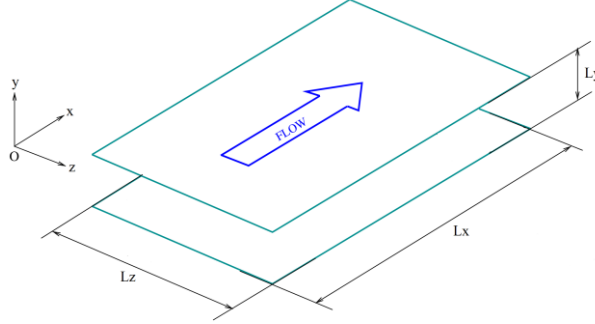


# Tutorial for the simulation of wall-bounded turbulent channel flow

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## 1. Introduction

This tutorial illustrates the step by step usage of Fourth-order **Channel3d** for wall-bounded turbulent channel flow at  $Re_\tau \approx 180$ . Here, periodic boundary conditions are used in streamwise (x) and spanwise (z) directions, while no-slip conditions are adopted for top and bottom walls in normal direction (y). The computational sketch is shown in Fig.1.



**Fig.1.** Sketch of computational domain for wall-bounded turbulent flows

The simulation has been initiated with a laminar parabolic Poiseuille velocity profile, plus a divergence-free large-amplitude sinusoidal perturbation:

$$u_0 = r_d u_b \bar{y} \exp\left(-\frac{9}{2} \bar{y}^2\right) \cos\left(\frac{\pi z}{100h} Re_\tau\right) + 3u_b \left(\bar{y} - \frac{1}{2} \bar{y}^2\right) \quad (1a)$$

$$w_0 = r_d u_b \bar{y} \exp\left(-\frac{9}{2} \bar{y}^2\right) \sin\left(\frac{\pi x}{250h} Re_\tau\right) \quad (1b)$$

$$v_0 = 0 \quad (1c)$$

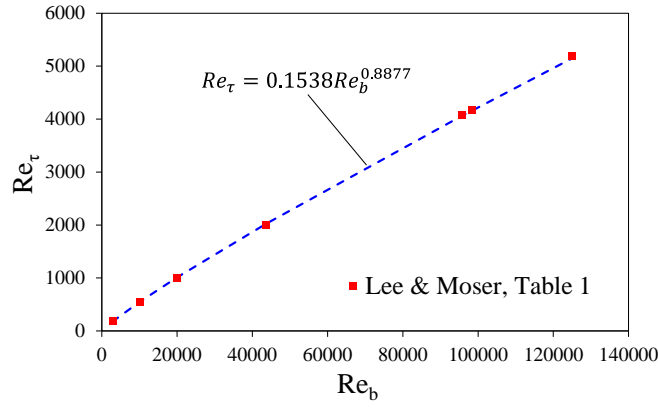
where  $\bar{y}$  is the dimensionless y coordinate defined as  $\bar{y} = \min(y, 2h - y)/h$ , and h denotes half channel height.  $r_d$  is a random number, ranging from 0.9 to 1.1, to further disturb the initial flow. The flow is driven by a uniform pressure gradient, which varies in time to maintain a constant mass flow rate. After several time unit, the laminar profiles transit to turbulent ones, and the pressure gradient begins to fluctuate near an almost constant value.

## 2. Preparation

The friction Reynolds number is defined as  $Re_\tau = u_\tau h / \nu$ , where  $\nu$  is the fluid kinematic viscosity, and  $u_\tau$  is friction velocity:  $u_\tau = \sqrt{(v \partial \bar{u} / \partial y)_{wall}} = \sqrt{(-dp/dx)h}$ .  $Re_b$  denotes the bulk Reynolds number:  $Re_b = u_b h / \nu$ , where  $u_b$  is the bulk velocity. For wall-bounded turbulence,  $Re_b$  and  $Re_\tau$  have the following empirical relationship:

$$Re_\tau = 0.1538 Re_b^{0.8877} \quad (2)$$

Eq.(2) is regressed from the data provided by Lee & Moser [1], and the regression result is also shown in Fig.2. If we want to simulate the case  $Re_\tau \approx 180$ , we can use Eq.(2) to get an approximate value for  $Re_b$ , i.e.  $Re_b \approx 2860$ . If we further set  $u_b = 2/3$  and  $h = 1$ , we can get the corresponding viscosity:  $\nu = (u_b h) / Re_b = 2.3310 \times 10^{-4}$ .



