Motion in 2 or 3 dimension

Chapter - 3

Topic	Section	Test Your Understanding	In-class Problems (Example)	Assignment (Exercise)
Position and Velocity Vectors, Acceleration Vector (Parallel & Perpendicular component of acceleration)	3.1, 3.2	TYU-3.1 TYU-3.2	3.1	3.4, 3.9, 3.31, 3.36
Projectile Motion	3.3	TYU-3.3	3.7	
Motion in a Circle, Relative Velocity	3.4	TYU-3.4	3.12	
Relative Velocity	3.5	TYU-3.5	3.14	

Position Vector

An overall position relative to the origin can have components in *x*, *y*, and *z* dimensions.

Position vector is: $\vec{r} = x\,\hat{i} + y\,\hat{j} + z\,\hat{k}$ and $|\vec{r}| = \sqrt{x^2 + y^2 + z^2}$

Velocity Vector

The change in position (the displacement) during the interval Δt is the **average velocity**. It is given by

$$\vec{v}_{av} = \frac{\vec{r}_2 - \vec{r}_1}{t_2 - t_1} = \frac{\overrightarrow{\Delta r}}{\Delta t}$$

Where, $\overrightarrow{\Delta r} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$

Instantaneous velocity is the limit of the average velocity as the time interval approaches zero, and it equals the instantaneous rate of change of position with time.
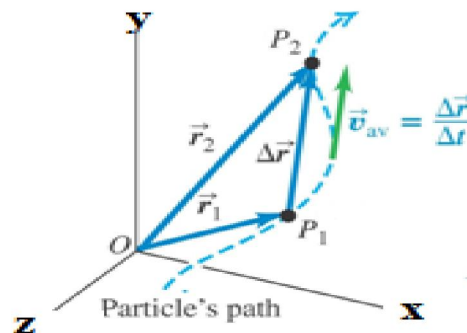
$$\vec{v} = \underset{\Delta t \to 0}{\text{Lim}} = \frac{\overrightarrow{\Delta r}}{\Delta t} = \frac{\overrightarrow{dr}}{dt}$$

$$\Rightarrow \vec{v} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{k}$$

$$\Rightarrow \vec{v} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$$

$$\Rightarrow |\vec{v}| = \sqrt{v_x^2 + v_y^2 + v_z^2}$$

and $\tan\alpha = \dfrac{v_y}{v_x}$

The Acceleration Vector

The acceleration vector can result in a change in either the magnitude OR the direction of the velocity. In the figure the instantaneous velocity of the car changes in both magnitude and direction for which the car accelerates by slowing while rounding a curve. The average acceleration can be obtained by finding the change in velocity $\Delta\vec{v}$. The average acceleration has the same direction of $\Delta\vec{v}$.

Car moving along a curved road	How to obtain the change in velocity	Direction of average acceleration
In the figure the instantaneous velocity of the car changes in both magnitude and direction for which the car accelerates by slowing while rounding a curve.	The average acceleration of the car between P₁ and P₂ is obtained by finding the change in velocity $\Delta\vec{v}$ ($\Delta\vec{v} = \vec{v}_2 - \vec{v}_1$)	The average acceleration has the same direction of $\Delta\vec{v}$

Average accelerating vector is given by:

$$\vec{a}_{av} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1} = \frac{\overrightarrow{\Delta v}}{\Delta t}$$

Instantaneous accelerating vector is given by:

$$\vec{a} = \underset{\Delta t \to 0}{\text{Lim}} = \frac{\overrightarrow{\Delta v}}{\Delta t} = \frac{\overrightarrow{dv}}{dt}$$

$$\Rightarrow \vec{a} = \frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j} + \frac{dv_z}{dt}\hat{k}$$

$$\Rightarrow \vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k} \Rightarrow |\vec{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2} \quad \text{and} \quad \tan\alpha = \frac{a_y}{a_x}$$

Projectile motion

A body is projected with a velocity V_0 by making an angle α_0 with the horizontal.

