# Dispersion Trading: Construction and Evaluation

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KTH Industrial Engineering
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# Dispersion Handel: Konstruktion och Utvärdering

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Examensarbete INDEK 2013:80
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Industriell ekonomi och organisation
SE-100 44  STOCKHOLM

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## Sammanfattning

Sedan införandet av derivat i den moderna finansiella marknadsplatsen har volatilitets baserade handelsstrategier blivit ett allt viktigt verktyg för kapitalförvaltare. Sen finanskrisen har en populär handelsstrategi varit dispersion handel, men få publicerade studier av dispersion handel existerar. Detta examensarbete syftar till att genomföra en studie av hur dispersion strategier presterar och undersöka deras egenskaper. Detta uppnås genom att identifiera enkla dispersion handelsstrategier, isolera och utvärdera deras egenskaper för att sedan dra slutsatser generellt om dispersion handel. Tre grundläggande dispersion strategier identifieras baserade på optionsspreadar och hur de presterar backtestas. Vi fann att strategierna levererade positiv avkastning med låg marknadskorrelation och acceptabel risk. Vi fann också att transaktionskostnader är en nyckelfaktor för att framgångsrikt använda dispersion handel. Således är det en viktig faktor att tänka på när en dispersion baserad handelsstrategi läggs upp. Ett intressant ämne för vidare forskning är hur handelssignaler såsom den indirekta korrelationen eller den indirekta volatilitetsskillnaden kan användas för att öka lönsamheten. Samt även att undersöka dispersion handels marknadspåverkan.



## Nyckelord

Dispersion handel, spårnings portfölj, indirekt volatilitet, backtesting, option baserad dispersion strategi, option spread.

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**Dispersion Trading: Construction and Evaluation**

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## Abstract

Since the introduction of derivatives into the modern financial market, volatility based trading strategies have emerged as important tools for asset managers. Since the financial crisis a popular trading strategy has been dispersion trading, however few published studies of dispersion trading exist. This thesis aim to perform a study of how dispersion strategies perform and their characteristics. This is achieved by finding basic common dispersion trading strategies, isolate and evaluate their attributes to then draw conclusions in general about dispersion trading. Three basic dispersion strategies are found based on vanilla option spreads and their performance is back-tested. It was found that the strategies delivered positive return with low market correlation and acceptable risk. It is also found that transaction costs is a key-factors to successfully use dispersion trading. Thus it is  a vital factor to consider when creating a dispersion based trading strategy. An interesting topic for further research is how trading signals such as the implied correlation and the implied volatility spread can be used to increase profitability. As well to model market impact from dispersion trading.



## Key-words

Dispersion trading, tracking portfolio, implied volatility, back-testing, option dispersion strategies, option spreads

## Acknowledgement

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# 1 Introduction

*This chapter will begin by introducing the reader to dispersion trading. Then the problems is formulated follow by delimitations and the process.*

## 1.1 Background

Volatility has become a vital area for both practitioners and academics alike. Option trading is not done in dollar but rather in volatility. A natural consequence of this is the increased focus on pure volatility based trading strategies, which usually are based on one of the following volatility properties (Marshall, 2008b):

- Implied volatility usually exceeds realised volatility
- Mean reversion of volatility
- Negative correlation with volatility and the market i.e. fear index
- Volatility smile/skew
- Time impact of volatility

A trading strategy called dispersion trading has in recent post financial crisis years become popular again amongst sophisticated hedge funds and market makers (Deng, 2008). The strategy is essentially based on an arbitrage assumption that realised volatility of an index and the realised volatility of the corresponding basket of underlying stocks should be equal. Previous studies such as Deng (2008) and Marshall (2008b) have identified that the implied relationship is far from satisfied in the market, which makes room for possible arbitrage opportunities. The most common mispricing is a positive dispersion where implied volatility on index options trade at a premium in relation to the basket of single stock options, allowing a trader to sell expensive index options and hedge the position using cheaper single stock options and thereby netting a small profit. This strategy is often able to deliver high return in relation to risk which is why it is favoured by participants able to execute it.

Common dispersion strategies relay on selling ATM[1] straddles[2] on the index and buying ATM straddles on 30% to 40% of the stocks that make up the index (Deng, 2008). Alternatively, strangles[3] can be used. However it is important to consider that this strategy does not represent an arbitrage in its purest form since the strategy is still associated with correlation risk[4] but rather a bet on a mispricing in the market (FDAXHunter Equity Derivatives, 2004).

A major disadvantage with dispersion trading is the wide reliance on quantitative models and need for executing several legs i.e. several transactions for

[1] At The Money.
[2] An option spread, see Chapter 3.9
[3] Another option spread, see Chapter 3.9
[4] Unless the implied correlation factor is above 1.

each trade. This stops many participants from executing the strategy as extensive research is needed. Combined with advanced trading systems to monitor positions, identify trading opportunities and execute multiple legs simultaneously. In addition to being able to trade with low transactions costs and in large sizes[1] which otherwise severely dampens the profitability of the strategy.

Dispersion trading and other volatility based strategies can be viewed as complex and quantitative strategies. However these strategies are based on the same logic as any other strategy, buy low and sell high. The only difference is that volatility is instead bought low and sold high. Volatility cannot be directly observed as stock quotes can, thus an extra layer of complexity is added since volatility needs to be estimated from other variables using models (Tsay, 2002; Sinclair, 2008).

## 1.2 Problem formulation

Increased availability of cheap market data and processing power in combination with growing derivatives markets is shifting the focus of institutional investors to quantitative trading strategies. One popular strategy is dispersion trading, however there is a gap in published academic research of investigations into practical dispersion trading strategies and their profitability. This is even more evident when only looking at the Swedish market. An important problem is then that *there is little research-based knowledge about dispersion trading strategies and their profitability.*

## 1.3 Research questions

The study is divided into two sections (1) identify basic dispersion based strategies and (2) test their performance. Two research questions are investigated:

**RQ1:** *What would possible strategies look like?*

**RQ2:** *What is the strategy performance after trading costs?*

## 1.4 Purpose

The purpose of this study is to evaluate the performance and characteristics of dispersion trading. This is to be achieved while taking a practical approach to provide results relevant for investors seeking to create a dispersion strategy.

## 1.5 Contribution

Little academic research is found on dispersion trading. One example is Marshall (2008b) who made a study in empirical evidence of the P&L of dispersion trading. The study is based on the American stock index S&P500. Our study

[1] Several thousand contracts at a time.

will instead be based on the Swedish equity index OMXS30 and will verify if the same results can be observed in the Swedish equity market.

Furthermore this study will not focus on proving the existence of trading opportunities as Marshall (2008b) in general, but rather test the hypothesis that profitable dispersion trading strategy exists. This will give the study a more practical trading focused approach to dispersion trading.

## 1.6 Delimitations

When developing a complex trading strategy such as dispersion trading there are many factors which will affect the P&L of the strategy. For example a whole study can be focused only on how correlation between stocks and the market is calculated. Therefore many simplifications and assumptions had to be made in order to reach the final goal of a complete strategy.

There are several types of dispersion trading strategies and it is outside the scope of this study to evaluate them all. Thus the study is limited to testing some of the most common types of dispersion based strategies. This is translated into a test of three basic strategies which are introduced in Chapter 3.9.

This study will not evaluate the use of different entry and exit signals and the strategies will instead always be active and have positions. Furthermore intra-day data is available to us but given the number of calculations needed and time available to optimise functions; end of day data will be used. The disadvantage with end of the day data is that this data only record the final state of the day and not any information about what took place inside the day.

It is decided to only delta hedge once a day. In a more realistic situation delta hedging should not be defined by a time condition but rather a delta deviation threshold and not only take delta into consideration but also other greeks such as gamma and gamma squared (Taleb, 1997).

Modelling market impact for strategies such as dispersion trading is a large problem in itself and is not included in this study.

It is also decided to only focus on the Swedish equity market and not to include other markets in the study.

## 1.7 Prior knowledge

It is assumed that the reader possess prior knowledge about basic equity derivatives and their pricing, different types of volatilities such as historical and implied, basic understanding of statistics and financial modelling.

## 1.8 Process

This study solve many different problems to arrive at the final goal of a working dispersion trading strategy. The aim of this study is not to always find the optimal solution, but rather develop plausible strategies which can work as examples for further studies to improve upon. Figure 1 illustrate all the necessary tasks identified to reach this goal.

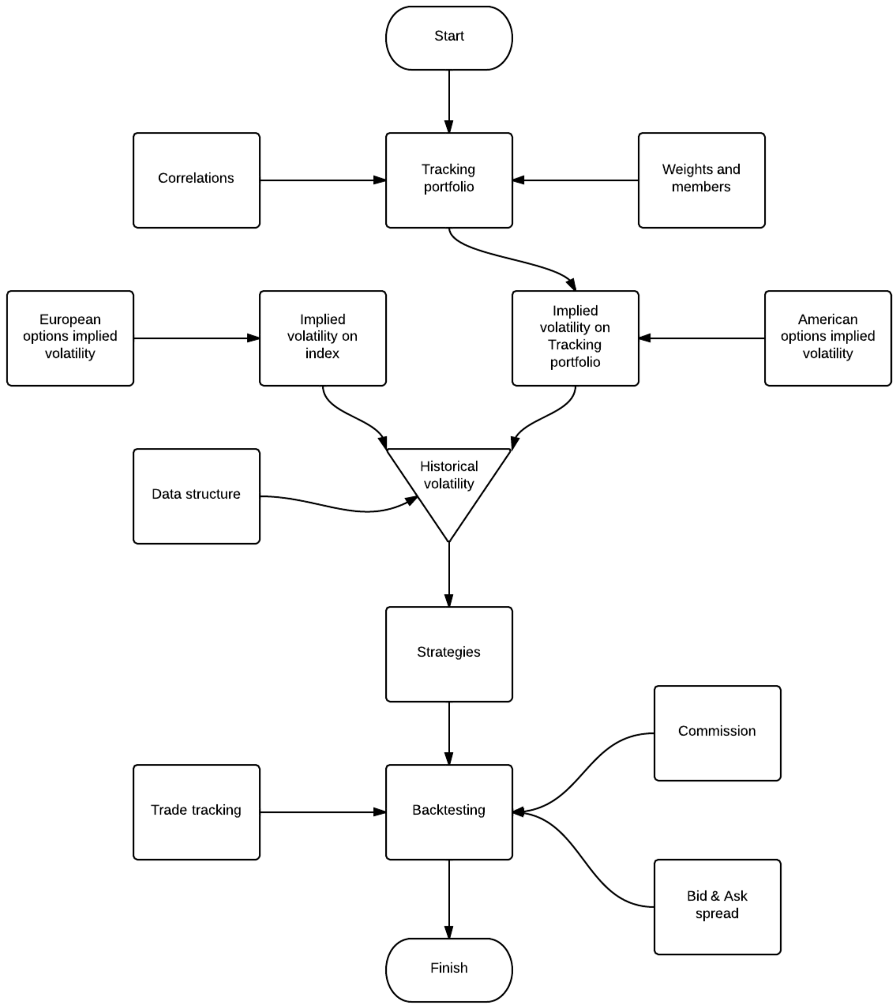


*Figure 1: The required main tasks and their interconnections which need to be finished in order to complete the study. All tasks are organised such that they need to be finished in a descending order. The study will start by developing a tracking portfolio. Then it is necessary to calculate the historical volatility smile and design strategies to test. Finally the strategies are tested and analysed to draw conclusions about dispersion trading.*

The first major step is to establish a tracking portfolio. The tracking portfolio is the portfolio of single stock options which will be used to hedge index positions. The tracking portfolio can be configured using different members and weights which determine how well it can track and thus hedge positions in the index. To interpret results of the strategy and understand what effects play a role, it is necessary to calculate the implied historical volatility smile surface. This is achieved by structuring options and calculating their implied volatility. Three prototype strategies will be tested in order to evaluated dispersion trading characteristics illuminate important aspects. They are tested by using a back-testing model and it is these results that will serve as a foundation for the conclusions.

## 1.9 Disposition

Following this introduction is a literature review, Chapter 2 covering other related and relevant research. This gives the reader a broader view of the subject and the current research situation. After the literature review relevant theory is given in Chapter 3. It covers all necessary knowledge required to perform this study. In the methodology section, Chapter 4, all methods and their implementation are described as well as how the results will be measured and verified. This is followed by an introduction and discussion of data-types and sources in Chapter 5. Then the results of the study is presented in Chapter 6 followed by the discussion in Chapter 7 of results. Finally the study is summarised in the conclusion, Chapter 8.

# 2 Literature review

*In this chapter prior studies on dispersion trading and related subjects are covered. It is divided into different relevant problems which are then discussed using the results from these studies.*

A literature study on dispersion trading is conducted. It shows that little first-hand published research exist on the subject. There are a few studies evaluating dispersion trading in general such as FDAXHunter Equity Derivatives (2004); Ganatra (2004); Deng (2008); Marshall (2008b). None of these studies take a holistic and practical approach to dispersion trading. Simplifications are made in order to investigate specific aspects of dispersion trading. However none develop a final and complete strategy, test the strategy and publish its results. This is not surprising as a successful study of that type would be kept secret to prevent others to profit from it. This study will have a less detail centred approach where simplifications and assumptions are made to allow this study to instead produce a complete strategy. Another difference between this study and others is that none other was found which investigate dispersion trading for the Swedish stock market[1].

## 2.1 Why the mispricing persist

It is not entirely clear why the dispersion mispricing persist but two main arguments are made: (1) a risk based hypothesis which argues that various risks, e.g. volatility risk and correlation risks motivates a premium for index options in relation to stock options. This hypothesis is argued by Bakshi et al. (2003) which relates the premium to the difference in risk-neutral skewness of their underlying distributions. Driessen et al. (2005) argues that index options have a risk premium which is absent in stock options because index options hedge correlation risk, which is particularly pronounced in index option puts. (2) market inefficiency where market supply and demand of index and stock options drive option premiums. This hypothesis is among other supported by Bollen and Whaley (2004) who according to them the net buying pressure of index options drive option premiums. This is also supported by Garleanu et al. (2009). They developed a model showing that option premiums increase with market demand. Furthermore Lakonishok et al. (2007); Garleanu et al. (2009) showed that end investors are net short single-stock options and net long index options which translates into a net demand for index options and net supply of single-stock options.

Thanks to major changes in the U.S. option markets around 1999 and 2000 an opportunity for a natural experiment arose which was investigated by Deng (2008). He showed that the profitability of dispersion trading was reduced after the 1999 to 2000 period which would confirm the supply and demand hypothesis. The changes in the market decreased the cost of arbitrage by introducing cross listing of options which led to a decreased bid-ask spread.

[1] Nasdaq OMX Stockholm

The study was performed by selling ATM straddles of the S&P500 index and buying ATM straddles on all single stocks comprising the same index. The results were that dispersion trading was profitable prior to the 1999/2000 changes and then the monthly profitability was reduced by 24% after the changes. These results support the supply and demand line of arguments. Other dispersion strategies were also tested in the study to verify the results.

The study support the use of dispersion trading, since if the mispricing of options can be motivated on a risk basis it follows that dispersion trading should not be profitable in the long run given hidden risk[1]. Deng (2008) also use implied correlation as an indicator for when to initiate dispersion trades.

## 2.2 Emperical evidence of dispersion trading

Marshall (2008b) develops empirical evidence for dispersion trading in the U.S. options market. She was able to prove the tendency of positive dispersion, i.e. index options tend to trade at higher implied volatility in relation to the corresponding portfolio. The study is performed on the S&P500 index and underlying stocks for a two years period. End of day data is used and implied volatility for all stocks are obtained from synthetic VIX-indexes provided by Bloomberg. The study is highly relevant since it proves the existence of trading opportunities with dispersion trading. Unfortunately the use of synthetic VIX-indexes limits the applicability when designing a dispersion strategy intended to trade on. Because VIX-indexes calculate the average volatility of options on a certain instrument weighted to a 30-day to maturity period. The difference here for a trader is that he cannot trade all the strikes used to calculated the index but will rather buy and sell individual options. These options will trade at different volatilities than the averaged VIX-index depending on the then current volatility smile and term structure, see Chapter 3.7.1 and Chapter 3.7.2 for more information about the smile and the term structure.

Consequently P&L calculated in this study works as an indicator of how profitable dispersion trading can generally be, but it does not provide evidence for a plausible strategy and its performance.

## 2.3 Dispersion trading using variance swaps

Ganatra (2004) take another approach to dispersion trading by using variance swaps instead of vanilla options. Variance swaps are OTC derivatives sold by investment banks which return is determined by the variance of an asset. Variance/volatility swaps is a very popular approach for investors to build exposure to dispersion trading in a direct manner (Kolanovic et al., 2010).

Ganatra (2004) implements a strategy and show profits from using variance swaps in a dispersion strategy configuration for the Euro stoxx 50 index and its constituents. Variance swaps are interesting tools that allow a more direct approach to dispersion trading than using vanilla option spreads. Nevertheless after the financial crisis of 08/09 liquidity in single stock variance swaps

[1]In the long run equals enough time for the hidden risk events to take place.

have decreased (Martin, 2013) and thus investors are left with the vanilla option approach. Furthermore variance swaps are OTC instruments created by investment banks, thus a third party is inserted between the two investors or investment banks act as the counterpart. In both cases investments banks would want to be compensated for the risk they are taking by providing these products and so it is logical to assume that worse prices will be obtained using variance/volatility swaps than if the dispersion trader create the exposure her self using plain vanilla options.

# 3 Theory

*In the Theory chapter relevant theory for this study will be introduced. This will serve as a foundation for which the methods are then chosen on. First the concept of dispersion trading is given in greater detail followed by the theory needed to model it.*

## 3.1 Dispersion trading

Dispersion trading is a volatility based strategy seeking to profit from difference in implied volatility between similar instruments. The fundamental relationship usually builds on that given two instruments with the same cash flow their prices' should be the same (Berk and Demarzo, 2011). Translating this basic fact to the modern financial market there are cases where several different instruments share the same return, for example tracking the same index[1]. This is the case dispersion trading build on where index options are one type of instrument and single stock options are the other. In theory the cash flow of index options can be replicated by selecting the correct single stock options and combining them in a portfolio.

An index option is an option on an index. The index is constructed using a portfolio of underlying instruments which in this case consist of the 30 most actively traded stocks on the Stockholm stock exchange[2]. Consequently the index option can be viewed as an option on the portfolio of stocks used to calculate the index (FDAXHunter Equity Derivatives, 2004). Then it is possible to replicate the index option by selecting single stock options consisting of the same portfolio used to calculate the index, see Figure 2.

In the case of OMXS30 it means that it is possible to replicate the cash flow and thus the price of an OMXS30 index option by combining single stock options of the 30 most traded stocks i.e. the stocks constituting the OMXS30 index. Assuming that options on all 30 stocks are used in the weights corresponding to the index. In an efficient market the price of the index option and the portfolio of single stock options should then be the same. Interestingly it has been observed that this is not so. For example Marshall (2008b) were able to prove the existence of misspricings for the American S&P 500 index and Hardle and Silyakova (2010) identified the same occurrence in the German DAX index. The tendency is that index options are expensive in relation to the corresponding portfolio of single stock options. This give rise to a trading opportunity since it is possible to sell expensive index options and then buy a cheaper portfolio of single stock options, which is what dispersion trading is all about. In this context expensive equals that index options tend to trade at higher implied volatility than the corresponding portfolio of single stock options.

Unfortunately dispersion trading is not as easy as this sound. Several conditions complicate and limit the possibility to trade on the miss pricing.

[1] I.e. an index ETF and futures on the same index.

[2] The OMXS30-index

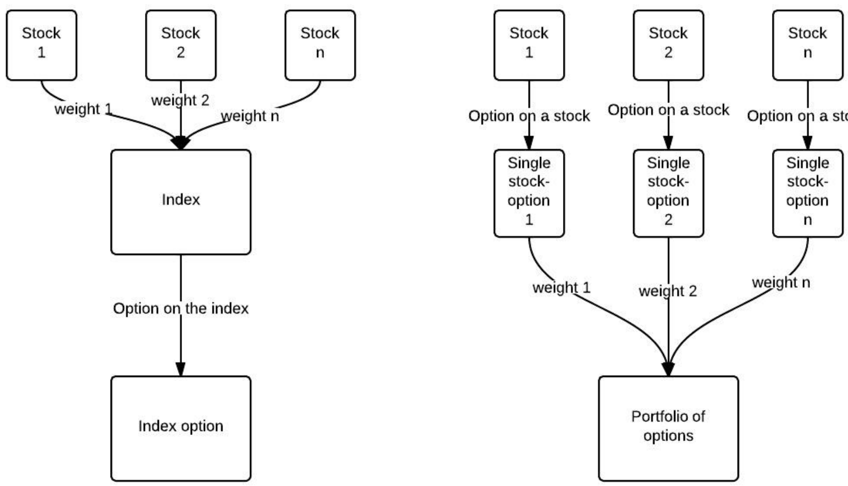


*Figure 2: The similarity between the income streams of an index option and a portfolio of options. An index option is an option on an index. Since the index itself is calculated from a portfolio of stocks using certain weights the index option can be replicated by using the very same weights directly on options of the stocks comprising the index. Both these methods should in theory yield the same return regardless of if it is an index option or a portfolio of single stock options.*

1. There are not always single stock options available to trade on all the stocks needed to construct the index.
2. Even if there are single stock options available for all stocks required to construct the portfolio it is not feasible to do it when taking transaction costs into consideration.
3. The miss pricing is small which mean that only sophisticated traders able to trade with small transaction costs, in large quantities and have access to trading systems able to handle the many individual trades can profit from dispersion trading.

These conditions is usually addressed by only trading some of the single stock options used when constructing the index rather than all of them called partial index tracking (FDAXHunter Equity Derivatives, 2004; Deng, 2008). This mean that there are fewer stocks to manage, which minimise the effect of (1), (2) and (3). See Figure 3.

The problem with partial index tracking is that then it is not possible to entirely replicate the return of the index. Hence dispersion trading is no longer risk free as the instruments used for hedging the index exposure cannot entirely hedged the risk. A correlation risk arises between the index position and the hedging portfolio of single stock options used[1]. Therefore a key problem to dispersion trading is to construct the tracking portfolio in such a way that it limits the correlation risk as much as possible.

## 3.2 Developing a dispersion trading strategy

The following problems need to be solved in order to reach a working dispersion strategy.

- A tracking portfolio of single stock options has to be optimised.
- A correlation matrix for the stocks has to be calculated.
- Implied volatility of index options have to be calculated.
- Implied volatility of the single stock options have to be calculated.

## 3.3 The tracking portfolio

When constructing a tracking portfolio it is necessary to determine which instruments should be used and which weights in the portfolio they should have. The complexity of the problem is different depending on which approached is taken. The problem can either be modelled as a linear problem for which many methods exist or as a non-linear problem. The later approach is more complex and computer intensive as it usually incorporate different numerical solutions

[1]The hedging portfolio is often referred to as a tracking portfolio or a proxy portfolio.

