QF604 Econometrics of Financial Markets

Group Assignment 3

Group Members:

Ankit RAWAT

Chenpeng LI

Chuanyu ZHANG

Yan YAN

Yinzi GAN

DECLARATION

Completion of this assignment is a result of group collaboration. About completion of the assignment, we:

1. *have clear and fair division of work;*
2. *have no complaint on other group members.*

I have read through and give personal consent to statements above.

SIGNATURE:

1 INTRODUCTION

The Fisher effect states that real interest rate is equal to the nominal interest rate minus the expected inflation rate (Fisher, 1930). This somewhat makes intuitive sense, as the cost of borrowing should inherently be directly affected by expected inflation rates, as lender would seek more compensation if the expectation of the future value of their money is lower due to high expected inflation.

The Fisher effect in equation form is as follows:

$$r = i_t - \pi_t$$

Where $i_t$is the nominal interest rate at time t and π_t is the expected inflation at time t.

Fama and Gibbons (Fama & Gibbons, 1984) presented a very similar equation showing the relationship of nominal interest rate and expected inflation as follows:

$$R_t = E_{t-1}[r_t] + E_{t-1}[I_t]$$

Where Rt is the nominal risk-free interest rate from period $(t-1)$ to (t), while the terms on the RHS are the expectations at time $(t-1)$ for real interest rate and inflation rate at time t.

However the relationship is often complicated by other factors. Hjalmarsson and Osterholm (Hjalmarsson & Osterholm, 2007) sought to examine the relationship between nominal interest rate and expected inflation rate through cointegration testing.

2 DATA AND METHODOLOGY

Our data for US nominal interest rate and CPI inflation comes from OECD.org. First we replicate the results of the Hjalmarsson & Osterholm paper using data from Jan-1974 to Oct-2006. Then we perform the same procedures using new data from Oct-2006 to Dec-2020. Lastly we perform the tests again for the whole period from Jan-1974 to Dec-2020.

The sequences of tests are as follows:

a) Unit root test for stationarity using the Augmented Dickey-Fuller test
b) Johansen cointegration test on raw data
c) Johansen cointegration test with constraints

3 RESULTS AND DISCUSSION

3.1 Sub-period Jan-1974 to Oct-2006

TABLE 1 shows the results from the Augmented-Dickey Fuller test performed on the two variables, on the null hypothesis that there exists a unit root in the time series. Looking at the p-values, we conclude that we cannot reject the null hypothesis that there is a unit root and hence that the time series is stationary.

We then proceed with the cointegration test. Before that, we need to select the optimal lag length, which is done by optimizing the Akaike Information Criterion (AIC), and we select a lag of 10 as recommended by Hjalmarsson & Osterholm.

TABLE 1 ADF test statistics for first sub-period

	τ_{ADF}	p-value	90% confidence level critical value	95% confidence level critical value	99% confidence level critical value
Inflation π_t	-2.149654	0.225023	-2.571	-2.869	-3.448
Interest rate i_t	-2.413003	0.138100	-2.571	-2.869	-3.448

TABLE 2 Johansen cointegration test statistics for first sub-period

Null hypothesis	J_{trace}	Jtrace critical values [90%, 95%, 99%]	J_{max}	J_{max} critical values [90%, 95%, 99%]
r = 0	23.858	[13.4294, 15.4943, 19.9349]	18.301	[2.7055, 3.8415, 6.6349]
r = 1	5.557	[12.2971, 14.2639, 18.52]	5.557	[2.7055, 3.8415, 6.6349]

