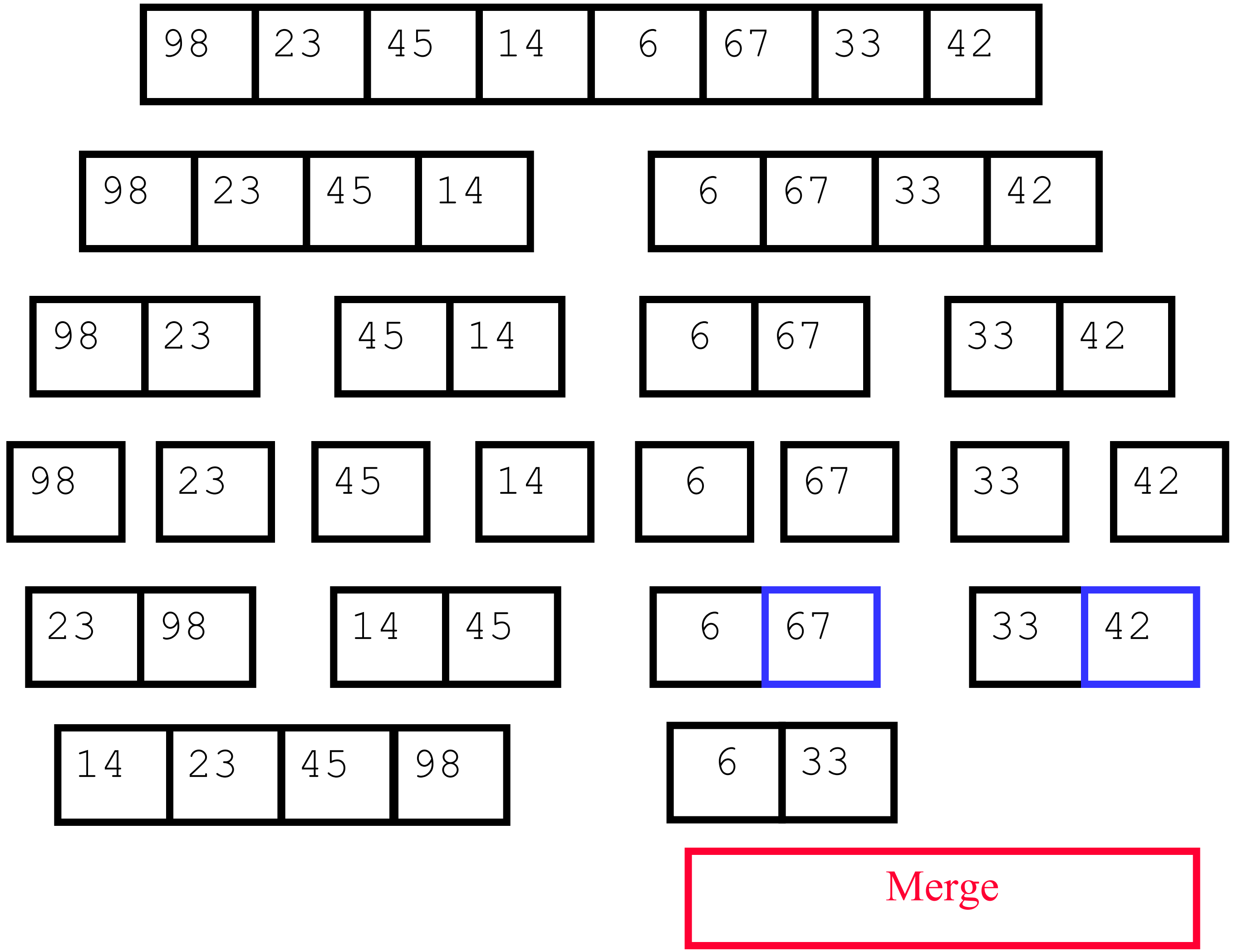
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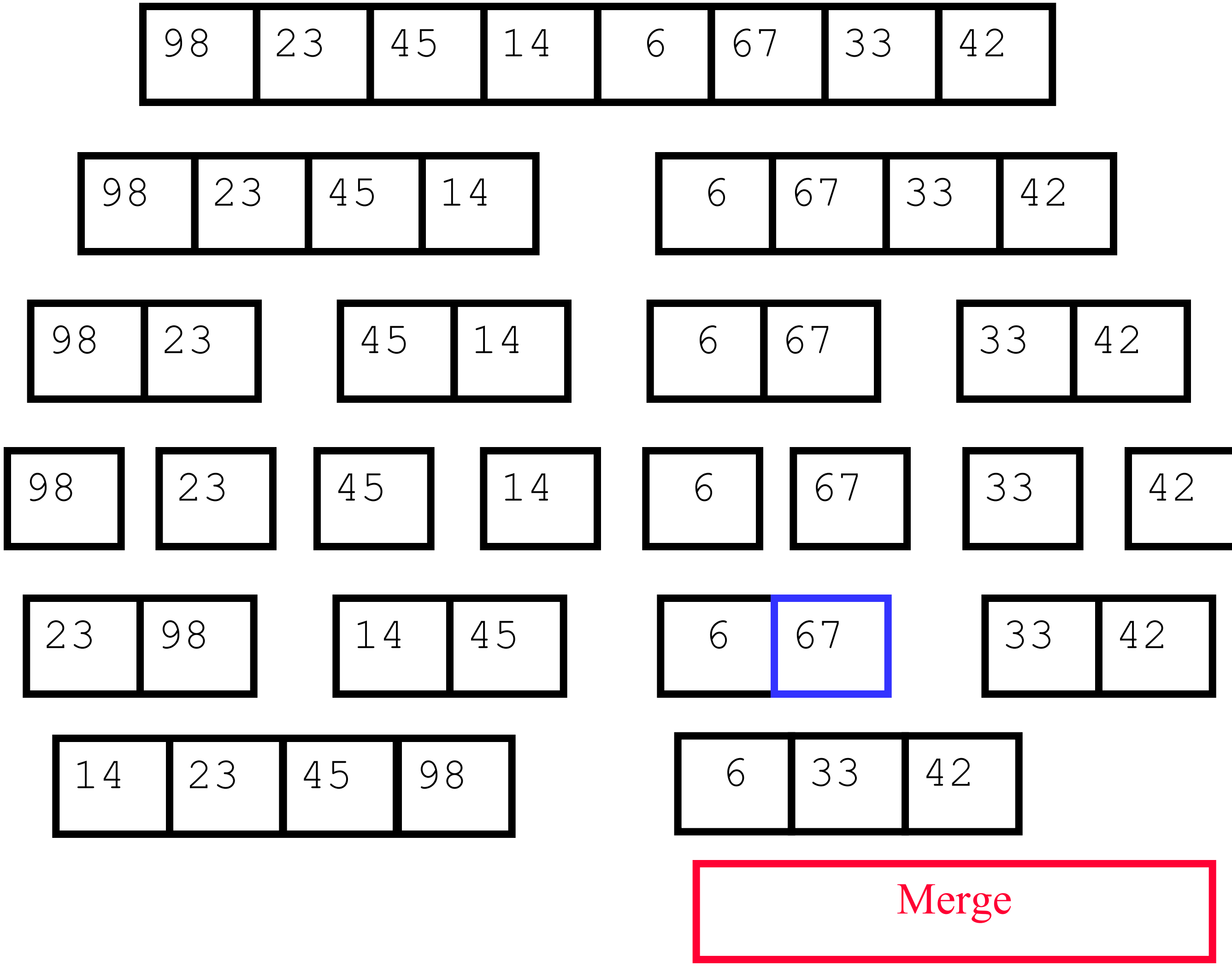
