FINAL EXAM: ECE 4554 / 5554

Fall 2022

By submitting your work for grading, you agree to this Honor System pledge:

I have neither given nor received unauthorized aid on this exam.

Instructions: This is an open-book, open-notes exam. You may use your laptop to access on-line reference material. Do not communicate or share exam-related materials with anyone during the exam, in-person or otherwise.

The exam is worth 100 points. Time limit: 120 minutes.

The exam consists of 9 problems on 9 pages.

The last problem is a Python/OpenCV programming problem. For that problem, you have been given a Jupyter notebook file and input image file.

For the written problems, prepare your answers in a PDF file. Show your work and clearly indicate your final answers. Handwritten answers are permitted, but they need to be easily legible to the grader. Partial credit is possible for most problems. If you do not have time to complete a problem, describe briefly how you would finish it.

Throughout this exam, the abbreviations "1D", "2D", and "3D" represent "one-dimensional", "two-dimensional", and "three-dimensional", respectively.

The exam has been designed so that you should not need to ask questions during the exam period. If you spot a mistake, or if anything seems unclear, please write the assumptions that you need and continue with the exam. If you do have a major question, you may contact the instructor through email. Do not post questions at Piazza during the exam period.

If the instructor needs to make announcements during the exam period, they will be posted as Canvas announcements.

Instructions for uploading your solutions are given on the last page of this exam. *Uploading to Canvas must be completed before the end of the exam period.* After you upload your files, please *verify that your submitted files are correct* by downloading them from Canvas and checking them. The files that you submit to Canvas are the files that will be graded.

Problem 1. (10 points) Consider two 2-dimensional kernels g and h, which are shown below.

$$\mathrm{g} = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} \qquad\qquad h = \begin{bmatrix} 2 & -2 \\ 0 & -1 \end{bmatrix}$$

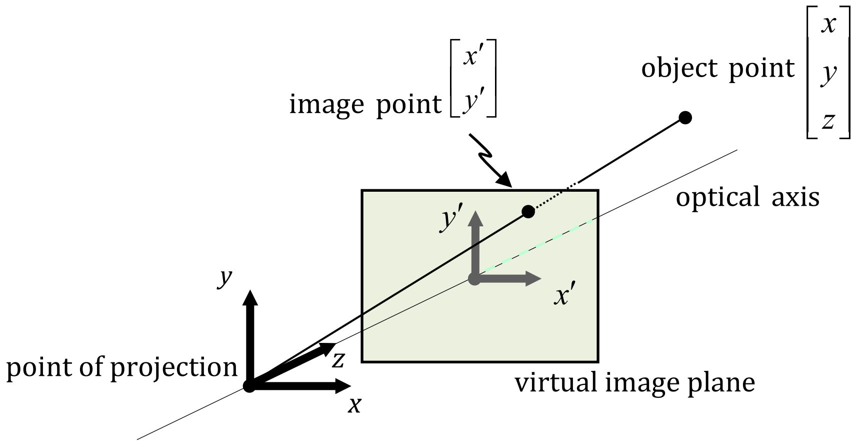
Let I represent a 2D image, and let "*" represent 2D convolution. Solve for a new kernel f that will satisfy the following relationship:

$$(I * g) * h = I * f$$

Problem 2. (10 points) Consider the 2D geometric transformation $\begin{bmatrix} x' \\ y' \end{bmatrix} = M \begin{bmatrix} x \\ y \end{bmatrix}$, where $M = \begin{bmatrix} A & -B \\ B & A \end{bmatrix}$. Assume that *A* and *B* refer to two scalar constants such that $A^2 + B^2 \neq 0$.

If possible, show that *M* can be decomposed into a sequence of 2 geometric transformations: rotation and scaling. If not possible, provide a clear explanation.

Problem 3. (12 points) Consider the common camera geometry that is shown below. For this problem, assume perspective projection.



