Lecture Summary: Introduction to Linear Mappings – Part 1

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Key Points

- **Introduction to Linear Mappings:**
  - A linear mapping represents a function that transforms inputs to outputs while preserving the properties of linear combinations.
  - Example: Computing the cost of goods in a grocery shop using quantities and prices as inputs.
- **Example: Grocery Shop Cost Function:**
  - Prices at Shop A:
    Rice: 45 Rs/kg, Dal: 125 Rs/kg, Oil: 150 Rs/liter.
  - Cost for given quantities x_1 (rice), x_2 (dal), x_3 (oil):
    $$c_A(x_1, x_2, x_3) = 45x_1 + 125x_2 + 150x_3.$$
  - This is a linear combination of x_1, x_2, x_3 with coefficients $45, 125, 150$.
  - Example computations:
    $$c_A(1, 2, 1) = 45 \cdot 1 + 125 \cdot 2 + 150 \cdot 1 = 445\,\text{Rs}.$$
    $$c_A(2, 1, 2) = 45 \cdot 2 + 125 \cdot 1 + 150 \cdot 2 = 515\,\text{Rs}.$$
- **Generalization:**
  - A cost function $c_A : \mathbb{R}^3 \to \mathbb{R}$ is defined as:
    $$c_A(x_1, x_2, x_3) = 45x_1 + 125x_2 + 150x_3.$$
  - Inputs are represented as vectors:
    $$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix},$$
    and the cost function is equivalent to a matrix-vector multiplication:
    $$c_A(x) = \begin{bmatrix} 45 & 125 & 150 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}.$$
- **Properties of Linear Mappings:**
  - Linearity ensures that combinations of inputs can be simplified:
    $$c_A(a \cdot x + b \cdot y) = a \cdot c_A(x) + b \cdot c_A(y).$$

- This property allows for simplified computations in real-world scenarios.

- **Case Study: Caterer's Orders:**
  - Office 1 requires 20 kg rice, 10 kg dal, 4 liters oil:

$$c_A(20, 10, 4) = 2750\,\text{Rs}.$$

  - Office 2 requires 30 kg rice, 12 kg dal, 2 liters oil:

$$c_A(30, 12, 2) = 3150\,\text{Rs}.$$

  - For combined orders (Wednesday):

$$c_A\left(\frac{1}{2}(20, 10, 4) + \frac{5}{4}(30, 12, 2)\right) = 5312.5\,\text{Rs}.$$

  - Linearity allows splitting computations into contributions from Monday and Tuesday orders.

- **Key Observations:**
  - A linear function can be interpreted as a combination of contributions weighted by inputs.
  - Linearity simplifies real-world calculations like those in commerce and logistics.

Simplified Explanation

Example: Linear Cost Function Prices: Rice = 45 Rs/kg, Dal = 125 Rs/kg, Oil = 150 Rs/liter.

- For quantities $x_1 = 2$ (rice), $x_2 = 1$ (dal), $x_3 = 2$ (oil):

$$c_A(x_1, x_2, x_3) = 45 \cdot 2 + 125 \cdot 1 + 150 \cdot 2 = 515\,\text{Rs}.$$

- Combined inputs maintain the linearity:

$$c_A\left(a \cdot x_1 + b \cdot x_2\right) = a \cdot c_A(x_1) + b \cdot c_A(x_2).$$

Conclusion

In this lecture, we:

- Introduced linear mappings as functions preserving linear combinations.
- Illustrated linearity through examples like cost functions and catering orders.
- Highlighted the computational advantages of linearity in real-world scenarios.

This sets the foundation for exploring more advanced applications of linear mappings in data science and mathematics.