Horizontal component of the velocity is: $V_{0x} = V_0 \cos\alpha_0$

Vertical component of the velocity is: $V_{0y} = V_0 \sin\alpha_0$

Horizontal acceleration $a_x = 0$

Vertical acceleration $a_y = -g$ during ascend

Vertical acceleration $a_y = +g$ during descend

Solving for **x-motion** we get:

$V_x = V_{0x}$ and $x - x_0 = V_{ox} t$

Solving for **y-motion** we get:

$V_y = V_{0y} - gt$ and $y - y_0 = V_{oy} t - \frac{1}{2} g t^2$

If we set $x_0 = y_0 = 0$, the equations describing projectile motion are:

$x = V_{ox} t$ and $y = V_{oy} t - \frac{1}{2} g t^2$

Thus the projectile equations are:

$$x = \left(v_o \cos\alpha_0 \right) t \qquad \text{and} \qquad y = \left(v_o \sin\alpha_0 \right) t - \frac{1}{2} g t^2 \quad \text{------- (1)}$$

$$v_x = v_o \cos\alpha_0 \qquad \text{and} \qquad v_y = v_o \sin\alpha_0 - g t \qquad \text{------- (2)}$$

Putting the value of 't' from eq^n (1) and solving we get

$$y = \left(\tan\alpha_0 \right) x - \left(\frac{g}{2 v_0^2 \cos^2\alpha_0} \right) x^2$$

This is the equation of a parabola. Thus, the trajectory of a projectile is always a parabola

Maximum height of a projectile

At the maximum height position, $V_y = 0$

Using the equation $V^2_y - V^2_{0y} = 2(-g) h$ we get

$0 - V^2_0 \sin^2\alpha_0 = 2(-g) h$

$V^2_0 \sin^2\alpha_0 = 2 g h$

$$h = \frac{v_0^2 \sin^2\alpha_0}{2g}$$

This is the maximum height attain by a projectile.

Time of flight (T)
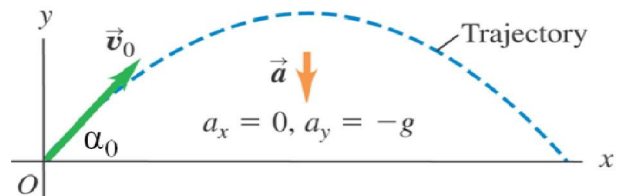
t = time to reach the maximum height.

Then T = 2t

At the maximum height position, $V_y = 0$

Using the equation $V_y = V_{0y} - gt$ we get

$0 = V_{0y} - gt => gt = V_{0y}$

$$t = \frac{V_{0y}}{g} = \frac{V_0 \sin\alpha_0}{g}$$

Total time of flight is T = 2t

$$T = 2t = \frac{2V_0 \sin\alpha_0}{g}$$

This represents the total time of flight

Horizontal Range

Let R = horizontal range

$$R = (V_{0x})\, T$$

$$R = \left(V_0 \cos\alpha_0\right)\left(\frac{2V_0 \sin\alpha_0}{g}\right) \Rightarrow R = \frac{V_0^2 \sin 2\alpha_0}{g}$$

## <u>Motion of a particle in a Circular path</u>

Uniform circular motion

A particle moves a distance Δs at constant speed along a circular path. v_1 and v_2 are the velocities at P_1 and P_2 respectively.

The corresponding vector triangle is drawn.

Δv represents the change in velocity.

The direction is along the radius and towards the center.

Body moves in circular path   Vector triangle   Acceleration is directed to center
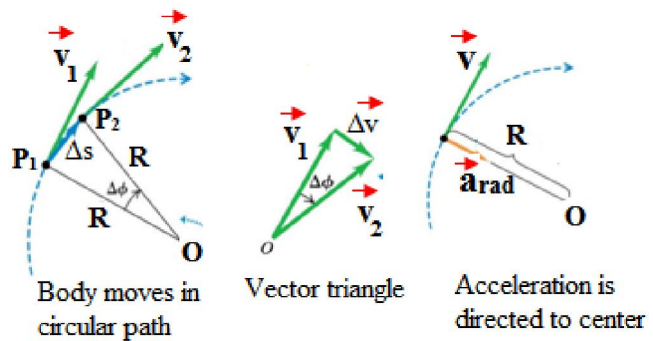
We know that $\bar{a} = \dfrac{\Delta \vec{V}}{\Delta t}$

So the direction of instantaneous acceleration in uniform circular motion always points toward the center of the circle.

