ECE 4554 / 5554: Computer Vision: Homework 1

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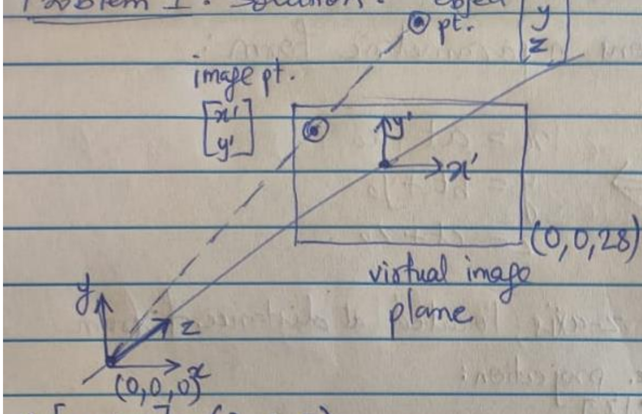
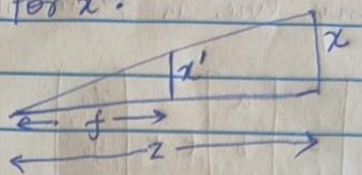
Program : *MS CPE (Non-Thesis)*

Problem 1 : Solution :

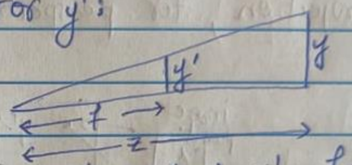


For x':



$$\frac{x'}{f} = \frac{x}{z} \Rightarrow x' = \frac{fx}{z}$$

For y':



$$\frac{y'}{f} = \frac{y}{z} \Rightarrow y' = \frac{fy}{z}$$

a) $[x, y, z] = (3, 4, 5)$

$x' = 28\,mm \times \frac{3m}{5m} = 16.8\,mm$

$y' = 28\,mm \times \frac{4m}{5m} = 22.4\,mm$

$(x', y', z) = (16.8, 22.4, 28)\,mm$

b) $[x, y, z] = (6, 8, 10)$

$x' = 28\,mm \times \frac{6m}{10m} = 16.8\,mm$

$y' = 28\,mm \times \frac{8m}{10m} = 22.4\,mm$

$(x', y', z) = (16.8, 22.4, 28)\,mm$

c) $[x, y, z] = (-1, -2, 6)$

$x' = 28\,mm \times \frac{-1\,m}{6\,m} = -4.67\,mm$

$y' = 28\,mm \times \frac{-2\,m}{6\,m} = -9.33\,mm$

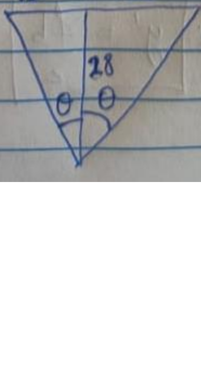
$(x', y', z) = (-4.67, -9.33, 28)\,mm$

d) $[x, y, z] = (0, 0, 6)$

$x' = 28\,mm \times \frac{0\,m}{6\,m} = 0\,mm$

$y' = 28\,mm \times \frac{0\,m}{6\,m} = 0\,mm$ $\quad (x', y', z) = (0, 0, 28)\,mm$

e) Horizontal field of view :



$$\Rightarrow 2\tan^{-1}\left(\frac{12}{28}\right) = 46.39°$$

Problem 2: Solution

$$A^TA = \begin{bmatrix}2&5\\3&6\\4&7\end{bmatrix}\begin{bmatrix}2&3&4\\5&6&7\end{bmatrix} = \begin{bmatrix}29&36&43\\36&45&54\\43&54&65\end{bmatrix}$$

Solving for eigen values & eigen vectors for A^TA: $\det(A^TA-\lambda I)=0$

$$\begin{vmatrix}29-\lambda&36&43\\36&45-\lambda&54\\43&54&65-\lambda\end{vmatrix} \Rightarrow \boxed{\lambda = \frac{139-\sqrt{19105}}{2},\ \frac{139+\sqrt{19105}}{2},\ 0}$$