**Fig.2.** Relationship between bulk and friction Reynolds number

### 3. Parameter setting

The computational domain sizes are set to  $4\pi h$ ,  $2h$ ,  $2\pi h$  in  $x$ -,  $y$ - and  $z$ -directions respectively. Details about the flow parameters and grid mesh are listed in Table 1. The input parameter text can be found in file `./doc/TurbulentChannel_4th_direct.prm`. The illustrations for every input parameters can be found in file `./doc/channel4th_prm.md`.

**Table 1**

List of parameters for turbulent channel flow cases. Here  $N_x$ ,  $N_y$ , and  $N_z$  are the number of grid points in streamwise, wall-normal and spanwise directions, respectively.  $\Delta x^+$  and  $\Delta z^+$  denote grid spacings in wall-parallel directions.  $\Delta y_w^+$  and  $\Delta y_c^+$  represent the grid spacing at the wall and the centerline in wall-normal direction respectively.  $Tu_\tau/h$  is the time averaging window without transition.

Flow Case	$Re_\tau$	$Re_b$	$N_x$	$N_y$	$N_z$	$\Delta x^+$	$\Delta y_w^+$	$\Delta y_c^+$	$\Delta z^+$	$Tu_\tau/h$
Cha180	182.3	2860	384	192	256	5.97	0.0244	2.98	4.48	20.40

### 4. Running the code

If the executable file `./channel4th` and `./Tool/interpolateField/interpolateField` has not been compiled, you should compile it firstly, following the guidance written in `./README.md`, Installation part. The simulation results and restart files will be stored in the folder `./CFD/Results/` and `./CFD/Restart`, respectively.

Now, you can run this test case directly by typing:

```
1  chmod a+x ./channel4th
2  mpirun -n 8 ./channel4th ./Input/TurbulentChannel_4th_direct.prm
```

Alternatively, we can run the case on a coarse grid firstly, and then interpolate the resulting flow field to the fine mesh. If so, we can do as follows.

#### 4.1 Running on the coarse mesh

Firstly, we run on a coarse grid where  $N_x \times N_y \times N_z = 180 \times 132 \times 120$ . Here we choose an arbitrary coarse mesh configuration. The evolution of  $Re_\tau$  for coarse mesh is shown in Fig.3.

```
1  mpirun -n 8 ./channel4th ./Input/TurbulentChannel_4th_1.prm
```

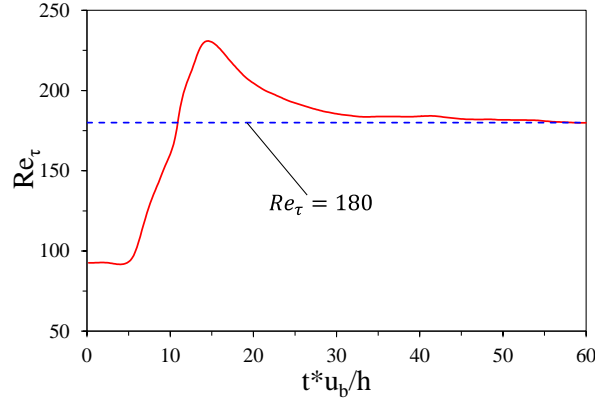


Fig.3. Time evolution of the friction Reynolds number  $Re_\tau$  for coarse mesh

#### 4.2 Interpolating flow field from coarse mesh to fine mesh

Now, we need to interpolate flow field from coarse mesh to fine mesh, using the executable file `./Tool/interpolateField/interpolateField`:

```
1 cd ./Tool/interpolateField/
2 chmod a+x interpolateField
3 mpirun -n 8 interpolateField interp1.prm
4 cd ../../
```

After that, New restart file (`./CFD/RestartForCha180_2_0000003000`) will be generated.