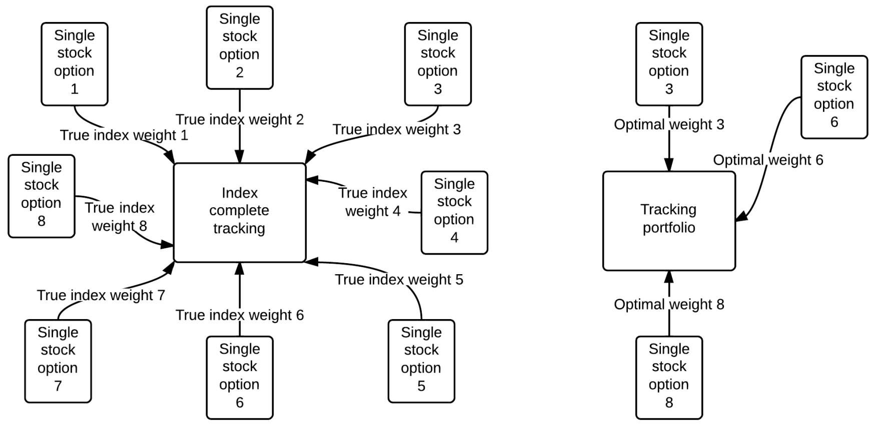


*Figure 3: The difference between complete index tracking and partial index tracking. In this example an index consisting of 8 stocks is completely replicated to the left using the true index weights and to the right a partial tracking is performed, only using 3 of the stocks from the index. Note that in the latter case new weights have been optimised to minimise the tracking error.*

but it also enables us to optimise with non-linear constraints in mind, such as transaction costs.

The problem with creating a tracking portfolio is well researched e.g. Rudolf et al. (1998); Beasley et al. (2002); Blume and Edelen (2002); Corielli and Marcellino (2005); Roland and Berg (2008); Barro and Canestrelli (2008); Krink et al. (2009). The reason for this is that the same problem is faced by index tracking funds that represent a large portion of assets under management (Blume and Edelen, 2002).

Different methods are associated with different advantages and disadvantages. Fundamentally the main difference is if we are to treat the problem as a simplified linear problem or create a more advanced model able to take non-linear constraints into consideration. Popular linear approaches are based on linear regression analysis or principle component analysis while popular non-linear approaches are based on genetic algorithm (Maringer and Oyewumi, 2007). Numerous implementations exist for these methods with different characteristics and improvements such as using weighted data or incorporating faster converging solutions. The fundamental characteristics of the two approaches are summarised in Table 1.

Krink et al. (2009) use differential evolution which is a version of genetic algorithm to successfully construct a tracking portfolio. Another example is Beasley et al. (2002) who are able to use evolutionary heuristics which is also based on the idea of genetic algorithm to construction an index tracking portfolio. The major advantage with these methods over regression or PCA is that they are able to individually model transaction cost and other possible costs

| Name | Advantage | Disadvantage |
|---|---|---|
| Genetic Algorithm | Non-linear constraints can be used such as, transaction costs, maximum number of stocks allowed. | Computational ineffective and complicated to implement. |
| Regression or PCA | More computational effective. | Cannot take non-linear constraints into consideration. |

*Table 1: The advantages and disadvatanges of using linear or non-linear models for the tracking portfolio.*

associated with adding a specific stock.

## 3.4 The correlation matrix

The next component for the dispersion trading strategy is to establish the correlation matrix. Since the goal is to calculate the volatility of the tracking portfolio and the portfolio consists of instruments with different cross correlations all those need to be calculated. For example E.q. 1 illustrate how the variance of a portfolio is calculated consisting of three stocks (Berk and Demarzo, 2011).

$$\begin{aligned} \sigma_p^2 = w_A^2\sigma_A^2 + w_B^2\sigma_B^2 + w_C^2\sigma_C^2 + 2w_Aw_B\sigma_A\sigma_B\rho_{AB} + \\ 2w_Aw_C\sigma_A\sigma_C\rho_{AC} + 2w_Bw_C\sigma_B\sigma_C\rho_{BC} \end{aligned} \tag{1}$$

Where:

1. $\sigma_p^2$ = Variance of portfolio
2. $w_A$ = Weight of asset A
3. $\sigma_A^2$ = Variance of asset A
4. $\rho_{AB}$ = correlation asset A and B

The equation was originally developed by Markowitz (1952) where he showed how in its general form, E.q. 2, the variance for a portfolio with n stocks is calculated.

$$\sigma_p^2 = \sum_{i=1}^{n}\sum_{j=1}^{n} w_iw_j\sigma_i\sigma_j\rho_{i,j} \tag{2}$$

E.q. 2 is then used to populate the correlation matrix. The problem with a correlation matrix is that it grows quickly. For example in the case of three assets a, b and c in a portfolio; the correlation matrix is $3 \times 3$ large and look as E.q. 3.

$$\begin{bmatrix} \rho_{a,a} & \rho_{a,b} & \rho_{a,c} \\ \rho_{b,a} & \rho_{b,b} & \rho_{b,c} \\ \rho_{c,a} & \rho_{c,b} & \rho_{c,c} \end{bmatrix} \tag{3}$$

Thus for a portfolio of only three stocks the correlation matrix contains nine cross correlations. Fortunately the correlation matrix can be compressed. First of all, the diagonal of the matrix represent the correlation between the asset and itself which naturally is one. Secondly the correlation of asset a and b is the same as the correlation of b and a i.e. the order does not matter. This means that the number of correlation factors which needs to be calculated can be reduced for n stocks to E.q 4.

$$\frac{(n \times n - n)}{2} \tag{4}$$

The problem with dispersion trading is that it often requires many stocks in the tracking portfolio. S&P500 is a popular dispersion trading index. It contains 500 stocks for which the compressed correlation matrix would contain 124 750 elements. Furthermore to calculate the correlation for one element thousands of samples are used which translate into millions of calculations for the whole matrix.

This is quite cumbersome but it can be further simplified as shown by Marshall (2008a) by assuming that the portfolio contains zero unsystematic risk. Risk can be divided into two categories systematic and unsystematic. Systematic risk is the risk that is shared by the market and the only risk left in a market portfolio. Unsystematic risk is the risk from one stock, i.e. the risk that can be diversified when incorporated into a portfolio. In this case the aim of the tracking portfolio is to replicate an index i.e. a market portfolio. Consequently it is safe to assume that the correlation between the tracking portfolio and the market portfolio is close to one and thus it contains very little unsystematic risk. Now it is possible to calculate the standard deviation on the portfolio only by taking the systematic risk into consideration, which translates into E.q. 5.

$$\sigma_m = \sum_{i=1}^{n} w_i \sigma_i \rho_{i,m} \tag{5}$$

*Note that the portfolio is now notated with m i.e. it is a market portfolio.* Using this equation the number of cross correlations are greatly reduced to only n. Since now it is only required to calculate the return correlation between stocks in the portfolio and the market portfolio.

## 3.5 Implied volatility of index options

Calculating the implied volatility of index options is a fairly conventional process as they are of European type. European options are the most basic type of option, consequently there are no troublesome issues such as early exercise or Asian tails to take into account when valuing them. Thus effective analytical solutions such as the Black-Scholes-Merton model[1] are available to us (Hull, 2008). Not much effort will be used to explain the Black-Scholes model as there are numerous well written sources on the subject e.g. Hull (2008) but essentially it is a differential equation that is satisfied by the derivative's price.

One of the inputs to the differential equation is the volatility of the underlying instrument. Unfortunately the Black-Scholes differential equation cannot directly be solved for the volatility. Instead the implied volatility is backed out by guessing at an initial volatility, calculating the resulting price using the Black-Scholes model and then compare it to the current market price. Then guesses at a newer better implied volatility and repeats the process (Hull, 2008).

This process is much more computational effective than other numerical solutions such as binomial tree, Monte Carlo simulations or finite difference methods.

The original Black-Scholes model assumes no dividends being paid on the underlying instrument. Since the OMX30 index is not a total return index i.e. dividends being paid is not compensated for in the index calculations this would be a problem. To clarify, many stocks in the OMXS30 index tend to pay out dividends in March and April each year. This means that a valuation being performed prior to those months using OMXS30 as underlying on an option expiring after the dividend period would be incorrect. Since the future value of the index adjusted at the risk free rate is not the anticipated future value of the index[2]. In practice though, this is not a major problem as the true underling instrument of index options are not the index but futures on the index. These futures trade at the correct future value of the index at expiration and thus by using them as the underlying security we circumvent the problem.

The yield to use as the risk free rate is another factor when valuing options. One important factor for the rate is the yield curve. The yield curve show how the yield of bonds on the same issuer change depending on time to maturity (Nyberg et al., 2012). It is created by arranging and then plotting rates of bonds of the same issuer depending on its time to maturity left, an example can be viewed in Figure 4.

The yield curve can look very different from one instance to another but one tendency is for it to trade in contango where rates further out are higher than rates closer to maturity (Nyberg et al., 2012).

Since the yield can change depending on time to maturity it is important to use the correct yield when valuing options. Thus an option with 30 days to maturity should be valued with the yield on the yield curve corresponding to 30

[1]Also known as the Black-Scholes model.

[2]We know that the stocks constituting the index will pay out dividends and thus fall in price.

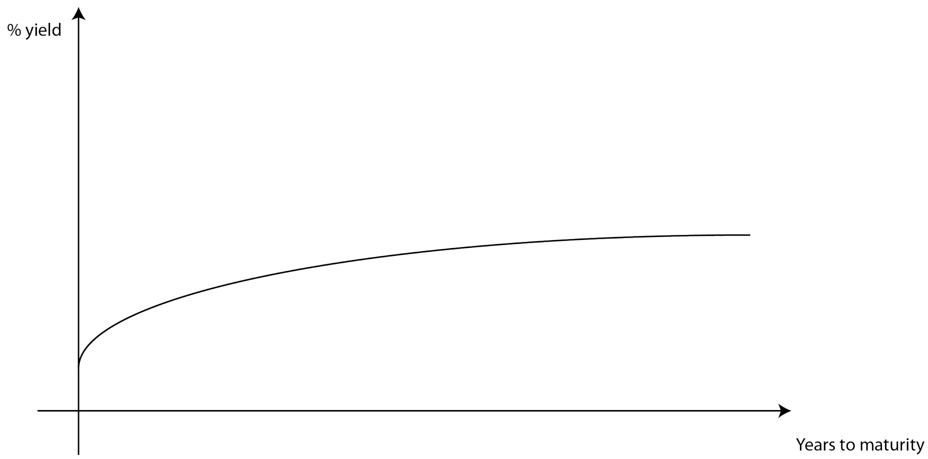


*Figure 4: The yield curve for an issuer. The yield is plotted on the y-axis depending on time left to maturity on the x-axis. In this case the yield curve is in contango, where bonds with more time to maturity has a higher yield.*

days to maturity and an option with one year to maturity should respectively be valued with a yield that has one year to maturity.

## 3.6 Implied volatility of single stock options

As was seen in the previous chapter, calculating implied volatility for European style options can be accomplished using closed form solutions such as the Black-and-Scholes formula. Unfortunately no analytical evaluation approach exist for American options which is the type single stock options are.

The difference between an American and a European option is that an American option can be exercised whenever during the option's life time as opposed to European options that can only be exercised on the maturity date. This is called early exercise. In the case of no dividends it is normally not profitable to exercise before expiration (Hull, 2008). The reason for early exercise being unprofitable is as follows. Imagine an American call option with one month to expiration when the stock price is $50 and then strike price is $40. This option is deep in the money and an investor could be tempted to use his right to early exercise. However if the investor is planning to hold the stock for longer than one month it would instead be better to wait to maturity as he will earn the risk free rate till that date. Since the investor now control the stock with the option but does not need to pay for it until exercise. Furthermore if the stock drops below the $40 exercise price the investor will be happy to have kept the option and if the stock continue to increase in value so will the option by as much. Thus it will always be profitable to keep an American call option to maturity. However in the case of American put options it can sometime be better to use early exercise. Imagine again an American option but this time it is a put option with the exercise price at $10 and the corresponding stock price is almost zero. In this case the put option is deep in the money. If an investor choose to wait to maturity he will lose the risk free rate till then as it is better to receive the money from selling the stock now rather than later. Furthermore as the stock price cannot become negative the investor do not stand to profit more from the put option and can only lose if the stock appreciate in value (Hull, 2008). This is only the case when the put option is deep in the money which will almost never happen in a dispersion trading scenario and so is of little consequence to this study.

However this reasoning assume no dividends during the one month life time of the option which will not always hold true. The problem with discrete dividends is that if the ex-dividend is just prior to the maturity date and the dividend is large enough it will be better to early exercise (Hull, 2008). Consequently the problems has constraints which are hard to address in an analytical solution which is why American options are usually valued using numerical approaches.

Frequently one of these three types are used to value American options:

- Binomial trees
- Monte Carlo simulations
- Finite difference method

The finite difference methods employ a method where the derivative's value is calculated by solving the differential equation that the derivative satisfies by in turn converting it to a difference equation (Hull, 2008). Monte Carlo simulation involves taking random numbers from a distribution and sample many different paths the variables underlying the derivative can take. Then the pay-off is calculated for all paths and the arithmetic average is calculated and finally discounted with the risk free rate. Binomial trees divide the time till maturity into discreet intervals of time for which the stock can either go up or down which creates a tree. A derivative is then calculated by starting at the end of the tree and trace back through it. Normally Monte Carlo simulations are used to calculate the price if the derivative's price is dependent on its past history i.e. path dependent as Monte Carlo simulations does not calculate the price backwards as trees and finite difference does (Hull, 2008).

As was explained in the last paragraph the binomial tree approach involves dividing the options life into discreet time intervals. During this time interval it is assumed that the underlying asset S can either move up or down to the new values Su or Sd, see Figure 5. This procedure can then be extended into many levels which create a binomial tree, see Figure 6.

A option's value is calculated by starting at the end of the tree when time is equal to T[1], at this point the value of an option is:

- Put: $K - S_T$

[1] Time of maturity.

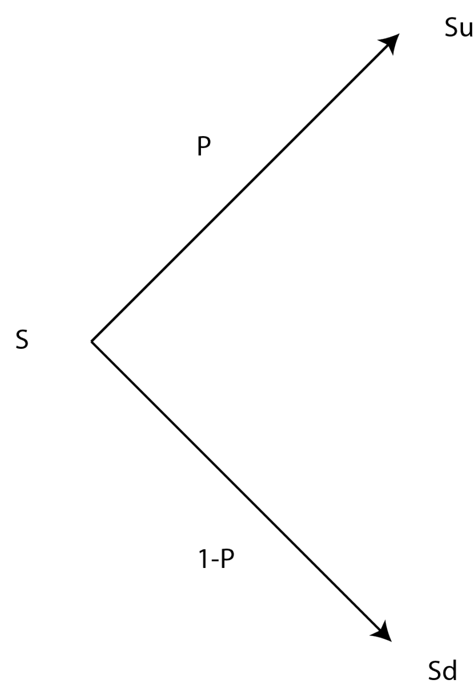


*Figure 5: A one step binomial tree. It shows how the price of the underling instrument can move up or down in price during a discrete time period. The chance of the price move being upward is P and consequently the chance of the move being downward is 1-P. The price move is calculated by multiplying the original price S with the factor u for upward and d for downward resulting in the price Su and Sd respectively.*

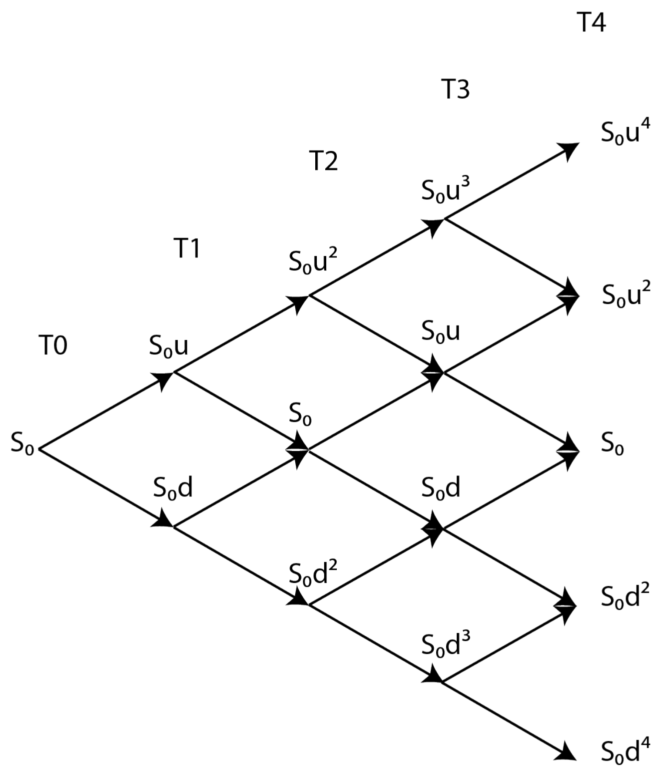


*Figure 6: A four level binomial tree. A price jump can either be upwards size u or downwards size d. There is a boundary condition that the option price at the end of the tree is equal to $K - S_T$ for a put or $S_T - K$ for a call. This is then used to step backwards through the tree until the starting point is reached. The price at every node is calculated using the two prior prices, their probabilities and is then discounted at the risk free rate.*

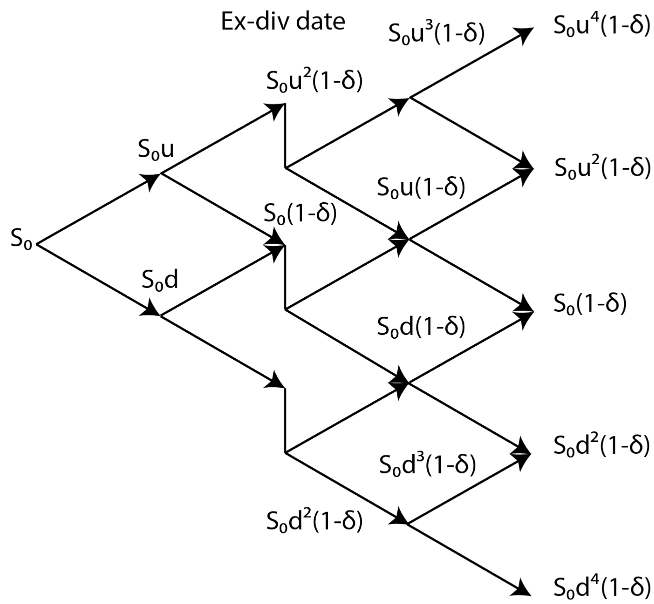


*Figure 7: The binomial tree for a stock during a period where it pays dividend. The tree is adjusted downward on the ex-dividend day by the corresponding dividend amount.*

- Call: $S_T - K$

Where $S_T$ is the underlying asset price at time T and K is the strike price of the option. Then one time step delta t is taken backwards in time and that price is calculated using the two previous prices and probabilities discounted to the risk free rate (Hull, 2008). This process is repeated till the price at all nodes has been calculated. Finally a check is performed to see if it is more profitable to early exercise the option than to hold it one more time period delta t.

The tree method also enable the implementation of adjustment for discrete dividends. In the case of index options the underlying asset is the future which is adjusted by the market for dividends as previously explained, but in the case of single stock options it is the stock that is the underlying instrument and it is not adjusted for dividends. Instead the stock is traded down by the dividend amount on the ex-dividend day. This is called a discrete dividend where the price of the stock is adjusted between two discreet time interval rather than continuously. This creates a problem when valuing options since the model does not know the stock is trading down because of dividends. One solution to this is to simply adjust all stock prices for dividends. Another approach is to adjust the tree structure where it is shifted downwards on the ex-dividend date, see Figure 7.

The binomial tree model calculates the price of options but it is implied volatility that we are interested in. The translation into implied volatility is then accomplished in the same way as for European options by an iterative process.

The volatility that minimise the difference between the price calculated using the Cox-Ross-Rubinstein tree model and the real observed price in the market is found by repeatedly guessing at volatilities (Hull, 2008).

## 3.7 Term structure and option strikes

In theory, options with the same underlying instrument should trade at the same implied volatility regardless of the options strike and maturity (Hull, 2008). This was mainly true in the early days of derivatives trading but since then traders have become aware of other factors affecting an option's price depending on its strike and maturity. This becomes a problem when back-testing dispersion trading strategies since changes in time and strikes will affect the P&L of the strategy. Consider a dispersion spread that is initiated with exactly 30 days to maturity. If one day pass and all things else being equal the spread will be affected by the options term structure since it is now only 29 days left to maturity. Thus it will be necessary to commit to a fixed time-structure and strike price.

### 3.7.1 The volatility smile-skew

The volatility smile is the tendency for options to trade at different implied volatilities depending on their strike price. In a perfect world all options of the same underlying and maturity should trade at the same implied volatility regardless of their strike price. However options with lower strike prices usually trade at higher implied volatility and deep out of the money or deep in the money options also tend to trade at higher implied volatility (Hull, 2008). By plotting options' implied volatilities on a graph depending on their strike prices on the x-axis it would result in a curve similar to a smile, hence the name *volatility smile*. The volatility smile looks a bit different depending on the asset class. In forex markets it often look like a smile, see Figure 8 but in the equity markets it usually has more of a skew tendency, see Figure 9.

No clear explanation exist as to why the volatility smile exist but a possible explanation can stem from that market participants assume another return distribution than a log-normal one, which is assumed in the Black-Scholes-Merton model (Hull, 2008). Prior to the stock market crash of October 1987 equity options traded at about the same volatility but after the crash the smile-skew behaviour became more dominant. This imply that traders assume a fat-tail distribution, particularly the left tail, i.e. participants are aware of the risk of new statistical anomalies. Another explanation can come from leverage. When a stock decline in value the company's leverage increase which translates into a higher risk (Hull, 2008). Thus option sellers require a higher implied volatility to compensate for the increased risk.

