System of Linear Non-Homogenous Equations.

$$AX = b.$$

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & & \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ \vdots \\ \vdots \\ b_n \end{bmatrix}.$$

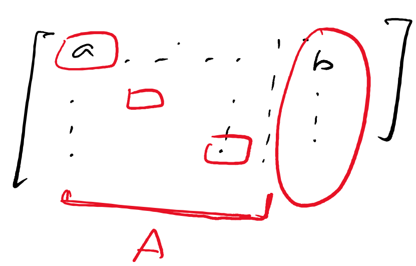
Augmented Matrix.

$$[A : b] = \left[\begin{array}{cccc|c} a_{11} & a_{12} & \cdots & a_{1n} & b_1 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} & b_n \end{array}\right].$$

Note: After reducing this matrix to its echelon form if the augmented column becomes a free coloumn, then it means augmented column will be spanned by pivot columns i.e. colums of A.

Consistency Condition:-

$$\text{Rank}\,[A : b] = \text{Rank}\ A.$$



Inconsistency case:-

$$\text{Rank}\,[A : b] > \text{Rank}\,[A]$$

$$\text{Rank}[A : b] > \text{Rank}[A].$$

A

Check whether the system of equations is consistent or inconsistent.

$$\begin{aligned} x + y + z &= -3 \\ 3x + y - 2z &= -2 \\ 2x + 4y + 7z &= 7. \end{aligned}$$

$$\text{Rank}[A : b] = ? \quad \begin{bmatrix} 1 & 1 & 1 & -3 \\ 3 & 1 & -2 & -2 \\ 2 & 4 & 7 & 7 \end{bmatrix}$$

$R_2 \to R_2 - 3R_1$

$R_3 \to R_3 - 2R_1$

$$\sim \begin{bmatrix} 1 & 1 & 1 & -3 \\ 0 & -2 & -5 & 7 \\ 0 & 2 & 5 & 13 \end{bmatrix} \underset{\sim}{\xrightarrow{R_3 \to R_3 + R_2}} \underbrace{\begin{bmatrix} \underbrace{\boxed{1} & 1 & 1}_{2} & \vdots & -3 \\ 0 & \boxed{-2} & -5 & \vdots & 7 \\ 0 & 0 & 0 & \vdots & \boxed{20} \end{bmatrix}}_{3}$$

So, $\text{Rank}[A : b] = 3$

$\text{Rank}[A] = 2$

$\therefore \text{Rank}[A : b] > \text{Rank}[A]$, Hence inconsistent!.

Check their consistency!

$x + y + z = 6$

$x + 2y + 3z = 14$

$x + 4y + 7z = 30.$

$\Rightarrow$ Consistent!

Investigate for what value of λ, μ, the simultaneous equations:

$x + y + z = 6$

$x + 2y + 3z = 10$

$x + 2y + \lambda z = \mu.$

have (i) no solution, (ii) a unique solution (iii) an infinite number of solutions.

$\Rightarrow$

i)

$$\begin{bmatrix} \boxed{1} & 1 & 1 & : & 6 \\ 0 & \boxed{1} & 2 & : & 4 \\ 0 & 0 & \underset{0 \ \boxed{1}}{\lambda - 3} & : & \underset{\neq 0}{\mu - 10} \end{bmatrix}$$

$\mu \neq 10$

$\text{Rank}[A : b] > \text{Rank}[A]$

$\text{Rank}[A : b] \to 3$, $\text{Rank}[A] \to 2$

$\lambda = 3$

$\mu \neq 10$

Infinite solutions

$\sim$

Infinite solutions

$\underline{Rank\,[A : b] = Rank\,[A]}$

$< n$

no. of columns of A.

ii) $\lambda \neq 3$

iii) $\left[\begin{array}{ccc:c} \boxed{1} & 1 & 1 & 6 \\ 0 & \boxed{1} & 2 & 4 \\ 0 & 0 & \lambda-3 & \mu-10 \end{array}\right]$

$\lambda = 3, \quad \mu = 10$

$Rank\,[A : b] = Rank\,[A] = 2 < 3$