Lecture Summary: Partial Derivatives

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Key Points

- **Definition of Partial Derivatives:**
  - For a scalar-valued function $f(x_1, x_2, \ldots, x_n)$ defined on a domain $D \subset \mathbb{R}^n$, the partial derivative with respect to x_i at a point $\vec{a} \in \mathbb{R}^n$ is:
  $$\frac{\partial f}{\partial x_i}(\vec{a}) = \lim_{h \to 0} \frac{f(\vec{a} + h\vec{e}_i) - f(\vec{a})}{h},$$
  where $\vec{e}_i$ is the ith standard basis vector.
  - Partial derivatives measure the rate of change of f with respect to one variable, keeping all other variables constant.
- **Interpretation:**
  - Partial derivatives generalize the concept of derivatives from single-variable calculus to multivariable functions.
  - They capture the slope of the function in the direction of the ith coordinate axis.
- **Examples of Partial Derivatives:**
  - Example 1: For $f(x, y) = x + y$:
  $$\frac{\partial f}{\partial x} = 1, \quad \frac{\partial f}{\partial y} = 1.$$
  - Example 2: For $f(x, y, z) = xy + yz + zx$:
  $$\frac{\partial f}{\partial x} = y + z, \quad \frac{\partial f}{\partial y} = x + z, \quad \frac{\partial f}{\partial z} = x + y.$$
  - Example 3: For $f(x, y) = \sin(xy)$:
  $$\frac{\partial f}{\partial x} = y\cos(xy), \quad \frac{\partial f}{\partial y} = x\cos(xy).$$
- **Computing Partial Derivatives:**
  - To compute $\frac{\partial f}{\partial x_i}$:
    1. Treat x_i as the only variable.
    2. Treat all other variables as constants.
    3. Differentiate f with respect to x_i using standard differentiation rules.
  - For well-behaved (smooth) functions, this process is straightforward. For piecewise or non-smooth functions, limits may be required.
- **Applications of Partial Derivatives:**
  - Analyze the behavior of multivariable functions, such as rates of change in physical systems.
  - Compute gradients and optimize multivariable functions.
  - Formulate equations in physics, engineering, and economics.

Simplified Explanation

What Are Partial Derivatives? Partial derivatives measure how a multivariable function changes when only one variable changes, keeping the others fixed.

How to Compute Them? Treat one variable as the only variable and differentiate as usual, treating all others as constants.

Example: For $f(x, y) = x^2 + 3y$:

$$\frac{\partial f}{\partial x} = 2x, \quad \frac{\partial f}{\partial y} = 3.$$

Conclusion

In this lecture, we:

- Defined partial derivatives as a natural extension of single-variable derivatives.
- Explored examples to compute partial derivatives.
- Highlighted their applications in analyzing multivariable systems.

Partial derivatives are foundational in multivariable calculus, enabling detailed exploration of functions with several variables.