#### 4.3 Running on the fine mesh

Lastly, we run on the fine mesh where  $N_x \times N_y \times N_z = 384 \times 192 \times 256$ . The evolution of  $Re_\tau$  for fine mesh is shown in Fig.4.

```
1 mpirun -n 8 ./channel4th ./Input/TurbulentChannel_4th_2.prm
```

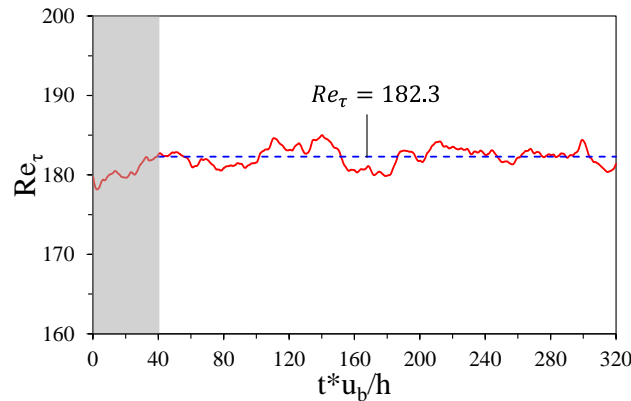


Fig.4. Time evolution of the friction Reynolds number  $Re_\tau$  for fine mesh. Gray area indicates the adjustment stage from coarse mesh to fine mesh, and the corresponding time interval will be excluded from the statistic calculation.

### 5. Visualization

One can use Paraview for visualization. An interface file ended with the suffix `.xmf` is included in the folder `./CFD/Results/` (`./CFD/Results/VisuForCha180_2_.xmf` here), and this file can be read by Paraview directly.

## 6. Post-processing

Post-processing might be the most interesting part for wall-bounded turbulence. Here I will illustrate how to get the desired statistic data using **Channel3d**.

### 6.1 Data storage

The canonical statistic data include mean velocity profile, turbulent intensity, Reynolds stress, vorticity correlation, skewness, flatness, turbulent budget, energy spectra and so on.

If  $a$ ,  $b$ ,  $c$ , and  $d$  denote four variables. The second-order correlation can be defined as:

$$\langle a'b' \rangle = \langle [a - \langle a \rangle][b - \langle b \rangle] \rangle \quad (3)$$

While Eq.(3) can not be used directly, due to the fact that the exact  $\langle a \rangle$  and  $\langle b \rangle$  are not available until end of the simulation. So alternately, we can use the following expression instead:

$$\langle a'b' \rangle = \langle ab \rangle - \langle a \rangle \langle b \rangle \quad (4)$$

Similarly, the third-order and fourth-order correlations read:

$$\langle a'b'c' \rangle = \langle abc \rangle - \langle a \rangle \langle bc \rangle - \langle b \rangle \langle ac \rangle - \langle c \rangle \langle ab \rangle + 2\langle a \rangle \langle b \rangle \langle c \rangle \quad (5)$$

$$\begin{aligned} \langle a'b'c'd' \rangle = & \langle abcd \rangle - 3\langle a \rangle \langle b \rangle \langle c \rangle \langle d \rangle \\ & + \langle a \rangle \langle b \rangle \langle cd \rangle + \langle b \rangle \langle c \rangle \langle da \rangle + \langle c \rangle \langle d \rangle \langle ab \rangle \\ & + \langle d \rangle \langle a \rangle \langle bc \rangle + \langle a \rangle \langle c \rangle \langle bd \rangle + \langle b \rangle \langle d \rangle \langle ac \rangle \\ & - \langle a \rangle \langle bcd \rangle - \langle b \rangle \langle cda \rangle - \langle c \rangle \langle dab \rangle - \langle d \rangle \langle abc \rangle \end{aligned} \quad (6)$$

