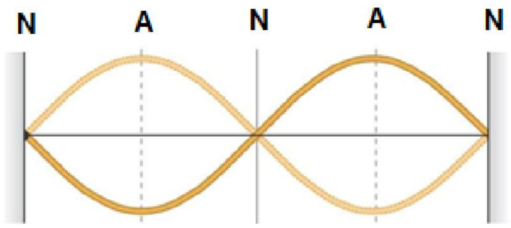
Chapter 15
Mechanical Waves

Topic	Conceptual Problems (To be Discussed in Class)	In Class Problems	Assignment Problems
15.7 Standing waves on a string 15.8 Normal modes of a string	TYU- 15.7	Example- 15.7	15.37, 15.40

15.7 Standing waves on a string:

Waves traveling in opposite directions on a taut string interfere with each other. The resulting wave is called the standing wave which does not move on the string.
The standing waveform must have nodes at both ends. Destructive interference occurs where the wave displacements cancel, and constructive interference occurs where the displacements add. At the nodes no motion occurs, and at the antinodes the amplitude of the motion is greatest.



N = Nodes: points at which the string never moves
A = Antinodes: points at which the amplitude of string motion is greatest

Analytical treatment:
When two sinusoidal waves coming from opposite directions superimposed upon each other standing waves are formed.

The waves can be written as:

$y_1 = A\cos(kx - \omega t)$,

$y_2 = A\cos(kx + \omega t)$,

The standing wave is the superposition of the above two waves. i.e

$y = y_1 + (-y_2)$

$y = A\cos(kx - \omega t) - A\cos(kx + \omega t)$

$y = A\,[2\sin kx \ \sin\omega t]$

$y = [2A \ \sin kx]\ \sin\omega t$

$y = A_{sw}\sin\omega t$

Where, $A_{sw} = 2A \ \sin kx$ = Amplitude of the standing wave

Nodes : Minimum amplitude position of a standing wave are called as nodes
For nodes we must have
$A_{sw} = 2A \ \sin kx$ = minimum
$\sin kx$ = minimum => $\sin kx = 0 = \sin n\pi$

$$kx = n\pi \quad => \quad \frac{2\pi}{\lambda}x = n\pi \quad \Rightarrow \quad x = n\left(\frac{\lambda}{2}\right)$$

Thus nodes points from the origin are

$$x = 0,\ \frac{\lambda}{2},\ 2\left(\frac{\lambda}{2}\right),\ 3\left(\frac{\lambda}{2}\right),........$$

Antinodes: Maximum amplitude position of a standing wave are called as antinodes

For antinodes we must have

$A_{sw} = 2A \sin kx = \text{maximum}$

$\sin kx = \text{maximum} \quad \Rightarrow \quad \sin kx = \pm 1 = \sin (2n+1)\pi/2$

$$\Rightarrow \quad kx = (2n+1)\frac{\pi}{2}$$

$$\Rightarrow \quad \frac{2\pi}{\lambda} x = (2n+1)\frac{\pi}{2} \Rightarrow \quad x = (2n+1)\left(\frac{\lambda}{4}\right)$$

Thus antinodes points from the origin are

$$x = \frac{\lambda}{4}, \quad 3\left(\frac{\lambda}{4}\right), \quad 5\left(\frac{\lambda}{4}\right), \ldots\ldots$$

Adjacent **nodes** distance = Adjacent **antinodes** distance = $\lambda/2$

A standing wave, unlike a traveling wave, does not transfer energy from one end to the other. The two waves that form it would individually carry equal amounts of power in opposite directions. There is a local flow of energy from each node to the adjacent antinodes and back, but the average rate of energy transfer is zero at every point.

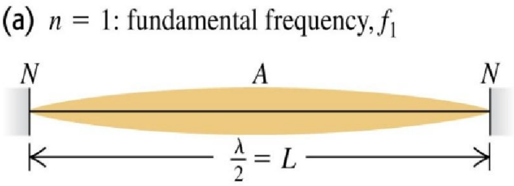
15.8 Normal modes of a string

Let's consider a string of length L, rigidly fixed at both ends.

