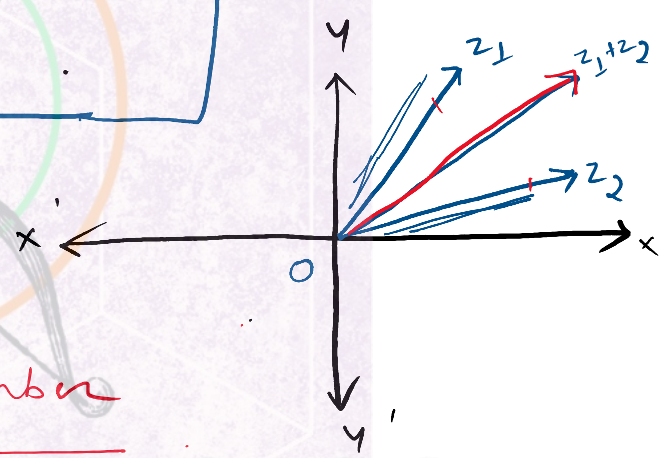
Last Lecture Recap:

i) What is complex no?
ii) Properties.
iii) Modulus.
iv) complex no → Geometric intution.
v) Conjugate.

Triangle Inequality

z_1, z_2 be two complex number, then

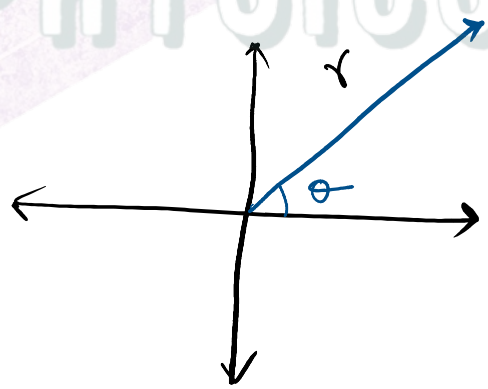
$$\boxed{|z_1 + z_2| \leq |z_1| + |z_2|}.$$



Polar Representation complex Number

$Z = a + ib$.

$a = r\cos\theta \qquad b = r\sin\theta$.



'θ' → Argument of the complex number. /amplitude.

$z = a + ib$

$= r\cos\theta + i(r\sin\theta)$

r → modulus of complex number.

$= r\cos\theta + i(r\sin\theta)$

$= \underline{r}(\cos\theta + i\sin\theta)$

$r \rightarrow$ modulus of complex number.

$r > 0$.

Note :- Zero complex number has no polar form

$$\boxed{-\pi < \theta \leq \pi} \rightarrow \text{Principal values}$$

All the values of θ are expressed as Arg Z

If α be the value of θ satisfying

$\cos\theta = \frac{a}{r}$, $\sin\theta = \frac{b}{r}$

$$\boxed{\text{Arg } Z = \alpha + 2n\pi.}$$

eg: $\alpha = \frac{3\pi}{2}$ $\quad Z = -i$ $\quad$ arg. $\quad -\pi < \theta \leq \pi$.

$\text{Arg } Z = \frac{3\pi}{2} + 2n\pi$, $n \in \mathbb{Z}$

$\pi - \frac{3\pi}{2} = \frac{2\pi - 3\pi}{2} = -\pi/2$

#1 $Z = -1 + i$,

i) Find mod Z and arg Z

ii) Express it in polar form.

$$z = r(\cos\theta + i\sin\theta)$$
$$= \sqrt{2}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right).$$

#2 $z = -1 - i$, express it in polar form.

$\theta = \frac{5\pi}{4}$, but it does not lie in the range of Principal values.

$$\therefore \quad \operatorname{Arg} z = -2\pi + \frac{5\pi}{4} \qquad n = -1$$
$$= \frac{-8\pi + 5\pi}{4} = -\frac{3\pi}{4}$$

$$z = \sqrt{2}\left(\cos\left(-\frac{3\pi}{4}\right) + i\sin\left(-\frac{3\pi}{4}\right)\right) \therefore$$

Theorem 1 :- If z_1 and z_2 be two non-zero complex number, if θ_1 is the arg. of z_1 and θ_2 be the arg of z_2 then, the arg. of (z_1+z_2) is $(\theta_1+\theta_2)$.

Thm 2 :- $\theta_1 \to z_1$, $\theta_2 \to z_2$. $\theta_1 - \theta_2 \to z_1 - z_2$ $\Big\}$ Arg.

Thm 3 :- z_1, z_2 be non-zero complex number,

then $\arg(z_1 z_2) = \arg(z_1) + \arg(z_2) + 2k\pi$

where $k = 0$ if $-\pi < \arg z_1 + \arg z_2 \le \pi$.

where, $K = 0$, if $-\pi < \arg z_1 + \arg z_2 \leq \pi$.

$K = 1$, if $\arg z_1 + \arg z_2 \leq -\pi$

$K = -1$ if $\arg z_1 + \arg z_2 > \pi$.

Thm^4 :- $z_1, z_2 \rightarrow$ non-zero complex number.

$$\arg(z_1/z_2) = \arg(z_1) - \arg(z_2) + 2k\pi$$

$$\left.\begin{array}{l} K = 0, \text{ if } -\pi < \arg z_1 - \arg z_2 \leq \pi \\ K = 1, \text{ if } \arg z_1 - \arg z_2 \leq -\pi \\ K = -1, \text{ if } \arg z_1 - \arg z_2 > \pi \end{array}\right\}.$$

#3. $z = 1 + i\tan\left(\frac{3\pi}{5}\right)$, Find $\arg z$.

$z = r(\cos\theta + i\sin\theta)$. $\qquad \frac{3 \times \cancel{180}^{36}}{\cancel{5}} = 108$.

$r\cos\theta + ri\sin\theta = 1 + i\tan\frac{3\pi}{5}$

$r\cos\theta = 1$, $\qquad r\sin\theta = \tan\frac{3\pi}{5}$.

$r^2 = \sec^2\frac{3\pi}{5}$. $\Rightarrow r = \sec\frac{3\pi}{5}$.

$r\cos\theta = 1$.

$$\Rightarrow \sec\frac{3\pi}{5}\cdot\cos\theta = 1$$

$$\Rightarrow \cos\theta = \cos\frac{3\pi}{5}$$

$$\Rightarrow \theta = \frac{3\pi}{5}.$$

$$\boxed{\arg z = \frac{3\pi}{5}}$$

Integral and Rational Powers of complex no.

Let z be a complex number and n be a integer.

i) $z^0 = 1$.

ii) $z^n = z^{n-1}\cdot z$, $n > 0$

iii) $z^{-n} = (z^{-1})^n$ if $z \neq 0$, $n > 0$.

iv) $z^m \cdot z^n = z^{m+n}$.

v) $(z^m)^n = z^{mn}$.

vi) $z_1^m \cdot z_2^m = (z_1 z_2)^m$.

De Moivre's Theorem

$$(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin(n\theta)$$

$$\boxed{e^{i\theta} = \cos\theta + i\sin\theta}$$

$$z = r(\cos\theta + i\sin\theta)$$
$$= \boxed{r\cdot e^{i\theta}}$$

$n = \sqrt{2}$, this will not work.