Motion along a Straight line

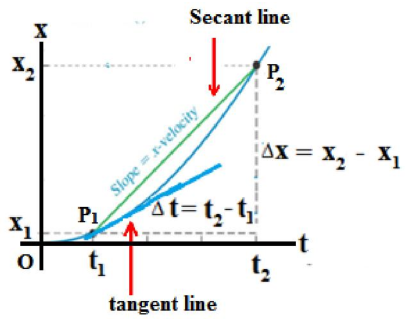
Chapter-2

<table>
<tr><th>Topic</th><th>Section</th><th>Test Your Understanding</th><th>In-class Problems (Example)</th><th>Assignment (Exercise)</th></tr>
<tr><td>Motion along a straight line: Displacement, Average and Instantaneous Velocity, Average and Instantaneous Acceleration (v-t, x-t graphs explanation)</td><td>2.1, 2.2,2.3</td><td rowspan="2">TYU-2.1,
TYU-2.3,
TYU-2.5,
TYU-2.6</td><td rowspan="2">2.4,
2.7,
2.9</td><td rowspan="2">2.10,
2.18,
2.38</td></tr>
<tr><td>Motion along a straight line: Motion with Constant Acceleration, Freely falling bodies, Velocity and Position by integration</td><td>2.4, 2.5, 2.6</td></tr>
</table>

Displacement, time, and average velocity

The average x-velocity of the particle is

$$v_{av\text{-}x} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t}$$



Instantaneous velocity

Instantaneous velocity is the average velocity in the limit as the time interval becomes infinitesimally small and is given by

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\overrightarrow{\Delta x}}{\Delta t} = \frac{\overrightarrow{dx}}{dt}$$

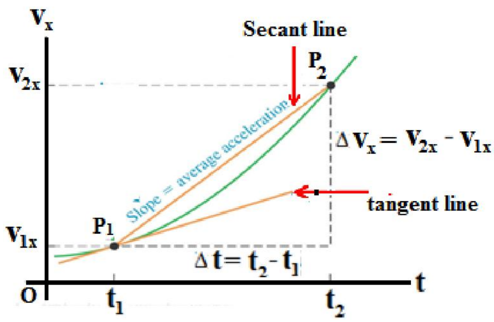
Position-time graph:

- Slope of the Secant line of the x~t graph represent the average velocity
- Slope of the tangent line of the x~t graph represent the instantaneous velocity

Average and instantaneous acceleration

- Acceleration describes the change of velocity with time.
- The average x-acceleration is

$$\vec{a}_{av\text{-}x} = \frac{\vec{v}_{2x} - \vec{v}_{1x}}{t_2 - t_1} = \frac{\Delta \vec{v}_x}{\Delta t}$$



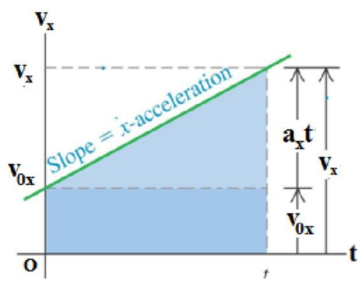
- The instantaneous acceleration is

$$a_x = \frac{dv_x}{dt} = \frac{d^2x}{dt^2}$$

- Slope of secant line of $v_x \sim t$ curve gives the average x-acceleration between two points.
- Slope of tangent to $v_x \sim t$ curve at a given point gives the instantaneous x-acceleration at that point.

Motion with constant acceleration

For a particle moving with constant acceleration, the velocity changes at the same rate throughout the motion.

