**ELECTRICAL & ELECTRONICS ENGINEERING**

# DIGITAL SIGNAL PROCESSING LABORATORY PROJECT REPORT


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**Project Subject :**

**IIR NOTCH FILTER DESIGN**

**Due date :** **23 -05 -2016**

## INTRODUCTION

A Notch filter is a bandstop filter that eliminates only one frequency component. Notch filters are very useful in the medical field especially in Electrocardiography. During ECG measurement there are interferences produced by the electrical field from the power lines and the magnetical field due to the current in the power line which introduce noise into the ECG signal. In this case, a Notch filter is used to eliminate the 60Hz (or 50Hz) and its harmonics. Thus producing a more reliable ECG signal in order to ensure a more error free diagnosis from doctors.

## Design of 2nd Order IIR Notch Filter

For this type of filter, we used the pole and zero placement method. Poles are complex conjugate, with the magnitude r controlling the bandwidth and the angle theta controlling the center frequency.The zeros are placed on the unit circle with the same angles with respect to poles. This will improve passband performance. The magnitude and the angle of the complex conjugate poles determine the 3 dB bandwidth and center frequency, respectively.

Design formulas for bandstop filters are given in the following equations:

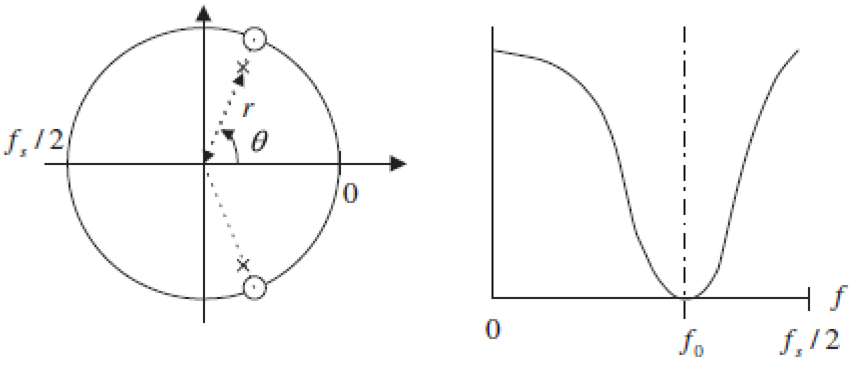
$$r \approx 1 - (BW_{3dB}/f_s) \times \pi, \text{ good for } 0.9 \leq r < 1$$

$$\theta = \left(\frac{f_0}{f_s}\right) \times 360^\circ$$

$$H(z) = \frac{K\left(z - e^{j\theta}\right)\left(z + e^{-j\theta}\right)}{\left(z - re^{j\theta}\right)\left(z - re^{-j\theta}\right)} = \frac{K\left(z^2 - 2z\cos\theta + 1\right)}{\left(z^2 - 2rz\cos\theta + r^2\right)}$$

The scale factor to adjust the bandstop filter so it has a unit passband gain is given by

$$K = \frac{\left(1 - 2r\cos\theta + r^2\right)}{\left(2 - 2\cos\theta\right)}$$




Pole-zero placement for a second-order notch filter.