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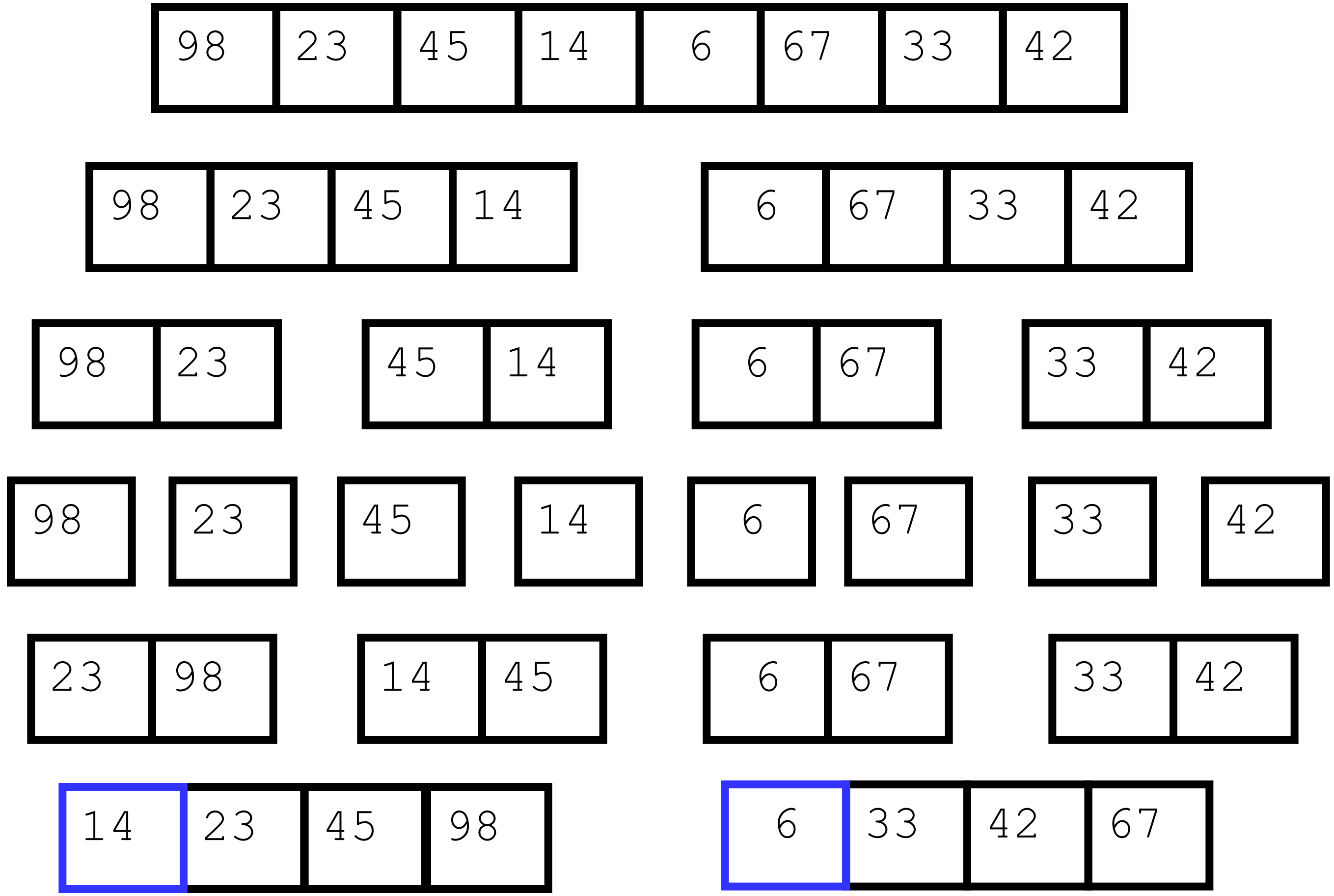

