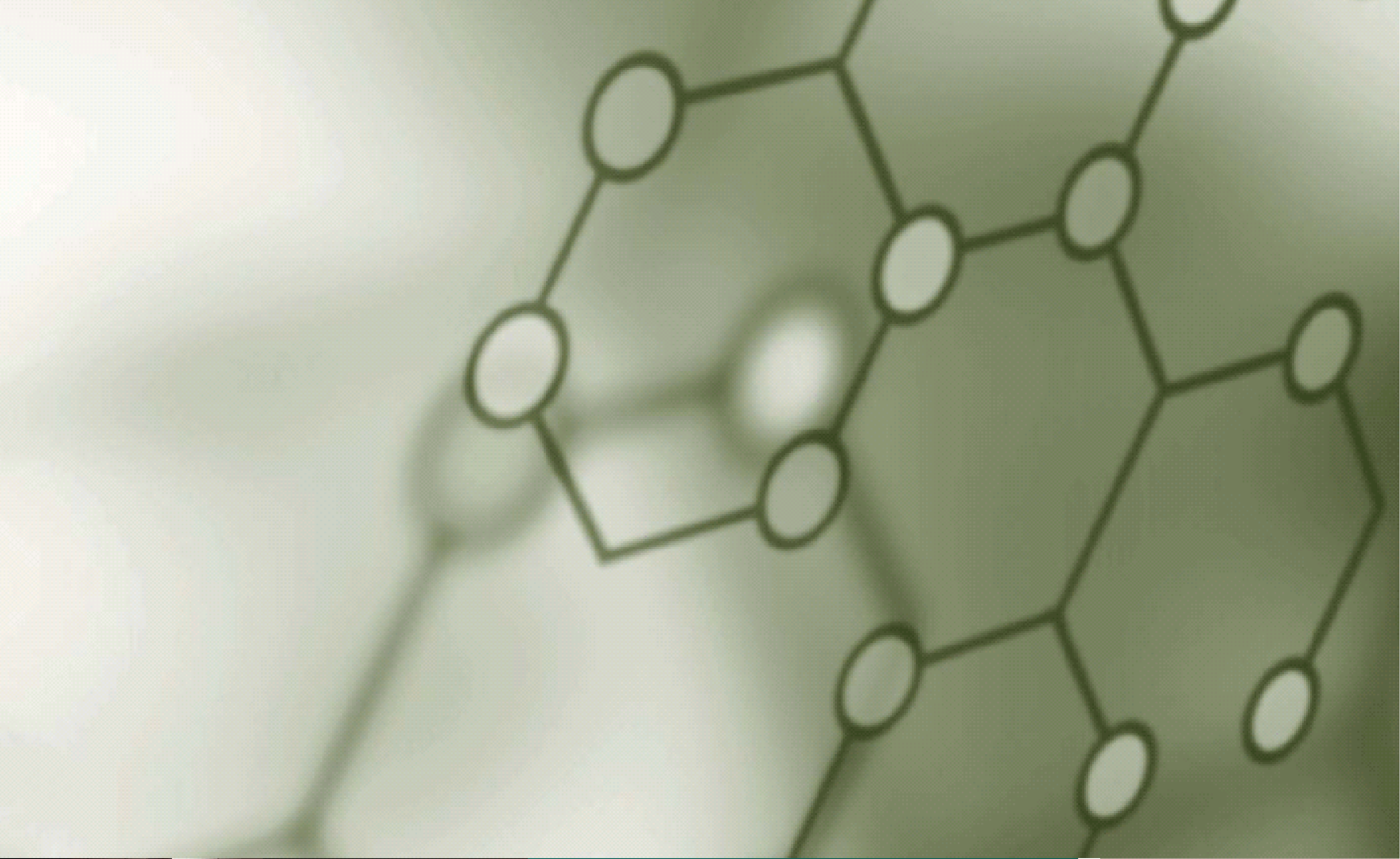

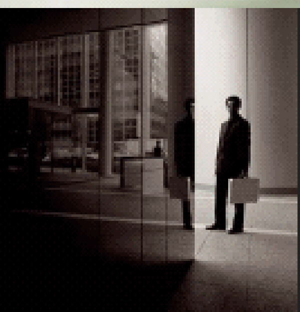

SUNGARD ADAPTIV

Risk Management
and Operations
Solutions

Adaptiv

Finance Guide:
Part Two - Equities

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Last updated 08 March 2010.

Contents

Issue parameters for equities that affect calculations

Settlement rule or account periods

This affects forward price and discounting calculations – the action of currency interest rates and stock repo rates is applied from the spot settlement date to the forward settlement date. For some account period types, the settlement for the spot and the forward will be the same date – This means that the forward price will be equal to the spot price unless any dividends become ex-dividend in the intervening period.

Contract size

Contract size acts as a value multiplier.

Price Factor

Designed to allow the user to specify the use of different units of currency for the issue price – e.g. 0.01 on a GBP stock means price in pence not pounds, 0.01 on a USD stock means price in cents not dollars. Prices and dividends on the market parameters screen are interpreted as being quoted in this way.

Basket components

A basket or index where the components are defined will affect calculations in a number of ways. Firstly it allows the user the possibility of calculating aggregate market parameters instead of entering entirely new ones for the basket. Secondly, it is used to determine the splitting of sensitivity values by underlying components.

Basket divisor

This is a way of adjusting the “quantity” values of the components in the basket without editing all of them in turn. It will affect the calculated basket price.

Modelling market parameters for equities

Currency yield curves and exchange rates

Panorama Equity takes all its currency yield curves from Panorama. Each issue potentially uses a different Panorama curve, and issues that have underlying issues – e.g. options on equities, warrants, futures etc. potentially use different curves to their underlying. An issue is attached to a Panorama "market" which can be used as an indirection to an actual curve.
N.B. When a deal is entered against an issue, it sets the market of the deal according to the issue, but this will not change if the market in the issue is then changed.

Pricing rules

Panorama supports multiple sets of market data, labelled under "pricing rules". Panorama Equity supports this also, with the potential for many sets of equity market parameters stored under the different Panorama pricing rule names. A Panorama pricing rule specifies an equity market data "Parameter set".

Spot Price

The spot price is either entered directly or as implied from a near future and a spread on the market parameters screen. It may also be calculated from components in the case of a basket or index.

Dividends, dividend growth and tax

Dividends are modelled as discrete payments. The user may choose to enter as many specific dividend payments as he wants, but can also enter projection parameters for dividends beyond those entered. The dividends definition is made up of three spreadsheet type controls:

Dividends – this is where you input the explicit dividends and view calculated projections

Announce Date	=	The date on which the amount of the dividend will be made public. This is not currently used in calculations.
Ex-Div	=	The ex-dividend date for this dividend. This is the trade date from which a purchaser of the equity is not entitled to the dividend.
Payment	=	The date on which this dividend will be paid.
Amount	=	The amount of the dividend. The currency is assumed to be that of the Equity Issue.
Currency	=	The currency of the dividend.
T Return	=	Flag for basket or index calculations to indicate that the dividend is to be re-invested in the value of the basket or index – e.g. DAX index.
Tax Credit	=	The date on which the tax credit will be received.
TC Amount	=	The amount of the tax credit. This is usually a calculated field and cannot be changed directly – except in the case of a basket where the user has chosen to edit the dividends manually.

Tax credit – this is where you specify tax credit rates

Term	=	The first date on which the associated tax credit rate applies. The last date that the rate applies is the Term date of the next row if one exists.
Tax credit	=	The tax credit rate for the associated term. Any dividend with a payment date in the defined term will have a tax credit calculated according to this rate.

Dividend growth – this is where you specify projection parameters for calculating beyond the explicit dividends

Term	=	The first date on which the associated dividend growth rate applies. The last date that the rate applies is the Term date of the next row if one exists.

Dvd Growth	=	The last full year of dividends are expected to repeat on the same dates in following years. Each repeat, a growth rate is picked from the term containing the payment date to create the new projected dividend.

Repo Rates

Repo rates distort the theoretical forward price arbitrage. They are entered as annualised Act'l/365 zero coupon yields (i.e. there is no sophisticated bootstrapping yield curve). You can enter as few or as many repo rates for different terms as you wish, and repo rates may be implied by setting the basis field (basis = deviation from theoretical forward), or the forward price directly.
The "Repo Rates" tab in the market parameters screen is also a useful place to see the components of the forward price calculation in action and to check the numbers.

Entering repo rates

Term	=	Either enter a specific date ("21/10/1999") or a tenor type input ("6M") or a contract code ("H99"). Any explicit dates represent the forward trade date, not the forward settlement date.
Days	=	The number of days to the forward date. (Non-editable)
Settle	=	The settlement date for the forward
Int. Rate	=	The discount factor from Panorama quoted as an annual act'l/365 zero coupon yield. (Non-editable)
Dvd Yld	=	The dividends expressed as a yield. (Non-editable)
PV(Dvds)	=	The present value of the dividends earned by holding the stock to the forward date. (Non-editable)
Repo Rate	=	The repo rate quoted as an annualized zero coupon act'l/365 yield. The repo rate is assumed to apply from the spot settlement date to the forward settlement date.
Basis	=	The difference between the theoretical forward price and the actual forward price created by the repo rate. Can be calculated from the repo rate or used to imply it.
Forward	=	The actual forward price calculated. Can be calculated from the repo rate or used to imply it.

Volatilities

Sometimes, e.g. for quanto options, FX volatilities are required. These are taken directly from Panorama. Equity volatilities are defined on the market parameters page, except in the case of convertible bonds and warrants where the user has the opportunity to over-ride the underlying equity volatility structure with a single value volatility for the issue. The volatility grid is two dimensional, with the potential for using a double x-axis – i.e. individual rows within the volatility grid can be set to use one or other of the x axes.

Definition of the x axes – the volatility smile

Strike Centre	=	The central value of the strikes on the strike axis.
+/- Step Size (strike)	=	The step size in the strikes on the strike axis.
Number	=	The number of points on the x axes.
Float Centre	=	The central value of the floating x axis. 0 indicates at-the-money spot (strike = spot), +10 means strike = spot + 10, +10% means strike = spot * 110% (see Float Type)
+/- Step Size (Float)	=	The step size in the floating axis values.

Float Type	=	Either PERCENT or SPREAD PERCENT = axis values taken to mean percentage spreads over spot. SPREAD = axis values taken to mean absolute spreads over spot.

The volatility grid

Term	=	Either enter a specific date ("21/10/1999") or a tenor type input ("6M") or a contract code ("H99").
Date	=	Calculated from the Term field (non-editable)
Days	=	The number of days from today to Date.
Float Smile	=	Switch to determine which x axis to use. "No" means to use the strike axis – this means volatility remains constant for a given fixed strike option as spot moves. "Yes" means use the floating axis values. This can be set differently for each row in the grid.
Values	=	These can be filled as direct inputs or specified to be interpolated from neighbours in a horizontal line.

Correlations (quanto options)

Correlations between stock prices and exchange rates are required for pricing quanto options. On a market parameters page for a given underlying, you can add correlation figures against a range of currencies. These should be interpreted as the correlation as it is to be used in the quanto calculation - This resolves the issue of whether to input a positive or negative correlation here. Positive correlations tend to increase the value of quanto options and negative ones decrease them.

Beta

Beta is used in the spot / vol matrix tab. It represents the percentage change in the stock price given a percentage point change in the underlying "benchmark" of that stock.

Basket market parameters

In principle there is no need for independent basket market parameters at all. All could be generated from the market parameters of the components. In some cases this is simple – such as for the spot price, but calculating a basket volatility structure from the component volatility structures and correlations gets very complicated. At the other end of the scale, a basket could be treated in much the same way as a normal equity, and have its own completely independent set of parameters with no connection to the components. Panorama supports various configurations between these two extremes. Use the "extra" tab in the market parameters to decide which basket parameters you wish to use manual values for and which you wish to be calculated from the components. The parameters that can be switched to be calculated automatically, are shown in the table below.

Parameter	*Calculated*	*Explanation*
Spot price	Y	The spot price can easily be calculated from the components if desired.
Dividends	Y	The dividends can easily be calculated from the components if desired. N.B. If automatic dividends are used, the tax rate and growth rates on the market parameters screen become irrelevant. If "manual" dividends are used, then the tax rate only applies to projections unlike single stocks where the tax rates apply to all dividends.
Forward prices	Partial	Calculating the forward of the basket from the forwards of the components is too expensive in performance terms. The forward price is calculated from the spot, dividends and repo rates. If all three of these are set to automatic, then this corresponds to almost perfect theoretical forward price, except for the slight approximation between repo dates (which can be addressed by adding repo points)
Repo rates	Partial	If the user specifies automatic repo rates, then the system will calculate perfect theoretical repo rates for *the repo dates entered only*. Other repo rates will be interpolated from these values rather than calculated from scratch. This very small inaccuracy can be lessened or removed by adding more repo date points.

Volatilities	Y	The volatility can be calculated from the component volatilities, using correlations from the VaR statistics set. This is not recursive, so for a basket of two baskets, the statistics set must contain the correlations between the two sub-baskets, and not just the correlations of their component stocks.
Beta	Partial	Exhibits the same behaviour as the quanto currency correlations, i.e. it is obtained from the VaR statistics set if one has been specified, or is entered manually otherwise. Currently it is not calculated independently from the basket component correlations with the index.

Equity forward price calculations

Arbitrage equation

$$F = \left(S \cdot df_0 - pvd^* - pvtc^*\right) \times \frac{rdf_F}{df_F}$$

Where:

F	=	Forward price of equity on the forward date.
S	=	Spot price of equity
pvd_i	=	Present value of dividend payment, i, that is earned by holding stock from today to the forward date
$pvtc_i$	=	Present value of tax credit payment, i, that is earned by holding stock from today to the forward date
rdf_{X_i}	=	Discounting effect due to the repo rate at the i[th] ex-dividend date
$pvd = \sum_{i=1}^{N} pvd_i$	=	Present value of dividend payments that are earned by holding stock from today to the forward date
$pvtc = \sum_{i=1}^{N} pvtc_i$	=	Present value of tax credit payments that are earned by holding stock from today to the forward date
$pvd^* = \sum_{i=1}^{N} \frac{pvd_i}{rdf_{X_i}}$	=	Present value of dividend payments that are earned by holding stock from today to the forward date adjusted to incorporate repo factors at ex-dividend dates. This ensures that drop in the forward price at the ex-dividend date is exactly the sum of discounted payment and tax credit amounts.
$pvtc^* = \sum_{i=1}^{N} \frac{pvtc_i}{rdf_{X_i}}$	=	Present value of tax credit payments that are earned by holding stock from today to the forward date adjusted to incorporate repo factors at ex-dividend dates. This ensures that drop in the forward price at the ex-dividend date is exactly the sum of discounted payment and tax credit amounts.
rdf_F	=	Discounting effect due to the repo rate at the forward settlement date.
df_0	=	Currency discount factor for settlement date of spot deal, calculated from today and knowledge of the settlement rule or accounting periods for the issue.
df_F	=	Currency discount factor for settlement date of forward deal, calculated from the forward settlement date and knowledge of the settlement rule or accounting periods for the issue.

In terms of N individual dividends between today and the forward date, where for an absolute i[th] dividend:

$d_i = \textit{the absolute amount of the dividend}$

$D_i = 0$,

while if the i[th] dividend is proportional:

$d_i = 0$

$D_i = \textit{the proportion of the forward price on the ex-dividend date that will serve as the dividend},$

the forward price is given by

$$F = \left[S \cdot df_0 \cdot g_0 - \sum_{i=1}^{N} \frac{g_i \cdot d_i}{rdf_{X_i}} \left(df_{P_i} + tc_i \cdot df_{T_i} \right) \right] \times \frac{rdf_F}{df_F}$$

where

$$g_i = \prod_{j=i+1}^{N} \left(1 - \frac{D_j}{df_{X_j}} \left(df_{P_j} + tc_j \cdot df_{T_j} \right) \right)$$

tc_i	=	Tax credit percentage at the i[th] dividend date.
df_{P_i}	=	Currency discount factor for payment date of the i[th] dividend.
df_{T_i}	=	Currency discount factor for tax credit date of the i[th] dividend.

Assumption

It is assumed that the absolute amount of proportional dividends is set on the ex-div-date.

Calculation of Rdf

A repo *rate* is interpolated from the repo rate structure store with the market parameter. The interpolation date is the forward trade date (not settlement) yet the repo is taken to apply from the spot settlement date to the forward settlement date. A very simple discount calculation is used.

$$rdf = (1 + R)^{-T}$$

Where:

R	=	Repo rate interpolated from the market parameters repo rates, for the forward trade date.
T	=	Time in years from the spot settlement date to the forward settlement date.

Sensitivities

$$\frac{\partial F}{\partial S} = df_0 \times g_0 \times \frac{rdf_F}{df_F}$$

The sensitivity to a % shift in all dividends is non-linear when proportional dividends are used

$$\frac{\partial F}{\partial D} = \frac{1}{100} \times \left(S \cdot df_0 \cdot h_0 - \sum_{i=1}^{N} (h_i + g_i) \times \frac{d_i}{rdf_{X_i}} \times \left(df_{P_i} + tc_i \cdot df_{T_i} \right) \right) \times \frac{rdf_F}{df_F}$$

where $h_i = -g_i \sum_{k=i+1}^{N} \dfrac{\dfrac{D_k}{df_{X_k}} (df_{P_k} + tc_k \cdot df_{T_k})}{\left(1 - \dfrac{D_k}{df_{X_k}} \left(df_{P_k} + tc_k \cdot df_{T_k} \right) \right)}$

(D is a %age shift in all dividends)

Interest rate sensitivity of the forward price

The forward price will be sensitive to the discount factor at the forward settlement date, but also to discount factors on the dividend payment dates and the spot settlement date. To get a single figure, we can express the sensitivity of the forward price to a parallel shift of some sort in interest rates – We have chosen the annualised Act'l/365 yield measurement of interest rates to do this:

$$df = (1+Y)^{-T}$$

Where:

Y	=	Annualised yield measure of interest rates.
T	=	Time in years from the spot settlement date to the forward settlement date.

