Problem 1

Monday, September 26, 2022 9:04 PM

a) From the matrix expression, we can write equations for x' and y' as follows:

$$x' = \frac{h_{00}x + h_{01}y + h_{02}}{h_{20}x + h_{21}y + h_{22}} \qquad y' = \frac{h_{10}x + h_{11}y + h_{12}}{h_{20}x + h_{21}y + h_{22}}$$

Next, rearrange the terms:

$$-h_{20}xx' - h_{21}yx' - h_{22}x' + h_{00}x + h_{01}y + h_{02} = 0$$
$$-h_{20}xy' - h_{21}yy' - h_{22}y' + h_{10}x + h_{11}y + h_{12} = 0$$

These equations are linear with respect to the parameters h_{ij}.
If we have at least 4 pairs of corresponding points (x, y) <--> (x', y'), then we can solve for the parameters.
One approach is to set up the matrix equation **Q a** = **0** as follows:

$$\begin{bmatrix} x_1 & y_1 & 1 & 0 & 0 & 0 & -x_1x_1' & -y_1x_1' & -x_1' \\ 0 & 0 & 0 & x_1 & y_1 & 1 & -x_1y_1' & -y_1y_1' & -y_1' \\ x_2 & y_2 & 1 & 0 & 0 & 0 & -x_2x_2' & -y_2x_2' & -x_2' \\ 0 & 0 & 0 & x_2 & y_2 & 1 & -x_2y_2' & -y_2y_2' & -y_2' \\ x_3 & y_3 & 1 & 0 & 0 & 0 & -x_3x_3' & -y_3x_3' & -x_3' \\ 0 & 0 & 0 & x_3 & y_3 & 1 & -x_3y_3' & -y_3y_3' & -y_3' \\ x_4 & y_4 & 1 & 0 & 0 & 0 & -x_4x_4' & -y_4x_4' & -x_4' \\ 0 & 0 & 0 & x_4 & y_4 & 1 & -x_4y_4' & -y_4y_4' & -y_4' \\ & & & & & \vdots & & & \\ x_n & y_n & 1 & 0 & 0 & 0 & -x_nx_n' & -y_nx_n' & -x_n' \\ 0 & 0 & 0 & x_n & y_n & 1 & -x_ny_n' & -y_ny_n' & -y_n' \end{bmatrix} \begin{bmatrix} h_{00} \\ h_{01} \\ h_{02} \\ h_{10} \\ h_{11} \\ h_{12} \\ h_{20} \\ h_{21} \\ h_{22} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

b) For the point correspondences that are specified, here is the solution after normalization by h_{22}:

$$H = \begin{bmatrix} 8 & 3 & 3 \\ 4 & 4 & 2 \\ 2 & 1 & 1 \end{bmatrix}$$

Problem 2

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a) Given: $A = \begin{bmatrix} a_{00} & a_{01} & a_{11} \\ a_{10} & a_{11} & a_{12} \\ 0 & 0 & a_{22} \end{bmatrix}$

without showing all the steps here, we can solve for A^{-1}:

$$A^{-1} = \begin{bmatrix} \frac{a_{11}}{a_{00}a_{11} - a_{01}a_{10}} & \frac{-a_{01}}{a_{00}a_{11} - a_{01}a_{10}} & \frac{a_{01}a_{12} - a_{02}a_{11}}{a_{00}a_{11} - a_{01}a_{10}} \\ \frac{-a_{10}}{a_{00}a_{11} - a_{01}a_{10}} & \frac{a_{00}}{a_{00}a_{11} - a_{01}a_{10}} & \frac{a_{02}a_{10} - a_{00}a_{12}}{a_{00}a_{11} - a_{01}a_{10}} \\ 0 & 0 & 1 \end{bmatrix}$$

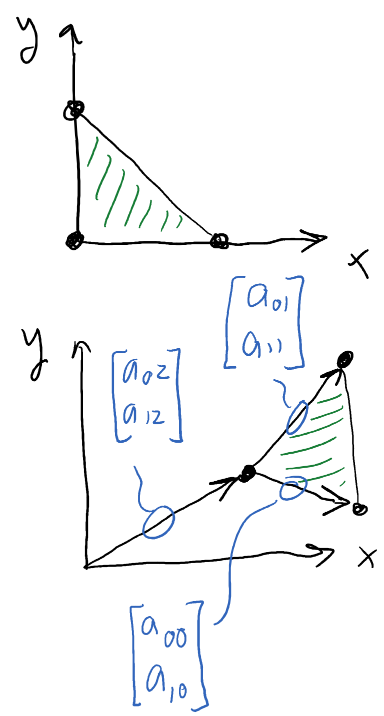
Because the bottom row is $[0 \quad 0 \quad 1]$, we conclude that A^{-1} is affine.

b)

$$\begin{bmatrix} a_{02} \\ a_{12} \\ 1 \end{bmatrix} = A \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} a_{00} + a_{02} \\ a_{10} + a_{12} \\ 1 \end{bmatrix} = A \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} a_{01} + a_{02} \\ a_{11} + a_{12} \\ 1 \end{bmatrix} = A \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$



c) I'll not show the product BA here, but if you multiply them you will find that the bottom row is [0 0 1], which means that we can conclude that the product BA also represents a 2D affine transform.

Problem 3

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Let p_1 and p_2 represent endpoints of the nail in 3D, with $p_1 = (x_1, y_1, z_c)$ and $p_2 = (x_2, y_2, z_c)$.

The image locations are

$$p_1' = \frac{f}{z_c}\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} \quad \text{and} \quad p_2' = \frac{f}{z_c}\begin{bmatrix} x_2 \\ y_2 \end{bmatrix}$$

The distance between p_1' and p_2' is

$$d = \left(\left[\frac{f}{z_c}(x_1 - x_2)\right]^2 + \left[\frac{f}{z_c}(y_1 - y_2)\right]^2 \right)^{1/2}$$

$$= \frac{f}{z_c}\left[(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_c - z_c)^2\right]^{1/2}$$

$$= \frac{f}{z_c} L,$$ where L is the length of the nail in 3D.

<u>Because $\frac{f}{z_c} L$ is constant</u>, we can conclude that the apparent (imaged) length of the nail does not depend on its position or orientation on the table.

Problem 4

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$$\frac{\partial I}{\partial x} \triangleq \lim_{\Delta x \to 0} \frac{I(x+\Delta x, y) - I(x,y)}{\Delta x}$$

$$\approx \frac{I(x+\Delta x, y) - I(x,y)}{\Delta x}$$

$$\frac{\partial^2 I}{\partial x^2} \triangleq \frac{\partial}{\partial x}\left(\frac{\partial I}{\partial x}\right) = \lim_{\Delta x \to 0} \frac{\frac{\partial I}{\partial x}(x+\Delta x, y) - \frac{\partial I}{\partial x}(x,y)}{\Delta x}$$

$$\approx \frac{\frac{I(x+2\Delta x, y) - I(x+\Delta x, y)}{\Delta x} - \frac{I(x+\Delta x, y) - I(x,y)}{\Delta x}}{\Delta x}$$

$$= \frac{1}{(\Delta x)^2}\left[I(x+2\Delta x, y) - 2I(x+\Delta x, y) + I(x,y) \right]$$

The template represents coefficients of these terms

1	-2	1

$I(x,y)$ $I(x+\Delta x, y)$ $I(x+2\Delta x, y)$

we conclude that this template is a good approximation to $\frac{\partial^2 I}{\partial x^2}$ when $\Delta x = 1$.