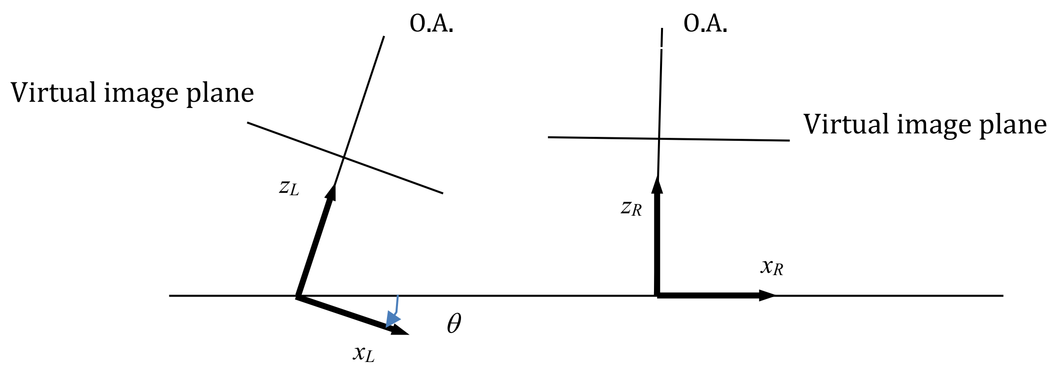
The focal length is $f = 0.03$ meters. Assume that a small insect travels along a path that can be described using the equation $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \\ 3 \end{bmatrix} + \begin{bmatrix} 8 \\ 7 \\ 6 \end{bmatrix} t$, where t represents time.

a) At time $t = 0$, the insect appears at location _____________ in the image.

b) Now assume that t becomes very large. In that case, the insect will appear at location _____________ in the image.

c) Consider a second insect that travels along a path that can be described as $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -5 \\ -4 \\ -3 \end{bmatrix} + \begin{bmatrix} 8 \\ 7 \\ 6 \end{bmatrix} t$, where t again represents time. As t becomes large, this second insect will appear at location _____________ in the image.

Problem 4. (10 points.) The diagram below shows a common stereo camera arrangement, as viewed from above. This arrangement is similar to the stereo imaging set-up that we have discussed in lectures, except that the camera on the left has rotated inward by an angle θ. The two optical axes (O.A.) are coplanar, and both cameras have the same focal length *f*. Vertical axes for the two cameras, y_L and y_R, are not shown, and are perpendicular to this page. When the left camera rotates, the baseline distance *B* does not change.



a) If possible, solve for the location of the left epipole. (If not possible, explain your reasoning.)

b) If possible, solve for the location of the right epipole. (If not possible, explain explain your reasoning.)

Problem 5. (10 points) Assume that image 1 and image 2 are a stereo pair, and the fundamental matrix for these images is $F = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ -1 & 0 & -1 \end{bmatrix}$. Let (x_1, y_1) refer to a location in image 1, and let (x_2, y_2) refer to a location in image 2.

a) Let P_2 refer to the point in image 2 that has coordinates (x_2, y_2) = (3, 4). Find an equation for the epipolar line in image 1 that is associated with P_2.

b) Let P_1 refer to the point in image 1 that has coordinates (x_1, y_1) = (2, 3). Find an equation for the epipolar line in image 2 that is associated with P_1.

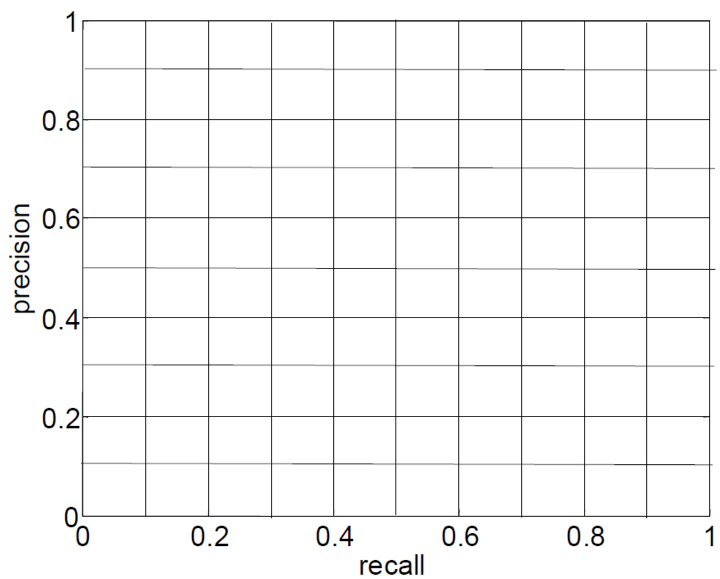
Problem 6. (10 points) Consider a triangle A that has vertices (x_1, y_1), (x_2, y_2), and (x_3, y_3). Provide a short, convincing explanation stating why (or why not) it is possible to find a 2D affine transformation that will map the vertices of triangle A onto the vertices of any new triangle B. (You do not need to write a detailed mathematical proof, but your reasoning must be clear. You do not need to consider degenerate cases, such as all vertices being colinear.)