For instance, if we want to determine pressure strain term  $\pi_{1,2} = \langle p'(\partial u'/\partial y + \partial v'/\partial x) \rangle$ , we can further expand it as follows:

$$\pi_{1,2} = \left\langle p \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \right\rangle - \langle p \rangle \left[ \frac{\partial \langle u \rangle}{\partial y} + \frac{\partial \langle v \rangle}{\partial x} \right] = \left\langle p \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \right\rangle - \langle p \rangle \frac{\partial \langle u \rangle}{\partial y} \quad (7)$$

So we need to calculate and store  $\langle p(\partial u/\partial y + \partial v/\partial x) \rangle$ ,  $\langle p \rangle$ , and  $\langle u \rangle$  in advance. [Table 2](#) lists the raw averaged data contained in file `./CFD/Results/stats*`. It is also worth nothing that due to the use of fully staggered grids, so averaged variables (e.g.  $\langle u \rangle$ ) are placed in the y-center layer, while others (e.g.  $\langle v \rangle$ ) are stored in the y-node layer. The definition of turbulent budget terms are listed in [Appendix A](#).

The raw averaged 1D energy spectra data are also stored in the file `./CFD/Results/Spec*`.

### 6.2 Statistic results calculation

The final statistic results can be calculated by the post-processing Matlab code, `./Tool/readStat_CH.m`. If Matlab is not available, one can use Octave instead:

```
1 cd ./Tool/
2 octave readStat_CH.m
3 cd ../
```

After running this Matlab code, the final statistic results will be generated in the folder `./StatOut/`.

Table 2

List of the raw averaged data contained in file `./CFD/Results/stats*`. Here 'location' column indicates where the variable is placed. 'C' means 'Center', and 'N' stands for 'Node'.

id	variable name	location	id	variable name	location
1	$\langle u \rangle$	C	19	$\langle uuu \rangle$	C
2	$\langle v \rangle$	N	20	$\langle www \rangle$	C
3	$\langle w \rangle$	C	21	$\langle uuuu \rangle$	C
4	$\langle p \rangle$	C	22	$\langle vvvv \rangle$	N
5	$\langle uu \rangle$	C	23	$\langle wwww \rangle$	C
6	$\langle vv \rangle$	N	24	$\langle p \partial u / \partial x \rangle$	C
7	$\langle ww \rangle$	C	25	$\langle p \partial v / \partial y \rangle$	C
8	$\langle pp \rangle$	C	26	$\langle p \partial w / \partial z \rangle$	C
9	$\langle uv \rangle$	C	27	$\langle p(\partial u / \partial y + \partial v / \partial x) \rangle$	N
10	$\langle vw \rangle$	C	28	$\langle u \nabla^2 u \rangle$	C
11	$\langle uw \rangle$	C	29	$\langle v \nabla^2 v \rangle$	N
12	$\langle up \rangle$	C	30	$\langle w \nabla^2 w \rangle$	C
13	$\langle vp \rangle$	C	31	$\langle \frac{\partial u}{\partial x} \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \frac{\partial v}{\partial y} \rangle$	C
14	$\langle wp \rangle$	C	32	$\langle (\partial u / \partial z)(\partial v / \partial z) \rangle$	N
15	$\langle uuv \rangle$	C	33	$\langle \omega_x \omega_x \rangle$	N
16	$\langle vvv \rangle$	N	34	$\langle \omega_y \omega_y \rangle$	C
17	$\langle wwv \rangle$	C	35	$\langle \omega_z \omega_z \rangle$	N
18	$\langle uvv \rangle$	C			

## 7. Statistic results validation

This section compare the statistic results with Lee & Moser [1], and Vreman & Kuerten [2] to for validation.

### 7.1 Basic profiles

Fig.5 shows some turbulent statistic profiles, i.e. mean velocity, velocity fluctuations, Reynolds stress, vorticity fluctuations, mean pressure, pressure fluctuation.

