LINEAR ALGEBRA

$ax + by + c = 0 \rightarrow$ It represents a st. line.

Introduction to vectors:

magnitude + direction.

e.g $\rightarrow$ Force, displacement etc.

i) Addition of Vectors: If we add two vectors, their sum is also a vector.

ii) Scalar Multiplication: If we multiply a vector by a number, we again get a vector.

Vector in 2D

$$V = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

$v_1 \rightarrow$ First component

$v_2 \rightarrow$ Second component.

$$\tilde{V} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \quad \tilde{\omega} = \begin{bmatrix} \omega_1 \\ \omega_2 \end{bmatrix}$$

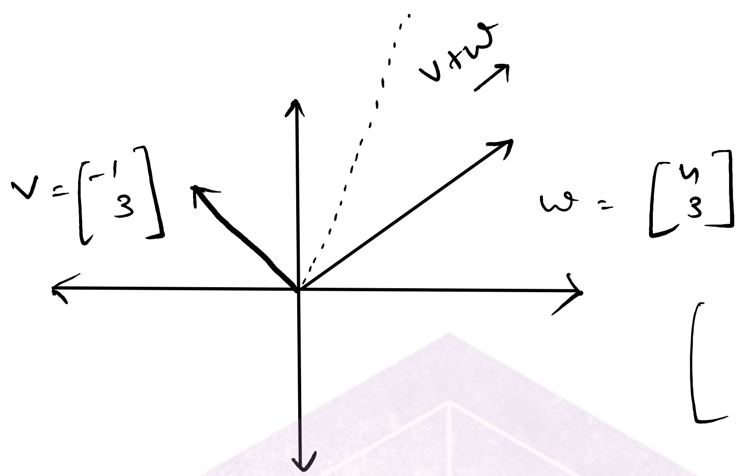
$\tilde{V}$

$\vec{V}$

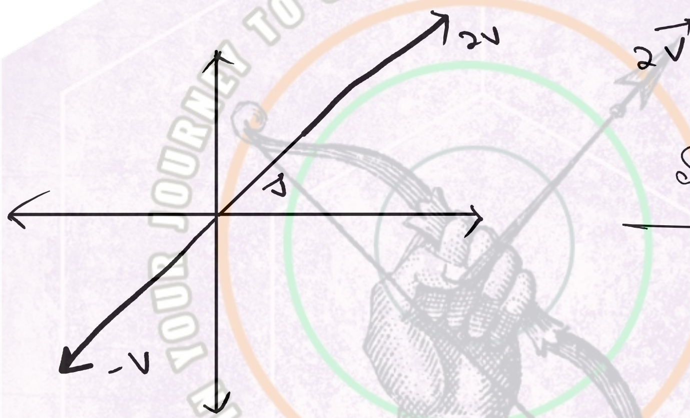
$V \tilde{V}$

By addition of matrix,

$$V + \omega = \begin{bmatrix} v_1 + \omega_1 \\ v_2 + \omega_2 \end{bmatrix} \quad V = \begin{bmatrix} -1 \\ 3 \end{bmatrix} \quad \omega = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$$

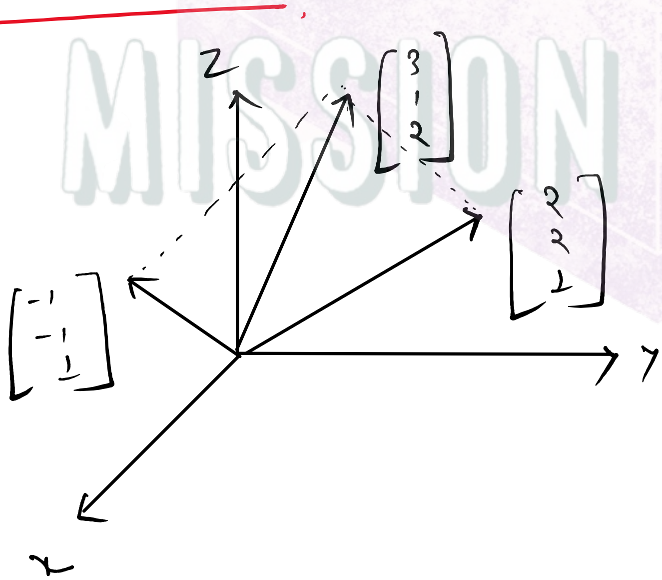


[Addition of vector.]