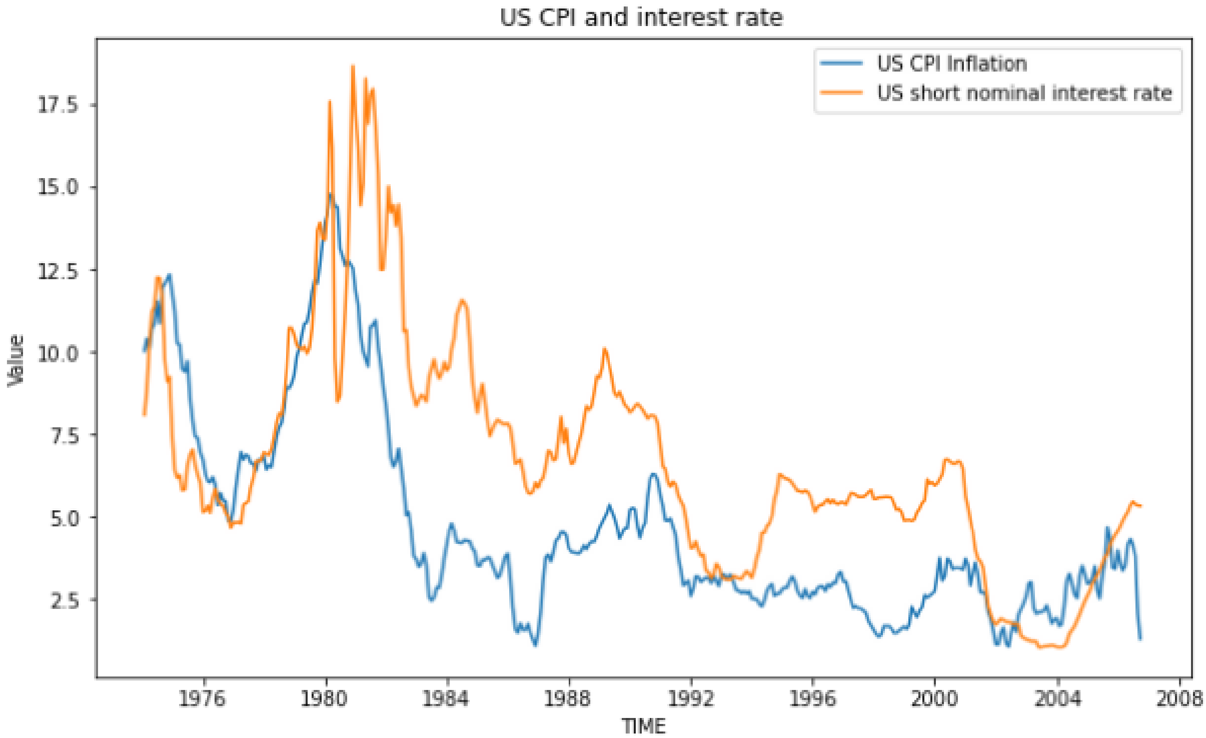
TABLE 2 shows the results from the Johansen cointegration tests. The null hypothesis of zero cointegrating vectors (r=0) is rejected by both trace statistic and the maximum eigenvalue statistic. However the null hypothesis of one cointegrating vector (r=1) cannot be rejected using the trace statistic even at 90% confidence interval. Based on this information, we might conclude that there exists a cointegrating relationship between CPI inflation and nominal interest rates.

To further ensure the validity of our results and avoid spuriously concluding that the variables are cointegrated, we also test whether the cointegrating vector satisfies the restrictions of β’ = (0 1) or β’ = (1 0) using a monte carlo study. *TABLE 3* shows the results of such a study. We find that the restriction β’ = (0 1) is rejected while β' = (1 0) is not. Hence we conclude that there is no true cointegration, instead the evidence suggests that CPI inflation and nominal interest rates are integrated of different orders.

TABLE 3 Cointegrating vector test for first sub-period

Restriction	Test statistic
$\beta' = (0\ 1)$	7.523 (0.006)
$\beta' = (1\ 0)$	0.654 (0.457)

GRAPH 1: US CPI inflation and nominal interest rate for first sub-period



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3.2 Sub-period Oct-2006 to Dec-2020

TABLE 4: ADF test statistics for first sub-period

	τ_{ADF}	p-value	90% confidence level critical value	95% confidence level critical value	99% confidence level critical value
Inflation π_t	-2.723199	0.070109	-3.473	-2.880	-2.577
Interest rate i_t	-4.376677	0.000326	-3.473	-2.880	-2.577