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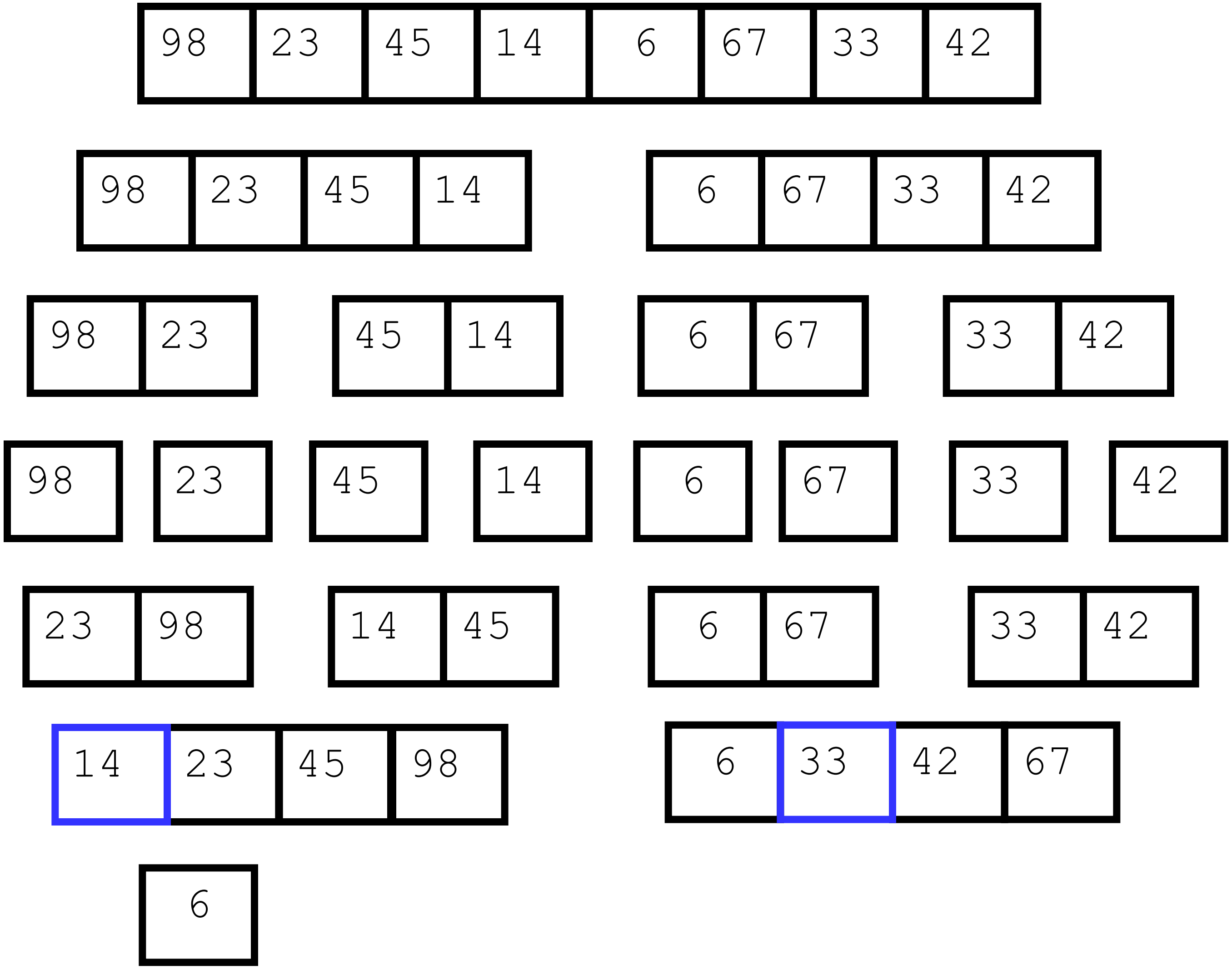
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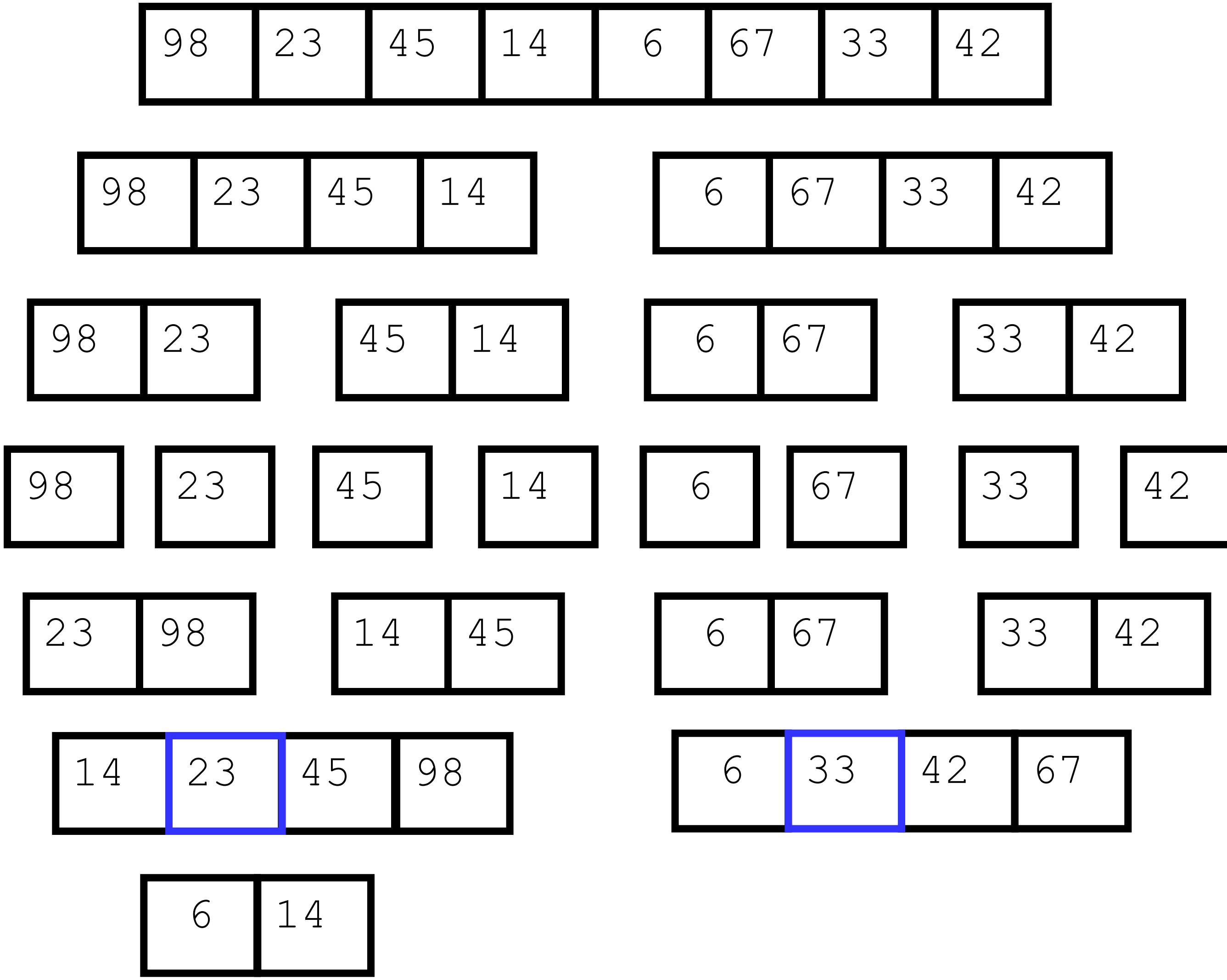
