# Lecture Summary: Maximum of Two Random Variables

## Source: Lecture 2.9.pdf

## Key Points

- **Introduction to the Maximum Function:**
  - The maximum of two random variables, $Z = \max(X, Y)$, represents the largest value between $X$ and $Y$.
  - This lecture focuses on $X$ and $Y$ being independent and identically distributed (i.i.d.) uniform random variables over $\{1, 2, \ldots, 6\}$.
- **Steps to Derive the PMF of $Z$:**
  1. **Determine the Range of $Z$:**
     - The range of $Z$ is $\{1, 2, \ldots, 6\}$, as the maximum of two values between 1 and 6 also lies in this range.
  2. **Count the Occurrences for Each Value of $Z$:**
     - For $Z = 1$, the only possible pair is $(1, 1)$.
     - For $Z = 2$, possible pairs are $(2, 1)$, $(2, 2)$, and $(1, 2)$.
     - For $Z = 3$, possible pairs are $(3, 1)$, $(3, 2)$, $(3, 3)$, $(2, 3)$, and $(1, 3)$.
     - This pattern continues for higher values of $Z$.
- **General Formula for PMF of $Z$:**
  - For $Z = z$, the count of pairs $(x, y)$ satisfying $\max(x, y) = z$ is:
$$2z - 1.$$
  - The probability is then:
$$P(Z = z) = \frac{2z-1}{36}, \quad z \in \{1, 2, \ldots, 6\}.$$
- **Visualization of the PMF:**
  - The PMF shows an increasing trend, with probabilities rising as $z$ increases.
  - Example values:
$$P(Z = 1) = \frac{1}{36}, \quad P(Z = 2) = \frac{3}{36}, \quad P(Z = 3) = \frac{5}{36}, \quad \ldots, \quad P(Z = 6) = \frac{11}{36}.$$
- **Extensions:**
  - For general $X, Y \sim \text{Uniform}\{1, 2, \ldots, n\}$, the range of $Z = \max(X, Y)$ is $\{1, 2, \ldots, n\}$.
  - The count of pairs $(x, y)$ where $\max(x, y) = z$ is $2z - 1$.
  - PMF:
$$P(Z = z) = \frac{2z-1}{n^2}, \quad z \in \{1, 2, \ldots, n\}.$$

## Simplified Explanation

**Maximum Function:** The maximum of two random variables $X$ and $Y$ is their largest value. For i.i.d. uniform distributions: - Range of $Z = \{1, 2, \ldots, n\}$. - Probability $P(Z = z) = \frac{2z-1}{n^2}$.

**Example for $n = 6$:**

$$P(Z = 1) = \frac{1}{36},\ P(Z = 2) = \frac{3}{36},\ P(Z = 3) = \frac{5}{36}, \ldots,\ P(Z = 6) = \frac{11}{36}.$$

## Conclusion

In this lecture, we:

- Derived the PMF of the maximum function for two uniform random variables.
- Used a systematic approach to count pairs satisfying $\max(X, Y) = Z$.
- Generalized the results for uniform distributions over $\{1, 2, \ldots, n\}$.

Understanding the maximum function is essential for probabilistic analysis and forms the basis for studying other functions of random variables.