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**Project Report**

**EE305- Digital Signal Processing**

***IMAGE PROCESSING***

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## **CONTENT**

<b>S. no.</b>	<b>Topic name</b>	<b>Page no.</b>
<b>1.</b>	Adaptive Median filter	<b>4</b>
<b>2.</b>	Adaptive Riesz Mean Filter	<b>6</b>
<b>3.</b>	Different Adaptive Modified Riesz Mean Filter	<b>9</b>
<b>4.</b>	Alpha Trimmed Mean Filter	<b>11</b>
<b>5.</b>	Geometric Mean Filter for Image Denoising	<b>13</b>
<b>6.</b>	Sector Rotation Filter	<b>15</b>
<b>7.</b>	Modified Median Filter	<b>19</b>
<b>8.</b>	Recursive Spline Interpolation Filter	<b>21</b>
<b>9.</b>	Basic Mean Filter	<b>23</b>
<b>10.</b>	Min/Max Filters	<b>25</b>
<b>11.</b>	Basic Median Filter	<b>27</b>
<b>12.</b>	Result	<b>29</b>
<b>13.</b>	Conclusion	<b>38</b>
<b>14.</b>	Reference	<b>46</b>

## PARAMETERS USED

Filters	Parameters
<b>Adaptive Median Filter</b>	$A_{ij}^k = k$ – Symmetric pad matrix $t = \max$ window size( 9 )
<b>Adaptive Riesz Mean Filter</b>	$[b_{ij}]_{m \times n} =$ Binary Matrix $A_{ij}^k = k$ – Symmetric pad matrix $ps(a_{ij}, a_{st}) =$ Pixel Singularity
<b>Different Adaptive Modified Riesz Mean Filter</b>	$[b_{ij}]_{m \times n} =$ Binary Matrix $A_{ij}^k = k$ – Symmetric pad matrix $pw(a_{ij}, a_{st}) =$ Pixel Weight
<b>Alpha Trimmed Mean Filter</b>	$(d, \alpha) =$ Item trimming parameters $(\alpha = d/2)$ $d = (1 - mn)$ $(m, n) =$ Window dimensions
<b>Geometric Mean Filter for Image Denoising</b>	$(m, n) =$ Window Dimensions
<b>Sector Rotation Filter</b>	$m =$ Square window Size
<b>Modified Median Filter</b>	$[d_{ij}]_{m \times n} =$ Binary Matrix of noisy image
<b>Recursive Spline Interpolation Filter</b>	$m =$ Square window size
<b>Basic Mean Filter</b>	No Such Parameters
<b>Min/Max Filters</b>	No Such Parameters
<b>Basic Median Filter</b>	No Such Parameters

# Adaptive Median Filter

*Non-linear spatial filter*

## INTRODUCTION

It uses median filtering to remove impulsive noise. The window used for filtering here is adaptive i.e it starts from a window size of  $3 \times 3$  (for  $k=1$ ) and increases the window size if the adaptivity conditions are not met.

A window matrix is defined for filtering as  $A_{ij}^k$  of size  $(2k+1) \times (2k+1)$ , where  $i$  and  $j$  are the indices of the pixel  $a_{ij}$  being processed and  $k$  is the number of pixels taken around it.

$$A_{ij}^k = \begin{bmatrix} a_{(i-k)(j-k)} & \cdots & a_{(i-k)(j+k)} \\ \vdots & a_{ij} & \vdots \\ a_{(i+k)(j-k)} & \cdots & a_{(i+k)(j+k)} \end{bmatrix}_{(2k+1) \times (2k+1)}$$

The adaptivity condition used here checks if:

a)  $a_{ij} = \max(A_{ij}^k)$  **OR**  $a_{ij} = \min(A_{ij}^k)$

This checks if the center pixel is noisy and needs processing by comparing it to maximum and minimum of the window. If it is impulsive the algorithm moves on to the next check. If it is not impulsive then pixel does not need filtering and the moves on to the next pixel.

b)  $\min(A_{ij}^k) < \text{median}(A_{ij}^k) < \max(A_{ij}^k)$

This checks if the median value is impulsive or not. If it is impulsive, then the filter increases the window size and checks again. If it is not impulsive then the filter replaces the center pixel with the median.

## ALGORITHM: -

**Input:** noisy matrix,  $A := [a_{ij}]_{m \times n}$     **Output:** Denoised matrix,  $A := [a_{ij}]_{m \times n}$

### Algorithm :-

Initialize  $t = 9$

Compute  $\bar{A}_t$

**For all  $i$  and  $j$**

**If**  $a_{ij} = \max(A_{ij}^k)$  **OR**  $a_{ij} = \min(A_{ij}^k)$

**For**  $k$  from 1 to  $t$

**If**  $\min(A_{ij}^k) < \text{median}(A_{ij}^k)$  **AND**  $\text{median}(A_{ij}^k) < \max(A_{ij}^k)$

$a_{ij} \leftarrow \text{median}(A_{ij}^k)$

**Break**

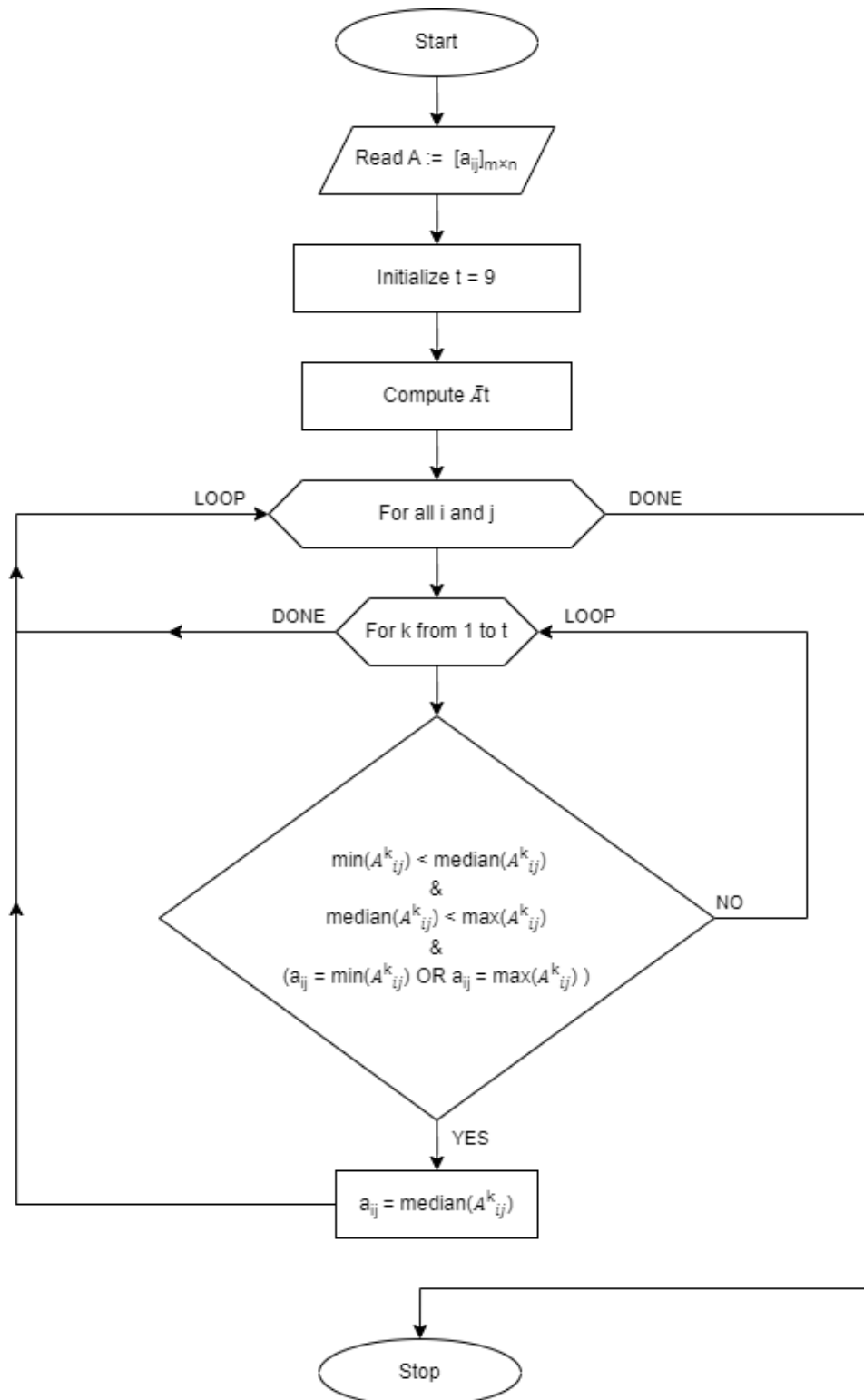
End if

End for

End if

End for

FLOWCHART: -



$A^k_{ij}$  = k – Symmetric pad matrix

# **Adaptive Riesz Mean Filter**

*Non-linear spatial filter*

## **INTRODUCTION**

In this section, two linear spatial window filters are introduced that are designed for high density impulse noise the Adaptive Riesz mean Filter (ARmF) and the Different Adaptive Modified Riesz mean Filter (DAMRmF). They work on pixels of the image that have extreme values (i.e. 0 or 255).

The ARmF calculates a Riesz mean of the mask window based on the pixel similarity of each pixel and replaces the noisy pixel. It has an adaptivity condition for a completely noisy window to prevent error in the results.

The DAMRmF works similar to the ARmF but uses a modified version of the Riesz Mean in its calculations and employs an adaptivity condition based on the Adaptive Median Filter.

## **DETAIL**

### **Adaptive Riesz mean Filter**

To apply filtering to the pixels at the edge of the image, first the matrix were symmetrically padded with the required number of pixels. The padded image matrix for an image  $A := [a_{ij}]_{m \times n}$  is represented by  $\bar{\bar{A}}_t$  where  $t$  is the number of pixels padded around the image.

To process the image, the noisy pixels have to be determined. A binary matrix for that purpose was defined. The binary matrix of image  $A$  is defined as  $B := [b_{ij}]_{m \times n}$  where,

$$b_{ij} = \begin{cases} 0, & \text{if } a_{ij} \text{ is a noisy entry of } A \\ 1, & \text{otherwise} \end{cases}$$

A window matrix was defined for filtering as  $A_{ij}^k$  of size  $(2k+1) \times (2k+1)$ , where  $i$  and  $j$  are the indices of the pixel  $a_{ij}$  being processed and  $k$  is the number of pixels taken around it.

$$A_{ij}^k = \begin{bmatrix} a_{(i-k)(j-k)} & \cdots & a_{(i-k)(j+k)} \\ \vdots & a_{ij} & \vdots \\ a_{(i+k)(j-k)} & \cdots & a_{(i+k)(j+k)} \end{bmatrix}_{(2k+1) \times (2k+1)}$$

Adaptive Riesz mean Filter (ARmF) works with adaptive windowing i.e. it starts with a window size of  $3 \times 3$  (for  $k=1$ ) and increases the window size if all the entries in the window are considered noisy before processing it.

The filter works on the idea of a weighted mean filter in which the weight of each pixel is determined by the pixel similarity of the pixel to the center pixel. The pixel similarity between two pixels is defined as

$$ps(a_{ij}, a_{st}) = \left( \frac{1}{1 + |i - s| + |j - t|} \right)^2$$

For pixels that are closer to each other, the value of pixel similarity will be closer to 1. The mean calculated using this method is called the Riesz mean of the window.

$$\text{Riesz mean of } (A_{ij}^k) = \frac{\sum_{(s,t)} a_{st} \times ps(a_{(k+1)(k+1)}, a_{st})}{\sum_{(s,t)} ps(a_{(k+1)(k+1)}, a_{st})}$$

Here, (s,t) is the set of all s and t such that  $a_{st}$  is a non-noisy entry of  $A_{ij}^k$  and  $a_{(k+1)(k+1)}$  is the center pixel. The noisy center pixel is then replaced by the Riesz mean calculated this way.