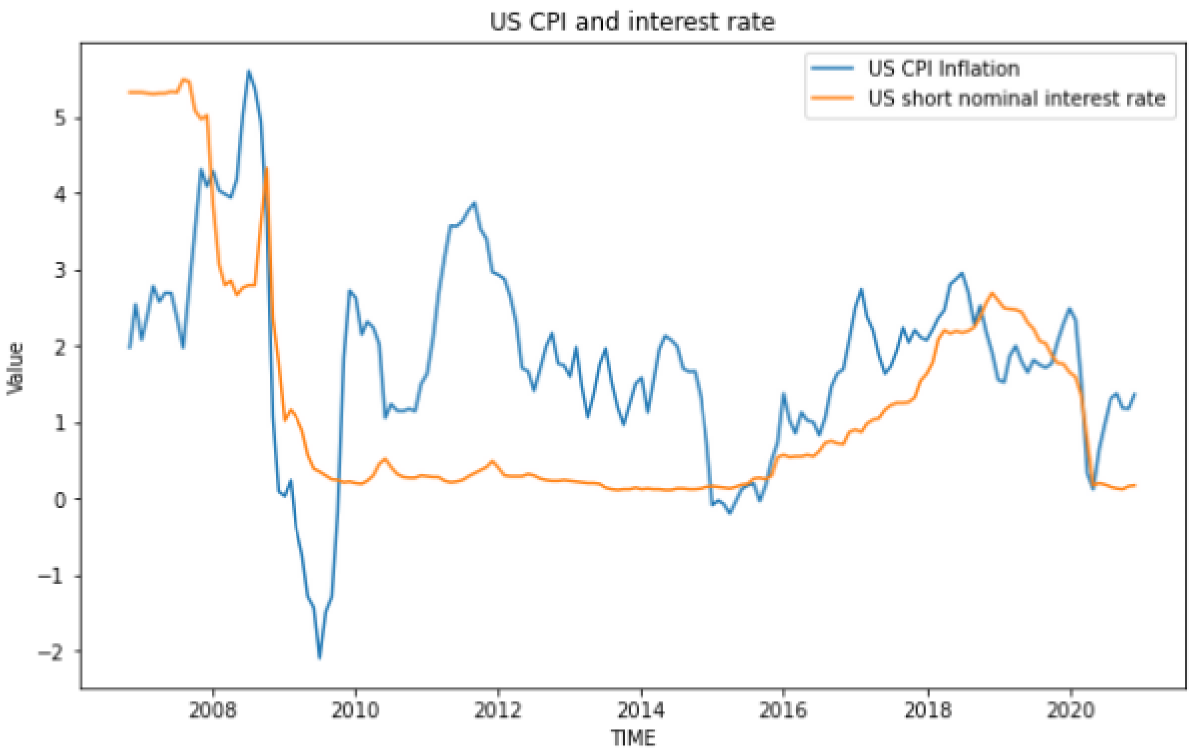
TABLE 5: Johansen cointegration test statistics for first sub-period

Null hypothesis	Jtrace	Jtrace critical values [90%, 95%, 99%]	Jmax	Jmax critical values [90%, 95%, 99%]
r = 0	37.685	[13.429, 15.494, 19.935]	21.072	[12.297, 14.264, 18.52]
r = 1	16.613	[2.706, 3.842, 6.635]	16.613	[2.706, 3.842, 6.635]

For the more recent sub-period, we can see from *TABLE 4 t*hat while we cannot reject the null for CPI inflation, we reject the null for interest rate, and conclude that the time series is stationary.

For *TABLE 5*, we see that the Johansen cointegration test rejects the null of zero cointegrating vectors (r=0), but also rejects the null of one cointegrating vector (r=1). This implies that the matrix **Π** has full rank and therefore that there is no cointegrating relationship.

GRAPH 2: US CPI inflation and nominal interest rate for second sub-period



3.3 Full period Jan-1974 to Dec-2020

TABLE 6: ADF test statistics for first sub-period

	τ_{ADF}	**p-value**	**90% confidence level critical value**	**95% confidence level critical value**	**99% confidence level critical value**
Inflation π_t	-2.628305	0.087287	-3.442	-2.867	2.570
Interest rate i_t	-1.986228	0.292644	-3.442	-2.867	2.570