The vector triangle and $\Delta P_1 P_2 O$ are similar. So

$$\frac{\Delta V}{V} = \frac{\Delta s}{R} \quad\Rightarrow\quad \Delta V = \frac{V}{R}\Delta s \quad\Rightarrow\quad \frac{\Delta V}{\Delta t} = \frac{V}{R}\frac{\Delta s}{\Delta t}$$
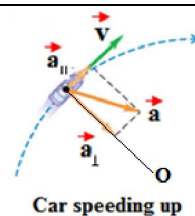
$$\Rightarrow \left(\lim_{\Delta t \to 0}\frac{\Delta V}{\Delta t}\right) = \frac{V}{R}\left(\lim_{\Delta t \to 0}\frac{\Delta s}{\Delta t}\right) \quad\Rightarrow\quad a = \frac{V}{R}V \quad\Rightarrow\quad a = \frac{V^2}{R}$$

For uniform circular motion, the speed is constant and the acceleration is perpendicular to the velocity with magnitude v^2/R.
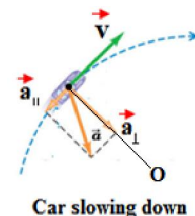
Non-uniform circular motion

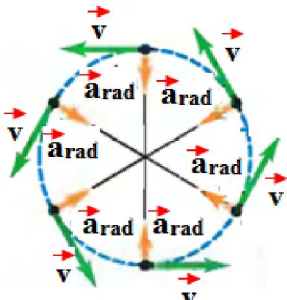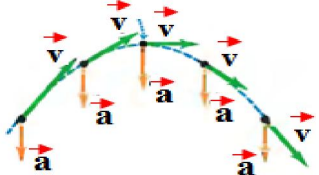| Car speeding up along a circular path. • $\vec{a}_{\parallel}$ is along the direction of $\vec{v}$ • $\vec{a}_{\parallel}$ is changes car's speed. • $\vec{a}_{\perp}$ is changes car's direction.	 Car speeding up
Car slowing down along a circular path. • $\vec{a}_{\parallel}$ is along the opposite direction of $\vec{v}$ • $\vec{a}_{\parallel}$ is changes car's speed. • $\vec{a}_{\perp}$ is changes car's direction.	 Car slowing down

Car moves with constant speed	
• Acceleration is exactly perpendicular to velocity and has constant magnitude. • No parallel component. $a_\parallel = 0$	
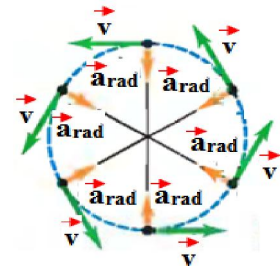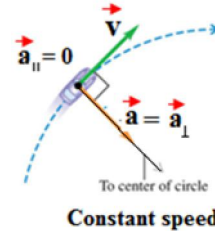
Uniform circular motion vs. projectile motion
Same: the magnitude of acceleration is the same at all times for both the cases.

	Uniform circular motion	Projectile motion
Difference:	Direction of acceleration changes continuously so that it always points toward the center of the circle.	Direction of the acceleration in projectile always points down
	Velocity is always perpendicular to acceleration	Velocity and acceleration are perpendicular only at the peak of the trajectory.

Relative velocity

The velocity of a moving body seen by a particular observer is called the velocity relative to that observer, or simply the relative velocity.

If point P is moving relative to reference frame A, we denote the velocity of P relative to frame A as $v_{P/A}$.

If P is moving relative to frame B and frame B is moving relative to frame A, then the x-velocity of P relative to frame A is $v_{P/A-x} = v_{P/B-x} + v_{B/A-x}$.

Test Your Understanding of Section 3.1:

In which of these situations would the average velocity vector $\vec{v}_{av}$ over an interval be equal to the instantaneous velocity $\vec{v}$ at the end of the interval?
  (i)   a body moving along a curved path at constant speed;
  (ii)   a body moving along a curved path and speeding up;
  (iii)   a body moving along a straight line at constant speed;
  (iv)   a body moving along a straight line and speeding up.
3.1 Answer: (iii)
If the instantaneous velocity $\vec{v}$ is constant over an interval, its value at any point (including the end of the interval) is the same as the average velocity $\vec{v}_{av}$ over the interval.
In (i) and (ii) the direction of $\vec{v}$ at the end of the interval is tangent to the path at that point, while direction of $\vec{v}_{av}$ points from the beginning of the path to its end (in the direction of the net displacement).
In (iv) and are both $\vec{v}$ and $\vec{v}_{av}$ directed along the straight line, but $\vec{v}$ has a greater magnitude because the speed has been increasing.

