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Lecture → 19

Condition for exactness of Higher ODE (Linear)

For a first order diff eq^n:

$$M(x,y)dx + N(x,y)\,dy = 0.$$

↳ It is exact if ∃ $F(x,y)$ such that:

$$\boxed{dF = \frac{\partial F}{\partial x}dx + \frac{\partial F}{\partial y}dy}$$

So, the condition will be:

$$\boxed{\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}}$$

Extension to Higher Order.

$$P\,dx + Q\,dy + R\,dz = 0.$$

It will be exact when:

$$\boxed{\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x},\ \frac{\partial Q}{\partial z} = \frac{\partial R}{\partial y},\ \frac{\partial R}{\partial x} = \frac{\partial P}{\partial z}}$$

Then the sol^n will be:

$$F(x,y,z) = C.$$

INTEGRATING FACTOR (IF).

$$M(x,y)\,dx + N(x,y)\,dy = 0.$$

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$$M(x,y)\,dx + N(x,y)\,dy = 0$$

not exact

$$\frac{\partial M}{\partial x} \neq \frac{\partial N}{\partial x}$$

Then we multiply a f^n $\mu(x,y)$ called Integrating Factor (IF).

such that :

$$\boxed{\mu M\,dx + \mu N\,dy = 0} \rightarrow \text{exact}$$

Equations of the Form $\dfrac{d^n y}{dx^n} = f(y)$ $\forall\, n \geq 2$

$$\frac{d^n y}{dx^n} = f(y) \quad , (n \geq 2)$$

For example:

$$\frac{d^2 y}{dx^2} = 6y \ , \quad \frac{d^2 y}{dx^2} = e^y$$

How will I solve this type of eq^n.

$$\frac{d}{dx} = p \cdot \frac{d}{dy} \qquad p = \frac{dy}{dx}$$

$$\frac{d^2 y}{dx^2} = \frac{dp}{dx} = \frac{dp}{dx} \cdot \frac{dy}{dx}$$

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$$\frac{d^2y}{dx^2} = \frac{dp}{dx} = \frac{dp}{dy}\cdot\frac{dy}{dx}$$

$$= p\cdot\frac{dp}{dy}.$$

So, the equation becomes :

$$p\cdot\frac{dp}{dy} = f(y).$$

$$\Rightarrow p\,dp = f(y)\,dy.$$

Integrate :

$$\Rightarrow \frac{p^2}{2} = \int f(y)\,dy + C.$$

$$\Rightarrow \left(\frac{dy}{dx}\right)^2 = 2\int f(y)\,dy + C$$

$$\Rightarrow \frac{dy}{dx} = \sqrt{2\int f(y)\,dy + C}$$

$$\Rightarrow dx = \frac{dy}{\sqrt{2\int f(y)\,dy + C}}$$

Integrate and get the sol^n :

Problems :

1) Solve : $\frac{d^2y}{dx^2} = 6y.$

$y = e^{mx}$ ⟵

$c_1 e \quad + c_2 e$

$$c_1 e \quad + c_2 e$$

$$p\,dp = 6y\,dy.$$

Integrate:

$$\frac{p^2}{2} = 3y^2 + c.$$

$$\Rightarrow dx = \frac{dy}{\sqrt{6y^2+c}}.$$

$$\Rightarrow x + c_1 = \int \frac{dy}{\sqrt{6y^2+c}}.$$

#2 Solve:

$$\frac{d^2y}{dx^2} = 1/y.$$

$$\hookrightarrow dx = \frac{dy}{\sqrt{2\ln|y|+c}}.$$

$$\boxed{x + c_1 = \int \frac{dy}{\sqrt{2\ln|y|+c}}}.$$

$$\left(\frac{dy}{dx}\right)^2 = 2\ln|y| + c.$$

#3 Solve: $\frac{d^2y}{dx^2} = y^3 + y.$

$$\left(\frac{dy}{dx}\right)^2 = y^4 + y^2 + c$$

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$$\left(\frac{dy}{dx}\right) = y + y + c$$

$$\Rightarrow \quad dx = \frac{dy}{\sqrt{y^4/2 + y^2 + c}}.$$

$$\Rightarrow \quad x + c_1 = \int \frac{dy}{\sqrt{y^4/2 + y^2 + c}}.$$

Doubts

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