For each discount factor used in the forward price calculation the sensitivity of the forward price to that discount factor can be expressed as a sensitivity to a parallel shift in annualised yields as follows:

$$\frac{\partial F}{\partial Y} = \sum_i \left(\frac{\partial F}{\partial df_i} \times -T_i \times df_i^{\,1+\frac{1}{T_i}} \right)$$

These are summed across all discount factor sensitivities to give the total yield sensitivity.

$$\frac{\partial F}{\partial Y} = \frac{\partial F}{\partial df} \times \frac{\partial df}{\partial Y} = \frac{\partial F}{\partial df} \times -T \times (1+Y)^{-T-1} = \frac{\partial F}{\partial df} \times -T \times df^{\,1+\frac{1}{T}}$$

In calculating the interest rate sensitivities to the forward price it is assumed that: $\frac{df_{P_i}}{df_{X_i}} \approx 1$ and $\frac{df_{T_j}}{df_{X_i}} \approx 1$.

This approximation is only made in the sensitivity calculation and not when calculating the forward price. Consequently there are sensitivities to the spot discount factor, df_0, the discount factor for the settlement date of forward deal, df_F and the discount factors at the dividend payment, df_{P_i}, and tax credit dates, df_{T_i}.

For a proportional dividend, i,

$$\frac{\partial F}{\partial (df_{P_i})} = 0 \text{ and } \frac{\partial F}{\partial (df_{T_i})} = 0.$$

While for an absolute dividend, i

$$\frac{\partial F}{\partial (df_{P_i})} = -\frac{g_i \cdot d_i \cdot rdf_F}{rdf_{X_i} \cdot df_F} \text{ and } \frac{\partial F}{\partial (df_{T_i})} = -\frac{g_i \cdot d_i \cdot tc_i \cdot rdf_F}{rdf_{X_i} \cdot df_F}.$$

Repo rate sensitivity of the forward price

The forward price will not only be sensitive to the repo rate discount factor at the forward settlement date, but also to repo factors on the ex-dividend dates. As with interest rates, to get a single figure, we can express the sensitivity of the forward price to a parallel shift of some sort in repo rates.

For each repo factor used in the forward price calculation the sensitivity of the forward price to that repo factor can be expressed as sensitivity to a parallel shift in annualised yields.

These are summed across all repo factor sensitivities to give the total yield sensitivity.

$$\frac{\partial F}{\partial R} = \sum_i \left(\frac{\partial F}{\partial (rdf_i)} \times -T_i \times df_i^{\,1+\frac{1}{T_i}} \right)$$

For a proportional dividend, i, $\frac{\partial F}{\partial (rdf_{X_i})} = 0$,while for an absolute dividend, i,

$$\frac{\partial F}{\partial (rdf_{X_i})} = \frac{g_i \cdot d_i \cdot (df_{P_i} + tc_i \cdot df_{T_i}) \cdot rdf_F}{(rdf_{X_i})^2 \cdot df_F}.$$

Dividend projection

Basic method

Any explicit dividends entered into the market parameters page are stored in the database and reloaded into memory. Whenever a forward price is computed, we check whether the dividends project far enough. Does the ex-dividend date of the last dividend extend beyond the forward date being calculated? If this is not the case, the dividends are projected using the method described below, and the projection is stored in memory for performance reasons. The projection is only extended if a forward price is needed that extends beyond the projected dividends. This projection in memory is cleared when the cache of market parameters is cleared, i.e. at the beginning of a portfolio calculation.

Method of projection

Panorama Equity projects dividends by repeating the last full year’s worth of dividends, but using growth rates to change the size of those dividends into the future, and using the tax credit parameters to potentially use different tax credit rates. There are, of course, some assumptions to be made about how exactly ex-dividend, payment dates and tax credit dates fall in the calendar one year on.

Reading the “last full year”

There is a problem with the above. If a stock pays a dividend on , say, the 1st Friday in March every year, there will be cases where two dividends are less than a year apart – Say the first of the two falls on the 6th March, and the second of the two falls on 2nd March. Simply measuring the “last full year” as 12 months will tend to include both dividends in the same year. In order to avoid this, we have used the last 11 months before the last entered dividend to create the projection dividends. This works because dividends are not paid more frequently than quarterly in reality. There may be some problems that need to be manually worked around when a stock changes its dividend payment date.

Projection without growth

The last year’s worth of explicit dividends are projected simply by simply adding 1 to the year (except for leap year 29th February, which projects to 28th February. This is done for Announce date, ex-dividend date, payment date and tax credit date.

Application of growth rates

Each dividend is grown using the growth rate functionality as it is projected. The payment date of the dividend to be projected 1 year ahead is used to find a value on the table of growth rates (dates before the first growth rate will use the first growth rate and dates after the last will use the last). Interpolation is not used. This is simply a look up table where the growth rate applies from the date next to it to either just before the next date or infinitely in the future. You might find it a good idea to enter a long dated zero element in the growth table to stop the forward price drifting below zero for long maturity instruments. When a growth rate g is obtained, the new dividend is simply the same size as the old one multiplied by a factor of (1+g).

Application of tax credit rates

Tax credit rates work in a similar way to growth rates in that it is simply a look-up table, not an interpolation table. The projected dividend payment date is the date used to look up on this table.

Special dividends

Special dividends are not projected

Basket calculations

Spot price
A basket spot price is defined by the quantities of the components (interpreted as the number of contracts of the component) and the fx conversion mechanisms (if components are quoted in a different currency to the basket) – which can use either a fixed or a floating rate.

$$S_{basket} = \frac{\sum_{components}\left(S_i \cdot df_i \cdot P_i \cdot Q_i \cdot C_i \cdot X_i\right)}{D_{basket} \cdot df_{basket} \cdot P_{basket}}$$

Where

S_i	=	Spot price of the component
df_i	=	Component currency discount factor for the settlement date of the component spot deal
P_I	=	Price factor of component issue
Q_i	=	Quantity of the component in the basket as number of contracts
C_i	=	Contract size of the component issue
X_i	=	Exchange rate to convert into the basket currency if required (fixed or float)
S_{basket}	=	Spot price of basket
D_{basket}	=	Basket divisor
df_{basket}	=	Basket currency discount factor for the settlement date of the basket spot deal
P_{basket}	=	Basket issue price factor

Forward price
The forward price of the basket can be calculated using a formula similar to the above, using forward prices of components instead of spot prices. This needs to be adjusted if the basket has “total return” components, i.e. components whose dividends are going to be reinvested in the basket / index (e.g. the DAX). If the components are in a foreign currency and the FX conversion is at a fixed rate the adjustment is made in a similar way to the quanto forward adjustment.

$$F_{basket} = \frac{\sum_{components}\left(\left(F_i \cdot U_i + R_i\right) \cdot df_i \cdot P_i \cdot Q_i \cdot C_i \cdot X_i\right)}{D_{basket} \cdot df_{basket} \cdot P_{basket}}$$

Where

F_i	=	Forward price of the component
U_i	=	Either 1 or Quanto adjustment for fixed exchange rate foreign components
df_i	=	Component currency discounting factor for the settlement date of the component forward deal to the forward
P_I	=	Price factor of component issue
Q_i	=	Quantity of the component in the basket as number of contracts
C_i	=	Contract size of the component issue
X_i	=	Exchange rate to convert into the basket currency if required (fixed or float)
S_{basket}	=	Spot price of basket
D_{basket}	=	Basket divisor
df_{basket}	=	Basket currency discount factor for the settlement date of the basket spot deal
P_{basket}	=	Basket issue price factor

Quanto adjustment for component quoted in different currency but converted at fixed rate:

$$F' = F \times e^{q}$$
$$q = \rho_{stock,fx} \times \sigma_{stock} \times \sigma_{fx} \times T$$

Implied repo curve

Because of performance considerations, the basket forward price is not calculated in full each time. Instead, an implied repo curve is calculated from forward prices calculated in full on the repo dates defined in the market parameters screen and then those implied repo rates are interpolated on to create other forward prices during risk calculations. This is an approximation because it will ignore potential structure between those dates, but accuracy can be improved as desired by adding more repo date points – It should be noted that we use the full spot calculation and aggregate dividend calculation each time.

Volatility calculations

To calculate the basket volatility, we need to calculate the component weights, volatilities and correlations in the basket currency.

Suppose we have a basket made up of N components with spot prices S_i^* in the component currencies and exchange rates FX_i with the basket currency. Then the normalised weights of the components are given by

$$w_i = \frac{S_i}{\sum_{j=1}^{N} S_j}$$

where $S_i = FX_i S_i^*$.

For a component *a* denominated in currency *x*, with spot price *A* and exchange rate *X* with the basket currency, the volatility is given by

$$\sigma_{AX} \approx \sqrt{\sigma_A^2 + \sigma_X^2 + 2\rho_{A,X}\sigma_A\sigma_X}$$

where we have ignored some higher order cross terms.

For the correlations, suppose we have two stocks, *a* and *b*, denominated in currencies *x* and *y* respectively. Let the relevant spot prices and exchange rates with the basket currency be denoted by *A*, *B*, *X* and *Y* respectively. We wish to find the correlation between the basket currency values of the two stocks, *i.e.* between *AX* and *BY*. Again neglecting some higher order cross product terms, we obtain the expression

$$\rho_{AX,BY} \approx \frac{\rho_{A,B}\sigma_A\sigma_B + \rho_{A,Y}\sigma_A\sigma_Y + \rho_{B,X}\sigma_B\sigma_X + \rho_{X,Y}\sigma_X\sigma_Y - \rho_{A,X}\sigma_A\sigma_X\rho_{B,Y}\sigma_B\sigma_Y}{\sigma_{AX}\sigma_{BY}}$$

Then the basket covariance matrix $\mathbf{V}$ has elements $V_{ij} = \rho_{ij}\sigma_i\sigma_j$ and the basket volatility is given by the expression

$$\sigma = \sqrt{\mathbf{w}^{\mathrm{T}}.\mathbf{V}.\mathbf{w}}$$

Depositary receipts

Depositary Receipt as a single component basket

From a pricing point of view, a depositary receipt is like a basket with a single component, denominated in a different currency to the component. The stock is paid for in a foreign currency and will pay dividends in a foreign currency, although these are probably the standard stock dividends converted to the foreign currency at floating fx rates. As with baskets, Adaptiv offers the user the chance to use manual or calculated figures for the spot price, dividend stream and repo rates of the instrument. Similarly, volatilities may be entered manually or calculated from the component volatility.

Calculation of spot price

$$S_{dr} = \frac{C_{dr} \times S_u \times df_u \times X_{dr,u}}{df_{dr}}$$

Where:

S_{dr}	=	Spot price of the depositary receipt
C_{dr}	=	Conversion ratio of depositary receipt
S_u	=	Spot price of underlying
df_u	=	Discount factor for the underlying currency and underlying spot settlement date
$X_{dr,u}$	=	Current exchange rate from the underlying currency to the depositary receipt currency
df_{dr}	=	Discount factor for the depositary receipt currency and depositary receipt spot settlement date

Definition of EqDelta

EqDelta is defined as the amount of stock purchased today (settling on the spot settlement date) that has the same spot price sensitivity as the instrument – i.e. it indicates the amount that you should purchase or sell today to neutralise your spot sensitivity. To calculate this we need to calculate the ratio between the sensitivity of the instrument and the sensitivity of a purchase of 1 share today to the spot price of the share.

PV and sensitivity of a spot deal done today

A spot deal done today settles on the spot settlement date. If we were to sell it today, we would make a profit or loss that would be realised on the spot settlement date. So the formula for PV of a spot deal done today is as follows:

$$PV_{spotdeal} = (Spot - Tradeprice) \times df_{spot}$$

$$\frac{\partial PV_{spotdeal}}{\partial Spot} = df_{spot}$$

EqDelta as a spot hedge calculation

We are trying to match spot price sensitivities with a spot deal done today, so the EqDelta for an instrument is as follows:

$$EqDelta = \frac{\partial PV_{instrument}}{\partial S} \div \frac{\partial PV_{spotdeal}}{\partial S} = \frac{\partial PV_{instrument}}{\partial S} \div df_{spot}$$

PV of a settled spot position

If we have a settled spot position then this is different to a spot deal done today because we can repo the shares from today to the spot settlement date. So the formula for PV of a settled spot position is as follows:

$$PV_{spotposition} = (Spot) \times \frac{df_{spot}}{rdf}$$

Where rdf is the repo discount effect from today to the spot settlement date.

$$\frac{\partial PV_{spotposition}}{\partial Spot} = \frac{df_{spot}}{rdf}$$

$$EqDelta_{spotposition} = \frac{1}{rdf}$$

Likewise any spot trade that settles before the current spot settlement allows the opportunity for some repo, so can be adjusted in a similar fashion by rdf.

Currently, Adaptiv collects settled and unsettled equity positions into a single position, and so the small repo effect before the spot settlement date cannot be properly accounted for in our risk calculations. We have opted to assume that the aggregate position is one that has risk as though it settled on the current spot settlement date.

Calculation of futures hedge equivalence

Definition

The futures hedge is intended to be a calculation of the hedge using a liquid benchmark future as opposed to the underlying equity. The calculation takes into account the beta factor (as defined above) and also the exact maturity of the future.

We need to match the sensitivities to the equity stock price such that

$$FH \times \frac{\partial PV_{future}}{\partial S_{equity}} = \frac{\partial PV_{instrument}}{\partial S_{equity}}$$

Where:

FH	=	Futures hedge as a number of futures contracts
S_{equity}	=	Equity spot price

We already know the instrument sensitivity to the equity spot price – This is related to EqDelta although not exactly the same because of the settlement period.

The sensitivity of a future contract to the spot price of the equity can be written as follows:

$$\frac{\partial PV_{future}}{\partial S_{equity}} = \frac{\partial PV_{future}}{\partial S_{benchmark}} \div \frac{\partial S_{equity}}{\partial S_{benchmark}}$$

We know the sensitivity of the future to its own underlying spot price (assume that the future is margined)

$$\frac{\partial PV_{future}}{\partial S_{benchmark}} = contractsize_{future} \times \frac{\partial Forward_{benchmark}}{\partial S_{benchmark}}$$

And beta gives us the second term

$$\frac{\partial S_{equity}}{\partial S_{benchmark}} = \beta_{equity} \times \frac{S_{equity}}{S_{benchmark}}$$

Option Greeks and other outputs

The following Greeks are available for individual positions and aggregated for portfolios containing equity option based positions. Where relevant, results are available both as a ratio and as a cash amount. Some of the outputs are only relevant for convertibles and are marked as such.

Accrued interest (Convertibles Only)

This is the amount of the first coupon to which the potential bond purchaser will not be entitled to should they enter the trade.
Accrued interest (if any) = V_I

BondOnlyPrice (Convertibles Only)

Clean value of the straight bond = V_B

Contracts

This is the number of contracts making up each position. May show zero if position closed.

CashHedge

This is simply the EqDelta figure multiplied by the current spot price of the underlying equity.
CashHedge = EqDelta × spot price

EqDelta

For a full description read the chapter "Definition of EqDelta" – It is the sensitivity of the PV of the option to a unit change in the underlying spot price, divided by the discount factor to the underlying spot settlement.

EqGamma

There are two alternative market conventions for this. One is the change in delta for a unit change in the spot price of the underlying. The other is the change in delta for a 1 percent change in the spot price of the underlying. Adaptiv currently uses the first convention.

FuturesContract

This indicates the contract of futures for which the future hedge has been calculated.

OptionOnlyPrice (Convertibles Only)

Value of option component of convertible bond (unscaled):
V_O = value of conversion option + value of call option + value of put option

EqPrice

EqPrice is intended to be the fair unscaled premium for the option. It is the value of the option quoted in the appropriate currency and units (price factor of 0.01 for the option issue indicates a GBP option is quoted in pence), and calculated as for the option settlement date, i.e. it is a present value compounded to the option settlement date.
Clean value of the convertible bond:

$$V_C = V_B + V_O$$

DirtyPrice (Convertibles Only)

This is the total value of all the cash flows generated by the bond, and the embedded options. Therefore, it can also be expressed as the instrument price plus the accrued interest on the next bond coupon. Dirty value of the convertible: $V_D = V_C + V_I$

FuturesHedge

This is a calculated hedge in terms of futures on the benchmark of the underlying. This is only calculated if the appropriate definitions have been made.

ValuePerShare (Convertibles Only)

Value of convertible on a per share basis:
$V_S = V_C \div$ current conversion ratio $\times$ nominal of bond $\div$ 100

ValuePerShareDelta (Convertibles Only)

Δ_S = sensitivity of V_S to change in price of underlying equity

EqDeltaRatio (Convertibles Only)

The unscaled version of EqDelta – i.e. position specific scaling factors removed.
Δ_C = sensitivity of V_C to change in price of underlying equity

EqDivSens

This is the change in PV for an increase of 1% in each discrete dividend used in the calculation. This is shown either in the currency of the instrument or in the base currency of the portfolio, depending on the settings in the portfolio's Pricing Rules dialog.

EqGammaRatio (Convertibles Only)

The unscaled version of EqGamma – i.e. position specific scaling factors removed.
Γ_S = sensitivity of Δ_S to change in equity price

EqTheta

This is the rate of change of the option price with respect to the change in the time to option maturity measured in years.
Θ = Sensitivity of V_O to change in time to maturity

OptionOnlyRho (Convertibles Only)

ρ_O = sensitivity of V_O to a parallel shift in the interest rate curve used for discounting

EqPriceRho (Convertibles Only)

ρ_C = sensitivity of V_C to a parallel shift in the interest rate curve used for discounting.

EqRepoSens

This is the change in PV for an increase of a +100 basis point parallel shift of the repo curve.

EqVega

This is the rate of change of the option price with respect to a change in the asset volatility. Some people also refer to this as lambda or kappa.
Vega = sensitivity of V_O to a change in equity price volatility.

ImpliedVol

For options and warrants this is shown as the volatility implied from the current price of the contract (shown in the position list).

Chi (Convertibles Only)

This is the rate of change of the option price with respect to a change in the foreign exchange rate. This feature is relevant only in the cases with the cross-currency feature turned on.
χ = Sensitivity of V_O to a change in the spot foreign exchange rate

Parity (Convertibles Only)

This is the value obtained if it were possible to convert the bond into stock on the valuation date.
V_P = value of bond if converted now
= stock price $\times$ current conversion ratio $\times$ FX Rate (Bond per share)

PremiumOverParity (Convertibles Only)

P $= (V_C \div V_P) - 1$

PV

This is the sum of the discounted expected values of the future cashflows.