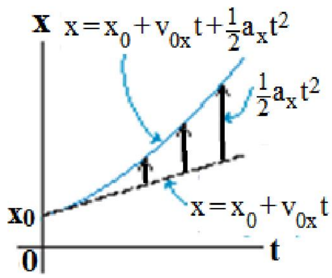


A body attains a velocity v_x after any instant't' with initial velocity v_{0x}. If the body moves with constant acceleration then it is given by

$$a_x = \frac{v_x - v_{0x}}{t - 0}$$

$$\Rightarrow \quad v_x = v_{0x} + a_x t$$

- For a particle moving with constant acceleration, $v_x \sim t$ curve is straight line
- Total area under $v_x \sim t$ graph = $x - x_0$ = displacement.
- $x \sim t$ graph for uniform acceleration is shown in the figure.



The equations of motion with constant acceleration

1. Derive $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$

Let us consider a body moving with constant acceleration.

Let, x_0 = Initial displacement
x = Final displacement
v_{0x} = Initial velocity
v_x = Final velocity
a_x. = Constant acceleration

Since the body is moving with constant acceleration, the average velocity can be written as

$$v_{av} = \frac{v_x + v_{0x}}{2} \quad --------(1)$$

We know that

$$v_x = v_{0x} + a_x t \quad --------(2)$$

Putting eqn (1) in (2) we get

$$v_{av} = \frac{(v_{0x} + a_x t) + v_{0x}}{2}$$

$$v_{av} = \frac{2v_{0x} + a_x t}{2} \quad --------(3)$$

Again, the average velocity can be written as

$$v_{av} = \frac{x - x_0}{t} \quad --------(4)$$

Comparing eqn (3) and (4) we get

$$\frac{x - x_0}{t} = \frac{2v_{0x} + a_x t}{2}$$

$$x - x_0 = \frac{2v_{0x}t + a_x t^2}{2}$$

$$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2 \quad --------(5)$$

2. Derive $v_x^2 - v_{0x}^2 = 2a_x(x - x_0)$

We know that

$$v_x = v_{0x} + a_x t$$

$$t = \frac{v_x - v_{0x}}{a_x} \quad --------(1)$$

Again we know that

$$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2 \quad ------(2)$$

Putting eqn (1) in (2) we get

$$x - x_0 = v_{0x}\left(\frac{v_x - v_{0x}}{a_x}\right) + \frac{1}{2}a_x\left(\frac{v_x - v_{0x}}{a_x}\right)^2$$

$$x - x_0 = \frac{v_{0x}\left(v_x - v_{0x}\right)}{a_x} + \frac{\left(v_x - v_{0x}\right)^2}{2a_x}$$

$$2a_x\left(x - x_0\right) = 2v_{0x}\left(v_x - v_{0x}\right) + \left(v_x - v_{0x}\right)^2$$

$$v_x^2 - v_{0x}^2 = 2a_x\left(x - x_0\right) \quad ------(3)$$

The equations of motion with constant acceleration using integration method

1. Derive $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$

We know that

$$v_x = \frac{dx}{dt} \quad \Rightarrow \quad dx = v_x dt \quad \Rightarrow \quad dx = \left(v_{0x} - a_x t\right)dt$$

$$\int_{x_0}^{x} dx = \int_0^t \left(v_{0x} + a_x t\right)dt$$

$$\left[x\right]_{x_0}^{x} = v_{0x}\left[t\right]_0^t + a_x\left[\frac{t^2}{2}\right]_0^t$$

$$x - x_0 = v_{0x}\left(t - 0\right) + a_x\left(\frac{t^2}{2} - \frac{0^2}{2}\right)$$

$$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$$

2. Derive $v_x^2 - v_{0x}^2 = 2a_x\left(x - x_0\right)$

We know that

$$v_x = \frac{dx}{dt} \quad \Rightarrow \quad v_x = \frac{dx}{dt}\frac{dv}{dv} \quad \Rightarrow \quad v_x = \frac{dx}{dv}a_x$$

$$\Rightarrow a_x dx = v_x dv_x \qquad \Rightarrow \int_{x_0}^{x} a_x dx = \int_{v_{0x}}^{v_x} v_x dv_x$$

