Lecture Summary: Comparison of MME and MLE Across Examples

Lecture: 9.8 - Finding MME and ML Estimators

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Key Points

- **Objective:**
  - Compare Method of Moments Estimation (MME) and Maximum Likelihood Estimation (MLE) across various distributions.
  - Highlight similarities, differences, and challenges in their application.
- **Examples:**
  1. **Exponential Distribution:**
     - PDF:
     $$f_X(x;\lambda) = \lambda e^{-\lambda x}, \quad x > 0.$$
     - MME:
     $$\hat{\lambda}_{\text{MME}} = \frac{1}{\bar{X}}.$$
     - MLE:
     $$\hat{\lambda}_{\text{MLE}} = \frac{1}{\bar{X}}.$$
     - Observation: Both methods yield the same estimator due to the simplicity of the distribution.
  2. **Discrete Distribution with Values $\{1, 2, 3\}$:**
     - Probabilities: p_1, p_2, p_3 with $p_1 + p_2 + p_3 = 1$.
     - MME:
       * Two sample moment equations solve for p_1 and p_2, with $p_3 = 1 - p_1 - p_2$.
     - MLE:
     $$\hat{p}_i = \frac{\text{Count of } i \text{ in samples}}{n}, \quad i = 1, 2, 3.$$
     - Observation: MLE provides intuitive and simpler solutions compared to MME, which involves solving nonlinear equations.
  3. **Uniform Distribution $[0, \theta]$:**
     - MME:
     $$\hat{\theta}_{\text{MME}} = 2\bar{X}.$$
     - MLE:
     $$\hat{\theta}_{\text{MLE}} = \max(X_1, X_2, \ldots, X_n).$$
     - Observation: MME can produce unrealistic results (e.g., $\hat{\theta} < \max(X_i)$), whereas MLE aligns with the observed data.

4. **Gamma Distribution:**
   - PDF involves parameters α and β.
   - MME:
     * Closed-form solutions relate moments to parameters.
   - MLE:
     * Requires solving equations numerically, as closed-form expressions are unavailable.
   - Observation: MME is simpler to compute, while MLE requires computational tools.
5. **Binomial Distribution (N, p):**
   - MLE for p:
   $$\hat{p} = \frac{\text{Number of successes}}{n}.$$
   - MLE for N (if unknown):
     * Solving for N involves optimizing a likelihood function, which is complex.
   - Observation: When both N and p are unknown, MLE can become computationally intensive.

- **General Observations:**
  - MME is often computationally simpler but may lack intuitive alignment with data.
  - MLE provides estimators that maximize data likelihood, offering consistency and efficiency but can involve complex calculations.
  - In cases like uniform and discrete distributions, MLE often yields more intuitive results compared to MME.
  - For complex distributions (e.g., Gamma, Binomial with unknown N), numerical methods are required for MLE.

Simplified Explanation

Key Idea: MME equates sample moments to theoretical moments, while MLE maximizes the likelihood of observed data.

Examples: - For exponential distributions, both MME and MLE yield the same estimator. - For uniform and discrete distributions, MLE often provides simpler and more realistic estimators.

Why It Matters: Understanding these methods helps choose the appropriate estimation technique based on the distribution and computational resources.

Conclusion

In this lecture, we:

- Compared MME and MLE across a range of distributions.
- Highlighted cases where MME and MLE align or diverge.
- Discussed practical challenges in applying MLE to complex distributions.

Both MME and MLE have their strengths and weaknesses, and the choice depends on the specific problem and computational feasibility.