RunningYield (Convertibles Only)

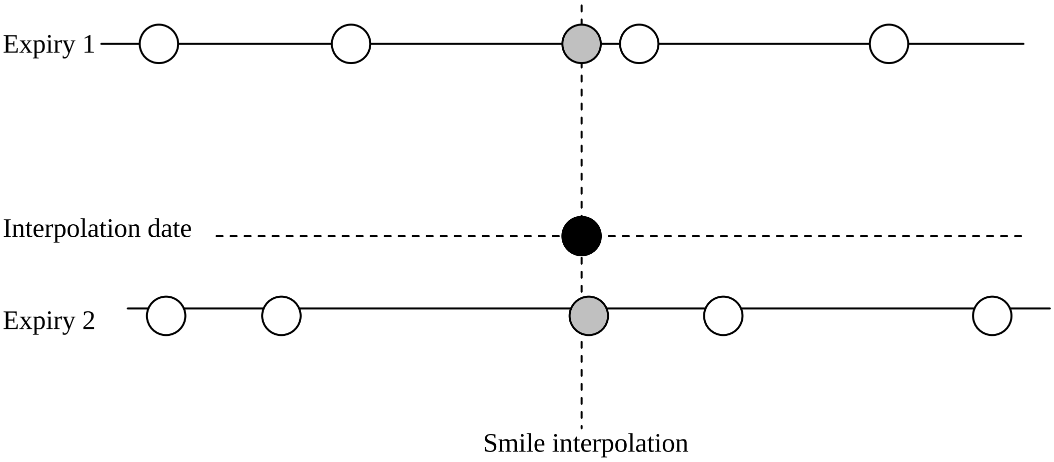
Running yield = bond coupon $\div V_C$

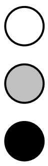
YieldAdvantage (Convertibles Only)

Yield advantage = (bond coupon – dividend yield) $\div V_C$

Interpolation on a volatility surface

The volatility surface is defined as two-dimensional – Expiry horizon as one axis and a function of strike and/or spot as the other. Volatilities are interpolated using cubic splines on the strike / spot axis and linearly between expiry horizons. Cubic splines are used to smooth interpolations on the strike / spot axis because of the importance of the slope of the volatility smile in delta adjustments for options.





= Original input points.

= Cubic spline interpolations on volatility smile at adjacent expiry horizons.

= Linear interpolation in time and volatility used.

Cubic spline method used

To interpolate between points n and n+1, we fit a cubic equation to the gap between those points that passes through the two points and has a gradients at each end set arbitrarily as follows:

$$\left(\frac{\partial y}{\partial x}\right)_{x=x_n} = \frac{y_{n+1} - y_{n-1}}{x_{n+1} - x_{n-1}}$$

$$\left(\frac{\partial y}{\partial x}\right)_{x=x_{n+1}} = \frac{y_{n+2} - y_n}{x_{n+2} - x_n}$$

This ensures continuous gradients at the points.
In the penultimate points, and alternative gradient fixing is used which doesn't use the non-existent extra point – i.e. in the above, y_{n+1}, x_{n+1} is used where y_{n+2}, x_{n+2} doesn't exist and y_n, x_n is used where y_{n-1}, x_{n-1} doesn't exist.

Extrapolation

In both linear and cubic spline interpolation and in both dimensions, extrapolation is always purely flat from the nearest point

Adjustments for quanto and compo options

Quanto

A *quanto* option is one in which the payout is in cash and the formula for the payout is the same as for a normal option. The exception being that the payout is in a different currency – e.g. a call option struck at $100 expiring when the market is at $120 has a normal cash payout of $20. A quanto option may have been specified to pay this quantity in sterling – i.e. £20. Obviously there is a conversion to be done to reflect the fact that £20 is more valuable than $20 – this is a multiplication of the option values by the exchange rate from the stock currency to the quanto currency. There is also a more subtle adjustment that needs to be taken into account. This is an adjustment to the forward price fed into the option formula that takes into account the volatility of the quanto FX rate and the correlation between the quanto FX rate and the underlying stock price.

Adjust the forward price for correlation effects as follows:

$$F' = F \times e^{q}$$

$$q = \rho_{stock,fx} \times \sigma_{stock} \times \sigma_{fx} \times T$$

Where:

F	=	The true forward price
F'	=	The correlation adjusted forward price
$\rho_{stock,\ fx}$	=	The correlation between the stock price and the fx rate between the stock currency and the quanto currency. The fx rate is the one quoted as units of quanto currency per stock currency. You can also use a negative correlation with the alternative quotation, i.e. such that a positive correlation has the effect of increasing the forward price used and hence increasing the value of quanto call options and quanto forwards.
σ_{stock}	=	The volatility of the stock price
σ_{fx}	=	The volatility of the fx rate between the stock currency and the quanto currency
T	=	The time in years from today to the forward date

And multiply any forward values by a forward fx rate into the quanto currency

Compo

A *compo* option is one where the strike is quoted in terms of another currency – e.g. an option on a sterling based stock being quoted as an option on the US-dollar price of the stock. In this case the predicted forward price needs to be multiplied by the forward exchange rate from the stock currency to the compo currency to give a forward price in the compo currency. The volatility of the stock price in the compo currency needs to be calculated using the volatility of the FX conversion rate and the correlation between the FX conversion rate and the stock price in the original currency.

Adjust the forward price, simply using the arbitrage with an FX forward rate

Adjust the volatility of the option struck in the compo currency as follows:

$$\sigma_{compo} = \sqrt{\sigma_{stock}^{2} + \sigma_{fx}^{2} + 2 \times \rho_{stock,fx} \cdot \sigma_{fx} \cdot \sigma_{stock}}$$

where definitions are as above.

N.B. Although Adaptiv does support a term structure of FX volatilities, only the fx volatility at expiry is used in the American option model which does use a time-structured equity volatility. This is done for performance reasons.

Calculations for equity options, warrants and index futures and options

European style

For european style options we are using a standard Black model with the forward price calculated as described and a volatility interpolated as described. The standard Black delta, after conversion to a spot equivalent figure, is adjusted for volatility smile effects as follows in the cases where the volatility smile is defined in terms of relation to the spot price:

$$\frac{\partial V}{\partial Spot} = \left(\frac{\partial V}{\partial Spot}\right)_{ConstVol} + \frac{\partial V}{\partial Vol} \cdot \frac{\partial Vol}{\partial Spot}$$

At present, no volatility smile adjustment is made to the standard Black gamma figure – although there is a slight effect due to the curvature of the volatility smile.

The present value of the option is quoted as a value *today*, whereas the EqPrice figure is a fair price for the *settlement* date of the option – i.e. the PV of the option is not precisely equal to the price we quote multiplied by the size of the deal.

American style

For options with possible early exercise, we are using a binomial tree calculation with 30 time steps as default. A term structure of volatility is allowed for by using non-equal time steps. The tree is drawn with 2 extra "wings", as is the standard market practice for calculating delta and gamma figures. Vega and theta are calculated by using new tree calculations.

For further information on the binomial tree calculation – see the section "binomial tree geometry". Volatility smile effects are allowed for in the same way as for European style options.

The binomial tree approach forces us to assume proportional dividends so that the tree nodes recombine and is also less flexible than other numerical techniques, so we have developed a finite difference approach (encompassing trinomial tree techniques) that is currently used for barrier options.

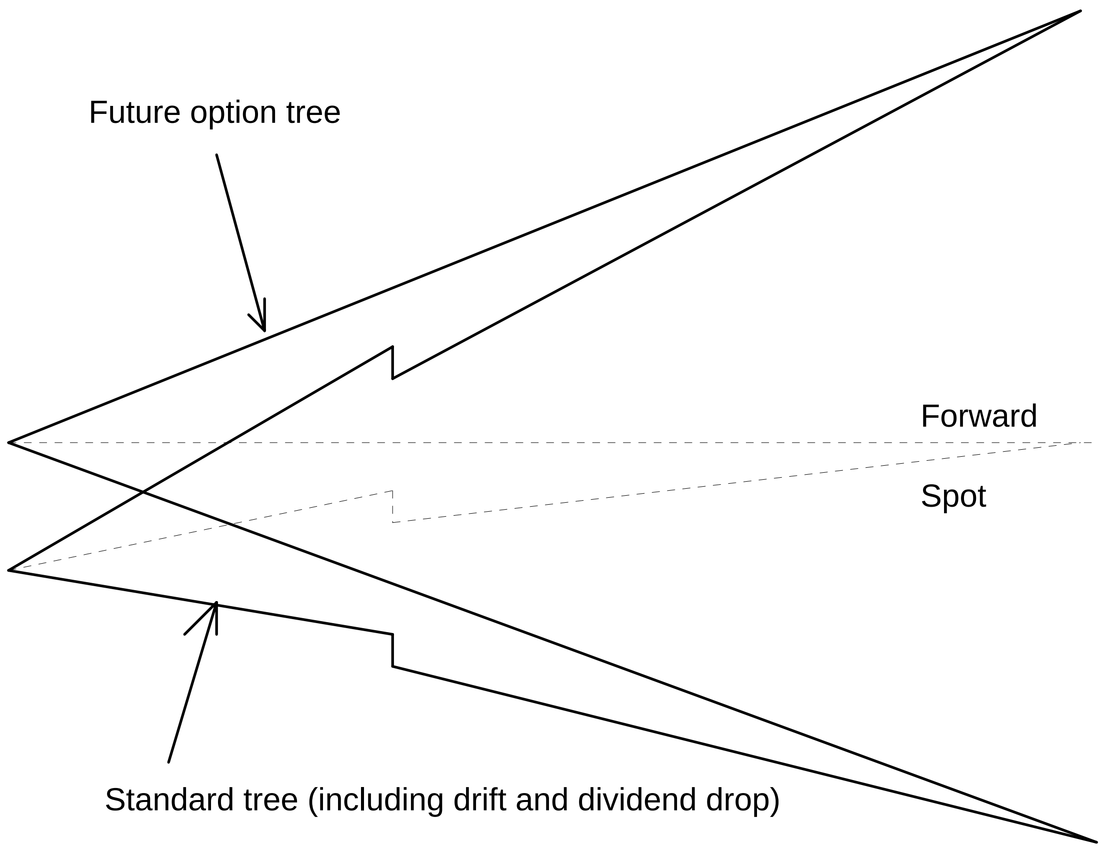
Index futures and options

The Adaptiv market parameters model gives the user the ability to define and calculate an index in terms of its components, effectively as a basket – Alternatively manual overrides can be used, such as entering a strip of futures prices to define an implied repo curve for the index. If you wish to simulate a simple continuous dividend yield type calculation, simply set the repos and dividends to manual; enter no dividends, and either enter the dividend yield into the repo column or enter futures prices to imply a dividend yield. Once this is done, index futures and options are priced in exactly the same way as single stock futures and options.

Options on futures

Early exercise and margining

Options on futures where the option premium is non-margined (i.e. paid on purchase with no variation margin), and the underlying future is margined (i.e. can close out and receive full intrinsic value on the close out date), and the exercise style is American do sometimes have an early exercise premium. This happens when the option is so far enough in the money that, the discounted intrinsic value plus the time value is less than the non-discounted intrinsic value alone. For these options (the majority of options on futures), Adaptiv use a binomial tree approach but where the underlying asset value is the future not the spot. If you want to force a Black model, then you should set the exercise style to “European”.



LIFFE options

LIFFE options have their premium paid effectively on the expiry date – This is indicated in the Adaptiv software by setting the option issue “Margined” flag to true. This means that there is no benefit to early exercise and so the analytical black model is used. It also means that the option premium does not need to be discounted from the expiry date.

Delta and Gamma adjustment

Because the options are priced with the future as the underlying, the delta and gamma need to be adjusted to be spot values.

Compound options

N.B. Adaptiv use compound option methodologies for instalment warrants, where there is just one remaining unpaid instalment.

General solution

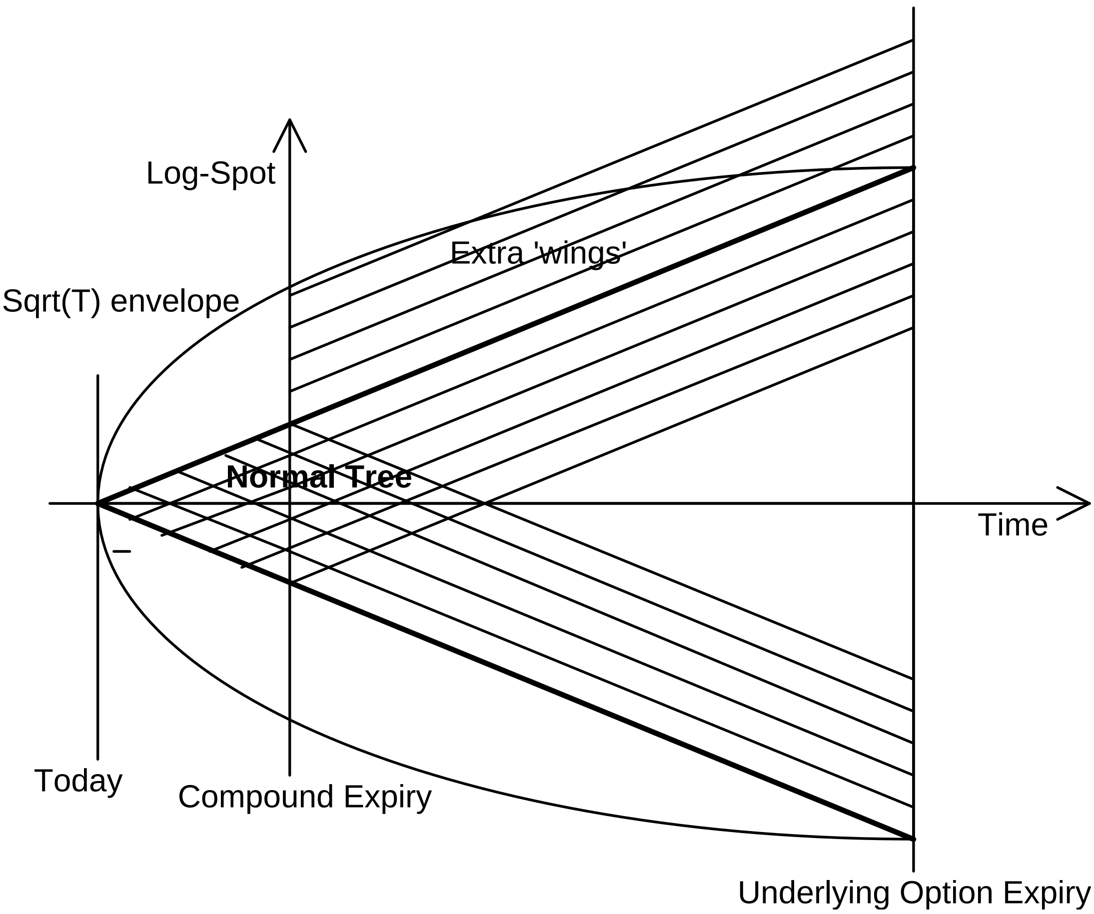
The general solution procedure is to calculate the expected value of the underlying option value on the compound expiry date. Assuming the spot price is log-normally distributed at the compound expiry, the compound price for a European option on an underlying option is then given by

$$\text{CompoundPrice} = \frac{DF}{\sigma\sqrt{2\pi T}}\int_0^\infty e^{-\left(\log(F/S_c)-\sigma^2 T/2\right)^2/2\sigma^2 T} UnderlyingOptionValue(S_c)\frac{dS_c}{S_c}$$

We split the above integral into two sections, for example for a call on a call, in the form

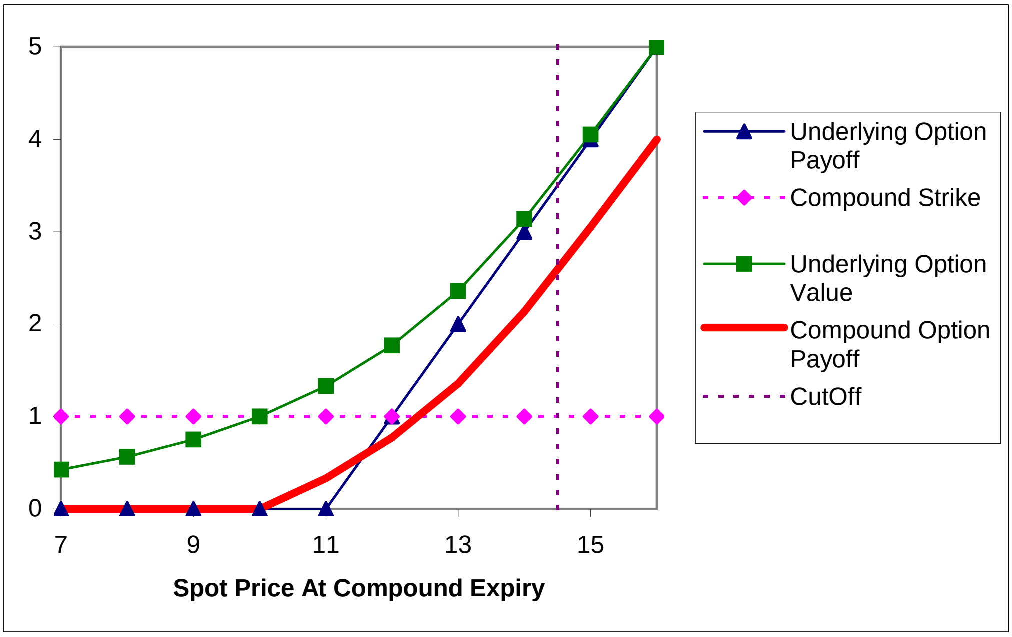
$$\int_0^\infty = \int_0^{S^*} + \int_{S^*}^\infty$$

where S^* is the level of the highest node. The first integral is evaluated numerically using the node values from the tree, and the second part may be done analytically.