Particle situated at two ends of the string has east scope for vibration. Therefore we get node permanently at two ends of the string.

(a) **Fundamental mode of vibration**

The minimum frequency that can be produced is called the fundamental frequency or 1st harmonics.
In fundamental mode of vibration number of nodes are **two** and **one** Antinode between the two nodes.
It is shown in the figure.
From the figure it is clear that

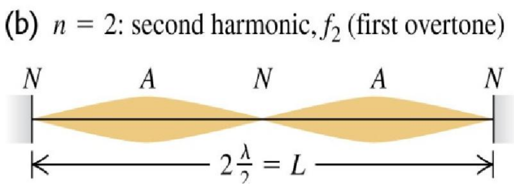


$$\lambda_1 = 2L \Rightarrow \quad L = \frac{\lambda_1}{2}$$

Frequency is given by

$$f_1 = \frac{v}{\lambda_1} = \frac{v}{2L}$$

(b) **Second Harmonics (1st Overtone)**



Next mode of vibration is that where number of nodes are **three (3)** and antinodes are **two (2)**.
It is shown in the figure.
From the figure it is clear that

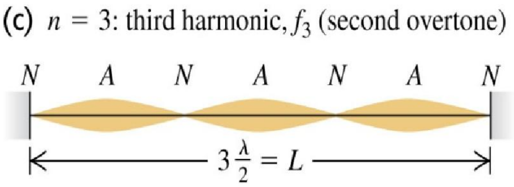
$$\frac{2\lambda_1}{2} = L \quad \Rightarrow \quad \lambda_1 = L$$

Frequency is given by

$$f_2 = \frac{v}{\lambda_2} = \frac{v}{L} = 2\left(\frac{v}{2L}\right) = 2f_1$$

Here the frequency is called 2^{nd} Harmonics or 1^{st} Overtone.

(c) Third Harmonics (2^{nd} Overtone)



Next mode of vibration is that where number of nodes are **four (4)** and antinodes are **three (3)**.
It is shown in the figure.
From the figure it is clear that

$$\frac{3\lambda_3}{2} = L \quad \Rightarrow \quad \lambda_3 = \frac{2L}{3}$$

Frequency is given by

$$f_3 = \frac{v}{\lambda_3} = \frac{v}{2L/3} = 3\left(\frac{v}{2L}\right) = 3f_1$$

Here the frequency is called 3^{rd} Harmonics or 2^{nd} Overtone.

From the above we can write the general form as below.

- For a taut string fixed at both ends, the possible wavelengths are

$$\lambda_n = \frac{2L}{n}, \qquad \text{where } n = 1, 2, 3, \ldots$$

$$f_n = \frac{v}{\lambda_n} = n\left(\frac{v}{2L}\right) = n\, f_1 \qquad \text{where } n = 1, 2, 3, \ldots$$

Test Your Understanding of Section 15.7

Suppose the frequency of the standing wave in Example 15.6 were doubled from 440 Hz to 880 Hz. Would all of the nodes for f = 440 Hz also be nodes for f = 880 Hz?. If so, would there be additional nodes for f = 880 Hz? If not, which nodes are absent for f = 880 Hz?

Answer:

Doubling the frequency makes the wavelength half as large. Hence the spacing between nodes (equal to $\lambda/2$) is also half as large. There are nodes at all of the previous positions, but there is also a new node between every pair of old nodes.

Example- 15.7: A giant bass viol

In an attempt to get your name in Guinness World Records, you build a bass viol with strings of length 5.00 m between fixed points. One string, with linear mass density is 40 g/m, is tuned to a 20.0-Hz fundamental frequency (the lowest frequency that the human ear can hear). Calculate (a) the tension of this string, (b) the frequency and wavelength on the string of the second harmonic, and (c) the frequency and wavelength on the string of the second overtone.