Scalar Multiplication

Vector in 3D



Visualizing Linear Equations

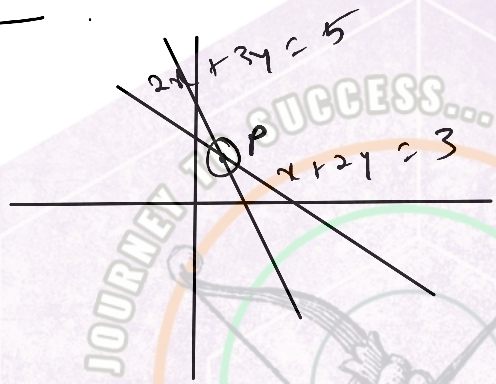
$\begin{bmatrix} 1 & 2 \end{bmatrix}\begin{bmatrix} x \\ \end{bmatrix}$

Visualizing Linear Equations

$$\begin{matrix} x+2y = 3 \\ 2x+3y = 5 \end{matrix} \Bigg\} < \begin{matrix} \text{Row} \\ \text{column} \end{matrix}$$

$$\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$$

Row Picture :



Column Picture :

$Ax = b$. $A = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$, $x = \begin{bmatrix} x \\ y \end{bmatrix} \in \mathbb{R}^2$.

$$Ax = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} x + \begin{bmatrix} 2 \\ 3 \end{bmatrix} y .$$

$\downarrow c_1$ $\quad \downarrow c_2$

'Ax' $\rightarrow$ linear combination of columns of A.

$b = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$.

$c_1 x_1 + c_2 x_2$

$$\begin{bmatrix}1\\2\end{bmatrix}x + \begin{bmatrix}2\\3\end{bmatrix}y = \begin{bmatrix}3\\5\end{bmatrix}$$

$$(x, y) = (1, 1)$$

$$\cancel{6} \quad 1 \cdot \begin{bmatrix}1\\2\end{bmatrix} + 1 \cdot \begin{bmatrix}2\\3\end{bmatrix} = \begin{bmatrix}3\\5\end{bmatrix}$$

$$x + y + z = 3$$
$$x + 2y + z = 4$$
$$y + z = 2$$

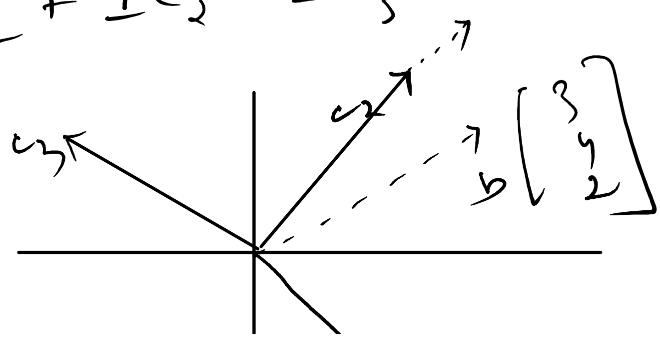
$$Ax = b$$

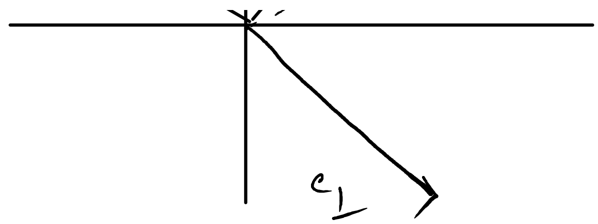
$$\begin{bmatrix}1 & 1 & 1\\1 & 2 & 1\\0 & 1 & 1\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}3\\4\\2\end{bmatrix}$$

$$x\begin{bmatrix}1\\1\\0\end{bmatrix} + y\begin{bmatrix}1\\2\\1\end{bmatrix} + z\begin{bmatrix}1\\1\\1\end{bmatrix} = \begin{bmatrix}3\\4\\2\end{bmatrix}$$

$$(x, y, z) = (1, 1, 1).$$

$$1c_1 + 1c_2 + 1c_3 = b$$





$Ax = b$ .... ?