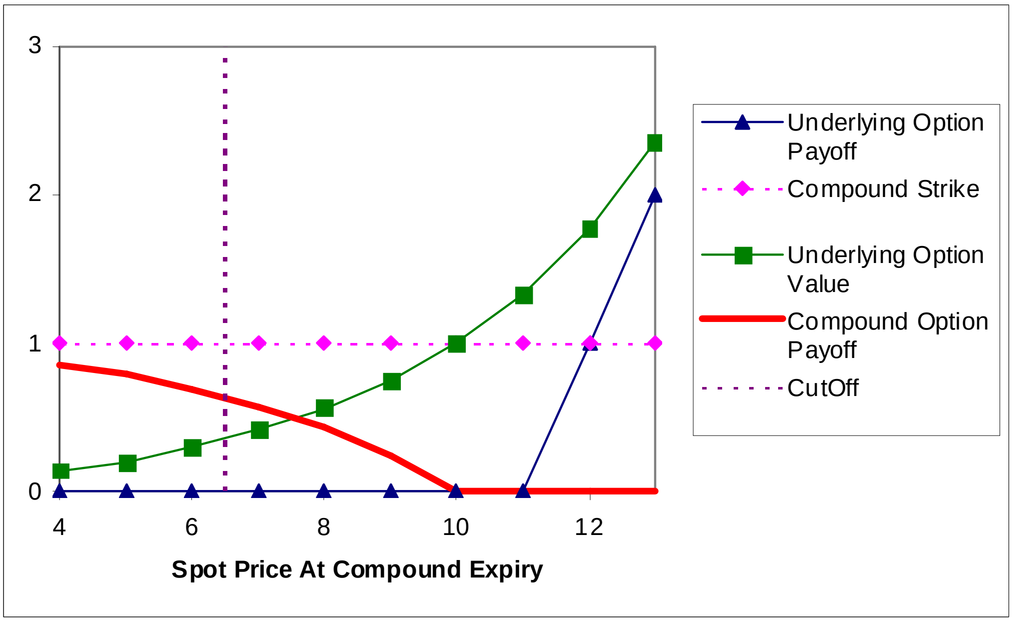
Integration

For the first part of the integral, we calculate the option values on the compound expiry using the binomial tree, with extra 'wings' above and below the standard tree to get enough points to cover the correct range of likely spot prices and make the integration accurate. Options on European options are done in the same way but with early exercise switched off in the tree.



We then interpolate onto a finer grid and integrate in log-spot space using Simpson's rule or other high order method.

For the second part, where the option value vs. spot price graph is sufficiently close to a straight line, the area under the lognormal distribution curve may still be significant. The product of the two functions may be integrated analytically, and in the case of a call on a call is effectively the Black-Scholes price of a plain option and a digital option struck at the cut-off level S^*. For a put on a call for example, the graph looks as follows, and the area below the cut-off level at $S = 6.5$ is approximated by a digital put struck at the cut-off point.



Barrier options

General Methodology

We use a Crank-Nicolson finite difference method, giving a combination of precision and stability, as described extensively in Wilmott (1998).

Grid Generation

In the direction in which there is no barrier, the upper or lower limits of the grid are set to roughly six standard deviations in volatility, with modifications due to the forward price and dividends. Specifically,

$$S_{\max} = R_T S$$

and

$$S_{\min} = S / R_T$$

where the variation ratio R_T is given by

$$R_T = \left(1 + \frac{F}{S}\right) \times \left(1 + \frac{PVD}{S}\right) \times \left(1 + \sigma\sqrt{T}\right)^6$$

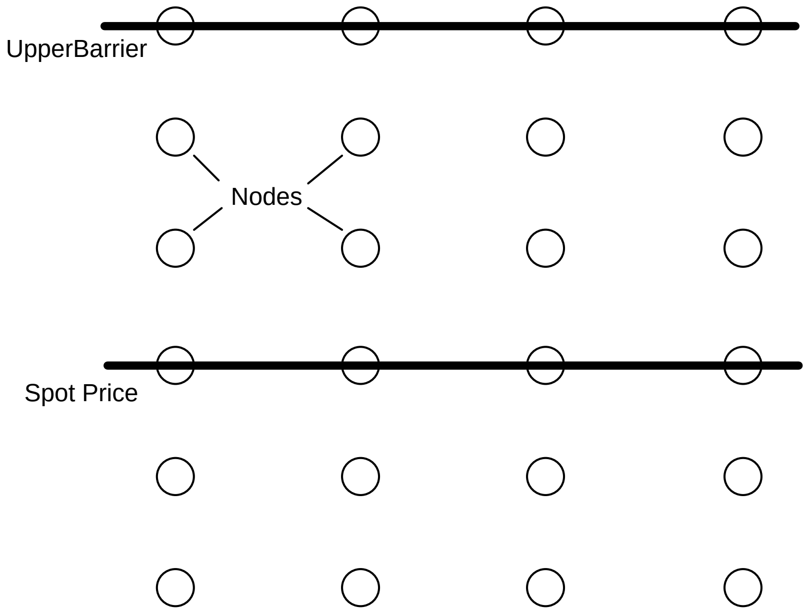
Here F, PVD and σ are the forward price, present value of the dividends and volatility respectively. Thus the grid extends as far out as the worst-case scenario in each direction, for example a high forward price, high volatility, and a large dividend drop just before expiry.

The default spot step size is defined as

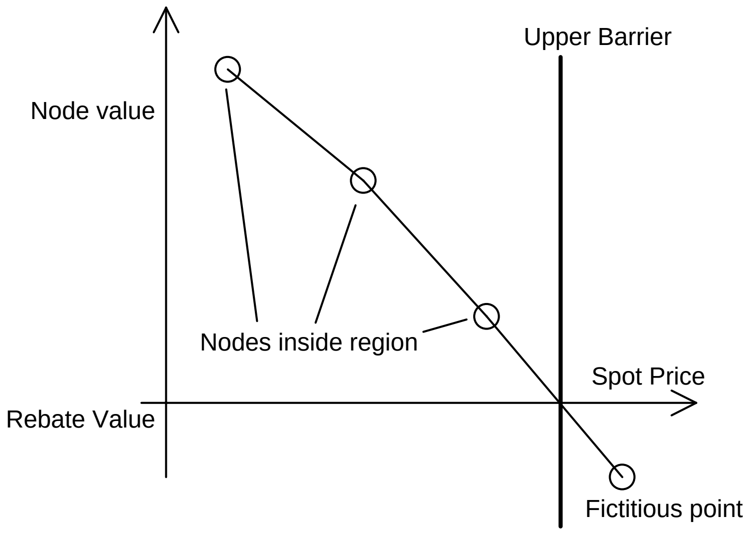
$$\Delta S_{theo} = \sigma S \sqrt{\Delta t}$$

which is then adjusted downwards to ensure that the following constraints on the positioning of the nodes is satisfied.

A common feature of grid-based models is the presence of stepping in the calculated price as parameters such as spot or barrier level are changed. To remove as many of these effects as possible, and to ensure the best accuracy, we align the grid such that the spot price and at least one of the barriers is on a node. In the case of single barrier options, this is sufficient - we can always choose the spacing between nodes to ensure that there are an integer number of them between the spot price and the barrier.



For double barrier options, we position a node on the spot price and one on the barrier with the highest intrinsic option value, so that the errors introduced at the barrier not on a node will be as small as possible. For example, a double knockout put option will have nodes on the spot price and on the lower barrier. At the other barrier, we insert a fictitious point as described in Wilmott (1998) so that the option value matches up with the rebate value at the boundary.



Dividends

Dividends, if small compared with the spot step size, are incorporated into the effective step forward rate. If larger than the spot step size, we shift the node and interpolate onto the new grid.

Americans

We simply replace the option value at each node with the intrinsic value if greater. This is not strictly correct, as the Crank-Nicolson method is implicit in nature, so we should actually replace the values at the same time as we calculate the next time step. However, this adds very little extra accuracy compared to the reduction in the speed of the calculation, so we have chosen not to implement it.

Knock-Ins

European knock-in barrier options are priced using the property that a portfolio consisting of a knockout and a knock-in barrier with the same barrier levels and strikes has the same value as a plain option. This is because if one of the barrier-options knocks in, then the other knocks out, thus one always ends up with only one option at expiry. American knock-in barrier options cannot be handled currently.

Asian options

Currently Asian Options are priced using the Black model, with the forward price and the volatility adjusted to take account of the averaging.

Variance

The variance of the underlying in the remaining life of the option is given by

$$Variance = \frac{1}{N^2}\sum_{k=1}^{N}\left(2(N-k)+1\right)w_k\sigma_k^2T_k$$

where N , w_k , σ_k and T_k are the number of remaining observations, the normalised weight, the forward volatility to that observation and the time between now and the observation respectively.

Note that for the case of one observation at expiry, this expression reduces to the usual form $Variance = \sigma^2T$.

Similarly, it can be shown that for a large number of observations the above expression reduces to the closed form result for a continuous average option,

$$\lim_{N\to\infty} Variance = \frac{\sigma^2T}{3}$$

The forward price used is the weighted average of the known fixings and the forward prices at the unknown observation dates:

$$Fwd = \frac{1}{N_{Fixed}+N}\left[\sum_{k=1}^{N_{Fixed}} v_k Fixing_k + \sum_{k=1}^{N} w_k Fwd_k\right]$$

with v_k the weights of the known fixings.

Quanto Effects

For quantos we adjust each of the unknown forwards by the quanto currency correlation factor

$$\exp(\sigma_{FX}\sigma_{Stock}\rho_{FX,Stock}T$$

Instalment receipts (part-paid equities)

General approach

Instalment receipts are part-paid equities, where a series of future obligatory instalments are used to purchase shares. It may be the case that the holder is not entitled to 100% of any dividend payments until the contract is fully paid. Each instalment may entitle the user to a greater percentage of the dividends.

Pricing

Instalment receipts can be priced using the underlying equity price and subtracting the present value of any future instalments and any lost dividend rights and any lost repo income.

$$I = E - \sum_{j} \left(S_j + (1 - f_j) \cdot d_j \right) - R$$

where:

I = Present value of instalment receipt

E = Present value of underlying equity

j = Instalment counter

f_j = Fraction of dividend income (including tax credits) earned by instalment receipt holder between the last instalment and this instalment. It is assumed that the last instalment guarantees 100% of the dividends from then on. i.e. the instrument turns into regular stock.

d_j = Present value of dividend income earned by holding the underlying stock from either the previous instalment or today if there is no previous instalment after today to the instalment j

R = Lost repo income (assumed zero at present)

S_i = Present value of Instalment payment j

Sensitivities

Because we have assumed above that R, the lost repo income, is zero, the delta of the contract turns out to be the same as that for the underlying equity. There is, however, a negative dividend risk and a positive interest rate risk due to the lost dividend income.

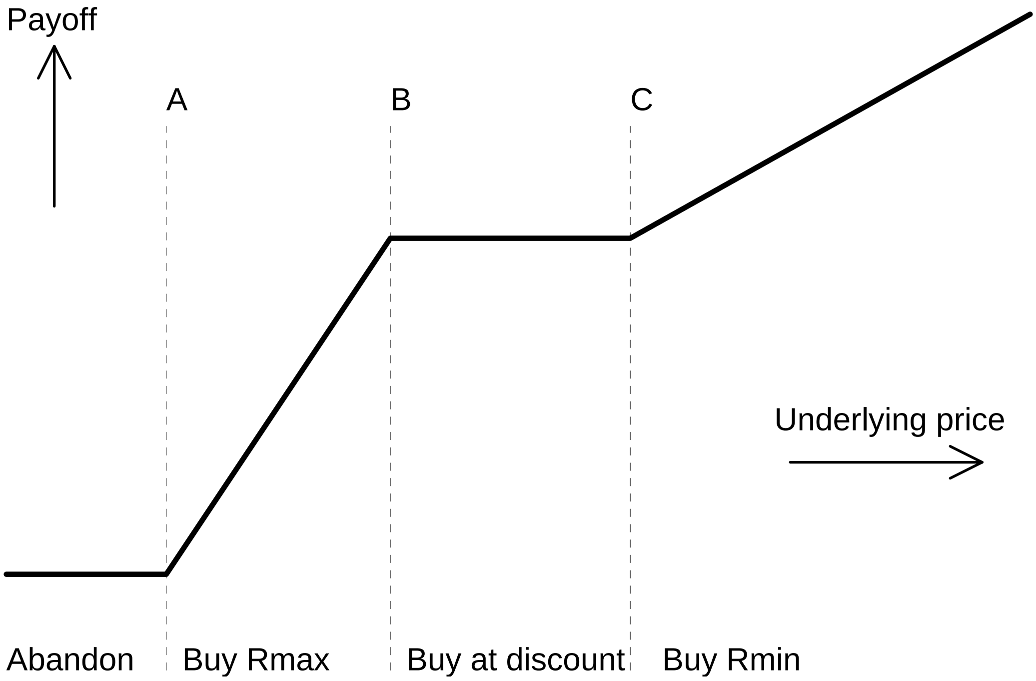
Discount options

Description

These securities entitle the holder to purchase a variable number of shares at a discount to the market price at maturity. The number shares on exercise is based on the ordinary price at maturity *S*, a specific discount *d* and the strike price *K*. The number of ordinary shares issued is $K/(S(1-d))$ subject to maximum and minimum bounds, R_{Max} and R_{Min}. Hence the value on exercise (which is optional) is $\max(S\max(\min(K/(S(1-d)), R_{Max}), R_{Min}) - K, 0)$. Issues have been known to include a number of discount options with different exercise dates in the one traded security.

Pricing

The payoff described above looks like this:



Point A is where S = K / R_{Max}
Point B is where S = K / (R_{Max} * (1-d))
Point C is where S = K / (R_{Min} * (1-d))

Below point A, the payoff is zero because the option is abandoned.

Between points A and B the gradient is R_{Max}, where the option holder buys a capped number of shares R_{Max} at a price of K / (R_{Max} * (1-d))

Between points B and C the gradient is zero, where the option holder buys a variable number of shares at a discount.

Above point C the gradient is R_{Min}, where the option holder buys a minimum number of shares R_{Min} at a price of K / (R_{Min} * (1-d))

This payoff profile can be simulated using the following 3 vanilla options:
Long R_{Max} call options struck at K / R_{Max}

Short R_{Max} call options struck at K / (R_{Max} * (1-d))
Long R_{Min} call options struck at K / (R_{Min} * (1-d))

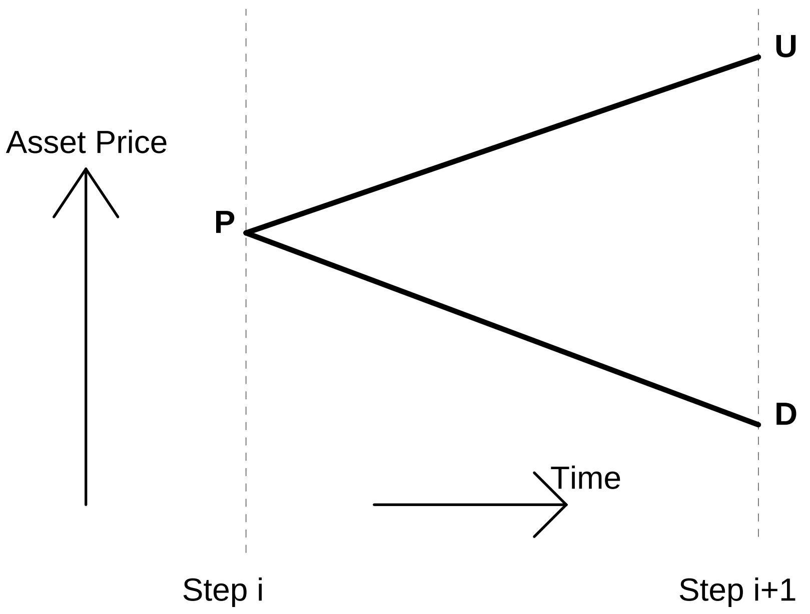
In the case of numerical methods, a new intrinsic value calculation is all that is required.

Sensitivities

Because the option is just a simple sum of other options the sensitivities sum also.

Binomial tree geometry

Single Decision



Node spacing in the asset price dimension

In order for the binomial tree to recombine at every time step and at all asset price levels the following must be true for all decision nodes in the tree:

$$R = \frac{U}{D} = e^{k} \qquad k = 2 \times V \times \sqrt{\frac{T}{N}}$$

Where:

R	=	A constant for the whole tree – the ratio of adjacent asset price nodes
U	=	The asset price value for the upward move in the diagram
D	=	The asset price value for the downward move in the diagram
V	=	The annualised volatility to the expiry of the option
T	=	The time to expiry of the option in years
N	=	The number of decision steps used in the tree

We have taken the decision to use 50/50 probabilities for the up and down steps, which means that the calculation of the up step ratio and down step ratio need to take into account the drift of the mean of the distribution. These ratios must be the same for all asset prices within a time step but will vary between time steps:

$$\frac{U}{P} = \frac{M_{i+1}}{M_i} \times \frac{2R}{R+1} \qquad D = \frac{U}{R}$$

Where:

M_{i+1}	=	The mean of the distribution (asset forward price) at the next time step i+1
M_I	=	The mean of the distribution (asset forward price) at this time step I
P	=	The asset price at the decision node
U	=	The asset price if the move is an upward one

The second term in this equation is related to the fact that the mean of a lognormal distribution will naturally drift upwards as time goes by, so needs to be corrected for before forcing the mean to follow our calculated forward price.

Node spacing in the time dimension

As stated before, a combination binomial tree forces us to have constant variance increase at each step in the time dimension. To provide a model that uses the volatility term structure, we can adjust the time jump of each step as follows:

1. Start by assuming time steps represent equal time intervals and interpolate volatility for each step. Calculate a variance at each of these time steps.
2. Now, for each step, calculate what the variance has to be to be consistent with the above asset price spacing. This is simply equal to the variance at expiry multiplied by the step number over the total number of steps.
3. Using the variance curve calculated in step 1, interpolate to find a time value for the time step that has the desired variance.

Custom product pricing using Monte Carlo simulation

These products are defined using the Custom Product issue definition screen, where arbitrary numbers of underlyings and observation dates can be defined. The final payoff of the option is defined in an implementation of the ICustomPayoff COM interface, referenced on the issue definition screen.

Currently the model is limited to European style options with a single final payout, and deterministic interest rates for the purposes of forward rate estimation and future cashflow discounting. We can therefore use a standard Monte Carlo simulation model, very similar to those described in elementary finance texts such as Hull (1997).

The underlyings supported are:
All valid Equity type underlyings – Equity, Index, Basket and ADR
FX rates
Interest rates

For equity and FX underlyings the paths are generated assuming a log-normal distribution, with the drift rates and volatilities taken from either the equity market parameters object or the FX forward rate curve and FX vol surface as appropriate. For interest rates, the paths use a normal model, with drift rates and volatilities taken from the appropriate Yield Curve and Cap vol surface specified for the given Market in the Pricing Rules. The limitations of this approach to the interest rate products will be similar to those of pricing interest rate caplets with a normal Black model. With that in mind, pricing of complex interest rate products should not be attempted.

In the discussion that follows, the term option is frequently used, but it should be emphasised that the products that can be priced are determined by the payoff that is specified and as such are not limited to option type products.

Overall approach

The total simulation is made up of 10 sub simulations, each containing N scenarios, where N is the value specified in the issue definition screen.

In each scenario, correlated risk-neutral paths for each underlying are generated using either the built in pseudo-random number generator, or an external one, whichever is specified. These paths are then sent to the appropriate CustomPayoff object, which returns the appropriate payoff for that set of paths.

