Descarte's Rule of signs:

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_0$$

$$\{a_n, a_{n-1}, \dots, a_1, a_0\}.$$

$$\{1, 3, -2, 0, -3, 0, 4, 0, 0, 7\}$$

C V V V V C

2C 3V.

$$\{1, 3, -2, 0, -3, 0, 4, 0, 0, 7\}.$$

$$x^{10} + 3x^9 - 2x^8 + 0.x^7 - 3x^6 + 0.x^5 + 4x^4 + 0.x^3 + 0.x^2$$

3 continuation.

2 variation.

Thm: The number of positive roots of an equation $f(x) = 0$, with real coeff does not exceed the number of variations of signs in the sequence of the coeff of $f(x)$ and if less, if less, it is less by an even number.

$V \rightarrow$ Variations

$r \rightarrow$ positive real root

$$\boxed{V = r + 2h} \qquad h > 0$$

Note :- what will be the case in case of negative roots?

$f(x) = 0$

$\hookrightarrow f(-x) = 0$

#1) Apply Descarte's Rule to examine the nature of roots:

i) $x^4 + 2x^3 + 3x - 1 = 0$. $\quad \underline{1}$

$+ \quad + \quad + \quad -$ $\quad \underline{1}$

$\hookrightarrow \exists$ one positive real root. $= 0$

ii) $\boxed{x^6 + x^4 + x^2 + x + 3 = 0}$.

$\rightarrow f(-x) = x^4 - 2x^3 - 3x - 1 = 0$

$+ \quad - \quad - \quad -$

real.

$\hookrightarrow$ 1 negative root.

Roots = 1 +ve real

1 -ve real

Roots = 1 +ve .
1 −ve real
2 complex roots (complex conjugates).

$f(-x) = x^6 + x^4 + x^2 - x + 3$

$+ \quad + \quad + \quad - \quad +$ $\boxed{\text{2 variations}}$

2 negative roots.

Strum's Method for location of Roots :

Case I : All roots are unequal.

$f(x) \rightarrow$ polynomial, with real coeff.

$a, b \in \mathbb{R}$, $(a < b)$. The no. of real roots of the eq^n $f(x) = 0$ lying between a and b is equal to the excess of the number of changes of signs in the sequence of Strum functions $f(x), f_1(x), \ldots f_r(x)$ when $x = a$ over the number of changes of signs in the seq., when $x = b$.

Example: Find the no. and position of the real roots of the eq^n $x^3 - 3x + 1 = 0$.

$f(x) = x^3 - 3x + 1$. Reduced $f'(x)$

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$\uparrow$

$f_1(x) = 3x^2 - 3$ (Removing factor 3). $f_1(x)$

$= x^2 - 1.$

$f_2(x) = 2x - 1$

$2x-1 \overline{)\, x^2 + 0.x - 1} ($

$f_3(x)$

$x^2 - 1 \overline{)\, \cancel{x^3} - 3x + 1} \;(x$

$\cancel{x^3} \quad - x$

$- \quad +$

$-2x + 1.$

$2x - 1 \overline{)\, x^2 + 0x - 1} \;(\frac{1}{2}x + \frac{1}{4}$

$x^2 - \frac{1}{2}x$

$- \quad +$

$\frac{1}{2}x - 1.$

$\frac{1}{2}x - \frac{1}{4}$

$- \quad +$

$\frac{1}{4} - 1$

$= \frac{1-4}{4} = -\frac{3}{4}.$

$f(x) = x^3 - 3x + 1.$

$f_1(x) = x^2 - 1$ (Derivative of $f(x)$)

$f'(x) = 3x^2 - 3 = 3(x^2 - 1)$

$f_2(x) = $ $f_1(x)\overline{)f(x)}($

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$f_3(x) = $ $f_2(x)\overline{)f_1(x)}($

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	$f(x)$	$f_1(x)$	$f_2(x)$	$(f_3(x))$	Changes of sign
a = $(-\infty)$	$-$	$+$	$-$	$+$	3
0	$+$	$-$	$-$	$+$	2
$(+\infty)$ = b	$+$	$+$	$+$	$+$	0

$\boxed{3 - 0 = 3}$

$-\infty = -M$

$f(x) = x^3 - 3x + 1$

$f_1(x) = x^2 - 1$

$f_2(x) = 2x - 1$

$f_3(x) = 3.$

The eq^n has 3 real roots $\boxed{\text{one negative and two positive.}}$

$f(x) = 0$

$f(-x) = 0.$

$f(x) = 0$

$f(-x) = 0.$

$-x^3 + 3x + 1$

$- \quad + \quad +$

$\smile \longrightarrow$ 1 var $\longrightarrow$ one negative

$f(x) = 0$

$-\infty = -M$

$+\infty = M$