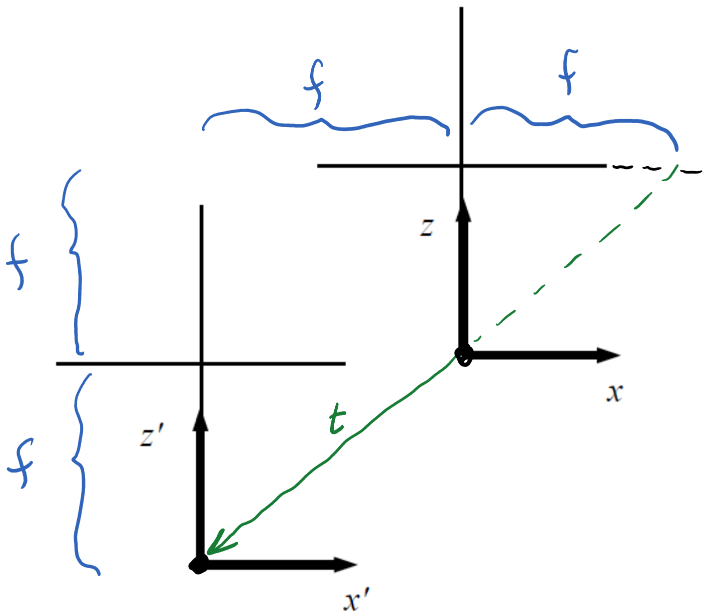
Problem 1

Sunday, November 6, 2022 10:43 AM



$$t = \begin{bmatrix} -f \\ 0 \\ -f \end{bmatrix}$$

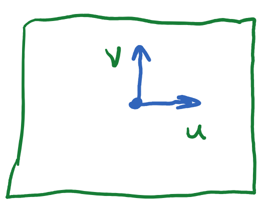
No rotation, so

$$R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

a) $E = [t]_x R = [t]_x = \underline{\begin{bmatrix} 0 & f & 0 \\ -f & 0 & f \\ 0 & -f & 0 \end{bmatrix}}$ or $\underline{\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}}$

because we assume that $\mathcal{E}$ is known only up to a scale factor.

b) I'll use these virtual image coordinates:



$$\mathcal{E}\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} 0 & f & 0 \\ -f & 0 & f \\ 0 & -f & 0 \end{bmatrix}\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\pm fv = 0 \Rightarrow v = 0$

$\underbrace{-fu + f = 0 \Rightarrow u = 1}$

Answer, up to a scale factor:

$$\underline{\begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}}$$

Looking at the diagram, we can solve directly: $\underline{\begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} f \\ 0 \end{bmatrix}}$

Problem 2

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$$t = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} \qquad R = \begin{bmatrix} \cos(45°) & -\sin(45°) & 0 \\ \sin(45°) & \cos(45°) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

a) $$E = [t]_x R = \begin{bmatrix} 0 & -1 & 2 \\ 1 & 0 & -3 \\ -2 & 3 & 0 \end{bmatrix} \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \underline{\underline{\begin{bmatrix} -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 2 \\ \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & -3 \\ \frac{\sqrt{2}}{2} & \frac{5\sqrt{2}}{2} & 0 \end{bmatrix}}} \approx \begin{bmatrix} -0.707 & -0.707 & 2 \\ 0.707 & -0.707 & -3 \\ 0.707 & 3.536 & 0 \end{bmatrix}$$