#### ALGORITHM:

**Input:** noisy matrix,  $A := [a_{ij}]_{m \times n}$

**Output:** Denoised matrix,  $A := [a_{ij}]_{m \times n}$

Convert A from uint8 to double form

**For** t from 5 to 1

    Compute the binary matrix  $B := [b_{ij}]_{m \times n}$

    Compute  $\bar{A}_t$  and  $\bar{B}_t$

**For** all i and j

**If**  $b_{ij} = 0$

**For** k from 1 to t

**If**  $B_{ij}^k$  is not a zero matrix

$a_{ij} \leftarrow (\text{Riesz mean of } A_{ij}^k)$

**Break**

**End if**

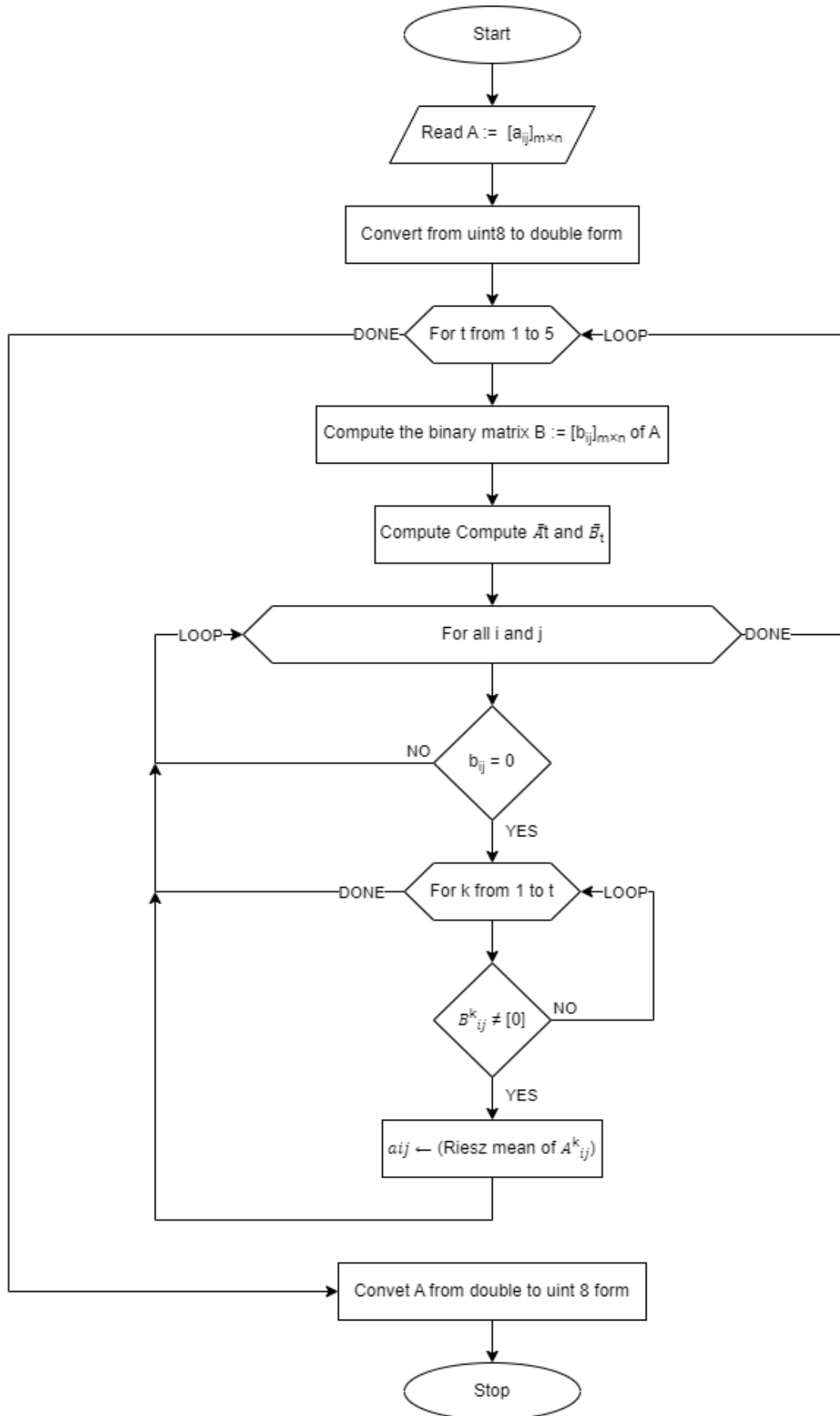
**End for**

**End if**

**End for**

**End for**

## FLOWCHART: -



$[b_{ij}]_{m \times n}$  = Binary Matrix  $A^k_{ij}$  =  $k$  – Symmetric pad matrix  $ps(a_{ij}, a_{st})$  = Pixel Singularity



## Different Adaptive Modified Riesz mean Filter:-

### *Non-linear spatial filter*

The Different Adaptive Modified Riesz mean Filter works similar to the Adaptive Riesz Mean Filter explained in the previous section. The initial padding and finding of the noisy pixels is the same as ARmF. The adaptive window condition in this filter first checks whether the median of the window is noisy or not. If median is noisy, the algorithm keeps moving on to a larger window around the noisy pixel until the median is found non-noisy or the maximum window size is reached. If median is not noisy, we replace the center pixel with the Modified Riesz mean of the window explained below. This filter also works on weighted mean principal but here the pixel similarity factor is changed with pixel weight defined below.

$$pw(a_{ij}, a_{st}) = \left( \frac{1}{1 + (i - s)^2 + (j - t)^2} \right)^2$$

So the weight of every pixel in the mean is decided by its pixel weight with respect to the center pixel of the window  $A_{ij}^k$ . This mean is called the Modified Riesz mean.

$$\text{Modified Riesz mean of } (A_{ij}^k) = \frac{\sum_{(s,t)} a_{st} \times pw(a_{(k+1)(k+1)}, a_{st})}{\sum_{(s,t)} pw(a_{(k+1)(k+1)}, a_{st})}$$

Here, (s,t) is the set of all s and t such that  $a_{st}$  is a non-noisy entry of  $A_{ij}^k$  and  $a_{(k+1)(k+1)}$  is the center pixel. The noisy center pixel is then replaced by the Modified Riesz mean calculated this way.

### **ALGORITHM: -**

**Input:** noisy matrix,  $A := [a_{ij}]_{m \times n}$     **Output:** Denoised matrix,  $A := [a_{ij}]_{m \times n}$

Convert A from uint8 to double form

**For** t from 5 to 1

    Compute the binary matrix  $B := [b_{ij}]_{m \times n}$

    Compute  $\bar{A}_t$  and  $\bar{B}_t$

**For** all i and j

**If**  $b_{ij} = 0$

**For** k from 1 to t

**If**  $0 < \text{median}(A_{ij}^k) \& \text{median}(A_{ij}^k) < 255$  **AND** ( $a_{ij} = 0$  **OR**  $a_{ij} = 255$ )

$a_{ij} \leftarrow (\text{Modified Riesz mean of } A_{ij}^k)$

**Break**

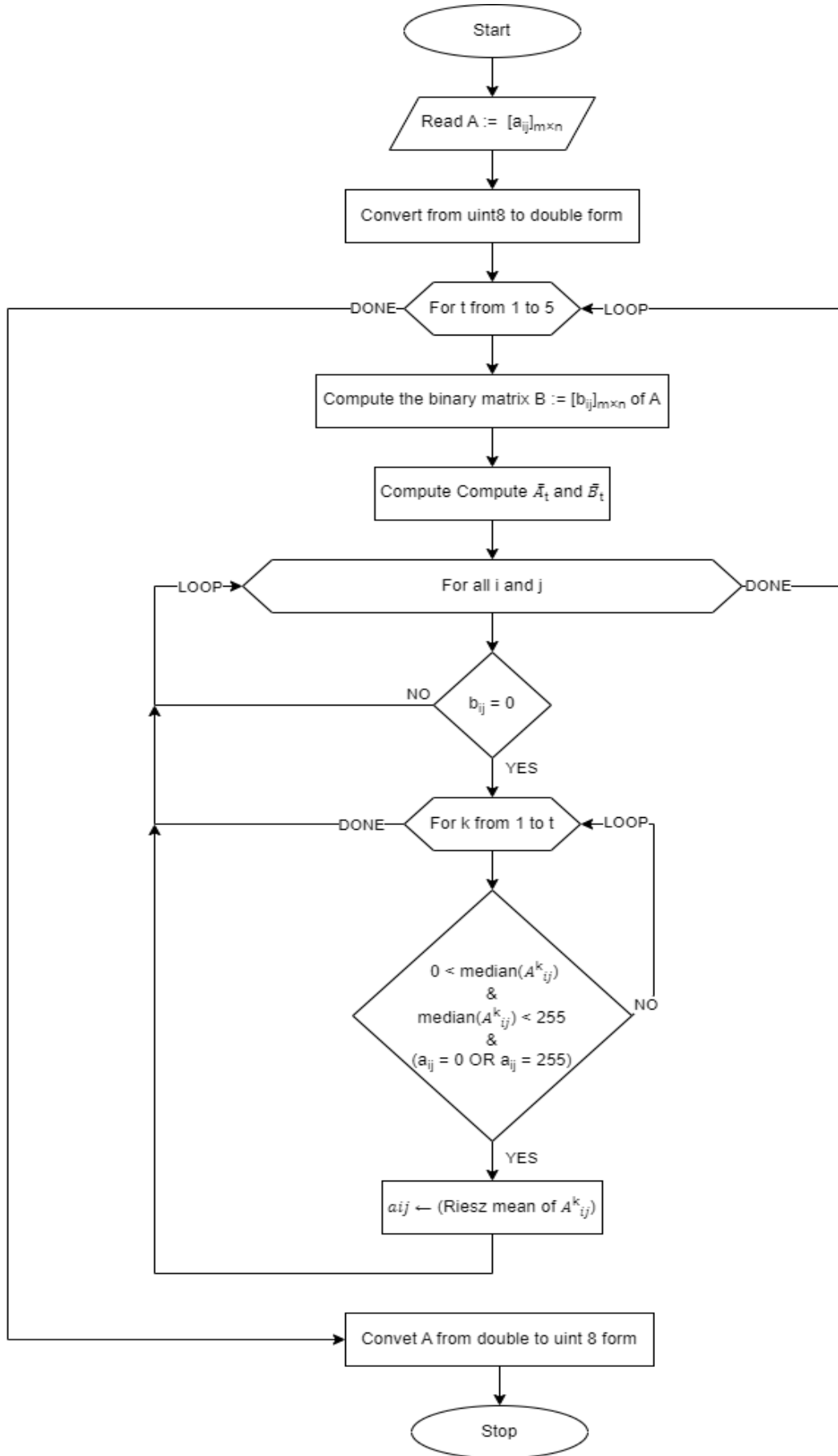
**End if**

**End for**

**End if**

**End for**

## FLOWCHART: -



$[b_{ij}]_{m \times n}$  = Binary Matrix  $A^k_{ij}$  =  $k$  - Symmetric pad matrix  $ps(a_{ij}, a_{st})$  = Pixel Singularity

## Alpha Trimmed Mean Filter for Image Denoising

### *Non-linear spatial filter*

#### **Introduction:**

It is nonlinear windowed filter that uses the median and mean filters. The idea behind this filter is to look at the area around each pixel and remove the pixels that has the highest and lowest value, and use the remaining elements to get the mean value. The alpha parameter controls how many elements are removed.

#### **Background:**

The idea is to sort the items, select the first and last element out of the sorted set, and then calculate the mean value using the remaining elements. For example below is the representation of how alpha trimmed mean is calculated

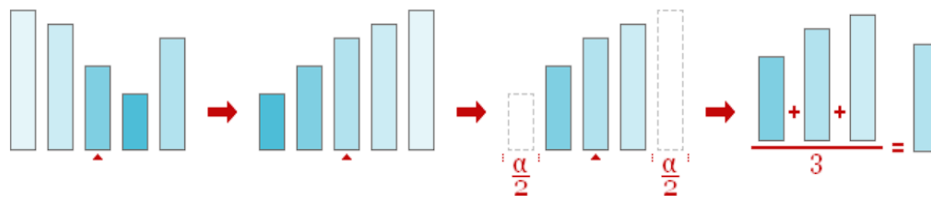


Fig. 1. Alpha-trimmed mean calculation.

To obtain an alpha-trimmed mean, items are arranged in the ascending/descending order and elements from first and last are removed from the collection, and then mean of the remaining elements is taken.

The formula used here to calculate alpha trimmed mean for a given noisy pixel which is to be replaced with the calculated value:

$$f(x,y) = \sum_{(s,t) \in S_{x,y}} g(s,t)$$

#### **Where:**

**m,n** : Window dimensions

**d**: This parameter is used for calculating alpha trimmed mean which is equal to  $2\alpha$

and  $\alpha$  is the  $d/2$  highest and  $d/2$  lowest intensity values of the window array.

$\sum g_r(s,t)$ : Summation of the remaining elements of the window array for finding the mean.

A few remarks concerning the **alpha parameter**, which controls item trimming. The number of elements that must be removed is referred to as alpha; in this filter, **d is 5**

which means  $\alpha$  is 2.5 but here  $\alpha$  is taken as 2 . As the filter is symmetric, alpha should be an even nonnegative integer smaller than the size of the filter window.

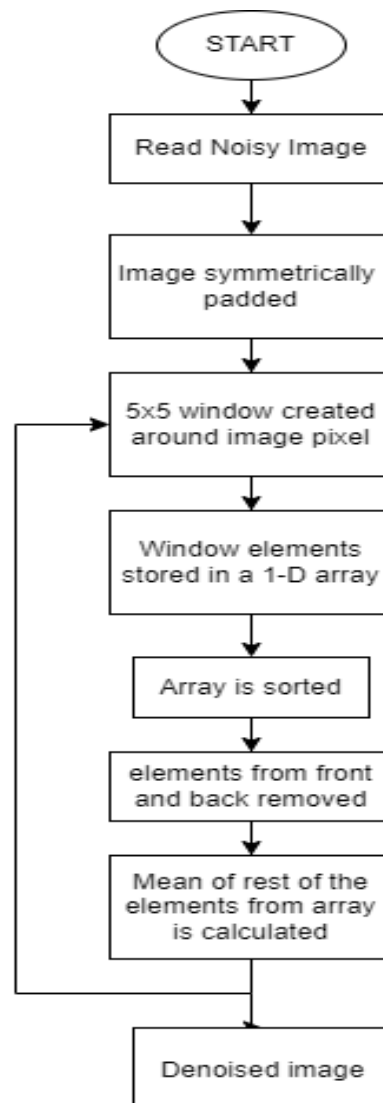
The alpha-trimmed mean filter degenerates into the mean filter when the alpha parameter's minimum value is 0. As alpha reaches its maximum value, which is equal to the filter window size minus one, the filter degenerates into a median filter.

### Algorithm:

1. Window is placed over each pixel.
2. form a 1-D window array.
3. Sort the elements in the window array.
4. Elements are removed from front and at the back of the arranged set.
5. Calculate the mean of the remaining elements of the window array

Note: The widow dimensions can be 3x3,5x5,7x7 according to which value of d and  $\alpha$  will change, but based on the observation 5X5 window gives the best result.

### Flow chart:



## Geometric Mean Filter for Image Denoising

### *Non-linear spatial filter*

#### **Introduction:**

It is a type of image filter used to filter noises and smooth out images. The idea is to find the geometric mean of the window and replace the element with geometric mean. The geometric mean's output picture,  $G(x,y)$ , is provided by

$$G(x,y) = [\prod_{i,j \in S} S(i,j)]^{1/mn}$$

Where:

**$G(x,y)$ :** Calculated Geometric Mean which will be replaced with the center pixel of the selected window.

**$m,n$ :** Window dimensions.

**$\prod(S(i,j))$ :** Product of Original Image Elements within the selected window.

#### **Background:**

Geometric mean of the elements present in the geometric mean window is calculated and the center pixel of the window is replaced with result. The pixel  $(x,y)$  in the output image, for example, product of all 8 of the surrounding pixels raised to the  $1/9$  power is calculated when the window size is set to 3 by 3.

