Lecture Summary: Multiple Random Variables and Joint PMFs

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Key Points

- **Transition to Multiple Random Variables:**
  - Random variables simplify probability spaces by mapping complex outcomes to numerical values.
  - Multiple random variables often arise in experiments with interconnected outcomes, requiring tools to analyze their relationships.
- **Examples of Multiple Random Variables:**
  - **Coin Toss Experiment:**
    - Toss a fair coin three times, defining random variables X_1, X_2, and X_3 to indicate whether the first, second, and third tosses are heads.
    - These are indicator random variables, taking values 1 for heads and 0 for tails.
    - Events based on X_1 (e.g., $X_1 = 1$) are independent of events based on X_2 or X_3.
  - **Two-Digit Lottery:**
    - Random variable X: Units digit of the number.
    - Random variable Y: Remainder when the number is divided by 4.
    - X and Y are uniformly distributed over their respective ranges, but they are not independent.
    - Example: If $X = 1$, $Y = 0$ is impossible because numbers ending in 1 are not multiples of 4.
  - **IPL Powerplay Data:**
    - Random variable X: Total runs scored in an over.
    - Random variable Y: Number of wickets lost in the over.
    - A relationship exists: Overs with more wickets typically have fewer runs.
- **Modeling Multiple Random Variables:**
  - Relationships between random variables help model complex experiments (e.g., IPL data).
  - Dependencies are modeled probabilistically rather than deterministically.
- **Joint PMFs (Probability Mass Functions):**
  - For discrete random variables X and Y defined on the same probability space:

$$f_{XY}(t_1, t_2) = P(X = t_1 \text{ and } Y = t_2).$$

  - Properties:
    - $f_{XY}(t_1, t_2) \geq 0$ for all t_1, t_2.
    - $\sum_{t_1, t_2} f_{XY}(t_1, t_2) = 1$.
- **Examples of Joint PMFs:**

- **Coin Toss:**
  - X_1: First toss indicator, X_2: Second toss indicator.
  - Joint PMF: $f_{X_1X_2}(t_1, t_2) = P(X_1 = t_1 \text{ and } X_2 = t_2) = (1/2) \cdot (1/2) = 1/4$ for independent tosses.
- **Two-Digit Lottery:**
  - X: Units digit, Y: Remainder modulo 4.
  - Example: $f_{XY}(0, 0) = P(X = 0 \text{ and } Y = 0)$ includes numbers like 00, 20, 40, ..., totaling 5 outcomes: $5/100 = 1/20$.

- **Properties of Joint PMFs:**
  - All probabilities in the PMF lie between 0 and 1.
  - The sum of all probabilities in the PMF equals 1, covering all possible outcomes.
  - PMFs can be represented as tables or matrices for easier interpretation and computation.

Simplified Explanation

Multiple Random Variables: These arise when experiments have interconnected outcomes. Examples include: - Coin tosses (independent variables). - Two-digit lottery numbers (dependent variables). - IPL data (complex dependencies).

Joint PMFs: Functions that assign probabilities to pairs of values taken by two random variables, satisfying:

$$f_{XY}(t_1, t_2) \geq 0, \quad \sum f_{XY}(t_1, t_2) = 1.$$

Conclusion

In this lecture, we:

- Explored multiple random variables and their applications.
- Defined joint PMFs and derived examples.
- Discussed how dependencies between variables help model real-world scenarios.

Joint PMFs are crucial for analyzing and modeling relationships between random variables in complex experiments.