b) SVD shows that the rank of E is $\underline{\underline{2}}$

Problem 3

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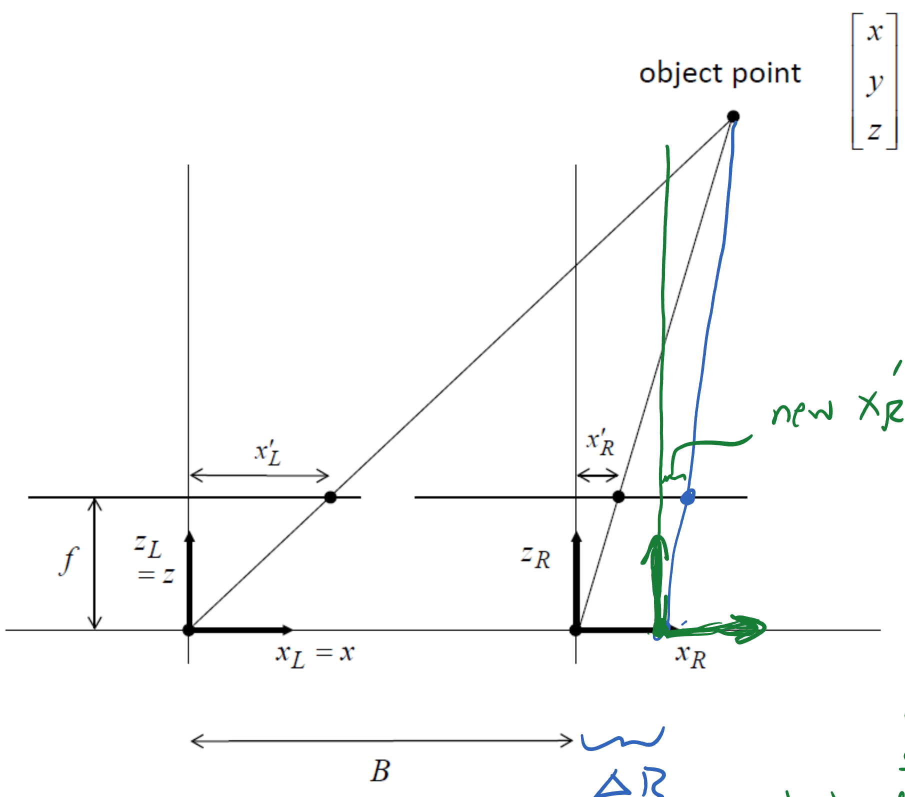
Given:

$$(x'_L, y'_L) = (0.005, 0.0) \quad \text{and} \quad (x'_R, y'_R) = (0.003, 0.0)$$

a) $$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{B}{d} \begin{bmatrix} x_L' \\ y_L' \\ f \end{bmatrix} \quad \text{where } d \triangleq x_L' - x_R'$$

$$= \frac{0.15}{0.005 - 0.003} \begin{bmatrix} 0.005 \\ 0.0 \\ 0.025 \end{bmatrix} = \underline{\underline{\begin{bmatrix} 0.375 \\ 0.0 \\ 1.875 \end{bmatrix}}} \text{ (meters)}$$

b) First, for this stereo geometry, notice that disparity <u>increases</u> slightly for any 3D point, if B increases a little.



Here, assume that the only change is the right camera moving to the right by ΔB. The new x_R' value is <u>smaller</u>, which means that $d \triangleq x_L' - x_R'$ is <u>larger</u>.

Problem 3 (continued)

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Next, let's assume that the computed z value always has some error that results from a small error in computed disparity.

$$d_{comp} = d + d_{err}$$

signed error in disparity

true disparity value

$$z_{comp} = z + z_{err}$$

signed error in computed range

true range value

Solve for z_{err}:

$$z_{err} = z_{comp} - z$$

$$= \frac{Bf}{d_{comp}} - \frac{Bf}{d}$$

$$= \frac{Bf}{d + d_{err}} - \frac{Bf}{d}$$

$$= \frac{Bfd - Bf(d + d_{err})}{(d + d_{err})\, d}$$

$$= \frac{-Bf\, d_{err}}{d\,(d + d_{err})}$$

$$= -z \frac{d_{err}}{(d + d_{err})}$$

So the absolute error is $|z_{err}| = \left| z \frac{d_{err}}{d + d_{err}} \right|$

Of these terms, let's assume that z and d_{err} are not affected by a small change in B.

On the previous page, we argued that d increases as B increases, which means that $|z_{err}|$ is expected to decrease.

Problem 4

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First, let's introduce some notation for this problem.

$$\underbrace{x'}_{\text{image point}} = \underbrace{K}_{\text{intrinsic parameters}} [\underbrace{R \mid t}_{\text{extrinsic parameters}}] \underbrace{X}_{\text{a point in 3D world coordinates}}$$

$$\begin{bmatrix} s \cdot u \\ s \cdot v \\ s \end{bmatrix} = \overbrace{\begin{bmatrix} f & 0 & u_0 \\ 0 & f & v_0 \\ 0 & 0 & 1 \end{bmatrix}}^{\text{intrinsic parameters}} \overbrace{\begin{bmatrix} r_{00} & r_{01} & r_{02} & t_x \\ r_{10} & r_{11} & r_{12} & t_y \\ r_{20} & r_{21} & r_{22} & t_z \end{bmatrix}}^{\text{extrinsic parameters}} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

The goal is to solve for position and orientation of the camera, which are represented by the extrinsic parameters in the equations above.

It is reasonable to assume that the detected lane lines are parallel in 3D space, and therefore it may be helpful to consider the vanishing point in the image for these lane lines.