Test Your Understanding of Section 3.2:
A sled travels over the crest of a snow-covered hill. The sled slows down as it climbs up one side of the hill and gains speed as it descends on the other side. Which of the vectors (1 through 9) in the figure correctly shows the direction of the sled's acceleration at the crest? (Choice 9 is that the acceleration is zero.)
Answer: vector 7
At the high point of the sled's path, the speed is minimum. At that point the speed is neither increasing nor decreasing, and the parallel component of the acceleration (that is, the horizontal component) is zero. The acceleration has only a perpendicular component toward the inside of the sled's curved path. In other words, the acceleration is downward.
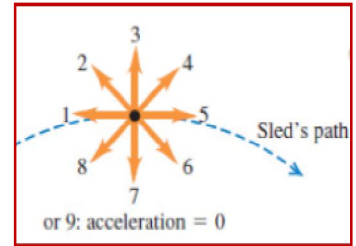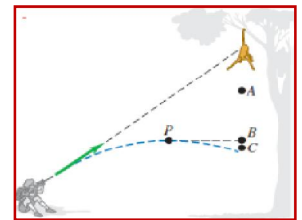
Test Your Understanding of Section 3.3:
Suppose the tranquilizer dart as a relatively low muzzle velocity so that the dart reaches a maximum height at a point P before striking the monkey, as shown in the figure. When the dart is at point P, will the monkey be (i) at point A (higher than P), (ii) at point B (at the same height as P), or (iii) at point C (lower than P)? Ignore air resistance.

Answer: (i)
If there were no gravity (g=0), the monkey would not fall and the dart would follow a straight-line path (shown as a dashed line). The effect of gravity is to make the monkey and the dart both fall the same distance $\frac{1}{2}gt^2$ below their positions. Point A is the same distance below the monkey's initial position as point P is below the dashed straight line, so point A is where we would find the monkey at the time in question.

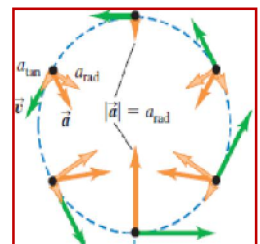**Test Your Understanding of Section 3.4:**
Suppose that the particle in Figure experiences four times the acceleration at the bottom of the loop as it does at the top of the loop. Compared to its speed at the top of the loop, is its speed at the bottom of the loop (i) $\sqrt{2}$ times as great; (ii) 2 times as great; (iii) $2\sqrt{2}$ times as great; (iv) 4 times as great; or (v) 16 times as great?

3.4 Answer: (ii)
At both the top and bottom of the loop, the acceleration is purely radial and is given by

$$a_{rad} = \frac{V^2}{R}$$

The radius R is the same at both points, so the difference in acceleration is due purely to differences in speed. Since a_{rad} is proportional to the square of the v, speed must be twice as great at the bottom of the loop as at the top.

Test Your Understanding of Section 3.5:
Suppose the nose of an airplane is pointed due east and the airplane has an air speed of 150 km/h. Due to the wind, the airplane is moving due north relative to the ground and its speed relative to the ground is 150 km/s. What is the velocity of the air relative to the earth?
(i)   150 km/h from east to west;
(ii)   150 km/h from south to north;
(iii) 150 km/h from southeast to northwest;
(iv)   212 km/h from east to west;
(v)   212 km/h from south to north;
(vi)   212 km/s from southeast to northwest;
(vii) there is no possible wind velocity that could cause this.

3.5 Answer: (vi)
The effect of the wind is to cancel the airplane's eastward motion and give it a northward motion. So the velocity of the air relative to the ground (the wind velocity) must have one 150-kmh component to the west and one 150-kmh component to the north. The combination of these is a vector of magnitude $\sqrt{(150\text{km/h})^2 + (150\text{km/h})^2} = 212$ km/h that points to the northwest.

Example Problems
Example:3.1
A robotic vehicle, or rover, is exploring the surface of Mars. The stationary Mars lander is the origin of coordinates, and the surrounding Martian surface lies in the *xy*-plane. The rover, which we represent as a point, has *x*- and *y*-coordinates that vary with time:
$x = 2.0\text{ m} - (0.25\text{ m/s}^2)t^2$
$y = (1.0\text{ m/s})t + (0.025\text{ m/s}^3)t^3$
(a)   Find the rover's coordinates and distance from the lander at *t* = 2 s
(b)   Find the rover's displacement and average velocity vectors for the interval t = 0 s to t = 2 s
(c)   Find a general expression for the rover's instantaneous velocity vector $\vec{v}$. Express $\vec{v}$ at t = 2s in component form and in terms of magnitude and direction.
Solution:

(a)   At t =2.0s the rover's coordinates are
$x = 2.0\text{ m} - \left(0.25\text{m/s}^2\right)t^2$

$x = 2.0\text{ m} - \left(0.25\text{m/s}^2\right)\left(2s\right)^2 = 1.0\text{m}$

$y = \left(1.0\text{m/s}\right)t + \left(0.025\text{m/s}^3\right)t^3$

$y = \left(1.0\text{m/s}\right)t + \left(0.025\text{m/s}^3\right)\left(2s\right)^3 = 2.2\text{m}$

The rover's distance from the origin at time t =2.0s is
$r = \sqrt{x^2 + y^2} = \sqrt{\left(1.0\text{ m}\right)^2 + \left(2.2\text{ m}\right)^2} = 2.4\text{m}$

(b)   Position vector is:
$\vec{r} = x\,\hat{i} + y\hat{j} = \left[2.0\text{ m} - \left(0.25\text{m/s}^2\right)t^2\right]\hat{i} + \left[\left(1.0\text{m/s}\right)t + \left(0.025\text{m/s}^3\right)t^3\right]\hat{j}$

At t = 0.0s the position vector ($\vec{r}_0$) is
$\vec{r}_0 = 2.0\text{ m}\,\hat{i}$

At t = 2.0s the position vector ($\vec{r}_2$) is
$\vec{r}_2 = \left(1.0\text{ m}\right)\hat{i} + \left(2.2\text{m}\right)\hat{j}$

The displacement from t = 0.0s to t = 2.0s is:
$\Delta\vec{r} = \vec{r}_2 - \vec{r}_0 = \left[\left(1.0\text{ m}\right)\hat{i} + \left(2.2\text{m}\right)\hat{j}\right] - \left(2.0\text{ m}\right)\hat{i} = \left(-1.0\text{ m}\right)\hat{i} + \left(2.2\text{m}\right)\hat{j}$

During the interval from t = 0.0s to t = 2.0s the rover moves 1.0 m in the negative x-direction and 2.2 m in the positive y-direction.
Now the average velocity over this interval is
$\vec{v}_{av} = \frac{\overrightarrow{\Delta r}}{\Delta t} = \frac{\left(-1.0\text{ m}\right)\hat{i} + \left(2.2\text{m}\right)\hat{j}}{2.0\text{s} - 0.0\text{s}} = \left(-0.5\text{ m/s}\right)\hat{i} + \left(1.1\text{m/s}\right)\hat{j}$

The x-component and y-component of the average velocity are
$\vec{v}_{av\text{-}x} = \left(-0.5\text{ m/s}\right)\hat{i}$ and $\vec{v}_{av\text{-}y} = \left(1.1\text{m/s}\right)\hat{j}$

(c) The components of instantaneous velocity are

$$v_x = \frac{dx}{dt} = \frac{d}{dt}\left[2.0\,m - \left(0.25m/s^2\right)t^2\right] = -\left(0.25m/s^2\right)2t = -\left(0.5m/s^2\right)t \text{ and}$$

$$v_x = \frac{dx}{dt} = \frac{d}{dt}\left[(1.0m/s)t + \left(0.025m/s^3\right)t^3\right] = 1.0m/s + \left(0.025m/s^3\right)3t^2 = 1.0m/s + \left(0.075m/s^3\right)t^2$$

So the instantaneous velocity vector is

$$\vec{v} = v_x\hat{i} + v_y\hat{j} = \left[-\left(0.5m/s^2\right)t\right]\hat{i} + \left[1.0m/s + \left(0.075m/s^3\right)t^2\right]\hat{j}$$

At t=2.0s the instantaneous velocity vector is

$$\vec{v}_2 = v_x\hat{i} + v_y\hat{j} = \left[-\left(0.5m/s^2\right)(2.0s)\right]\hat{i} + \left[1.0m/s + \left(0.075m/s^3\right)(2.0s)^2\right]\hat{j}$$

$$\Rightarrow \vec{v}_2 = -(1.0m/s)\hat{i} + (1.3m/s)\hat{j}$$

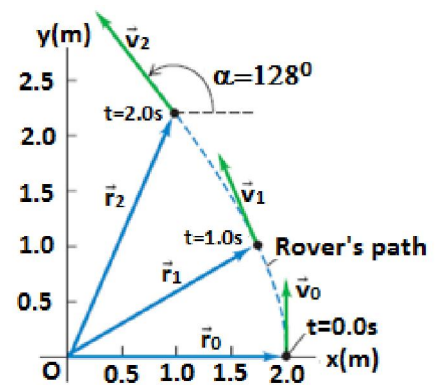$$\Rightarrow v_2 = \sqrt{\left[-(1.0m/s)\right]^2 + \left[1.3m/s\right]^2} = 1.6m/s$$