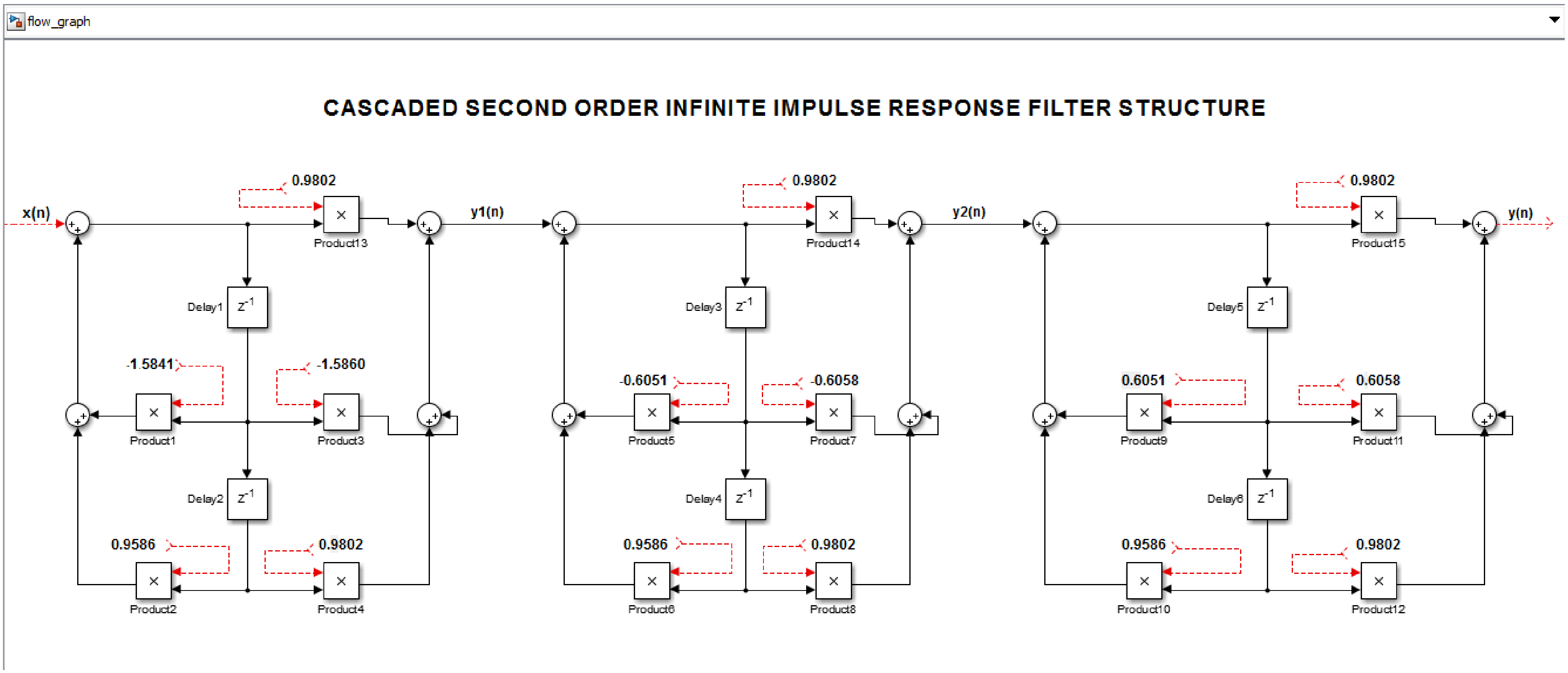
The above method was used to design the filter in MATLAB. The codes and simulations are shown below.

## MATLAB CODES for 3-CASCADED 2nd ORDER IIR NOTCH FILTER DESIGN

```
%%
% MUSTAFA GÜÇLÜ - 05110000994
% BACHIR TCHANA TANKEU - 05120000820
% 3-Cascaded Second Order IIR Notch Filter Design

clear all
close all
clc

fs=600; % Sampling rate
f1=60; % fundamental harmonic
f2=120; % second harmonic
f3=180; % third harmonic
BW=4; % 3 dB bandwidth
r=1-BW/fs*pi; % the required magnitude of the poles

% Notch filter with a notch frequency of 60 Hz ==> H1(z)
theta1=f1/fs*360*pi/180; %the center frequency to obtain the angle of the pole..
…location

K1=(1-2*r*cos(theta1)+r^2)/(2-2*cos(theta1)); % the unit-gain scale factor
b1=K1*[1 -2*cos(theta1) 1] % coefficients for the numerator
a1=[1 -2*r*cos(theta1) r^2] % coefficients for the denominator

% Notch filter with a notch frequency 120 Hz ==> H2(z)
theta2=f2/fs*360*pi/180; %the center frequency to obtain the angle of the pole..
…location

K2=(1-2*r*cos(theta2)+r^2)/(2-2*cos(theta2)); % the unit-gain scale factor
b2=K1*[1 -2*cos(theta2) 1] % coefficients for the numerator
a2=[1 -2*r*cos(theta2) r^2] % coefficients for the denominator

% Notch filter with a notch frequency of 180 Hz ==> H3(z)
theta3=f3/fs*360*pi/180; %the center frequency to obtain the angle of the pole..
…location

K3=(1-2*r*cos(theta3)+r^2)/(2-2*cos(theta3)); % the unit-gain scale factor
b3=K1*[1 -2*cos(theta3) 1] % coefficients for the numerator
a3=[1 -2*r*cos(theta3) r^2] % coefficients for the denominator

%%
% MUSTAFA GÜÇLÜ - 05110000994
% BACHIR TCHANA TANKEU - 05120000820

load 'ecgbn.dat'; % Load noisy ECG recording
T=1/600; % Sampling interval
t=0:T:1499*T; % Recover time

b=conv(conv(b1,b2),b3); %convolution for the numerators
a=conv(conv(a1,a2),a3); %convolution for the denominators
[H w]=freqz(b,a);
```