These payoffs are then collected, discounted and averaged to calculate the price for the product.

The sensitivities are calculated by shifting each of the underlyings in turn and doing a full recalculation. Clearly this may be quite slow if you have many underlyings. The deltas and gammas are calculated using a central difference scheme, the vega using a forward difference.

Random number generation

The built in generator uses a method based on the ran2 method of L’Ecuyer (1998). The model allows the user to specify their own sequence generator. In this case, this generator must return an array of independent random numbers uniformly distributed on the interval [0, 1). By implementing the IMonteCarloSequence method then, substantial increases in performance may be produced by the use of quasi-random sequences such as those due to Sobol’ or Fauré for example, described in Press et al (1992)

Whichever method of random sequence generation is used, Box-Muller transformation is then used to convert the values into normal form. Finally the Cholesky decomposition of the underlyings correlation matrix is used to generate correlated steps in the usual way.

Path passing to the payoff calculation object

There are three options for path passing:
At end of each path

This is probably the most common way, where the set of underlying paths are passed to the payoff object at the end of each scenario. The payoff object must return the payout value for this path.

Step-by-step
Useful for barrier options or other types where it may be useful to stop the scenario path generation before it has finished the path if some sort of condition is reached. The payoff object can check at each step whether the scenario should be stopped then, or a payoff returned.

All paths at once
All the scenarios are generated in one go and passed all at once to a separate method of the payoff object. It is then the payoff objects responsibility to loop over the scenarios calculating the payoffs and returning an array of these to the model. This may lead to a slight performance improvement in some circumstances since it reduces the numbers of COM method calls significantly, albeit at the expense of much larger data objects being stored.

Variance reduction techniques

Antithetic variables
This is the simplest additional method where for each scenario it's equivalent 'anti-scenario' is generated using the negative of all the random variables used in its generation. This is effectively ensures that the mean of the random variable distribution is exactly zero.

Quadratic resampling
This is similar but ensures that the second moment of the random variables is unity. This is only applied for the built in random number generator. If an external generator is used, it is the responsibility of that generator to ensure that the sequences passed to the model have exactly unit variance.

Control variate
A common technique when a similar option to that being calculated has an exact analytical solution. An example would be using the Monte Carlo model to price an arithmetic average asian option. In this instance, an exact analytical formula is available for the price of a similar geometric average option. When the actual payoff is calculated for each path, the payoff object can return the equivalent payout from the geometric option. An addition method on the interface allows the payoff object to calculate the exact price of the geometric option. The model then uses this to calculate a better approximation to the actual option in question, on the assumption that the difference between the true price and simulated price of both options are equal.

This leads to significantly increased accuracy, or equivalently a reduction in calculation time for a given accuracy, in many cases, even with control options that are significantly different from those being calculated.

Outputs

Equity
The deltas, gammas and vegas are quoted in the same way as for standard equity products.

FX
Delta is change in PV for a unit change in the FX rate expressed in the quote currency, i.e. the way specified in the FX grid.
Gamma is the change in Delta for a unit change in the FX rate as above.
Vega is the change in PV for a 1% change in volatility.

Interest rate
Delta is change in PV for a 1% parallel shift in the yield curve.
Gamma change in Delta for a 1% parallel shift in the yield curve.
Vega is change in PV for a 1% shift in volatility.

Value at Risk methodologies

The Adaptiv pricing models are integrated with the Adaptiv value at risk methodologies – e.g. historical or Monte Carlo generated scenario based VAR.

Risk factors

It is impractical to use every Adaptiv market parameter - e.g. dividends, repo rates, volatility surface points as risk factors to be perturbed in VAR analysis. It is necessary to simplify so that the VAR calculation is manageable. We have chosen to make the following simplifications:

1. The important risk factors for equity instruments are the spot price and the implied volatilities.
2. Because the implied volatilities are represented by a 2-dimensional surface, we have opted to only allow log-parallel shifts in this surface, using the 90-day at-the-money volatility as the benchmark from which a historical data sequence is stored.

Baskets

Where a basket or index is defined in terms of its components, the price of that basket is not an independent stochastic variable – The shift in the index or basket spot price can be easily calculated from the shifted components. So, although the basket spot price is stored in the archived data, and will be present in the statistics set, Adaptiv will use the component shifts (recursively if necessary) instead.

Calculations and sensitivities for equity swaps, CFDs and stock lending deals

Scope

The general title "equity swaps" is used here to describe equity swaps, CFDs and stock lending deals. All of these are composed of an "equity leg", where the payments are made according to a return on either an amount of money invested in stock or a return on a number of stocks, and an "interest" or "collateral" leg. Some interest legs have payments that are calculated exactly as with interest legs that are components of interest rate swaps – These are not discussed here. Other interest legs, i.e. those where the instrument is based on a number of shares rather than an amount of money, are equity price dependent. This is because the principal amount of the interest period is calculated from the number of shares and an equity price fixing at the beginning of the period.

Open-ended deals

CFDs and stock lending deals are often open-ended – i.e. there is no maturity, and the deal rolls forward each fixing. Although these deals are modelled with arbitrary maturity dates, it is important not to value any cash flows that are not committed to. The measurement of whether a cash flow is committed to can be complicated. It is not the same as which cash flows are fixed. CFDs can have "lagged" interest rate flows that represent the funding of the equity leg, and when an equity leg flow is fixed, the associated interest leg funding flow is committed to even though it is not yet fixed.

Instrument	Equity Leg	Interest Leg
Equity swap	Shares or notional. Return on shares (+ percentage of dividend return).	Fixed or floating interest. Actual or notional exchange of principals. If equity leg is shares based, principal amount based on share price fixed at beginning of period.
CFD	Shares. Price and dividend return. Often open-ended.	Notional exchange of principals. Principal amount based on share price fixed at beginning of period.
Stock lending	Shares. Dividend Return only. Often open-ended. Actual exchange of "principals" in the sense that equity flows out and back in at maturity.	Actual exchange of principals. Principal amount based on share price fixed at beginning of period, but also multiplied by collateral margin – e.g. 105%

Collateral adjustments

In the case of interest legs based on a number of shares, and where there is actual exchange of principals, when the equity price is re-fixed at the beginning of a calculation period, there is also a requirement to adjust the collateral upwards or downwards with an additional principal payment. In practice, the collateral is only adjusted when the value travels outside of agreed boundaries, but we have modelled the pricing as though the collateral adjustment is made on every fixing.

Stock lending deals equity "principals"

In the case of stock lending deals, both the interest leg and the equity leg include actual exchange of principals. In the case of the equity leg, this represents the outward and return flows of the stock. This is important for valuation. The flows are valued as short cash equity and long forward equity. If repo rates are zero, the leg is worthless, with the discounted forward value and dividend income exactly cancelling the cash equities value. If repo rates are non-zero, this does not cancel, and the leg has a negative net present value, representing the opportunity cost of lending the stock. To make a fair deal, this must be compensated with a positive net present value on the interest leg, which is achieved with the stock lending fee being used as a spread under LIBOR – i.e. representing the repo rate.

Value of interest leg payments based on shares

These are calculated simply as the value of the interest payment, multiplied by the share price either fixed or predicted for the start date of the interest period. If actual principal exchange occurs, the collateral adjustment is added to this calculation.

$$VIR = F_1 \cdot P \cdot IR + P \cdot \left(F_0 - F_1\right)$$

Where:

VIR	=	Value of an interest rate payment based on shares
F_1	=	The equity fixing price at the start of the interest period.
P	=	The principal amount as a number of shares
IR	=	The interest payment for 1 unit of currency calculated in the normal fashion
F_0	=	The previous equity fixing price – i.e. the one that governed the collateral level before refixing.

Value of equity leg cashflows

$$VN = df_{pay} \times P \times \left(\frac{F_2 + D_{1,2}}{F_1} - 1 \right)$$

$$VS = df_{pay} \times P \times (F_2 - F_1 + D_{1,2})$$

Where:

VN	=	Present value of roll payment if principal is a constant notional amount
VS	=	Present value of roll payment if principal is a constant number of shares
Df_{pay}	=	Discounting factor to the payment date
P	=	Principal amount
F_2	=	Forward on end date
F_1	=	Forward on start date
$D_{1,2}$	=	Dividend returns between start date and end date. This will be scaled by a percentage.

$$F_2 = (S \cdot df_0 - pvd_2 - pvtc_2) \times \frac{rdf_2}{df_2} \qquad \frac{\partial F_2}{\partial S} = \frac{df_0 \cdot rdf_2}{df_2}$$

Where:

S	=	Spot
Pvd_2	=	Pv of dividend payments that are earned by holding stock to end date
$Pvtc_2$	=	Pv of tax credits that are earned by holding stock to end date
Rdf_2	=	Discounting factor due to repo rate to end date
Df_0	=	Currency discount factor for settlement date of spot deal
Df_2	=	Currency discount factor for settlement date of forward deal F_2

Similarly:

$$F_1 = (S \cdot df_0 - pvd_1 - pvtc_1) \times \frac{rdf_1}{df_1} \qquad \frac{\partial F_1}{\partial S} = \frac{df_0 \cdot rdf_1}{df_1}$$

Delta

$$\frac{\partial VN}{\partial S} = df_{pay} \times P \times \left(\frac{1}{F_1} \cdot \frac{\partial F_2}{\partial S} - \frac{F_2 + D_{1,2}}{F_1{}^2} \cdot \frac{\partial F_1}{\partial S} \right)$$

$$\frac{\partial VS}{\partial S} = df_{pay} \times P \times \left(\frac{\partial F_2}{\partial S} - \frac{\partial F_1}{\partial S} \right)$$

Gamma

$$\frac{\partial^2 VN}{\partial S^2} = df_{pay} \times P \times \left(\frac{-1}{F_1{}^2} \cdot \frac{\partial F_1}{\partial S} \cdot \frac{\partial F_2}{\partial S} - \frac{1}{F_1{}^2} \cdot \frac{\partial F_1}{\partial S} \cdot \frac{\partial F_2}{\partial S} + \frac{2 \times (F_2 + D_{1,2})}{F_1{}^3} \cdot \frac{\partial F_1}{\partial S} \cdot \frac{\partial F_1}{\partial S} \right) = \frac{-2}{F_1} \times \frac{\partial V}{\partial S} \cdot \frac{\partial F_1}{\partial S}$$

$$\frac{\partial^2 VS}{\partial S^2} = 0$$

Dividend sensitivity

Dividend sensitivities come from 3 sources – the sensitivities to the two forward prices F_1 and F_2, and the sensitivity to the dividend return $D_{1,2}$.

$$\frac{\partial V}{\partial D} = \frac{\partial V}{\partial F_1} \cdot \frac{\partial F_1}{\partial D} + \frac{\partial V}{\partial F_2} \cdot \frac{\partial F_2}{\partial D} + \frac{\partial V}{\partial D_{1,2}} \cdot \frac{\partial D_{1,2}}{\partial D}$$

Where:
D is a percentage increase in dividends

$$\frac{\partial VN}{\partial F_1} = -df_{pay} \times P \times \frac{F_2 + D_{1,2}}{F_1{}^2} \qquad \frac{\partial VS}{\partial F_1} = -df_{pay} \times P$$

$$\frac{\partial VN}{\partial F_2} = df_{pay} \times P \times \frac{1}{F_1} \qquad \frac{\partial VS}{\partial F_2} = df_{pay} \times P$$

$$\frac{\partial VN}{\partial D_{1,2}} = df_{pay} \times P \times \frac{1}{F_1} \qquad \frac{\partial VS}{\partial D_{1,2}} = df_{pay} \times P$$

$$\frac{\partial F_1}{\partial D} = \frac{\partial F_1}{\partial (pvd_1 + pvtc_1)} \cdot \frac{\partial (pvd_1 + pvtc_1)}{\partial D} = \frac{rdf_1}{df_1} \cdot \frac{(pvd_1 + pvtc_1)}{100}$$

$$\frac{\partial F_2}{\partial D} = \frac{\partial F_2}{\partial (pvd_2 + pvtc_2)} \cdot \frac{\partial (pvd_2 + pvtc_2)}{\partial D} = \frac{rdf_2}{df_2} \cdot \frac{(pvd_2 + pvtc_2)}{100}$$

Now $D_{1,2}$ has a two possible calculations that result in dividend sensitivity:

$$D_{1,2} = \frac{pd}{df_{pay}} \times \sum \left(d_i \times df_i \right)$$

Where

Pd	=	Dividend multiplier
d_i	=	A dividend payment amount earned in the swap period
df_I	=	Discounting factor for the payment date of the dividend

i.e. this is a calculation that compounds the dividends to the end date of the period.

An alternative calculation of $D_{1,2}$ is to simply sum the dividends:

$$D_{1,2} = pd \times \sum d_i$$

In both cases, increasing the amount of the dividends by 1% increases the dividend return calculated:

$$\frac{\partial D_{1,2}}{\partial D} = \frac{D_{1,2}}{100}$$

When it comes to setting the risk equivalent cashflows (Zpos in Adaptiv terminology), the two methods are different. We need to look at the value of the roll's sensitivity to an individual dividend payment di. To turn the sensitivity into an equivalent cash flow, we simply need to divide by the discount factor on the sensitivity date.

$$\frac{\partial V}{\partial d_i} = \frac{\partial V}{\partial F_1} \cdot \frac{\partial F_1}{\partial d_i} + \frac{\partial V}{\partial F_2} \cdot \frac{\partial F_2}{\partial d_i} + \frac{\partial V}{\partial D_{1,2}} \cdot \frac{\partial D_{1,2}}{\partial d_i}$$

$$\frac{\partial F_1}{\partial d_i} = -df_i \times \frac{d_i + tc_i}{d_i} \times \frac{rdf_1}{df_1}$$

$$\frac{\partial F_2}{\partial d_i} = -df_i \times \frac{d_i + tc_i}{d_i} \times \frac{rdf_2}{df_2}$$

when the dividend is earned in the forward period.

Now the dividend return sensitivity depends on which method you use:

For the compounding method:

$$\frac{\partial D_{1,2}}{\partial d_i} = pd \times \frac{df_i}{df_{pay}}$$

And for the simple sum method:

$$\frac{\partial D_{1,2}}{\partial d_i} = pd$$

N.B. In the case where the multiplier is 1 and a compounding method is used, this makes the equivalent Zpos for 1 share to be simply the dividend payment amount. In the case where the simple sum method is used, the equivalent Zpos is multiplied by a ratio of the discount factor at the payment date to the discount factor at the dividend date.

Repo sensitivity

Repo sensitivity stems purely from the forward price sensitivities

$$\frac{\partial V}{\partial R} = \frac{\partial V}{\partial F_1} \cdot \frac{\partial F_1}{\partial rdf_1} \cdot \frac{\partial rdf_1}{\partial R} + \frac{\partial V}{\partial F_2} \cdot \frac{\partial F_2}{\partial rdf_2} \cdot \frac{\partial rdf_2}{\partial R}$$

$$\frac{\partial F_1}{\partial rdf_1} \cdot \frac{\partial rdf_1}{\partial R} = \frac{(S \cdot df_0 - pvd_1 - pvtc_1)}{df_1} \times -T_1 \times rdf_1 \times rdf_1^{\frac{1}{T_1}} \quad = \quad F_1 \times -T_1 \times rdf_1^{\frac{1}{T_1}}$$

Where T_1 is the time in years to the forward date

Similarly for F_2:

$$\frac{\partial F_2}{\partial rdf_2} \cdot \frac{\partial rdf_2}{\partial R} = \frac{(S \cdot df_0 - pvd_2 - pvtc_2)}{df_2} \times -T_2 \times rdf_2 \times rdf_2^{\frac{1}{T_2}} \quad = \quad F_2 \times -T_2 \times rdf_2^{\frac{1}{T_2}}$$

Calculation for cliquets with floating strikes (non-asian)

Value

$$VN = df_{pay} \times \frac{P}{F_1} \times B(v, (k \cdot F_1), F_2 + D_{1,2} \cdots)$$

$$VS = df_{pay} \times P \times B(v, (k \cdot F_1), F_2 + D_{1,2} \cdots)$$

Where:

VN	=	Present value of roll payment if principal is a constant notional amount
VS	=	Present value of roll payment if principal is a constant number of shares
Df_{pay}	=	Discounting factor to the payment date
P	=	Principal amount
$B(v,F_1,F_2+D_{1,2}\ldots)$	=	Black model valuation of option with volatility = v, strike = F_1, Forward = $F_2+D_{1,2}$
F_2	=	Forward on end date
F_1	=	Forward on start date
k	=	Strike multiplier – e.g. 100% or 110%
$D_{1,2}$	=	Dividend return between start date and end date

$$F_2 = (S \cdot df_0 - pvd_2 - pvtc_2) \times \frac{rdf_2}{df_2} \qquad \frac{\partial F_2}{\partial S} = \frac{df_0 \cdot rdf_2}{df_2}$$

Where:

S	=	Spot
Pvd_2	=	Pv of dividend payments that are earned by holding stock to end date
$Pvtc_2$	=	Pv of tax credits that are earned by holding stock to end date
Rdf_2	=	Discounting factor due to repo rate to end date
Df_0	=	Currency discount factor for settlement date of spot deal
Df_2	=	Currency discount factor for settlement date of forward deal F_2

Similarly:

$$F_1 = (S \cdot df_0 - pvd_1 - pvtc_1) \times \frac{rdf_1}{df_1} \qquad \frac{\partial F_1}{\partial S} = \frac{df_0 \cdot rdf_1}{df_1}$$

Delta (with volatility in Black model constant as spot moves)

$$\frac{\partial VN}{\partial S} = df_{pay} \times \frac{P}{F_1} \times \left(\frac{\partial F_2}{\partial S} \cdot \frac{\partial B(v, (k \cdot F_1), F_2 + D_{1,2} \cdots)}{\partial F_2} + \frac{\partial F_1}{\partial S} \cdot k \cdot \frac{\partial B(v, (k \cdot F_1), F_2 + D_{1,2} \cdots)}{\partial (k \cdot F_1)} - \frac{1}{F_1} \cdot \frac{\partial F_1}{\partial S} \cdot B(\cdots) \right)$$

$$\frac{\partial VS}{\partial S} = df_{pay} \times P \times \left(\frac{\partial F_2}{\partial S} \cdot \frac{\partial B(v, (k \cdot F_1), F_2 + D_{1,2} \cdots)}{\partial F_2} + \frac{\partial F_1}{\partial S} \cdot k \cdot \frac{\partial B(v, (k \cdot F_1), F_2 + D_{1,2} \cdots)}{\partial (k \cdot F_1)} \right)$$

See the following pages for adjustment of delta taking into account volatility smiles against strike (“fixed”) and spot (“floating”).