$$\Rightarrow a_x\left[x\right]_{x_0}^{x} = \left[\frac{v_x^2}{2}\right]_{v_{0x}}^{v_x} \quad \Rightarrow \quad a_x\left(x - x_0\right) = \frac{v_x^2}{2} - \frac{v_{0x}^2}{2}$$

$$v_x^2 - v_{0x}^2 = 2a_x\left(x - x_0\right)$$

Freely falling bodies

- Free fall is the motion of an object under the influence of only gravity.
- The velocity change is the same in each time interval, so the acceleration is constant.
- The constant acceleration of a freely falling body is called the **acceleration due to gravity.**
- The acceleration of a body in free fall is always downward.
- Near the earth's surface, $g = 9.81\ m/s^2$.

The equations of motion for freely falling body are

$$v_y = v_{0y} + gt$$

$$y = v_{0y}t + \frac{1}{2}gt^2$$

$$v_y^2 - v_{0y}^2 + 2gy$$

Where,
y = Vertical displacement
v_{0y} = Initial velocity
v_y = Final velocity
g . = acceleration due to gravity

Test Your Understanding: 2.1, 2.3, 2.5, 2.6

Test Your Understanding of Section 2.1

Each of the following automobile trips takes one hour. The positive *x*-direction is to the east.

(i) Automobile *A* travels 50 km due east.
(ii) Automobile *B* travels 50 km due west.
(iii) Automobile *C* travels 60 km due east, then turns around and travels 10 km due west.
(iv) Automobile *D* travels 70 km due east.
(v) Automobile *E* travels 20 km due west, then turns around and travels 20 km due east.
(a) Rank the five trips in order of average *x*-velocity from most positive to most negative.
(b) Which trips, if any, have the same average *x*-velocity?
(c) For which trip, if any, is the average *x*-velocity equal to zero?

Answer:

(a): (iv), (i) and (iii) (tie), (v), (ii);
(b): (i) and (iii);
(c): (v)

Explanation: $\Delta t = 1$hous for all the trips

(a) the average x-velocity is

(i) $\Delta x = +50\text{km},\ v_{av\text{-}x} = \frac{\Delta x}{\Delta t} = +50\text{km/h}$ (ii) $\Delta x = -50\text{km},\ v_{av\text{-}x} = \frac{\Delta x}{\Delta t} = -50\text{km/h}$

(iii) $\Delta x = 60\text{km} - 10\text{km} = +50\text{km},\ v_{av\text{-}x} = \frac{\Delta x}{\Delta t} = +50\text{km/h}$

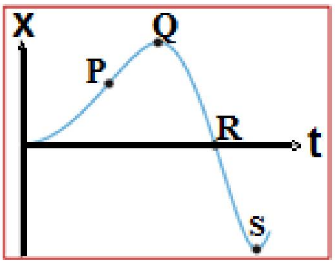
(iv) $\Delta x = 70\text{km},\ v_{av\text{-}x} = \frac{\Delta x}{\Delta t} = +70\text{km/h}$ (v) $\Delta x = -20\text{km} + 20\text{km} = 0,\ v_{av\text{-}x} = \frac{\Delta x}{\Delta t} = 0$

(iv) $\Delta x = 70\text{km},\ v_{av\text{-}x} = \frac{\Delta x}{\Delta t} = +70\text{km/h}$

(b) In option (i) and (iii) the average x-velocity is +50km/h

Test Your Understanding of Section 2.3

Look at the x-t graph in the adjoining Figure.



(a) At which of the points P, Q, R, and S is the x-acceleration a_x positive?
(b) At which points is the x-acceleration negative?
(c) At which points does the x-acceleration appear to be zero?
(d) At each point state whether the velocity is increasing, decreasing, or not changing.