We gain some insight if we divide both sides of the equation by z:

$$\frac{s}{z} \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = K \left[R \mid t \right] \begin{bmatrix} x/z \\ y/z \\ 1 \\ 1/z \end{bmatrix}$$

In the limit as z approaches infinity, we can solve as follows:

$$\underbrace{\begin{bmatrix} b_1 c_2 - b_2 c_1 \\ a_2 c_1 - a_1 c_2 \\ a_1 b_1 - a_2 b_1 \end{bmatrix}}_{\text{vanishing point, from an earlier homework}} = K \left[R \mid t \right] \underbrace{\begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}}_{\text{A representation of the "point at infinity" in the z direction}}$$

Problem 4 (continued)

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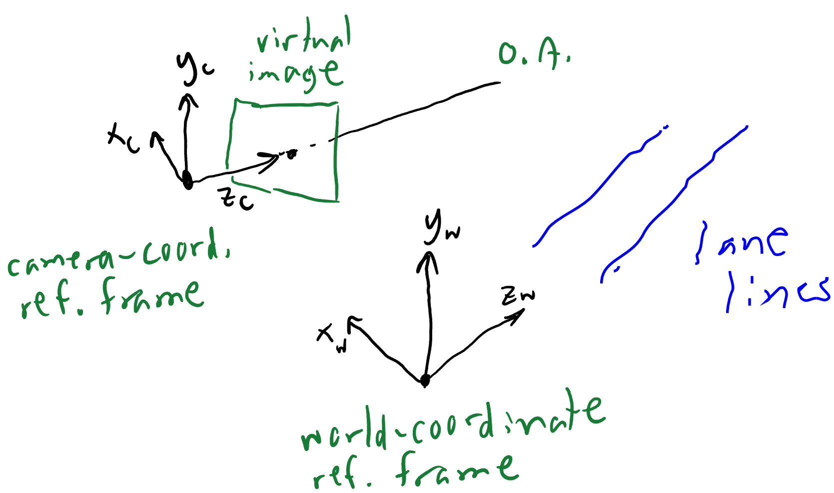
We can multiply both sides by K^{-1}:

$$K^{-1}\begin{bmatrix} b_1 c_1 - b_2 c_1 \\ a_2 c_1 - a_1 c_2 \\ a_1 b_1 - a_2 b_1 \end{bmatrix} = \begin{bmatrix} R \mid t \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

$$\frac{1}{f}\begin{bmatrix} 1 & 0 & -u_0 \\ 0 & 1 & -v_0 \\ 0 & 0 & f \end{bmatrix}\begin{bmatrix} b_1 c_2 - b_2 c_1 \\ a_2 c_1 - a_1 c_2 \\ a_1 b_1 - a_2 b_1 \end{bmatrix} = \begin{bmatrix} r_{00} & r_{01} & r_{02} & t_x \\ r_{10} & r_{11} & r_{12} & t_y \\ r_{20} & r_{21} & r_{22} & t_z \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

$$\frac{1}{f}\begin{bmatrix} b_1 c_2 - b_2 c_1 - u_0 a_1 b_1 - u_0 a_2 b_1 \\ a_2 c_1 - a_1 c_2 - v_0 a_1 b_1 - v_0 a_2 b_1 \\ f a_1 b_1 - f a_2 b_1 \end{bmatrix} = \begin{bmatrix} r_{02} \\ r_{12} \\ r_{22} \end{bmatrix}$$

All of the values on the left-hand side of the equation are assumed to be known.
So, we can solve for some of the rotation parameters using knowledge of the vanishing point.
An interpretation of the vector on the right is that it points in the direction of the world-coordinate z axis, as represented with respect to the camera-coordinate reference frame.



Problem 4 (continued)

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The problem statement allowed us to make more assumptions for this problem, such as having the Optical Axis parallel to the road. We could make the stronger assumption that y_c is parallel to y_w and allow rotations about the y_w axis only. (This rotation is often called yaw or pan.)

In this case we have the rotation matrix as follows, with θ as the yaw angle:

$$\begin{bmatrix} r_{00} & r_{01} & r_{02} \\ r_{10} & r_{11} & r_{12} \\ r_{20} & r_{21} & r_{22} \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$$

Following from the previous page, we have this expression that allows us to solve for the yaw angle under these assumptions:

$$\frac{1}{f}\begin{bmatrix} b_1 c_2 - b_2 c_1 - u_0 a_1 b_1 - u_0 a_2 b_1 \\ a_2 c_1 - a_1 c_2 - v_0 a_1 b_1 - v_0 a_2 b_1 \\ f a_1 b_1 - f a_2 b_1 \end{bmatrix} = \begin{bmatrix} \sin\theta \\ 0 \\ \cos\theta \end{bmatrix}$$

To solve for translation parameters, more information is needed.
For example, if we assume that the 3D separation distance for the lane lines is known, then it may be possible to recover some of the translation parameters.