ECE 4554 / 5554: Computer Vision: Homework 2

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Problem 1: Solution:

a)
$$g \otimes h = \begin{bmatrix} -1 & -4 & 7 \\ -2 & 5 & 8 \\ 0 & 6 & 0 \end{bmatrix} \otimes \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$$

Padding g with the required number of 0's,

$$= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & -4 & 7 & 0 \\ 0 & -2 & 5 & 8 & 0 \\ 0 & 0 & 6 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \otimes \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$$

Dot product:

1) $\begin{matrix} 0\times1 & 0\times3 \\ 0\times2 & -1\times4 \end{matrix} = -4$

similarly move h across g and get the other values.

2) $\begin{bmatrix} 0\times1 & 0\times3 \\ -1\times2 & -4\times4 \end{bmatrix} = -18$

3) $\begin{bmatrix} 0\times1 & 0\times3 \\ -4\times2 & 7\times4 \end{bmatrix} = 20$

$$= \begin{bmatrix} -4 & -18 & 20 & 14 \\ -11 & 3 & 59 & 23 \\ -6 & 37 & 41 & 8 \\ 0 & 18 & 6 & 0 \end{bmatrix}$$

b)
$$h \otimes g = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} \otimes \begin{bmatrix} -1 & -4 & 7 \\ -2 & 5 & 8 \\ 0 & 6 & 0 \end{bmatrix}$$

Padding h with the required number of 0's,

$$= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 3 & 0 & 0 \\ 0 & 0 & 2 & 4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \otimes \begin{bmatrix} -1 & -4 & 7 \\ -2 & 5 & 8 \\ 0 & 6 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 6 & 18 & 0 \\ 8 & 41 & 37 & -6 \\ 23 & 59 & 3 & -11 \\ 14 & 20 & -18 & -4 \end{bmatrix}$$

c) $g * h = \boxed{g} \otimes \boxed{y}$

$$= \begin{bmatrix} -1 & -4 & 7 \\ -2 & 5 & 8 \\ 0 & 6 & 0 \end{bmatrix} \otimes \begin{bmatrix} 4 & 2 \\ 3 & 1 \end{bmatrix}$$

Padding g with the required number of 0's :

$$= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & -4 & 7 & 0 \\ 0 & -2 & 5 & 8 & 0 \\ 0 & 0 & 6 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \otimes \begin{bmatrix} 4 & 2 \\ 3 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & -7 & -5 & 21 \\ -4 & -13 & 21 & 52 \\ -4 & 8 & 54 & 32 \\ 0 & 12 & 24 & 0 \end{bmatrix}$$

Problem 2 : Solution :

a) 2D linear filter is seperable if :

$$h(x,y) = h_1(x) * h_2(y)$$

$$h_1(x) = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} ; \quad h_2(x) = \begin{bmatrix} -1 & 0 & 1 \end{bmatrix}$$

$$h(x,y) = h_1(x) * h_2(y)$$ → flipping from top to bottom and left to right

$$= \boxed{h_1(x)} \otimes \boxed{(h)^{5}}$$

Padding with required number of 0's

$$= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \otimes \begin{bmatrix} 1 & 0 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$$ ← Sobel Edge detection filter

b) 1D Gaussian filter : $g = \frac{1}{14}\begin{bmatrix} 1 & 3 & 6 & 3 & 1 \end{bmatrix}$

$$g^T = \frac{1}{14}\begin{bmatrix} 1 \\ 3 \\ 6 \\ 3 \\ 1 \end{bmatrix}$$

$$g * g^T = \frac{1}{14}\begin{bmatrix} 1 & 3 & 6 & 3 & 1 \end{bmatrix} * \frac{1}{14}\begin{bmatrix} 1 \\ 3 \\ 6 \\ 3 \\ 1 \end{bmatrix}$$

$$\boxed{g} \otimes \boxed{f_1} = \frac{1}{14}\begin{bmatrix}1 & 3 & 6 & 3 & 1\end{bmatrix} \otimes \frac{1}{14}\begin{bmatrix}1\\3\\6\\3\\1\end{bmatrix}$$

Padding with required number of 0's:

$$= \frac{1}{14}\begin{bmatrix} & \vdots & \vdots & \vdots & & & \vdots & \\ & 0 & 0 & 0 & 0 & 0 & 0 & 0 \cdots \\ \cdots & 0 & 0 & 0 & 0 & 0 & 0 & 0 \cdots \\ \cdots & 0 & 0 & 0 & 0 & 0 & 0 & 0 \cdots \\ \cdots & 0 & 0 & 0 & 0 & 0 & 0 & 0 \cdots \\ \cdots & 0 & 1 & 3 & 6 & 3 & 1 & 0 \cdots \\ \cdots & 0 & 0 & 0 & 0 & 0 & 0 & 0 \cdots \\ \cdots & 0 & 0 & 0 & 0 & 0 & 0 & 0 \cdots \\ \cdots & 0 & 0 & 0 & 0 & 0 & 0 & 0 \cdots \\ \cdots & 0 & 0 & 0 & 0 & 0 & 0 & 0 \cdots \\ & & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{bmatrix} \otimes \frac{1}{14}\begin{bmatrix}1\\3\\6\\3\\1\end{bmatrix}$$