↳ Is it solvable for every b ?

$$x + y + 2z = 3$$
$$x + 2y + 3z = 4$$
$$y + z = 2$$

$$\begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 2 \end{bmatrix}$$

$$x \underset{c_1}{\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}} + y \underset{c_2}{\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}} + z \underset{c_3}{\begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}} = \begin{bmatrix} 3 \\ 4 \\ 2 \end{bmatrix}.$$

$\boxed{Ax = \bar{b}}$

$$\boxed{c_3 = c_1 + c_2}.$$

$Ax = b$, will be solvable only when b lies in the plane formed by first two colum of A.

in the plane formed by first two column of A.

If b does not lie in the plane formed by col 1 and col 2 of A, then no linear comb of A i.e. Ax will give you b.

Note: If columns of A are independent then every vector b can be written as unique linear combination of these 3 columns.

Question → How to check dependent or independent?

RANK of a Matrix

Sub-matrix: $A \to m \times n$

Matrix obtained by leaving some rows and columns of A is called sub-matrix of A.

$$A = \begin{bmatrix} 1 & 5 & 6 & 9 & 10 \\ 4 & 5 & 2 & 1 & 6 \\ 6 & 1 & 2 & 4 & 2 \\ -1 & 0 & 5 & 3 & 2 \end{bmatrix} \quad 4 \times 5$$

↓

$$A^* = \begin{bmatrix} 1 & 5 & 6 & 9 \\ 4 & 5 & 2 & 1 \\ 6 & 1 & 2 & 4 \\ -1 & 0 & 5 & 3 \end{bmatrix} \longrightarrow \text{Sub-matrix of } A \quad 4 \times 4$$

$$\begin{bmatrix} \vdots & \vdots & \vdots & \vdots \\ -1 & 0 & 5 & 3 \end{bmatrix}_{4\times 4}$$

Def^n of rank: A number 'r' is said to be the rank of a matrix 'A' if it posess the following Properties:

i) There is atleast one square sub-matrix of A of order r, whoose determinant is not equal to zero.

ii) If the matrix A contains any square sub-matrix of order '$r+1$', the det. value of every square sub matrix of order '$r+1$' must be zero.

Some more Properties:

a) The rank of r of an $m\times n$ matrix can at most be equal to smaller of the numbers m and n

$$\text{rank}(A) \leq m, n$$

b) Rank of every non-singular matrix of order n is n

c) Rank of every non-zero matrix ≥ 1.

d) Rank of every null matrix is zero.

Echelon form of Matrix

A matrix is said to be in Echelon form if

a) Every row of A which has all its entries zero ... non-zero entry

a) Every row of A which has all its entries zero occurs below every row which has a non-zero entry.

b) The number of zeros before the first non-zero element in a row is less than the number of such zeros in next row.

Note: The rank of a matrix in Echelon form is equal to the number of non-zero rows of the matrix.

$$\begin{bmatrix} \boxed{2} & 6 & 3 \\ \boxed{0} & \boxed{5} & 4 \\ 0 & 0 & \boxed{5} \end{bmatrix}$$

$\square \rightarrow$ Pivot element

$$\begin{bmatrix} 0 & \boxed{7} & 0 & 3 & 6 \\ 0 & 0 & 0 & \boxed{1} & 5 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} \boxed{4} & 6 & 2 \\ 0 & \boxed{1} & 5 \\ 0 & 0 & \boxed{7} \\ 0 & 0 & 0 \end{bmatrix}$$

Elementary Operation.

i) Interchange of any two rows or colums.

ii) Multiplication of the elements of any row (or column) by any non-zero number.

iii) Addition to + Multipliation.

Symbol used :

1) $R_i \leftrightarrow R_j$, $c_i \leftrightarrow c_j$.

2) $R_i \longrightarrow kR_i$, $c_i \longrightarrow kc_i$.

3) $R_i \longrightarrow R_i + kR_j$, $c_i \rightarrow c_i + kc_j$ ..

A/w

a) $\begin{bmatrix} 1 & 3 & 4 & 3 \\ 3 & 9 & 12 & 9 \\ -1 & -3 & -4 & -3 \end{bmatrix}$ b) $\begin{bmatrix} -2 & -1 & -3 & -1 \\ 1 & 2 & 3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$.

find the rank of these two matrix .