As previously mentioned the volatility looks a bit different depending on what type of asset class is plotted. Another tendency for the smile is to change depending on what type of market it is i.e. bull or bear market. For example

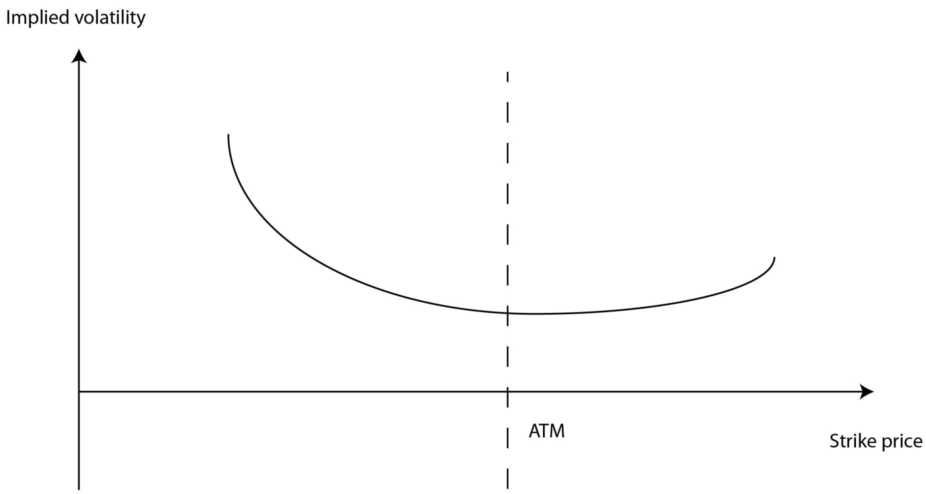


*Figure 8: The volatility smile. Options' implied volatility is plotted depending on their strike prices' that result in a curve reminding of a smile, hence its name. Deep Out of the money (OOM) options are more expensive than at the money or even slightly in the money options. This pattern is usually even more pronounce for put options than for call options. Implied volatility is plotted on the y-axis depending on the strike price on the x-axis.*

if the market is currently experiencing a crash the smile become steeper as can be seen in Figure 9 and further increase the skew.

These changes in the volatility smile can be used to further increase profits from dispersion trading. One possible strategy is to sell strangles[1] instead of straddles[2] on the index leg. This allow a investor to sell more expensive OOM options using the volatility smile and buy cheaper protection on the tracking portfolio using ATM straddles. Then the ATM straddles should be cheaper not only from the dispersion effect but also from the volatility smile.

### 3.7.2 The term structure

Another factor is that options of the same underlying tend to trade at different implied volatilities depending on their time till maturity (Hull, 2008). The volatility term structure illustrates this effect by arranging options by time to maturity and plotting their implied volatility on the y-axis. The tendency for this structure is to trade in contango i.e. the front month being cheaper than further out in the time structure, see Figure 10.

In a bear market the term structure usually breaks over into backwardation i.e. options close to maturity trade at higher implied volatility than options further out, see Figure 11.

[1] See Chapter 3.9.2.
[2] See Chapter 3.9.1.

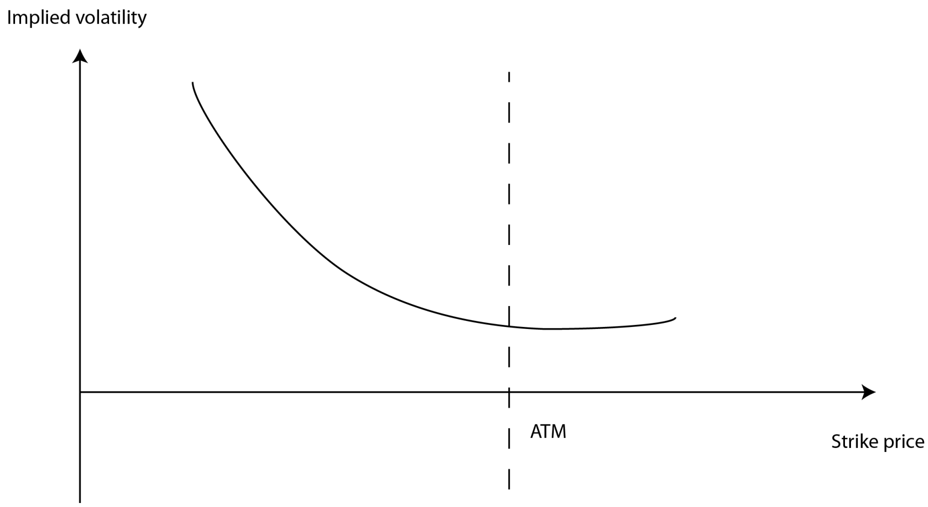


*Figure 9: The volatility skew. Here options of lower strikes are priced at even higher implied volatility, that result in a steeper sloping curve. This is often referred to as a volatility skew instead of a volatility smile which is more pronounced in equity markets. The tendency is also for the skew to steepen in bear markets. Implied volatility is plotted on the y-axis depending on the strike price on the x-axis.*

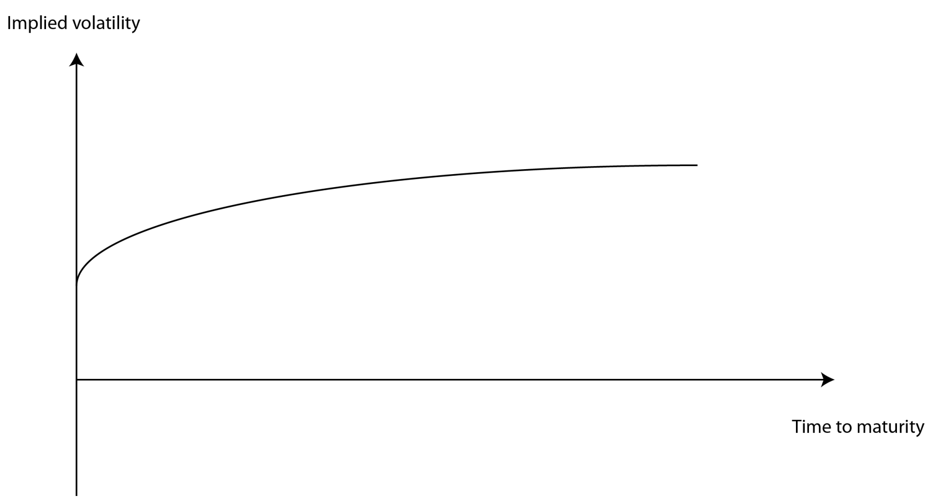


*Figure 10: The implied volatility term structure in contango. As can be viewed, options closer in time till maturity trade at lower implied volatility levels than options further out in the term structure. Implied volatility is plotted on the y-axis against time left to maturity on the x-axis.*

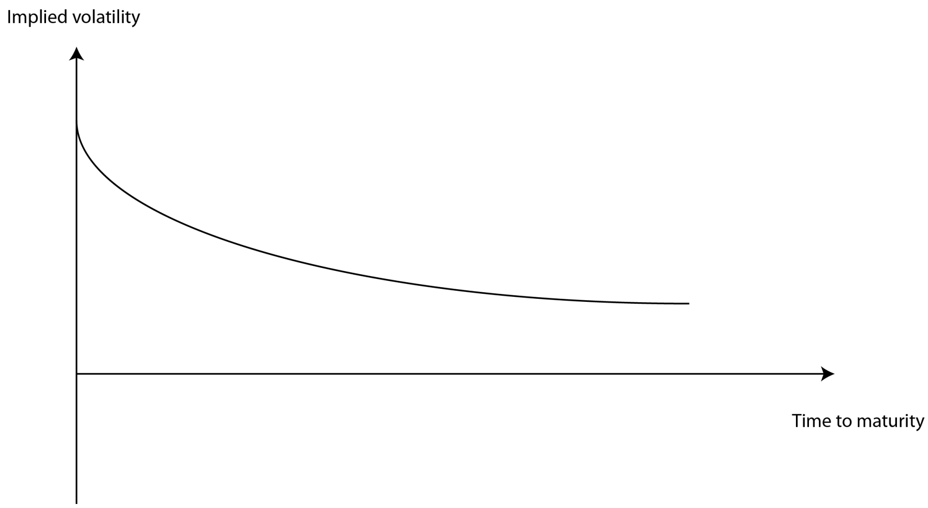


*Figure 11: The implied volatility term structure but now in backwardation. I.e. options closer to maturity tend to trade at higher implied volatility than options with longer time till maturity. Implied volatility is plotted on the y-axis against time left to maturity on the x-axis.*

There is no clear reason for this effect but one possible explanation can come from the fact that volatility exhibits mean reversion characteristics (Natenberg, 1994). So when implied volatility is high compared to historical volatility the market assume this will not persist and price future implied volatility lower compared to current implied volatility. Consequently when implied volatility is historically low the expectation is that it will increase in the future (Hull, 2008).

## 3.8 Dispersion trading as average implied correlation

A complementary way to view dispersion trading is through implied correlation. Correlation can be used to illustrate how stocks constituting the index are behaving (FDAXHunter Equity Derivatives, 2004).

For example in Figure 12, the index is trading on an average day of low correlation. Stocks can individually have high or low returns but the index itself is average out and its return is smaller since the correlation between the stocks are close to zero. For this day dispersion would have been profitable since we would have sold volatility on the index leg and it only moved 0,125% while we had bought volatility on the stock leg and it moved a lot more. Though on a day of high correlation as in Figure 13 being long dispersion would not be as profitable. A dispersion spread would be profitable on the long volatility leg for the four stocks but be unprofitable at the index leg. In this case the correlation is high as the stocks move in conjunction with the index. Utilising this concept it is possible to translate the current market conditions into correlation, i.e. implied correlation. In the first case the market is trading at a low correlation

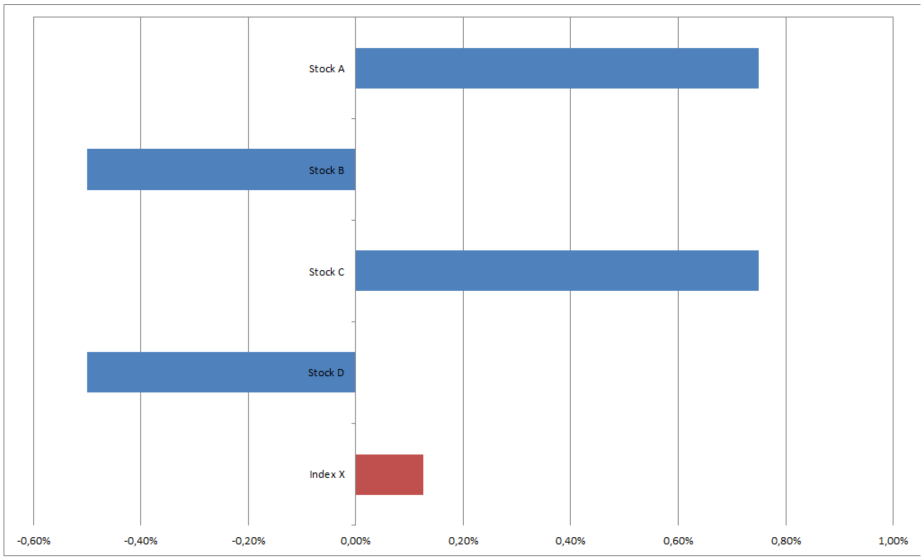


*Figure 12: The return of an index and individual returns of the index's four constituents on a day of low correlation. Some stocks increased in value and others decreased which resulted in an average close to zero. Thus it is profitable to be long volatility on the stocks and short volatility on the index, i.e. long dispersion.*

close to zero as the stocks do not move correlated with each other. In the latter case the implied correlation factor is instead close to one since all stocks move close to each other. The implied correlation factor can be calculated by assuming that the implied volatility of the stock portfolio should be equal to the implied volatility of the index options as in E.q. 6

$$\sigma_p = \sigma_{index} \tag{6}$$

Then it is possible to find the average correlation factors which satisfy E.q. 5 by solving it for average correlation in the portfolio, E.q. 7.

$$\bar{\rho}_{p,m} = \frac{\sigma_{index}}{\sum_{i=1}^{n} w_i \sigma_i} \tag{7}$$

This means that initiating a dispersions trade is equal to selling correlation. When implied correlation is close to one it could be a good signal to sell correlation i.e. buy the dispersions spread. When implied correlation is close to zero it could even be good to buy correlation i.e. selling the dispersions spread. Thus implied average correlation could work as a good indicator of when to initiate a dispersion trade and complement the absolute difference in implied volatility to further improve on the strategy and better select trading opportunities (Ganatra, 2004).

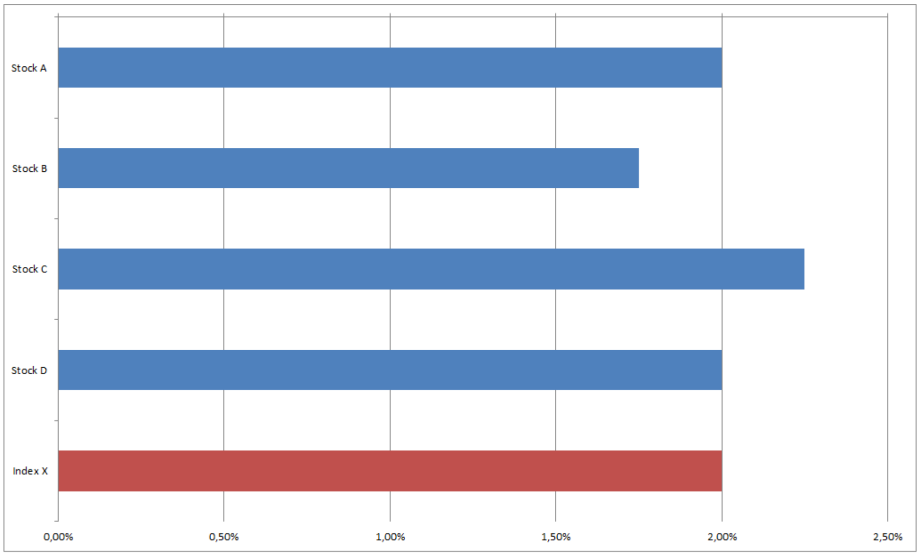


*Figure 13: The index return and the returns of the index's four constituents but now on a day of high correlation. This time the stocks have moved in union and the resulting average index move is large. Thus being long dispersion is not very good as one will lose more on the index leg than on the stock legs.*

## 3.9 Strategies

Strategies can be designed in many ways and there are multiple instruments available. The most straight forward process is to buy and sell variance swaps on the tracking portfolio and index respectively (Ganatra, 2004). However the liquidity in single stock variance swaps has drastically decreased during recent years and investors are instead forced to use plain vanilla options[1] (Martin, 2013).

Dispersion trading is a non-directional strategy, the exposure is in the volatility of the underlying and not its direction. Thus if one is to use vanilla options in dispersion trading it is necessary to hedge the directional i.e. delta risk. Straddles and strangles are two basic volatility spreads which are suitable for these requirements since they give exposure to volatility with limited delta (Natenberg, 1994). This make the use of straddle and strangles appropriate for constructing dispersion exposure which is also often the case (Kolanovic et al., 2010).

### 3.9.1 The straddle

A straddle is an option spread where one buy (sell) both put and call options of the same strike price and maturity on the same underlying. The pay-off graph

[1] Due to the convex nature of variance swaps combined with the tendency of single stocks to jump in a market crash few dealers quote single stock variance swaps after the events of the 2008 financial crisis.

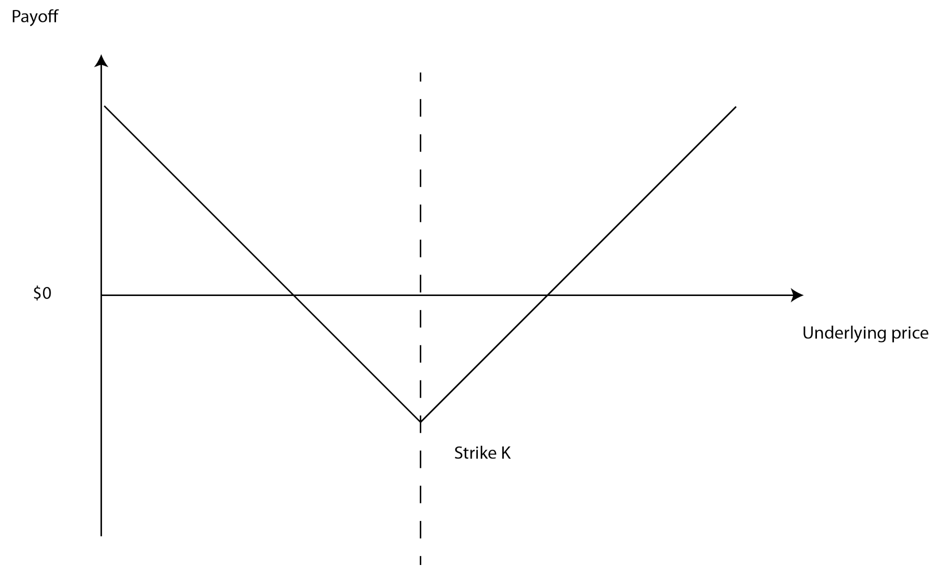


*Figure 14: Pay-off graph of a long straddle position with the strike price K. Pay-off is plotted against the final underlying price. If the final price ends up at the strike price both options will expire worthless and maximum loss is realised. If the underlying price ends up far down or up from the strike price a profit is realised. The loss is limited to the price paid for the straddle and the profit is unlimited. A short straddle has the opposite P&L where maximum profit is realised if the final price ends up at the strike price and the loss increased as the final price moves away from the strike price.*

can be viewed in Figure 14.

#### 3.9.2 The strangle

A strangle option spread is similar to a straddle. One still buy (sell) both put and call options with the same maturity and on the same underlying but now different strikes are used. The investor use put options which have a strike below the call options and vice versa. The pay-off graph for a strangle can be viewed in Figure 15.

Practically which type of dispersion strategy to use is often determined by circumstances. Dispersion trading is done in very large quantities since the difference in implied volatility is often very small. Consequence liquidity is of major concern when choosing a strategy and so if some other party is showing interest in one type of option this could for example serve as a good starting point for one certain type of dispersion trade.

#### 3.9.3 Delta hedging

To clarify, delta exposure is the directional prise change in an option depending on the underlying security, i.e. how the option price change depending on price

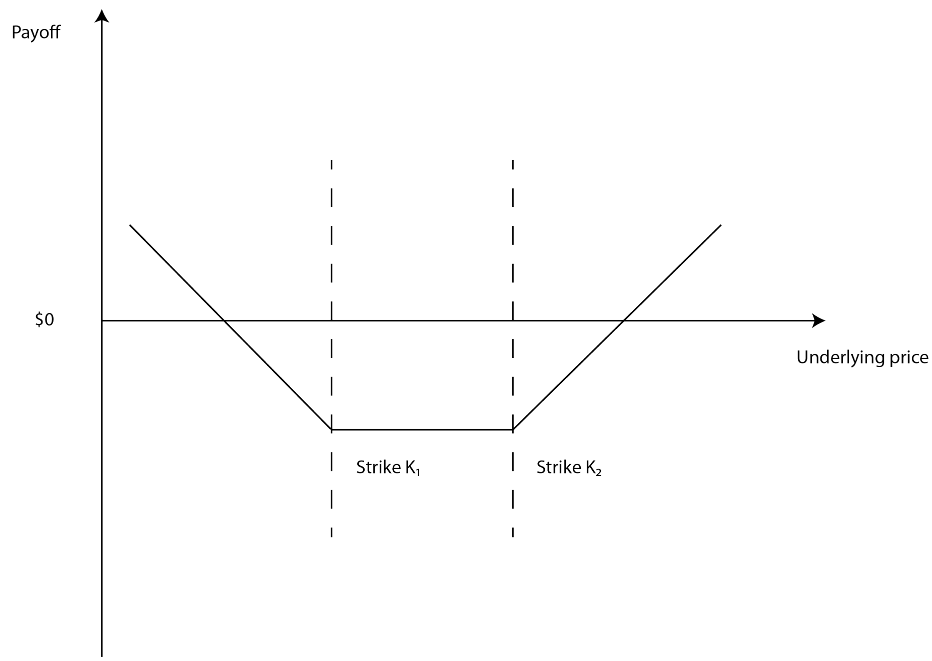


*Figure 15: Pay-off graph of a long strangle position. In this case two different strike prices are used one lower for the put option, K1, and one higher for the call option, K2. The characteristics of a long strangle position is similar to the long straddle illustrated in Figure 14 but instead of specific maximum loss point the strangle has an area for where maximum loss is realised.*

changes in the underlying (Hull, 2008). Dispersion trading is a delta neutral strategy, which means that it is not affected by directional price moves in the underling security. Both straddles and strangle option spreads are almost delta neutral if they are perfectly ATM or in the case of a strangle perfectly centred around ATM. Nonetheless they are not perfectly delta neutral depending on other factors such as the volatility smile. Furthermore, as soon as an ATM straddle or strangle is bought or sold it will only be delta neutral around what the ATM price was at the time the spread was initiated. Thus when the market continues to move after the trade has been initiated delta exposure will arise. This exposure is not desired since dispersion trading should be delta neutral and hence needs to be hedged. Finally it is also necessary to delta hedge in order to realise the actual volatility for which straddles and/or strangles are bought cheaper and sold more expensive against.

Delta hedging can generally be performed in two main ways, (1) static delta hedging or (2) dynamic delta hedging (Taleb, 1997). (1) static delta hedging is the process of finding a second instrument which cash flow is equal to that generated from the delta exposure to be hedged. Then this instrument is sold and the delta risk is netted (Taleb, 1997). An example of static delta hedging is to buy a Bear OMXS30 index ETF against a long OMXS30 index tracking fund position. The resulting aggregated cash flow should be very close to zero. (2) dynamic delta hedging is instead the procedure of continuously re-hedging the delta exposure using an instrument. The difference is that the instrument used does not follow the delta risk on an one to one basis [1] and such the hedge needs to be readjusted at discrete intervals (Taleb, 1997). Dynamic delta hedging is necessary for dispersion trading since no other suitable instrument exist which follow the delta exposure of options other than other options.

[1] The exposure is non-delta one.

# 4 Methodology

*In this chapter it is explained what methods are used and how they are implemented. First the general method is chosen for how to do the financial modelling. Then a more detailed explanation of every method required follows.*

## 4.1 General method

The general method of the study will be based on a back-testing process, where old recoded financial data is used to evaluate a strategy or a portfolio (Ni and Zhang, 2005). Back-testing together with stochastic simulation, e.g. Monte Carlo simulations are the two most common types of methods used to evaluate a portfolio or a strategy (Ni and Zhang, 2005; Mandelbrot and Hudson, 2008; Taleb, 2010). The main difference between these two methods are that back-testing is based on true market data and that stochastic simulations use randomly generated data from a distribution e.g. normal distribution. The two different types of methods are inherent with different general advantages and disadvantages (Kahn, 1990; Ni and Zhang, 2005; Mandelbrot and Hudson, 2008), see Table 2.

| Name | Advantage | Disadvantage |
|---|---|---|
| Back-testing | - Real world data is used which show how the strategy would have turned out in the past and might indicate future performance. | - Tested time period is limited to available data. - Resolution and available instruments are limited to the available data. - Too old data might not hold any relevance to future events. - Use of too little data might result in missing significant events. |
| Stochastic simulations | - Data can quickly be generated as desired. | - The financial market is a complex system and can by definition not be successfully described by a distribution. |

*Table 2: Advantages and disadvantages of back-testing and stochastic simulations (Kahn, 1990; Ni and Zhang, 2005; Mandelbrot and Hudson, 2008).*

As both types of methods have different disadvantages it would be beneficial to test the trading strategy using both methods though this is seldom done as it would double the work required. This is also the case in this study and thus it will only be based on back-testing.