$$= \frac{1}{14 \times 14}\begin{bmatrix} 1 & 3 & 6 & 3 & 1 \\ 3 & 9 & 18 & 9 & 3 \\ 6 & 18 & 36 & 18 & 6 \\ 3 & 9 & 18 & 9 & 3 \\ 1 & 3 & 6 & 3 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.00510, & 0.01530, & 0.03061, & 0.01530, & 0.00510 \\ 0.01530, & 0.04591, & 0.09183, & 0.04591, & 0.01530 \\ 0.03061, & 0.09183, & 0.18367, & 0.09183, & 0.03061 \\ 0.01530, & 0.04591, & 0.09183, & 0.04591, & 0.01530 \\ 0.00510, & 0.01530, & 0.03061, & 0.01530, & 0.00510 \end{bmatrix}$$

Problem 3: Solution

a) For $G(x,y)$ to be seperable:

$$G(x,y) = F(x) \times H(y) \rightarrow \text{1-D Gaussian}$$

2-D Gaussian Function 1-D Gaussian

$$F(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{x^2}{2\sigma^2}} \quad \left[\text{Assuming } \mu = 0\right]$$

$$H(y) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{y^2}{2\sigma^2}} \quad \left[\text{Assuming } \mu = 0\right]$$

$$F(x) \times H(y) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-x^2/2\sigma^2} \cdot \frac{1}{\sqrt{2\pi}\,\sigma} e^{-y^2/2\sigma^2}$$

$$= \frac{1}{2\pi\sigma^2} e^{-\frac{(x^2+y^2)}{2\sigma^2}}$$

$$= G(x,y)$$

$\therefore$ $G(x,y)$ is seperable into 2 1-D Gaussian functions.

b) $G(x,y) = \frac{1}{2\pi\sigma^2} e^{-\frac{(x^2+y^2)}{2\sigma}}$ ⇒ 2-D Gaussian Function

To get Laplacian of a Gaussian, we have to compute $\underline{\nabla^2 G(x,y)}$

$$\nabla^2 G(x,y) = \frac{\partial^2}{\partial x^2}\left[G(x,y)\right] + \frac{\partial^2}{\partial y^2}\left[G(x,y)\right]$$

$$= \frac{\partial^2}{\partial x^2}\left[\frac{1}{2\pi\sigma^2} e^{-\frac{(x^2+y^2)}{2\sigma^2}}\right] + \frac{\partial^2}{\partial y^2}\left[\frac{1}{2\pi\sigma^2} e^{-\frac{(x^2+y^2)}{2\sigma^2}}\right]$$

Using Chain rule on the below eqn.:

$$= \frac{1}{2\pi\sigma^2}\left[\frac{\partial}{\partial x}\left\{e^{-\frac{(x^2+y^2)}{2\sigma^2}}\right\}\left\{\frac{-2x}{2\sigma^2}\right\}\right] + \frac{1}{2\pi\sigma^2}\left[\frac{\partial}{\partial y}\left\{e^{-\frac{(x^2+y^2)}{2\sigma^2}}\right\}\left\{\frac{-2y}{2\sigma^2}\right\}\right]$$

$$= \frac{1}{2\pi\sigma^2}\left[-\frac{\partial}{\partial x}\frac{xe^{-\frac{(x^2+y^2)}{2\sigma^2}}}{\sigma^2} - \frac{\partial}{\partial y}\frac{ye^{-\frac{(x^2+y^2)}{2\sigma^2}}}{\sigma^2}\right]$$

$$= \frac{1}{2\pi\sigma^4}\left[-e^{-\frac{(x^2+y^2)}{2\sigma^2}} - xe^{-\frac{(x^2+y^2)}{2\sigma^2}}\left(\frac{-x}{\sigma^2}\right) - e^{-\frac{(x^2+y^2)}{2\sigma^2}} - ye^{-\frac{(x^2+y^2)}{2\sigma^2}}\left(\frac{-y}{\sigma^2}\right)\right]$$

$$= \frac{1}{2\pi\sigma^4}\left[\left(\frac{x^2+y^2}{\sigma^2}\right)e^{-\frac{(x^2+y^2)}{2\sigma^2}} - 2e^{-\frac{(x^2+y^2)}{2\sigma^2}}\right]$$

$$\therefore \nabla^2 G(x,y) = \frac{e^{-\frac{(x^2+y^2)}{2\sigma^2}}}{2\pi\sigma^4}\left[\left(\frac{x^2+y^2}{\sigma^2}\right) - 2\right]$$

$$\boxed{\nabla^2 G(x,y) = \frac{e^{-\frac{(x^2+y^2)}{2\sigma^2}}}{\pi\sigma^4}\left[\left(\frac{x^2+y^2}{2\sigma^2}\right) - 1\right]}$$