Lecture Summary: Linear Mappings – Part 2

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Key Points

- **Review of Linear Mappings:**
  - A linear mapping is a function $f : \mathbb{R}^n \to \mathbb{R}^m$ that satisfies:
  $$f(a \cdot x + b \cdot y) = a \cdot f(x) + b \cdot f(y),$$
  where $a, b \in \mathbb{R}$ and $x, y \in \mathbb{R}^n$.
  - Linear mappings preserve the operations of addition and scalar multiplication.
- **Examples: Cost Functions in Multiple Shops:**
  - Shop A prices:
  $$c_A(x_1, x_2, x_3) = 45x_1 + 125x_2 + 150x_3.$$
  - Shop B prices:
  $$c_B(x_1, x_2, x_3) = 40x_1 + 120x_2 + 170x_3.$$
  - Shop C prices:
  $$c_C(x_1, x_2, x_3) = 50x_1 + 130x_2 + 160x_3.$$
  - These functions can be written in matrix form:
  $$c(x_1, x_2, x_3) = \begin{bmatrix} 45 & 125 & 150 \\ 40 & 120 & 170 \\ 50 & 130 & 160 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}.$$
- **Using Linear Mappings for Decision-Making:**
  - To compare costs across shops, evaluate the cost functions for given quantities of rice, dal, and oil.
  - Example: For $x_1 = 2$ kg (rice), $x_2 = 1$ kg (dal), $x_3 = 2$ liters (oil):
  $$c_A(2, 1, 2) = 515, \quad c_B(2, 1, 2) = 540, \quad c_C(2, 1, 2) = 550.$$
  - Shop A is the most economical in this scenario.
- **General Form of Linear Mappings:**
  - A general linear mapping $f : \mathbb{R}^n \to \mathbb{R}^m$ has the form:
  $$f(x_1, x_2, \ldots, x_n) = \begin{bmatrix} \sum_{j=1}^n a_{1j}x_j \\ \sum_{j=1}^n a_{2j}x_j \\ \vdots \\ \sum_{j=1}^n a_{mj}x_j \end{bmatrix},$$
  where $a_{ij} \in \mathbb{R}$ are constants.

- This can be expressed as:
$$f(x) = A \cdot x,$$
where A is an $m \times n$ matrix and x is a column vector.

- **Linearity of Mappings:**
  - Linearity ensures that:
  $$f(x + c \cdot y) = f(x) + c \cdot f(y),$$
  where $x, y \in \mathbb{R}^n$ and $c \in \mathbb{R}$.
  - This property simplifies computations, allowing for decomposition and scaling of inputs.

Simplified Explanation

Example: Cost Vector for Three Shops Prices for rice, dal, and oil:

$$\text{Shop A: } [45, 125, 150], \quad \text{Shop B: } [40, 120, 170], \quad \text{Shop C: } [50, 130, 160].$$

- For $x_1 = 2$ (rice), $x_2 = 1$ (dal), $x_3 = 2$ (oil):
$$c_A(2, 1, 2) = 515, \quad c_B(2, 1, 2) = 540, \quad c_C(2, 1, 2) = 550.$$

- Matrix form:
$$c(x) = \begin{bmatrix} 45 & 125 & 150 \\ 40 & 120 & 170 \\ 50 & 130 & 160 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}.$$

Conclusion

In this lecture, we:

- Explored linear mappings through real-world cost functions in multiple shops.
- Defined the general form of a linear mapping using matrices.
- Highlighted the computational advantages of linearity in comparing and analyzing functions.

Linear mappings are powerful tools for representing and solving problems involving linear relationships in various dimensions.