Answer:

(a) S, where the x-t graph is curved upward (concave up).
(b) Q, where the x-t graph is curved downward (concave down).
(c) P and R, where the x-t graph is not curved either up or down.
(d) At P, $a_x = 0$ (velocity is **not** changing); at Q, $a_x < 0$ (velocity is decreasing, i.e., changing from positive to zero to negative); at R, $a_x = 0$ (velocity is not changing); and at S, $a_x > 0$ (velocity is increasing, i.e., changing from negative to zero to positive)..

Test Your Understanding of Section 2.5

If you toss a ball upward with a certain initial speed, it falls freely and reaches a maximum height ‘h’ a time ‘t’ after it leaves your hand.

(a) If you throw the ball upward with double the initial speed, what new maximum height does the ball reach?

(i) $h\sqrt{2}$ (ii) $2h$; (iii) $4h$; (iv) $8h$; (v) $16h$.

(b) If you throw the ball upward with double the initial speed, how long does it take to reach its new maximum height?

(i) t/2 (ii) $t/\sqrt{2}$ (iii) t; (iv) $t\sqrt{2}$ (v) $2t$.

Answers:

(a) (iii)

We know that $v_y^2 = v_{0y}^2 - 2\,g\,y$

Here, $y = h$, $v_y = 0$

So, $0 = v_{0y}^2 - 2\,g\,h \quad \Rightarrow \quad h = \dfrac{v_{0y}^2}{2\,g}$

If the initial *y*-velocity is increased by a factor of 2, the maximum height increases by a factor of 2^2=4 and the ball goes to height $4h$.

(b) (v)

We know that $v_y = v_{0y} - g\,t$

The *y*-velocity at the maximum height is v_y=0, so

$0 = v_{0y} - g\,t \quad \Rightarrow \quad t = \dfrac{v_{0y}}{g}$

If the initial y-velocity is increased by a factor of 2, the time to reach the maximum height increases by a factor of 2 and becomes 2t

Test Your Understanding of Section 2.6

If the *x*-acceleration a_x is increasing with time, will the v_x –t graph be (i) a straight line, (ii) concave up (i.e., with an upward curvature), or (iii) concave down (i.e., with a downward curvature)?

Answer: (ii)

The acceleration a_x is equal to the slope of the v_x –t graph. If a_x is increasing, the slope of the v_x –t graph is also increasing and the graph is concave up.

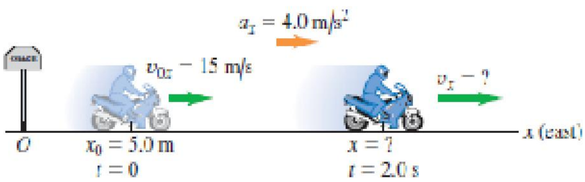
Example Problems: 2.4, 2.7, 2.9

Example – 2.4:

A motorcyclist heading east through a small town accelerates at a constant 4.0m/s^2 after he leaves the city limits. At time t = 0 he is 5.0 m east of the city-limits signpost, moving east at 15 m/s.

(a) Find his position and velocity at t = 2.0s

(b) Where is he when his velocity is 15m/s?



Solution:

(a) We know that, $x = x_0 + v_{0x}t + \dfrac{1}{2}a_x t^2$

$$\Rightarrow \; x = 50m + (15m/s)(2.0s) + \frac{1}{2}(4.0m/s^2)(2.0s)^2 = 43m$$

Again, $v_x = v_{0x} + a_x t$

$$\Rightarrow v_x = 15 m/s + (4.0 m/s^2)(2.0 s) = 23 m/s$$

(b) We know that $v_x^2 = v_{0x}^2 - 2a_x(x - x_0)$

$$\Rightarrow v_x^2 - v_{0x}^2 = 2a_x(x - x_0)$$

$$\Rightarrow x = x_0 + \frac{v_x^2 - v_{0x}^2}{2a_x}$$

$$\Rightarrow x = 50 m + \frac{(25 m/s)^2 - (15 m/s)^2}{2(4.0 m/s^2)} = 55 m$$

Example – 2.7: Up-and-down motion in free fall

You throw a ball vertically upward from the roof of a tall building. The ball leaves your hand at a point even with the roof railing with an upward speed of 15.0 m/s; the ball is then in free fall. On its way back down, it just misses the railing. Find

a) the ball's position and velocity 1.00 s and 4.00 s after leaving your hand;
b) the ball's velocity when it is 5.00 m above the railing;
c) the maximum height reached;
d) the ball's acceleration when it is at its maximum height.

