Chapter 6

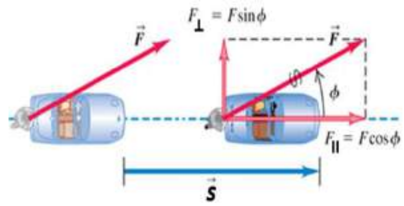
Work and Kinetic Energy

	Conceptual Problems (To be Discussed in Class)	In Class Problems	Assignment Problems
6.1 Work 6.2 Kinetic Energy and the Work–Energy Theorem 6.3 Work and Energy with Varying Forces 6.4 Power	TYU-6.1 TYU-6.2	Example- 6.2, 6.3, 6.10	Exercise 6.2, 6.20, 6.47

Work

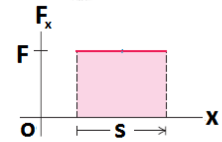
The work done by a constant force acting at an angle ϕ to the displacement is

$W = F s \cos\phi. = \vec{F} \cdot \vec{s}$



The area under the force ~ displacement graph represents the work done.

Area under F~ x graph = W =F s



Positive work (Force (F) has a component in direction of displacement)	$0^0 < \phi < 90^0$	• Gravity does +ve work during free fall of a body. • Work done on a spring during stretching.
Negative work (Force (F) has a component opposite to the direction of displacement)	$90^0 < \phi < 180^0$	• Work done by the hands of a person on a ball on his hand. • Work done by the gravitational force on a body moving vertically upward direction. • Work done by the frictional force.
Zero work (Force (F) has a component perpendicular to the direction of displacement)	$\phi = 90^0$	• Person walking with constant velocity on a level floor while carrying a book on his head. • a body rotating along a circular path.

Work done by a constant force:

A constant force (F) is applied on a body so that the so that its gets displaced from x_1 to x_2 and the respective velocities are v_1 and v_2 from x_1 to x_2 .
The work done is give by

$$W = F s \cos\phi. \quad \Rightarrow \quad W = m a s \cos 0. \quad \Rightarrow \quad W = m\left(\frac{v_2^2 - v_1^2}{2s}\right) s$$

$$\Rightarrow \quad W = \frac{1}{2} m v_2^2 - \frac{1}{2} m v_2^2 \quad \Rightarrow \quad W = K_2 - K_1 = \Delta K$$

Thus work done is the change in kinetic energy of the body.

Work done by a variable force

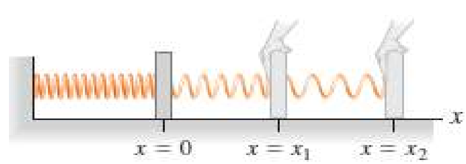
Let 'dw' be the work done for the very small displacement dx, then,

$dW = F_x\, dx$ where, F_x = force applied

$$\Rightarrow \quad W = \int_{x_1}^{x_2} F_x\, dx = F_x \int_{x_1}^{x_2} dx = F_x(x_2 - x_1)$$

Work done in stretching a spring

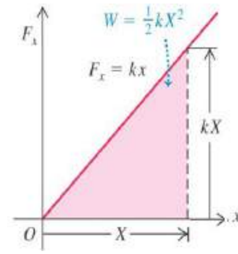
Force of a stretched spring, are not constant. The force needed to stretch an ideal spring is proportional to the spring's elongation. It is given by



F_x = kx.

Where, k = spring constant

Units of k = Newton/meter
k = Large, for tight spring and k = Small, for Loose spring
The work done by this force when the elongation goes from x_1 to x_2 is



$$W = \int_{x_1}^{x_2} F_x dx \quad \Rightarrow \quad W = \int_{x_1}^{x_2} k\, x\, dx$$

$$\Rightarrow \quad W = k\left[x^2\right]_{x_1}^{x_2} \quad \Rightarrow \quad W = \frac{1}{2}k\, x_2^2 - \frac{1}{2}k\, x_1^2$$

When an external force is applied on the spring then the work done on the spring is **positive** $\left(W = +\frac{1}{2}k\, x^2\right)$

But work done by the spring is **negative**. In this case the work is done by the restoring force ($F_x = -kx$).