As can be seen in the Table 2 a major pitfall which is hard to address when back-testing is choosing a suitable data period. A too short data period result in great risk of missing major events which would have great impact on the performance of the strategy or portfolio. On the other hand a too long data period results in including old data with little relevance to the current market situation. In practice historical intra-day financial data is hard to come by and the data used is often govern by what is available. Another important aspect of back-testing is that even though the historical data represents a true data path it is only one path and there are no guarantees that the future will turn out as in the past. Consequently it is important not to over optimise the strategy for in-sampling data. Thus it is important to not use all data as training data[1] but to save some of the historical data to an out of sample verification process. By back-testing the strategy on old financial data it is possible to develop proof of how the strategy would have performed if used on the data time period. This allows us to understand how different changes in the strategy and market conditions will impact the P&L.

## 4.2 The tracking portfolio

The tracking portfolio is constructed using a stepwise linear regression method. This is a linear model which cannot handle transaction costs, i.e. optimise the portfolio while taking transaction costs into account. The reason for choosing this over a non-linear alternative such as evolutionary heuristics is that it is good enough for the purpose and later studies can investigate any increased performance from using a better model.

Stepwise multiple linear regression is used to first arrange all predictors i.e. stocks in order of their explanation power i.e. $R^2$. Then stocks are removed from the portfolio till a satisfactory amount of stocks are left while still maintaining a high enough $R^2$.

Another important factor is which data is used when optimising the tracking portfolio. The OMXS30 index is itself re-weighted once every six months. The obvious goal should be to incorporate the historical data in the model which best describe the imminent future. The problem with this is that it is impossible to know which data set will best fit the future. With a too short data period the model will become over-fitted and have a great in sample performance but low out of sample performance. With a too long data period the model will be fitted to too old financial data which probably will have little prediction ability. It is more of an art to select the correct data period and to further complicate the matter it is possible to assign different weights to the data.

The tracking capability of the tracking portfolio will be tested by performing the regression on different sections of the data. Where for the first iteration all data is used in one regression. Then on the next iteration the data is divided

[1] The section of the dataset the model is optimised on.

into two parts where the regression analysis is repeated twice, one for each part of the total data. This process is repeated 40 times i.e. for the final iteration the total dataset is arranged into 40 sub-datasets and a new regression is performed for each set[1] . The total regression sum of squares and the average $R^2$ is plotted for the number of iterations performed.

Finally a cross-validation test is performed. Cross-validation for financial time series are a bit tricky since many cross-validation techniques assumes no correlation between time steps e.g. k-fold. In this case an in and out of sample analysis of the regression model could be performed by training the linear regression on the first five years of data and then test it on the remaining five years of data. However this will not work very well given that the market has changed during those ten years and little descriptive power exist in the earliest data and also half of the data will be lost to verification. To address this a rolling forecasting origin model will be used (Hyndman and Athanasopoulos, 2002). It works by first training the model using the minimum necessary data which in this study will be 252 samples. Then the model is used to forecast the next data point $\hat{y}_i$ for which a tracking error is estimated compared to $y_i$, the true value of y. Finally the training data set is moved one step forward and the whole process is repeated. The mean squared error (MSE) and the mean absolute scaled error (MASE) is then calculated. MSE is useful to interpret how well the model performed out of sample. However it is a scale-dependent measurement, i.e. the size of the value of the time series matter. Thus the MASE will be used to indicate how good the model is (Hyndman and Athanasopoulos, 2002) while MSE is used to compare results from the same data set. A naïve version of MASE is used for non-seasonal time series. Here the tracking error is compared to a naïve forecasting model. A value below one indicate that the model is better than the average naïve forecast and respectively a value above one indicate that the model is worse than the average naïve forecast. The rolling forecasting origin model is also very data efficient since only one data point is lost for verification. A One-Sample Kolmogorov-Smirnov test is performed on the error term in order to verify if there is a systematic error or not. The test is performed against a normal distribution with a significance level of 99,9%. The stability of members and weights is also investigated by measuring the standard deviation of weights and how many different portfolios are used as the tracking portfolio.

## 4.3 Implied volatility calculations

Implied volatility is calculated in two different ways; (1) index options that are European options are based on the Black-Scholes-Merton method and (2) the single stock options which are American are based on the Cox-Ross-Rubinstein method. The reason for using different methods is that single stock options has discrete dividends while index options is priced using adjusted underlying

[1] The data set correspond to ten trading years, or 2516 data points. Subsequently 40 subsets equal a re-weighting every quarter.

securities. Another reason is that the American, single stock options, can be early exercised.

### 4.3.1 American implied volatility

The Cox-Ross-Rubinstein (CRR) tree model is used to calculate American option prices. It is a binomial based tree model described in Chapter 3.6 A modified version of the built in Matlab function for the CRR model, *crrprice*, is developed. The reason not to use the included function is that it is too slow. The CRR method calculate the price of an American option. To take the step to implied volatility the *fzero* solving function in Matlab is used to back out the volatility from the CRR model. It finds the input volatility which minimise the difference in price between the CRR model and the price observed in the market.

### 4.3.2 European implied volatility

As described in Chapter 3.5 the Black-and-Scholes pricing model is used to calculate the implied volatility for European options. The built in *blsimpv* function in Matlab is discarded for an adjusted version that is faster. The adjusted function finds implied volatility in the same way as is done for the American implied volatility calculation, i.e. a Matlab solver is used to back out the implied volatility.

## 4.4 Strategies

Three types of strategies are tested (1) the Straddle strategy, (2) the Strangle strategy and (3) the Combination strategy. These are very basic strategies and would not serve as a final strategy to trade. However testing these will enable us to research and draw conclusions on important aspects of dispersion trading and serve as early prototypes for a working strategy. The strategies were selected since they are some of the most common and basic vanilla options dispersion strategies which best illustrate isolated characteristics. The three strategies are tested for the following attributes:

- Is the strategy profitable or not and during which conditions is this true?
- What are the risks associated with using the strategy?
- What are the key problems to implement the strategy?

### 4.4.1 The Straddle strategy

The Straddle strategy is a dispersion strategy based on the use of straddle options spreads, see Chapter 3.9.1 about a straddle spread. To initiate a dispersion trade using straddles one would sell a straddle on the OMXS30 index using a ATM put and call option. This position would then need to be hedged by buying ATM straddles on the tracking portfolio, see Figure 16.

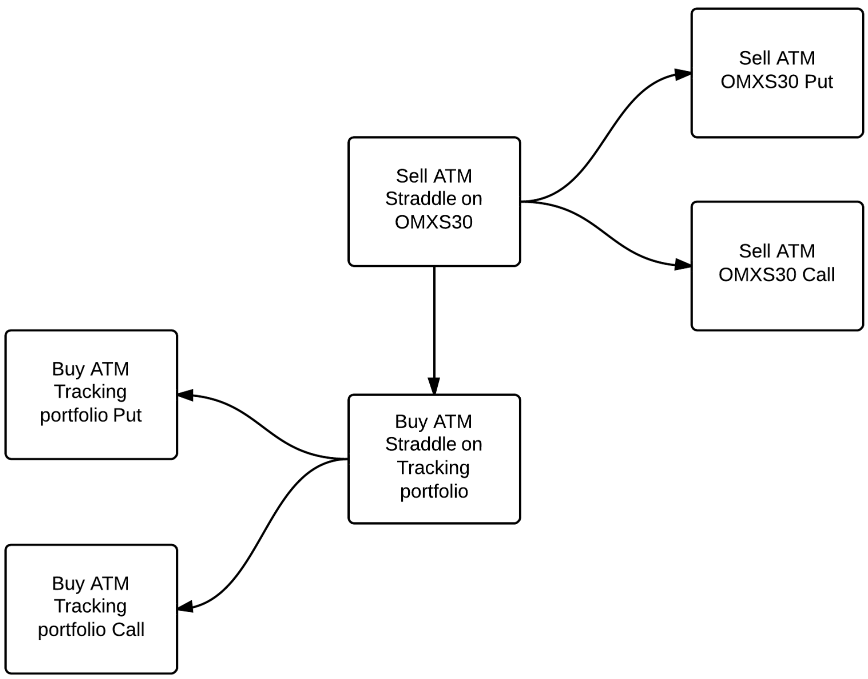


*Figure 16: First a position on the OMXS30 index is sold and it is then hedged using the tracking portfolio. The position consist of selling a ATM OMXS30 put and a ATM OMXS30 call, buy ATM tracking portfolio put and buy ATM tracking portfolio call.*

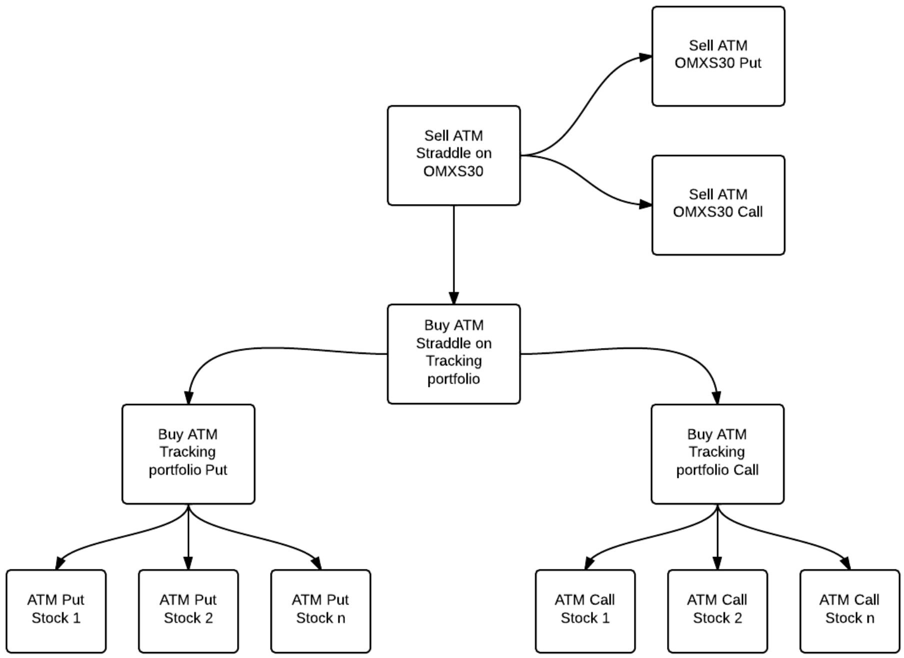


*Figure 17: Buying a ATM straddle on the tracking portfolio is accomplished by buying both ATM puts and calls on all stocks included in the tracking portfolio using the weights from the tracking portfolio.*

Since it is plain vanilla options that are used it is impossible to directly buy options on the tracking portfolio but rather we need to buy ATM put and call options on all the individual stocks in the tracking portfolio with the same weights as in the tracking portfolio, see Figure 17.

Now the volatility smile structure explained in Chapter 3.7.1 becomes a problem since options are very seldom exactly ATM. For example on the OMXS30 index options are quoted in increments of 10 with strikes below 1000 and in increments of 20 above 1000 i.e. 970, 980, 990, 1000, 1020, 1040, etc. Such it would only be possible to trade exactly ATM straddles whenever the underlying is trading at one of these strikes.

This is overcome by using linear interpolation between the two strikes which are currently closest from above and below to ATM[1]. E.g. if the underlying security is trading at 998 the closest strike from below is 990 and from above it is 1000. Consequently to create one ATM option two quoted options are needed which gives us the next structure of the trade, see Figure 18.

Next the term structure needs to be address. In Chapter 3.7.2 it was explained that an option's price also depends on the time till maturity. Thus it

[1] Linear interpolation in this case is essentially the arithmetic mean between the two strikes.

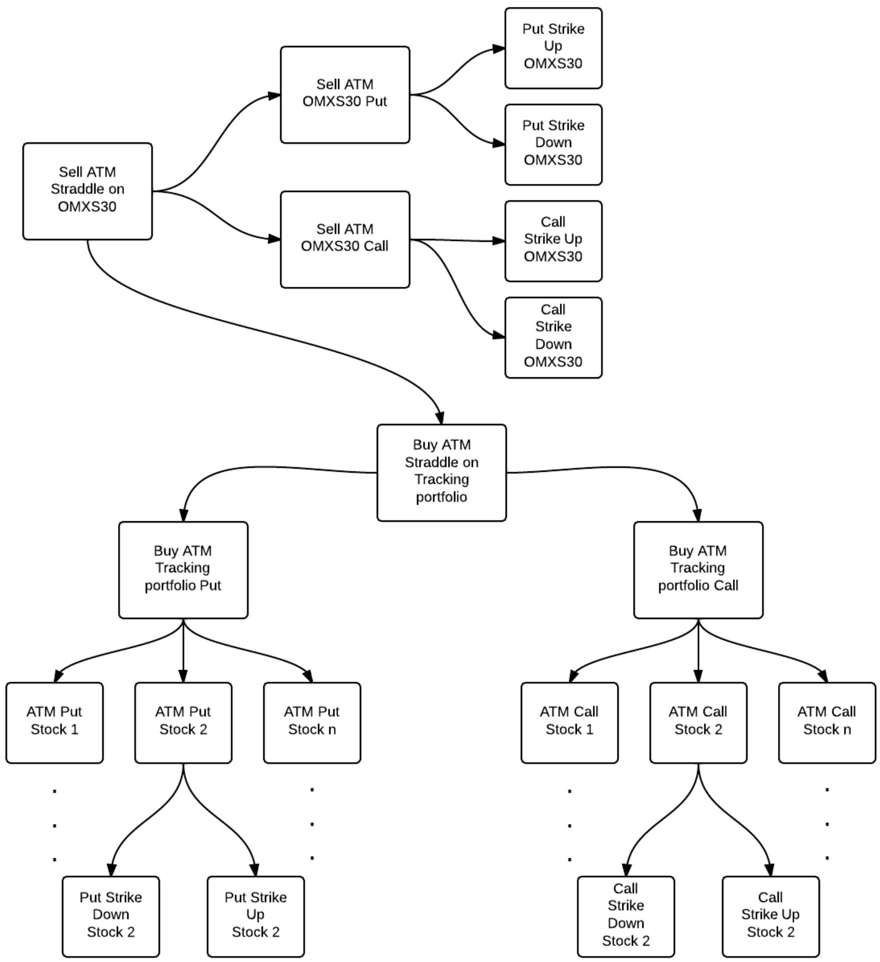


*Figure 18: To manufacture a ATM option two options are combined, one below (down) in strike price and one above (up). This is done for all ATM put and calls being sold and bought. Interpolation is used to calculate the required weights.*

is necessary to hold a constant term structure in order to isolate the dispersion effect.

The term structure will be held constant at 30 days using the same interpolation (sometimes extrapolation) process as CBOE[1] do for their VIX-indexes (Chi, 2009). In this process a combinations of options with less than 30 days to maturity[2] and options with more than 30[3] days to maturity are combined together to form an option with exactly 30 days to maturity. Weights for the front and back options are determined using an interpolation method. In this process options with less than seven days to maturity is never used. The reason being is that other factors such as supply or demand squeezes can heavily affect the price of options that are very close to maturity. Such the front and back months are rolled when the front month has seven days left to maturity. This means that sometimes both the front and the back month can have more than 30 days to maturity, for which case extrapolation is used i.e. one of the weights become negative and the other is greater than one. Consequently for each option in Figure 18 two options of different maturities are needed which is illustrated in Figure 19.

The term structure effect and strike structure effect result in that for every option used in the straddle four options are needed to hold the time and strike constant. Every straddle incorporate two options and in the case of a tracking portfolio consisting of six instruments; seven[4] straddles are used resulting in a total of 56 individual options to trade.

ATM straddle options spreads are at inception almost delta neutral, nevertheless small delta exposure will arise and these needs to be hedged since they will otherwise be scaled up when one is trading the dispersion trade in large quantities. Furthermore, profits from the dispersion effect come from that the market overvalue index volatility and undervalue single stock options. One part to realise the gain from exploring these incorrect valuations is to individually delta hedge all legs.

Dynamic delta hedging[5] is in itself a complex process and not the focus of this study. Hence a simple delta hedging procedure is selected to hedge once a day at 17:07. This procedure is certainly not the most efficient and other practical solutions should rather focus on absolute deviation of the delta, gamma, even shadow gamma[6] etc. from the desired target (Taleb, 1997).

Trading all 56 options and perform delta hedging enable us to isolate the dispersion effect and therefore investigate its pure P&L.

[1]Chicago Board Options Exchange A major options exchange.

[2]Called the front month, the front month is defined as the month with the least time till maturity

[3]Called the back month, the back month is defined as the month with the most time till maturity

[4]Six straddles for the tracking portfolio leg and one straddle for the index leg.

[5]Dynamic delta hedging is the process of continuously re-hedge a position as oppose to static hedging where another instrument is able to completely hedge a position without and recalibration (Taleb, 1997).

[6]Shadow gamma is the derivative of delta with respect to the volatility. I.e. how the delta change depending on sigma (Taleb, 1997).

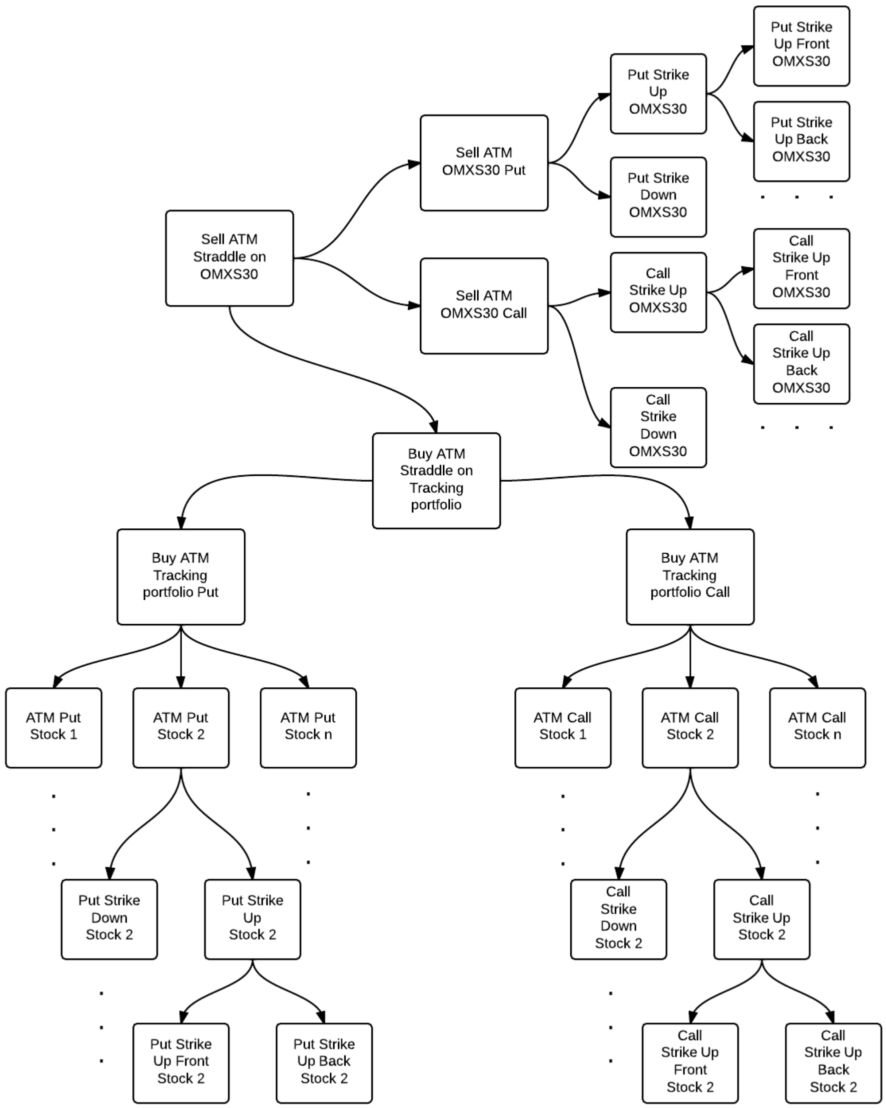


*Figure 19: On the next level one option is used in the front month and one option is used in the back month for every option on the previous level. Interpolation is again used to create the constant term structure.*

There are many possible ways to size the positions[1], in this study we chose to use a self-finance condition. It is selected so that at first 1 index straddle is sold and a premium is received. Then a constant k is calculated which is the number of straddles on the tracking portfolio to buy. It is calculated by dividing the premium from the index leg with the price for a straddle on the tracking portfolio. Consequently no cash is credited or paid when taking the positions.

Transaction cost will not be addressed at first and will instead be addressed in a separate analysis. Such, it is assumed that the full amount can be executed in the middle of the spread.

### 4.4.2 The Strangle strategy

The Strangle strategy is similar to the Straddle strategy in almost every way. The difference is that strangles are used instead of straddles, see Chapter 3.9.2 about strangles. Thus there are instead two strike price levels which are defined as 5% above and 5% below ATM. E.g. if ATM is at 1000 then the lower strike is 950 and the higher strike is 1050, see Figure 20.

The difference between using strangle spreads instead of straddles spreads is that they have less delta since OOM options are used and also that they trade at other implied volatilities than ATM options. Such it is interesting to see how the P&L of using strangles instead of straddles is affected.

### 4.4.3 The Combination strategy

The Combination strategy is a combination of the Straddle and the Strangle strategy. This strategy test if it is possible to enhance return by also trade the volatility smile, see Chapter 3.7.1. This is done by buying volatility where it is cheap which usually is ATM and selling volatility where it is expensive usually OOM. A more realistic approach would be to switch between the different strategies depending on the current volatility smile. For simplicity though we will instead evaluate the Combination strategy which instead use that on average volatility is lower ATM and higher OOM.

It is important to notice that this strategy will increase return by adding more risk in the form of untimely changes in the volatility smile. Such this strategy does not only produce return from the dispersion effect but also from the volatility smile effect. The combination strategy will sell 5% OOM index strangles and buy ATM tracking portfolio straddles, see Figure 21 for further details.

## 4.5 Tracking P&L and other financial metrics

P&L of the strategy will be tracked in combination with several financial metrics. The following financial metrics will be used.

1. Sharpe ratio

[1]Self-financed, delta neutral, gamma neutral etc.