For instance, given below is a 3x3 window of the center pixel:

5	16	22
6	3	18
12	3	15

Gives the result:  $(5*16*22*6*3*18*12*3*15)^{(1/9)} = 8.77$

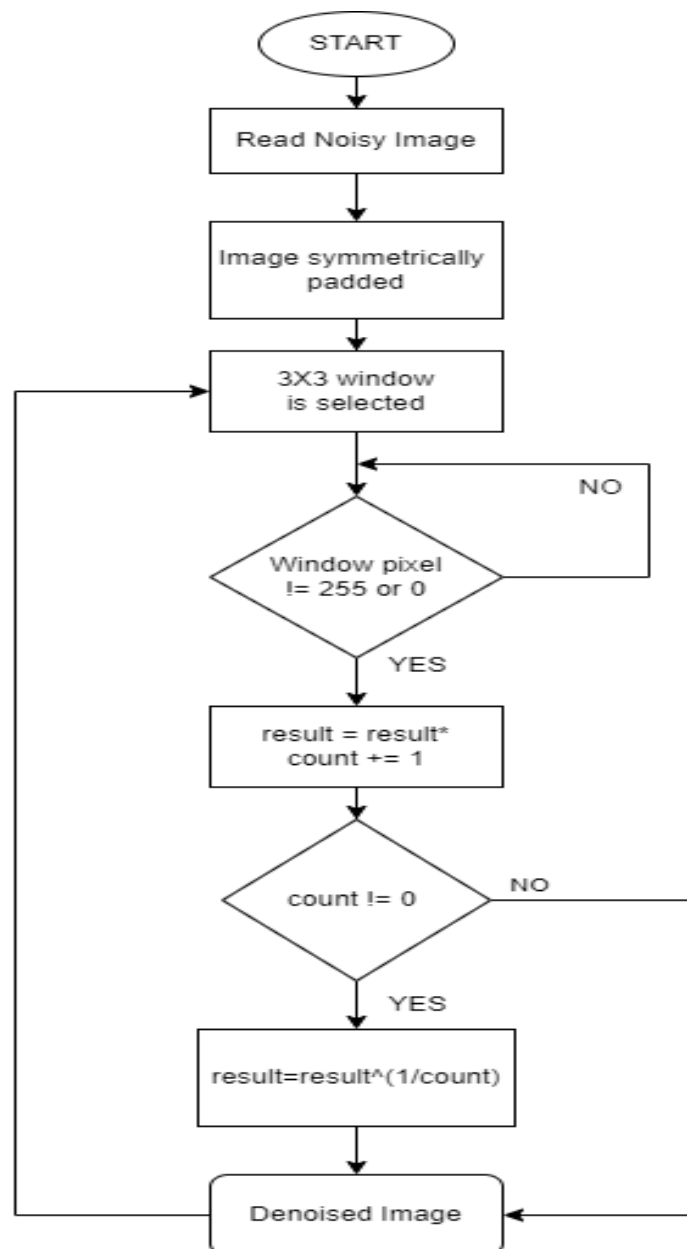
The centre element of the widow (i.e 3) will get replaced with the above calculated value i.e 8.77.

### Algorithm:

1. Read the noisy image.
2. Symmetrically pad the noisy image matrix.
3. Select the window size.
4. If window pixel  $\neq 255$  or  $0$  product of pixel value is stored in result array and element count is incremented.
5. Find the geometric mean of result array.
6. Replace the centre element of the window with calculated geometric mean.
7. Do it for all elements of the noisy image matrix.

**Note:** Noisy Image has been symmetrically padded for the smoothening of the edges. For a 3x3 window, padding of 1 unit should be provided .It increases with the increase in the window size.

### Flow chart:



## **Sector Rotation Filter**

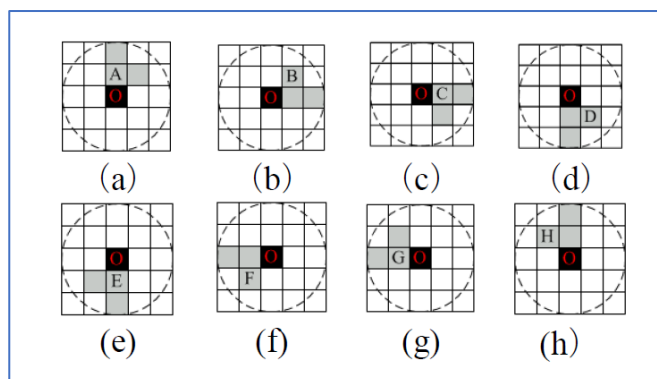
### *Non-linear spatial filter*

#### **INTRODUCTION: -**

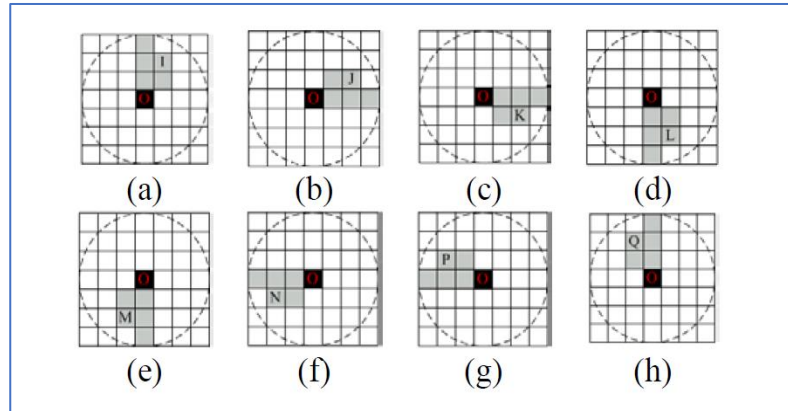
Image processing is the process of removing noise pollution which is caused due to several environmental and equipment factors of image acquisition. These noise pollution cause interference to image processing. Different noises are added to the image to check the working of the filter and comparison is being made with the help of certain parameters. Image denoising with the help of standard median filter somewhat fails to clarify the borders of a noisy image and thus resulting the image becoming blurrier. Adaptive median filtering technique, which is done from calculating and checking the weight of a pixel and perform certain mathematical operation on them, and then calculates the median and store it as the output. This methodology is a superior version of basic noise filtering technique, but the corners of certain change in weighted pixels remain in same noise condition with the inefficient time taken over the basic technique. Further on certain algorithms were developed and filtering effect on low density noise has been improved till some extent. Moreover, when there are more extreme points in the selected window, the value of blurred coefficient and fuzzy variables are still noisy, so the resultant picture obtained is still having relatively high noise. So, to fulfil this fault, an algorithm is developed to increase the window dimensions variably according to the noise found on the sector of the window selected of the image. Here, the value of center of the window in all possible dimensions is calculated to have the relation between the points with large difference and that are the ones taken in the result.

#### **IMPROVEMENT IN THE ALGORITHM: -**

For image with low noise window of smaller size would work significantly better than the one with large size. The rotation rule for a window of 5x5 window is shown below.



There are the 8 difference points considered according to the size of the window. The fan region with a small difference indicates a high difference relation with the pixel value of the non-noisy area. If all the difference points are found at corner or the higher value, then the noise percentage is higher, hence a window of significantly larger size is selected. The window of higher size is shown below.



If the same condition is found which lead to expansion of window size is the above condition, then the same would be applied here as well and a window of higher dimension will be selected.

Formulate the value of difference in the 8-difference point in the fan selected in the window.

$(x-2, y-2)$	$(x-2, y-1)$	$(x-2, y)$	$(x-2, y+1)$	$(x-2, y+2)$
$(x-1, y-2)$	$(x-1, y-1)$	$(x-1, y)$	$(x-1, y+1)$	$(x-1, y+2)$
$(x, y-2)$	$(x, y-1)$	O $(x, y)$	$(x, y+1)$	$(x, y+2)$
$(x+1, y-2)$	$(x+1, y-1)$	$(x+1, y)$	$(x+1, y+1)$	$(x+1, y+2)$
$(x+2, y-2)$	$(x+2, y-1)$	$(x+2, y)$	$(x+2, y+1)$	$(x+2, y+2)$

$$a(x-1, y) = 2*f(x-1, y) - f(x-2, y) - f(x-1, y+1)$$

$$b(x-1, y+1) = 2*f(x-1, y) - f(x-2, y) - f(x-1, y+1)$$

$$c(x, y+1) = 2*f(x-1, y) - f(x-2, y) - f(x-1, y+1)$$

$$d(x+1, y+1) = 2*f(x-1, y) - f(x-2, y) - f(x-1, y+1)$$

$$e(x+1, y) = 2*f(x-1, y) - f(x-2, y) - f(x-1, y+1)$$

$$f(x+1, y-1) = 2*f(x-1, y) - f(x-2, y) - f(x-1, y+1)$$

$$g(x, y-1) = 2*f(x-1, y) - f(x-2, y) - f(x-1, y+1)$$

$$h(x-1, y-1) = 2*f(x-1, y) - f(x-2, y) - f(x-1, y+1)$$

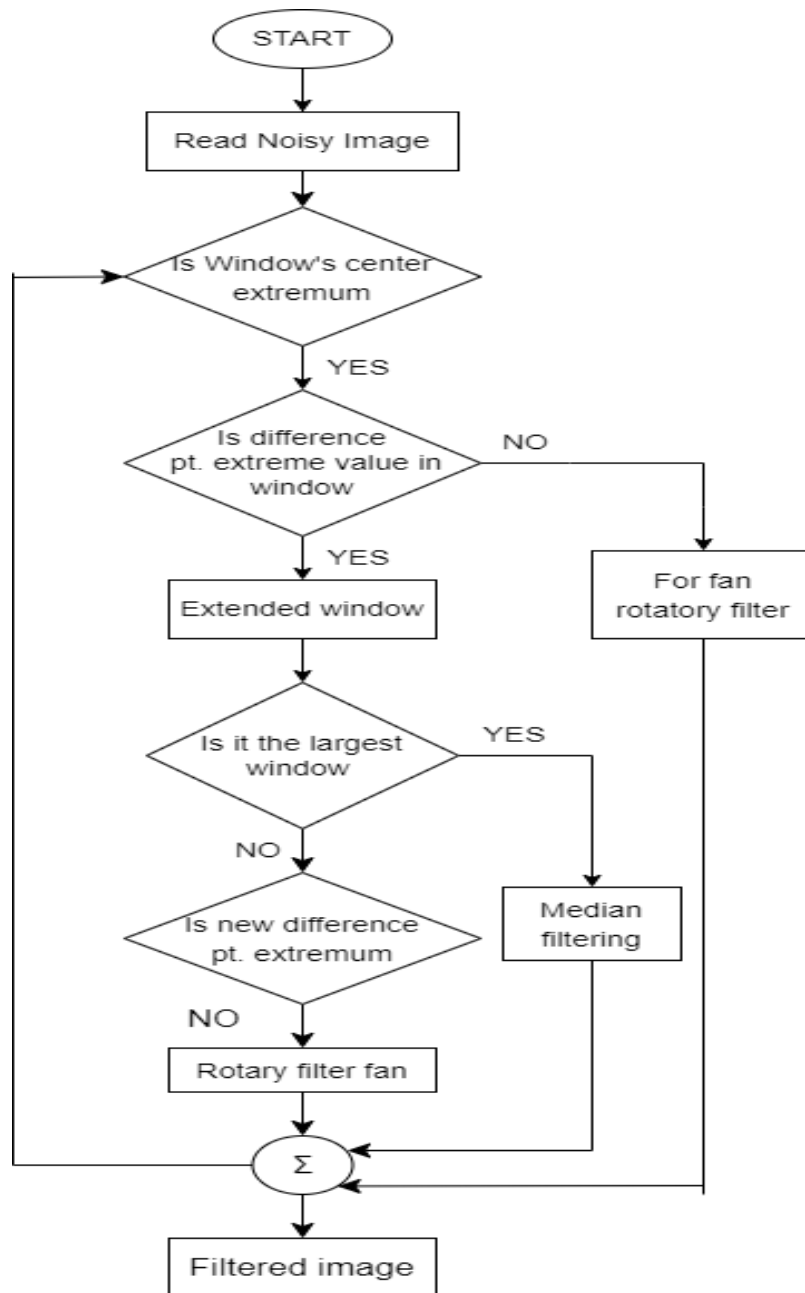
### ALGORITHM: -

1. Convert the input image to grayscale, if necessary.
2. Choose a center point for the rotation. This could be the center of the image or any other point of interest.
3. Define the size of the filter to determine the size of the sectors to be rotated.



4. For each pixel in the image:
  - a. Calculate the distance from the pixel to the center point.
  - b. Calculate the angle between the pixel and the center point.
  - c. Rotate the sector by an amount proportional to its distance from the center point.
  - d. Assign the rotated pixel value to the original pixel location in the output image.
5. For the maximum size of the window (above 13×13 window), the median of the window is directly calculated for output.
6. Add the pixel without filtering and the pixel points after applying operation to obtain output image.

**FLOWCHART: -**



# **Modified Median Filter**

*Non-linear spatial filter*

## **INTRODUCTION: -**

An image is a composition of squared pixels arranged in row and column. It is captured by capturing devices like camera or scanner and stored in the mass storage of the computer system. A captured image is subjected to many kinds of distortion during the stages of processing, storing, compressing, transmitting and others. During the transfer of the image, noise may be added along the actual information. The unwanted information that added along the required information is due to certain environmental variations and faulty locations during transfer. It is an improvement on the standard median filter, which is a commonly used technique for noise reduction. The modified median filter extends the concept of the median filter by using additional information about the local characteristics of the signal to improve noise reduction performance.