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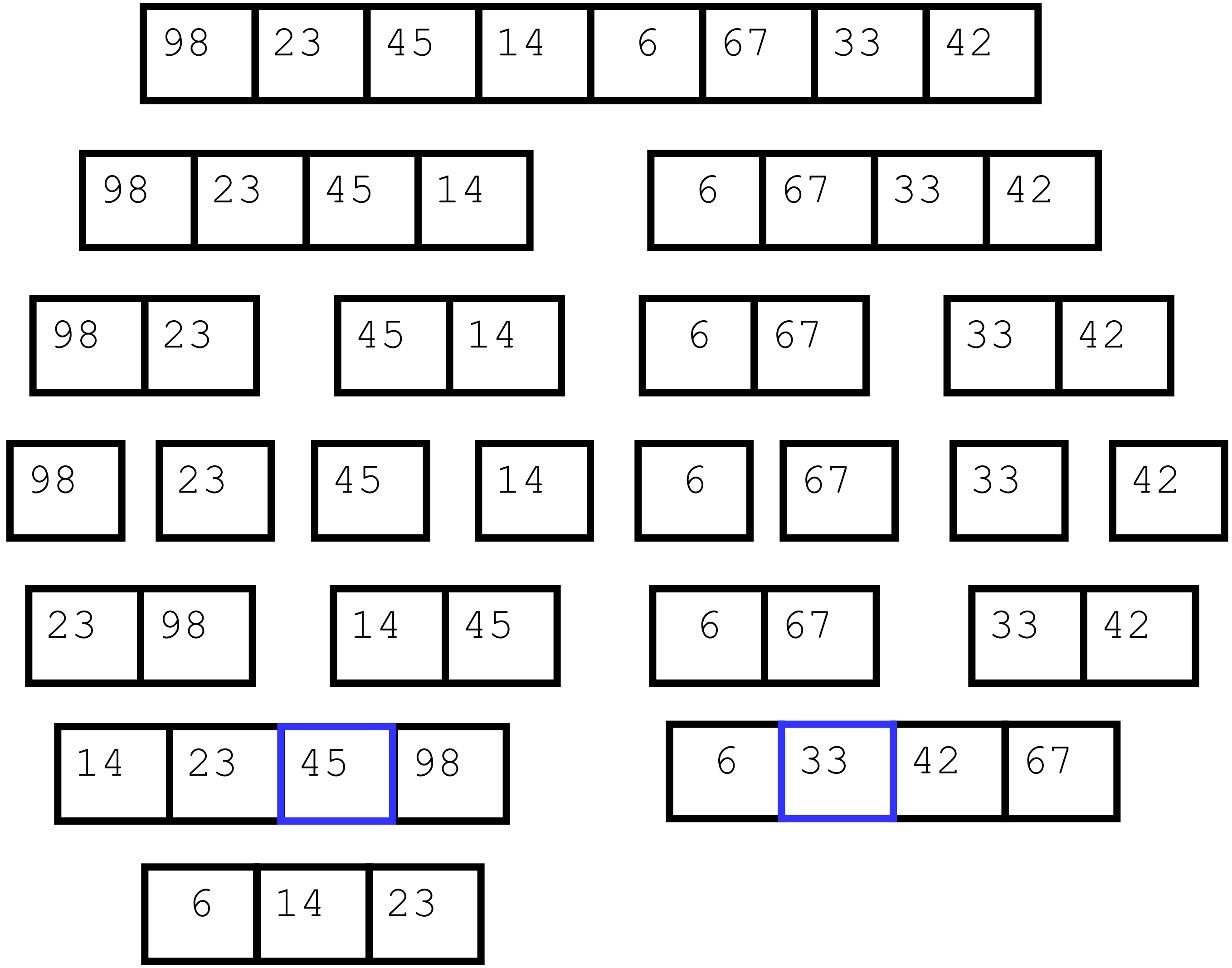
Merge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