In the adjoining figure the direction of the velocity vector $\vec{v}_2$, which is at an angle α between 90^0 and 180^0 with respect to the positive x-axis is

$$\tan^{-1}\frac{v_y}{v_x} = \tan^{-1}\frac{1.3m/s}{-1.0m/s} = 52^0$$

This is off by 180^0;
So, the correct value of the angle is
$\alpha = 180^0 - 52^0 = 128^0$ or 38^0 west of north.

Example:3.7
A batter hits a baseball so that it leaves the bat at speed $v_0 = 37$ m/s, at an angle $\alpha_0 = 53.1^0$, at a location where g = 9.8 m/s^2.
   a) Find the position of the ball and its velocity at t = 2.0 s
   b) Find the time when the ball reaches the highest point of its flight, and its height **h** at this time.
   c) Find the horizontal range of the ball.

Solution:
a) $v_0 = 37$ m/s $\alpha_0 = 53.1^0$

Components of the initial velocity of the ball are

$v_{0x} = v_0 \cos\alpha_0 = (37.0$ m/s$) \cos 53.1° = 22.2$ m/s

$v_{0y} = v_0 \sin\alpha_0 = (37.0$ m/s$) \sin 53.1° = 29.6$ m/s

x-co-ordinate of displacement after t = 2s is

x = v_{0x} t = (22.2 m/s)(2s) = 44.4 m/s

y-co-ordinate of displacement after t = 2s is

$$y = v_{0y}t + \frac{1}{2}a_yt^2 = (29.6 \text{ m/s})(2s) + \frac{1}{2}(-9.8m/s^2)(2s)^2 = 39.6 \text{ m}$$

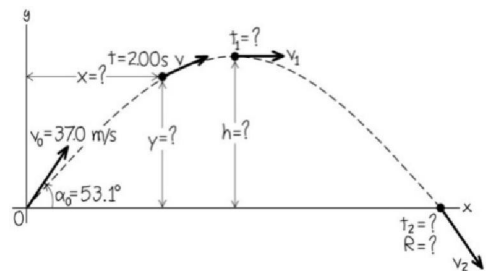Displacement of the body after t = 2s is

$$r = \sqrt{x^2 + y^2} = \sqrt{(44.4 \text{ m})^2 + (39.6 \text{ m})^2} = 59.49 \text{ m}$$

x-component of velocity after t = 2s is

$v_x = v_{0x} = 44.4$m/s

y-component of velocity after t = 2s is

$$v_y = v_{0y} + g\,t = 29.6 \text{ m/s} + \left(-9.8 \text{ m/s}^2\right)(2s) = 10 \text{ m/s}$$

The y-component of velocity is positive at t = 2 s, so the ball is still moving upward.

The magnitude of the velocity is

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(22.2 \text{ m/s})^2 + (10 \text{ m/s})^2} = 24.4 \text{ m/s}$$

The direction angle β of $\vec{v}$ with respect to the positive x-axis is

$$\tan\beta = \frac{v_y}{v_x} \implies \beta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{10}{22.2}\right) = 24.2^0$$

(b)   At the highest point, the vertical velocity $v_y = 0$.

t_1 = time to reach the highest point

$$v_y = v_{0y} + g\,t \implies 0 = 29.6 \text{ m/s} + \left(-9.8 \text{ m/s}^2\right)t_1 \implies t_1 = \frac{39.6 \text{ m/s}}{9.8 \text{ m/s}^2} = 3.02s$$

h = height at the highest point

$$h = v_{0y}t_1 + \frac{1}{2}g\,t_1^2 = (29.6 \text{ m/s})(3.02s) + \frac{1}{2}\left(-9.8 \text{m/s}^2\right)(3.02s)^2 = 44.7 \text{ m}$$

(c)   T = total time of flight = 2 t_1 = 2 x (3.02 s) = 6.04 s

R = horizontal range

R = v_{0x} T = (22.2 m/s)(6.04 s) = 134 m

Example:3.12

Passengers on a carnival ride move at constant speed in a horizontal circle of radius 5.0 m, making a complete circle in 4.0 s. What is their acceleration?

