ECE 4554 / 5554: Computer Vision: Homework 2

Fall 2022

Instructions

- This assignment is due at Canvas on Sept. 17 before 11:59 PM. Late submissions are allowed at the cost of 1 token per 24-hour period. A submission received only a minute after the deadline will cost 1 token.
- Again, please review the Honor Code statement in the syllabus. You may discuss general approaches to solving these problems with other students, as well as software libraries and syntax. Beyond that point, you must work independently. The work that you submit for a grade must be your own.
- The assignment consists of 3 analytical problems, which are presented here, and 3 "machine problems", which require work using Colab. Notice that one of the analytical problems is required for 5554 students, but is optional (extra credit) for 4554 students.
- For the analytical problems, prepare an answer sheet that contains all of your written answers in a single file named `Homework1_Problems1-3_USERNAME.pdf`. (Use your own VT Username.) Show your work. Handwritten solutions are allowed, but they must be easily legible to the grader. For the machine problems, you must provide a Jupyter notebook file and an associated pdf file. Details are provided at the end of this assignment.
- For machine problems, the notebook file that you submit must be compatible with Google Colab. The grader should be able to execute your code after making only one change to your file, which is the location of the working directory. If the notebook file does not execute, then the grader will be tempted to assign a grade of zero for the machine problems.
- After you have submitted to Canvas, it is your responsibility to download the files that you submitted and verify that they are correct and complete. *The files that you submit to Canvas are the files that will be graded.* You may submit multiple times, and we will grade the last files that you submit.

Problem 1. (10 points) Consider two 2-dimensional kernels g and h, which are shown below.

$$g = \begin{bmatrix} -1 & -4 & 7 \\ -2 & 5 & 8 \\ 0 & 6 & 0 \end{bmatrix} \qquad h = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$$

a) Solve for $g \otimes h$, where $\otimes$ refers to cross-correlation.

b) Solve for $h \otimes g$.

c) Solve for $g * h$, where * refers to convolution.

Problem 2. (10 points) A 2D linear filter $h(x,y)$ is called *separable* if it can be decomposed into the convolution of two 1D filters: $h(x,y) = h_1(x) * h_2(y)$, where * represents convolution. For example, the 3×3 box filter can be decomposed as follows:

$$\frac{1}{9}\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix} = \frac{1}{3}\begin{bmatrix}1\\ 1\\ 1\end{bmatrix} * \frac{1}{3}[1 \quad 1 \quad 1]$$

a) Show that a familiar edge-detection filter is separable by considering $\begin{bmatrix}1\\ 2\\ 1\end{bmatrix}$ and $[-1 \quad 0 \quad 1]$.

b) Consider this rough approximation to a 1D Gaussian filter: $g = \frac{1}{14}[1 \;\; 3 \;\; 6 \;\; 3 \;\; 1]$. Assume that 2D Gaussian filters are separable. Construct a 2D Gaussian filter in the discrete domain using the filter g and its transpose, g^{T}.

Problem 3. (10 points) For 5554 students, this problem is required. For 4554 students, this problem is optional and can be submitted for extra credit.

Consider the two-dimensional, continuous-domain Gaussian function, which is given by

$$G(x,y) = \frac{1}{2\pi\sigma^2} e^{\frac{-(x^2+y^2)}{2\sigma^2}}$$

a) Show, analytically, that this function is separable.

b) Write an analytical expression for the Laplacian of Gaussian, $\nabla^2 G(x,y)$, in the continuous domain. (Show the steps needed to arrive at your answer, of course.) Simplify your expression.

Machine Problems.
You have been given a Jupyter notebook file `Homework2_USERNAME.ipynb` and two image files. Replace "`USERNAME`" with your Virginia Tech Username. Then upload those files to Google Drive. Open the `ipynb` file in Google Colab. Follow the instructions that you will find inside the notebook file.

What to hand in: After you have finished, you will have created the following 3 files. Upload these 3 files to Canvas before the deadline. Do not combine them in a single ZIP file.

`Homework2_Problems1-3_USERNAME.pdf` ← Your solutions to problems 1 through 4

`Homework2_Code_USERNAME.zip` ← Your zipped Jupyter notebook file

`Homework2_Notebook_USERNAME.pdf` ← A PDF version of your Colab session