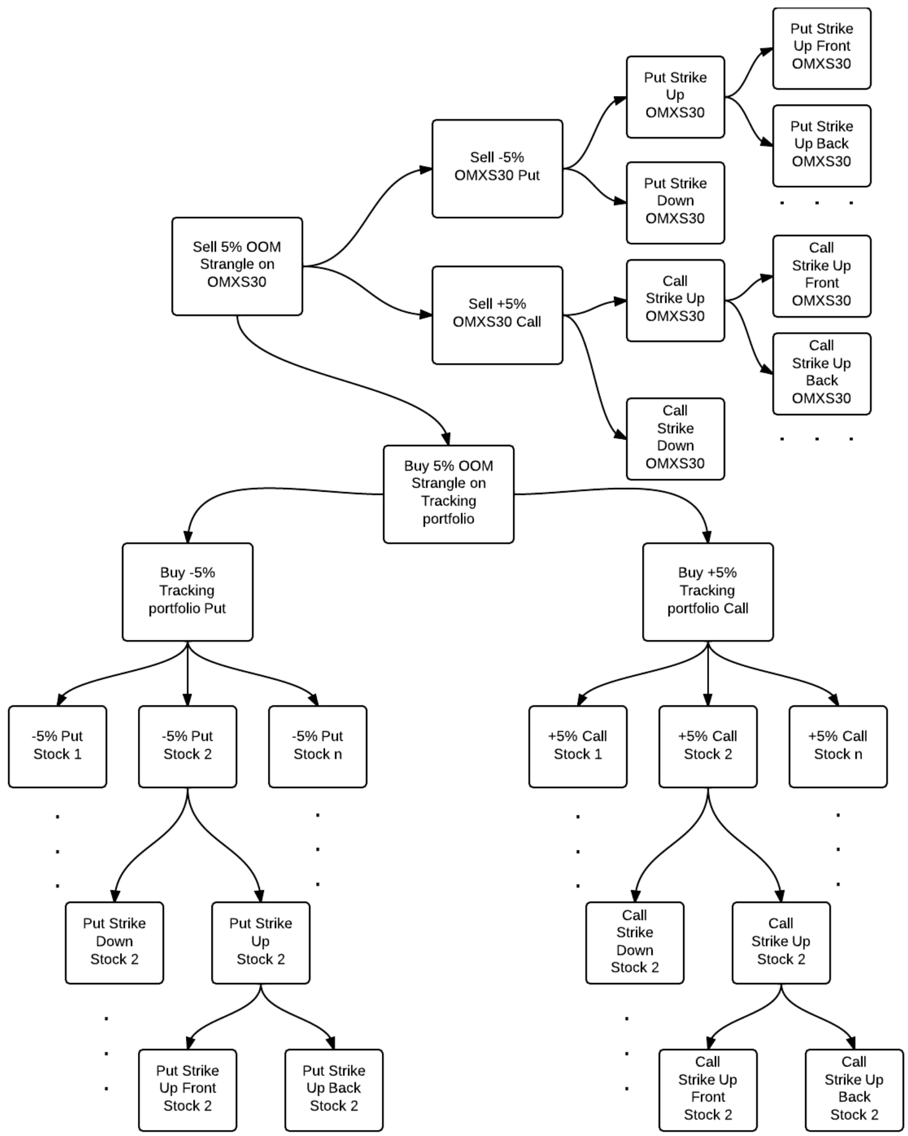


*Figure 20: The Strangle strategy. Now options that are 5% OOM are used both for puts and calls instead of ATM options. The difference between this strategy and the Straddle strategy is that what was the ATM price is now adjusted 5% up for call options and 5% down for put options.*

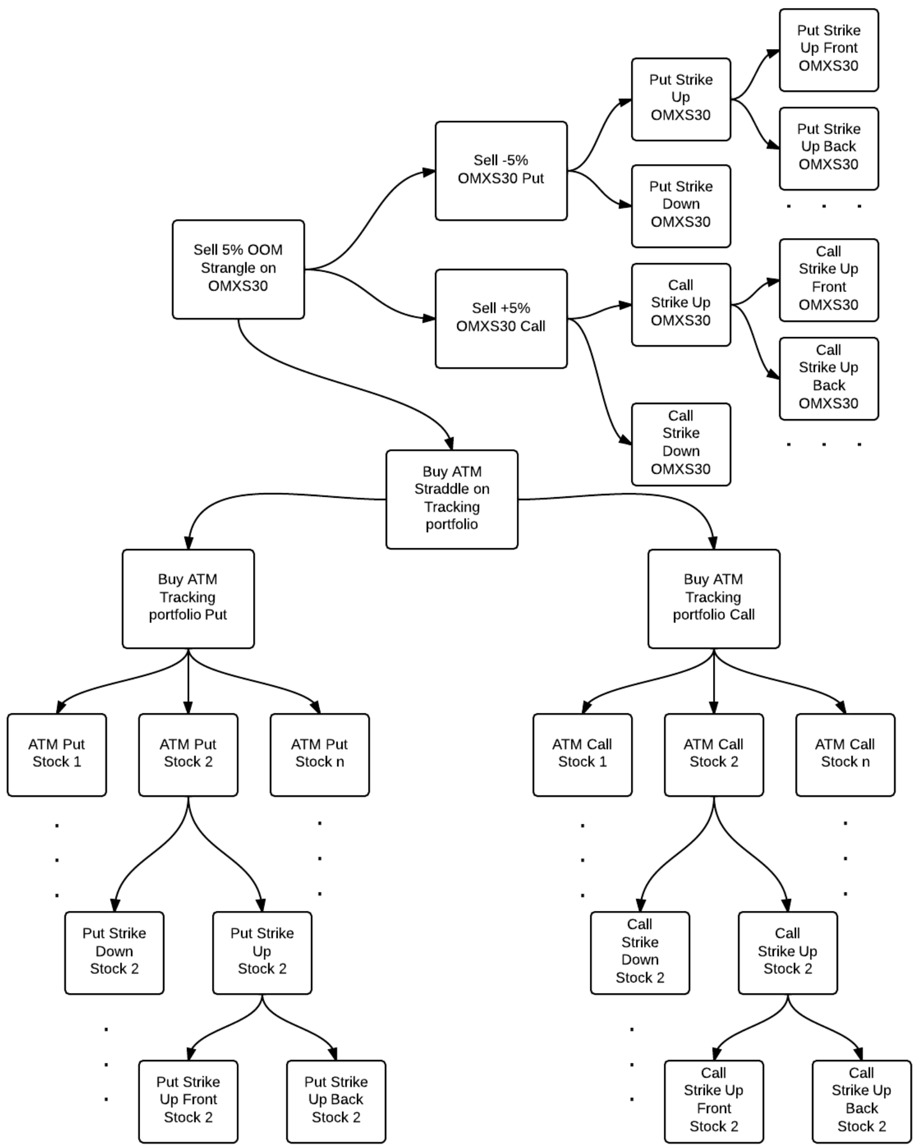


*Figure 21: The complete Combination strategy. Here the Straddle and the Strangle strategy is combined in order to use the volatility smile. ATM options are on average cheaper and thus ATM straddles are bought and expensive OOM strangles are sold.*

2. Maximum draw-down
3. Market correlation

The Sharpe ratio is a financial metric which can be calculated for portfolios or strategies. It measures the ratio of reward to volatility over the market. I.e. how return is received in relation to the risk (Berk and Demarzo, 2011). Thus it reflects if the achieved return was efficiently created. The Sharpe ratio is defined in E.q. 8:

$$\text{Sharpe ratio} = \frac{\text{Portfolio Excess Return}}{\text{Portfolio Volatility}} = \frac{E\left[R_p\right] - r_f}{SD\left(R_p\right)} \tag{8}$$

OMXS30 is used as a proxy for market returns. Since the strategy is self-financed it becomes complicated to calculate the Sharpe ratio, because it is hard to define what capital is used in order to create the recorded return. In this study the premium received from the index option leg is assumed to be the capital used[1].

Maximum draw-down (MDD) is a great compliment to the Sharpe ratio as it focus on what the worst loss was (Bughardt et al., 2003). The Sharpe ratio might misstate the performance of a strategy or portfolio if there are fat tails in the return distribution. MDD is the maximum loss from the global maximum peak to the following lowest local low, see E.q. 9.

$$\text{MDD}(T) = \underset{\tau \in (0,T)}{max}\left[\underset{t \in (0,\tau)}{max} X(t) - X(\tau)\right] \tag{9}$$

Market correlation is the correlation factor between the return of the strategy and market return. It will be denoted as $\beta$. It indicates how the strategy's return moves with the general market.

To further investigate how the return distribution of the strategies look, return will be plotted both in a histogram and a Box-and-Whiskers plot. Furthermore a one sample Kolmogorov-Smirnov test will also be used in order to test if the return distribution is normal distributed or not.

The historical volatility smile is plotted using both put and call options, see Chapter 3.7.1 for information about the historical volatility smile. Options are arranged depending on their strike price on the x-axis and their implied volatility is then plotted on the z-axis for every time step on the y-axis.

Transaction costs will be tested for the Straddle strategy. The strategy will from the beginning assume no commission fees on transactions and all trades can be executed at the mid-price. Then the average bid-ask spread will be investigated and the strategy will be retested first with paying the full spread instead of half the spread and then also by adding commission[2]. The average

[1] Note that this value is equal to the premium paid for the tracking portfolio premium since the portfolio is self-financed.

[2] Commission is based on the average commission paid by a large investment company

bid-ask spread will be calculated for both the index leg and the tracking portfolio separately. Only the Straddle strategy is tested for transaction costs because it is believed that results will be similar between the three strategies.

Both the implied volatility spread and the implied correlation spread will be plotted. The implied volatility spread is the difference between implied volatility on the index and on the tracking portfolio. The reader is referred to Chapter 3.8 for more information about implied correlation.

## 4.6 Tools

Matlab version R2012b will be used to solve the different computational task as well as performing the back-test. Excel is used for basic calculations, graphing and light data handling. Adobe Illustrator is used to create graphs and Lucidcharts are used to create flow charts. Word and LaTeX is used for word processing.

# 5 Data

*In the Data chapter all data used in the study is discussed. First all data types in the study is covered. Then an explanation to how and where it was collected is given. Finally how the data is organised and stored is also explained.*

There are numerous data sources available for historical financial data e.g. Bloomberg, Reuters, Yahoo finance and Google finance. However when evaluating strategies based on derivatives the field becomes limited. The problem from a data standpoint with derivatives is that they have a finite life time, so when back-testing instruments change frequently. Furthermore additional data is required than just the bid-ask price of an option. E.g. underlying price, strike price, time to maturity, interest rates etc. at the time of the bid-ask option quote. Consequently, a much more complex data structure is require which can handle all this information than in the case of a more traditional equity stock strategy.

The dataset is an important part of a successful back-test. The back-test can never become better than the data used in it. To not introduce unknown biases in the data we are collecting the data our self rather than buying it, in order to control the whole data process.

## 5.1 Data type and time period

There are several types of calculation methods described in the methodology chapter used to evaluate the dispersion trading strategy. Different methods require different types of data. Variables can mainly be categorised into two types:

1. Intra-day data variables
2. Long term data variables

Intra-day variables could for example be the current bid and ask price of an option. This information can drastically change from one day to another and thus it would be interesting to sample the bid and ask on an intra-day basis. On the other hand a long term variable such as the historical correlation between a stock and the index will not change from day to day as one day is a small portion of the historical average. Thus it is not necessary to use intra-day data when calculating historical averages. In this study intra-day data is both available and preferable. However it is impractical to test the strategy on much higher resolution than end-of-day data. Thus a compromise had to be made to only sample intra-day data once a day.

In this study the following data is used:

1. End-of-day stock prices of all stocks present in the OMXS30 index and the OMXS30 index itself from 2003-01-31 to 2013-01-31.

2. Once a day bid and ask prices of all options at that time quoted on the OMXS30 index and similarly all stock options quoted with underlying instrument being a member of the OMXS30 index from 2010-09-10 to 2012-10-12.

3. At the time of an option bid-ask sample current bid-ask of the underlying security, current time and date, interest rate and characteristics of the option in question i.e. call or put, strike price, maturity date is also collected.

Stock and index prices are collected at the end of each trading day and are used to determine the characteristics of the tracking portfolio as well as calculating the correlations between stocks and the index. These are call auction prices determined in the closing call held at 17:25-17:30 every trading day.

Option bid and ask quotes are also sampled once a day but end of day data is not used since options do not go through a closing procedure as stocks do, instead the data is sampled at 17:07 every day. This time is selected because it does not coincide with scheduled market news, it is usually an active part of the trading day but it is not to close to the closing where temporary supply and demand patterns can distort the price. If the current quote is older than two hours the data is discarded[1]. The data is also discarded in the case of only a bid or if only a ask is quoted i.e. not a complete spread is quoted.

A time period of ten years is used for the data intended for the long term variables while a data period of two years is used for the intra-day variables.

## 5.2 Data sources

As previously mentioned there are many available data sources though not as many collect data on options while maintaining other data such as expiration date, strike price etc. The primary data source is provided by a Swedish investment company. The data comes from a SQL database built and maintained by them-self. The database collect and store tick for tick data from the NASDAQ OMX Stockholm exchange. The major advantage of using this data over data provided from other vendors is that all modifications that are made to the data is known to us.

Complementing data is obtained from a Bloomberg Terminal and an Online Trader. Those data sources are used for finding corporate events such as ex-dividends dates and historical daily stock turnover.

Data is imported into Matlab by either the use of Excel or by directly storing the data in Matlab file format (.mat).

## 5.3 Data structure

The main data structure is built up in a tree format. At the top of the tree are the underlying instruments in our case the OMXS30 index and its constituting stocks. All stocks and the index is in turn divided up into options from

[1] In this case the quote is considered to old to pair with other current data.

different expiration dates, i.e. options expiring in October 2010, options expiring November 2010 etc. All maturities are in turn divided up into puts, calls and underlying instrument as well as related information to the maturity such as expiration date. Puts and calls are structured in the same way; for which first all calls and puts are respectively listed. Under every specific call (or put) there are bid, ask, strike, implied volatility, delta and ID. Some of these are constant for all trading days and thus are just one single value e.g. ID. Others are hold in vectors containing a value for every trading day. See Figure 22 for a visualisation of the structure.

Other minor data structures are also used in order to store other types of data e.g. stock dividends, ex-dividends dates and historical returns.

## 5.4 Missing data

There are 15 days[1] with missing data points in the data set. Three reasons for the missing data are identified: (1) the trading day was a half-day and the market closed at 13:00 therefore no available data existed at 17:07. (2) no bid and ask quotes younger than two hours existed at 17:07 for the instrument. (3) There was an error in the data collection process and no data points for the day exist.

The frequency of missing data points is rather low but since many instruments are traded every day and one missing data point is enough to discards a whole day the problem is significant. The issue is addressed by filling in missing data points using interpolation. The simplest method is linear interpolation which essentially fill in missing data points by calculating the arithmetic mean of the two adjacent data points. A higher order interpolation method is cubic Hermite interpolation which instead uses low degree polynomial to fill in the missing data (Fritsch and Carlson, 1980; Lalescu, 2009). This process result in a smother interpolation and is also the method employed in this study using the *interp1* function in Matlab with the method set to *cubic*.

## 5.5 Market conditions

The 500 trading days data period selected in the study was chosen because of the different types of market conditions present. There are three different market sections in the data set.

1. Bull market - September 2010 to May 2011.
2. Bear market - May 2011 to December 2011.
3. Bull market - December 2011 - September 2012.

The data will enable us to explore how the strategy performs given different market conditions, see Figure 23 for a time line of the three different markets.

[1]Out of 500 days.

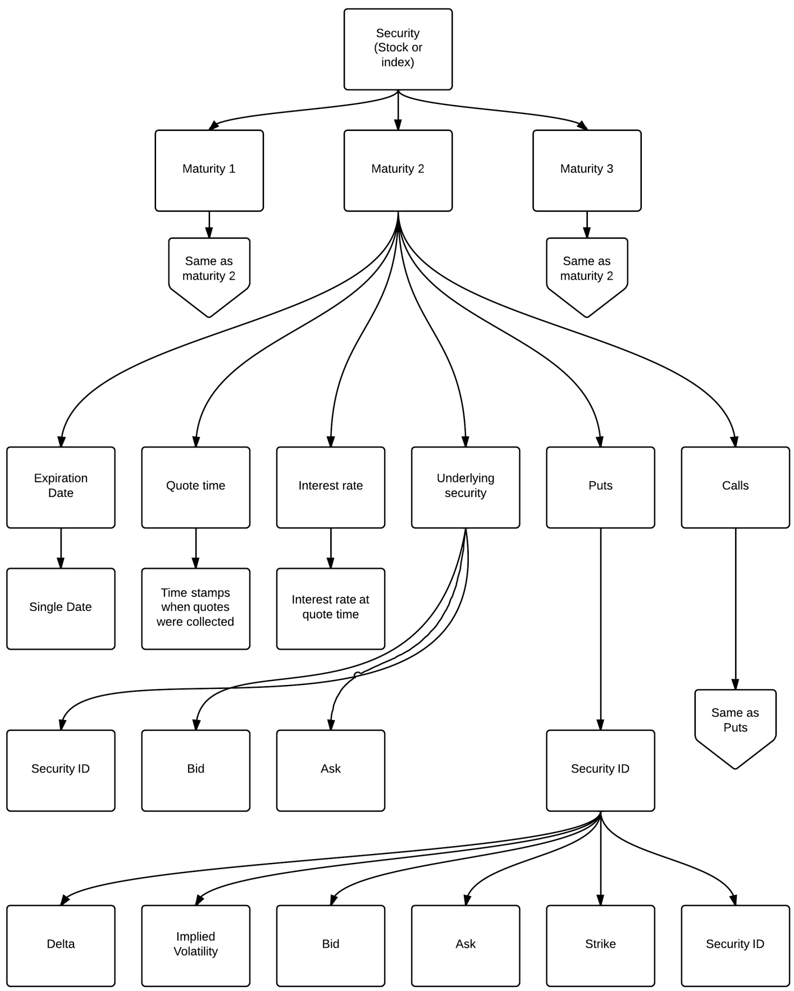


*Figure 22: The main tree data structure. The Figure only shows one branch but all branches are identical. The data structure starts at the top where the underlying security is chosen followed by; maturity, puts, calls or underlying security information, specific options and finally data of that option.*

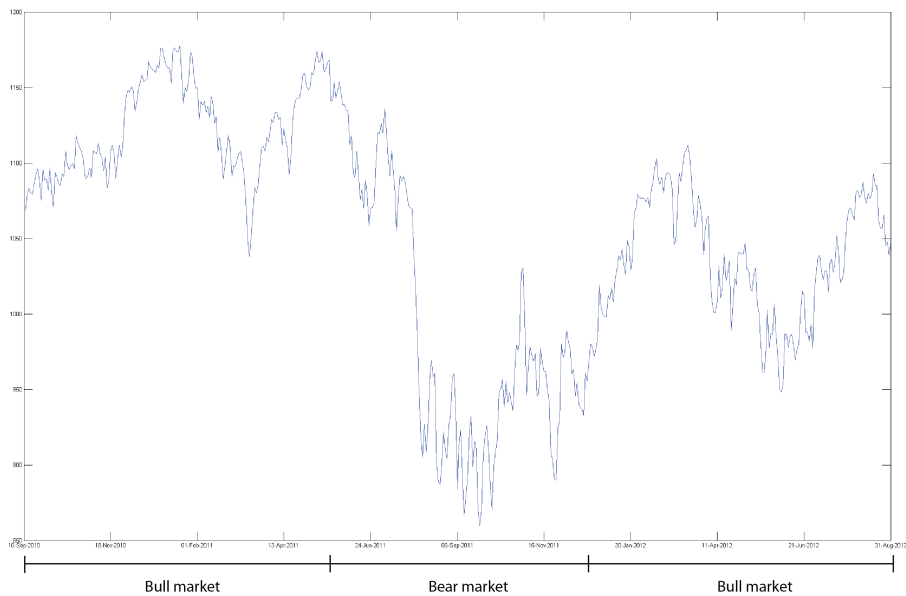


*Figure 23: There are three different markets conditions present in the data set. The first period in the data is a bull market between September 2010 to May 2011. Then there is a bear market between May 2011 to December 2011. Followed by a second bull market between December 2011 to September 2012. The figure show daily OMXS30 index levels for the test period.*

# 6 Results

*Here the results of the study is given. We begin by showing how the three strategies performed in the back-test. Then follows the results used to verify the validity of the model and finally the results from the transaction cost analysis can be found and the historical volatility smile.*

## 6.1 Strategy returns

First return and return-metrics is presented for the three strategy scenarios and can be found in Table 3. Again there are three strategies; the Straddle-, the Strangle- and the Combination- strategy however many of the results were very similar between the strategies thus some were moved to Appendix A. Returns were positive for all strategies for the two year period. Since the positions are self-financed it is hard to determine the fractional return of the strategies. Nonetheless the size of the index leg is used as a proxy where the daily return is divided by the index leg premium.

| Strategy | P&L | % | Stdv | Sharpe | MDD | % | $\beta$ |
|---|---|---|---|---|---|---|---|
| Straddle | 100,04 | 182,9% | 23,72% | -1,278 | 28,49 | 26% | -0,44 |
| Strangle | 87,96 | 95,8% | 17,17 % | -0,175 | 16,3 | 25% | -0,3 |
| Combination | 88,45 | 88,1% | 22,68% | -0,180 | 21,81 | 34% | -0,25 |
| OMXS30 | - | -1,9% | 15,48% | - | 317,66 | 36% | 1 |

*Table 3: Absolute return and financial return-metrics for the three different strategies. The first column is the cumulative return for the strategies, i.e. the sum of money generated by the strategy during the trading period, starting at zero. The second column is the return for the period in percent based on the size of the index premium. The third column is the standard deviation of daily log return. The fourth column is the annualised Sharpe ratio. The fifth and sixth columns are the absolute maximum draw down and percentage of maximum draw down in relation to at that time cumulative return. The final column is the beta, i.e. correlation between return and the market.*

Next the return graph of the three strategies is plotted and can be seen in Figure 24 together with the return of the OMXS30 index for the period. All three strategies had steadily increasing profits during the evaluation period. The Combination strategy performed worse than the Strangle strategy during the summer-autumn of 2011 but was then able to recover in late 2011. This coincide with the bear market of 2011 and an increase in the volatility smile and subsequent decrees in the smile.

## 6.2 Implied volatility

Implied volatility for both the OMXS30 index and the tracking portfolio were calculated and can be viewed in Figure 25 for the Straddle strategy. Implied volatility levels begun and ended the period at around the same levels but arose sharply during the bear market in the summer and autumn of 2011.

