Lecture Summary: Conditioning with Multiple Discrete Random Variables

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Key Points

- **Relationship Between Marginals, Conditionals, and Joint PMFs:**
  - Marginalization reduces complexity by focusing on smaller subsets of variables.
  - Conditional PMFs bridge marginals and the joint PMF, enabling detailed analysis.
  - Key formula:

$$f_{X_1,X_2,\ldots,X_n}(t_1,t_2,\ldots,t_n) = f_{X_1}(t_1)\prod_{i=2}^{n} f_{X_i|X_1,\ldots,X_{i-1}}(t_i \mid t_1,\ldots,t_{i-1}).$$

- **Basic Conditioning:**
  - For two variables X_1 and X_2:

$$f_{X_1|X_2}(t_1 \mid t_2) = \frac{f_{X_1,X_2}(t_1,t_2)}{f_{X_2}(t_2)}.$$

  - This formula generalizes to multiple variables:

$$f_{X_1,X_2|X_3}(t_1,t_2 \mid t_3) = \frac{f_{X_1,X_2,X_3}(t_1,t_2,t_3)}{f_{X_3}(t_3)}.$$

- **Examples of Conditioning:**
  - **Example 1: Pairwise Conditioning:**
    * $X_1 \mid X_2 = t_2$: Conditional PMF is derived as:

$$f_{X_1|X_2}(t_1 \mid t_2) = \frac{f_{X_1,X_2}(t_1,t_2)}{f_{X_2}(t_2)}.$$

  - **Example 2: Conditional on Multiple Variables:**
    * For $X_1 \mid (X_2 = t_2, X_3 = t_3)$:

$$f_{X_1|X_2,X_3}(t_1 \mid t_2,t_3) = \frac{f_{X_1,X_2,X_3}(t_1,t_2,t_3)}{f_{X_2,X_3}(t_2,t_3)}.$$

  - **Example 3: Joint Conditional PMF:**

$$f_{X_1,X_3|X_2,X_4}(t_1,t_3 \mid t_2,t_4) = \frac{f_{X_1,X_2,X_3,X_4}(t_1,t_2,t_3,t_4)}{f_{X_2,X_4}(t_2,t_4)}.$$

- **Factoring the Joint PMF:**

– Any joint PMF can be factored iteratively:

$$f_{X_1,X_2,\ldots,X_n}(t_1,t_2,\ldots,t_n) = f_{X_n}(t_n)\prod_{i=1}^{n-1} f_{X_i|X_{i+1},\ldots,X_n}(t_i \mid t_{i+1},\ldots,t_n).$$

– This flexibility allows reordering variables, e.g., $f_{X_3|X_1,X_2}$ or $f_{X_1|X_2,X_3}$.

- **Applications:**
  – Conditioning simplifies computations in high-dimensional problems.
  – Factorization helps analyze dependencies and decompose complex distributions.
  – Practical examples include evaluating conditional probabilities in tabular data.

Simplified Explanation

Conditioning and Marginalization: Conditioning relates individual and joint distributions, while marginalization reduces variables.

Factoring: Break down a joint PMF into products of marginals and conditionals, simplifying analysis.

Example: For $f_{X_1,X_2,X_3}(t_1,t_2,t_3)$:

$$f_{X_1,X_2,X_3}(t_1,t_2,t_3) = f_{X_1}(t_1)\cdot f_{X_2|X_1}(t_2 \mid t_1)\cdot f_{X_3|X_1,X_2}(t_3 \mid t_1,t_2).$$

Conclusion

In this lecture, we:

- Defined conditional PMFs for multiple random variables.
- Demonstrated the utility of factoring joint PMFs into simpler components.
- Highlighted the flexibility of variable ordering in factorization.

These principles are foundational for probabilistic analysis in high-dimensional spaces.