Solution:

$f_1 = 20.0$ Hz, $L = 5.0$ m, $\mu = 40.0$ g/m

a) The string tension F is given by

$$F = 4\,\mu\,L^2\,f_1^2 = 4\left(40.0 \times 10^{-3}\ \text{kg/m}\right)\left(5.0\ \text{m}\right)^2\ \left(20.0\ \text{s}^{-1}\right)^2 = 1600\ \text{N}$$

b) The frequency and wavelength of the second harmonic (n=2) are

$$f_2 = 2\ f_1 = 2\ (20.0\ \text{Hz}) = 40.0\ \text{Hz}$$

$$\lambda_2 = \frac{2L}{2} = \frac{2(5.0\ \text{m})}{2} = 5.0\ \text{m}$$

b) The second overtone is the third harmonics, n=3
Its frequency and wavelength are

$$f_3 = 3\ f_1 = 3\ (20.0\ \text{Hz}) = 60.0\ \text{Hz}$$

$$\lambda_3 = \frac{2L}{3} = \frac{2(5.0\ \text{m})}{3} = 3.33\ \text{m}$$

Assignment problem: 15.37, 15.40

Exercise-15.37

Standing waves on a wire are described by $y(x,t) = (A_{sw}\sin kx)\sin\omega t$, with $A_{sw} = 2.5\ \text{mm}, \omega = 942\ \text{rad/s}$ and $k = 0.75\pi\ \text{rad/m}$. The left end of the wire is at $x = 0$. At what distances from the left end are (a) the nodes of the standing wave and (b) the antinodes of the standing wave?

Solution:

$A_{sw} = 2.5\ \text{mm},\ \omega = 942\ \text{rad/s}$ and $k = 0.75\pi\ \text{rad/m}$.

(a) At a node $y = 0$ for all t. This requires that

$\sin kx = 0$

$\Rightarrow kx = n\pi,\ n = 0, 1, 2, \ldots$

$$\Rightarrow x = \frac{n\pi}{k} = \frac{n\pi}{0.75\pi\ \text{rad/m}} = (1.33\text{m})\ n,\ \ n = 0,1,2...$$

(b) At a antinodes $y = \text{maximum}$ for all t. This requires that

$\sin kx = \pm 1$

$$\Rightarrow kx = (2n+1)\frac{\pi}{2}$$

$$\Rightarrow x = (2n+1)\frac{\pi}{2k} = \left(n + \frac{1}{2}\right)\frac{\pi}{0.75\pi\ \text{rad/m}} = (1.33\text{m})\left(n + \frac{1}{2}\right),\ \ n = 0,1,2...$$

Exercise-15.40

A 1.5 m long rope is stretched between two supports with a tension that makes the speed of transverse waves 48 m/s What are the wavelength and frequency of

a) the fundamental;
b) the second overtone;
c) the fourth harmonic?

Solution: $L = 1.5\ \text{m},\ \ v = 48\ \text{m/s}$

a) For a string fixed at both ends, the wavelength of the standing wave is given by

$$\lambda_n = \frac{2L}{n}$$

and the frequency is given by $f_n = \dfrac{v}{\lambda_n} = n\left(\dfrac{v}{2L}\right)$

For the fundamental, n =1,

So, $\lambda_1 = \dfrac{2L}{1} = 2(1.5\text{m}) = 3.0\text{m}$ and $f_1 = \dfrac{v}{2L} = \dfrac{48 \text{ m/s}}{2(1.5\text{m})} = 16 \text{ Hz}$

b) For the second overtone, n = 3.

So, $\lambda_3 = \dfrac{2L}{3} = 1.0\text{m}$ and $f_3 = 3\,x\,(16 \text{ Hz}) = 48 \text{ Hz}$

c) For the fourth harmonic, n = 4.

So, $\lambda_4 = \dfrac{2L}{4} = 0.75\text{m}$ and $f_4 = 4\,x\,(16 \text{ Hz}) = 64 \text{ Hz}$