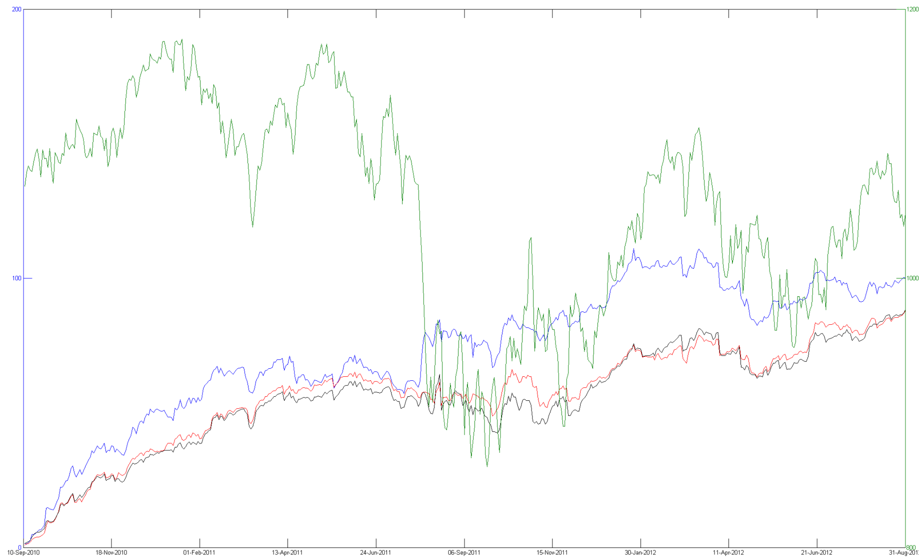


*Figure 24: Cumulative return for the three strategies, the Straddle in blue, the Strangle in red and the Combination in black. The return is achieved for 500 trading days which roughly equals two calendar years. OMXS30 index return was also plotted in green. Portfolio returns are plotted on the left y-axis and index return on the right y-axis in different scales.*

The two implied volatilities traded closer during calm markets i.e. prior to and after the market crash in the summer-autumn 2011. The graphical representation for the Strangle, Figure A.1, and the Combination strategy, Figure A.2, were very similar to the Straddle strategy and can be found in Appendix A.

The implied volatility spread was also calculated i.e. what the differences was in implied volatility between the index and the tracking portfolio. It can be viewed in Figure 26 for the Straddle strategy together with index return. Again the graphical result for the Strangle, Figure A.3, and the Combination strategy, Figure A.4, were similar to the Straddle strategy and can be found in Appendix A.

The implied volatility spread was on most days positive but there were some days with a negative spread. Again it is possible to see that the spread increases during the large summer-autumn crash in 2011. It was also found that the implied volatility spread was negative correlated with the market index.

Correlation between the implied volatility spread and the OMXS30 index was tested and the average value of the implied volatility spread for the different strategies was also calculated and can be found in Table 4.

Implied volatility spreads for the three strategies were all negative correlated with the OMXS30 index. The mean spread for the Straddle and Strangle strategies were the same with a small increase in mean spread for the Combination

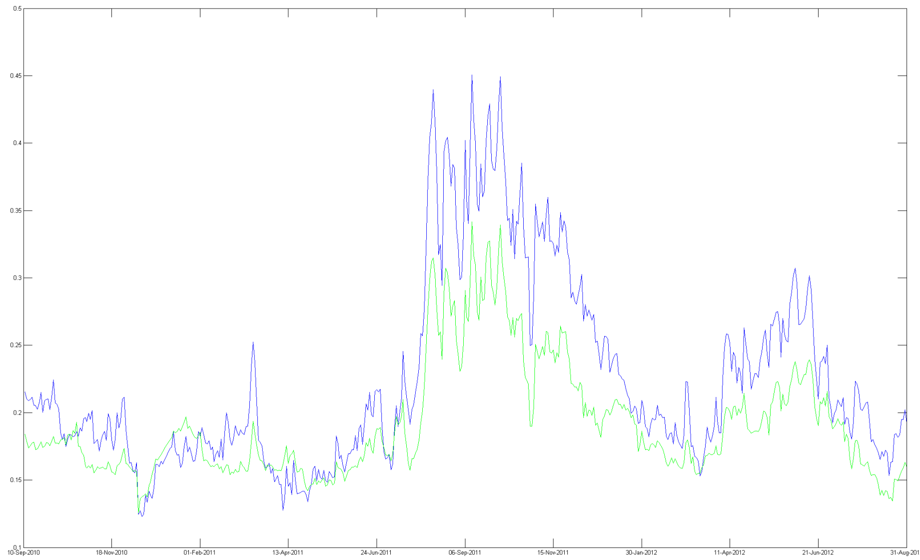


*Figure 25: Implied volatility for both index options on the OMXS30 and stock options on the tracking portfolio for the Straddle strategy. Index implied volatility is in blue and is slightly more expensive than volatility on the tracking portfolio in green. It is this difference between implied volatilities that is profited on by dispersion trading, i.e. the blue line is sold and the green line is bought.*

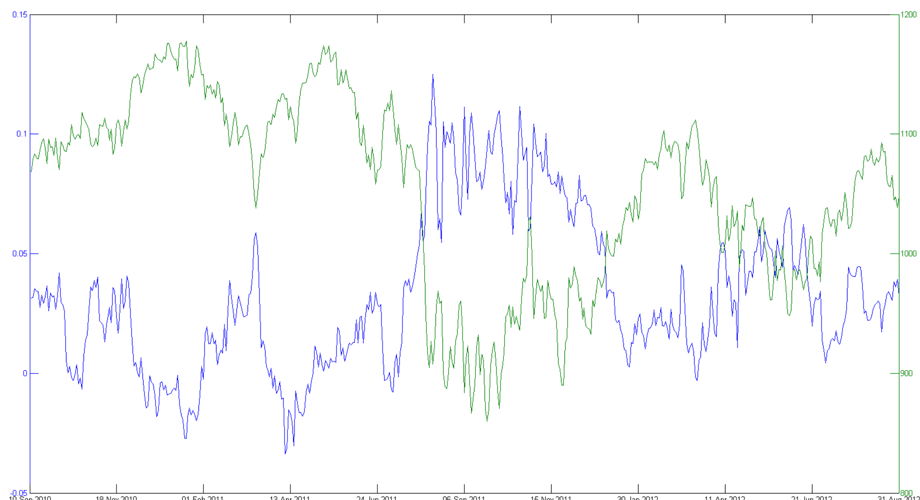


*Figure 26: The implied volatility spread for the Straddle strategy. The blue line is the difference in implied volatilities shown in figures 25 and is plotted on the blue y-axis to the left. OMXS30 index return is plotted using another scale on the y-axis to the right in green to illustrate the high negative correlation.*

| Strategy | Mean spread | Correlation with index |
|---|---|---|
| Straddle | 3,30% | -0,9 |
| Strangle | 3,30% | -0,89 |
| Combination | 3,70% | -0,88 |

*Table 4: Show the mean implied volatility difference between index options and the tracking portfolio in the back-test for the three strategies. It also show the correlation factor between the implied volatility spread and index. The strong negative correlation present in all three strategies indicate that the spread widens when the market goes down i.e. dispersion trading is more profitable at the peak and just after a market collapse.*

strategy.

## 6.3 Implied correlation

Implied correlation was also calculated and can be found in Figure 27, for the Straddle strategy. It is plotted together with a two standard deviation 20 periods Bollinger band.

Implied correlation traded mostly within two standard deviations Bollinger band. Implied correlation increased during the bear market in summer-autumn of 2011 and then decreased when the market recovered. During the worst periods of the bear market in 2011 it occasionally traded above 1 which implies pure arbitrage. The graphical representation of the implied correlation for the Strangle, Figure A.5, and the Combination, Figure A.6, strategy were also similar to the Straddle strategy and can be found in Appendix A.

## 6.4 Return distribution

The daily return of the Straddle strategy was plotted in a histogram, Figure 28, with a superimposed normal distribution line to illustrate the kurtosis and skewness of the return distribution. A second box-and-whisker plot, Figure A.7, can be found in Appendix A.

The mean return for the Straddle strategy was 0,2 and the median return was 0,22. 290 of the trading days had positive return and the remaining 210 days had negative return. The daily return was tested with a one-sample Kolmogorov-Smirnov test and the null hypothesis that the return come from a normal distribution was rejected at a 99,9% confidence level.

## 6.5 Tracking portfolio

The in-sample results from the multiple linear regression analysis on the whole ten year data set can be found in Table 5.

ABB was the first stock to be include in the tracking portfolio both for log returns and squared log returns see Table 5. However after ABB there was no real consensus on the order of stocks to include. Consistent higher $R^2$ was

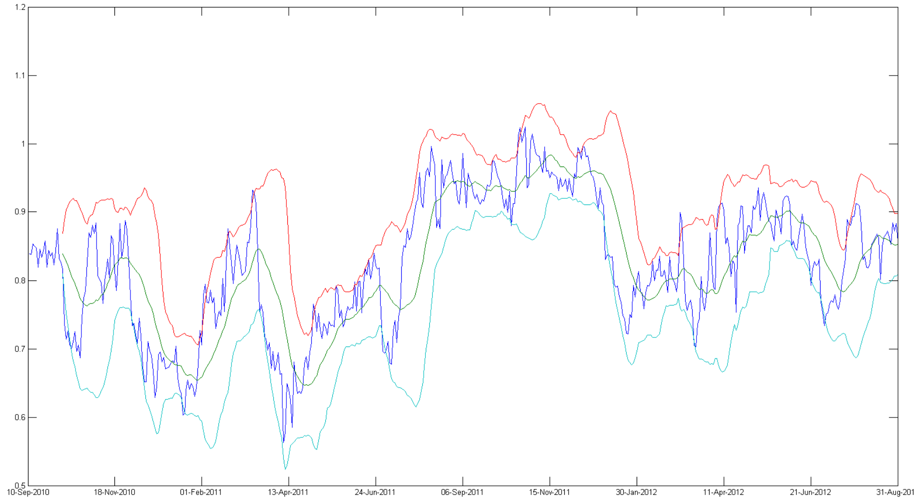


*Figure 27: Implied correlation plotted with a two standard deviations Bollinger band for the Straddle strategy. The blue line is implied correlation, the green line its 20 period simple moving average, the red and cyan line are the two standard deviations confidence level calculated on the moving average. Implied correlation is on average rather high and during the bear market of 2011 it peaks with some days even over one.*

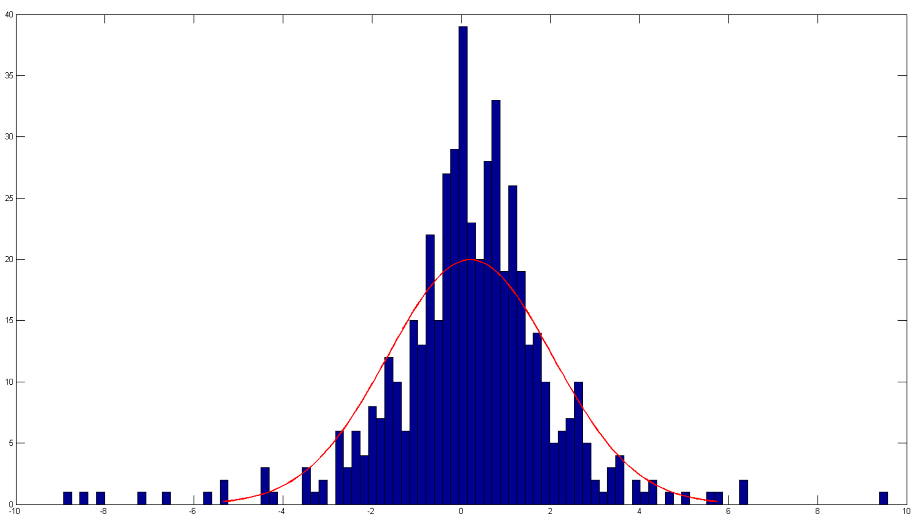


*Figure 28: A histogram plot of daily return whit a normal distribution line. The blue bars are the number of occurrences of returns with the correspond return size on the x-axis. The red line is a superimposed normal distribution line using the mean and standard deviation of the returns. The true return distribution seem to have fat tails especially to the left.*

| | Log returns | | Squared log returns | |
|---|---|---|---|---|
| #Stocks | Members | $R^2$ | Members | $R^2$ |
| 1 | ABB | 0,3998 | ABB | 0,2146 |
| 2 | HM B | 0,6259 | INVE B | 0,6899 |
| 3 | ERIC B | 0,7498 | NDA | 0,7959 |
| 4 | NDA | 0,8869 | HM B | 0,8169 |
| 5 | TLSN | 0,9139 | ATCO A | 0,8577 |
| 6 | AZN | 0,9186 | SKA B | 0,8648 |
| 7 | SAND | 0,9571 | TLSN | 0,8683 |
| 8 | ATCO A | 0,9663 | SHB A | 0,8724 |
| 9 | VOLV B | 0,9742 | SAND | 0,8784 |
| 10 | SHB A | 0,9802 | SCV B | 0,8808 |

*Table 5: These are the results from the in-sample linear regression analysis. The regression analysis was performed both on log returns and squared log returns. No clear stock selection order was found between the two. The log returns method had an overall higher $R^2$ than the squared log return method. The first column show the order of which stocks should be included, i.e. ABB is the first stock to have. Then HM B is the second stock to include in case of log returns and INVE B for squared log returns. The members column show which stock to add to the portfolio.*

obtained from using log returns than from using squared log returns. Using 29 of the 30 stocks in the index yielded the maximum in-sample $R^2$ of 0,99395[1] and 12 stocks could be removed before the in-sample $R^2$ dropped below 0,99. The maximum in-sample $R^2$ using squared log returns was 0,89107 using 29 stocks.

The tracking error for the tracking portfolio was tested by varying the re-weighing period from 1 to 40 times, the result can be found in Figure 30. In the first case the whole ten year data period of 2516 data points were used to optimise the tracking portfolio and in the last case the tracking portfolio was re-weighted every third month[2]. The trend is that in-sample $R^2$ increases and SSR decreases with the number of intervals although the improvements were small. Note that this is in-sample $R^2$ and SSR and therefore these values should increase as the model is closer fitted to the data.

When using five stocks in the tracking portfolio the rolling forecasting origin cross-verification test resulted in a MSE of $1,6437 \times 10^{-5}$, correlation between the forecast values and the true values of 0,9633 and a MASE of 0,1878. Again the MSE value is not scaled and hence hard to interpret outside of the same data set. The MASE value indicate that the model is better than the naïve model. However the One-Sample Kolmogorov-Smirnov test rejected the null hypothesis that the error term come from a normal distribution at a 99,9% confidence level and for all 29 tracking portfolios[3]. This indicate that the model can be improved

[1] ATCO A and ATCO B are both in the OMXS30 index.

[2] Note that the whole data set is still used, just that 40 individual regressions was performed instead of 1 large regression on the whole data set.

[3] 29 different tracking portfolios from using $1, 2, \ldots, 29$ stocks.

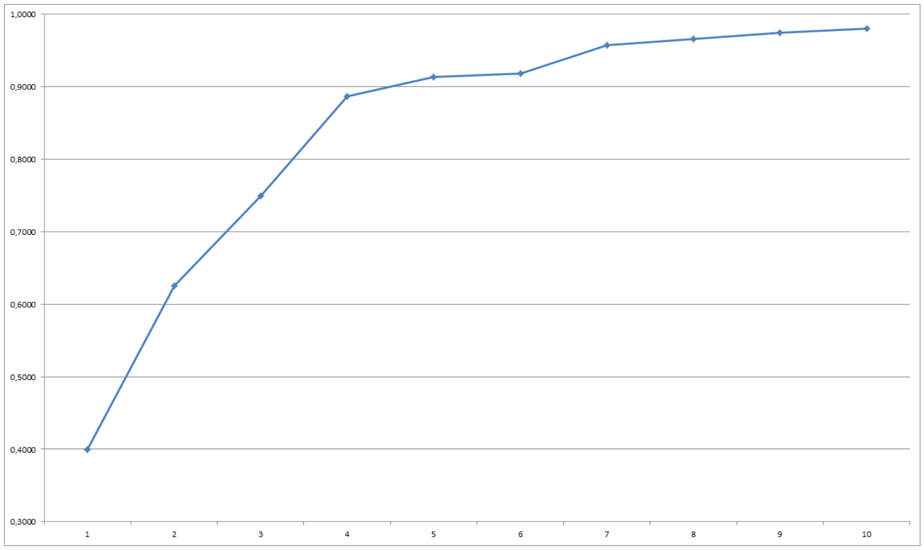


*Figure 29: The in-sample $R^2$ depending on number of stocks in the tracking portfolio using log returns. The improvement in $R^2$ is greatly reduced after having around five to eight stocks in the tracking portfolio, i.e. the slope is decreasing.*

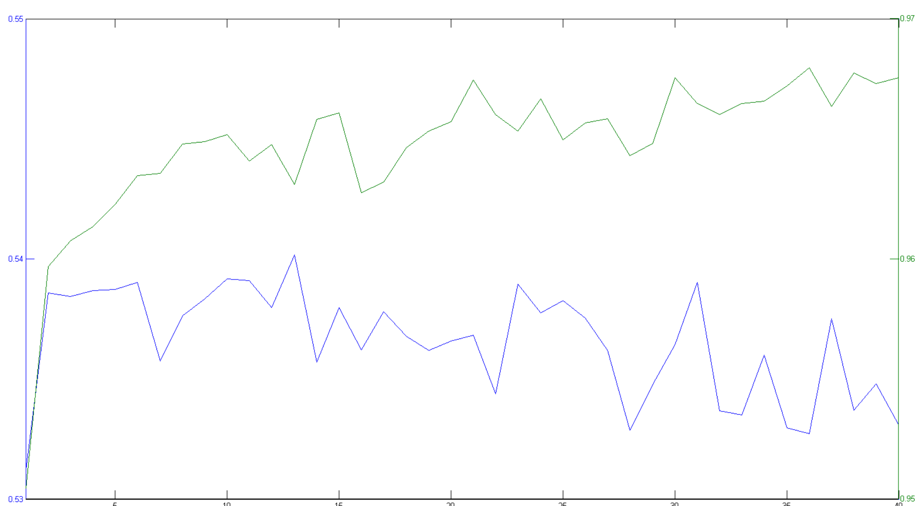


*Figure 30: The in-sample $R^2$ in green and SSR in blue depending on how many sub-datasets are used for the ten year regression analysis.*

| # Portfolio | Portfolio | members | | | | Prc of days |
|---|---|---|---|---|---|---|
| 1 | ABB | AZN | ERIC B | NDA | TLSN | 0,62% |
| 2 | ABB | AZN | ERIC B | HM B | TLSN | 3,62% |
| 3 | ABB | ERIC B | HM B | NDA | TLSN | 95,76% |

*Table 6: These are the three tracking portfolios which were chosen by the regression analysis. This show that the composition of the tracking portfolio was very stable where the last portfolio was used most of the time. The total number of trading days in the data set was 2263.*

| # Portfolio | Std w | | | | |
|---|---|---|---|---|---|
| 1 | 0,29% | 0,21% | 0,85% | 0,34% | 0,51% |
| 2 | 0,23% | 0,99% | 1,28% | 0,86% | 0,44% |
| 3 | 5,62% | 4,18% | 4,11% | 3,69% | 5,41% |

*Table 7: The standard deviation of weights in the three different portfolios selected by the regression analysis. The third portfolio had a higher standard deviation of weights which could be related to that it was used for a much longer time period and therefore had to adapt to larger changes in market conditions.*

as there are consistent biases present in the error term regardless of how many stocks are used.

Members of the tracking portfolio were stable with the allocation found in Table 6.

No other portfolio was selected by the regression model with the constraint of max five stocks.

The standard deviation of the weights for the three portfolios presented in Table 6 can be found in Table 7.

The first two portfolios had lower standard deviation for their weights. This is probably related to that those samples were very small with 14 days for the first portfolio and 82 days for the second portfolio. Results from the third portfolio is based on 2167 days and thus needed to adapt to larger changes in market conditions.

MSE, MASE and correlation from the cross-verification for 1 to 29 stocks in the tracking portfolio can be found in Figure 31, 32 and 33. They all consistently show that the maximum benefit of adding another stock to the tracking portfolio is reached at around five to six stocks.

## 6.6 Transaction costs

The first step to analyse the impact of transaction costs was to see how the return was impacted by paying the full bid-ask spread instead of half the spread[1] Thus the bid-ask spread for all options used was determined[2]. It was found that

[1] Mid-price is half the bid-ask spread.

