Lecture Summary: Marginal PMFs and Independence of Events

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Key Points

- **Marginal PMF Definition:**
  - For random variables X and Y with a joint PMF $f_{XY}(x, y)$:
  $$f_X(x) = \sum_y f_{XY}(x, y), \quad f_Y(y) = \sum_x f_{XY}(x, y).$$
  - Marginal PMFs represent the probabilities of individual random variables, extracted from the joint PMF.
- **Marginalization Process:**
  - Sum over rows of the joint PMF table to get $f_X(x)$.
  - Sum over columns of the joint PMF table to get $f_Y(y)$.
  - Marginal PMFs are unique for a given joint PMF.
- **Examples of Marginalization:**
  - **Coin Toss:**
    * Joint PMF table for two tosses has $P(X = x, Y = y) = 1/4$ for all x, y.
    * Marginal PMFs:
    $$f_X(x) = \sum_y f_{XY}(x, y) = 1/2, \quad f_Y(y) = \sum_x f_{XY}(x, y) = 1/2.$$
  - **Two-Digit Lottery:**
    * Random variables X (units digit) and Y (remainder modulo 4).
    * Joint PMF table can be marginalized to obtain $f_X(x)$ and $f_Y(y)$.
- **Non-Uniqueness of Joint PMFs:**
  - Different joint PMFs can produce the same marginal PMFs.
  - Example: A joint PMF table with uniform values can produce the same marginal PMFs as one with non-uniform entries.
- **Independence of Events:**
  - Random variables X and Y are independent if:
  $$P(X = x \text{ and } Y = y) = P(X = x) \cdot P(Y = y) \quad \forall x, y.$$
  - Independence is determined using the joint and marginal PMFs.
  - Example:

* Joint PMF: $f_{XY}(0,0) = 1/20$.
* Marginals: $P(X = 0) = 1/10$, $P(Y = 0) = 1/5$.
* Check:

$$P(X = 0 \text{ and } Y = 0) \neq P(X = 0) \cdot P(Y = 0).$$

Hence, X and Y are not independent.

Simplified Explanation

Marginal PMFs: Marginal PMFs give probabilities for individual random variables and are derived by summing rows or columns of a joint PMF.

Independence: Two random variables are independent if their joint probability equals the product of their individual probabilities.

Example: If $f_{XY}(0,0) \neq f_X(0) \cdot f_Y(0)$, then X and Y are not independent.

Conclusion

In this lecture, we:

- Defined marginal PMFs and explained how to compute them.
- Discussed the non-uniqueness of joint PMFs.
- Highlighted the criteria for independence using examples.

Marginal PMFs and independence are foundational tools for analyzing relationships between random variables.