ECE 4554 / 5554: Computer Vision: Homework 1

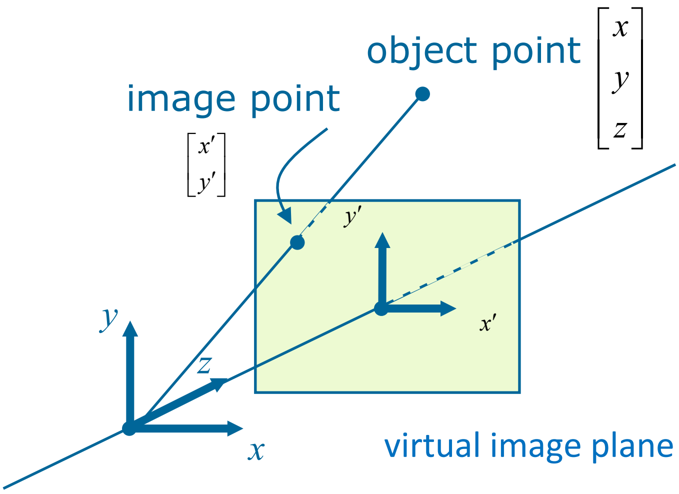
Fall 2022

Instructions

- This assignment is due at Canvas on Sept. 3 before 11:59 PM. As described in the syllabus, late submissions are allowed at the cost of 1 token per 24-hour period. A submission received only a minute after the deadline will cost 1 token.
- Please review the Honor Code statement in the syllabus. For this assignment, you may discuss general approaches to solving the problems with other students. You may discuss software libraries and syntax. Beyond that point, you must work independently. The work that you submit for a grade must be your own.
- The assignment consists of 4 analytical problems, which are presented here, and 2 "machine problems", which require work using Colab. Notice that one of the analytical problems is required for 5554 students, but is optional (extra credit) for 4554 students.
- For the analytical problems, prepare an answer sheet that contains all of your written answers in a single file named `Homework1_Problems1-4_USERNAME.pdf`. (Use your own VT Username.) Handwritten solutions are allowed, but they must be easily legible to the grader. For the machine problems, you must provide a Jupyter notebook file and an associated pdf file. Details are provided at the end of this assignment.
- For machine problems, the notebook file that you submit must be compatible with Google Colab. The grader should be able to execute your code after making only one change to your file, which is the location of the working directory. If the notebook file does not execute, then the grader will be tempted to assign a grade of zero for the machine problems.
- After you have submitted to Canvas, it is your responsibility to download the files that you submitted and verify that they are correct and complete. *The files that you submit to Canvas are the files that will be graded.*

Problem 1. (10 points) Consider the diagram below, which was introduced in a recent lecture. A camera with a <u>left-handed</u> (x, y, z) coordinate system is centered at the point of projection. The z axis lies on the optical axis, and a virtual image plane is located at a distance f (the focal length) from the point of projection. For this problem, locations within the image are indicated using metric units (i.e., meters, not pixels) with respect to coordinate system (x', y') as shown. Assuming ideal perspective projection, find the image locations that correspond to the following 3-dimensional scene points (given in meters). Assume f = 28 mm. For full credit in this problem, provide units (such as m, cm, or mm) with your answer.

a) $(x, y, z) = (3, 4, 5)$
b) $(x, y, z) = (6, 8, 10)$
c) $(x, y, z) = (-1, -2, 6)$
d) $(x, y, z) = (0, 0, 6)$



e) Assume that the sensor array for this camera is of size 18 mm × 24 mm (height × width). Determine the horizontal field of view for this camera. (The answer should be an angle, in degrees, that corresponds to the width of the sensor array.)

Problem 2. (10 points)

Consider the matrix equation $\mathbf{Ax} = \mathbf{0}$, where $\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}$, $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$, and $\mathbf{0} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$.