So, work done by the spring is $W = -\frac{1}{2}k\, x^2$

If $x_1 = 0$ and $x_2 = x$, then

$$W = \frac{1}{2}k\, x^2 - \frac{1}{2}k\,(0)^2 \quad \Rightarrow \quad W = \frac{1}{2}k\, x^2$$

The area under the graph represents the work done on the spring as the spring is stretched from x = 0 to a maximum value 'x' is

Work–Energy Theorem for straight line motion

Let us a body applied with a varying force F_x. Then work done on the body is given by

$W = \int_{x_1}^{x_2} F_x\, dx$ where, x_1 and x_2 are the initial and final displacement respectively.

$$\Rightarrow \quad W = \int_{x_1}^{x_2} m\, a_x\, dx = m\int_{x_1}^{x_2} \frac{dv_x}{dt} dx = m\int_{v_1}^{v_2} v_x\, dv_x$$

$$\Rightarrow \quad W = m\left[\frac{v^2}{2}\right]_{v_1}^{v_2} \quad \Rightarrow \quad W = \frac{1}{2}m\, v_2^2 - \frac{1}{2}m\, v_1^2$$

$$\Rightarrow \quad W = K_2 - K_1 = \Delta K$$

Thus work on a body moving in a straight line is the change in kinetic energy. This is wok-kinetic energy theorem.

Power (P)

- Power is the time rate at which work is done.
- Average power is $P_{av} = \frac{\Delta W}{\Delta t}$
- Instantaneous power is $P = \frac{dW}{dt}$
- $P = \vec{F} \cdot \vec{v}$
- SI unit of power is watt (1 Watt = 1 J/s)
- Bigger unit of power is horsepower (1 hp = 746 W)
- The kilowatt-hour (kwh) is the usual commercial unit of electrical energy.
- 1 kW h = (103 J/s)(3600 s) = 3.6 x 10^6 J = 3.6 MJ

Test your Understanding (TYU)

Test Your Understanding of Section 6.1:

An electron moves in a straight line toward the east with a ***constant speed*** of 8 x 10^7 m/s. It has electric, magnetic, and gravitational forces acting on it. During a 1-m displacement, the total work done on the electron is

(i) positive; (ii) negative;

(iii) zero; (iv) not enough information given to decide.

Answer: (iii)

The electron has constant velocity, so its acceleration is zero.

The net force F_{net} on the electron is also zero. (by Newton's second law)

Therefore the total work done by all the forces = $F_{net} \cdot x = 0 \cdot x = 0$

The individual forces may do nonzero work.

Test Your Understanding of Section 6.2:

Rank the following bodies in order of their kinetic energy, from least to greatest.

(i) a 2.0-kg body moving at 5.0 m/s;

(ii) a 1.0-kg body that initially was at rest and then had 30 J of work done on it;

(iii) a 1.0-kg body that initially was moving at 4.0 m/s and then had 20 J of work done on it;

(iv) a 2.0-kg body that initially was moving at 10 m/s and then did 80 J of work on another body.

Answer: (iv), (i), (iii), (ii)

(i) m = 2 kg, v = 5 m/s. Kinetic energy, $K = \frac{1}{2} m v^2 = \frac{1}{2}(2\text{kg})(5\,\text{m/s})^2 = 25\,\text{J}$

(ii) m = 1 kg, v_{0x}= 0, W = 30 J. $K_2 = W + K_1 =$ 30J + 0 = 30J

(iii) m = 1 kg, v_{0x}= 4m/s, W = 30 J. $K_2 = W + K_1 =$ 20J + (½)mv^2_{0x} = 20J + 8J = 28 J

(iii) m = 2 kg, v_{0x}= 10m/s, K_1 = (½)mv^2_{0x} = 100 J

When the body did 80 J of work on another body, the other body did (- 80 J) of work on 1st body.

So, K_1 = 80 J. $K_2 = K_1 + W$ = 100J + (- 80 J) = 20J

In-class problem (Example Problems): 6.2, 6.3, 6.10

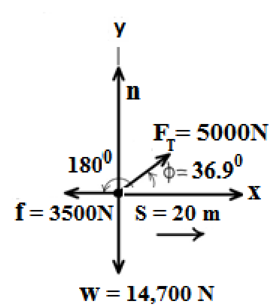
Examle-6.2: Work done by several forces

A farmer hitches her tractor to a sled loaded with firewood and pulls it a distance of 20 m along level ground (shown in the figure). The total weight of sled and load is 14,700 N. The tractor exerts a constant 5000-N force at an angle 36.9^0 of above the horizontal. A 3500-N friction force opposes the sled's motion. Find the work done by each force acting on the sled and the total work done by all the forces

Solution:

Forces acting on the sled along with the load are shown in the free body diagram.