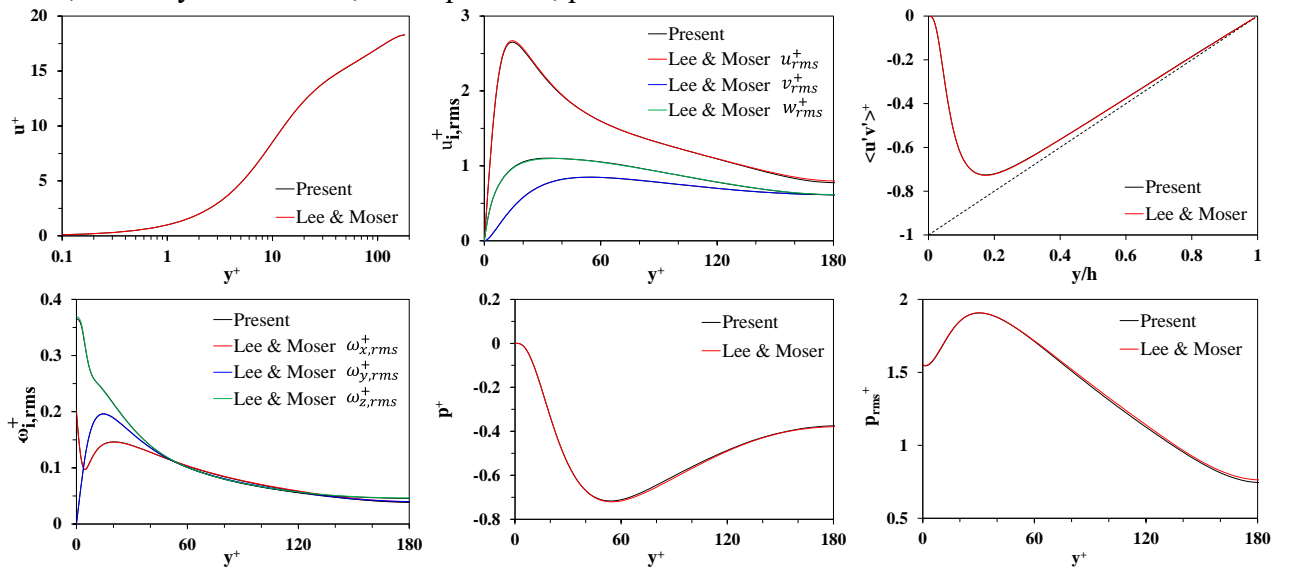


Fig.5. Basic turbulent statistic profiles

## 7.2 Budget profiles for TKE

Fig.6 shows the budget profiles for TKE.