Solution:

a) Displacement is given by

$$y = y_0 + v_{0y} t + \frac{1}{2} a_y t^2 \quad \Rightarrow \quad y = y_0 + v_{0y} t + \frac{1}{2}(-g) t^2$$

$$\Rightarrow y = 0 + (15 m/s) t + \frac{1}{2}(-9.8 m/s^2) t^2$$

$$\Rightarrow y = 15t - 4.9t^2$$

Now the velocity is given by

$$v_y = v_{0y} + a_y t \quad \Rightarrow \quad v_y = v_{0y} + (-g) t$$

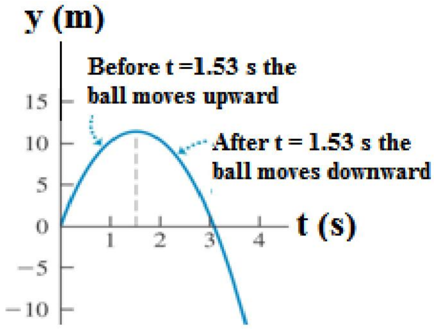
$$\Rightarrow v_y = 15 - 9.8t$$

- When t = 1.00 s, then

  $y = 15t - 4.9t^2 = 10.1 m$ and

  $v_y = 15 - 9.8t = +5.2 m/s$

  That is, the ball is 10.1 m above the origin (y is positive) and moving upward with a speed of 5.2 m/s (v_y, is positive). This is less than the initial speed because the ball slows as it ascends.



- When t = 4.00 s, then

  $y = 15t - 4.9t^2 = -18.4 m$

and

$$v_y = 15 - 9.8\,t = - 24.2 \text{ m/s}$$

The ball has passed its highest point and is 18.4 m below the origin (vy is negative).
It is moving downward (is negative) with a speed of 24.2 m/s .

The ball gains speed as it descends; it is moving at the initial 15 m/s speed as it moves downward past the ball's launching point, and continues to gain speed as it descends further.

y-t graph for a ball thrown upward with an initial speed of 15 m/s. (curvature is downward because $a_y = -g$ is negative)

Any free-fall body can be verified from y-t and v_y-t graphs

y-t graph is plotted by using equation $y = y_0 + v_{0y}\,t + \frac{1}{2} a_y\, t^2$

b) Velocity is given by

$$v_y^2 = v_{0y}^2 + 2a_y(y - y_0) \quad \Rightarrow \quad v_y^2 = v_{0y}^2 + 2(-g)(y - 0)$$

$$\Rightarrow \quad v_y^2 = (15.0 \text{ m/s})^2 + 2\,(-9.80 \text{ m/s}^2)\, y$$

When the ball is 5.00 m above the origin we have y = +5 m, so

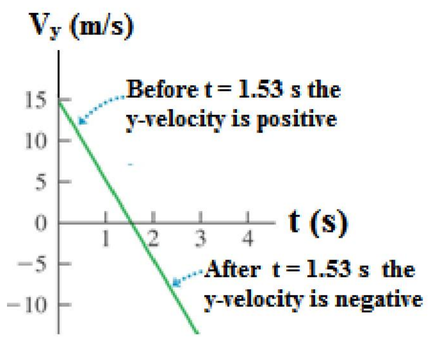
$$v_y^2 = (15.0 \text{ m/s})^2 + 2\,(-9.80 \text{ m/s}^2)\,(+5.0 \text{ m})$$

$$\Rightarrow \quad v_y^2 = 127 \text{ m}^2/\text{s}^2 \quad \Rightarrow \quad v_y = \pm 11.3 \text{ m/s}$$