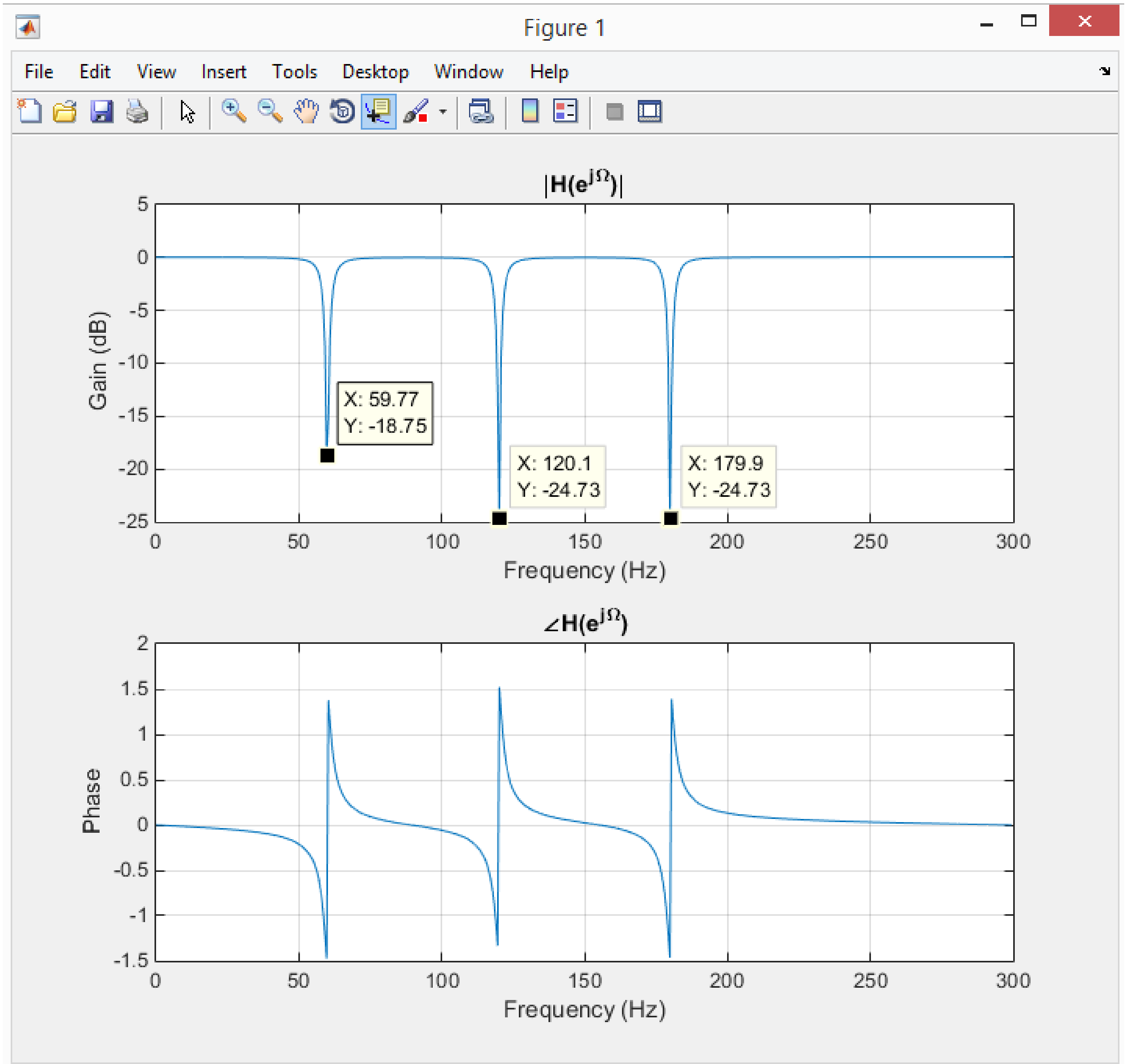
```
subplot(2,1,1);
plot(w*fs/(2*pi),20*log10(abs(H))); %frequency domain
grid on;
xlabel('Frequency (Hz)');
ylabel('Gain (dB)');
title('\midH(e^j^\Omega)\mid');
subplot(2,1,2);
plot(w*fs/(2*pi),angle(H)); %frequency domain
grid on;
xlabel('Frequency (Hz)');
ylabel('Phase');
title('\angleH(e^j^\Omega)');

y=filter(b,a,ecgbn); % filtering for ECG Signal with Noise
figure;
subplot(2,1,1);
plot(t,ecgbn); % Input Signal with noise
grid on;
xlabel('time (s)');
ylabel('Amplitude');
title('ECG Signal with Noise');
subplot(2,1,2);
plot(t,y); %Filtered Signal at 60Hz, 120Hz, and 180Hz
grid on;
xlabel('time (s)');
ylabel('Amplitude');
title('The Signal after 3-Notch Filter');
```