Adjustment of delta to take into account volatility smiles

Forward unfixed cliquet periods are less sensitive to the spot price of their underlying shares than a fixed strike option and so their deltas are quite small. However, their vegas are still large – probably larger than normal options because they are always close to the money where vega is at a maximum, and also because the maturity of the options can be quite far out. Because of volatility smiles, delta adjustments need to be made as with normal fixed strike options, but because of the small delta returned from the Black model, and the large vega, these delta adjustments can be very large compared to the original delta. In fact they tend to dominate, with most of the delta being due to these smile effects.

Forward volatilities

Another complication arises due to the fact that the volatility time period starts on a forward date and we therefore need to use forward volatilities, calculated from “spot” volatilities using the arbitrage calculation below:

$$V_{1,2} = \sqrt{\frac{V_2^{\,2} \cdot T_2 - V_1^{\,2} \cdot T_1}{T_2 - T_1}}$$

Where:

$V_{1,2}$	=	Forward volatility from T1 to T2
V_1	=	“spot” volatility from T0 to T1
V_2	=	“spot” volatility from T0 to T2
T_1	=	Start date of forward period
T_2	=	Time from now to end date of forward period

Black model can give the following values:

$B(v,F_1,F_2+D_{1,2}\ldots)$	=	Undiscounted expectation value of the option. When discounted this becomes the fair premium.
DeltaForward	=	Derivative wrt F2, *assuming constant volatility*
DeltaStrike	=	Derivative wrt F1, *assuming constant volatility*
Vega	=	Derivative wrt the input (forward) volatility

Volatility smiles in Adaptiv Equity

In Adaptiv Equity, it is possible to create volatility smiles as varying by strike (“fixed”) or varying by relation to an at-the-money level (“floating”). If volatility smiles are purely fixed, then for normal options there is no adjustment to make because there is a fixed strike. However, for cliquets where the strike is dependent on the spot price, this does lead to a delta component. If volatility smiles are purely floating, this affects all option deltas.

Adaptiv Equity also allows different terms in the volatility structure to use different models. In the case where volatility requires an interpolation between two dates with different models, the volatility is first interpolated for each date on the smile for that date and then interpolated on date to give a final value. This can result in a mixed model – one that has some volatility sensitivity to strike and some to spot. The routines that give an interpolated volatility give the following outputs:

Vol	=	Interpolated volatility value
DvolBydSpot	=	Rate of change of the volatility value wrt Spot – This is zero if the interpolation uses a purely fixed smile.
DVolBydStrike	=	Rate of change of the volatility value wrt strike – This is zero if the interpolation uses a purely floating smile.

The easiest way for us to account for the volatility smiles is to simply adjust the values for Delta and Deltastrike obtained from the Black model using the more general:

$$\frac{\partial V}{\partial Strike} = \left(\frac{\partial V}{\partial Strike}\right)_{ConstVol} + \frac{\partial V}{dVol} \cdot \frac{\partial Vol}{\partial Strike}$$

$$\frac{\partial V}{\partial Spot} = \left(\frac{\partial V}{\partial Spot}\right)_{ConstVol} + \frac{\partial V}{\partial Vol} \cdot \frac{\partial Vol}{\partial Spot}$$

Or

$$DeltaStrike = DeltaStrike + Vega * dFwdVolBydStrike$$

(this will get the repo and dividend risks correct on the correct dates)

and then we need another adjustment to the delta spot as shown above:

$$DeltaSpot = DeltaSpot + Vega * dFwdVolBydSpot$$

Where:

Vega	=	An adjusted value (see below) for a shift in volatilities starting today rather than forward volatilities
DfwdVolBydSpot	=	The rate of change of the forward volatility wrt spot
DfwdvolBydStrike	=	The rate of change of the forward volatility wrt strike

$$V_{1,2} = \sqrt{\frac{V_2{}^2 \cdot T_2 - V_1{}^2 \cdot T_1}{T_2 - T_1}}$$

$$\frac{\partial V_{1,2}}{\partial V_1} = \frac{-1}{V_{1,2}} \cdot \frac{V_1 \cdot T_1}{(T_2 - T_1)}$$

$$\frac{\partial V_{1,2}}{\partial V_2} = \frac{1}{V_{1,2}} \cdot \frac{V_2 \cdot T_2}{(T_2 - T_1)}$$

So vega multiplier is simply the sum of these 2.

And to get the DfwdVolBydSpot, and dFwdVolBydStrike

$$\frac{\partial V_{1,2}}{\partial Spot} = \frac{\partial V_{1,2}}{\partial V_1} \cdot \frac{\partial V_1}{\partial Spot} + \frac{\partial V_{1,2}}{\partial V_2} \cdot \frac{\partial V_2}{\partial Spot}$$

$$\frac{\partial V_{1,2}}{\partial Strike} = \frac{\partial V_{1,2}}{\partial V_1} \cdot \frac{\partial V_1}{\partial Strike} + \frac{\partial V_{1,2}}{\partial V_2} \cdot \frac{\partial V_2}{\partial Strike}$$

Gamma

Assuming that there is no gamma due to the volatility smile:

$$VN = df \times \frac{P}{F_1} \times B(\sigma, k \cdot F_1, F_2, ...)$$

$$\frac{\partial VN}{\partial S} = df \times \frac{P}{F_1} \times \left[\frac{\partial (k \cdot F_1)}{\partial S} \cdot \frac{\partial B}{\partial (k \cdot F_1)} + \frac{\partial F_2}{\partial S} \cdot \frac{\partial B}{\partial F_2} - \frac{B}{F_1} \cdot \frac{\partial F_1}{\partial S} \right] = df \times \frac{P}{F_1} \times [A]$$

$$\frac{\partial^2 VN}{\partial S^2} = df \times P \times \left[\frac{1}{F_1} \cdot \frac{\partial A}{\partial S} - \frac{A}{F_1^{\,2}} \cdot \frac{\partial F_1}{\partial S} \right]$$

$$\frac{\partial A}{\partial S} = \frac{\partial X}{\partial S} + \frac{\partial Y}{\partial S} - \frac{\partial Z}{\partial S} \quad where\ X = \frac{\partial (k \cdot F_1)}{\partial S} \cdot \frac{\partial B}{\partial (k \cdot F_1)} \quad Y = \frac{\partial F_2}{\partial S} \cdot \frac{\partial B}{\partial F_2} \quad Z = \frac{B}{F_1} \cdot \frac{\partial F_1}{\partial S}$$

$$\frac{\partial X}{\partial S} = k \cdot \frac{\partial F_1}{\partial S} \cdot \left(k \cdot \frac{\partial F_1}{\partial S} \cdot \frac{\partial^2 B}{\partial (kF_1)^2} + \frac{\partial F_2}{\partial S} \cdot \frac{\partial^2 B}{\partial (kF_1) \partial F_2} \right) + k \cdot \frac{\partial^2 F_1}{\partial S^2} \cdot \frac{\partial B}{\partial (kF_1)}$$

$$\frac{\partial Y}{\partial S} = \frac{\partial^2 F_2}{\partial S^2} \cdot \frac{\partial B}{\partial F_2} + \frac{\partial F_2}{\partial S} \cdot \left(k \cdot \frac{\partial F_1}{\partial S} \cdot \frac{\partial^2 B}{\partial F_2 \partial (kF_1)} + \frac{\partial F_2}{\partial S} \cdot \frac{\partial^2 B}{\partial F_2^{\,2}} \right)$$

$$\frac{\partial Z}{\partial S} = \frac{B}{F_1} \cdot \frac{\partial^2 F_1}{\partial S^2} - \frac{B}{F_1^{\,2}} \cdot \left(\frac{\partial F_1}{\partial S} \right)^2 + \frac{1}{F_1} \cdot \frac{\partial F_1}{\partial S} \cdot \left(k \cdot \frac{\partial F_1}{\partial S} \cdot \frac{\partial B}{\partial (kF_1)} + \frac{\partial F_2}{\partial S} \cdot \frac{\partial B}{\partial F_2} \right)$$

Dividend sensitivity

Dividend sensitivities come from 3 sources – the sensitivities to the two forward prices F_1 and F_2, and the sensitivity to the dividend return $D_{1,2}$.

$$\frac{\partial V}{\partial D}=\frac{\partial V}{\partial F_1}\cdot\frac{\partial F_1}{\partial D}+\frac{\partial V}{\partial F_2}\cdot\frac{\partial F_2}{\partial D}+\frac{\partial V}{\partial D_{1,2}}\cdot\frac{\partial D_{1,2}}{\partial D}$$

Where:
D is a percentage increase in dividends

$$\frac{\partial VN}{\partial F_1}=-df_{pay}\times P\times\frac{F_2+D_{1,2}}{F_1^{\ 2}}\times DeltaStrike \qquad \frac{\partial VS}{\partial F_1}=-df_{pay}\times P\times DeltaStrike$$

$$\frac{\partial VN}{\partial F_2}=df_{pay}\times P\times\frac{1}{F_1}\times DeltaForward \qquad \frac{\partial VS}{\partial F_2}=df_{pay}\times P\times DeltaForward$$

$$\frac{\partial VN}{\partial D_{1,2}}=df_{pay}\times P\times\frac{1}{F_1}\times DeltaForward \qquad \frac{\partial VS}{\partial D_{1,2}}=df_{pay}\times P\times DeltaForward$$

$$\frac{\partial F_1}{\partial D}=\frac{\partial F_1}{\partial(pvd_1+pvtc_1)}\cdot\frac{\partial(pvd_1+pvtc_1)}{\partial D}=\frac{rdf_1}{df_1}\cdot\frac{(pvd_1+pvtc_1)}{100}$$

Now $D_{1,2}$ has a two possible calculations that result in dividend sensitivity:

$$\frac{\partial F_2}{\partial D}=\frac{\partial F_2}{\partial(pvd_2+pvtc_2)}\cdot\frac{\partial(pvd_2+pvtc_2)}{\partial D}=\frac{rdf_2}{df_2}\cdot\frac{(pvd_2+pvtc_2)}{100}$$

$$D_{1,2}=\frac{pd}{df_{pay}}\times\sum\left(d_i\times df_i\right)$$

Where

Pd	=	Dividend multiplier
d_i	=	A dividend payment amount earned in the swap period
df_I	=	Discounting factor for the payment date of the dividend

i.e. this is a calculation that compounds the dividends to the end date of the period.

An alternative calculation of $D_{1,2}$ is to simply sum the dividends:

$$D_{1,2}=pd\times\sum d_i$$

In both cases, increasing the amount of the dividends by 1% increases the dividend return calculated:

$$\frac{\partial D_{1,2}}{\partial D}=\frac{D_{1,2}}{100}$$

When it comes to setting the risk equivalent cashflows (Zpos in Adaptiv terminology), the two methods are different. We need to look at the value of the roll’s sensitivity to an individual dividend payment di. To turn the sensitivity into an equivalent cash flow, we simply need to divide by the discount factor on the sensitivity date.

$$\frac{\partial V}{\partial d_i}=\frac{\partial V}{\partial F_1}\cdot\frac{\partial F_1}{\partial d_i}+\frac{\partial V}{\partial F_2}\cdot\frac{\partial F_2}{\partial d_i}+\frac{\partial V}{\partial D_{1,2}}\cdot\frac{\partial D_{1,2}}{\partial d_i}$$

$$\frac{\partial F_1}{\partial d_i} = -df_i \times \frac{d_i + tc_i}{d_i} \times \frac{rdf_1}{df_1}$$

$$\frac{\partial F_2}{\partial d_i} = -df_i \times \frac{d_i + tc_i}{d_i} \times \frac{rdf_2}{df_2}$$

when the dividend is earned in the forward period.

The dividend return sensitivity depends on which method you use:

For the compounding method:

$$\frac{\partial D_{1,2}}{\partial d_i} = pd \times \frac{df_i}{df_{pay}}$$

And for the simple sum method:

$$\frac{\partial D_{1,2}}{\partial d_i} = pd$$

N.B. In the case where the multiplier is 1 and a compounding method is used, this makes the equivalent Zpos for 1 share to be simply the dividend payment amount. In the case where the simple sum method is used, the equivalent Zpos is multiplied by a ratio of the discount factor at the payment date to the discount factor at the dividend date.

Repo sensitivity

Repo sensitivity stems purely from the forward price sensitivities

$$\frac{\partial V}{\partial R} = \frac{\partial V}{\partial F_1} \cdot \frac{\partial F_1}{\partial rdf_1} \cdot \frac{\partial rdf_1}{\partial R} + \frac{\partial V}{\partial F_2} \cdot \frac{\partial F_2}{\partial rdf_2} \cdot \frac{\partial rdf_2}{\partial R}$$

$$\frac{\partial F_1}{\partial rdf_1} \cdot \frac{\partial rdf_1}{\partial R} = \frac{(S \cdot df_0 - pvd_1 - pvtc_1)}{df_1} \times -T_1 \times rdf_1 \times rdf_1^{\frac{1}{T_1}} \quad = \quad F_1 \times -T_1 \times rdf_1^{\frac{1}{T_1}}$$

Where T_1 is the time in years to the forward date

Similarly for F_2:

$$\frac{\partial F_2}{\partial rdf_2} \cdot \frac{\partial rdf_2}{\partial R} = \frac{(S \cdot df_0 - pvd_2 - pvtc_2)}{df_2} \times -T_2 \times rdf_2 \times rdf_2^{\frac{1}{T_2}} \quad = \quad F_2 \times -T_2 \times rdf_2^{\frac{1}{T_2}}$$

Adaptiv Equity convertible bond model

Overview

The Adaptiv Equities convertible bond model has the following characteristics:

- Single stochastic factor (underlying stock price)
- Binomial decision tree
- Term structured volatility
- Term structured interest rates and stock repo rates.
- Proportional discrete dividends
- Credit spread modelled as a single figure rather than a term structure
- Adjustments for volatility "smile"

The model uses the same binomial tree to represent movements in the underlying stock price as is used for pricing American style options. This decision tree itself, but not how it used for convertible bonds, is described in detail in the chapter "Binomial Tree Geometry" of this document.

Using the binomial tree

As with any tree model, the value is obtained by working backwards from the last "time slice" to today, setting the value at each node according to the options available at that time. At each node, we need to consider the following possibilities:

1. The holder could convert the bond to underlying stock.
2. If the bond has a put feature active, the holder could put the bond.
3. The holder could leave the bond as a bond and receive coupons, and finally the redemption. The holder is also leaving his options open by doing this, which also affects the value of this course of action.
4. If the bond has a call feature active, the *issuer* could call the bond.

The value of conversion

A node in the tree is a point in time and underlying stock price. You need first to consider whether the time is within an exercise window allowed by the issue. If not, the value of conversion is zero. If the time is within an exercise window, for simple convertibles the value of conversion at the node is the conversion ratio allowed at the time multiplied by the stock price of the node, minus any extra amount to be paid on conversion (this is a clause in some convertibles). The value maybe further modified by loss of accrued interest, forfeit of coupon if converted on a coupon date etc. The conversion value for some hybrids such as DECS, PEPS etc. is more complicated, with the conversion ratio being allowed to vary between minimum and maximum values. These more complicated payoffs are described in more detail below.

The value of the put option

The value of putting the bond is simply to be paid the strike price of the put option.

The value of leaving the bond as a bond

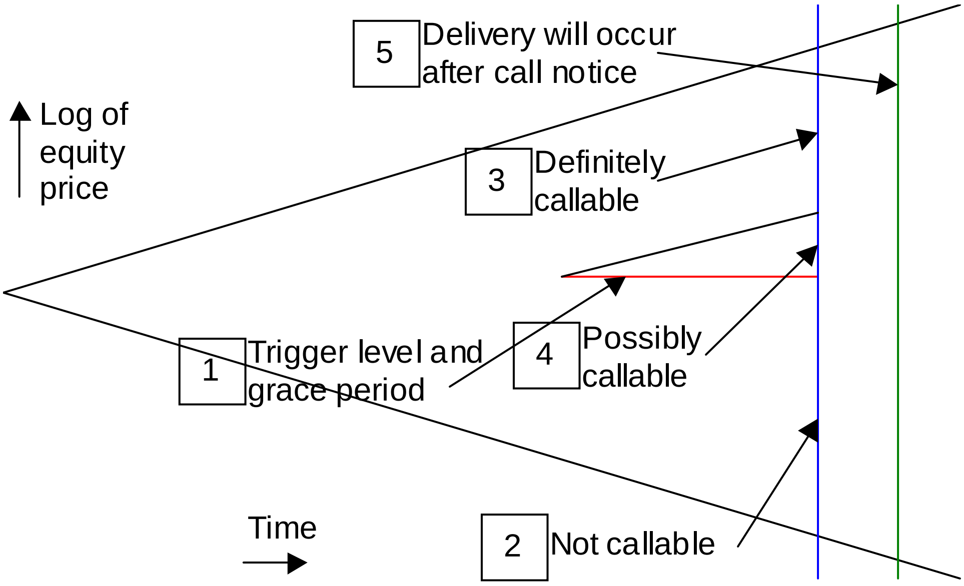
As a binomial tree is being used, with a 50/50 probability distribution in the tree geometry, part of the value of leaving the bond as a bond is the average of the two adjacent nodes in the next time slice of the tree discounted back from that time to the current node time. This part of the value can be seen as the value of leaving the holder's options open. The other part of the value is the sum of the cash flows to be earned by holding the bond from this time slice to the next. On the last time slice, this includes the redemption payment (only if the holder is allowed to leave the bond as a bond at the end – Mandatory conversion contracts do not allow this)

The value of the call option

The call option is held by the *issuer*, not by the bondholder, and is therefore used differently in the tree calculations. It is further complicated by the fact that call options can have quite complex clauses, including "triggers" and "grace periods". These mean that the issuer can only call the bond if the bond price has been above a certain level (the "trigger" level) for a certain period beforehand (the grace period). For some convertibles, the bond only needs to be above the trigger for a certain fraction of the days before calling. This means that whether or not the issuer has a right to call the bond at any node is not governed by simple values in the tree, but by a probability. This cannot be properly accounted for in decision tree techniques because of the path dependency. However, rather than resort to a completely different technique such as Monte Carlo, it is usual to provide an approximate solution. See below for how we have handled this issue.