We get two values of because the ball passes through the point twice, once on the way up (so is positive) and once on the way down (so is negative)

v_y-t graph is plotted by using equation $v_y = v_{0y} + a_y t$

v_y-t graph for a ball thrown upward with an initial speed of 15 m/s.(straight line with negative slope because $a_y = -g$ is constant and negative)



c) At the instant at which the ball reaches its maximum height y_1 , its y-velocity is momentarily zero i.e $v_y = 0$.
With $v_y = 0$, $y_0 = 0$ and $a_y = -g$, we get

$$v_y^2 = v_{0y}^2 + 2a_y(y - y_0) \quad \Rightarrow \quad 0 = v_{0y}^2 + 2(-g)(y_1 - 0)$$

$$\Rightarrow \quad 0 = v_{0y}^2 - 2\,g\,y_1 \quad \Rightarrow \quad y_1 = \frac{v_{0y}^2}{2g} = \frac{(15.0 \text{ m/s})^2}{2\,(9.80 \text{ m/s}^2)} = +11.5 \text{ m}$$

d) A free-fall misconception:
It's a common misconception that at the highest point of free-fall motion, where the velocity is zero, the acceleration is also zero.
If the acceleration is also zero, then ball will suspended in midair once it reaches the highest point.
But the acceleration is due to the force due to gravity and act continuously. At every point, including the highest point, and at any velocity, including zero, the acceleration in free fall is always $a_y = -g = -9.80 \text{ m/s}^2$

Example 2.9 Motion with changing acceleration

Sally is driving along a straight highway in her 1965 Mustang. At t = 0, when she is moving at 10 m/s, in the positive x-direction, she passes a signpost at x = 50 m. Her x-acceleration as a function of time is $a_x = 2.0\ \text{m/s}^2 - (0.10\ \text{m/s}^3)t$

a) Find her x-velocity and position x as functions of time.
b) When is her x-velocity greatest?
c) What is that maximum x-velocity?
d) Where is the car when it reaches that maximum x-velocity?

Solution:

(a) At t = 0 Sally's position is x_0 = 50 m and her x-velocity is v_{0x} = 10 m/s

$$v_x = v_{ox} + \int_0^t a_x dt = 10\ \text{m/s} + \int_0^t \left[2.0\ \text{m/s}^2 - (0.10\ \text{m/s}^3)\ t\right] dt$$

$$= 10\ \text{m/s} + \left(2.0\ \text{m/s}^2\right) t - \frac{1}{2}(0.10\ \text{m/s}^3)\ t^2 \qquad \text{-----}(1)$$

$$x = x_0 + \int_0^t v_x dt = 50\ \text{m} + \int_0^t \left[10\ \text{m/s} + \left(2.0\ \text{m/s}^2\right) t - \frac{1}{2}(0.10\ \text{m/s}^3)\ t^2\right] dt$$

$$= 50\ \text{m} + (10\ \text{m/s})\ t + \frac{1}{2}\left(2.0\ \text{m/s}^2\right) t^2 - \frac{1}{6}(0.10\ \text{m/s}^3)\ t^3 \qquad \text{------}(2)$$

(b) The maximum value of v_x occurs when the x-velocity stops increasing and begins to decrease. At that instant $a_x = 0$, So we set the expression for a_x equal to zero and solve for t:

$$a_x = 2.0\ \text{m/s}^2 - (0.10\ \text{m/s}^3)t$$

$$\Rightarrow\ 0 = 2.0\ \text{m/s}^2 - (0.10\ \text{m/s}^3)\ t$$

$$t = \frac{2.0\ \text{m/s}^2}{0.10\ \text{m/s}^3} = 20\ \text{s}$$

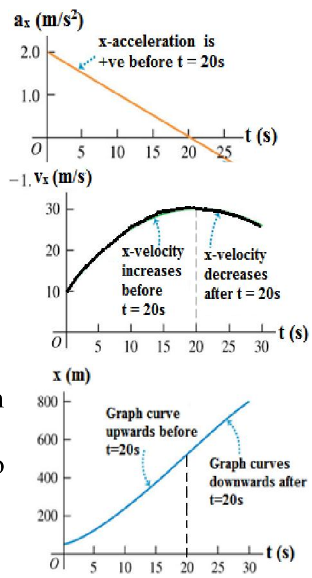
(c) From part (b) we know that the maximum x-velocity obtained at t = 20 s. Thus, maximum velocity can be obtained by putting t = 20s in eqn (1). Thus,

$v_{max\text{-}x}$ = 10 m/s + (2.0 m/s^2)(20 s) – ½ (10.10 m/s^3) (20 s)^2 = 30m/s

(d) Car's position at t = 20 s can be obtained by putting t = 20s in eqn (2). Thus,

$$x = 50\ \text{m} + (10\ \text{m/s})(20\text{s}) + \frac{1}{2}\left(2.0\ \text{m/s}^2\right)(20\ \text{s})^2 - \frac{1}{6}(0.10\ \text{m/s}^3)(20\ \text{s})^3 = 517\text{m}$$

Graphical representation



a_x-t graph: a_x is positive between t = 0 and t = 20s
a_x is negative t > 20s.
a_x = o at t = 20s

v_x-t graph: v_x is maximum at t = 20s (here a_x = 0).
The car speeds up until t = 20s (because and have the same sign)
The car slows down after t = 20s (because and have opposite signs).

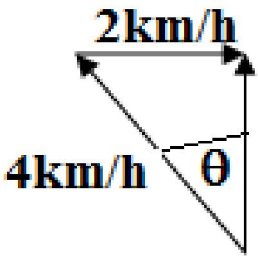
x-t graph: Since is maximum at t = 20s , the x-t graph has its maximum positive slope at this time.
Note that the x-t graph is concave up (curved upward) from t = 0 to t = 20s when a_x is positive.

The graph is concave down (curved downward) after t = 20 s, when a_x is negative.

Assignment Problems: 2.10, 2.18, 2.38

2.10: A man wishes to swim across a river 600m wide. If he can swim at the rate of 4km/hr in still water and the river flows at a rate 2km/hr, then in what direction must he swim to reach a point exactly opposite to the starting point and when will he reach it? Ans- t=624.3s

Solution:



First of all,
4kph = 1.1m/sec.
2kph = 0.55m/sec.
To cross directly across the river, he must point upstream.
This means he will be trying to follow the hypotenuse of an imaginary right triangle, and will be doing his 1.1m/sec. along it.
Velocity of the man along the vertical direction is

$$v_y = \sqrt{(1.1\ \text{m/s})^2 - (0.55\ \text{m/s})^2} = 0.953\ \text{m/s}$$

Time to cross d= 600m wide river is

$$t = \frac{d}{v_y} = \frac{600\ \text{m}}{0.953\ \text{m/s}} = 630\ \text{s}$$

From the figure $\sin\theta = \dfrac{2\text{km/h}}{4\text{km/h}} = \dfrac{1}{2} \Rightarrow \theta = 60^0$

2.18: The position of the front bumper of a test car under microprocessor control is given by $x(t) = 2.17\text{m} + (4.8\text{m/s}^2)t^2 - (0.1\text{m/s}^6)t^6$.

(a) Find its position and acceleration at the instants when the car has zero velocity.

