Lecture Summary: Jointly Continuous Random Variables

Source: Lecture 5.3.docx

Key Points

- **Introduction:**
  - Jointly continuous random variables describe the relationships between two or more continuous variables.
  - These models are crucial for analyzing phenomena involving simultaneous variation of multiple variables, such as sepal and petal dimensions in the Iris dataset.
- **Joint Density Function:**
  - A joint density function $f_{X,Y}(x, y)$ satisfies:
    1. $f_{X,Y}(x, y) \geq 0$ for all (x, y).
    2. $\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f_{X,Y}(x, y)\, dx\, dy = 1$.
  - The probability of (X, Y) falling within a region A is:

$$P((X, Y) \in A) = \int_A f_{X,Y}(x, y)\, dx\, dy.$$

  - Support is the region where $f_{X,Y}(x, y) > 0$.
- **Uniform Joint Distribution Example:**
  - Uniform density in a unit square: $f_{X,Y}(x, y) = 1$ for $0 \leq x, y \leq 1$.
  - Probability of a subregion is proportional to its area.
  - Example:
    * For region A with $x, y \in [0.1, 0.5]$:

$$P((X, Y) \in A) = \int_{0.1}^{0.5} \int_{0.1}^{0.5} 1\, dx\, dy = 0.16.$$

- **Non-Uniform Joint Density Example:**
  - Example density: $f_{X,Y}(x, y) = x + y$ for $0 \leq x, y \leq 1$.
  - Validation:

$$\int_0^1 \int_0^1 (x + y)\, dx\, dy = 1.$$

  - Probability of region A: $x, y \in [0, 0.5]$:

$$P((X, Y) \in A) = \int_0^{0.5} \int_0^{0.5} (x + y)\, dx\, dy = 0.125.$$

- **Visualization:**

- 2D and 3D plots help visualize joint densities, showing support and density variations.
  - Example: Uniform density forms a flat "plate" over the unit square; non-uniform density creates slopes.

- **Applications:**
  - Useful for modeling correlated continuous phenomena, such as:
    * Sepal length and width in flowers.
    * Temperature and humidity.
  - Simplifies probability calculations for regions of interest.

Simplified Explanation

Key Idea: Jointly continuous random variables describe two or more variables with a shared density.
Uniform Example: For $f_{X,Y}(x, y) = 1$ over a unit square, probability is the area of the region.
Non-Uniform Example: For $f_{X,Y}(x, y) = x + y$, higher values occur near $(1, 1)$, lower near $(0, 0)$.
Applications: Modeling real-world phenomena with interdependent variables.

Conclusion

In this lecture, we:

- Defined joint densities and their properties.
- Explored uniform and non-uniform examples.
- Highlighted applications in data science and probability.

Jointly continuous random variables provide powerful tools for modeling and analyzing multidimensional continuous data.