Problem 2. MT16121

1. The time complexity of Fleury's algorithm is $O(|E|^2)$ with the bottleneck present at bridge identification component. Optimizing this can push the bounds and improve the worst case time complexity. Other efficient approaches include Hierholzer's algorithm.

2. As the graph is Eulerian,
⇒ there exists an eulerian circuit in the graph (by definition of eulerian graphs). As the eulerian circuit starts and ends at the same vertex, therefore, every node has 2 edges incident over it.
Hence, G is eulerian iff every vertex has even degree. → G with 'm' edges

3. As the graph is Eulerian,
⇒ there exists an eulerian circuit.
The eulerian circuit can be represented by $(V_1, V_2), (V_2, V_3), \ldots (V_m, V_1)$.
The cycle may be of the form $(V_j, V_{j+1}), \ldots, V(j+k-1, V_{j+k})$ where $V_j, V_{j+1}, \ldots V_{j+k-1}$ are different & unique whereas $V_j = V_{j+k}$ for cycle completion.
Removing the smallest cycle (with minimum value of k), the modified graph cannot have $> m-1$ edges.
∴ By inductive hypothesis, the modified graph can be decomposed into disjoint cycles.
Hence, proved.