Lecture Summary: Parameter Estimation - Concepts and Examples

Lecture: 9.2 - Introduction to Parameter Estimation

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Key Points

- **Introduction to Parameter Estimation:**
  - Parameter estimation is a procedure to determine unknown parameters in a statistical model using data samples.
  - Involves identifying a function (estimator) that derives the parameter from iid samples.
- **Examples of Parameter Estimation:**
  1. **Bernoulli Trials:**
     - Samples: $X_1, X_2, \ldots, X_n \sim \text{Bernoulli}(p)$.
     - Goal: Estimate the success probability p from observed data.
     - Example Data: $1, 0, 0, 1, 0, 1, 1, 1, 0, 0$ (10 samples).
     - Larger sample sizes (e.g., 500 samples) improve confidence in the estimate.
  2. **Radioactive Decay (Poisson Distribution):**
     - Model: Number of alpha particles emitted in a fixed time follows Poisson(λ).
     - λ depends on the radioactive substance (e.g., uranium, plutonium).
     - Data: Observations of particle counts over 2700 intervals.
     - Goal: Estimate λ from the observed counts.
  3. **Electronic Noise (Gaussian Distribution):**
     - Model: Noise in voltage or current follows $N(\mu, \sigma^2)$.
     - Data: Repeated iid measurements of voltage (e.g., $1.07, 0.91, 0.88$).
     - Goal: Estimate μ and σ^2 from the measurements.
- **Estimation Process:**
  - Estimation involves finding a function (estimator) $\hat{\theta}$ that maps samples to an estimate of the parameter θ.
  - Example for Bernoulli trials:
$$\hat{p} = \frac{X_1 + X_2 + \cdots + X_n}{n}.$$
- **Estimator Properties:**
  - The parameter θ is a constant, but the estimator $\hat{\theta}$ is a random variable dependent on the samples.
  - Different sample realizations produce different estimates.
  - Good estimators provide values close to θ with high probability.

- **Illustrative Example:**
  - Estimators for Bernoulli(p):
    1. $\hat{p}_1 = \frac{1}{2}$ (ignores samples).
    2. $\hat{p}_2 = \frac{X_1+X_2}{2}$ (uses only two samples).
    3. $\hat{p}_3 = \frac{\sum_{i=1}^{n} X_i}{n}$ (uses all samples).
  - $\hat{p}_3$ is more effective as it incorporates all available data.
- **Goals of Parameter Estimation:**
  - Design estimators whose distributions are concentrated around the true parameter.
  - Provide guarantees on the estimator's performance (e.g., bias, variance).

Simplified Explanation

Key Idea: Parameter estimation uses sample data to approximate unknown parameters in a statistical model.

Examples: 1. Estimating p in Bernoulli trials. 2. Finding λ in radioactive decay models. 3. Determining μ and σ^2 for electronic noise.

Key Insight: Good estimators make full use of sample data to provide reliable approximations of the true parameter.

Conclusion

In this lecture, we:

- Introduced parameter estimation as a core statistical procedure.
- Explored real-world examples involving Bernoulli, Poisson, and Gaussian distributions.
- Discussed the design and properties of estimators.

Understanding parameter estimation is foundational for statistical modeling and inference.