# Lecture Summary: Functions of Two Random Variables (Sum, Max, and Min)

## Source: Lec 2.6.pdf

## Key Points

- **Introduction to Functions of Two Random Variables:**
  - Expands on functions of one random variable to consider combinations of two variables.
  - Common functions include the sum, maximum, and minimum, which often arise in data analysis.
- **Table Method for Small-Sized Examples:**
  - For discrete random variables with few values, the table method remains effective.
  - Example: $X, Y \sim$ iid Uniform$\{0, 1\}$, with $Z = X + Y$:
    - Joint PMF table:

| $(X, Y)$ | $Z$ | $P(X, Y)$ |
|---|---|---|
| $(0, 0)$ | 0 | $\frac{1}{4}$ |
| $(0, 1)$ | 1 | $\frac{1}{4}$ |
| $(1, 0)$ | 1 | $\frac{1}{4}$ |
| $(1, 1)$ | 2 | $\frac{1}{4}$ |

    - $P(Z = 0) = \frac{1}{4}$, $P(Z = 1) = \frac{1}{2}$, $P(Z = 2) = \frac{1}{4}$.
  - The method involves identifying repetitions and summing probabilities for each unique value of $Z$.
- **Example for Maximum Function:**
  - Define $Z = \max(X, Y)$ with different distributions for $X$ and $Y$.
  - Tabular computation includes joint PMFs and evaluations of $\max(X, Y)$.
  - Example:

| $(X, Y)$ | $Z$ | $P(X, Y)$ |
|---|---|---|
| $(0, 0)$ | 0 | $\frac{1}{2}$ |
| $(0, 1)$ | 1 | $\frac{1}{4}$ |
| $(1, 0)$ | 1 | $\frac{1}{8}$ |
| $(1, 1)$ | 1 | $\frac{1}{16}$ |
| $(0, 2)$ | 2 | $\frac{1}{16}$ |

  - Sum probabilities for repetitions to find:

$$P(Z = 0) = \frac{1}{2}, \quad P(Z = 1) = \frac{11}{32}, \quad P(Z = 2) = \frac{5}{32}.$$

- **Challenges for Larger Examples:**
  - When $X$ and $Y$ have many possible values, the table method becomes cumbersome.

- Example: Two dice rolls produce 36 combinations, manageable but error-prone.
- Larger scales, such as $X, Y \in \{1, 2, \ldots, 100\}$, yield $10^4$ combinations, making tabular methods impractical.

- **Next Steps for Large Scenarios:**
  - Transition to visualization and analytical techniques.
  - Understanding how functions map $(X, Y)$ to specific outcomes becomes essential for simplification.

## Simplified Explanation

**Functions of Two Variables:** Sum, max, and min are common operations. Small datasets can be managed with tables; larger ones require alternative methods.

**Example:** For $X, Y \sim \text{Uniform}\{0, 1\}$ and $Z = X + Y$: - $P(Z = 0) = \frac{1}{4},\ P(Z = 1) = \frac{1}{2},\ P(Z = 2) = \frac{1}{4}$.

**Challenges:** Tables become impractical for large datasets, necessitating smarter techniques.

## Conclusion

In this lecture, we:

- Applied the table method to functions of two random variables.
- Explored examples for sum and max functions.
- Discussed limitations of the table method for larger datasets.

Future discussions will focus on analytical and visualization approaches for handling large-scale random variable functions.