Solution:

Radial acceleration of the pas is passenger is

$$a_{rad} = \frac{v^2}{R} = \left(\frac{2\pi R}{T}\right)^2\left(\frac{1}{R}\right) = \frac{4\pi^2 R}{T^2} = \frac{4\pi^2(5.0 \text{ m})^2}{(4.0s)^2} = 12\text{m/s}^2 = 1.3g$$

Example:3.14

An airplane's compass indicates that it is headed due north, and its airspeed indicator shows that it is moving through the air at 240 km/h. If there is a 100-km h wind from west to east, what is the velocity of the airplane relative to the earth?
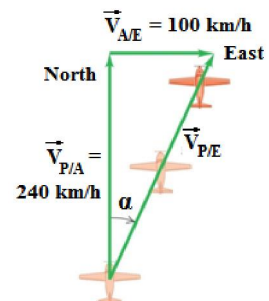
Solution:

$\vec{v}_{P/A}$ = 240 km/h   due north

$\vec{v}_{A/E}$ = 100 km/h   due east

$$\vec{v}_{P/E} = \vec{v}_{P/A} + \vec{v}_{A/E} \implies \left|\vec{v}_{P/E}\right| = \sqrt{v_{P/A}^2 + v_{A/E}^2}$$

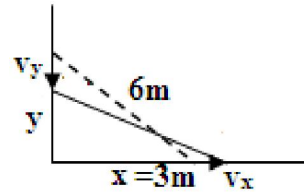$$\implies \left|\vec{v}_{P/E}\right| = \sqrt{(240 \text{ km/h})^2 + (100 \text{ km/h})^2} = 260 \text{ km/h}$$

$$\tan\alpha = \frac{v_{A/E}}{v_{P/A}} \implies \alpha = \tan^{-1}\left(\frac{100 \text{ km/h}}{240 \text{ km/h}}\right) = 23^0 \text{ E of N}$$

Solution to the Assignment Problems

Exercise-3.4 A ladder of 6 m length, which is in contact with a vertical wall and horizontal ground slides down the vertical plane. When the lower end is at a distance of 3 m from the wall, its velocity is 4 m/s. What is the velocity of the upper end at that instant?

Solution:

$x^2 + y^2 = 6^2$

$y = \sqrt{6^2 - x^2} = \sqrt{36 - 9} = 3\sqrt{3}\,\text{m}$

Again

$x^2 + y^2 = 6^2$

$2x\dfrac{dx}{dt} + 2y\dfrac{dy}{dt} = 0$

$x\,V_x + y\,V_y = 0$

$V_y = -\dfrac{x}{y}\,V_x = -\dfrac{3\text{m}}{3\sqrt{3}\text{m}}\left(4\text{m/s}\right) = -2.039\,\text{m/s}$

Exercise-3.9 Two particles are thrown up simultaneously with a velocity of 30 m/s, one thrown vertically and another at 45^0 with respect to the horizon. Find out the distance between them at t = 1.5s. (34.44m)

Solution: Particle thrown vertical upward

Height reached after t = 1.5s is

$h = v_{0y}t + \dfrac{1}{2}a_y t^2 = \left(30 \text{ m/s}\right)\left(1.5\text{s}\right) + \dfrac{1}{2}\left(-9.8\text{m/s}^2\right)\left(1.5\text{s}\right)^2 = 33.975\text{m}$

<u>Particle thrown at 45^0 with respect to the horizon</u>

$v_0 = 30$ m/s $\alpha_0 = 45^0$

Components of the initial velocity of the ball are

$v_{0x} = v_0 \cos \alpha_0 = (30\text{m/s}) \cos 45° = 21.21\,\text{m/s}$

$v_{0y} = v_0 \sin \alpha_0 = (30\text{m/s}) \sin 45° = 21.21\,\text{m/s}$
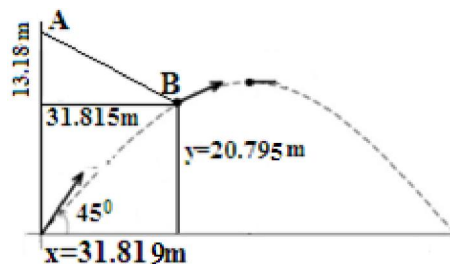
x-co-ordinate of displacement after t = 1.5s is