Problem 7. (12 points) Suppose that Tom has developed a system that performs *image retrieval* from a collection of images based on the contents of the images. For example, if Tom presents a photograph of a cat as a query image, he expects the system to return images of cats from the set of images. Use the following information to create a *precision-recall curve* that characterizes the performance of his system.

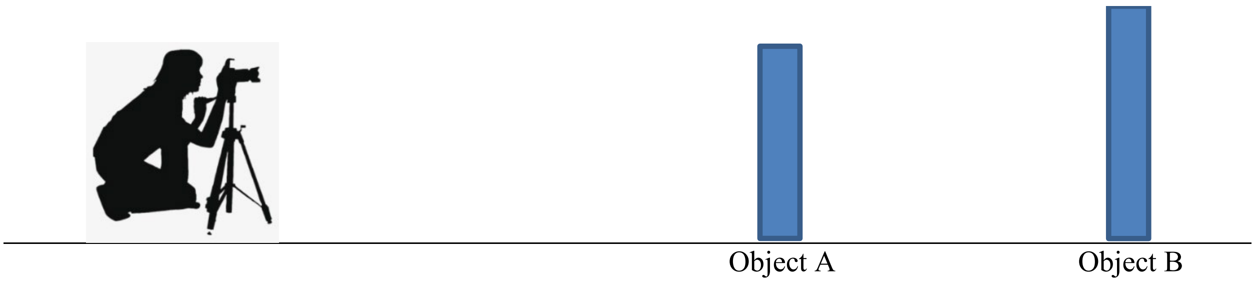
The entire data collection consists of 10 images. In response to a query image, the system returns images from the collection as shown below, ranked left-to-right in order of decreasing similarity. The abbreviations "TP" and "FP" refer to "True Positive" and "False Positive", respectively.

TP, FP, TP, FP, TP, FP, FP, TP, FP, FP

Determine the points on the diagram below that are needed to create a precision-recall curve. Show your work, and indicate the final precision-recall curve.



Problem 8. (6 points) Mary would like to use a range-from-focus method to estimate distances from her position to Object A and to Object B, as illustrated below. These are outdoor objects, and she positions herself so that she can see both objects. (Occlusion is not a problem.)



Mary will need to consider depth of field as she does her work. For each statement below, circle the one underlined choice inside brackets that makes the sentence true.

a) Mary can expect better accuracy if she uses a [<u>telephoto</u> / <u>wide-angle</u>] lens.

b) Mary can expect better accuracy if she moves [<u>farther away from</u> / <u>closer to</u>] the objects by a small amount.

c) If Mary uses a lens that has an automatic aperture control, she can expect better accuracy on a day that is [<u>sunny</u> / <u>cloudy</u>].

Problem 9. (20 points.) You have been given an image file **`image1.png`** and a Jupyter notebook file named **`Exam_Code_USERNAME.ipynb`**. Replace "USERNAME" with your Virginia Tech Username, and then upload both files to Google Drive. Open the `ipynb` file in Google Colab, and follow the instructions that you will find inside the notebook file.

What to hand in: After you have finished, download your Notebook file and place it in a `zip` file. Then upload the following 2 files to Canvas before the deadline.

`Exam_USERNAME.pdf` ← Your solutions to problems 1 through 9

`Exam_Code_USERNAME.zip` ← Your zipped Jupyter notebook file
(You do not need to submit a PDF output from Colab)
