De-Moivre's thm

$$(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin(n\theta).$$

n^{th} roots of Unity

$$x^3 = 1,$$

$$x = 1, \omega, \omega^2$$

$$\boxed{1+\omega+\omega^2 = 0}$$

$$\omega^3 = 1.$$

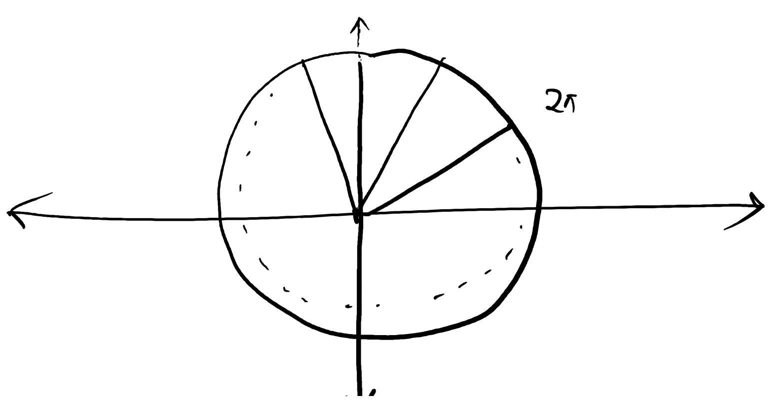
When, $z = 1$, mod $z = 1$ and $\arg z = 0$.

Then the n^{th} roots of unity are –

$$z = \cos\frac{2k\pi}{n} + i\sin\frac{2k\pi}{n}, \quad k = 0, 1, \dots n-1.$$

$$|z| = \sqrt{\cos^2\frac{2k\pi}{n} + \sin^2\frac{2k\pi}{n}} = 1.$$

$$\arg z = 0, \frac{2\pi}{n}, \frac{4\pi}{n}, \dots \left(2\pi - \frac{2\pi}{n}\right).$$



When, n is odd, the roots are,

$$1, \quad \cos\frac{2k\pi}{n} + i\sin\frac{2k\pi}{n}, \quad k = 1, 2, \dots \frac{n-1}{2}.$$

When, n is even, the roots are,

$$\pm 1, \quad \cos\frac{2k\pi}{n} \pm i\sin\frac{2k\pi}{n}, \quad k = 1, 2, \dots \frac{n}{2} - 1.$$

$$z^3 = 1, \quad \boxed{z^n - 1 = 0}$$

$$\omega = \cos\frac{2k\pi}{n} + i\sin\frac{2k\pi}{n}, \quad \text{when } k = 1$$

$$= \cos\frac{2\pi}{n} + i\sin\frac{2\pi}{n}$$

$$z^n - 1 = 0$$

$$\boxed{z = 1, \omega, \omega^2, \dots\dots \omega^{n-1}}$$

$$z^n - 1 = (z-1)\,\Pi(\quad)\,(\quad)$$

Eg. 1 Find the cube roots of 1.

$$z^3 = 1.$$

$$1 = \cos 0 + i\sin 0$$

$$(\cos 0 + i\sin 0)^{1/3}.$$

$1 = \cos 0$

$$z = (\cos 0 + i \sin 0)^{1/3}.$$

$$z = \cos \frac{2k\pi}{n} + i \sin \frac{2k\pi}{n}, \quad k = 0, 1, 2.$$

So, $z_1 = 1$, $\quad z_2 = \cos \frac{2\pi}{n} + i \sin \frac{2\pi}{n}$

$$z_3 = \cos \frac{4\pi}{n} + i \sin \frac{4\pi}{n}.$$

if $\omega = \cos \frac{2\pi}{n} + i \sin \frac{2\pi}{n}$ $= \cos 2 \cdot \left(\frac{2\pi}{n}\right) + i \sin 2 \cdot \left(\frac{2\pi}{n}\right)$

$$= \left(\cos \frac{2\pi}{n} + i \sin \frac{2\pi}{n}\right)^2$$

$$= \omega^2$$

Hence, the sol^n is $\{1, \omega, \omega^2\}$.

Eg 3 : Find the fourth roots of 1.

$\rightarrow$ $z^4 = 1$

or, $z = 1^{1/4}$

$$= (\cos 0 + i \sin 0)^{1/4}$$

$$= \left(\cos \frac{2k\pi}{4} + i \sin \frac{2k\pi}{4}\right) \quad k = 0, 1, 2, 3.$$

$$= \left(\cos \frac{2\pi}{4} + i \sin \frac{2\pi}{4}\right)^k.$$

$$= \left(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2}\right)^k.$$

$$= \left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)^{\ldots}$$

$$= i^{K}$$

Solution set: $\{1, i, i^2, i^3\}$.

$= \{1, i, -1, -i\}$.

Eg 3: Solve the eq^n:

$$x^6 + x^5 + x^4 + x^3 + x^2 + x + 1 = 0.$$

Sol^n:

$$\frac{1(x^7 - 1)}{(x-1)} = 0$$

or, $x^7 - 1 = 0$

Sol^n set: $\cos\frac{2k\pi}{7} + i\sin\frac{2k\pi}{7}$

$k = 0, 1, \ldots 6.$

Eg. 4: Prove that: $\sqrt{i} + \sqrt{-i} = \sqrt{2}$.

Sol^n: $i = \cos\frac{\pi}{2} + i\sin\frac{\pi}{2}$

$-i = \cos\left(-\frac{\pi}{2}\right) + i\sin\left(-\frac{\pi}{2}\right).$

$\sqrt{i} = i^{1/2} = \left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)^{1/2}$

$\hookrightarrow \sqrt{i} = i^{1/2} = \left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)^{\sim}$

$\sqrt{-i} = -i^{1/2} = \ldots\ldots$

So, $\sqrt{i} + \sqrt{-i} = \sqrt{2}$.

$\boxed{\text{PROVED}}$

Some more applications of De-Moivre's theorem !

Find expansion of $\cos n\theta$.

$\cos 2\theta = 2\cos^2\theta - 1$

$\cos 3\theta = 4\cos^3\theta - 3\cos\theta$

$\vdots$

$\cos n\theta = \ldots\ldots$

$\boxed{n \rightarrow \text{Positive int}}$

$\Rightarrow \cos n\theta + i\sin n\theta$

$= (\cos\theta + i\sin\theta)^n$

$= \cos^n\theta + {}^nC_1 \cos^{n-1}\theta \cdot (i\sin\theta)$

$+ {}^nC_2 \cos^{n-2}\theta \cdot i^2 \sin^2\theta$

$+ \ldots + i^n \sin^n\theta$

$(a+b)^n = a^n + {}^nC_1 a^{n-1} b^1 + \ldots + {}^nC_n b^n$

$= (\cos^n\theta - {}^nC_2 \cos^{n-2}\theta \cdot \sin^2\theta + {}^nC_4 \cos^{n-4}\theta \sin^4\theta + \ldots)$ $\Leftarrow$ ||

$+ i({}^nC_1 \cos^{n-1}\theta \sin\theta - {}^nC_3 \ldots)$ $\Leftarrow$ ||

$i^6 = (i^2)^3 = (-1)^3 = -1$

$\tan n\theta = \dfrac{\sin n\theta}{\ldots} \rightarrow \underline{\text{tan fun}}$

$$\tan n\theta = \boxed{\frac{\sin n\theta}{\cos n\theta}} \rightarrow \text{tan func}$$

FOO

= Do it in home !

<u>Next day</u>: Trignometric Function
Exp "
Logarithmic "