

## Descartes's Rule of signs :

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_0$$

$$\{a_n, a_{n-1}, \dots, a_1, a_0\}$$

$$\{ \underbrace{1, 3, -2, 0}_{2C}, \underbrace{-3, 0, 4, 0, 0, 7}_{3V} \}$$

$$\{ 1, 3, -2, 0, -3, 0, 4, 0, 0, 7 \}$$

$$x^{10} + 3x^9 - 2x^8 + 0x^7 - 3x^6 + 0x^5 + 4x^4 + 0x^3 + 0x^2 + 7x + 7$$

3 continuations.  
2 variations.

Th<sup>m</sup> : The number of positive roots of an equation.  
 $f(x) = 0$ , with real coeff does not exceed  
 the number of variations of signs in the  
 sequence of the coeff of  $f(x)$  and if  
 less, if less, it is less by an even number.

$V \rightarrow$  Variations

$r \rightarrow$  positive real root -

$$V = r + 2h$$

$h > 0$

Note :- what will be the case in case of negative roots?

$f(x) = 0$

$\hookrightarrow f(-x) = 0$

#1> Apply Descartes's Rule to examine the nature of roots!

i>  $x^4 + 2x^3 + 3x - 1 = 0$

+ + + -

1

1

$= 0$

$\hookrightarrow$  1 positive real root.

ii>

$x^6 + x^4 + x^2 + x + 3 = 0$

$f(-x) = x^6 - 2x^3 - 3x - 1 = 0$

+ - - -  
real.

$\hookrightarrow$  1 negative root.

Roots = 1 +ve real  
1 -ve real

Roots = 1 real  
 1 -ve real  
 2 complex roots (complex conjugates).

$$f(-x) = x^6 + x^4 + x^2 - x + 3$$

+ + + - +

2 variations

2 negative roots

### Sturm's Method for location of Roots:

Case I: All roots are unequal.

$f(x) \rightarrow$  polynomial, with real coeff.

$a, b \in \mathbb{R}$ , ( $a < b$ ). The no. of real

roots of the eq<sup>n</sup>  $f(x) = 0$  lying between  $a$  and  $b$  is equal to the excess of the number of changes of signs in the sequence of

Sturm functions  $f(x), f_1(x), \dots, f_r(x)$  when

$x = a$  over the number of changes of signs in the seq., when  $x = b$ .

Example: Find the no. and position of the real roots of the eq<sup>n</sup>  $x^3 - 3x + 1 = 0$ .

$$f(x) = x^3 - 3x + 1.$$

Reduced  $f'(x)$



$$f(x) = x^3 - 3x + 1.$$

Reduced  $f'(x)$

$$f_1(x) = \begin{array}{l} 3x^2 - 3 \quad (\text{Removing factor } 3) \\ \rightarrow x^2 - 1 \end{array} \quad \uparrow f_1(x)$$

$$f_2(x) = 2x - 1$$

$$(2x - 1) x^2 + 0x - 1/$$

$$f_3(x)$$

$$\begin{array}{r} x^2 - 1 \overline{) x^3 - 3x + 1} \quad (x \\ \underline{2x^3} \phantom{+ 1} \\ -x \phantom{+ 1} \\ \underline{+} \phantom{+ 1} \\ -2x + 1 \end{array}$$

$$\begin{array}{r} 2x - 1 \overline{) x^2 + 0x - 1} \quad \left( \frac{1}{2}x + \frac{1}{4} \right. \\ \underline{x^2 - \frac{1}{2}x} \phantom{- 1} \\ \phantom{x^2} \frac{1}{2}x - 1 \end{array}$$

$$\begin{array}{r} \phantom{x^2} \frac{1}{2}x - 1 \\ \underline{\phantom{x^2} \frac{1}{2}x - \frac{1}{4}} \\ \phantom{x^2} - \frac{3}{4} \end{array}$$

$$\frac{1}{2} - 1$$

$$= \frac{1 - 4}{4} = -\frac{3}{4}.$$

$$f(x) = x^3 - 3x + 1$$

$$f_1(x) = x^2 - 1 \quad (\text{Derivative of } f(x))$$

$$f'(x) = 3x^2 - 3 = 3(x^2 - 1)$$

$$f_2(x) = f_1(x) \cdot f(x)$$

$$f_3(x) = f_2(x) \cdot f_1(x)$$

	$f(x)$	$f_1(x)$	$f_2(x)$	$f_3(x)$	changes of sign
$-\infty$	-	+	-	+	3
0	+	-	-	+	2
$+\infty$	+	+	+	+	0

$3 - 0 = 3$

$\therefore$  -M

$$f(x) = x^3 - 3x + 1$$

$$f_1(x) = x^2 - 1$$

$$f_2(x) = 2x - 1$$

$$f_3(x) = 3$$

The eq<sup>n</sup> has 3 real roots

$$f(x) = 0$$

$$f(-x) = 0$$

one negative and two positive.

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A simple hand-drawn smiley face consisting of two horizontal lines for eyes and a curved line for a smile.

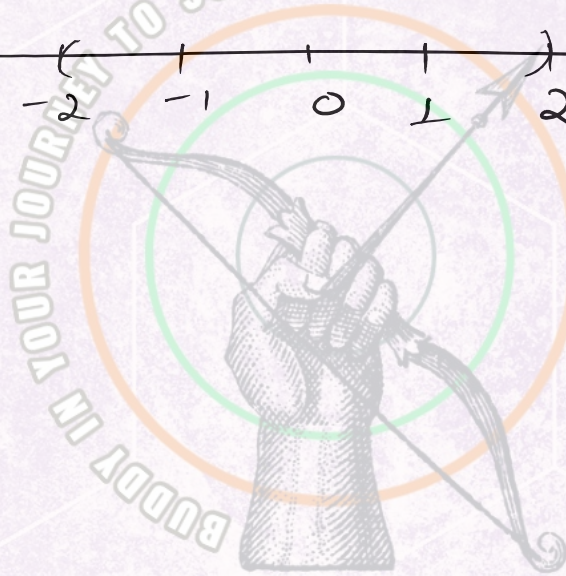
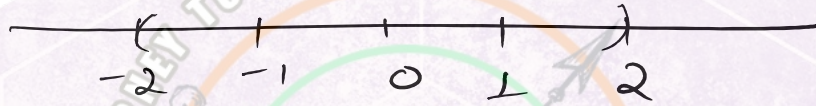
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van

on negative

$-\infty = -M$   
 $+\infty = M$

$$+\infty = (M)$$



# MISSION PHYSICS