CLASS_1_BASIC_ALGEBRA

23 March 2025 20:32

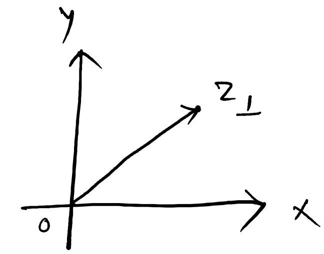
Tentative 2nd semester → July.

Complex Number
(way of Approaching)

Algebraic way.

$$\boxed{z = a + ib}$$

Geometric way.



$b \neq 0 \quad a \in \mathbb{R}$.

Each complex number can be represented as a Vector.

$$x^2 + 25 = 0$$
$$\text{or, } x = \pm\sqrt{-25}$$
$$\text{or, } x = \pm 5i$$

$$\boxed{\sqrt{-1} = i}$$

Real number → Magnitude.

Complex Number → Direction + Magnitude.

Complex number
(Perspective of Quadratic Equation)

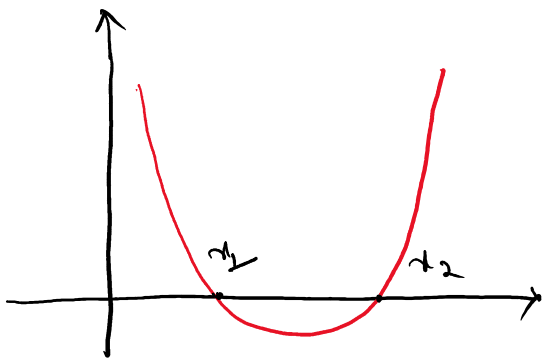
$$ax^2 + bx + c = 0$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

When, $b^2 - 4ac \geq 0$, then the roots are real.

$b^2 - 4ac < 0$ then the roots are complex.

When,

$b^2 - 4ac < 0$, then the roots are complex.



$$y = ax^2 + bx + c$$

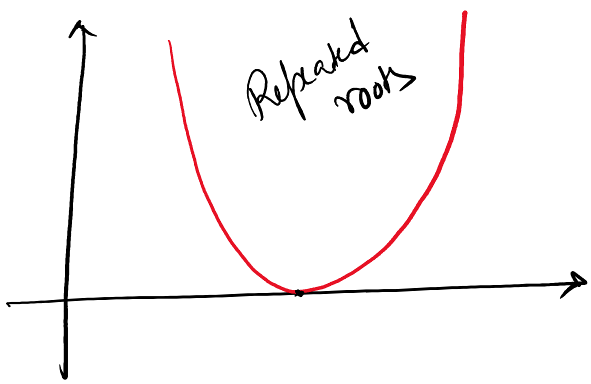
$$\Rightarrow y = a\left(x^2 + \frac{b}{a}x + \frac{c}{a}\right)$$

$$\Rightarrow y = a\left(x^2 + 2\cdot\frac{b}{2a}\cdot x + \frac{b^2}{4a^2} + \frac{c}{a} - \frac{b^2}{4a^2}\right)$$

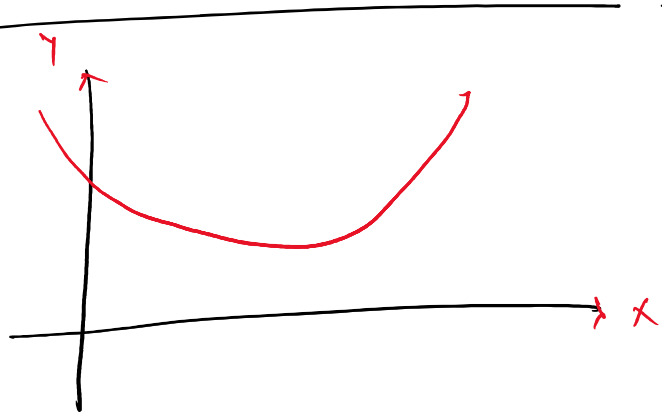
$$\Rightarrow y = a\left(x + \frac{b}{2a}\right)^2 + c - \frac{b^2}{4a}$$

$$\Rightarrow \boxed{y = a\left(x + \frac{b}{2a}\right)^2 + \frac{4ac - b^2}{4a}}$$

$$\hookrightarrow x^2 = 4ay$$



When no real roots are present,



Determine the nature of roots of following equations:

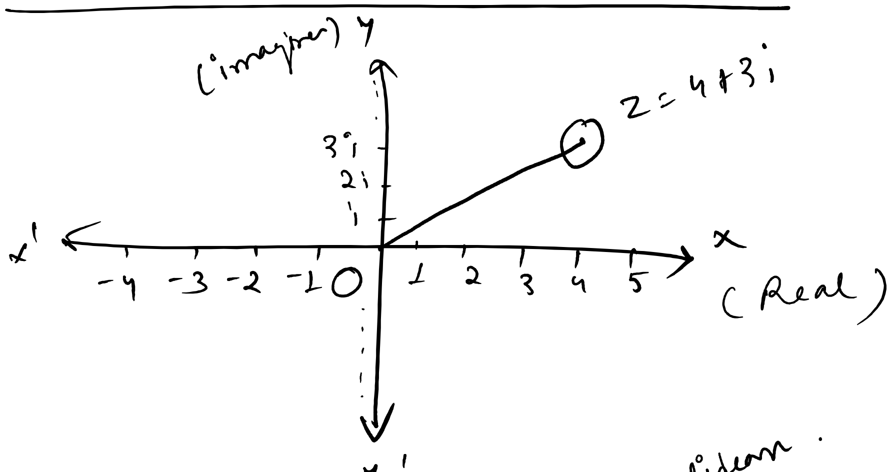
i) $3x^2 + 2x - 1 = 0$ $\longrightarrow$ Two real and distinct

ii) $2x^2 - 6x + 9 = 0$ $\longrightarrow$ Two real roots.

