

COSETS

left coset: let G be a group and H be a subgroup of G . let 'a' be an element of G . For all h in H , $ah \in G$.

The subset $\{ah : h \in H\}$ is called a left coset of H in G and is denoted by aH .

$b, c, \dots \in G$, $bh, ch, \dots \in bH, cH, \dots$

Note: In an additive group G , a left coset of H is denoted by $a + H = \{a + h : h \in H\}$

$$H + a$$

Example:

① $G = (\mathbb{Z}, +)$ and $H = (\overset{3n}{3\mathbb{Z}}, +)$

The left coset $0 + H = \{3n : n \in \mathbb{Z}\} = H$.

The left coset $1 + H = \{3n + 1 : n \in \mathbb{Z}\}$.

② $G = S_3$ and $H = \{P_0, P_3\}$.

$P_0 H = \{P_0, P_3\} \quad \{P_0, \dots, P_5\}$

$$P_0 H = \{P_0, P_3\}$$

$$\{P_0, \dots, P_5\}$$

$$P_1 H = \{P_1, P_5\}$$

$$P_5 H = \{ \quad \}$$

$$P_0 = e$$

$$P_4 = (123)$$

$$P_1 = (12)$$

$$P_5 = (132)$$

$$P_2 = (13)$$

$$P_3 = (23)$$

$$H = \{e, (23)\}$$

$$(132)e = (132)$$

$$(132)(23) = (13)$$

$$(P_5)$$

$$(132)$$

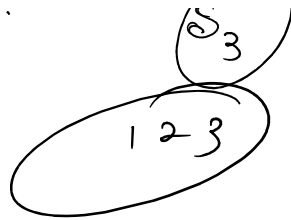
$$\begin{array}{ccc} \left. \begin{array}{l} 1 \rightarrow 1 \\ 1 \rightarrow 3 \end{array} \right\} & \left. \begin{array}{l} 2 \rightarrow 3 \\ 3 \rightarrow 2 \end{array} \right\} & \left. \begin{array}{l} 3 \rightarrow 2 \\ 2 \rightarrow 1 \end{array} \right\} \\ \hline 1 \rightarrow 3 & \hline 2 \rightarrow 2 & \hline 3 \rightarrow 1 \end{array}$$

$$(13)$$

$$\{P_5, P_2\}$$

$$S_3$$

$(5, 2)$



$\{2, 3\}$

$$\begin{array}{l} P_1 \quad (1 \ 2) \\ P_2 \quad (1 \ 3) \\ P_3 \quad (2 \ 3) \end{array}$$

$$P_4 = (1 \ 2 \ 3)$$

$$P_5 = (1 \ 3 \ 2)$$

Thm. let G be a group and H be a subgroup of G .
let $h \in H$. Then $hH = H$.

Thm. let G be a group and H be a subgroup of G .
Any two left cosets of H in G are either identical or they have no common element.

[If $a, b \in G$, then either $aH = bH$ or $aH \cap bH = \emptyset$]

Thm. let G be a group and H be a subgroup of G . let $a, b \in G$. Then $aH = bH$ if and only if $a^{-1}b \in H$.

$\Rightarrow aH = bH$, Then $ah_1 = bh_2 \quad \forall h_1, h_2 \in H$.
 $\therefore a^{-1}b = \underline{h_1 h_2^{-1}} \in H$, since H is a

subgroup.

conversely let $a^{-1}b \in H$. Then $a^{-1}b = h_3$ for

conversely, let $a^{-1}b \in H$, Then $a^{-1}b = h_3$ for some $h_3 \in H$.

$\therefore b = ah_3 \quad b \in aH$. But $\underline{b \in bH}$.

So, left coset of aH and bH have

a common element b

$aH = bH$. (by above thm).

Th^m: Any two left cosets of H in a group G have the same cardinality.

LAGRANGE'S Th^m:

The order of every subgroup of a finite group G is a divisor of the order of G .

$$\underline{o(H) \mid o(G)}.$$

FERMAT TH^m:

If p be a prime and a be an integer such that p is not a divisor of a , then

$$a^{p-1} \equiv 1 \pmod{p}.$$

Th^m: Every group of prime order is cyclic.

Th^m: Every group of prime order is cyclic.

Th^m: The order of each element in a finite group G is a divisor of $o(G)$.

Right coset

$$\{ha : h \in H\}$$

Additive: $H + a$.

Example

$$G = (\mathbb{Z}, +) \quad H = (3\mathbb{Z}, +).$$

$$H + 0 = \{3n\}$$

$$H + 1 = \{3n + 1\}$$

$$H + 2 = \{3n + 2\}.$$

Th^m: Let H be a subgroup of a group G .
Then the set of all distinct left cosets of H in G and the set of all distinct right cosets in G have same cardinality.

$$|L| = |R|.$$

Index: $[G : H]$ (Index of H in G)

$$= \frac{o(G)}{o(H)}.$$

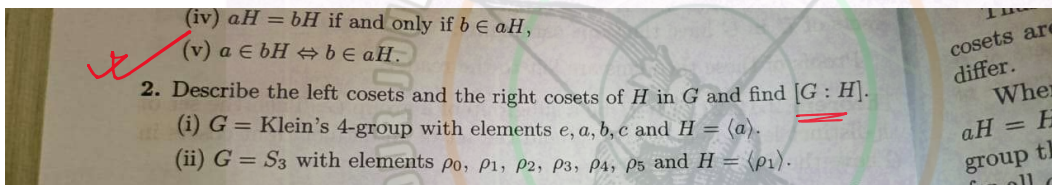
$$\frac{1}{o(H)}$$

$G = S_3 \quad H = A_3.$

$$[G : H] = \frac{o(G)}{o(H)} = \frac{6}{3} = 2.$$

$G = (\mathbb{Z}, +) \quad , \quad H = (2\mathbb{Z}, +).$

$$[G : H] =$$



Normalizer: For a subgroup H in G

the normalizer of H in G is

defined by:

$$N_G(H) = \{ g \in G : gHg^{-1} \subseteq H \}$$