Assume that all components of **A** are known, and you want to solve for the unknown vector **x**. When **A** satisfies certain conditions, the solution is the eigenvector associated with the smallest eigenvalue of $\mathbf{A}^T\mathbf{A}$. (The superscript *T* represents the transpose operation.) To avoid the trivial solution that all $x_i = 0$, it is common to impose the condition that **x** is a unit vector.

Solve for **x**, a unit vector, for the case $\mathbf{A} = \begin{bmatrix} 2 & 3 & 4 \\ 5 & 6 & 7 \end{bmatrix}$ using this eigenvector-based approach. As part of your answer, clearly show all eigenvalues of $\mathbf{A}^T\mathbf{A}$. (You may use any matrix solver to find the numerical values. For example, the NumPy functions `np.linalg.eig()` or `np.linalg.eigh()` might be used. If you use a matrix solver, cut and paste your code as part of your solution.)

Problem 3. (10 points) Two lines in the (*x, y*) plane can be described by the following pair of equations, where the symbols a_i, b_i, and c_i represent constant terms.

$$a_1x + b_1y + c_1 = 0$$
$$a_2x + b_2y + c_2 = 0$$

a) Assume that these 2 lines are <u>not parallel</u>. Solve for the point of intersection of these 2 lines. Express your answer in terms of the quantities a_1, b_1, c_1, a_2, b_2, and c_2.

b) For your answer in (a), give a numerical solution for the following case:

$a_1 = 3$, $b_1 = 4$, $c_1 = 5$, $a_2 = 6$, $b_2 = 7$, $c_2 = 8$

c) Mathematically, state the condition(s) that must be true when the 2 lines <u>are parallel</u>. Express your answer in terms of the quantities a_1, b_1, c_1, a_2, b_2, and c_2.

Problem 4. (10 points) For <u>5554</u> students, this problem is <u>required</u>. For <u>4554</u> students, this problem is optional and can be submitted for <u>extra credit</u>.

Notice that it is possible to represent an arbitrary line in 3 dimensions using the following parametric form:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \end{bmatrix} t + \begin{bmatrix} x_0 \\ y_0 \\ z_0 \end{bmatrix},$$

In this expression, $[x_0, y_0, z_0]^T$ is any reference point on the line, $[a,b,c]^T$ is a vector that indicates the direction of the line, and parameter *t* is an independent scalar variable. (To visualize this representation, consider $[x_0, y_0, z_0]^T$ and $[a,b,c]^T$ to be constants. Then $[x,y,z]^T$ represents a point on the line that depends on the choice of variable *t*. This representation is equivalent to vector addition in 3D space, with changes in *t* tracing different locations $[x,y,z]^T$ along the line.)

In general, lines that are parallel in 3D do not appear parallel in a 2D image. Instead, lines that are parallel in 3 dimensions will appear to converge at a single point in the image, known as the *vanishing point*. Assume perspective projection. As usual, assume that the virtual image plane is perpendicular to the z axis, and is located a distance f from the point of projection. Suppose that you are given two lines in 3D that are parallel to the line defined in the equation shown above. Show that you can write an equation for the location of the vanishing point in the image plane for these two 3D lines, in terms of the constants a, b, c, and the focal length f.

Machine Problems.
You have been given a Jupyter notebook file `Homework1_USERNAME.ipynb` and an image file `mandrill.tif`. Replace "`USERNAME`" with your Virginia Tech Username. Then upload both files to Google Drive. Open the `ipynb` file in Google Colab. Follow the instructions that you will find inside the notebook file.

What to hand in: After you have finished, you will have created the following 3 files. Upload these 3 files to Canvas before the deadline. Do not combine them in a single ZIP file.

`Homework1_Problems1-4_USERNAME.pdf` ← Your solutions to problems 1 through 4

`Homework1_Code_USERNAME.zip` ← Your zipped Jupyter notebook file

`Homework1_Notebook_USERNAME.pdf` ← A PDF version of your Colab session