[2] See Figure A.8, Figure A.9 and Figure A.10 in Appendix A for a graphical representation

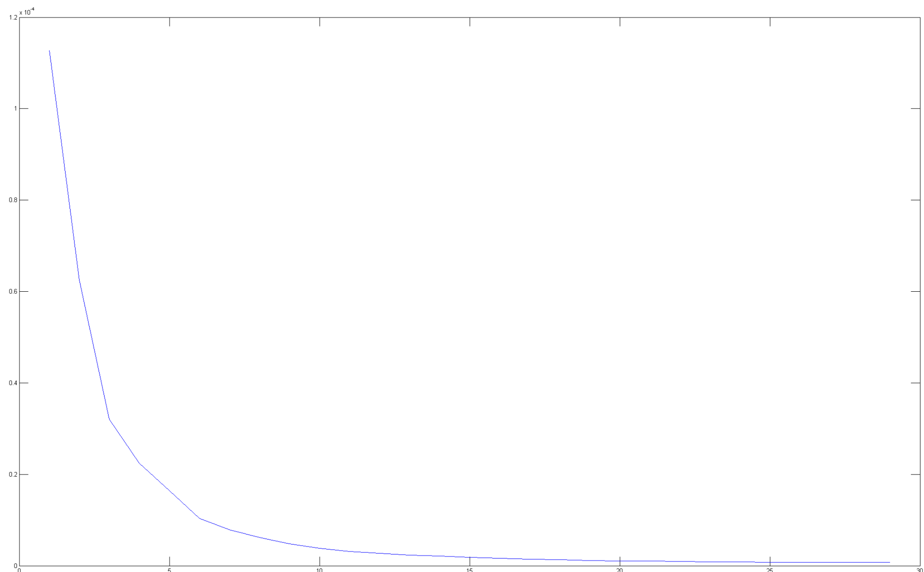


*Figure 31: Out-of-sample MSE is plotted on the y-axis against number of stocks in the tracking portfolio on the x-axis. In this case MSE is relevant since the same data set is used and the absolute, non-scaled error can be used to compare between the different portfolios. The MSE quickly diminish in the beginning by adding another stock. At around five to six stocks most of the decrease in MSE has happened.*

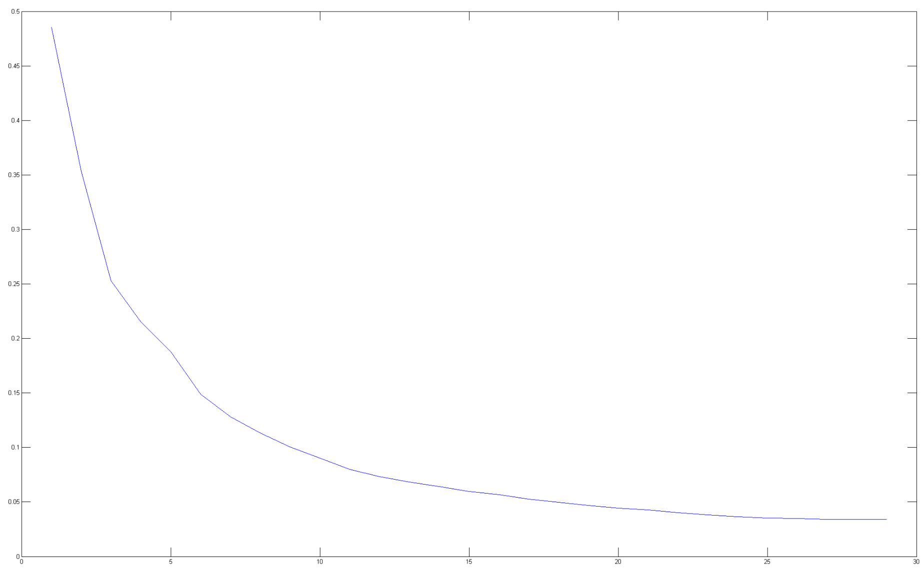


*Figure 32: Out-of-sample MASE is plotted on the y-axis and again the number of stocks on the x-axis. Now it is not necessary to use a scaled error term since it is compared for the same data set. Nonetheless the prediction quality increase very similar to the MSE result when compared to the naïve model.*

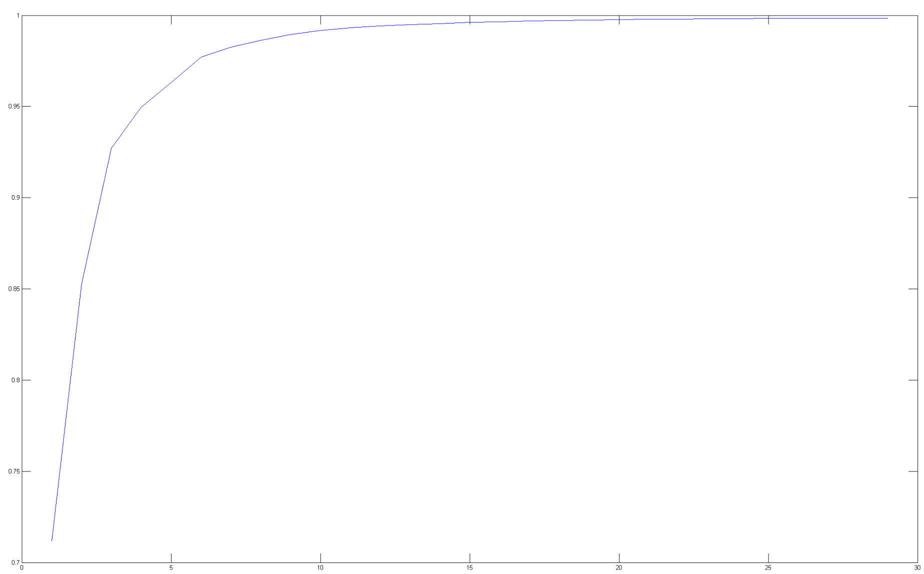


*Figure 33: Out-of-sample correlation is plotted on the y-axis and the number of stocks on the x-axis. The correlation factor between the predicted out-of-sample data and the true value quickly rose to 0.95 at around five to six stocks in the tracking portfolio. Then it starts to slowly converge towards one.*

| Call | | | | | | |
|---|---|---|---|---|---|---|
| ABB | ATCO A | ERIC B | HM B | NDA | TLSN | OMXS30 |
| 9,22% | 9,60% | 8,19% | 8,84% | 13,05% | 13,02% | 3,46% |
| Put | | | | | | |
| 9,29% | 8,08% | 7,96% | 8,08% | 12,14% | 10,59% | 3,35% |

*Table 8: Daily mean bid-ask option spreads. The spread is calculated as a percentage of the current mid price.*

| Price | P&L | Stdv | Sharpe | MDD | MDD% | $\beta$ |
|---|---|---|---|---|---|---|
| Mid-spread | 100,04 | 23,72% | -1,278 | 28,49 | 26% | -0,44 |
| Full-spread | 35,61 | 30,72 % | -2,358 | 37,99 | 59% | -0,07 |
| Commission | 29,17 | 33,15% | -2,464 | 38,94 | 65% | 0,03 |
| OMXS30 | - | 15,48% | - | 317,66 | 36% | 1 |

*Table 9: Results of the bid-ask spread analysis for the Straddle strategy. The first column is cumulative return, the second column is standard deviation of log returns. The third column is annualised Sharpe ratio. The fourth and fifth columns are maximum draw down and % maximum draw down. The last column is market beta. The first row hold the same results presented for the Straddle strategy in Table 3, the second row show results for the Straddle strategy paying full spread instead of the mid-price, the third row show results for the Straddle strategy paying both the full bid-ask spread and also commission. The final row show comparative results for the OMXS30 index during the period.*

the index option bid-ask spread tend to trade in the 2% to 4% interval[1]. The bid-ask spread was wider for the more illiquid single stock options which traded around at 10%. The mean bid-ask spreads for the six different single stock options and the index can be found in table 8.

The average bid-ask spread for the tracking portfolio was 9,77%. It was established using the mean bid-ask values found in Table 8 and combining them for the tracking portfolio, i.e. using its weights.

Then two P&L scenarios were established, one paying the whole bid-ask spread and one paying the whole bid-ask spread and commission for the Straddle strategy. These two scenarios were then compared to the P&L of the original Straddle strategy paying the mid price. The result can be found in Table 9 and in Figure 34.

[1]The spread is calculated as a percentage of the mid-price

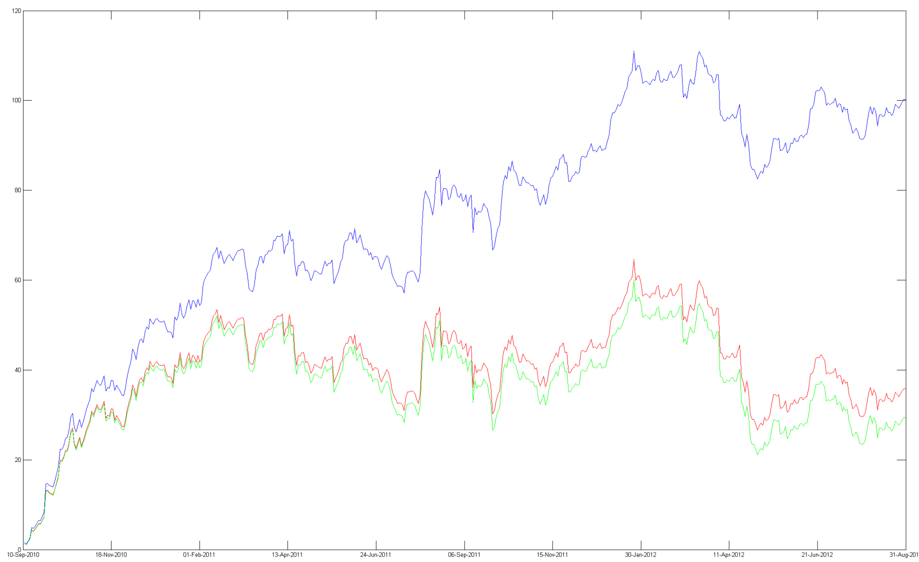


*Figure 34: Cumulative return for the Straddle strategy paying the mid-price in blue, paying the whole spread in red and paying the whole spread in green including commission. Return is negatively affected by the addition of the whole spread and then the commission. The major difference comes from paying the whole spread instead of half the spread.*

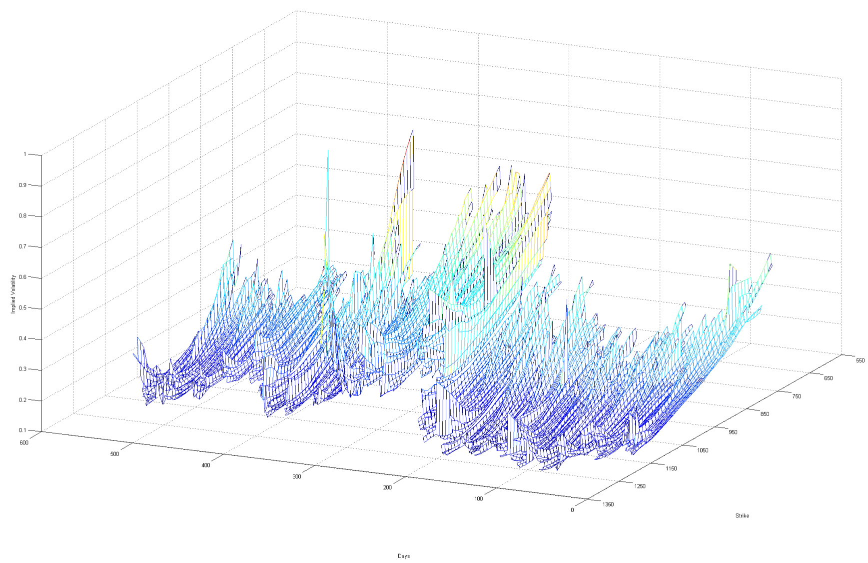


*Figure 35: The historical volatility smile for the trading period using both put and call options. Volatility is plotted on the z-axis, days on the x-axis and option strikes on the y-axis. Colder colours indicate lower implied volatility and hotter colours higher implied volatility. The skew is evident with lower implied volatility at higher strikes and higher implied volatility for lower strikes.*

## 6.7 Historical volatility smile

The historical volatility smile for the OMXS30 index was also plotted for the period in Figure 35 and Figure 36[1]. Both put and call options were used and missing data was interpolated with the same method as in the case of missing returns. It is possible to see how the OMXS30 index has changed during the period since the smile is centred around it. The smile appear to be warmer for OOM puts than for OOM calls and this pattern was strengthened when the marker crashed. This confirm that the smile became steeper during bear markets in the summer-autumn of 2011.

[1]The historical volatility smile surface is the same in both figures but the two figures are taken from two different angles of the 3D representation.

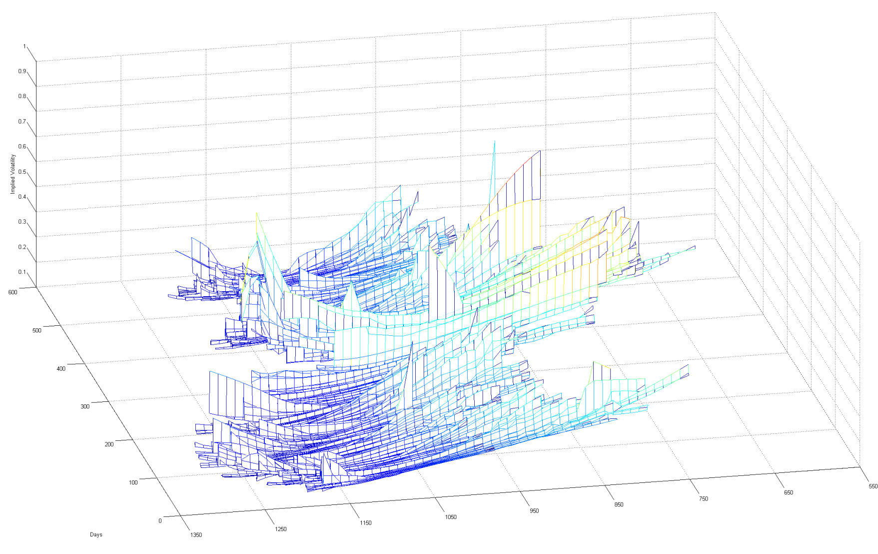


*Figure 36: The historical volatility smile surface for the trading period using both put and call options. Volatility is plotted on the z-axis, days on the x-axis and option strikes on the y-axis. Again colder colours indicate lower implied volatility and hotter colours higher implied volatility. From this angle it is easier to see how the market has moved during the period. The surface shifts left in the Figure or towards lower strikes since market makers stop quoting high strikes when the market move away from them and start quoting lower strikes when the market move towards them. It is also possible to see that the smile becomes more like a skew during the bear market i.e. lower strikes become hotter.*

# 7 Discussion

*In this chapter results are analysed. Firstly we evaluate the performance and characteristics of the strategies. Then follows an analysis of the tracking portfolio which then moves over into a discussion of assumptions, new findings and validity.*

## 7.1 Strategies

Firstly all three strategies had positive returns for the two year trading period[1], while the market index suffered a minor loss[2]. The Straddle strategy had the highest return but also the highest risk judging from the standard deviation and Sharpe ratio. Return for the Strangle and the Combination strategy were very similar, however the Combination strategy achieved the return with higher risk than the Strangle strategy. Thus it seems that continuously trading the smile show no increased profit but increased risk. The Strangle strategy had lower standard deviation than the Straddle strategy. A possible explanation to this could be that the Strangle strategy by default has a lower delta exposure than the other two which make the returns less sensitive to how well the exposure is delta hedged. The delta hedge process used in this study is far from optimal and thus a better delta hedging procedure could have a strong positive effect on the P&L or reduced risk for the Straddle and Combination strategy than for the Strangle strategy.

Sharpe ratios is usually a very good method to measure risk adjusted return for strategies or portfolios and is widely preferred (Chan, 2009). However calculating returns and Sharpe ratios become a complicated task when the strategy in question is self-financed. All three strategies had negative Sharpe ratios which mean that the return of the strategies were worse than the benchmark index, in this case the OMXS30. Interesting though the index had negative return and higher risk for the period while all strategies had positive returns. Thus it would seem that using the Index premium as required capital is too punitive for this kind of strategies. A margin model hence a modified version of the one used by NASDAQ OMX to calculate the required margin could better calculate the used capital (NAS, 2012). Furthermore the OMXS30 index could also be a bad benchmark index if it is to risky compared to dispersion trading. Although this is not outright supported by the findings since the standard deviation for the index were lower than return of the strategies.

Return of the Combination strategy was similar to the Strangle strategy but somewhat worse during the bear market. This could be attributed to that the skew steepened during the crash in the summer of 2011 and since the Combination strategy was short the skew it lost and then subsequently regain when the market recovered and the skew flattened by the turn of 2011. This can be confirmed in the historical volatility surface plot.

[1] 500 trading days, roughly equal 2 trading years.

[2] The OMXS30 equity index

An implied volatility index premium over the tracking portfolio was observed for all three strategies which confirm the existence of implied volatility discrepancies for the Swedish stock market. The average implied volatility spread was somewhat higher for the Combination strategy which would support the idea that the volatility smile-skew can be traded. However this was not confirmed in its P&L. The implied volatility spread increased in the bear market during the summer of 2011 and showed high negative correlation with the market. Thus dispersion strategies seem to become more profitable in stressed markets which also indicates that the market becomes less efficient during these periods. Implied correlation can further highlight the level of market inefficiency when during a few of the worst market days in the summer of 2011 crash implied correlation rose above one. Correlation can only trade in the range of [-1, 1] and therefore a correlation above one is pure arbitrage.

When looking at the distribution of returns, Figure 28 and Figure A.7, it seems that a large portion of returns fall outside the interval of what should be expected from a normal distribution. The one-sample Kolmogorov-Smirnov test confirm that returns are in fact not normal distributed. This is an important aspect since it should have great consequences for how the strategy is executed. For example the use of Sharpe ratio or standard deviation as a metric for risk or risk adjusted returns build on the assumption that returns are normal distributed. Thus MDD or other similar metrics that also incorporate fat tails should be emphasised. Outliers could also be caused by data errors and in all likelihood not all of the large return deviations are accurate. Nevertheless not all of the outliers can be explained by this.

An added benefit from being long the dispersion spread is that it could expose a investor to positive Black Swan events. Being long the dispersion spread equals buying tracking portfolio volatility and accordingly buying volatility on some of the index components. Hence company specific events resulting in high company specific volatility profit the long dispersion investor[1]. Consequently an investor should think twice about the risk he assume by shorting the dispersion spread. However an argument could be made that lighter stocks in the index i.e. those that will by definition not be included in the tracking portfolio are probably more prone to unexpected large company specific movements. Thus even though one is long the dispersion trade company specific events could have negative impact. Since they will still have a positive effect on index volatility[2], although a small one, and if the company is a light index weight it is probably not included in the tracking portfolio. Thus the investor fails to capture the increased single stock volatility leading to a loss.

Another risk with dispersion trading is that very large but hedged positions are initiated to capture small gains, but for example what happens if a hedge fail to work. Outright the positions are so large that even small errors can cause major losses. Therefore it is important to be prepared for what could go wrong

[1]Consider a company that report a year end result much lower than anticipated. This will probably lead to a spike in volatility for the stock but little will happen to index volatility because of diversification.

[2]Which a long dispersion investor is short.

if the model fail to predicted the market in order to prevent or at least limit the risk of Black Swan events such as what happened to LTCM (Lowenstein, 2002).

Transaction costs is of major concern when engaging in dispersion trading. The most important contribution to transaction costs is the bid-ask spread. The bid-ask spread vary greatly depending on what instrument is traded. For example the average spread for TLSN was almost 4 times as large as the average spread for OMXS30. When moving from paying the mid-price to paying the full spread final P&L was reduced by almost 65% which demonstrate the importance of incorporating transaction cost in the development process of the strategy. For this purpose it would been relevant to optimise the tracking portfolio not only for tracking error but also for transaction costs. The tracking portfolio is the major contributor to the bid-ask spread where the average spread for the index leg was 3,4% and the average spread for the tracking portfolio leg was 9,8%. Thus when developing a dispersion strategy one should pay attention to how the transaction cost can be minimised in the tracking portfolio.

Concluding this analysis of the strategies we have reached the following answers to the questions in Chapter 4.4:

- All three strategies were profitable and had low market correlation. They became more profitable during and immediately after market crashes, which was more evident for the Combination strategy.
- Given the low market correlation, risk assumed with dispersion trading is rather associated with the following characteristics:
  1. Risk for the implied volatility difference or implied correlation increase after the spread has been initiated.
  2. Inherent to the large positions, high sensitivity to model errors.
  3. Risk for the tracking portfolio fails to capture unusual market events.
  4. Return distribution exhibit fat tail behaviour and thus unforeseen outliers can have major impact on profitability.
- Transaction costs have major impact on profitability.

## 7.2 Tracking portfolio

The linear regression model showed good results and produced a tracking portfolio which was able to track the index with few members. As a comparison Kolanovic et al. (2010) constructed an *enhanced basket* for the Euro STOXX 50 index using 33 out of 50 stocks with a tracking error of 2,9% which is in-line with our results for the OMXS30 index. This indicate that the OMXS30 index is suitable for dispersion trading since few stocks can be used to capture most of the movement in the index.

It was also found that both members and weights were quite stable from day to day. This would imply that the model is robust since the index has changed during the ten years while the tracking portfolio could remain about the same.

The regression analysis can be performed on many different time horizons, updated daily using a trailing data window etc. It is rather hard to choose the best method since no guarantees exist that the future will play out as the past. Nevertheless the model saw improved tracking performance by shortening the data set and thus using more recent data. This indicate that the regression should be recalculated often and be based on current data. An alternative could be to also weigh the data where more recent data has a larger weight than older data. The trade-off of using weighted and shorter data sets is that important and infrequent events is missed out. These events could cause major losses if they are not included in the model. Thus closer fitted models result in better in-sample performance but this is achieved on the expense of out-of-sample results.

Cross-validation is a delicate process when performed on a financial time series since the descriptive quality of data decreases with time and also because returns are correlated. Such ordinary in-/out- of sample analysis is not very efficient (Hyndman and Athanasopoulos, 2002). The rolling forecasting origin method employed in this study try to address these problems while minimise the loss of data to the test set. The cross-validation test indicated a good out-of-sample prediction test using 5 stocks and a training period of the previous 252 trading days re-weighted daily. The cross-validation also showed that around five to six stocks in the tracking portfolio is the most optimal size. Adding more stocks than that led to little improvement in out-of-sample performance.

However the One-sample Kolmogorov-Smirnov test indicate that the error term is not random and therefore improvements to the model can be made since consistent bias exist in the error term.

## 7.3 The historical volatility smile

Plotting the historical volatility smile confirmed the tendency for the smile-skew to steepen in stressed markets. This imply that the Combination strategy could be advantageous to use after a market crash where both the implied volatility spread is high and the volatility smile is steep. This can be seen in the return graph of the Combination strategy Figure 24 where during the autumn of 2011 the Combination strategy outperform the Strangle strategy which otherwise were similar.

## 7.4 Assumptions and simplifications

It was assumed that the profit from dispersion trading arise from demand for index options and supply of single stock options. This is supported by studies such as Bollen and Whaley (2004) and Deng (2008) however there are advocates for the risk based hypothesis as well which if were proven would challenge the use of dispersion trading. Since then the premium for index options over single stock options has a fundamental explanation.

Another major assumption was that trading from the dispersion strategy has no market impact. This assumption is clearly incorrect as dispersion trading involves large positions in illiquid derivatives. There are methods to limit market impact when trading, for example the use of up to date trading algorithms which can trade securities while minimising market impact (Johnson, 2010). These algorithms can be programmed to gradually ease into the trade by trading fractions of the total amount. They usually start with illiquid legs and close the other legs as soon as fractions of the illiquid legs have been traded. Then the process is repeated while minimising market impact. Nevertheless these kind of strategies would quickly be spotted in the thin Swedish derivative market which is why dispersion trades often instead is block traded with OTC market makers[1].

The study was based on a back-testing methodology. The major assumption when back-testing is that the future will play out as the past. Less the use of a time machine it is impossible to know for sure how this assumption will affect results. Cross-verification was used in order to verify the tracking portfolio model. Nonetheless it is impossible to say if the strategy as a whole will perform in the future. Other stochastic simulation methods such as Monte Carlo simulations could be used to find further support for these results.

The delta hedging process used deserve more attention in any future studies. For example a process for when the current delta exposure becomes greater than a predetermined value the position is re-hedged could probably improve the hedging performance. It could be further enhanced by also take gamma and other greeks into consideration (Taleb, 1997).

The tracking portfolio was based on a linear regression model which was based on a few assumptions. Firstly it was assumed that the response variable[2] can be predicted using a linear model. This is not necessary true however as Ernie Chan puts it:

> *"Many years ago, a portfolio manager asked me in a phone interview: "Do you believe that linear or non-linear models are more powerful in building trading models?" Being a babe-in-the-woods, I did not hesitate in answering "Non-linear!" Little did I know that this is the question that separate the men from the boys in the realm of quantitative trading. Subsequent experiences showed me that non-linear models have mostly been unmitigated disasters in terms of trading profits."* Chan (2011).

Another assumption of linear regression was that the error term was homoscedastic i.e. have a constant variance. This is often not true and therefore it would be prudent to investigate. It was also assumed that error terms are independent which can be checked by testing for correlation with each other. All data in the model was assumed to be as meaningful regardless of how old it is. This is not true for stock returns as the market change which is why the selection of the data set is important.

The Black-and-Scholes option pricing method and the binomial tree pricing

[1] Dark pools can also be used.

[2] OMXS30 index return

method both make similar assumptions. They are widely accepted but should still be kept in mind (Hull, 2008):

1. Stock prices follow a Wiener process with constant expected return and standard deviation.
2. Short selling of securities are allowed with full use of the proceeds.
3. There are no transaction costs or taxes and all securities are perfectly divisible.
4. There are no dividends during the lifespan of the derivative.
5. There are no true arbitrage opportunities.
6. Trading is continuous.
7. The risk free rate is constant and the same for all maturities.