| | |
|---|---|
| Command Window<br>b1 =<br>0.9802 -1.5860 0.9802<br>a1 =<br>1.0000 -1.5841 0.9586<br>fx | $H_1(z) = \frac{0.9802-1.5860z^{-1}+0.9802z^{-2}}{1-1.5841z^{-1}+0.9586z^{-2}}$ |
| Command Window<br>b2 =<br>0.9802 -0.6058 0.9802<br>a2 =<br>1.0000 -0.6051 0.9586<br>fx | $H_2(z) = \frac{0.9802-0.6058z^{-1}+0.9802z^{-2}}{1-0.6051z^{-1}+0.9586z^{-2}}$ |
| Command Window<br>b3 =<br>0.9802 0.6058 0.9802<br>a3 =<br>1.0000 0.6051 0.9586<br>fx >> | $H_3(z) = \frac{0.9802+0.6058z^{-1}+0.9802z^{-2}}{1+0.6051z^{-1}+0.9586z^{-2}}$ |

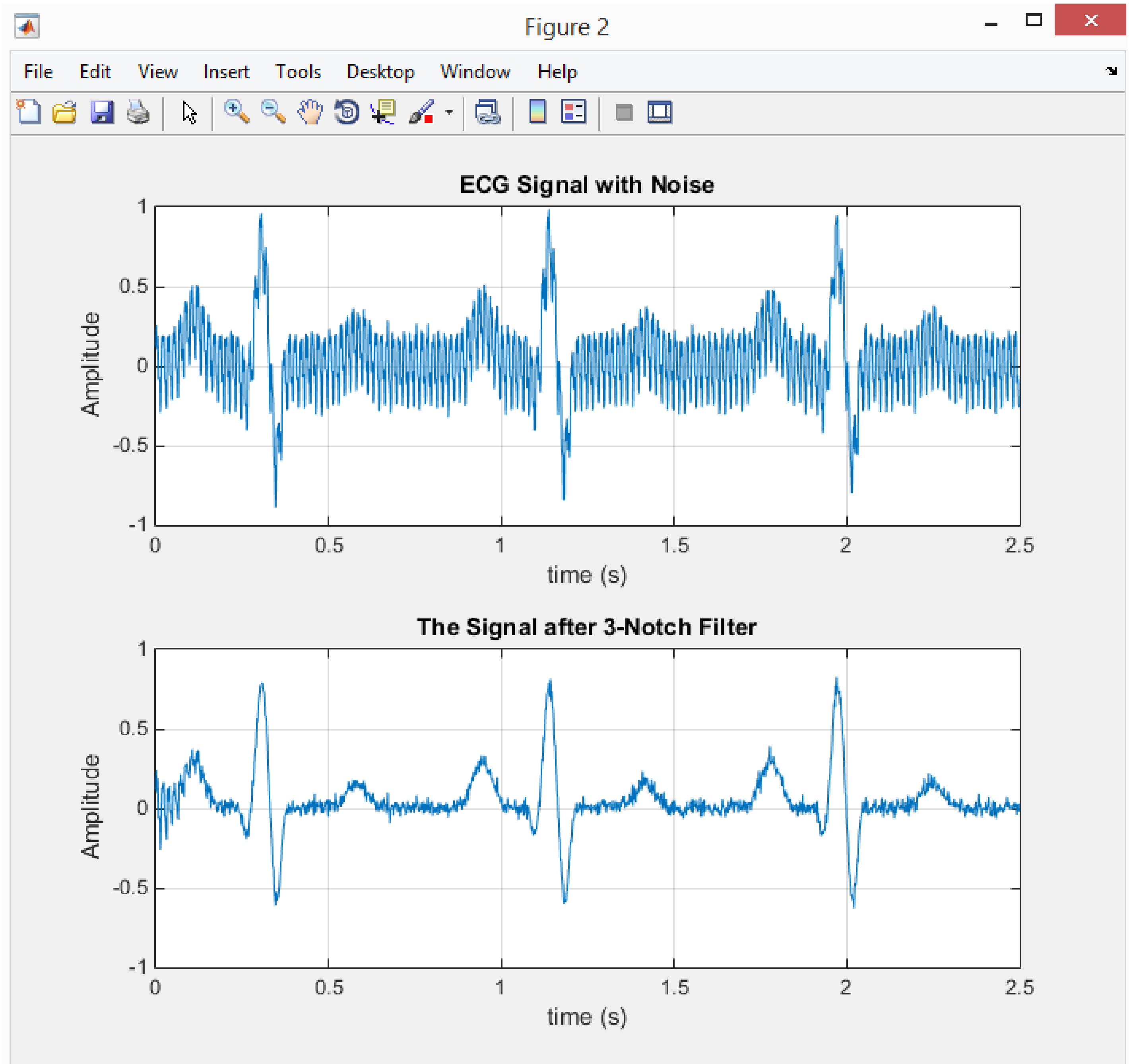


flow_graph
CASCADED SECOND ORDER INFINITE IMPULSE RESPONSE FILTER STRUCTURE
x(n)
y1(n)
y2(n)
y(n)
0.9802
Product13
Delay1
-1.5841
Product1
-1.5860
Product3
Delay2
0.9586
Product2
0.9802
Product4
0.9802
Product14
Delay3
-0.6051
Product5
-0.6058
Product7
Delay4
0.9586
Product6
0.9802
Product8
0.9802
Product15
Delay5
0.6051
Product9
0.6058
Product11
Delay6
0.9586
Product10
0.9802
Product12

## PLOTS OF MAGNITUDE AND PHASE OF THE FILTER



## PLOTS OF INPUT SIGNAL WITH NOISE AND FILTERED OUTPUT SIGNAL



# CONCLUSIONS

The codes for the 3 cascaded Notch filters were written in Matlab, the results of the simulations showed that the Cascaded Notch filter effectively removed the 60 Hz,120 Hz and 180 Hz frequency components of the noisy ECG signal.

In order to remove the other undesired frequency components, we can cascade a Bandpass filter with the Notch filter.

**References:**


1. TAN Li, Digital Signal Processing: Fundamentals and Applications, 2nd Ed., 2013.
2. AKKAYA Ibrahim, Laboratory Notes and Matlab Codes, 2016.
3. WANG Chun Mei, XIAO Wei Cai, Second-order IIR Notch Filter Design and implementation of digital signal processing system, Published by Atlantis Press, Paris, France, 2013.