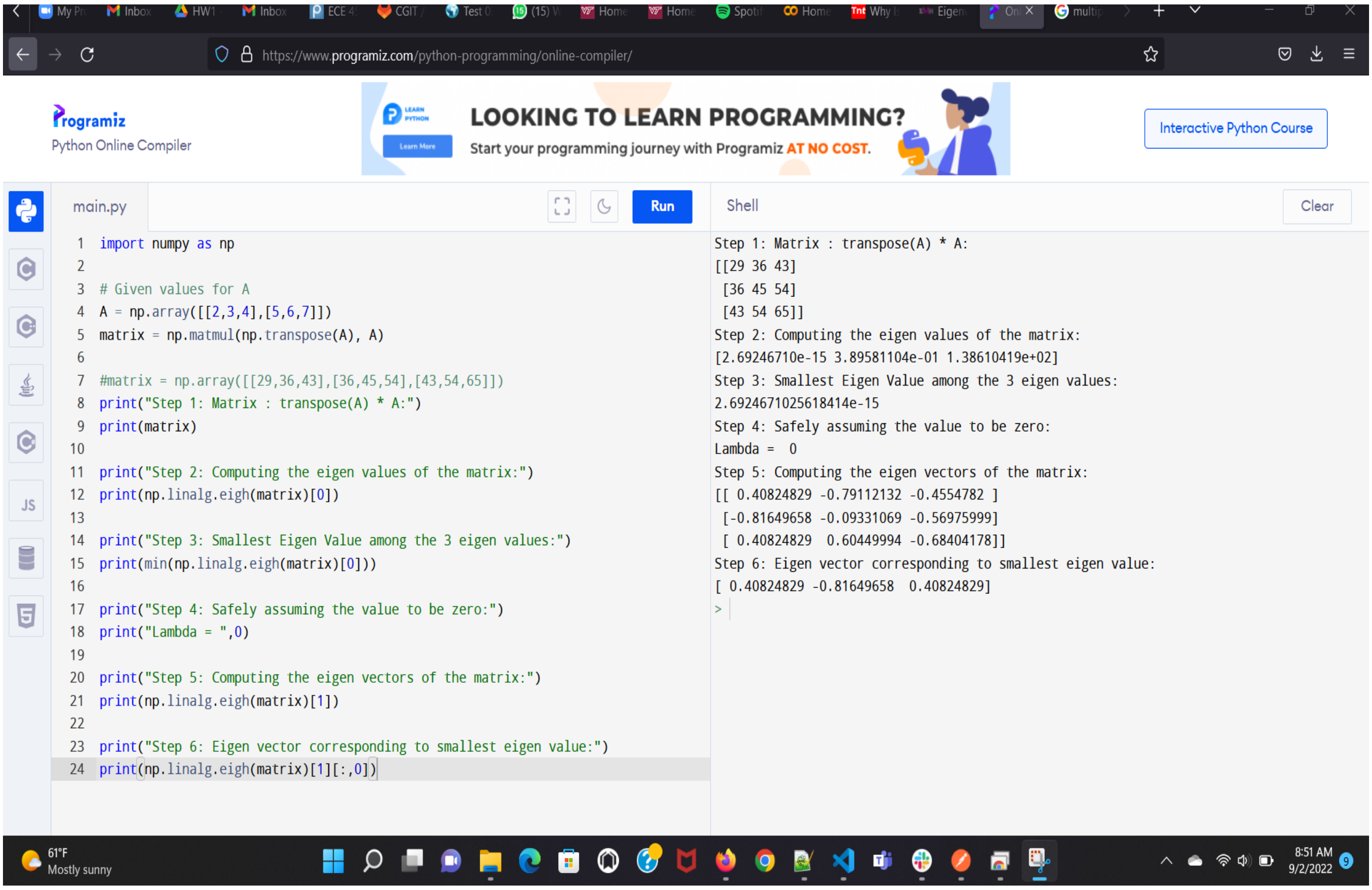
Used the following link to solve above: https://www.emathhelp.com/calculators to get fractional form

Smallest Eigen value $\boxed{\lambda=0}$ Eigen Vector for $\lambda=0 \Rightarrow X=[x,-2x,x]$

$\vec{v}_{\lambda=0} = [0.40824, -0.81649, 0.40824]$ Also called null space of matrix

$\vec{v}_{\lambda=0}$ is the solution unit vector.