$x = v_{0x}\, t = (21.21 \text{ m/s})(1.5\text{s}) = 31.819 \text{ m/s}$

y-co-ordinate of displacement after t = 1.5s is

$y = v_{0y}t + \dfrac{1}{2}a_y t^2 = \left(21.21 \text{ m/s}\right)\left(1.5\text{s}\right) + \dfrac{1}{2}\left(-9.8\text{m/s}^2\right)\left(1.5\text{s}\right)^2 = 20.795\text{m}$

Now, distance AB is

$AB = \sqrt{\left(31.819 \text{ m}\right)^2 + \left(13.18 \text{ m}\right)^2} = 34.44\text{m}$

Exercise-3.31 The radius of the earth's orbit around the sun (assumed to be circular) is 1.50 x 10^8 km, and the earth travels around this orbit in 365 days.
   (a) What is the magnitude of the orbital velocity of the earth, in m/s?
   (b) What is the radial acceleration of the earth toward the sun, in m/s²?
   (c) Repeat parts (a) and (b) for the motion of the planet Mercury (orbit radius = 5.79 x 10^7 km, orbital period = 88.0 days).

Solution:

Planets are assumed to be moving in circular orbits and therefore have acceleration

Radius (R) of the earth's orbit $= 1.50 \times 10^8$ km $= 1.50 \times 10^{11}$ m

Orbital period (T) of the earth $= 365$ days $= 3.16 \times 10^7$ s

(a)   Orbital velocity of the earth is $v = \dfrac{2\pi R}{T} = \dfrac{2\pi(1.50 \times 10^{11}\text{m})}{3.16 \times 10^7 \text{s}} = 2.98 \times 10^4 \text{m/s}$

(b)   Radial acceleration of the earth toward the sun is

$$a_{rad} = \dfrac{v^2}{R} = \dfrac{\left(2.98 \times 10^4 \text{ m/s}\right)^2}{(1.50 \times 10^{11}\text{m})} = 5.91 \times 10^{-3} \text{m/s}^2$$

(c)   Radius (R) of the Mercury's orbit $= 5.79 \times 10^7$ km $= 5.79 \times 10^{10}$ m

Orbital period (T) of the Mercury $= 88$ days $= 7.6 \times 10^6$ s

Orbital velocity of the Mercury is $v = \dfrac{2\pi R}{T} = \dfrac{2\pi(5.79 \times 10^{10}\text{m})}{7.6 \times 10^6 \text{s}} = 4.79 \times 10^4 \text{m/s}$

Radial acceleration of the Mercury is $a_{rad} = \dfrac{v^2}{R} = \dfrac{\left(4.79 \times 10^4 \text{ m/s}\right)^2}{(5.79 \times 10^{10}\text{m})} = 3.96 \times 10^{-2} \text{m/s}^2$

So, Mercury has a larger orbital velocity and a larger radial acceleration than earth.

Exercise-3.36   A canoe has a velocity of 0.4 m/s southeast relative to the earth. The canoe is on a river that is flowing 0.5 m/s east relative to the earth. Find the velocity (magnitude and direction) of the canoe relative to the river. (0.36m/s, 52.5⁰ west of south)

Solution:

$\vec{v}_{C/E}$ = velocity of canoe relative to the earth = 0.4 m/s due southeast

$\vec{v}_{R/E}$ = velocity of river relative to the earth = 0.5 m/s due east

$\vec{v}_{C/R}$ = velocity of canoe relative to the river

$\vec{v}_{C/E} = \vec{v}_{C/R} + \vec{v}_{R/E}$

$\Rightarrow \vec{v}_{C/R} = \vec{v}_{C/E} - \vec{v}_{R/E}$

The velocity components of $\vec{v}_{C/R}$ are

$- 0.50 \text{ m/s} + \dfrac{(0.40 \text{ m/s})}{\sqrt{2}} = - 0.50 \text{ m/s} + 0.283 \text{ m/s} = - 0.217 \text{ m/s}$ due east

$\dfrac{(0.40 \text{ m/s})}{\sqrt{2}} = 0.283 \text{ m/s}$ due south

$\left|\vec{v}_{C/R}\right| = \sqrt{(0.283\text{m/s})^2 + (- 0.217\text{m/s})^2}$

$\Rightarrow \left|\vec{v}_{C/R}\right| = 0.36\text{m/s}$

$\tan \alpha = \dfrac{0.283}{0.217} = 1.3 \Rightarrow \alpha = \tan^{-1}(1.3) = 52.43^0$ W of S