Handling call triggers

The issuer may give notice that the bond will be called only if the underlying price has been above a trigger level for a period of time called the grace period. This is included in the model as follows:



1. The red line represents the level of the trigger and length of the grace period stretching back from the current time, represented by the vertical blue line.
2. If the spot is below the trigger level currently then the bond is not callable.
3. The black line above the red line represents the spot price being at the trigger level at the start of the grace period and then rising as fast as the tree allows. If the current spot price is above where the black line meets the blue line then the underlying price has definitely been above the trigger level for the grace period and the bond is callable.
4. If the underlying price is between these two levels we approximate the probability that the bond is callable by linear interpolation:
5. If notice of call is given today the call will take place after a delay called the notice period. Hence the value if notice of call is given is the call payment discounted over the notice period.

Calculation of value at a node

The value at a node is calculated as follows:

$$Node = CP * Min(Max(Bond, Convert, Put), Call) + (1 - CP) * Max(Bond, Convert, Put)$$

Where:

Node	=	The node value
CP	=	The probability that the issuer can call the bond at this node.
Bond	=	The value of leaving the bond as a bond for the next time step, i.e. the value of leaving the options open plus any cash received. If the contract is mandatory conversion at maturity, this value is zero on the last time slice.

Convert	=	The value of conversion now at this node. Set to zero if the option is not available at this time.
Put	=	The values of putting the bond for the put strike price. Set to zero if the option is not available at this time.
Call	=	The value of being called, i.e. the call strike price at this time.

Vasicek Model

The Vasicek Model is one of several models, which addresses to a large extent the evolution of interest rates over time. One important aspect of interest rates is that they appear to be pulled back to some long-run average level over time. This phenomenon is known as *mean reversion.* There are compelling arguments in favour of mean reversion. For example, when rates are high, the economy tends to slow down and there are fewer requirements for funds on the part of borrowers. As a result, rates decline. When rates are low, there tends to be high demand for funds on the part of borrowers. As a result, rates tend to rise. A particular feature of the Vasicek model is that the *mean reversion* of short-term interest rates is incorporated. The rates have a Gaussian distribution about a mean level.

The evolution of the discount bond price P_T can be described by a stochastic differential equation as follows:

$$dP_T = \mu_T P_T dt + \sigma_{2T} P_T dZ$$

where:

T	is the bond maturity time
Z	is standard Brownian motion
μ_T	is the investor's belief of the rate of growth of the bond price
σ_{2T}	is a deterministic function given by

$$\sigma_{2T} = \frac{\sigma_2}{\alpha}(1 - e^{-\alpha(T-t)})$$ where α and α_2 are positive constants.

The associated short rate r_t satisfies the stochastic differential equation:

$$dr = \alpha(m - r)dt + \sigma_2 dZ$$

The short rate is pulled to to a level *m at rate a.* Superimposed upon this is a normally distributed stochastic term $\sigma_2 dZ$. The parameter α (Alpha) is known as the reversion rate and σ_2 (Sigma) the Vasicek volatility.

Coupon tax credits

To cope with coupon tax credits, Adaptiv have created a new document “Tax Regime”, which is there to be chosen from an issue screen to create tax credits against coupons, which will increase the present value of those coupons, but not affect any accrued interest calculations. This works in a very similar way to the tax credit functionality in the market parameters screen, and will probably be used there also in future versions.

DECS, PERCS, ELKS and PEPS

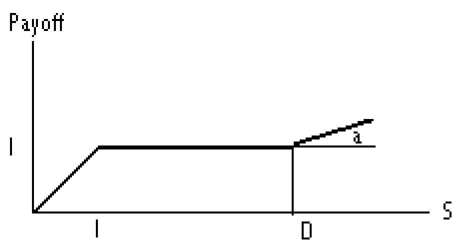
Adaptiv Equity DECS, PERCS, ELKS and PEPS are modelled as convertibles with additional embedded options and payoff conditions.

Payoff for DECS

The DECS contract is a mandatory conversion contract. Therefore, if the holder still owns the DEC at the maturity date, he/she must convert it into shares. The pay-off for the DECS contract is calculated in the following way:

S	if	S < I
I	if	I <= S <= D
aS	if	S > D

where
S is the underlying asset price at conversion
D is the conversion price



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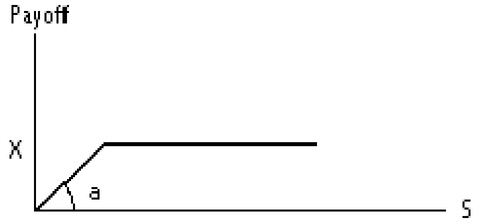
I is the issue price
a is the conversion ratio
I = aD

Payoff for PERCS

The pay-off for the PERCS contract is calculated in the following way:

aS if aS < X
X if X <= aS



where:
S is the underlying Share price at conversion. Share price is the current market value of a single share of the underlying asset.

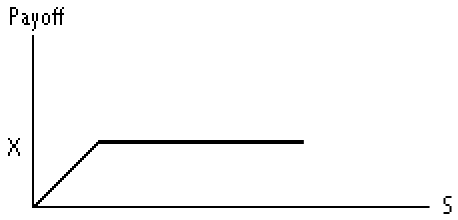
a is the Conversion ratio. Conversion ratio is the number of common shares per PERCS.

X is the Cap price. Cap price is the maximum possible payoff from the PERCS contract at maturity. This is entered as an absolute value. If the Share price is below this level, then the payoff is a single unit of stock. Otherwise, the payoff is the number of shares equivalent to the Cap price.

Payoff for ELKS

The pay-off for the ELKS contract is calculated in the following way:

S	if	S < X
X	if	X <= S



where
S is the underlying asset price at conversion,
X is the cap price.

It is worthwhile to note that this pay-off is exactly the same as for the PERCS except for the fact that an ELK is never callable, so we need only to specify the cap price.

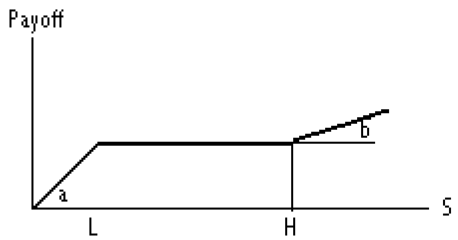
Payoff for PEPS

The PEPs contract is a mandatory conversion contract. Therefore, if the holder still owns the DEC at the maturity date, he/she must convert it into shares. There is a choice between the two alternative ways of calculating a PEPs contract: Payoff type *Simple* or *Complex*.

Payoff Simple:
aL = I = bH

Payoff Complex:

aS	if S < L
I	if L <= S <= H
bS	if S > H



where:

a is the maximum conversion ratio,
b is the minimum conversion ratio,
I is the issue price,
L is the lower price,
H is the higher price,
S is the underlying asset price at conversion.

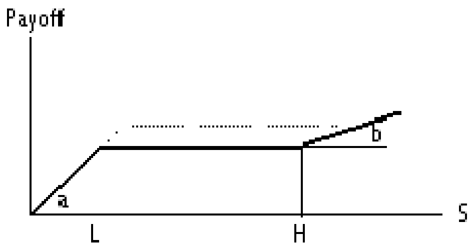
Simple case payoff

Converting Preference Shares

These securities are usually mandatory convertibles that convert into a number of shares based on the ordinary price at maturity *S*, a specific discount *d* and the par price *P* (or issue price). The number of ordinary shares issued is $P/(S(1-d))$ subject to maximum and minimum bounds, R_{Max} and R_{Min}. Hence the value on conversion is $S\max(\min(P/(S(1-d)), R_{Max}), R_{Min})$. This is a similar structure to PEPS but with the inclusion of the discount on conversion *d*. These securities are usually not redeemable. These securities have an attached "dividend" that is sometimes franked (tax imputation). Early conversion is sometimes at the option of the holder (and may be on a "one-for-one" basis) and sometimes at the option of the issuer. Prices are quoted "dirty".

Usually the discount, par price, maximum conversion price, minimum conversion price and dividend are fixed. There have been issues where they have changed over time.

The effect of the discount rate on the payoff is shown below – It moves the central constant level up and consequently L and H are moved to higher spot levels.



Implied volatility

Method

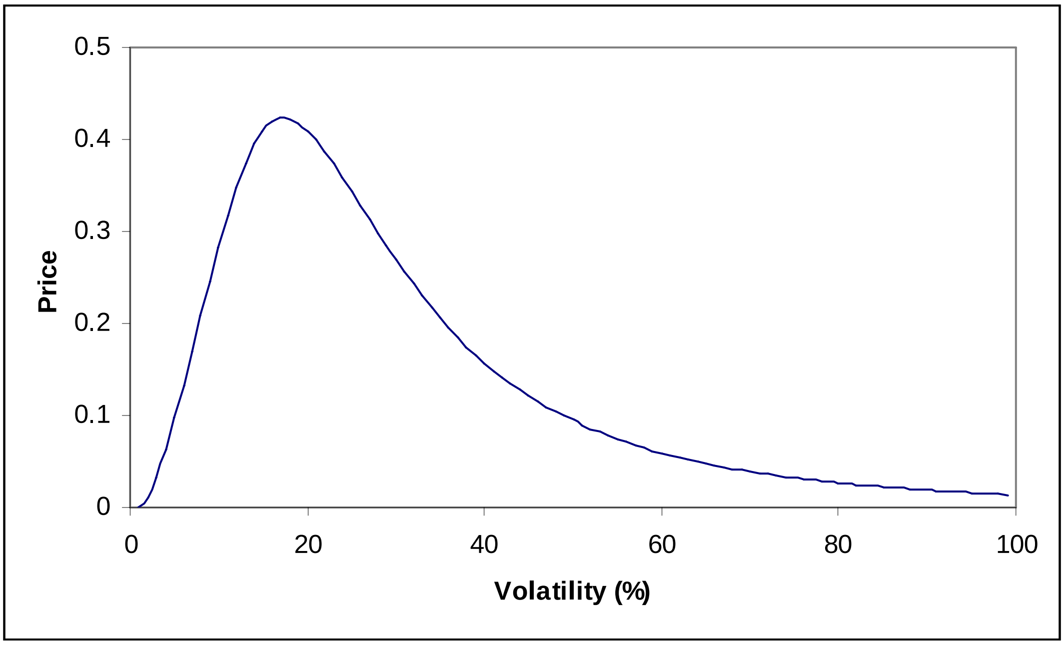
For most options, we use the Van Wijngaarden-Dekker-Brent algorithm as described in Press *et al.* (1992), which combines a bisection method for stability with inverse parabolic interpolation to attain the superlinear convergence desired for reasonable speed. The solution is found to a precision of 0.0001% in volatility.

Boundaries

The calculated implied volatilities for a warrant or convertible bond are limited to be between the lower and upper bound specified in the Pricing tab of the Issue screen. If these fields are left blank the lower bound defaults to 0 and the upper to 400%.

Barrier options

For barrier options, implied volatility calculations are more complex since in many cases the option price is not monotonic with volatility as it is for simple options. For example, a slightly out-of-the-money up-and-out call has a low price for low volatility (since the intrinsic option is of little value), a low price for high volatility (since it will almost certainly knockout before expiry) and a significant price for some volatility in between. Thus for any given price there will typically be either 0 or 2 values of the volatility which would produce such a price, as shown below. Clearly, implying a volatility for such an option depends on whether or not there is a solution and if so, which one is desired. We return the lower of the two solutions in such cases, or 0 if there is no solution.



Corporate action deal processing

Corporate Action Deal Processing With The Close/Open Methodology

The *Close/Open Method*ology is intended for use in processing corporate actions that affect a large number of deals. Instead of having to modify all existing deals in a database to reflect the appropriate changes, the *Close/Open Method* requires only one *closing* and one *opening* deal (per destination stock in each *bucket*[1]).

An *opening* deal will close out the existing position (within each *bucket*) and this will be followed by an *opening* deal (or deals for a *De-Merger* or *Spin-Off* corporate action) in the appropriate position (or positions) following the corporate action processing.

Bucketing, also referred to as partitioning, allows the *Close/Open Method* processing to act upon groups of deals with particular properties. For example, if bucketing by *TradingAreaID* is requested then the *Close/Open Methodology* will be applied on the deals grouped together with the same trading area. Therefore, if there is more than one variation in the trading areas of the deals then a *closing* and *opening* deal will be written for each bucket's variation.

Close/Open Methodology Summary

The *Close/Open Method* is currently implemented for the following corporate action types: De-Merger, Spin-Off, Merger, Bonus Issue, and Stock Split. It is not applicable for Rights Issue corporate actions.

Consider the following generic representation for the above corporate actions:

$$n \rightarrow \sum_{i=1}^{M} n'_i + c$$

where n is the number of original stock,
M is the number of new issues,
n'_i is the number of the i-th new stock held after the action,
c is new cash.

In determining the number of shares in the new stock and cash to be issued the above definition is scaled to the position held, i.e.:

$$N \rightarrow \sum_{i=1}^{M} N'_i + C$$

where $N = n \times s$ is the position held of original stock, i.e. the scaling factor is $s = {}^{N}\!/_{n}$,
$N'_i = n'_i \times s$ is the position held in the i-th new stock held after the action,
$C = c \times s$ is cash received according to the position held in the original stock, and

There are two constraints that can be placed upon this corporate action:

Preservation Of Value

[1] A *bucket* refers to a collection of deals with the same combination of specified properties, such as *TraderID*, *AccountingAreaID* etc.

The value held in the original stock value must be preserved in the transferral to the new stocks and cash component. This gives our first constraint:

Equation 1 - Preservation Of Value:

$$Np(t) = Cf^{C}(t) + \sum_{i=1}^{M} N_i' p_i'(t) f_i'(t)$$

where $p(t)$ is the price of the original stock,

$p_i'(t)$ is the price of the i-th new stock at the time of the corporate action,

$f'_i(t)$ is the exchange rate to convert $p_i'(t)$ to the currency of the original stock,

and $f^{C}(t)$ is the exchange rate to convert C to the currency of the original stock.

[IMPORTANT NOTE: To implement cross-currency[2] corporate actions the correct exchange rates should be used. The convention used is to quote the rates to convert the value in question to the same currency as that of the original issue.]

Preservation Of Profit And Loss

The P&L (including cash) must[3] be preserved through the corporate action, in the currency of the local stock:

Equation 2 - Preservation Of P&L:

$$N\left[p(t) - p(0)\right] = Cf^{C}(t) + \sum_{i=1}^{M} N_i'\left[p_i'(t) - p_i'(0)\right] f_i'(t)$$

where $p(0)$ is the average cost price of the original stock,

and $p_i'(0)$ is the book price for the i-th new stock.

By defining $w_i'(t)$ as the fraction of the original stock value assigned to the i-th stock, such that

$$\sum_{i=1}^{M} w_i'(t) = 1,$$

we can re-write Equation 1 and Equation 2 as follows:

Equation 3 - Preservation Of Value For Each Destination Stock:

$$w_i'(t)\left\{Np(t) - Cf^{C}(t)\right\} = N_i' p_i'(t) f_i'(t)$$

Equation 4 - Preservation Of P&L For Each Destination Stock:

$$w_i'(t)\left\{N\left[p(t) - p(0)\right] - Cf^{C}(t)\right\} = N_i'\left[p_i'(t) - p_i'(0)\right] f_i'(t)$$

Subtracting Equation 4 from Equation 3 gives the price to book the opening deals:

Equation 5 - Booked Price For Each Destination Stock:

$$\boxed{p_i'(0) = w_i'(t)\,\frac{N}{N_i'}\,\frac{p(0)}{f_i'(t)}}$$

[2] In context of corporate actions the phrase cross-currency means that either the any of the new stock issues or a cash component has a different currency from that of the original stock issue.
[3] The only exception to the preservation of *P&L* is when share rounding occurs; see the example on Stock Split With Share Rounding.

Note: the corresponding theoretical post-corporate action price is (from Equation 3):

Equation 6 - Non-Arbitrage Theoretical Opening Price For Each Destination Stock:

$$p_i'(t) = w_i'(t)\left[\frac{N}{N_i'}\frac{p(t)}{f_i'(t)} - \frac{C}{N_i'}\frac{f^C}{f_i'(t)}\right]$$

Trades to Book

When the corporate action is applied to the trades held, the corporate action needs to be scaled to the position held, i.e.:

To affect the corporate action, for every N original stock held:

- Book a closing sale trade of N shares in the original stock at $p(0)$, with a transaction cost of C.
- Book an opening purchase of N_i' new stock at $p_i'(0)$.

Cash Balance Adjustment

The trades that Adaptiv uses to affect a change in position due to the corporate action are “artificial”, i.e. these trades do not actually take place. Without correction (which we will implement with transaction costs) these trades can produce artificial movements in the cash balance. We can illustrate this most clearly with a cross-currency corporate action.

Consider an idealistic de-merger of stock A.L (London stock exchange) to 2 B.N (New York Stock exchange):

1. If, just prior to the corporate action, a trader is long 1,000 shares of A.L that were purchased at £10, then the cash balance is -£10,000.
2. Without any correction Adaptiv would processes the corporate action with:
   - A closing deal selling 1,000 shares of A.L at £10. This would cancel the existing cash balance leaving £0.
   - An opening deal purchasing 2,000 shares of B.N at of 2,000 at $7.5 (assuming USD/GBP = 1.5). The cash balance would then be -$15,000.

Thus, without correction the cash balance moves from -£10,000 to -$15,000.

As noted, we can correct this by writing transaction costs that are the reverse of the cost of the deals being generated by Adaptiv during the corporate action processing. So, with our example:

3. With cash balance adjustment transaction costs:
   - A closing deal selling 1,000 shares of A.L at £10, **with a transaction cost of -£10,000.** The transaction cost neutralises the £10,000 generated by the artificial closing deal, leaving the original £10,000 from the original, and real, trade on A.L.
   - An opening deal purchasing 2,000 shares of B.N at of 2,000 at $7.5 (assuming USD/GBP = 1.5), **with a transaction cost of +$15,000.** The transaction cost neutralises the cost from the opening deal, meaning the existing cash balance of -£10,000 is unaffected.

In principal these cash balance adjustment transaction costs only need to be written when the currency:market any of the new stocks is different from that of the original stock, however because merger corporate actions are effectively separate corporate actions these cash balance adjustments are written for the close and opening deals in all cases.

[Note, in the worked example below we will neglect discussion of the cash balance transaction costs as they will occur in all cases.]

