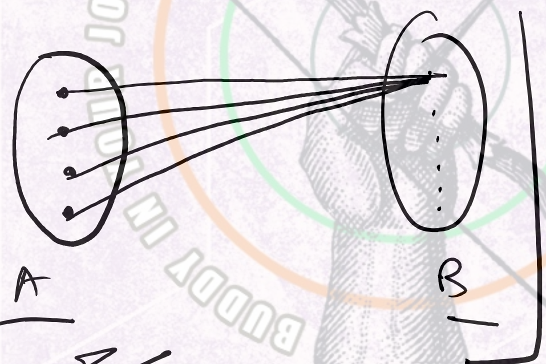
07 June 2025 21:35

Mapping or Function

$A \xrightarrow{f} B$ → non-empty sets.

Defn: Let A and B be two non-empty sets. A mapping f from A to B is a rule that assigns to each element x of A a definite element $y \in B$.



Many to one

One to one

Many to one

✗ One to Many

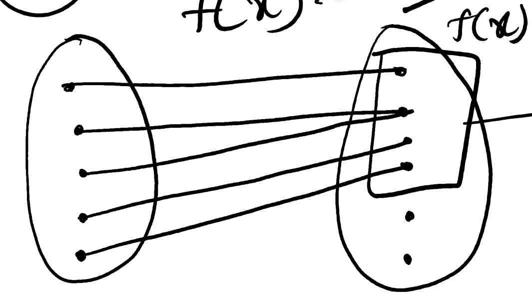
dom

$f : A \rightarrow B$ → co-dom.

Range $\subseteq$ Co-domain

x

$f(x)$ → $f(x)$



→ Range or Image set.

$f(A)$

A B

Examples:

$S = \{1, 2, 3, 4\}$, $T = \{a, b, c, d\}$. Let's examine the following relations f_1, f_2 between S and T

i) $f_1 = \{(1, a), (1, b), (2, c), (3, c), (4, d)\}$ X

ii) $f_2 = \{(1, b), (2, b), (3, c), (4, d)\}$. ✓

2. $f = \{(x, y) \in \mathbb{R} \times \mathbb{R} : y = \frac{1}{x}\}$. Let us examine if f is mapping from $\mathbb{R}$ to $\mathbb{R}$.

↳ Not a mapping because of zero.

Into mapping

$f : A \to B$.

$f(A) \subseteq B$.

Onto mapping

$f : A \to B$

$f(A) = B$.

$f : \mathbb{Z} \to \mathbb{Z}$
$f(x) = x + 1$. ; $y = x + 1$

$\Rightarrow x = y - 1$

$f : R \to R$

$f(x) = \sin x \qquad -1 \le f(x) \le 1$

$x = (4n+1)\frac{\pi}{2}$

Injective Mapping.

$f : A \to B$

$x_1, x_2 \in A, \quad f(x_1) = f(x_2)$

$\Rightarrow x_1 = x_2$

or,

$x_1 \ne x_2$

$\Rightarrow f(x_1) \ne f(x_2)$

Surjective Mapping.

$f : A \to B$. if $f(A) = B$.

Bijective Mapping

Surj

Injec

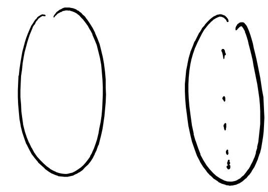
Ex: $f : \mathbb{Z} \to \mathbb{Z}$

$f(x) = 2x, \quad x \in \mathbb{Z}$, check bijectivity.

$y = 2x$

$\Rightarrow x = y/2$.

$\Rightarrow x = \pi/2$.



Z

$f : \mathbb{Z} \to \mathbb{Z}$

$f(x) = |x|$.

$f : \mathbb{Z} \to \mathbb{Z}$

$f(x) = x + 1$.

Equality of Mappings

$f : A \to B$, $g : A \to C$.

i) f and g have same domain.

ii) for all $x \in$ domain , $f(x) = g(x)$.

Ex: $S = \{x \in \mathbb{R} : x > 0\}$

$f : S \to \mathbb{R}$, defined by $f(x) = \dfrac{|x|}{x}$; $x \in S$.

and $g : S \to R$

$g(x) = 1$, $x \in S$, Then $f = g$?

$S = \{x \in \mathbb{R} : 0 \le x \le \frac{\pi}{2}\}$. Let $f : S \to R$

be defined by $f(x) = \cos x - \sin x$, $x \in S$.

$g : S \to R$, be defined by $g(x) = \sqrt{1 - \sin 2x}$

$g : S \to R$, be defined by $g(x) = \sqrt{1 - \sin 2x}$

$f = g$?

$\cos x - \sin x = \sqrt{1 - \sin 2x}$.

$= \sqrt{(\cos x - \sin x)^2} \,/\, \sqrt{(\sin x - \cos x)^2}$

$= \cos x - \sin x \,/\, \sin x - \cos x$.

$f(\pi/2) = -1$, $g(\pi/2) = \sqrt{1 - \sin 2\left(\frac{\pi}{2}\right)}$

$= \sqrt{1 - \sin(\pi)}$

$= \sqrt{1 - 0}$.

$= 1$.

$\pi/4$, $\pi/6$.

$\cos x - \sin x$

$\cos(\pi/6) - \sin(\pi/6) = \frac{\sqrt{3} - 1}{2}$.

$= \frac{\sqrt{3}}{2} - \frac{1}{2}$

$\sqrt{1 - \sin 2(\pi/6)} = \sqrt{1 - \sin \pi/3}$

$= \sqrt{1 - \frac{\sqrt{3}}{2}}$

$f \neq g$. for, $x \in \left(\pi/4, \pi/2\right]$

$= \sqrt{\frac{2 - \sqrt{3}}{2}}$

$= \sqrt{\frac{\quad}{2}}$

$f(x) = x^2$; $f: \mathbb{R} \to \mathbb{R}$

check bijectivity.

Composition of Mappings.

$f: A \to B$ and $g: C \to D$, such that $f(A)$ is a subset of C.

$h: A \to D$, $h(x) = g(f(x))$.

$\underline{g \circ f}$

$f \circ g \to f(g(x))$.

$\underline{f(g(x))}$ $\quad g: B \to C$

$\uparrow$

$g(x)$.

$f: \mathbb{R} \to \mathbb{R}$ and $g: \mathbb{R} \to \mathbb{R}$ be defined by

$f(x) = x + 1$, $g(x) = 3x$.

Find $f \circ g$ and $g \circ f$.

$f(g(x)) = g(x) + 1$
$= 3x + 1$.

$g(f(x))$
$= 3(x+1)$
$= 3x + 3$.

$f(x) = x$, $g(x) = 2x$

$f(g(x)) = 2x$.

$g(f(x)) = 2(x)$
$= 2x$.

$f(g(x)) = 2x$. || $g(f(x)) = 2(x) = 2x$.