Lecture Summary: Distribution of the Sum of Two Random Variables

Source: Lecture 2.8.pdf

Key Points

- **Overview:**
  - The focus is on deriving the probability mass function (PMF) of a function of two random variables, particularly their sum.
  - Process involves two key steps:
    1. Determine the range of possible values for $Z = X + Y$.
    2. Sum over the contours defined by $g(x, y) = Z$ in the joint PMF.
- **Steps to Derive the PMF of Z:**
  1. **Find the Range of Z:**
     - Identify all possible values Z can take based on the ranges of X and Y.
     - Example: If $X, Y \in \{1, 2, 3, 4, 5, 6\}$ (as in a dice roll), then $Z = X + Y$ can range from 2 to 12.
  2. **Sum Over Contours:**
     - For each possible value of Z, identify all pairs (x, y) such that $x + y = Z$.
     - Add the probabilities of these pairs using the joint PMF.
     - Visualization can aid this process, either graphically or by pattern recognition.
- **Example: Sum of Two Dice Rolls:**
  - **Step 1: Range:**
  $$Z = X + Y \in \{2, 3, \dots, 12\}.$$
  - **Step 2: Contours:**
    * $Z = 2$: $(1, 1)$.
    * $Z = 3$: $(1, 2), (2, 1)$.
    * $Z = 4$: $(1, 3), (2, 2), (3, 1)$.
    * Continue for all values up to $Z = 12$.
  - Compute PMF:
  $$P(Z = z) = \frac{\text{Number of pairs } (x, y) \text{ such that } x + y = z}{36}.$$
  - Example:
  $$P(Z = 2) = \frac{1}{36}, \quad P(Z = 3) = \frac{2}{36}, \quad P(Z = 4) = \frac{3}{36}.$$
  - Symmetry: $P(Z = z)$ increases to a peak at $Z = 7$ and decreases symmetrically.

- **Visualization:**
  - Graphical representation involves plotting x and y on a grid and identifying points where $x+y=Z$.
  - Contours corresponding to $x+y=Z$ are diagonal lines, each containing points that contribute to $P(Z=z)$.
- **Applications:**
  - This approach generalizes to sums of variables with other distributions or continuous cases.
  - The method emphasizes systematic counting and pattern recognition.

Simplified Explanation

Key Idea: The PMF of $Z=X+Y$ is derived by summing probabilities of all pairs (x,y) such that $x+y=Z$.

Example: For two dice: - $Z=2$: One pair $(1,1)$, so $P(Z=2)=\frac{1}{36}$. - $Z=3$: Two pairs $(1,2),(2,1)$, so $P(Z=3)=\frac{2}{36}$.

Visualization: Contours represent combinations of (x,y) with the same Z. The PMF is computed by counting points on each contour.

Conclusion

In this lecture, we:

- Discussed how to derive the PMF of the sum of two random variables.
- Used examples to illustrate the process of determining the range and summing over contours.
- Emphasized the role of visualization in simplifying the computation.

This method is foundational for understanding distributions of sums and other functions of random variables.