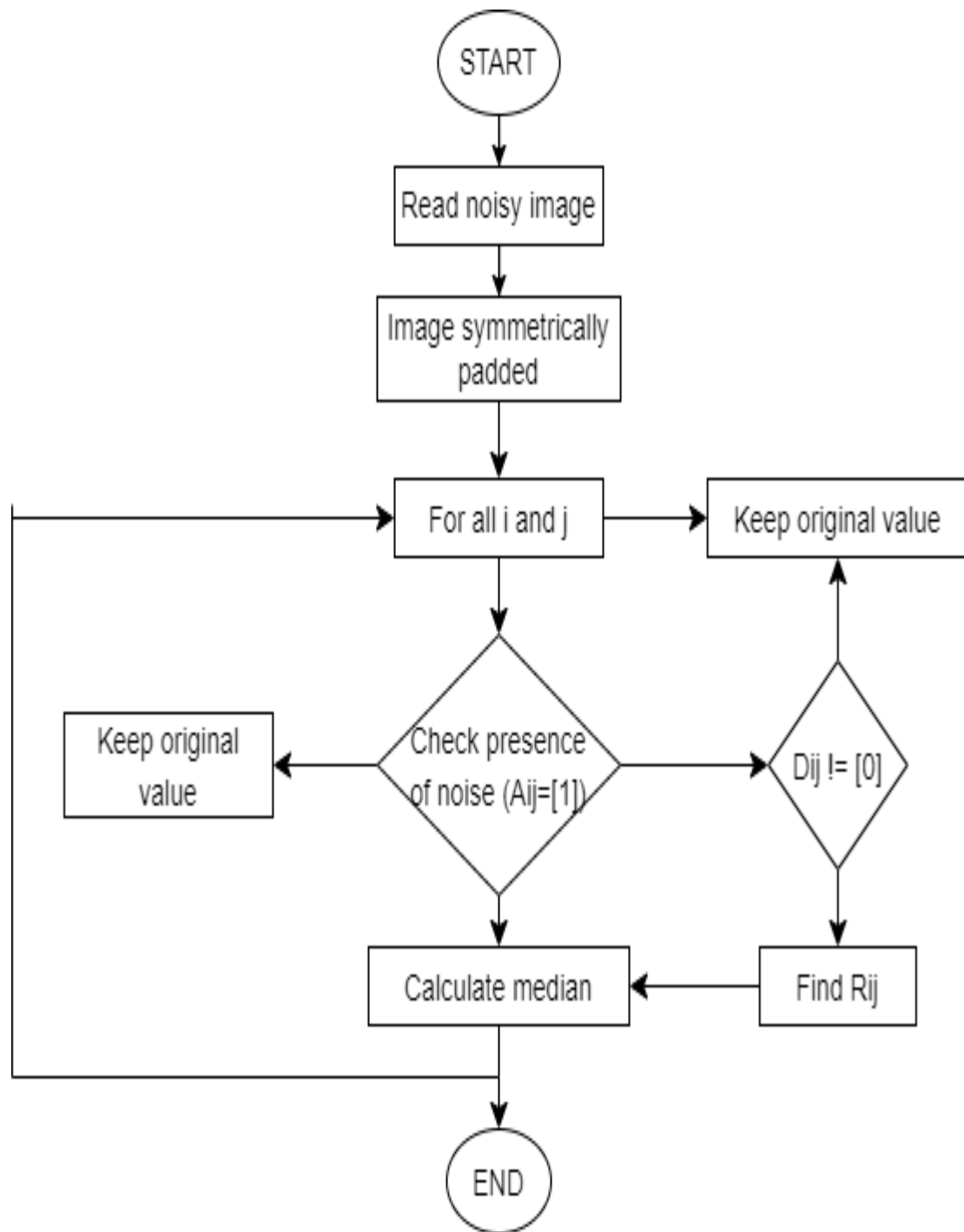
The modified median filter is particularly useful when dealing with non-Gaussian noise or when the signal is corrupted by large outliers. It works by examining the values of neighbouring pixels or samples in a signal and replacing the central pixel or sample with a value that is a function of the median of the neighbouring values. The exact function used to calculate the replacement value depends on the specific application and the nature of the noise present.

Overall, the modified median filter is a powerful tool for noise reduction in a wide range of applications, including image and audio processing, and can improve the quality and accuracy of signal analysis and interpretation

## **ALGORITHM: -**

1. Read the noise image matrix  $img = [a_{ij}]_{X \times Y}$ , where  $\{x, y\} > 1$ .
2. Obtain the binary matrix  $B = [d_{ij}]_{m \times n}$  of  $img$ .
3. Calculate and obtain the padded matrix.
4. For all  $i$  and  $j$ . If  $d_{ij}=1$ , then keep value of  $a_{ij}$  else  $B_{ij}!=0$  then
  - a) Find the value of  $R_{ij}$  for  $img_{ij}$ .
  - b) Evaluate the median of  $R_{ij}$ .
  - c) Replace this value to the  $c_{ij}$  else keep the value of  $a_{ij}$ .
5. Merge the median value and the original values together.

## FLOWCHART: -



# **Recursive Cubic Spline Interpolation Filter for Image Denoising**

*Non-linear spatial filter*

## **Introduction:**

Interpolation is defined as a technique to derive an equation with the help of given constant discrete points. For any higher order function defined over a range  $[x,y]$ , this method can be used to generate a lower order function. The more the data points are available to interpolate, the more accurate lower order function can be generated. The points/ data are connected through a smooth curve, which is being utilized by cubic spline interpolation filter.

## **Background:**

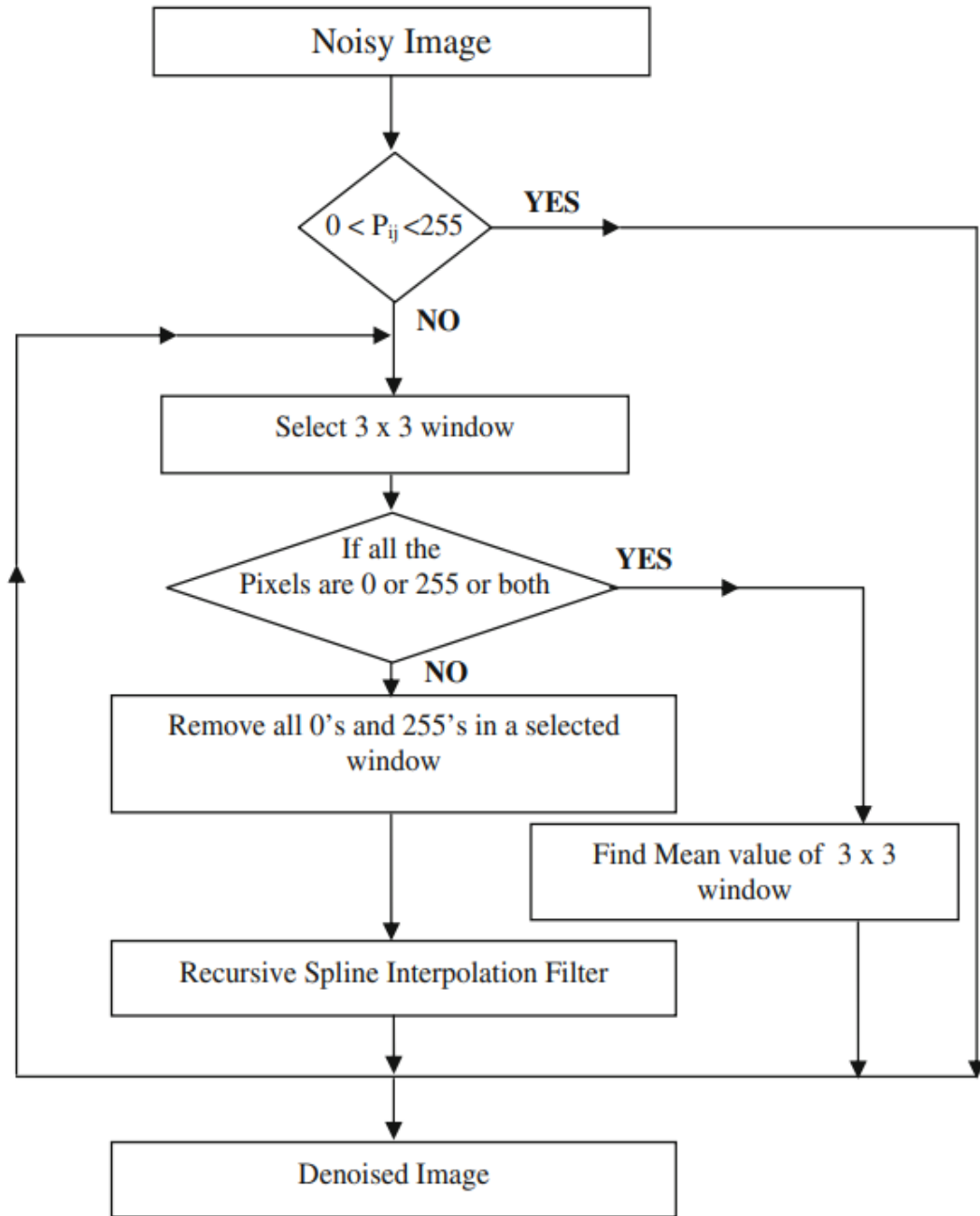
The filter is being used to remove Impulse noise. Basic idea behind this filter is to use the interpolation technique over the known noisy pixel. The filter would check each pixel to be either noise i.e. having a value of either 0 or 255, if it does it would take a 3x3 window around it, apply the interpolation method to give out the most accurate pixel value it is supposed to have. If the pixel is not found out to be noisy it would simply ignore it and move to next pixel.

## **Algorithm:**

1. Read the noisy image.
2. Symmetrically pad the noisy image matrix.
3. If the pixel is not noisy move to next pixel
4. If the pixel is noisy, Select a 3x3 window around the noisy image
  - a. If all the images are noisy in the 3x3 matrix take the mean value of the matrix
  - b. if not, ignore all the other noisy pixel present in the 3x3 matrix and apply the cubic spline interpolation on it
  - c. replace the generated value with the noisy pixel
5. Do it for all elements of the noisy image matrix.

Note: Noisy Image has been symmetrically padded for the smoothening of the edges. For a 3x3 window padding of 1 unit should be provided. It increases with the increase in the window size.

### Flow Chart:



❖  $P_{i,j}$  : It refers to the pixel value at location  $(i,j)$  of the matrix

# Basic Mean Filter for Image Denoising

## *Linear spatial filter*

### **Introduction:**

The basic mean filter is a type of image filter used to smear out noise and smooth out images. It is based on the algebraic mean in mathematics. The algebraic mean's output picture,  $f(x,y)$ , is provided by

$$f(x,y) = \frac{1}{mn} \sum_{(s,t) \in S_{x,y}} g(s,t)$$

Where:

**$f(x,y)$ :** Calculated Algebraic Mean which will be replaced with the center pixel of the selected window.

**$m,n$  :** Window dimensions.

**$\sum g(s,t)$ :** Sum of Original Image Elements within the selected window.

### **Background:**

The filter is defined to remove impulse noise. The filter iterates over symmetrically padded matrix, to check for pixels having a value impulse noise. If it does it would take a 3x3 window across it, find the mean of the window and replace the noisy pixel with the generated mean value. If the pixel is not found out to be noisy to be noisy it would simply move to the next pixel.

For instance, given below is a 3x3 window of the center pixel:

5	16	22
6	X	18
12	3	15

Here the centre pixel is noisy.

Replacing the centre pixel with the mean for the window

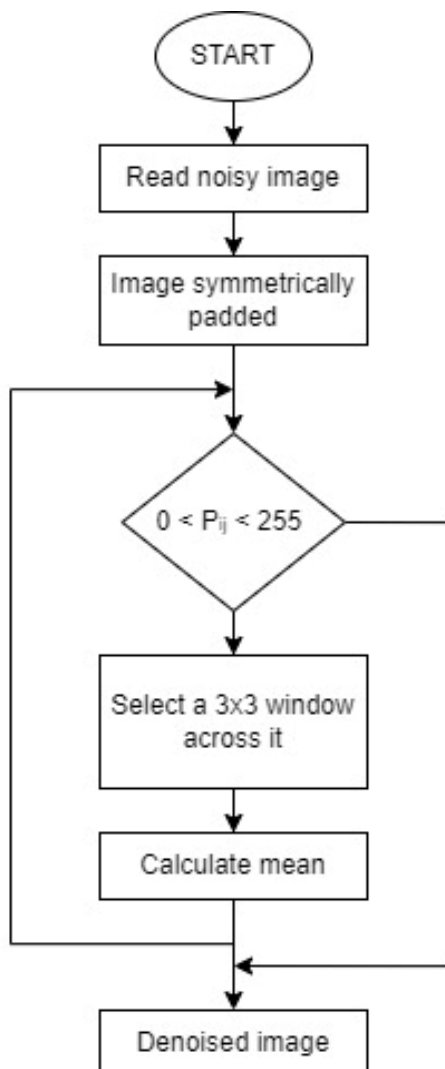
Gives the result:  $(5+16+22+6+3+18+12+3+15)/8 = 12.5$   
since the operation has been done in uint8, it would convert 12.5 to 13, and hence the centre pixel would be replaced by 13.

### Algorithm:

1. Read the noisy image.
2. Symmetrically pad the noisy image matrix.
3. If the pixel is not noisy move to next pixel
4. If the pixel is noisy, Select a 3x3 window around the noisy image
  - a. apply the mean filter on it and obtain algebraic mean of the 3x3 window
  - b. replace the generated value with the noisy pixel
5. Do it for all elements of the noisy image matrix.

Note: Noisy Image has been symmetrically padded for the smoothening of the edges. For a 3x3 window padding of 1 unit should be provided .It increases with the increase in the window size.

### Flow chart:



❖  $P_{ij}$ : It refers to the pixel value at location (i,j) of the matrix

# **Min/Max filters for Image Denoising**

## *Non-linear spatial filter*

### **Introduction:**

The Min-Max filter is a morphological filter which can be utilised for image denoising. Max and Min filter are used to process image data in spatial domain. Furthermore, they belong among order-statistic filters along with filter. It is a non-linear filter that, within a predetermined window size, replaces each pixel in an image with the lowest or maximum value of its neighbors' pixels. The smooth borders of the picture are successfully tracked by the min-max filter, and the smooth regions are successfully tracked by their average.