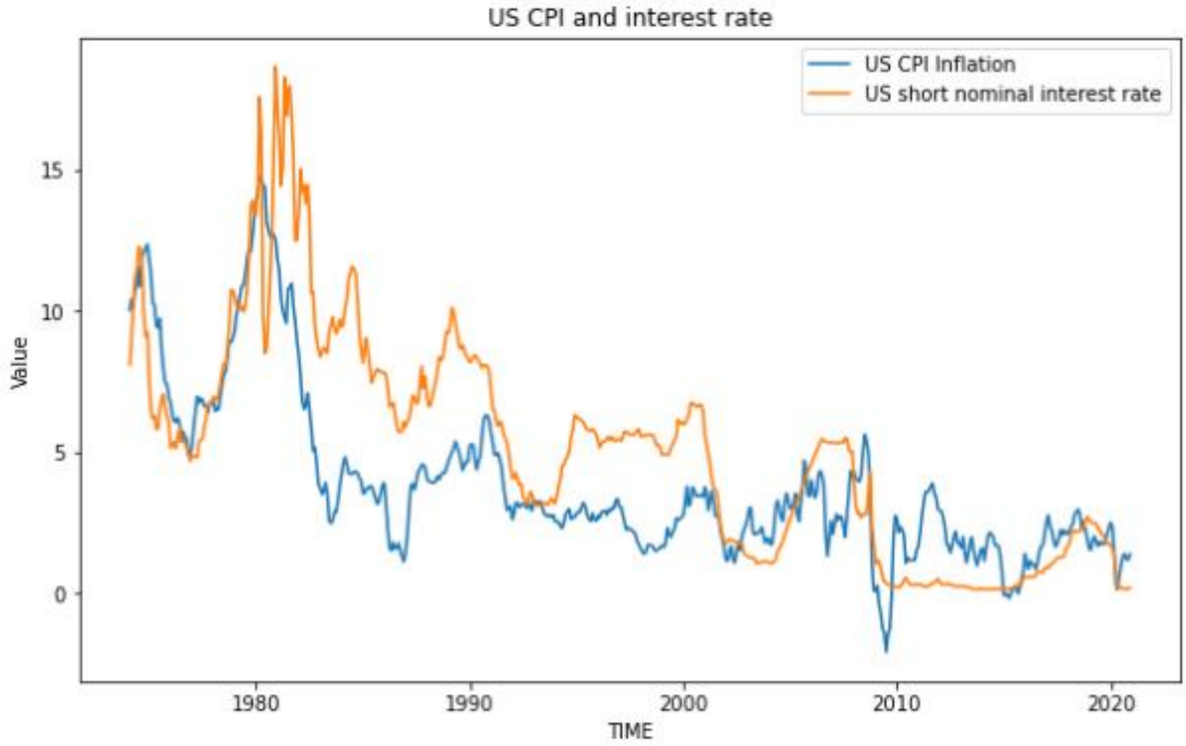
Finally, taking the full sample period, from *TABLE 6* we can see that the null of unit root cannot be rejected for both variables.

TABLE 7 shows that both the null hypothesis of zero and one cointegrating vectors can be rejected that the 5% significance level, indicating that there is no cointegrating relationship.

TABLE 7: Johansen cointegration test statistics for first sub-period

Null hypothesis	**Jtrace**	**Jtrace critical values [90%, 95%, 99%]**	**Jmax**	**Jmax critical values [90%, 95%, 99%]**
r = 0	28.076	[13.429, 15.494, 19.935]	23.6337	[12.297, 14.264, 18.52]
r = 1	4.442	[2.706, 3.842, 6.635]	4.4424	[2.706, 3.842, 6.635]

GRAPH 3: US CPI inflation and nominal interest rate for first sub-period



4 CONCLUSION

In the first test on the earlier sub-period, replicating the original work by Hjalmarsson and Osterholm, we confirmed their results that while on surface, just using the Johansen cointegration test, it appeared as if we would conclude that there is a cointegrating relationship between nominal interest rates and inflation rates. However digging deeper and applying certain constraints, we were able to show that we could not rule out the possibility of spurious cointegration and it is likely that inflation and nominal interest rates are integrated of different orders.

Extending this study to more recent data, we have shown that the two variables do not even pass the first hurdle of the Johansen cointegration test, leading to the conclusion that nominal interest rates and inflation rates are not cointegrated. This is in direct conflict with the Fisher effect and the inflation model proposed by Fama and Gibbons.

One possible explanation for this conflict is that interest rate changes tend to reflect the state of a country's monetary policy, rather than expected inflation alone. There are many factors that influence a country's central bank's monetary policy, especially in recent years with the downward trend in interest rates which have proven to be downwards-sticky. Visually, we can clearly see from *GRAPH 3* that the long-term trend in interest rates is downwards, and central banks have found it difficult to raise rates significantly without spooking the markets.

Given that we are likely to remain in a state of low interest rates relative to the past, and central bank's willingness to utilize monetary policy to achieve economic objectives, we believe that the Fisher effect and Fama and Gibbons' inflation model are somewhat outdated on the basis of insignificant cointegration between nominal interest rates and inflation rates.

5 REFERENCE LIST

Fama, E. M., & Gibbons, M. R. (1984). A comparison of inflation forecasts. *Journal of Monetary Economics*.

Fisher, I. (1930). *The Theory of Interest.* Clifton: Augustus M. Kelley.

Hjalmarsson, E., & Osterholm, P. (2007). *Testing for Cointegration Using the Johansen Methodology when Variables are Near-Integrated.* IMF: International Monetary Fund Working Paper.