Lecture Summary: Visualizing Functions of One Random Variable and One-to-One Functions

Source: Lecture 2.3.pdf

Key Points

- **Introduction to Functions of Random Variables:**
  - Functions of random variables frequently appear in statistical modeling.
  - Understanding how a function impacts a random variable's distribution is crucial for effective modeling.
- **Visualization of PMFs:**
  - Stem plots are a common method for visualizing probability mass functions (PMFs).
  - Example: For a uniform random variable X on $\{0, 1, \ldots, 10\}$, the PMF is:
$$f_X(x) = \frac{1}{11}, \quad x \in \{0, 1, \ldots, 10\}.$$
  - Stem plots can also represent distributions like Binomial$(10, 0.5)$, highlighting patterns such as peak probabilities near the mean.
- **One-to-One Functions:**
  - A one-to-one function maps each input value to a unique output value.
  - For such functions, probabilities remain unchanged, but the values are relabeled.
  - Example 1: $Y = X - 5$ for $X \sim$ Uniform$(\{0, 1, \ldots, 10\})$:
$$Y \in \{-5, -4, \ldots, 5\}, \quad P(Y = y) = \frac{1}{11}.$$
  - Example 2: $Y = 2X$ for $X \sim$ Binomial$(10, 0.5)$:
$$f_Y(y) = f_X(x), \quad y = 2x.$$
- **Impact of One-to-One Transformations:**
  - Linear transformations such as $Y = aX + b$ result in translated or scaled PMFs.
  - Stem plots illustrate how transformations affect the spread and alignment of distributions.
- **Monotonicity and Identification:**
  - Monotonic functions (increasing or decreasing) are always one-to-one.
  - A quick test for one-to-one behavior: Check that each horizontal line intersects the function plot at most once.
- **Applications of the Table Method:**
  - Create a table of values showing X, $Y = f(X)$, and their probabilities.
  - This method simplifies calculations for transformed distributions, especially for small datasets.

Simplified Explanation

What Happens with One-to-One Functions? These functions simply relabel the values of a random variable without altering probabilities. For example: - $X = 0 \to Y = -5$ under $Y = X - 5$. - $P(X = x) = P(Y = y)$, with $y = f(x)$.

Visualizing Transformations: Stem plots help visualize how the PMF shifts or stretches under transformations.

Conclusion

In this lecture, we:

- Explored one-to-one functions and their effects on random variables.
- Highlighted the use of stem plots for visualizing PMFs.
- Discussed methods for identifying and working with one-to-one transformations.

Understanding one-to-one functions is foundational for analyzing and modeling transformed random variables.