Formula used for calculating above-described pixel values:

$$f(x,y) = \max_{(s,t) \in S_{x,y}} \{g(s,t)\}$$

$$f(x,y) = \min_{(s,t) \in S_{x,y}} \{g(s,t)\}$$

### **For Max filter:**

**f(x,y):** Maximum value of the pixel present the selected window.

**g(s,t):** pixel intensity values of the selected window.

### **For Min filter:**

**f(x,y):** Minimum value of the pixels present the selected window.

**g(s,t):** pixel intensity values of the selected window.

### **Background:**

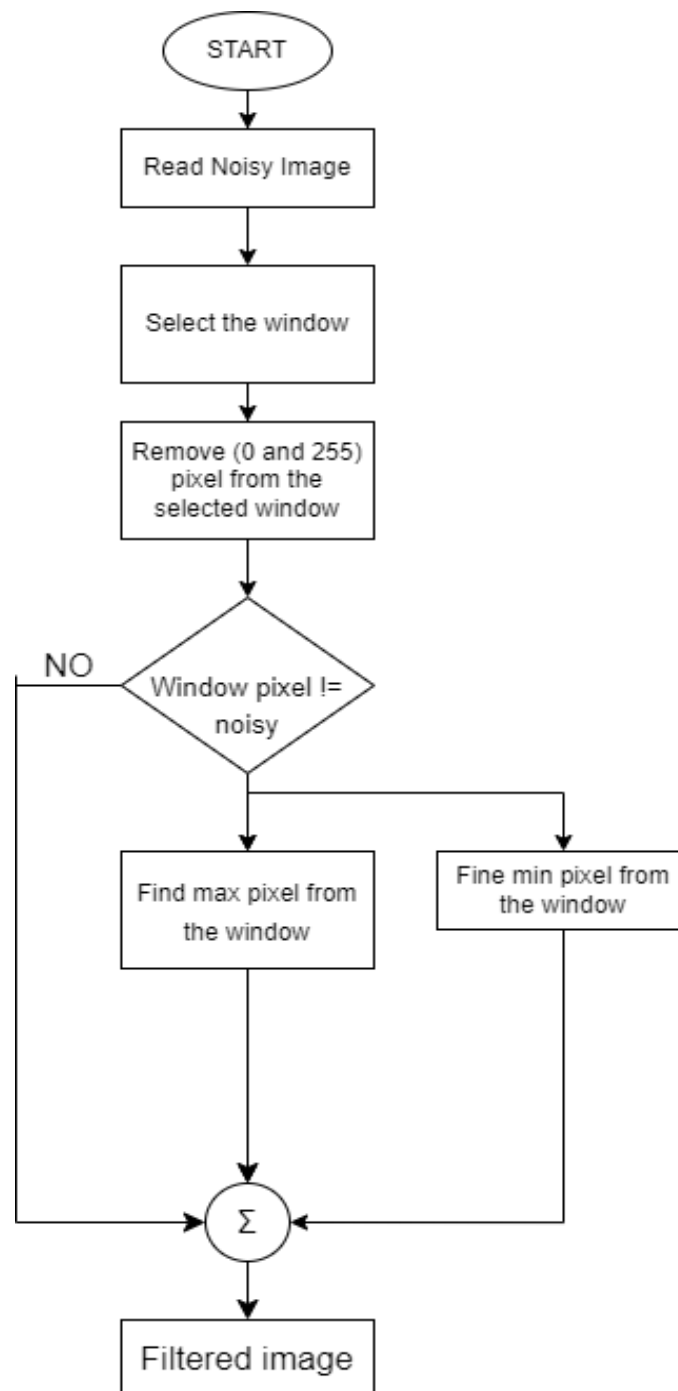
In Min/Max filter first a window is selected. Window size can be 3x3,5x5,7x7.. etc not necessarily the larger the window size the better the output cause it depends on how noisy the image is. In both Min/Max filter first 0 and 255 pixel values are eliminated from the window this made the filter to work on combination of salt & pepper noise rather than individual salt noise and pepper noise because in case of Max filter, the centre pixel will now get replaced with the maximum pixel value of the window which is not 255 thus preventing the image from becoming totally white, similarly for Min filter this prevents the image from becoming totally black



### Algorithm:

1. Read the noisy image.
2. Pad the image symmetrically.
3. Select the window.
4. Remove the 0 and 255 pixel values from the selected window.
5. (A) For Max filter: Find the maximum pixel value from the window.  
(B) For Min filter: Find the minimum pixel value from the window.
6. Replace the center pixel value of the window from the calculated Min/Max value.

### Flow chart:



# Basic Median Filter for Image Denoising

*Non-linear spatial filter*

## **Introduction:**

The basic median filter is a type of image filter used to smear out noise while preserving the of the images. It is based on the algebraic median in mathematics. The algebraic median output picture,  $f(x,y)$ , is provided by

$$f(x,y) = \text{median}_{(s,t) \in S_{x,y}} \{g(s,t)\}$$

Where:

**f(x,y):** Calculated Algebraic Median which will be replaced with the center pixel of the selected window.

**g(s,t):** pixel value of Image Elements within the selected window.

## **Background:**

The filter is defined to remove impulse noise. The filter iterates over symmetrically padded matrix, to check for pixels having a value impulse noise. If it does, it would take a 3x3 window across it, find the median value of the window and replace the noisy pixel with the generated mean value. If the pixel is not found out to be noisy to be noisy it would simply move to the next pixel.

For instance, given below is a 3x3 window of the center pixel:

5	16	22
6	X	18
12	3	15

Here the center pixel is noisy.

Replacing the center pixel with the median for the window

Gives the result: 13.5

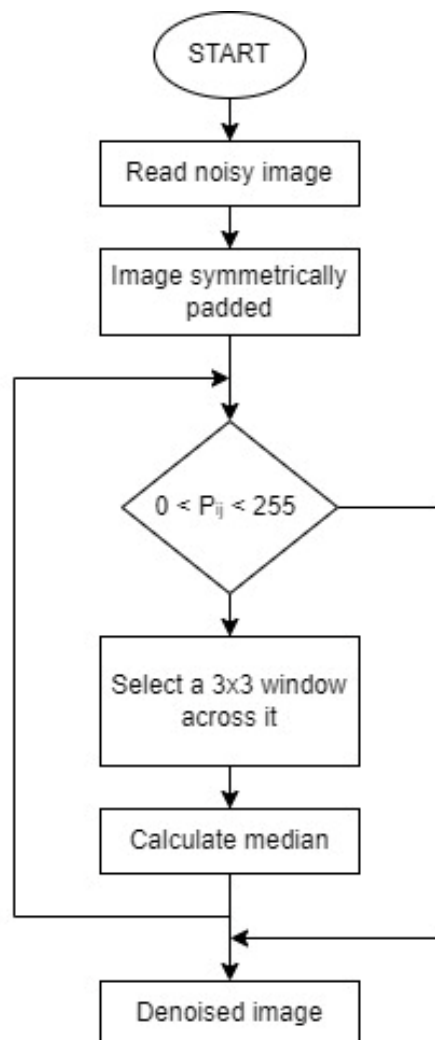
since operation has been done uint8, it would convert 13.5 to 14, and hence the center pixel would be replaced by 14.

### Algorithm:

1. Read the noisy image.
2. Symmetrically pad the noisy image matrix.
3. If the pixel is not noisy move to next pixel
4. If the pixel is noisy, Select a 3x3 window around the noisy image
  - a. apply the median filter on it and obtain algebraic median of the 3x3 window
  - b. replace the generated value with the noisy pixel
5. Do it for all elements of the noisy image matrix.

Note: Noisy Image has been symmetrically padded for the smoothening of the edges. For a 3x3 window padding of 1 unit should be provided .It increases with the increase in the window size.

### Flow chart:



❖  $P_{ij}$  : It refers to the pixel value at location (i,j) of the matrix

## RESULT :

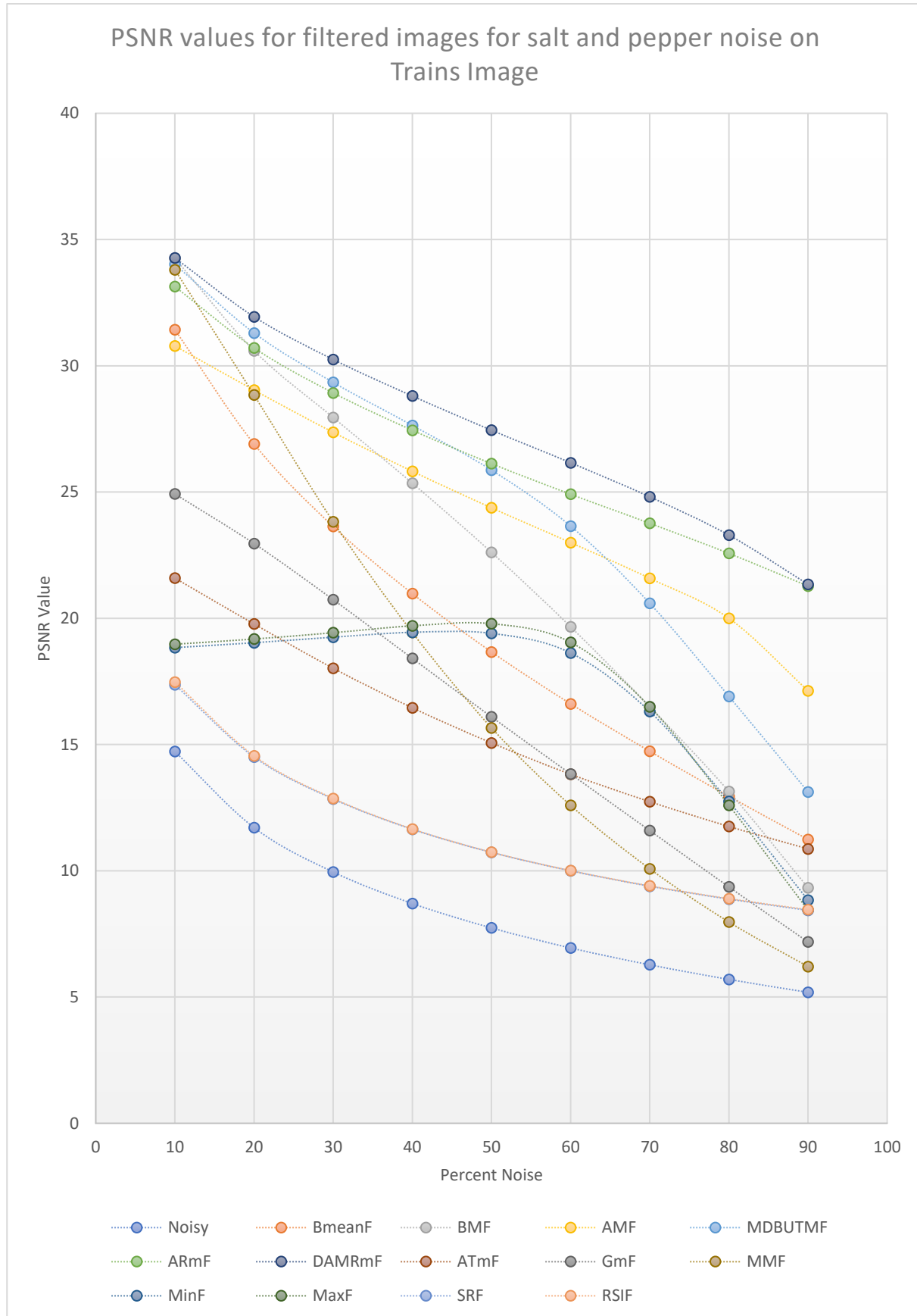
Below are the results of the performance of the various filters mentioned in this document:

50% salt and pepper noise was added to the given image and performance of the filter are as follows:

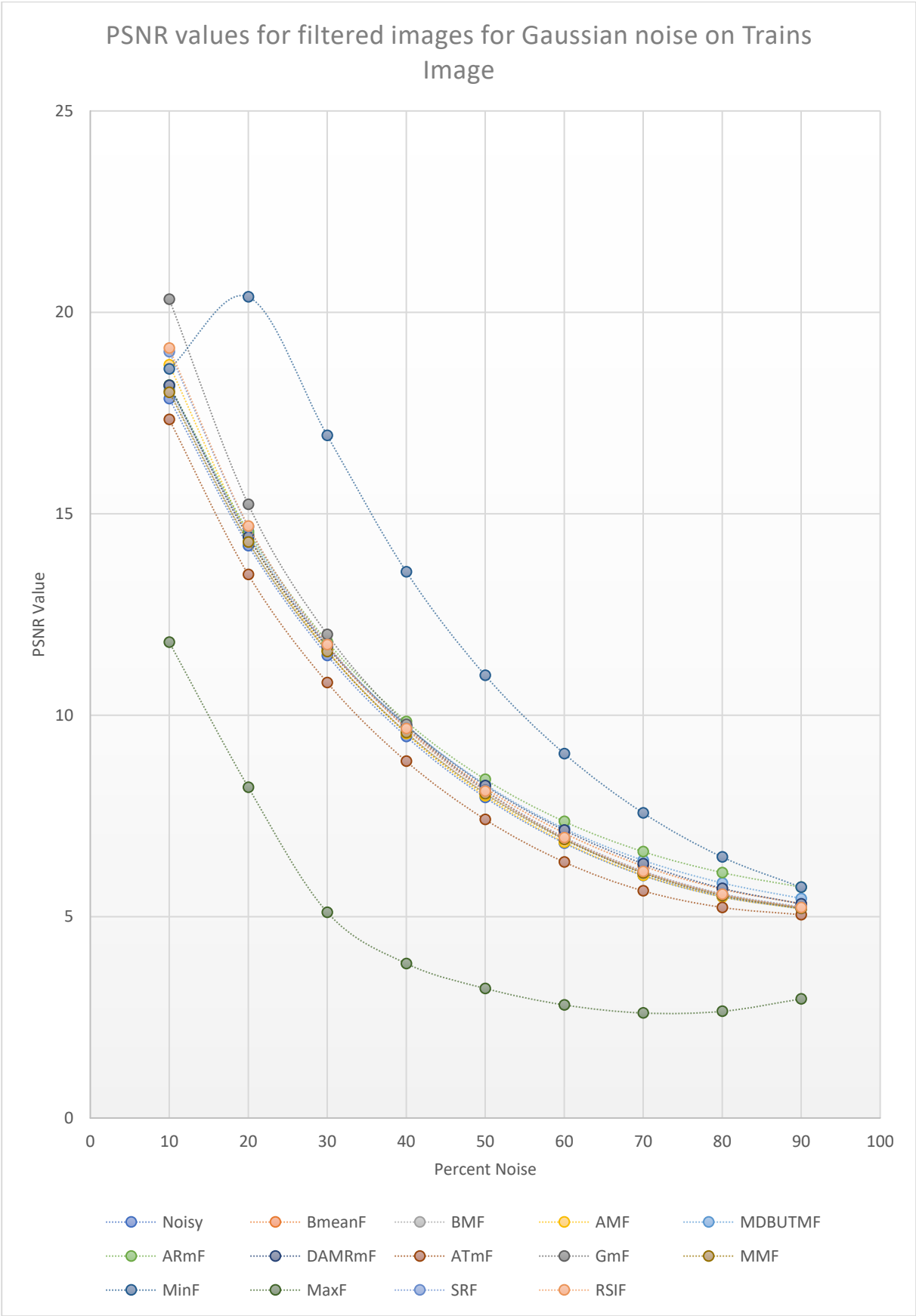
			
Original image	Noisy Image	Basic Mean Filter	Basic Median Filter
			
Adaptive Median Filter	Modified Decision Based Unsymmetrically Trimmed Median Filter	Adaptive Riesz Mean Filter	Different Adaptive Modified Riesz Mean Filter
			
