Lecture Summary: Conditional Distributions and Conditional PMFs

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Key Points

- **Introduction to Conditional PMFs:**
  - The conditional PMF of a random variable X given an event A is defined as:

$$P(X = t \mid A) = \frac{P(X = t \cap A)}{P(A)}.$$

  - Conditioning can change the range of the random variable. For example, values of X outside A may no longer be possible.
- **Extending to Two Random Variables:**
  - For two random variables X and Y with joint PMF $f_{XY}(t_1, t_2)$:

$$f_{Y|X}(t_2 \mid t_1) = \frac{f_{XY}(t_1, t_2)}{f_X(t_1)}.$$

  - The numerator is the joint PMF f_{XY}, and the denominator is the marginal PMF f_X.
- **Properties of Conditional PMFs:**
  - The conditional PMF is a valid PMF, meaning:

$$\sum_{t_2} f_{Y|X}(t_2 \mid t_1) = 1.$$

  - Conditional PMFs allow calculation of joint PMFs using:

$$f_{XY}(t_1, t_2) = f_{Y|X}(t_2 \mid t_1) \cdot f_X(t_1).$$

- **Examples:**
  - **Example 1: Joint PMF of X and Y:**
    * Joint PMF is given as a table.
    * Marginal PMFs f_X and f_Y are computed by summing rows and columns, respectively.
    * Conditional PMF $f_{Y|X}$ is computed by dividing joint probabilities by the marginal f_X.
  - **Example 2: Lottery Numbers:**
    * Joint PMF of X (units digit) and Y (remainder modulo 4).
    * Conditional PMF $f_{Y|X}$ reflects how Y depends on X.
  - **Example 3: General Calculation:**

    * From a joint PMF table, compute $f_{Y|X}(t_2 \mid t_1)$ for specific t_1 by normalizing the corresponding row or column.

- **Useful Identities:**
  - The relationship between joint, marginal, and conditional PMFs is:

$$f_{XY}(t_1, t_2) = f_{Y|X}(t_2 \mid t_1) \cdot f_X(t_1) = f_{X|Y}(t_1 \mid t_2) \cdot f_Y(t_2).$$

  - Summing conditional PMFs over the range of the conditioned variable gives 1:

$$\sum_{t_2} f_{Y|X}(t_2 \mid t_1) = 1.$$

Simplified Explanation

Conditional PMFs: Probability distributions of one random variable given specific information about another variable or event.

Example: If $f_{XY}(0,1) = 1/8$ and $f_X(0) = 3/8$, then:

$$f_{Y|X}(1 \mid 0) = \frac{f_{XY}(0,1)}{f_X(0)} = \frac{1/8}{3/8} = \frac{1}{3}.$$

Key Formula: The joint PMF can be reconstructed from the conditional and marginal PMFs:

$$f_{XY}(t_1, t_2) = f_{Y|X}(t_2 \mid t_1) \cdot f_X(t_1).$$

Conclusion

In this lecture, we:

- Defined conditional PMFs for single and multiple random variables.
- Explored examples to compute conditional probabilities.
- Highlighted the relationships between joint, marginal, and conditional PMFs.

Conditional PMFs are crucial for understanding dependencies between random variables and solving complex probabilistic problems.