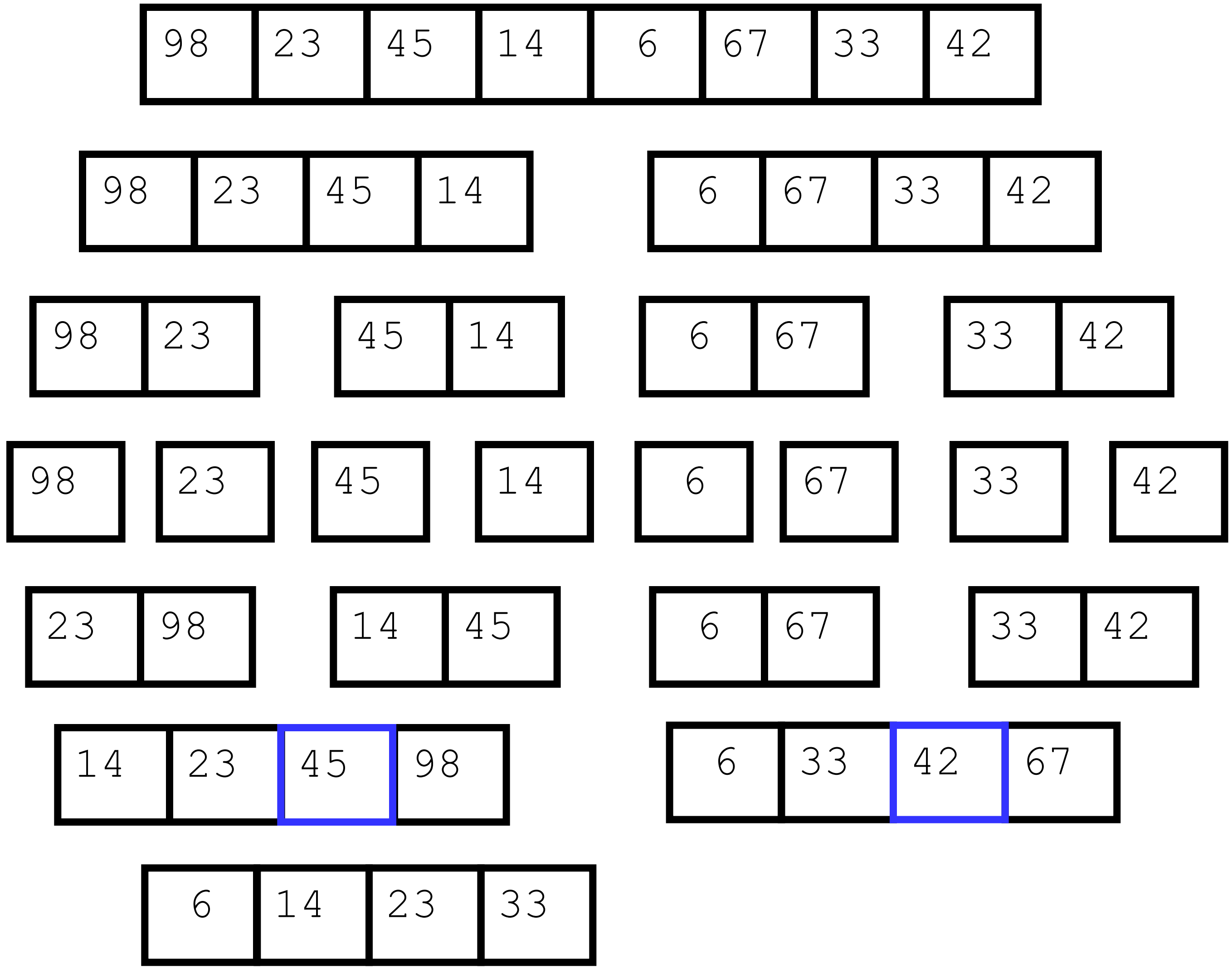
Merge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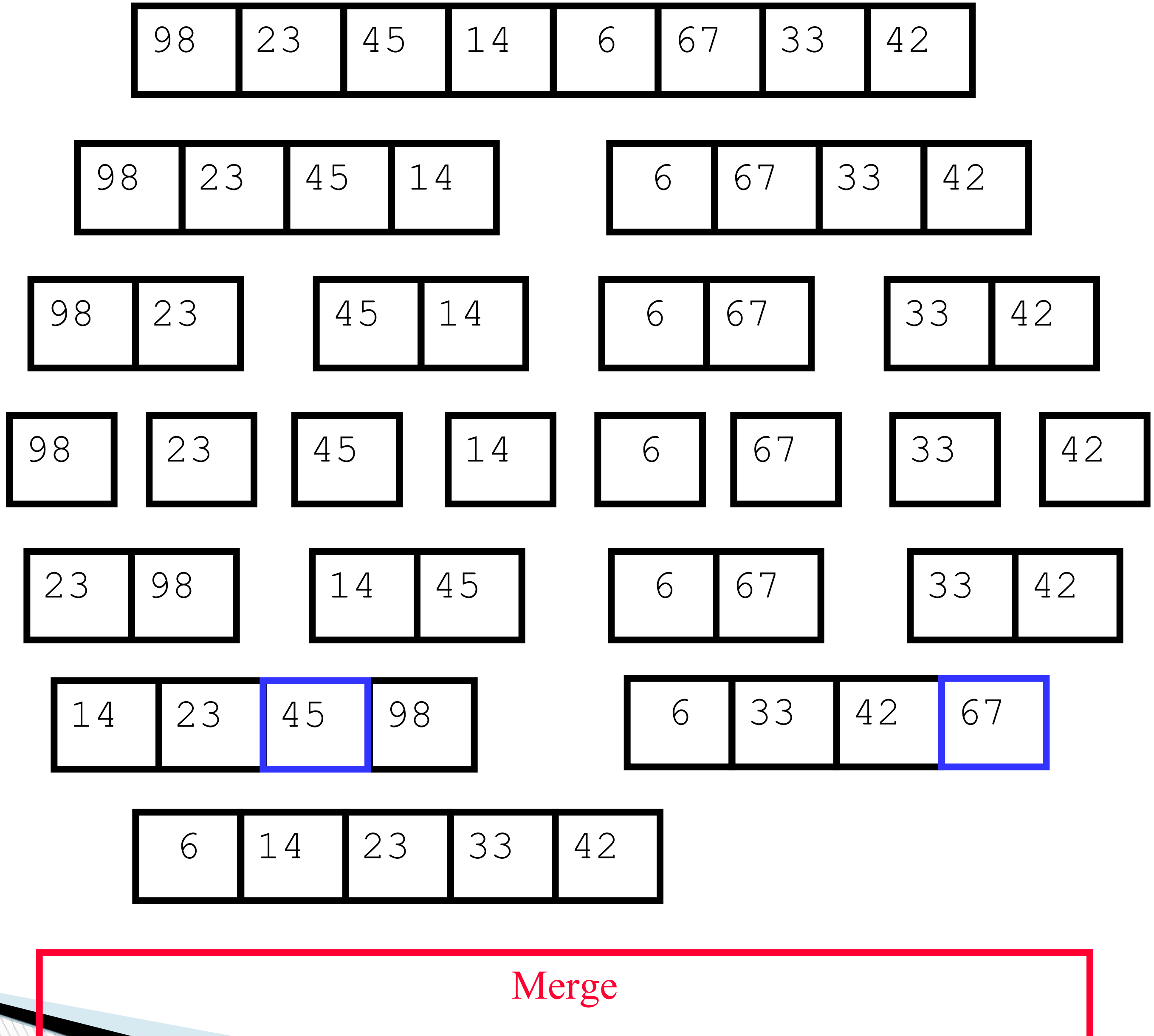
6 14 23 33

