Quaternion group

15 languages

Article Talk Read Edit View history Tools

From Wikipedia, the free encyclopedia

In group theory, the **quaternion group** Q_8 (sometimes just denoted by Q) is a non-abelian group of order eight, isomorphic to the eight-element subset $\{1, i, j, k, -1, -i, -j, -k\}$ of the quaternions under multiplication. It is given by the group presentation

$$Q_8 = \langle \bar{e}, i, j, k \mid \bar{e}^2 = e,\ i^2 = j^2 = k^2 = ijk = \bar{e} \rangle,$$

where *e* is the identity element and $\bar{e}$ commutes with the other elements of the group. These relations, discovered by W. R. Hamilton, also generate the quaternions as an algebra over the real numbers.

Another presentation of Q_8 is

$$Q_8 = \langle a, b \mid a^4 = e, a^2 = b^2, ba = a^{-1}b \rangle.$$

Like many other finite groups, it can be realized as the Galois group of a certain field of algebraic numbers.[1]

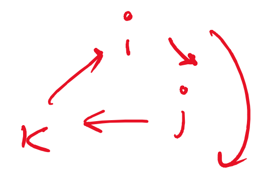
Quaternion group multiplication table (simplified form)

	1	**i**	**j**	**k**
1	1	i	j	k
i	i	−1	k	−j
j	j	−k	−1	i
k	k	j	−i	−1

Algebraic structure → Group theory

Group theory

$i \cdot j = k$



Is it commutative or non commutative

→ Because it's not symmetric.

Integral Powers of an element

$(G, o) \rightarrow$ Group.

$a, b, c \in G$.

$a \circ (b \circ c) = (a \circ b) \circ c$ 'o' is associative.

↳ $a \circ b \circ c$.

'a', $a \circ a \circ a$, $a \circ a \circ a \circ a \circ a \circ a \ldots a \in G$.

$a^n = \underbrace{a \circ a \circ \ldots\ldots a}$ (n factors).

$a^0 = e$.

$a^{-n} = a^{-1} \circ a^{-1} \circ \ldots\ldots a^{-1}$ (n factors).

Law of Indices in Groups

Let a be an element of a group $(G, \circ)$. Then for integers m and n.

i) $a^m \circ a^n = a^{m+n} = \underbrace{a \circ a \ldots\ldots \circ a}_{(m+n)}$.

ii) $(a^m)^n = a^{mn} = \underbrace{a \circ a \circ \ldots\ldots a}_{(mn)}$.

iii) $(a^n)^{-1} = a^{-n}$.

Order of an element

Let $(G, \circ)$ be a group and let $a \in G$, 'a' is said to be of finite order if there exists said to be of finite order if there exists a $n \in \mathbb{Z}^+$ such that $a^n = e_G$ holds.

$e_G \to$ identity element. (least n).

$a^n = e_G$

$a \circ a \circ a = e_G$

$\bar{0} \to$ identity element.

$(\mathbb{Z}_6, +)$

$$\mathbb{Z}_6 = \{\bar{0}, \bar{1}, \bar{2}, \bar{3}, \bar{4}, \bar{5}\}.$$

$$o(\bar{1}) = \underbrace{\bar{1} + \bar{1} + \dots + \bar{1}}_{} = \bar{6} \to \bar{0}$$

$$(\bar{1})^6 = e_G$$

So, $o(\bar{1}) = 6$.

$o(\bar{2}) = 3$. $o(\bar{3}) = 2$

$$o(\bar{4}) = \bar{4} + \bar{4} + \bar{4} = \overline{12} = \bar{6} + \bar{6}$$
$$= \bar{0} + \bar{0}$$

$$= \bar{0} + \bar{0}$$

$$= \bar{0}$$

$$o(\bar{5}) = \underbrace{\bar{5} + \bar{5} + \bar{5} + \bar{5} + \bar{5} + \bar{5}}_{6 \text{ times}}$$

So, $o(\bar{5}) = 6$.