- Force exerted by the tractor (F_T) at an angle 36.9^0 of above the horizontal
- Friction force (F_f) acting on the sled
- Weight (w) of sled and load
- Normal reaction (n) on the sled.



The works done by each force acting on the sled are:

Work W_T done by F_T is

$W_T = F_T\, s \cos\phi$ = (5000 N) x (120 m) x (0.80) = 80,000 N . m = 80 kJ

Work W_S done by F_S is

$W_S = F_S\, s \cos\phi$ = (3500 N) x (120 m) x (-1) = - 70,000 N . m = - 70 kJ

Work W_w done by weight (w) is

$W_w = w\ s\ \cos 90^0 = w\ s\ x\ (0) = 0$

Work W_n done by normal reaction (n) is

$W_w = w\ s\ \cos 90^0 = w\ s\ x\ (0) = 0$

The total work W_{tot} done on the sled by all forces is the algebraic sum of the work done by the individual forces:

$W_{tot} = W_T + W_S + 0 + 0 = 80\ kJ + (-\ 70\ kJ) = 10\ kJ$

Alternative method:

Net force exerted by the tractor is

$F = (5000N)\cos(36.9^0) - (3500N) = 500N$

Work done W_T by the force exerted by the tractor is

$W_T = F\ s = (500N)(20m) = 10000J$

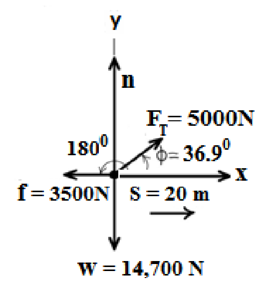
Examle-6.3:

A farmer hitches her tractor to a sled loaded with firewood and pulls it a distance of 20 m along level ground. The total weight of sled and load is 14,700 N. The tractor exerts a constant 5000-N force at an angle of 36.9 degrees above the horizontal. A 3500-N friction force opposes the sled's motion. Suppose sled moves at 2 m/s; what is speed after 20 m?

Solution:

Mass of sled and load is given by

$$m = \frac{w}{g} = \frac{147000\ N}{9.8\ m/s^2} = 1500\ kg$$



The initial kinetic energy K_1 is

$$K_1 = \frac{1}{2} m v_1^2 = \frac{1}{2}(1500\ kg)(2.0\ m/s)^2 = 3000\ J$$

The final kinetic energy K_1 is

$$K_2 = \frac{1}{2} m v_2^2 = \frac{1}{2}(1500\ kg) v_2^2 = 750\ v_2^2\ J$$

Net force exerted by the tractor is

$F = (5000N)\cos(36.9^0) - (3500N) = 500N$

Work done W_T by the force exerted by the tractor is

$W_T = F\ s = (500N)(20m) = 10000J$

The work–energy theorem is

$$W_{tot} = K_2 - K_1 = (750\ kg) v_2^2 - 3000\ J$$

$\Rightarrow$ $10000\ J = (750\ kg) v_2^2 - 3000\ J \Rightarrow (750\ kg) v_2^2 = 7000\ J \quad v_2 = 4.2\ m/s$

Example:6.10

A 50.0-kg marathon runner runs up the stairs to the top of Chicago's 443-m-tall Willis Tower, the tallest building in the United States. To lift herself to the top in 15.0 minutes, what must be her average power output? Express your answer in watts, in kilowatts, and in horsepower.

Solution:

Work done in lifting a mass 'm' against gravity to a height 'h' is

$W = m\ g\ h = (50.0\ kg)(9.80\ m/s^2)(443\ m) = 2.17\ x\ 10^5\ J$

Average power in a time 15.0 min = 900 s, is

$$P = \frac{W}{t} = \frac{2.17\ x\ 10^5\ J}{900\ s} = 241W = 0.241\ kW = 0.323\ hp$$