Worked Examples

Stock Split

Consider a "2 for 1" stock split of A.N (i.e. the number of shares issue by A.N has doubled). Suppose 10 shares are held in A.N that were bought at \$23.5, and at the time of the corporate action, t, are worth \$24:

- $s = 10/1 = 10$ (the scaling factor from the corporate action announcement to the position held),
- $N = 10$,
- $M = 1$,
- $N_1' = 20$,
- $C = \$0$,
- $p(0) = \$23.50$,
- $p(t) = \$24.00$,
- $f_1'(t) = 1.0$,
- $f^C(t) = 1.0$,
- $w_1'(t) = 1$.

Then, from Equation 5 we find the new book price for the A.N stock to be:

$$p_1'(0) = 1 \times \frac{10}{20} \times \frac{\$23.50}{1.0} = \$11.75$$

and from Equation 6 the corresponding market price is:

$$p_1'(t) = 1 \times \left[\frac{10}{5} \times \frac{\$24}{1.0} - \frac{\$0}{10} \times \frac{1.0}{1.0} \right] = \$12$$

We now check our two constraints:

- **Preservation of Value:** At time t Equation 1 gives:
  $10 \times \$24 = \$0 \times 1.0 + (20 \times \$12 \times 1.0)$
  i.e. $\$240 = \$240$ and so market value is preserved.
- **Preservation of P&L:** At time t Equation 2 gives:
  $10 \times [\$24.00 - \$23.50] = \$0 \times 1.0 + (20 \times [\$12 - \$11.75] \times 1.0)$
  i.e. $\$5 = \$5$ and so P&L is preserved.

Trades To Booked In Adaptiv

To affect this corporate action in Adaptiv the existing position in A.N is closed with a trade of 10 shares at \$23.50, and then re-opened with a new trade of 20 shares at \$11.75. To preserve *P&L* [4]the market parameter prices for A.N should be \$24 before the corporate action and \$12 afterwards.

De-Merger Into 2 Cross-Currency Stocks

Consider the de-merger of 5 A.N (quoted on the New York Stock Exchange) into 10 B.L (quoted on the London Stock Exchange), 1 C.N (NYSE) and £5 cash. Suppose 5 shares are held in A.N that were bought at \$23.5, and at the time of the corporate action, t, are worth \$24, and that the exchange rate to convert GBP to USD is 1.6 then:

- $s = 1$,

[4] When archiving *P&L* in Panorama deals should be added to the portfolio using a selection rule (to eliminate unsettled deals, i.e. in the *Dates* tab set-up so that deal *inclusive 0d Before TradeDates* and *inclusive 0d After* the *ExpiryDate*), rather than simply adding the deals in manually, otherwise the positions will be affected by deals that have yet to settle, disturbing the *P&L* calculation.

- $N = 5$,
- $M = 2$,
- $N'_1 = 10$ for B.L,
- $N'_2 = 1$ for C.N,
- $C = £5$.
- $p(0) = \$23.50$,
- $p(t) = \$24.00$,
- $f'_1(t) = 1.6$ as A.N is in quoted in \$ and B.L in £,
- $f'_2(t) = 1.0$ as both A.N and C.N are quoted in \$,
- $f^C(t) = 1.6$ as A.N is quoted in \$ and the cash component is in £.

Suppose it is also expected that the C.N will be worth twice that of A.N, i.e.:

- $w'_1(t) = \frac{2}{2+1} = 0.66\dot{6}$
- $w'_2(t) = \frac{1}{2+1} = 0.33\dot{3}$

Then, from Equation 5 we find the book price for the B.L stock to be:

$$p'_1(0) = 0.66\dot{6} \times \frac{5}{10} \times \frac{\$23.50}{1.6} = £4.895833$$

and from Equation 6 the corresponding market price is:

$$p'_1(t) = 0.66\dot{6} \times \left[\frac{5}{10} \times \frac{\$24}{1.6} - \frac{£5}{10} \times \frac{1.6}{1.6}\right] = £4.666667$$

Again from Equation 5 we find the book price for the C.N stock to be:

$$p'_2(0) = 0.33\dot{3} \times \frac{5}{5} \times \frac{\$23.50}{1.0} = \$7.833333$$

and from Equation 6 the corresponding market price is:

$$p'_2(t) = 0.33\dot{3} \times \left[\frac{5}{5} \times \frac{\$24}{1.0} - \frac{£5}{5} \times \frac{1.6}{1.0}\right] = \$7.466667$$

We now check our two constraints:

- **Preservation of Value:** At time t Equation 1 gives:
  $$5 \times \$24 = £5 \times 1.6 + (10 \times £4.666667 \times 1.6 + 5 \times \$7.466667 \times 1.0)$$
  i.e. $\$120 = \$120$ and so market value is preserved.
- **Preservation of P&L:** At time t Equation 2 gives:
  $$5 \times [\$24.00 - \$23.50] = £5 \times 1.6 + (10 \times [£4.666667 - £4.895833] \times 1.6 + 5 \times [\$7.466667 - \$7.833333] \times 1.0)$$
  i.e. $\$2.5 = \$2.5$ and so P&L is preserved.

<u>Trades To Booked In Adaptiv</u>

To affect this corporate action in Adaptiv the position in A.N is closed with a trade of 5 shares at \$23.50, and the cash is written in a transaction cost £5 on this deal; the position in B.L is then opened 10 shares at a priced at £4.90, and the position in C.N is opened with a deal of 1 shares at \$7.83. To preserve *P&L* the market parameter prices for A.N, B.L and C.N should be \$24, £4.67 (for a USD/GBP exchange rate of 1.6), and \$7.47, respectively.

Merger Of 2 Cross-Currency Stocks

It is important to note that a merger event is not strictly the reverse of a de-merger. In a de-merger the holder of the original stock will receive stock(s) in all of the specified new issue stocks. However, with a merger a holder of any (i.e. not necessarily all) of the original stocks will receive stock in the new issue. This means that, in terms of trade processing, a merger event may be considered a collection of associated stock split events (and currently Adaptiv actually effects a merger as a collection of separate events).

Consider the merger of 25 B.N (NYSE) with 8 C.L (LSE), producing 2 shares in A.N (NYSE) and $5 cash. As noted above this will be affected by two separate events for which the properties are as follows:

Merger component 1 for B.N to A.N:

Suppose the 100 shares of B.N were purchased at $12 that, at the time of the corporate action t, are worth $14, and that the exchange rate to convert GBP to USD is 1.6.

- $s = {}^{100}\!/_{25} = 4$,
- $N = 100$,
- $M = 1$,
- $N_1' = 8$ (i.e. $n_1' = 2$),
- $C = £20$ (i.e. $c = £5$),
- $p(0) = \$12.00$,
- $p(t) = \$14.00$,
- $f_1'(t) = 1.0$ as both B.N and A.N are quoted in $,
- $f^C(t) = 1.6$ as A.N is quoted in $ and the cash component is in £,
- $w_1'(t) = 1$ as there is only one new issue in this component of the merger event.

Then, from Equation 5 we find the book price for the A.N stock to be:

$$p_1'(0) = 1 \times \frac{100}{8} \times \frac{\$12}{1.0} = \$150$$

and from Equation 6 the corresponding market price is:

$$p_1'(t) = 1 \times \left[\frac{100}{8} \times \frac{\$14}{1.0} - \frac{£20}{8} \times \frac{1.6}{1.0}\right] = \$171$$

We now check our two constraints:

- **Preservation of Value:** At time t Equation 1 gives:

  $$100 \times \$14 = £20 \times 1.6 + (8 \times \$150 \times 1.0)$$

  i.e. $\$1400 = \$1400$ and so market value is preserved.
- **Preservation of P&L:** At time t Equation 2 gives:

  $$100 \times [\$14 - \$12] = £20 \times 1.6 + (8 \times [\$171 - \$150] \times 1)$$

  i.e. $\$200 = \$200$ and so P&L is preserved.

Merger component 2 for C.L to A.N:

Suppose the 4 shares of C.L were purchased at £10 that, at the time of the corporate action t, are worth £9, and that the exchange rate to convert GBP to USD is 1.6.

- $s = 4/8 = 0.5$ (i.e. the scaling factor from the corporate action announcement to the traded position),
- $N = 4$,
- $M = 1$,
- $N_1' = 1$ (i.e. $n_1' = 2$),
- $C = £2.50$ (i.e. $c = £5$),
- $p(0) = £10.00$,
- $p(t) = £9.00$,
- $f_1'(t) = 1/1.6 = 0.625$ to convert the A.N stock currency of USD to GBP for the original C.L stock,
- $f^C(t) = 1.0$ as the cash is in the same currency as the original C.L stock,
- $w_1'(t) = 1$ as there is only one new issue in this component of the merger event.

Then, from Equation 5 we find the book price for the A.N stock to be:

$$p_1'(0) = 1 \times \frac{4}{1} \times \frac{£10}{0.625} = \$64$$

and from Equation 6 the corresponding market price is:

$$p_1'(t) = 1 \times \left[\frac{4}{1} \times \frac{£9}{0.625} - \frac{£2.50}{1} \times \frac{1.0}{0.625} \right] = \$53.6$$

We now check our two constraints:

- **Preservation of Value:** At time t Equation 1 gives:

  $$4 \times £9 = £2.50 \times 1.0 + (1 \times \$53.6 \times 0.625)$$

  i.e. $£36 = £36$ and so market value is preserved.
- **Preservation of P&L:** At time t Equation 2 gives:

  $$4 \times [£9 - £10] = £2.50 \times 1.0 + (1 \times [\$53.6 - \$64] \times 0.625)$$

  i.e. $-£4 = -£4$ and so P&L is preserved.

<u>Trades To Book In Adaptiv</u>

To affect this merger in Adaptiv requires the execution of two corporate actions. For the first the position in B.N is closed with a trade of 100 shares at $12, and the cash is written in a transaction cost £20 on this deal; the position in the new A.N stock is then opened with 8 shares at $150. To preserve *P&L* the market parameter prices for B.N and A.N and C.N should be $14 and $171, respectively. The second action closes the position in C.L with a trade of 4 shares at £10, and a cash transaction cost of £2.50, and the position in the new A.N stock is generated with a trade of 1 share at a price of $64. To preserve *P&L* the market parameter prices for C.L and A.N and C.N should be $9 and $53.6 (for a USD/GBP exchange rate of 1.6), respectively. Both these corporate actions have to be executed for the completion of the merger event.

Stock Split With Share Rounding

Consider a “1 for 3” stock split of A.N (i.e. every three shares are replaced by one share). Suppose 10 shares are held in A.N that were bought at $23.5, and at the time of the corporate action, t, are worth $24:

- $s = 10/3 = 0.33\dot{3}$ (the scaling factor from the corporate action announcement to the position held),
- $N = 10$,
- $M = 1$,
- $N_1' = 3.33\dot{3}$,
- $C = \$0$,

- $p(0) = \$23.50$,
- $p(t) = \$24.00$,
- $f_1'(t) = 1.0$,
- $f^C(t) = 1.0$,
- $w_1'(t) = 1$.

Then, from Equation 5 we find the new book price for the A.N stock to be:

$$p_1'(0) = 1 \times \frac{10}{3.33\dot{3}} \times \frac{\$23.50}{1.0} = \$70.5$$

and from Equation 6 the corresponding market price is:

$$p_1'(t) = 1 \times \left[\frac{10}{3.33\dot{3}} \times \frac{\$24}{1.0} - \frac{\$0}{10} \times \frac{1.0}{1.0} \right] = \$72$$

We now check our two constraints:

- **Preservation of Value:** At time t Equation 1 gives:

  $$10 \times \$24 = \$0 \times 1.0 + (3.33\dot{3} \times \$72 \times 1.0)$$

  i.e. $\$240 = \$240$ and so market value is preserved.
- **Preservation of P&L:** At time t Equation 2 gives:

  $$10 \times [\$24.00 - \$23.50] = \$0 \times 1.0 + (3.33\dot{3} \times [\$72 - \$70.5] \times 1.0)$$

  i.e. $\$5 = \$5$ and so P&L is preserved.

However, because the theoretical number of shares we need to open the corporate action with is 3.3333 (recurring) share rounding needs to occur. In this simple case the stock price will be rounded down to 3 shares so the methodology now becomes:

- $N_1' = 3$ (rounded down from $N_1' = 3.33\dot{3}$).

Then, from Equation 5 we find the new book price for the A.N stock to be:

$$p_1'(0) = 1 \times \frac{10}{3} \times \frac{\$23.50}{1.0} = \$78.33\dot{3}$$

and from Equation 6 the corresponding market price is:

$$p_1'(t) = 1 \times \left[\frac{10}{3} \times \frac{\$24}{1.0} - \frac{\$0}{10} \times \frac{1.0}{1.0} \right] = \$80$$

We now check our two constraints:

- **Preservation of Value:** At time t Equation 1 gives:

  $$10 \times \$24 = \$0 \times 1.0 + (3 \times \$80 \times 1.0)$$

  i.e. $\$240 = \$240$ and so market value is preserved.
- **Preservation of P&L:** At time t Equation 2 gives:

  $$10 \times [\$24.00 - \$23.50] = \$0 \times 1.0 + \left(3 \times \left[\$80 - \$78.33\dot{3} \right] \times 1.0\right)$$

  i.e. $\$5 = \$5$ and so P&L is preserved.

Unfortunately, this is still not the end of the issue. The market price has been affected by our rounding of the share amount. Share rounding may also occur due to bucketing and the rounding in each bucket may be different. This will mean that we may have as many different market price calculations as we have buckets (and yet there can and should be only one market price in Adaptiv). Therefore we are forced to adopt the un-rounded market price.

We now check our *P&L* constraint using the un-rounded market price with rounded book price:

- **Preservation of P&L:** At time t Equation 2 gives:

  $$10\times\left[\$24.00-\$23.50\right]=\$0\times1.0+(3\times\left[\$72-\$78.33\dot{3}\right]\times1.0)$$

  i.e. $\$5=-\$19$ and so P&L is **not** preserved.

Thus, *P&L* cannot be preserved when share rounding occurs! However this is correct, as the trader/corporation will in reality only be issued with 3 shares, instead of the 3 and 1/3 shares they should theoretically receive, and so they will actually lost money on their position from this corporate action.

Trades To Booked In Adaptiv

To affect this corporate action in Adaptiv the existing position in A.N is closed with a trade of 10 shares at $23.50, and then re-opened with a new trade of 3 shares at $78.33333. The market price of A.N after the corporate action should be entered as $72, which results in a loss of *P&L*.

Rounding Positions

The above prescription means that occasionally non-integer stock positions will result from the scaling. A rigid rounding procedure (i.e. rounding all values up or down) will likely result in a discrepancy between the total numbers of shares in the buckets compared to that in the original position (after scaling). A procedure is required that conserves the total number of shares after bucketing.

The methodology implemented is designed to round *positions* to integer values, and also maintain the overall corporate holding. This is done by keeping track on the residual resulting from each position rounding step and then applying this residual to the next position to be rounded before it occurs.

The mathematical description for this methodology is:

$$N_i'=\mathrm{int}\left[\left(N_i+\varepsilon_{i-1}\right)+0.5\right],$$

where N_i is the position before rounding, N_i' is the position after rounding (to the nearest integer, which is accomplished by the addition of 0.5 before applying the **int** function), and ε_{i-1} is the residual from the previous rounding, where the residual is defined as:

$$\varepsilon_i=N_i'-N_i\,.$$

Example: Position Rounding Methodology

For illustration, consider the example in the following table:

Bucket	Original Position	Original Position Plus Previous Residual	**Rounded Position**	New Residual
Overall	10.0	10.0	**10.0**	N/A
Bucket #1	2.5	2.5	**3.0**	0.5
Bucket #2	1.2	1.7	**2.0**	0.3
Bucket #3	3.1	3.4	**3.0**	-0.4
Bucket #4	2.8	2.4	**2.0**	-0.4
Bucket #5	0.4	0.0	**0.0**	0.0
Bucket Total	10.0	N/A	**10.0**	N/A

A comparison of the Original Position in the “Overall” row with the “Bucket Total” row shows that the bucketing has conserved the total value, although the positions in the buckets are non-integer values. Applying the methodology prescribed above results in the numbers show in the “Rounded Position” column. The total from this column still agrees with that required from the “Overall” original position. [Note, a check is performed in the final bucket to ensure that the correct total will be obtained, despite the fact that the total should correct as keeping track of the residual before rounding means that next residual can never exceed 1.]

Adjusted Historic Prices for Equities

Corporate actions affect a stock's price even though the actual return made from investing in the company is unchanged. For example, the share price of a company that performed a stock split, giving two new shares for each old share, would halve, though the value of the investment that an investor had made would remain unchanged. Or, if a company pays a one dollar dividend, its share price falls by one dollar, even though company's prospects have not changed.

These factors differ from the changes in a company's stock price due to its performance or the state of the financial markets. However, in order to use price data to compare a given company's performance over time, one needs to have a record of the changes in price excluding the effects of corporate actions. So, we must create an adjustment to share prices to compensate for corporate actions.

More formally, the Stock Split Compensation Formula is: Adjusted Price = Price /Split Factor
and the Dividend Compensation Formula is: Adjusted Price = Price / (Price + Dividend).
The prices to be adjusted are only the records before than the corporate action, and a corollary of this is that on the final day in the archive, the Adjusted Price has to be the same as the unadjusted Price, simply because there are no later prices. The latest price is the reference point for the adjustment calculation.

Example: 2 for 1 stock split

Day	Price	Split Factor	Adjusted Price
Monday	40	1	**20**
Tuesday	25	**2**	25
Wednesday	20	1	20

Looking at the prices, it looks like a collapse in the value of the company happened on Tuesday. However, adjusting the prices for the stock split, shows us that the value of the company actually rose on
Tuesday, and there was no change between the Wednesday and Monday prices.

Example: $1 dividend

Day	Price	Dividend	Adjusted Price
Tuesday	25	0	**23.75**
Wednesday	20	0	**19**
Thursday	19	**1**	19

Again, it looks like a drop happened in the stock price between Wednesday and Thursday, but our Adjusted Price series tells a different story. The Adjusted Price for Tuesday is 19, which is calculated by 20 x (19/(19+1)). The Adjusted Price for Monday is 23.75, which is calculated by 25 x (19/(19+1)).

Adding together the adjustments for dividend and splits gives the algorithm that defines the historic equity price adjustment:

Start with all Adjusted Prices equal to Unadjusted Prices. Then
For each Date,
Multiply the Adjusted Prices for all previous dates by the Factor for this Date
Where Factor = Stock Adjustment x Dividend Adjustment
Where Stock Adjustment = 1 / Stock Split Factor for this Date
And Dividend Adjustment = Raw Price / (Raw Price + Dividend for this Date)
Where Raw Price = Unadjusted Price for this Date

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