(b) Draw v_x-t, and a_x~t graphs for the motion of the bumper between t = 0 and t = 2.0s

Solution:

$x(t) = 2.17\text{m} + (4.8\text{m/s}^2)t^2 - (0.1\text{m/s}^6)t^6$.

$$V_x = \frac{dx}{dt} = \frac{d}{dt}\left(2.17\text{m} + (4.8\text{m/s})t^2 - (0.1\text{m/s})t^6\right) = \left(9.6\text{m/s}^2\right)t - \left(0.6\text{m/s}^6\right)t^5$$

$$a_x = 9.6\text{m/s}^2 - \left(3.0\ \text{m/s}^6\right)t^4$$

(a) At velocity is zero i.e V_x=0, we have

$$0 = \left(9.6\text{m/s}^2\right)t - \left(0.6\text{m/s}^6\right)t^5$$

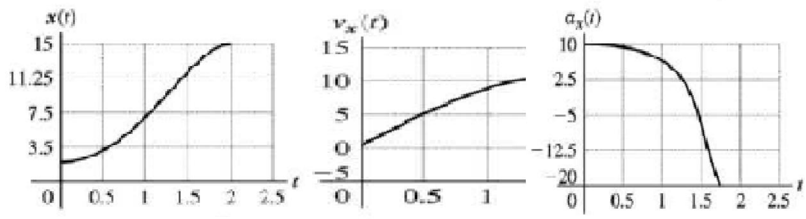
$$\Rightarrow t\left[\left(9.6\text{m/s}^2\right) - \left(0.6\text{m/s}^6\right)t^4\right] = 0$$

$$\Rightarrow t = 0 \text{ or, } \left(9.6\text{m/s}^2\right) - \left(0.6\text{m/s}^6\right)t^4 = 0$$

$$\Rightarrow \left(0.6\text{m/s}^6\right)t^4 = 9.6\text{m/s}^2 \Rightarrow t^4 = 16\text{s}^4 \Rightarrow t = 2\text{s}$$

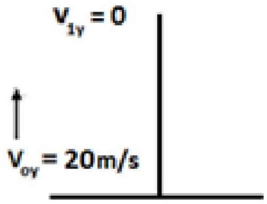
At t = 0, x = 2.17m and a_x = 9.6m/s^2.
At t = 0, x = 15m and a_x = -38.4m/s^2.

(b) For the entire time interval from t = 0 to t=2.00 s, The velocity v_x is positive and x increases. But the acceleration a_x is positive when the speed increases and a_x is negative for



the decease of speed.

2.38 A stone is thrown up vertically with a velocity of 20m/s. Find out the instances at which the magnitude of its, (i) momentum and (ii) kinetic energy will be half its initial value (g = 9.8m/s^2). (i) 0.5969s (ii) 3.4s



Solution: (i)

Initial momentum $= p_1 = m\, v_{0y}$

Final momentum $= p_2 = m\, v_{1y}$

Given: $p_2 = \frac{p_1}{2} \quad \Rightarrow \quad m\, v_{1y} = \frac{m\, v_{0y}}{2}$

$\Rightarrow \quad v_{1y} = \frac{v_{0y}}{2} = \frac{20m/s}{2} = 10m/s$

t_1 = time at which the momentum will be half the initial value

$v_{1y} = v_{oy} + g\ t_1$

$10 = 20 + (-9.8m/s^2)\, t_1$

$t_1 = 1.02s$

(ii) Initial K.E $= K.E_1 = \frac{1}{2} m\, v_{0y}^2$

Final momentum $= K.E_2 = \frac{1}{2} m\, v_{1y}^2$

Given: $K.E_2 = \frac{K.E_1}{2} \quad \Rightarrow \quad \frac{1}{2} m\, v_{1y}^2 = \frac{\frac{1}{2} m\, v_{0y}^2}{2} \quad \Rightarrow \quad v_{1y} = \frac{v_{0y}}{\sqrt{2}} = \frac{20m/s}{\sqrt{2}} = 10\sqrt{2}m/s$

t_2 = time at which the momentum will be half the initial value

$v_{1y} = v_{oy} + g\ t_2$

$10\sqrt{2} = 20 + (-9.8m/s^2)\, t_2$

$t_1 = 0.598s$