



Monte Carlo Simulation in Finance

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Monte Carlo simulation in finance is powerful when it comes to option pricing or risk management problems

Monte Carlo method can easily cope with high-dimensional problems where complexity and computational demand increase in linear fashion

Monte Carlo method is computationally demanding and needs huge memory for simple problems

Refer Python notebook attached



We highlight different implementation strategies in Python for Monte Carlo-based valuation of European options

All illustrations are based on model economy of Black-Scholes-Merton

Here risky underlying stock price or index level follows under risk neutrality, geometric Brownian motion with stochastic differential equation (SDE)

$$dS_t = rS_t dt + \sigma S_t dZ_t$$



$$C(S_t, K, t, T, r, \sigma) = S_t \cdot N(d_1) - e^{-r(T-t)} \cdot K \cdot N(d_2)$$

$$N(d) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^d e^{-\frac{1}{2}x^2} dx$$

$$d_1 = \frac{\log \frac{S_t}{K} + \left(r + \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}$$

$$d_2 = \frac{\log \frac{S_t}{K} + \left(r - \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}$$



S_t

Price/level of the underlying at time t

σ

Constant volatility (i.e., standard deviation of returns) of the underlying

K

Strike price of the option

T

Maturity date of the option

r

Constant riskless short rate



$$S_t = S_{t-\Delta t} \exp \left(\left(r - \frac{1}{2} \sigma^2 \right) \Delta t + \sigma \sqrt{\Delta t} z_t \right)$$

The variable z is standard normally distributed random variable $0 < \Delta t < T$ time interval

It also holds $0 < t \leq T$ with T as final time horizon



Considering benchmark values for Monte Carlo estimators we implement Monte Carlo valuation of European call option:

(a) Divide time interval $[0, T]$ in equidistant subintervals of length Δt

(b) Start iterating $i = 1, 2, \dots, I$:

1 For every time step $t \in \{\Delta t, 2\Delta t, \dots, T\}$, draw pseudorandom numbers z_t (i)

2 Determine time T value of index level $ST(i)$ by applying for pseudorandom numbers time step by time step to discretization scheme

3 Determine inner value h_T of European call option at T as $h_T(ST(i)) = \max(ST(i) - K, 0)$



$$C_0 \approx e^{-rT} \frac{1}{I} \sum_I h_T(S_T(i))$$