Lecture Summary: Examples on Functions of a Single Random Variable

Source: Lec 2.5.pdf

Key Points

- **Applying Functions to Discrete Random Variables:**
  - The function maps values of a random variable X to new values, creating a transformed variable Y.
  - A table-based method simplifies this transformation, especially for discrete variables.
- **Example 1: Uniform Random Variable on $\{-5, -4, \ldots, 5\}$:**
  - X is uniformly distributed over $\{-5, -4, \ldots, 5\}$ with $P(X = x) = \frac{1}{11}$.
  - Function: $Y = f(X)$, where:

$$f(X) = \begin{cases} 0, & X \leq 0, \\ X, & X > 0. \end{cases}$$

  - Table for Y:

Y	$P(Y = y)$
0	$\frac{6}{11}$
1	$\frac{1}{11}$
2	$\frac{1}{11}$
3	$\frac{1}{11}$
4	$\frac{1}{11}$
5	$\frac{1}{11}$

  - Y consolidates values of $X \leq 0$ into a single outcome ($Y = 0$).
- **Example 2: Large Uniform Range on $\{-500, -499, \ldots, 500\}$:**
  - X is uniformly distributed with $P(X = x) = \frac{1}{1001}$.
  - Function: $Y = \max(X, 5)$:

$$f(X) = \begin{cases} 5, & X \leq 5, \\ X, & X > 5. \end{cases}$$

  - Pattern in Y:
    * $Y = 5$ occurs for 506 values of X.
    * $Y = x$ for $x \in \{6, 7, \ldots, 500\}$.
  - Table for Y:

Y	$P(Y = y)$
5	$\frac{506}{1001}$
6	$\frac{1}{1001}$
7	$\frac{1}{1001}$
$\vdots$	$\vdots$
500	$\frac{1}{1001}$

- **Key Observations:**
  - Using a table to represent transformations helps manage even large datasets.
  - Patterns in Y can significantly reduce the effort needed to compute PMFs.

Simplified Explanation

Transformations of Random Variables: Functions like $f(X)$ consolidate or shift probabilities. Tables simplify deriving new PMFs, especially when patterns exist.

Examples: - $X \sim \text{Uniform}(-5, 5)$: $f(X)$ maps $X \leq 0$ to $Y = 0$. - $X \sim \text{Uniform}(-500, 500)$: $f(X) = \max(X, 5)$ creates a concentrated probability at $Y = 5$.

Conclusion

In this lecture, we:

- Explored how functions transform random variables.
- Highlighted the utility of tables for summarizing transformations.
- Showed how patterns simplify PMF computations for large datasets.

Understanding transformations and patterns enables efficient computation and visualization of transformed distributions.