ii) $2x^2 - 6x + 9 = 0 \longrightarrow$

iii) $-4x^2 + 7x - 9 = 0 \longrightarrow$ comple roots

Cartesian Coordinate system



$z = a + ib$

$\hookrightarrow x + iy.$

$z = 4 + 3i$

$(4, 3)$

$$|z| = \sqrt{x^2 + y^2}$$

$\hookrightarrow$ Geometric intution.

Modulus of complex no :- The euclidean distance of that complex number(z) from the origin.

For example, $z = x + iy.$

$$|z| = \sqrt{x^2 + y^2}.$$

Algebraic Operations on Complex Numbers :-

i) Addition

$z_1 = a + ib$

$z_2 = c + id.$

$$z_1 + z_2 = a + ib + c + id$$

$$= \underbrace{(a + c)}_{\text{Real part}} + \underbrace{\boxed{i(b + d)}}_{\text{imaginary part}}$$

ii) Subtraction

$z_1 = a + ib$

$z_2 = c + id.$

$$z_1 - z_2 = a + ib - (c + id)$$

ii) $z_2 = c + id$. $z_1 - z_2 = a + ib - (c + id)$

$= (a - c) + i(b - d)$.

iii) Multiplication $(a + ib)(c + id) = (ac - bd) + i(ad + bc)$

iv) Division $(a + ib) \div (c + id) = \dfrac{a + ib}{c + id}$

$\boxed{\text{Rationalization}}$

$= \dfrac{ac + bd + i(bc - ad)}{c^2 + d^2}$

$= \dfrac{ac + bd}{c^2 + d^2} + i\dfrac{(bc - ad)}{c^2 + d^2}$.

$\boxed{\text{Surds}} \rightarrow$ Rational Numbers

$i^? = 1$

$= -1$

$i^2 = -1$.

$(i^2)^2 = (-1)^2$

$\Rightarrow i^4 = 1$.

$$\boxed{\begin{aligned} i^{4k} &= 1 \\ i^{4k+1} &= i \\ i^{4k+2} &= -1 \end{aligned}}$$

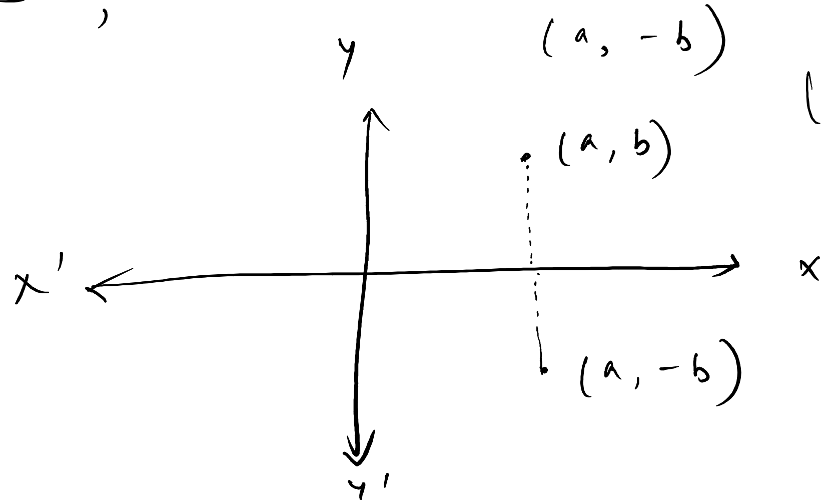
Algebraic identities $\xrightarrow[\text{maps}]{}$ complex identities

$$(z_1 + z_2)^2 = z_1^2 + z_2^2 + 2 z_1 \cdot z_2$$

Conjugate of complex number

$z = a + ib$, $\bar{z} = a - ib$.

(a, b) $(a, -b)$ (Reflection in x-axis).



* Multiplication of a complex number and its conjugate is always a real number.

$z = a + ib$, $\bar{z} = a - ib$

$$|\bar{z}| = \sqrt{a^2 + b^2} = |z|$$

Solve :-

i) $$\frac{1+i}{1-i} = \frac{(1+i)(1+i)}{(1-i)(1+i)} = \frac{(1+i)^2}{1-i^2}$$

$$\frac{\;}{1-i} \quad = \quad (1-i)(1+i) \quad 1-1$$

ii) i^{2000}

$\hookrightarrow = 1.$

$$= \frac{1+2i+i^2}{2}$$

$$= \frac{2i}{2} = i$$

<u>Next day</u> :- i) Polar representation (Geometric Intuition)

ii) De - Moivre's Theorem