Alpha Trimmed Mean Filter	Geometric Mean Filter	Modified Mean Filter	Min Filter
			
Max Filter	Sector Rotational Filter	Recursive Spline Interpolation Filter	

Below are the performance of the filters based on PSNR values in graphical form:

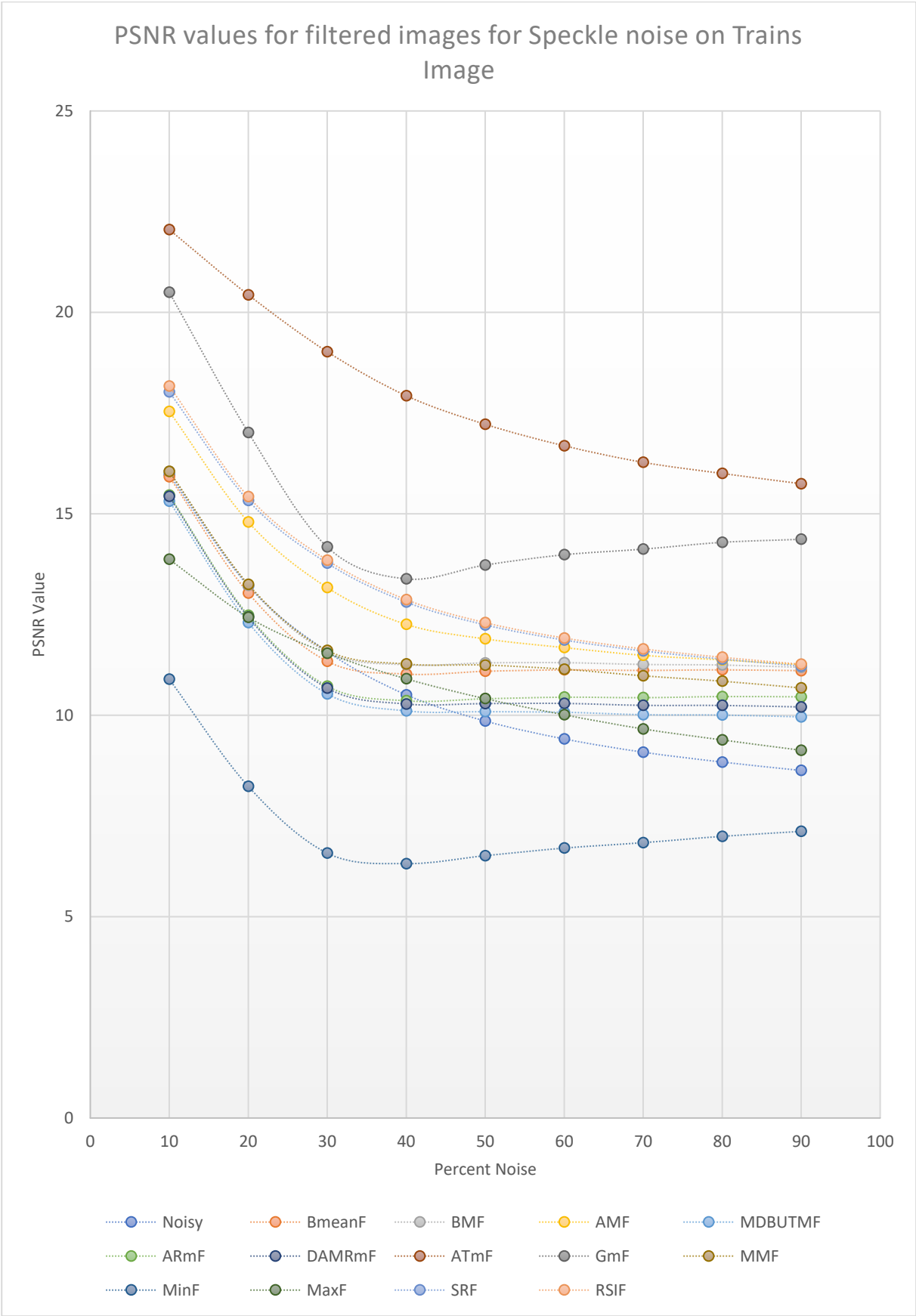
Noise: Salt and Pepper (From 10% to 90%);



Noise: Gaussian (From 10% to 90%);

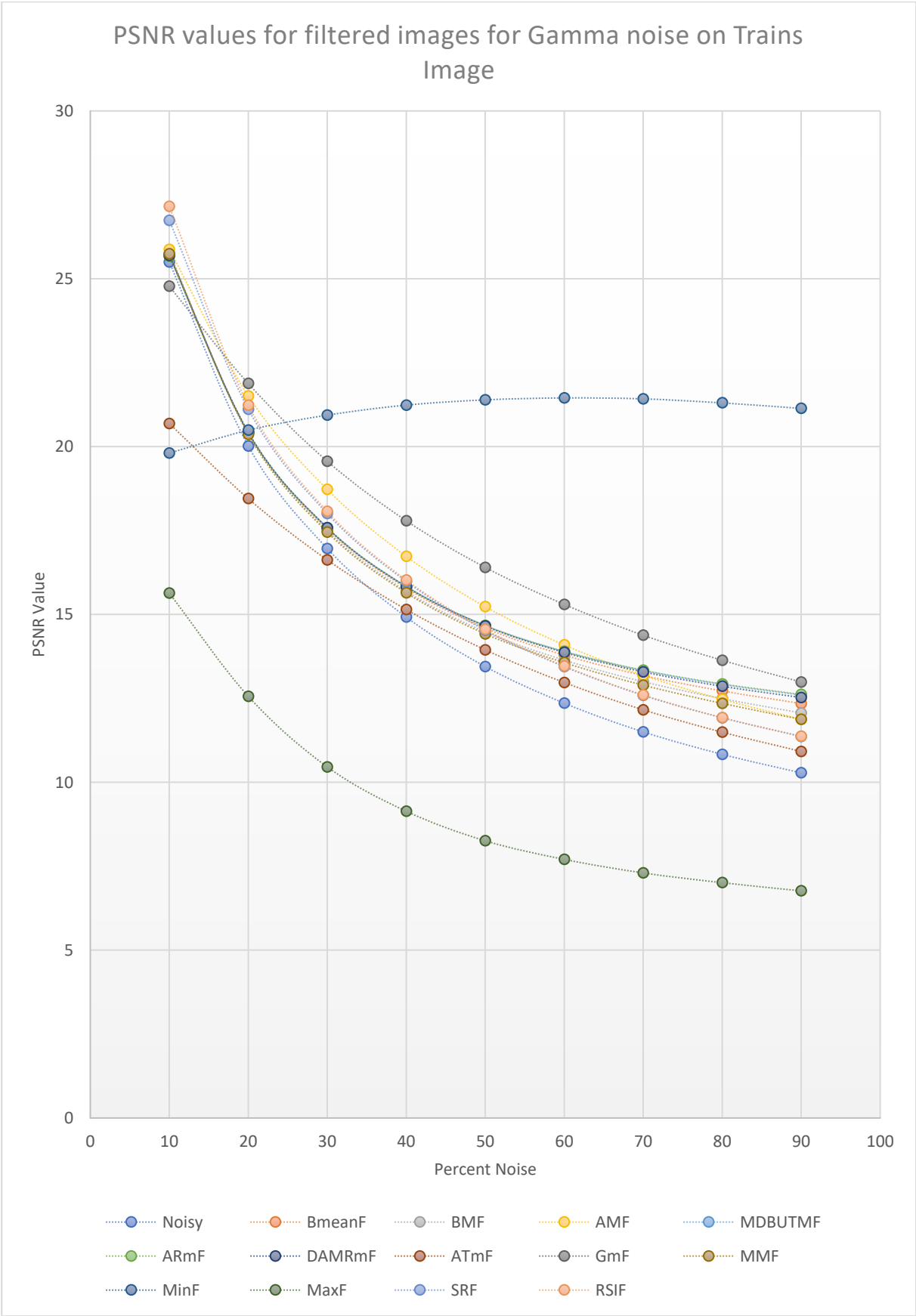


Noise: Speckle (From 10% to 90%);



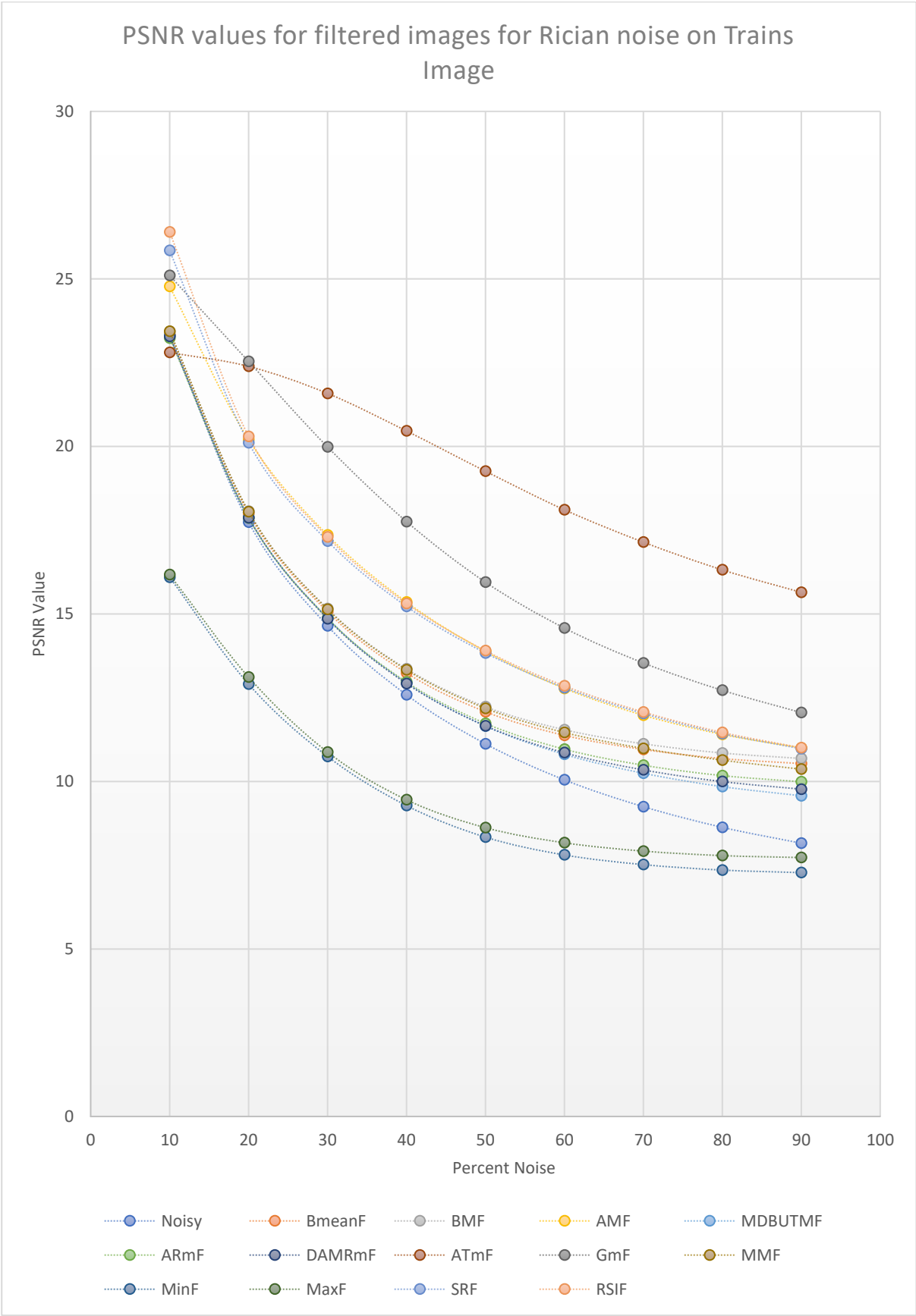


Noise: Gamma (From 10% to 90%);