This particular solution can also be solved completly through python code.




```python
import numpy as np

# Given values for A
A = np.array([[2,3,4],[5,6,7]])
matrix = np.matmul(np.transpose(A), A)

#matrix = np.array([[29,36,43],[36,45,54],[43,54,65]])
print("Step 1: Matrix : transpose(A) * A:")
print(matrix)

print("Step 2: Computing the eigen values of the matrix:")
print(np.linalg.eigh(matrix)[0])

print("Step 3: Smallest Eigen Value among the 3 eigen values:")
print(min(np.linalg.eigh(matrix)[0]))

print("Step 4: Safely assuming the value to be zero:")
print("Lambda = ",0)

print("Step 5: Computing the eigen vectors of the matrix:")
print(np.linalg.eigh(matrix)[1])

print("Step 6: Eigen vector corresponding to smallest eigen value:")
print(np.linalg.eigh(matrix)[1][:,0])
```

```
Step 1: Matrix : transpose(A) * A:
[[29 36 43]
 [36 45 54]
 [43 54 65]]
Step 2: Computing the eigen values of the matrix:
[2.69246710e-15 3.89581104e-01 1.38610419e+02]
Step 3: Smallest Eigen Value among the 3 eigen values:
2.6924671025618414e-15
Step 4: Safely assuming the value to be zero:
Lambda =  0
Step 5: Computing the eigen vectors of the matrix:
[[ 0.40824829 -0.79112132 -0.4554782 ]
 [-0.81649658 -0.09331069 -0.56975999]
 [ 0.40824829  0.60449994 -0.68404178]]
Step 6: Eigen vector corresponding to smallest eigen value:
[ 0.40824829 -0.81649658  0.40824829]
>
```

Problem 3: Solution

a) Point of intersection:

For x- co-ordinate

$$\frac{a_1x + c_1}{-b_1} = \frac{a_2x + c_2}{-b_2}$$

$$x = \frac{-c_1b_2 + c_2b_1}{a_1b_2 - a_2b_1}$$

$$\boxed{x = \frac{c_2b_1 - c_1b_2}{a_1b_2 - a_2b_1}}$$

For y- coordinate

$$\frac{b_1y + c_1}{-a_1} = \frac{b_2y + c_2}{-a_2}$$

$$\boxed{y = \frac{c_2a_1 - c_1a_2}{a_2b_1 - a_1b_2}}$$

b) Plugging values in above eqns.:

$$x = \frac{8\times4 - 5\times7}{3\times7 - 6\times4} \; ; \; y = \frac{8\times3 - 5\times6}{6\times4 - 3\times7}$$

$$\boxed{x = 1 \qquad y = -2}$$

c) For 2 lines to be parallel, their slopes should be equal [In 2D]

$$y = \underbrace{m}_{\text{slope}}x + c$$

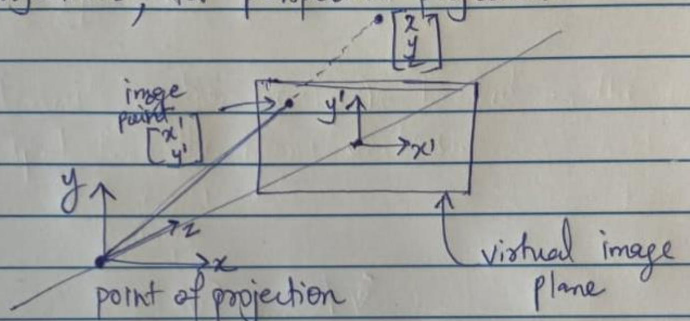
So for our case: $\frac{-a_1}{b_1} = \frac{-a_2}{b_2} \Rightarrow \boxed{\frac{a_1}{b_1} = \frac{a_2}{b_2}}$

Problem 4:

Equation of any line in 3-dimensions in parametric form:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \end{bmatrix} t + \begin{bmatrix} x_0 \\ y_0 \\ z_0 \end{bmatrix} \Rightarrow \begin{array}{l} x = at + x_0 \\ y = bt + y_0 \\ z = ct + z_0 \end{array}$$

Assuming virtual image plane ⊥r to the z-axis, located at distance f from point of projection, for perspective projection:



$$x' = \frac{fx}{z}$$

$$y' = \frac{fy}{z}$$

To compute the vanishing point $\begin{bmatrix} x' \\ y' \end{bmatrix}$ for parallel lines in 3 dimensions:

We need to get the perspective projection of the point:

$$x' = \frac{fx}{z} \quad ; \quad y' = \frac{fy}{z}$$

Replacing the above equations with parametric forms:

$$x' = f\left[\frac{at + x_0}{ct + z_0}\right] \quad ; \quad y' = f\left[\frac{bt + y_0}{ct + z_0}\right]$$

Since, it is a vanishing point, we can make $t \to \infty$

$$x' = \lim_{t\to\infty}\left[f\left(\frac{at + x_0}{ct + z_0}\right)\right] \; ; \; y' = \lim_{t\to\infty}\left[f\left(\frac{bt + y_0}{ct + z_0}\right)\right]$$

Using L-Hospital to solve the above $\boxed{x' = \frac{fa}{c}}$; $\boxed{y' = \frac{fb}{c}}$