Merge


98 23 45 14 6 67 33 42
98 23 45 14
6 67 33 42
98 23
45 14
6 67
33 42
98
23
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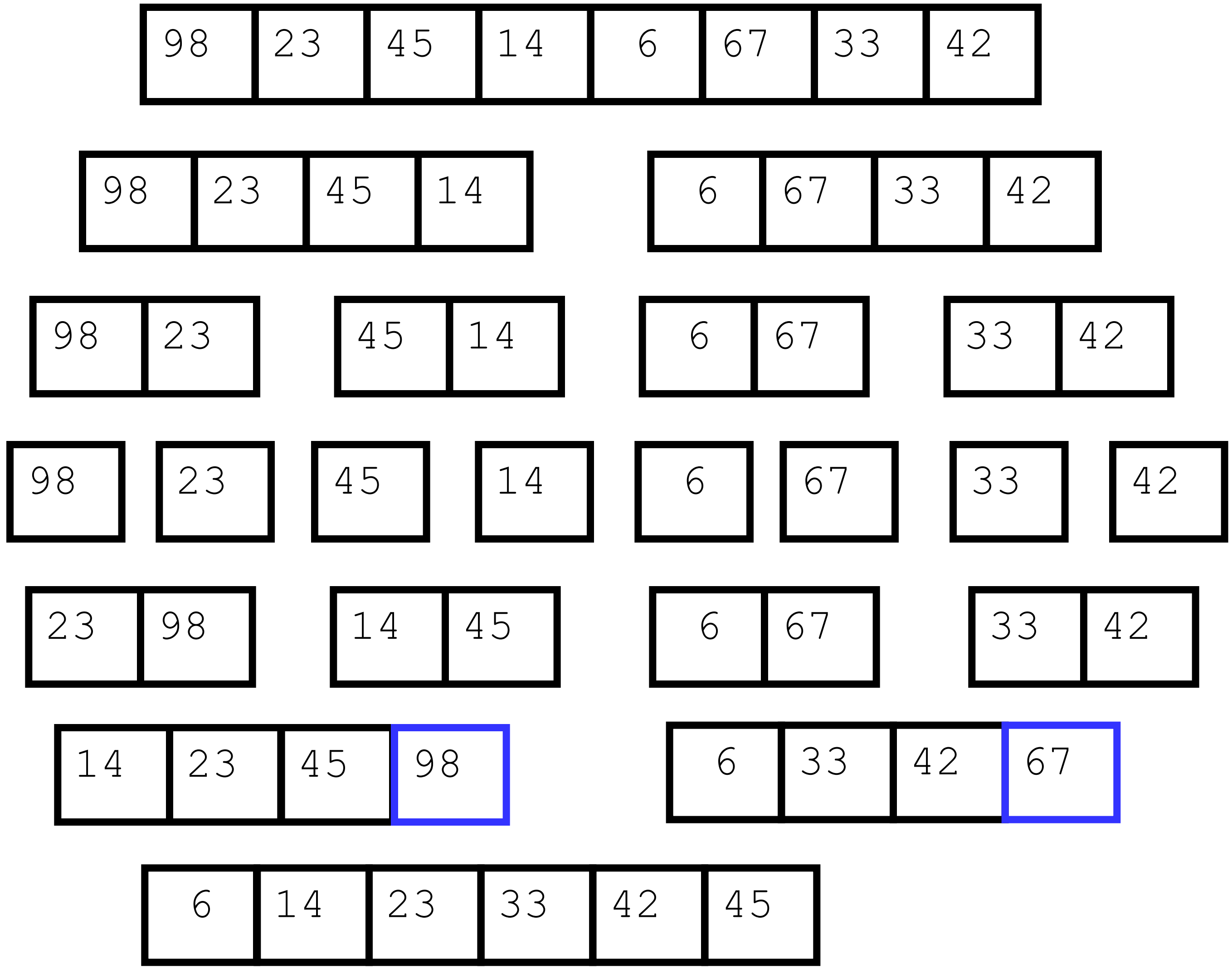
14
6
67
33
42
23 98
14 45
6 67
33 42
14 23 45 98
6 33 42 67
6 14 23 33 42
Merge