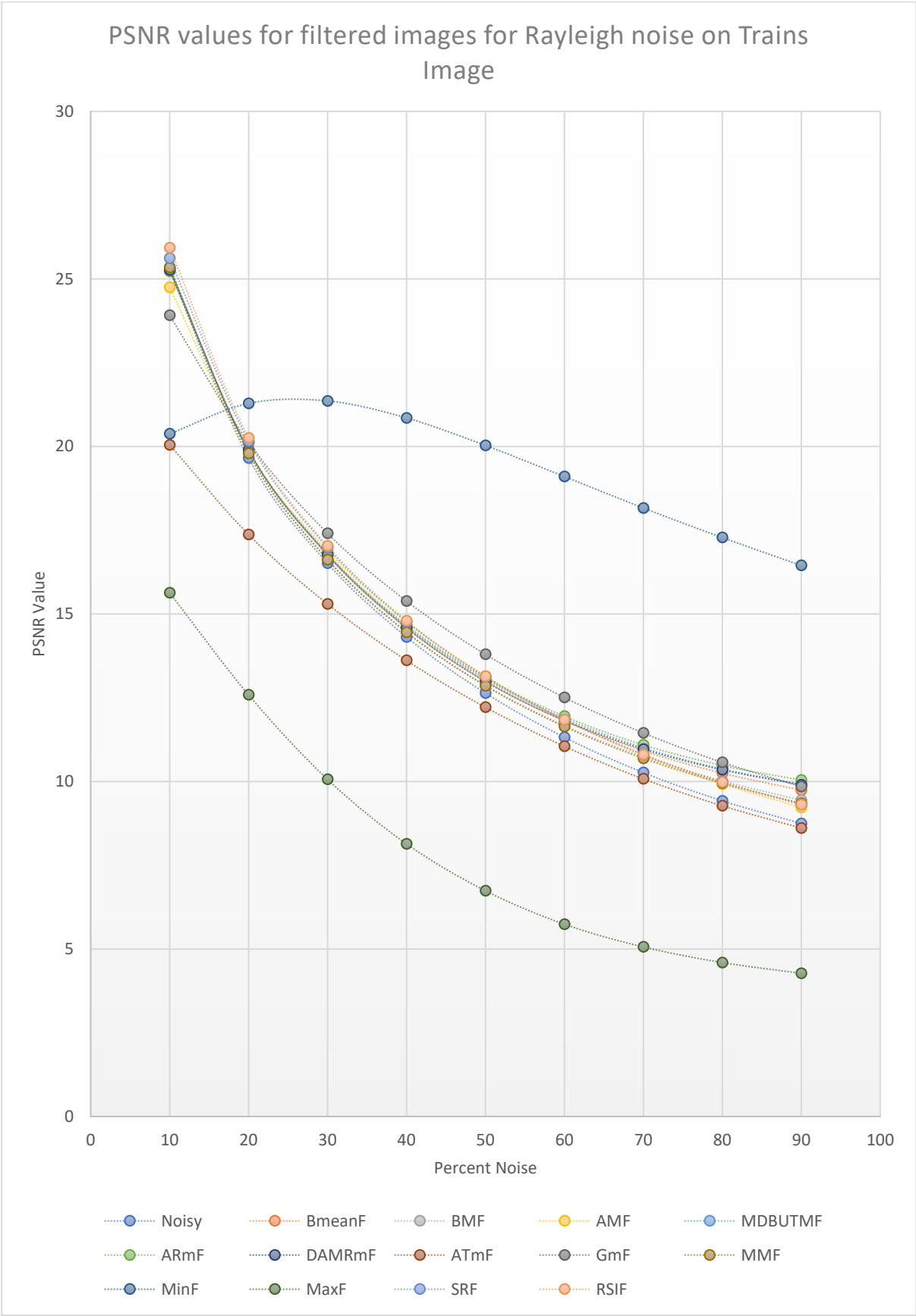




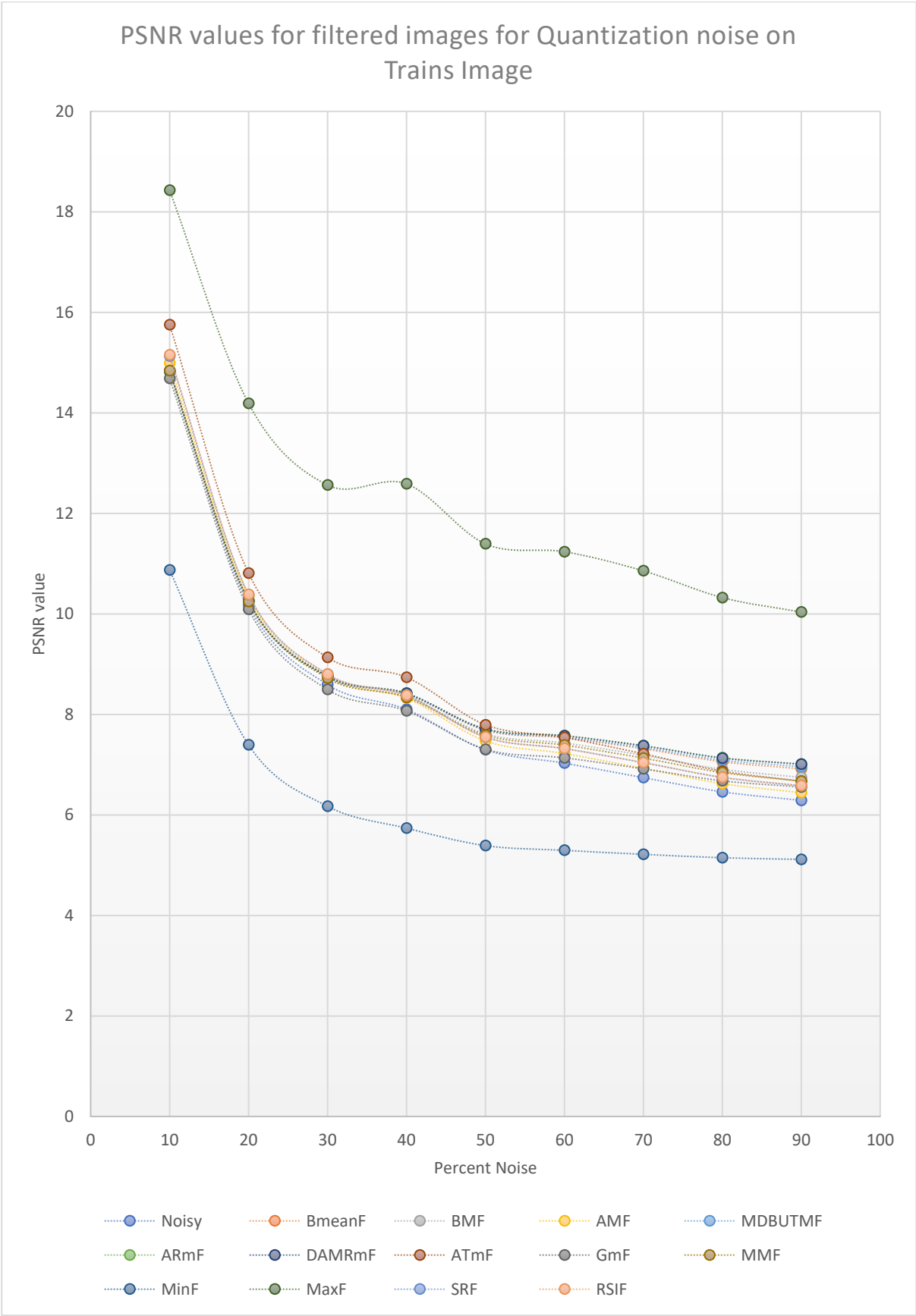
Noise: Rician (From 10% to 90%);



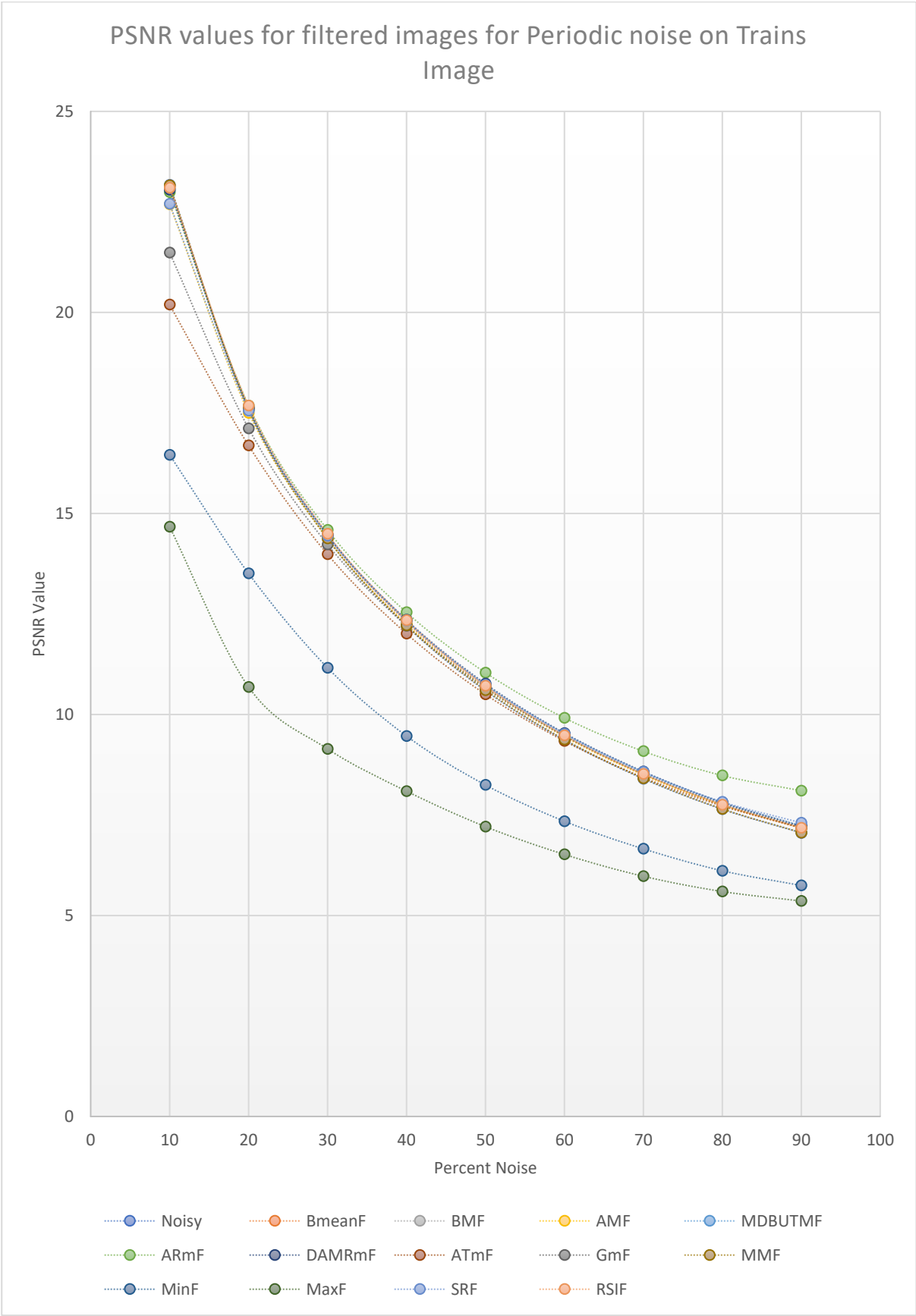
Noise: Rayleigh (From 10% to 90%):



Noise: Quantization (From 10% to 90%):



Noise: Periodic (From 10% to 90%):



## **CONCLUSION:**

From the above shown graphs it can be concluded that most of the filters enhanced the PSNR value of the image.

Details for the specific noises are given below:

### **1. Impulse noise (salt and pepper):**

noise %	Noisy	BmeanF	BMF	AMF	MDB UTM F	ARm F	DAM RmF	ATmF	GmF	MMF	MinF	MaxF	SRF	RSIF
10	14.72	31.43	34.26	30.79	34.02	33.14	34.27	21.59	24.92	33.80	18.84	18.97	17.37	17.47
20	11.71	26.90	30.60	29.04	31.30	30.71	31.94	19.77	22.96	28.84	19.03	19.18	14.50	14.55
30	9.95	23.63	27.95	27.36	29.34	28.93	30.25	18.02	20.74	23.82	19.25	19.43	12.84	12.86
40	8.70	20.98	25.35	25.82	27.64	27.45	28.80	16.45	18.41	19.44	19.44	19.70	11.64	11.65
50	7.74	18.67	22.61	24.37	25.87	26.12	27.45	15.06	16.09	15.66	19.40	19.79	10.73	10.73
60	6.94	16.61	19.66	23.00	23.65	24.91	26.16	13.82	13.83	12.59	18.63	19.05	10.00	10.01
70	6.28	14.73	16.50	21.58	20.59	23.76	24.81	12.74	11.59	10.08	16.31	16.50	9.39	9.40
80	5.70	12.95	13.14	20.00	16.91	22.57	23.29	11.76	9.36	7.97	12.74	12.59	8.88	8.89
90	5.19	11.24	9.33	17.13	13.12	21.28	21.35	10.87	7.18	6.21	8.84	8.44	8.43	8.46

- ❖ Adaptive Riesz Mean filter and Different Adaptive Modified Riesz Mean filter constantly performed well in all types of noise ranges for impulse noise.
- ❖ Modified Decision Based Unsymmetric Trimmed Median filter, Adaptive Median filter, Alpha trimmed mean filter and Basic Mean Filter failed to produce good PSNR values at high ranges of noise density, but performed well in all other noise values.
- ❖ Basic Median filter , Geometric mean filter, Modified Median filter, Min/ Max filter, Sector rotation filter and Recursive spline interpolation filter were able to produce high PSNR values only at low ranges of noise density.

