Lecture Summary: General and Continuous Random Variables

Source: Lecture 4.4.docx

Key Points

- **Discrete vs Continuous Random Variables:**
  - Discrete random variables are associated with a finite or countably infinite sample space.
  - Continuous random variables are defined over an interval of real numbers, simplifying modeling for large datasets.
- **Continuous Random Variables:**
  - Continuous random variables are modeled using a continuous cumulative distribution function (CDF).
  - A valid CDF satisfies:
    1. Non-decreasing behavior.
    2. Starts at 0 and ends at 1.
    3. Is right-continuous.
- **Probabilities Using Continuous CDFs:**
  - For any interval $a < X \leq b$:
$$P(a < X \leq b) = F_X(b) - F_X(a).$$
  - For a single value $X = c$, the probability:
$$P(X = c) = \begin{cases} F_X(c) - F_X(c^-), & \text{if there is a jump at } c, \\ 0, & \text{if } F_X(c) \text{ is continuous at } c. \end{cases}$$
- **Defining Continuous Random Variables:**
  - A random variable is continuous if its CDF is continuous at every point.
  - For continuous random variables:
$$P(X = c) = 0 \quad \forall c.$$
- **Examples of CDFs:**
  - Discrete random variable: CDF increases in jumps corresponding to probabilities of specific values.
  - Continuous random variable: CDF rises smoothly with no jumps.
  - Mixture: Combines discrete jumps and smooth increments.
- **Applications:**
  - Scenarios requiring continuous random variables:
    * Dartboard distance from the center.

    - Meteorite weights.
    - Heights or weights of humans.
    - Cricket bowling speeds.
    - Stock prices.
  - Continuous models are preferred when datasets are large and tracking every discrete value is impractical.

Simplified Explanation

Key Concept: Continuous Random Variables Continuous random variables are modeled using a smooth CDF, which simplifies probability calculations for large datasets or ranges.

Probabilities with CDFs: - Interval: $P(a < X \leq b) = F_X(b) - F_X(a)$. - Single value: $P(X = c) = 0$ if the CDF is continuous at c.

Examples: - Continuous: Smooth curves (e.g., bowling speeds). - Discrete: Stepwise jumps (e.g., die rolls).

Why Use Continuous Models? - Simplifies calculations for datasets with many values. - Models real-world phenomena efficiently.

Conclusion

In this lecture, we:

- Distinguished between discrete, continuous, and mixed random variables.
- Defined continuous random variables through their CDFs.
- Applied CDFs to compute probabilities for different events.
- Highlighted the importance of continuous models in practical scenarios.

Continuous random variables form a crucial foundation for advanced probability modeling, simplifying calculations and enhancing applicability.