98 23 45 14 6 67 33 42
98 23 45 14
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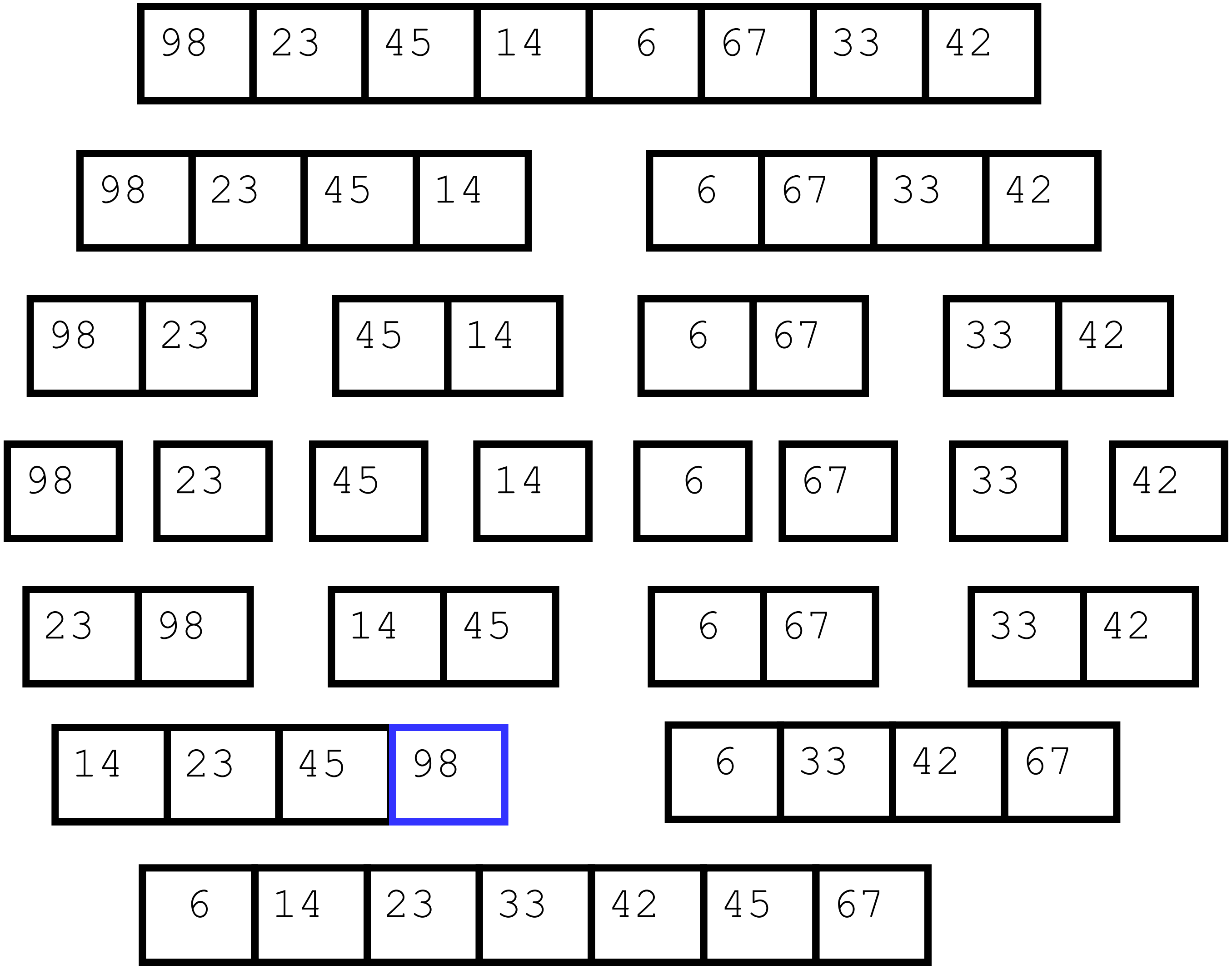
23 98
14 45
6 67
33 42
14 23 45 98
6 33 42 67
6 14 23 33 42 45

Merge


98 23 45 14 6 67 33 42
98 23 45 14
6 67 33 42
98 23
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42
23 98
14 45
6 67
33 42
14 23 45 98
6 33 42 67
6 14 23 33 42 45 67