**Therefore, from this comparison it can be concluded that Adaptive Riesz Mean filter and Different Adaptive Modified Riesz Mean filter are the best filters to remove Impulse noises from a given image.**

## 2. Gaussian noise:

noise %	Noisy	BmeanF	BMF	AMF	MDB UTM F	ARm F	DAM RmF	ATmF	GmF	MMF	MinF	MaxF	SRF	RSIF
10	17.86	18.18	18.06	18.70	18.16	18.19	18.19	17.34	20.33	18.01	18.59	11.81	19.0	17.86
20	14.21	14.45	14.31	14.47	14.50	14.56	14.42	13.49	15.23	14.30	20.38	8.21	14.6	14.21
30	11.48	11.69	11.59	11.61	11.73	11.83	11.70	10.81	12.00	11.58	16.94	5.11	11.7	11.48
40	9.47	9.69	9.58	9.54	9.75	9.84	9.74	8.86	9.77	9.56	13.56	3.84	9.67	9.47
50	7.95	8.19	8.07	8.00	8.28	8.41	8.25	7.41	8.14	8.05	10.99	3.22	8.13	7.95
60	6.82	7.07	6.93	6.85	7.19	7.36	7.15	6.35	6.93	6.91	9.05	2.81	6.97	6.82
70	6.02	6.25	6.10	6.03	6.40	6.61	6.31	5.64	6.08	6.09	7.57	2.61	6.12	6.02
80	5.49	5.68	5.55	5.49	5.84	6.09	5.70	5.23	5.52	5.53	6.48	2.65	5.56	5.49
90	5.19	5.32	5.22	5.19	5.45	5.73	5.32	5.05	5.20	5.21	5.73	2.96	5.23	5.19

**All of the filter designed are spatial filter hence they are not very effective against gaussian noise**

**From the PSNR table, it can be observed that none of the filters were able to enhance the image quality significantly and therefore it can be concluded that none of the designed filter is able to denoise the image having gaussian noise.**

### 3. Speckle noise:

noise %	Noisy	BmeanF	BMF	AMF	MDBUTMF	ARmF	DAMRmF	ATmF	GmF	MMF	MinF	MaxF	SRF	RSIF
10	15.97	15.92	16.05	17.54	15.31	15.46	15.43	22.05	20.50	16.05	10.89	13.87	18.0	15.97
20	13.21	13.03	13.22	14.80	12.30	12.48	12.43	20.44	17.02	13.25	8.23	12.43	15.3	13.21
30	11.58	11.34	11.57	13.17	10.53	10.72	10.67	19.02	14.17	11.60	6.58	11.54	13.7	11.58
40	10.50	11.02	11.25	12.25	10.11	10.36	10.28	17.93	13.38	11.27	6.31	10.90	12.8	10.50
50	9.85	11.09	11.30	11.89	10.09	10.41	10.29	17.22	13.73	11.24	6.51	10.41	12.2	9.85
60	9.41	11.12	11.30	11.68	10.07	10.45	10.29	16.69	13.98	11.14	6.70	10.01	11.8	9.41
70	9.08	11.11	11.26	11.49	10.01	10.43	10.24	16.28	14.12	10.98	6.84	9.66	11.5	9.08
80	8.84	11.12	11.24	11.37	10.00	10.46	10.24	16.00	14.29	10.84	6.99	9.39	11.3	8.84
90	8.63	11.11	11.19	11.25	9.96	10.46	10.20	15.75	14.37	10.68	7.12	9.13	11.2	8.63

- ❖ Alpha Trimmed mean filter and Geometric mean filter constantly performed well in all types of noise ranges for speckle noise.
- ❖ Basic Mean Filter, Adaptive Median filter, Basic Median filter Adaptive Riesz Mean filter, Different Adaptive Modified Riesz Mean filter, Sector rotation filter and Recursive spline interpolation filter failed to produce good PSNR values at high ranges of noise density, but performed relatively well in all other noise values.
- ❖ Modified Decision Based Unsymmetric Trimmed Median filter and Min/ Max filter were constantly unable to produce considerable PSNR values even at low noise density

**Therefore, it can be concluded that, Alpha Trimmed mean filter and Geometric mean filter showed improved performance at high density noise and can be considered to remove high density speckle noises.**

#### 4. Gamma noise:

noise %	Noisy	BmeanF	BMF	AMF	MDB UTM F	ARm F	DAM RmF	ATmF	GmF	MMF	MinF	MaxF	SRF	RSIF
10	25.50	25.71	25.75	25.88	25.72	25.66	25.69	20.69	24.78	25.75	19.81	15.64	26.7	25.50
20	20.02	20.42	20.40	21.51	20.43	20.41	20.42	18.45	21.88	20.37	20.49	12.56	21.1	20.02
30	16.96	17.56	17.50	18.72	17.58	17.58	17.58	16.63	19.56	17.45	20.94	10.46	18.0	16.96
40	14.92	15.79	15.70	16.73	15.83	15.83	15.83	15.15	17.79	15.65	21.24	9.14	15.9	14.92
50	13.45	14.60	14.48	15.23	14.67	14.67	14.65	13.95	16.40	14.42	21.39	8.26	14.5	13.45
60	12.36	13.79	13.64	14.09	13.89	13.90	13.87	12.97	15.30	13.57	21.45	7.70	13.4	12.36
70	11.50	13.18	13.00	13.19	13.33	13.34	13.29	12.16	14.38	12.89	21.42	7.30	12.5	11.50
80	10.83	12.72	12.49	12.47	12.92	12.93	12.86	11.49	13.64	12.35	21.30	7.01	11.9	10.83
90	10.28	12.35	12.07	11.88	12.60	12.62	12.52	10.92	12.99	11.87	21.14	6.77	11.3	10.28

- ❖ Minimum filter constantly performed well in all types of noise ranges for Gamma noise.
- ❖ Basic Mean Filter, Adaptive Median filter, Basic Median filter Adaptive Riesz Mean filter, Different Adaptive Modified Riesz Mean filter, Alpha trimmed mean filter, Sector rotation filter, Recursive spline interpolation filter and Modified Decision Based Unsymmetric Trimmed Median filter failed to produce good PSNR values at high ranges of noise density, but performed relatively well in all other noise values.
- ❖ Max filter was constantly unable to produce considerable PSNR values even at low noise density

**From the comparison table it can be observed that only Minimum filter can denoise the image containing gamma noise and therefore is an ideal option for gamma noise denoising.**



## 5. Rician noise:

noise %	Noisy	BmeanF	BMF	AMF	MDB UTM F	ARM F	DAM RmF	ATmF	GmF	MMF	MinF	MaxF	SRF	RSIF
10	23.31	23.33	23.43	24.77	23.30	23.23	23.28	22.80	25.10	23.43	16.10	16.17	25.84	26.40
20	17.74	17.99	18.06	20.20	17.89	17.86	17.87	22.39	22.54	18.05	12.90	13.11	20.10	20.29
30	14.64	15.07	15.16	17.35	14.87	14.87	14.85	21.58	19.98	15.14	10.75	10.88	17.17	17.29
40	12.59	13.24	13.36	15.35	12.94	12.97	12.92	20.46	17.76	13.33	9.28	9.45	15.22	15.31
50	11.12	12.08	12.23	13.88	11.65	11.73	11.66	19.26	15.95	12.18	8.34	8.62	13.83	13.90
60	10.05	11.38	11.54	12.78	10.81	10.96	10.86	18.11	14.58	11.46	7.81	8.17	12.79	12.85
70	9.25	10.95	11.12	11.97	10.24	10.48	10.34	17.14	13.54	10.99	7.52	7.92	12.02	12.07
80	8.63	10.68	10.85	11.40	9.84	10.17	10.00	16.31	12.72	10.63	7.35	7.79	11.42	11.46
90	8.16	10.53	10.68	11.00	9.57	9.99	9.77	15.64	12.05	10.37	7.28	7.73	10.97	11.00

- ❖ Alpha trimmed mean filter and geometric mean filter constantly performed relatively well in all types of noise ranges for Rician noise.
- ❖ Basic Mean Filter, Adaptive Median filter, Basic Median filter Adaptive Riesz Mean filter, Different Adaptive Modified Riesz Mean filter, Alpha trimmed mean filter, Sector rotation filter, Recursive spline interpolation filter and Modified Decision Based Unsymmetric Trimmed Median filter failed to produce good PSNR values at high ranges of noise density, but performed relatively well in all other noise values.
- ❖ Min /Max filter was constantly unable to produce considerable PSNR values even at low noise density

**Alpha trimmed mean filter relatively denoised the image. Geometric mean filter was also able to recover the image to some extent. The rest of the filters did not work.**

## 6. Rayleigh noise:

noise %	Noisy	BmeanF	BMF	AMF	MDB UTM F	ARm F	DAM RmF	ATmF	GmF	MMF	MinF	MaxF	SRF	RSIF
10	25.26	25.32	25.35	24.75	25.30	25.23	25.27	20.05	23.91	25.34	20.37	15.63	25.62	25.92
20	19.65	19.89	19.82	19.84	19.89	19.87	19.88	17.37	20.12	19.79	21.28	12.58	20.15	20.25
30	16.51	16.75	16.64	16.88	16.79	16.80	16.76	15.29	17.41	16.61	21.36	10.06	16.98	17.02
40	14.31	14.58	14.46	14.76	14.63	14.66	14.58	13.61	15.38	14.45	20.85	8.14	14.77	14.79
50	12.64	12.99	12.87	13.14	13.05	13.10	12.98	12.21	13.79	12.85	20.03	6.74	13.12	13.13
60	11.32	11.81	11.67	11.84	11.88	11.94	11.82	11.05	12.51	11.64	19.10	5.74	11.83	11.83
70	10.26	10.91	10.74	10.78	11.01	11.09	10.96	10.07	11.44	10.69	18.16	5.06	10.80	10.80
80	9.42	10.25	10.02	9.92	10.37	10.48	10.35	9.27	10.57	9.95	17.28	4.59	9.97	9.97
90	8.75	9.74	9.44	9.23	9.91	10.04	9.90	8.61	9.84	9.34	16.45	4.27	9.31	9.31

**All of the filter designed are spatial filter hence they are not very effective against Rayleigh noise**

**From the PSNR table, it can be observed that none of the filters were able to enhance the image quality and most of the even further deteriorated the image quality and therefore it can be concluded that none of the designed filter is able to denoise the image having Rayleigh noise.**

## 7. Quantization noise

noise %	Noisy	BmeanF	BMF	AMF	MDB UTM F	ARM F	DAM RmF	ATmF	GmF	MMF	MinF	MaxF	SRF	RSIF
10	14.82	14.83	14.84	14.99	14.83	14.83	14.83	15.75	14.68	14.84	10.87	18.43	15.13	15.15
20	10.18	10.26	10.25	10.34	10.25	10.26	10.26	10.81	10.09	10.25	7.39	14.19	10.38	10.38
30	8.59	8.75	8.73	8.76	8.75	8.76	8.76	9.13	8.50	8.72	6.17	12.56	8.79	8.80
40	8.10	8.41	8.36	8.32	8.41	8.42	8.42	8.74	8.07	8.34	5.74	12.59	8.38	8.38
50	7.30	7.68	7.61	7.47	7.69	7.71	7.71	7.79	7.31	7.58	5.39	11.39	7.54	7.54
60	7.03	7.53	7.43	7.22	7.55	7.58	7.57	7.54	7.14	7.39	5.30	11.24	7.31	7.32
70	6.74	7.32	7.19	6.93	7.34	7.38	7.37	7.22	6.92	7.13	5.22	10.86	7.04	7.04
80	6.46	7.06	6.91	6.62	7.09	7.14	7.13	6.87	6.68	6.84	5.15	10.32	6.74	6.74
90	6.29	6.92	6.75	6.44	6.96	7.01	7.01	6.67	6.56	6.67	5.12	10.04	6.57	6.58

- ❖ Maximum filter was able to enhance the image quality up to some extent.
- ❖ All the other filter were unable to enhance the image quality rather most of them even further deteriorated the image.

**From the PSNR table, it can be observed that only Maximum filters was able to enhance the image quality and rest of the filter deteriorated the image quality and therefore it can be concluded that only 'Maximum filter' of the designed filter can be used to denoise the image having Rayleigh noise.**

## 8. Periodic noise:

noise %	Noisy	BmeanF	BMF	AMF	MDB UTM F	ARm F	DAM RmF	ATmF	GmF	MMF	MinF	MaxF	SRF	RSIF
10	23.17	23.13	23.16	22.69	23.06	22.99	23.05	20.19	21.49	23.16	16.46	14.67	22.70	23.09
20	17.58	17.63	17.59	17.50	17.61	17.68	17.62	16.69	17.11	17.59	13.51	10.68	17.55	17.68
30	14.36	14.45	14.38	14.37	14.45	14.59	14.47	13.98	14.23	14.38	11.16	9.14	14.44	14.49
40	12.20	12.30	12.22	12.25	12.32	12.54	12.35	12.01	12.20	12.22	9.46	8.09	12.33	12.34
50	10.59	10.69	10.61	10.66	10.74	11.04	10.76	10.50	10.66	10.60	8.25	7.21	10.73	10.71
60	9.36	9.47	9.38	9.45	9.52	9.91	9.54	9.34	9.47	9.37	7.34	6.52	9.51	9.47
70	8.40	8.51	8.41	8.49	8.56	9.08	8.58	8.43	8.53	8.41	6.66	5.97	8.57	8.51
80	7.64	7.75	7.65	7.74	7.81	8.48	7.81	7.72	7.77	7.65	6.11	5.60	7.82	7.75
90	7.05	7.17	7.06	7.18	7.23	8.10	7.22	7.18	7.19	7.06	5.75	5.36	7.30	7.17

From the PSNR table, it can be observed that none of the filters were able to enhance the image quality and a few of them even further deteriorated the image quality and therefore can be concluded that none of the designed filters is able to denoise the image having Periodic noise.

## 9. Poisson noise:

noise %	Noisy	BmeanF	BMF	AMF	MDB UTM F	ARm F	DAM RmF	ATmF	GmF	MMF	MinF	MaxF	SRF	RSIF
10	27.11	26.99	27.17	27.48	26.95	26.80	26.90	22.94	25.79	27.17	17.32	17.46	29.17	30.68

- ❖ The addition of Poisson noise in the image nearly had no impact on the image quality as the PSNR of the noisy image is about 27.11.
- ❖ Min/Max filter degraded the image quality significantly and deteriorated the image.
- ❖ All the other filters gave a PSNR value nearly equal to the noisy PSNR value.
- ❖ Recursive spline interpolation filter and sector rotation filter marginally enhanced the PSNR value.

Therefore *Sector Rotation filter and Recursive spline interpolation filter* can be used to denoise the image having Poisson noise.

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- IMAGE DENOISING USING NEW ADAPTIVE BASED MEDIAN FILTER  
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