An intra-day sample point was used as the data point for a whole day of trading. This assumes that the average trading price for the whole day would be equal to that one quote. This is clearly not true and given the large position the execution price will almost always move against us.

## 7.5 New findings

Firstly, it was found that the same volatility spread detected in other markets (Marshall, 2008b) is also present in the Swedish market. It became stronger during stressed market conditions and was highly negative correlated with market returns. Secondly, implied correlation traded at unexpectedly high levels and even went higher than one during the most stressed market conditions. When implied correlation is above one it implies true risk free arbitrage. Thirdly, as anticipated, transaction costs have high impact on return of the strategy. Thus when devising dispersion strategies one should give much attention to minimising transaction costs. The major impact from adding transaction costs arise from paying the whole spread instead of the mid-price and accordingly this should be the area of focus.

## 7.6 Validity

Widely used and well documented methods were employed and thus it should not be too controversial. Implied correlation was found to be higher than anticipated but otherwise the findings are in-line with other studies (Ganatra, 2004; Marshall, 2008a) on other markets which also lends validity to results.

Nonetheless several improvements can be made to further increase validity. (1) the model can be tested on a more extensive dataset, both for a longer time period but more importantly on more frequent data e.g. minute samples instead of day samples. (2) stochastic methods can be used to verify results. (3) a better tracking portfolio method could be used in order to also optimise the tracking portfolio with transaction costs in mind. (4) a more realistic delta hedging process based on deviation of delta exposure while also looking at other greeks such as gamma. (5) market impact can be modelled to attain more realistic results.

The first four improvements are practical to implement while the fifth, modelling market impact is hard to achieve, because of the very large positions traded. Thus these results serve as an indication of how dispersion trading strategies behave and perform but they should not be directly translated into real P&L terms.

## 7.7 Entry and exit signals

The three strategies tested in this study is always in the market and thus use that on average index implied volatility is higher than tracking portfolio implied volatility. It was shown that this can be profitable but more practical strategies would use entry and exist signals to determine when to initiate and close dispersion trades. There are mainly two signals which can be used, implied correlation and the implied volatility spread between index options and the tracking portfolio. These are essentially very similar and indicate a similar picture, i.e. how much more expensive index options currently are than single stock options. In short dispersion strategies using entry and exit signals try to forecast the dispersion spread instead of using that being long dispersion on average pay off[1].

Dispersion strategies can be based on mean reversion where the fundamental idea is that the spread will converge back to its mean (Timmer, 2010). Thus if the spread is found to be far away from its historical mean it would be reasonable to assume that it will revert and thus the cheap leg should be bought and the expensive leg should be sold. Entry and exit signals can be generated by comparing implied volatility or implied correlation with historical volatility and historical correlation respectively. One method is the two standard deviation, 20 periods Bollinger band plotted with implied correlation (see Figure 27, A.5 and A.6). Consequently a possible strategy to investigate could be to buy the dispersion spread when implied correlation is equal to or below the lower two standard deviation confidence band and sell the spread when it reverts back to the moving average (Ganatra, 2004).

Another method is to instead of trusting that the spread will mean revert directly forecast it. There are many models which can be used to this purpose but one model in particular used for volatility forecasting is the Generalised Autoregressive conditional heteroskedasticity model (GARCH) (Sinclair, 2008). A GARCH model can be used to predict future volatility levels and therefore invest in the dispersion spread when the current value diverge from the forecast value.

[1] Very similar to how long only investors use that the equity market on average appreciate in value.

# 8 Conclusions

*In this chapter the report is concluded. The analysis is summarised and the implications of the study is then given. Finally recommendations for improvements and further studies are also included.*

This study was initiated in order to evaluate the performance and characteristics of dispersion trading. To accomplish this three basic strategies were tested and evaluated. All three strategies showed positive performance and low market correlation. A simple delta hedging procedure was used, a more realistic delta hedging method could probably enhance results. It was also found that transaction costs played a pivotal role when using dispersion trading. Thus transaction costs should be kept in mind when designing a dispersion strategy. For example a strategy can be to use dispersion trading to respond to other participants interest in selling single stock volatility, i.e. function as a flow market maker. By receiving the majority of the spread in a few of the single stock derivatives it will be possible to pay the spread in the remaining single stock/index derivatives to complete the dispersion spread and be profitable. Furthermore dispersion trading become highly profitable during stressed market conditions such as market crashes. The results even indicate that pure arbitrage situations can arise during those conditions.

Further research can both improve on the methods used in this study, such as using non-linear tracking portfolio optimisation models to also incorporate transaction costs but also focus on other areas left out of this study. Two main sections was not touched upon: (1) designing a strategy with entry and exit signals. (2) modelling market impact from the trading. The later addition is hard to accomplish given the very large positions required for dispersion trading. All three strategies tested in this study were always present in the market but a more realistic strategy could have entry and exist signals for when to initiate and close dispersion positions. Both the implied volatility spread and implied correlation can be used as trading signals.

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# A Appendix, complementing results

Complementing results can be found in this appendix. Many of the results were visually similar between the three strategies and therefore only one of the three strategies were presented in the Result chapter. Here results for the remaining two strategies are presented. The same presentation order is followed as in the Result chapter.

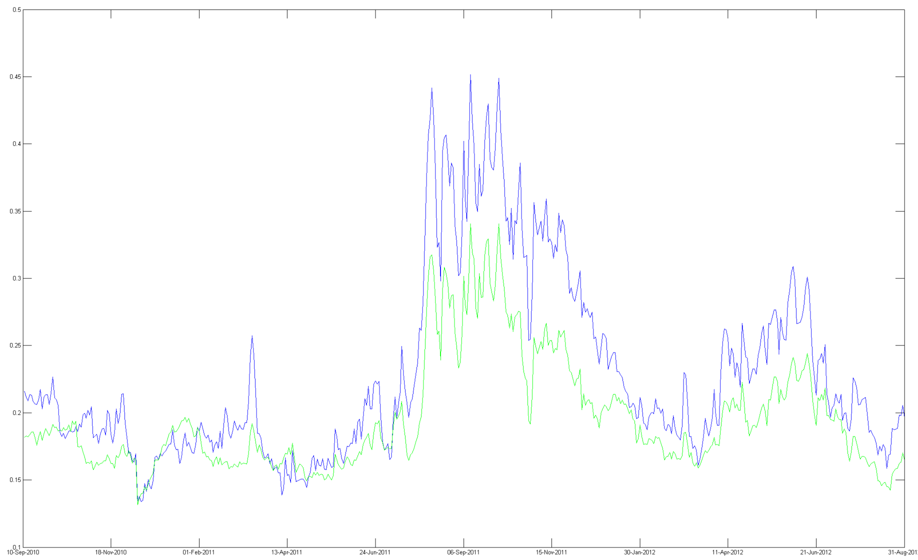


*Figure A.1: Implied volatility for both index options on the OMXS30 and stock options on the tracking portfolio for the strangle strategy. Index implied volatility is in blue and is slightly more expensive than volatility on the tracking portfolio in green.*

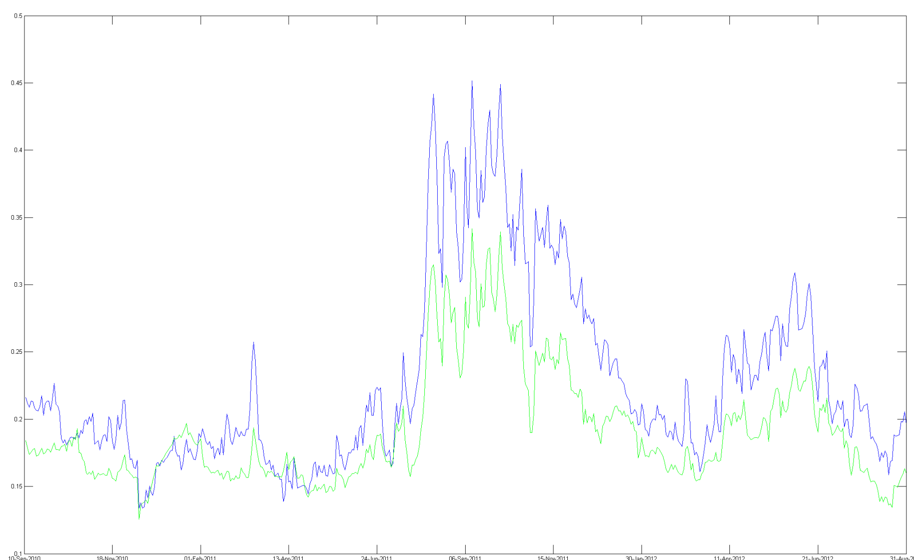


*Figure A.2: Implied volatility for both index options on the OMXS30 and stock options on the tracking portfolio for the combination strategy. Index implied volatility is in blue and is slightly more expensive than volatility on the tracking portfolio in green.*

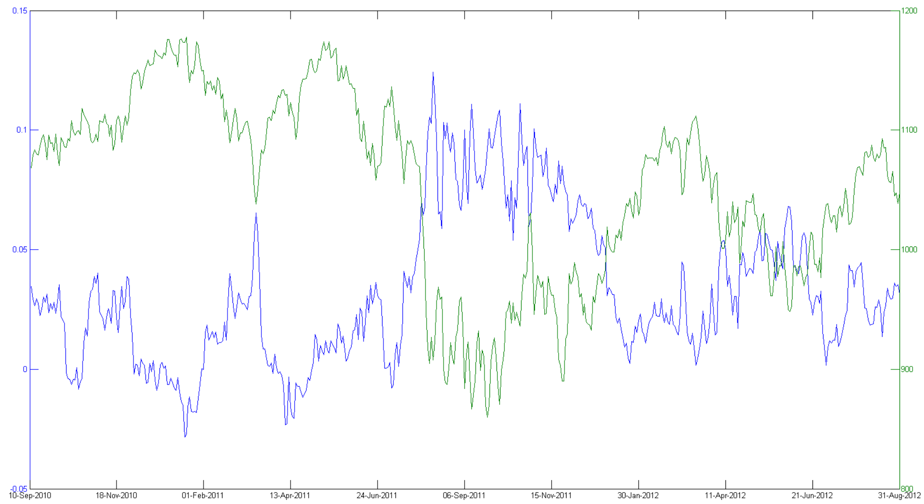


*Figure A.3: Implied volatility spread for the Strangle strategy. It is the difference in implied volatility of the index and the tracking portfolio. The implied volatility spread is plotted in blue and on the left y-axis and OMXS30 index return in green on the right axis using a different scale.*

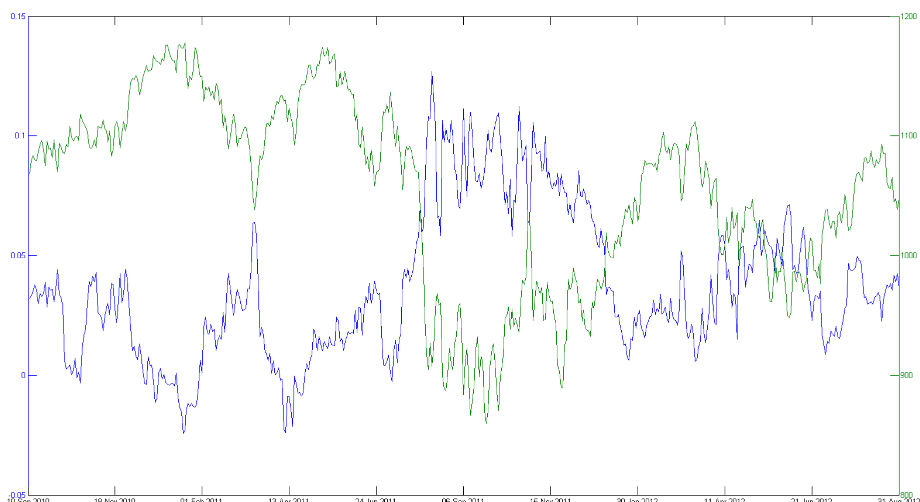


*Figure A.4: Implied volatility spread for the Combination strategy. It is the difference in implied volatility of the index and the tracking portfolio. The implied volatility spread is plotted in blue and on the left y-axis and OMXS30 index return in green on the right axis using a different scale.*

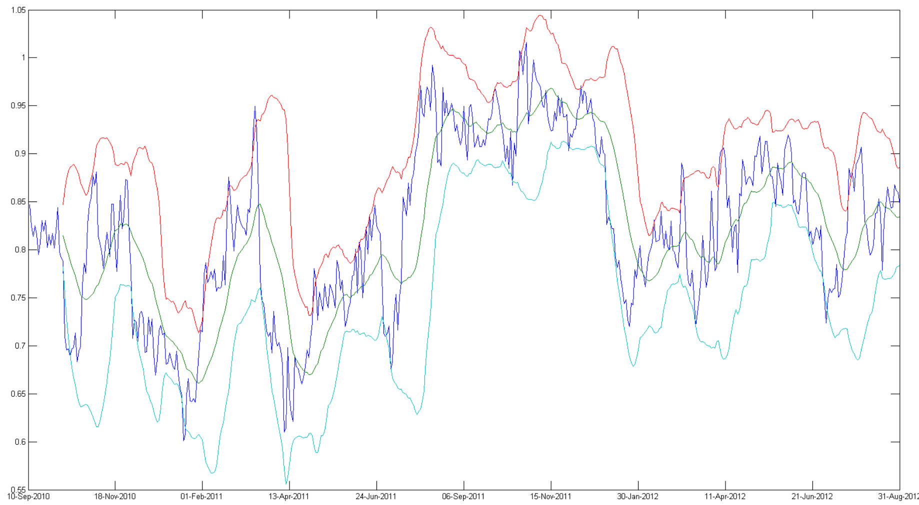


*Figure A.5: The implied correlation plotted with a two standard deviations Bollinger band for the strangle Strategy. The blue line is implied correlation, the green line its 20 period simple moving average, the red and cyan line are the two standard deviations confidence level calculated on the moving average. Implied correlation is on average rather high and during the bear market of 2011 it peaks with some days even over one.*

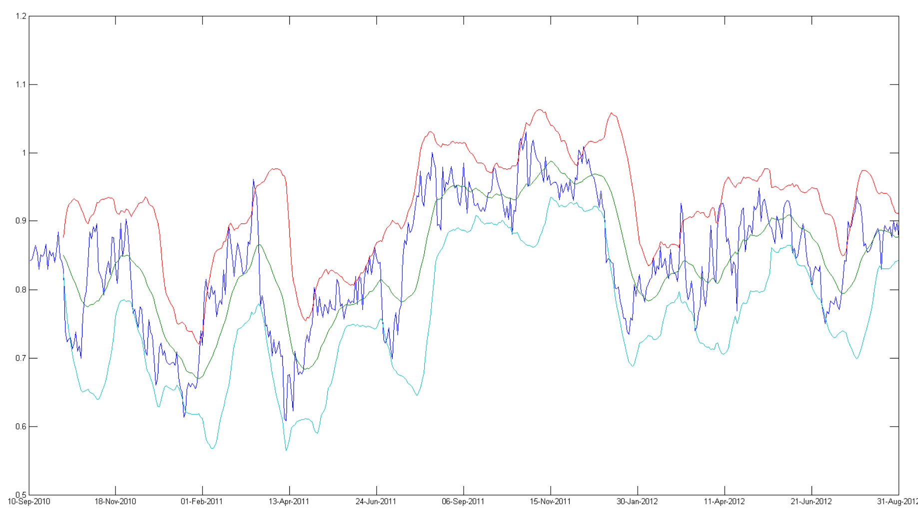


*Figure A.6: The implied correlation plotted with a two standard deviations Bollinger band for the Combination strategy. The blue line is implied correlation, the green line its 20 period simple moving average, the red and cyan line are the two standard deviations confidence level calculated on the moving average. Implied correlation is on average rather high and during the bear market of 2011 it peaks with some days even over one.*

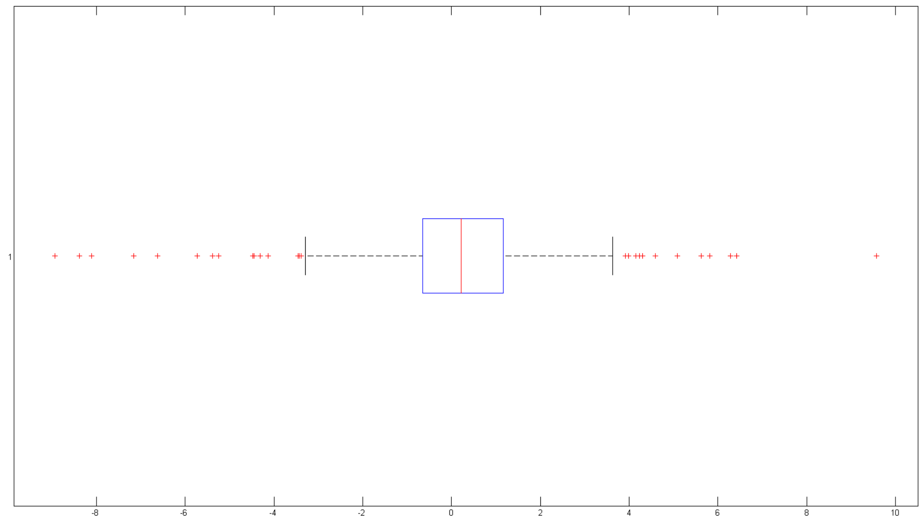


*Figure A.7: A box-and-whisker plot of daily return for the Straddle strategy. The box extends from the 25th to the 75th percentiles and the whiskers between the 0,7th and 99,3th percentile. Such 98,6% of all returns should fall within the whiskers which is not so. This also illustrate the non-normal distribution of returns.*

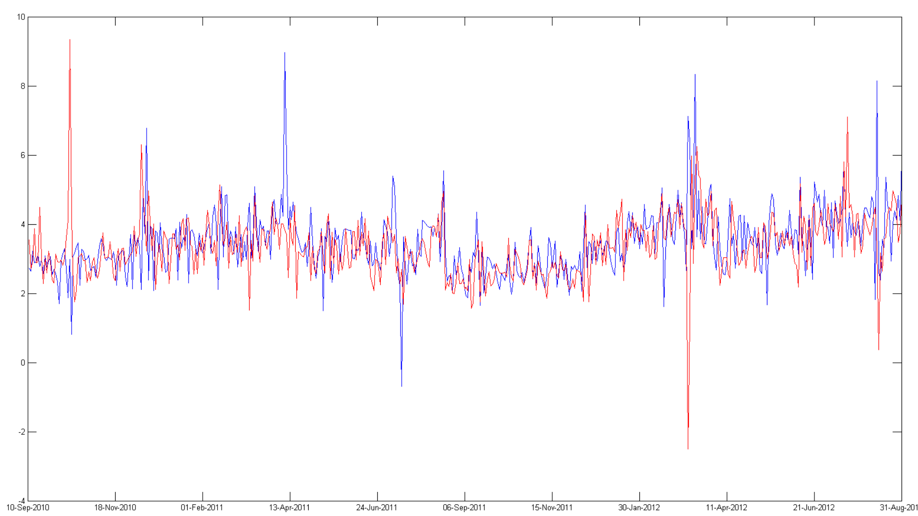


*Figure A.8: Daily bid-ask spread of ATM index options. The blue line represent call options and the red line put options. The spread as a percentage of the current mid-price is on the y-axis. No clear difference between put and call bid-ask spread was observed. The negative spread values are inherent to data errors as negative spreads can clearly not exist on the exchange since they would be instantly executed instead.*

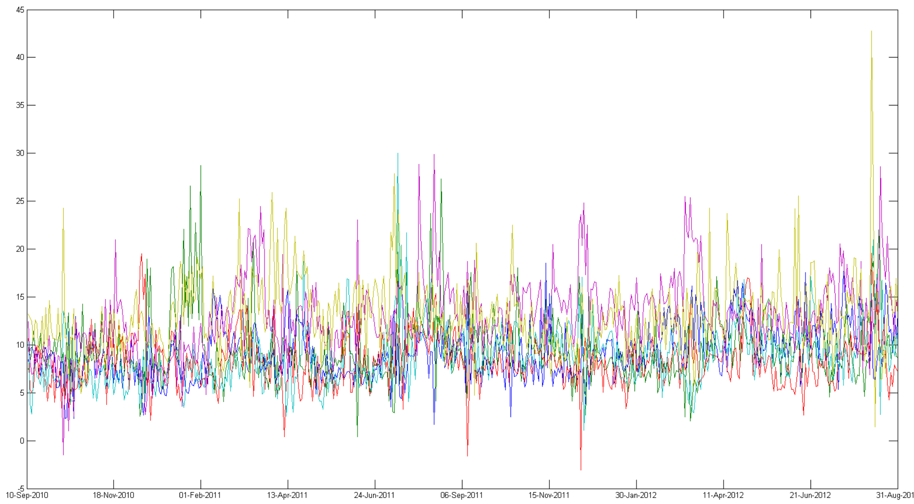

*Figure A.9: Daily bid-ask spread of ATM stock call options on the six stocks included in the tracking portfolio. The spread as a percentage of current mid-price is on the y-axis. The average spread vary for the individual stocks although they seem not to indicate any drift during the time period. Again the negative spread values are inherent to data errors as negative spreads can not exist on the exchange.*

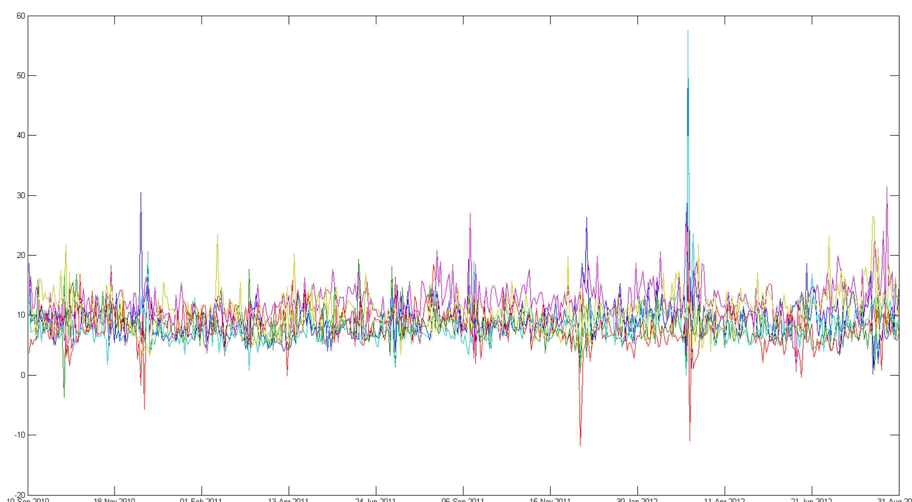

*Figure A.10: Daily bid-ask spread for the six ATM stock put options with the spread as a percentage of the current mid-price on the y-axis. The spread values are rather noisy in all cases but is on average greater for stock options than for index options.*