Merge

98	23	45	14	6	67	33	42

98	23	45	14

6	67	33	42

98 23   45 14   6 67   33 42

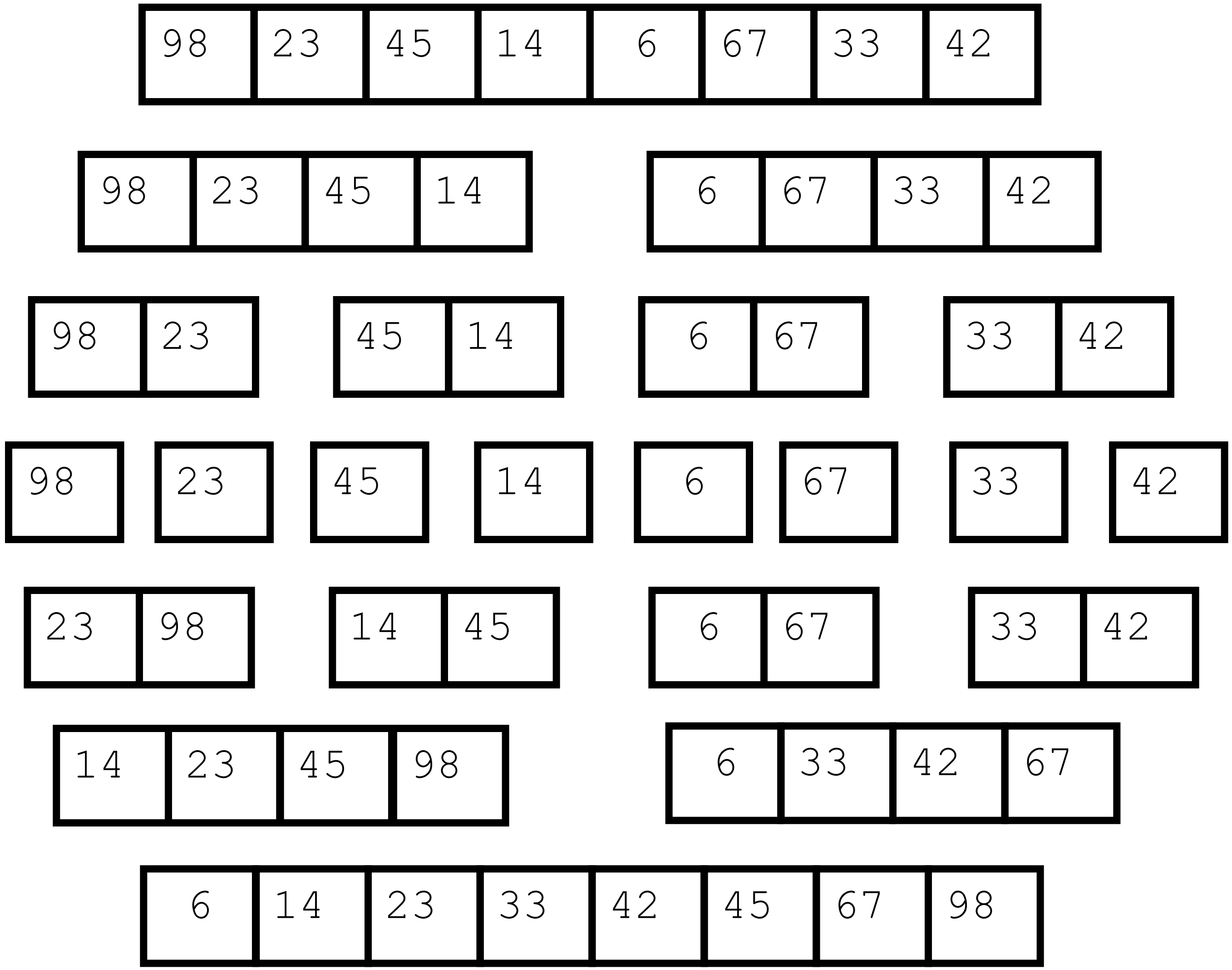
98   23   45   14   6   67   33   42

23 98   14 45   6 67   33 42

14 23 45 98   6 33 42 67

6	14	23	33	42	45	67	98

Merge


98 23 45 14 6 67 33 42
98 23 45 14
6 67 33 42
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23 98
14 45
6 67
33 42
14 23 45 98
6 33 42 67
6 14 23 33 42 45 67 98

98	23	45	14	6	67	33	42

6	14	23	33	42	45	67	98

Quick Sort

- Quicksort, like merge sort, applies the divide-and-conquer paradigm.
- three-step divide-and-conquer process for sorting a typical subarray A[p…r]:
- **Divide:**Partition (rearrange) the array A[p…r] into two (possibly empty) subarrays A[p…q-1] and A[q+1…r] such that each element of A[p…q-1] is less than or equal to A[q], which is, in turn, less than or equal to each element of A[q+1…r]. Compute the index q as part of this partitioning procedure.
- **Conquer:** Sort the two subarrays A[p…q-1] and [q+1…r] by recursive calls to quicksort.
- **Combine:** Because the subarrays are already sorted, no work is needed to combine them: the entire array A[p…r] is now sorted

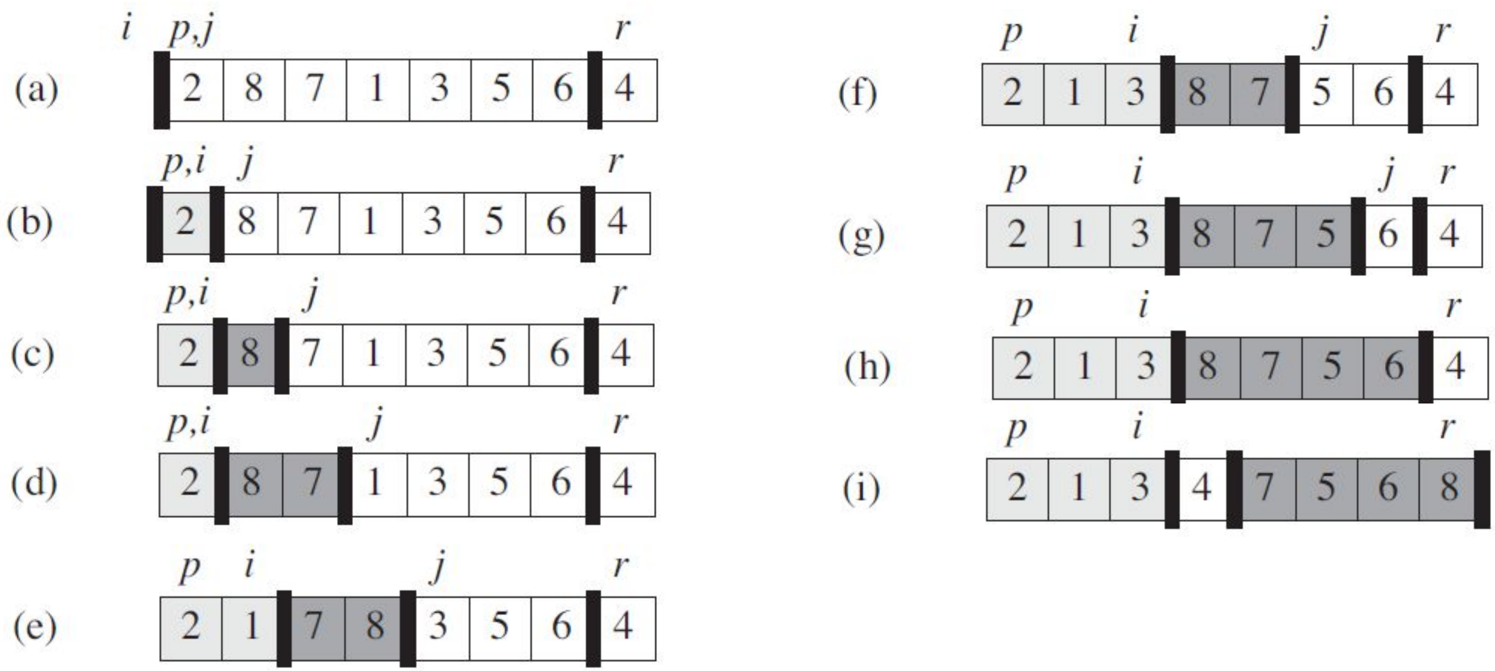
Algorithm

```
QUICKSORT(A, p, r)
if(p < r)
    q=PARTITION(A, p, r)
    QUICKSORT(A, p, q-1)
    QUICKSORT(A, q+1,r)
```

```
PARTITION(A, p, r)
x=A[r]   //pivot element
i = p-1
for j = p to r - 1
    if A[j]<=x
        i =i+1
        exchange A[i] with A[j]
exchange A[i+1]with A[r]
return i +1
```

To sort an entire array A, the initial call is QUICKSORT(A, 1, A.*length*)

Example



(a) i p,j r 2 8 7 1 3 5 6 4
(b) p,i j r 2 8 7 1 3 5 6 4
(c) p,i j r 2 8 7 1 3 5 6 4
(d) p,i j r 2 8 7 1 3 5 6 4
(e) p i j r 2 1 7 8 3 5 6 4
(f) p i j r 2 1 3 8 7 5 6 4
(g) p i j r 2 1 3 8 7 5 6 4
(h) p i r 2 1 3 8 7 5 6 4
(i) p i r 2 1 3 4 7 5 6 8