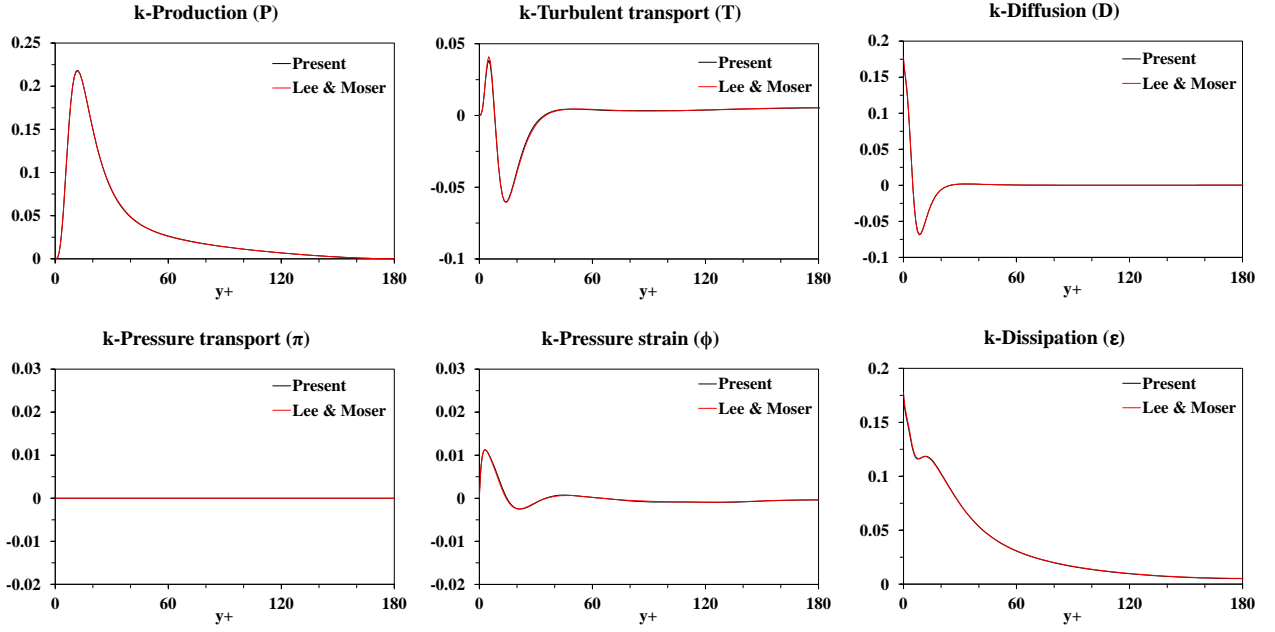


Fig.6. Budget profiles for TKE

## 7.3 Budget profiles for $\langle u'u' \rangle$

Fig.7 shows the budget profiles for  $\langle u'u' \rangle$ .

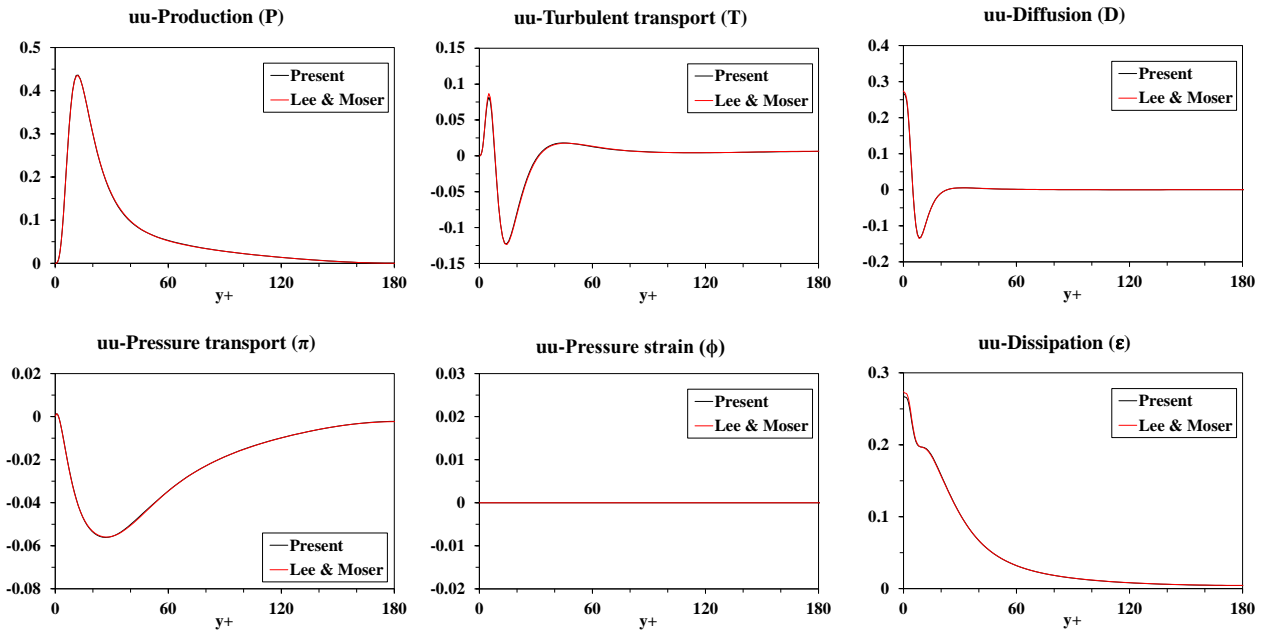


Fig.7. Budget profiles for  $\langle u'u' \rangle$

### 7.4 Budget profiles for $\langle v'v' \rangle$

Fig.8 shows the budget profiles for  $\langle v'v' \rangle$ .

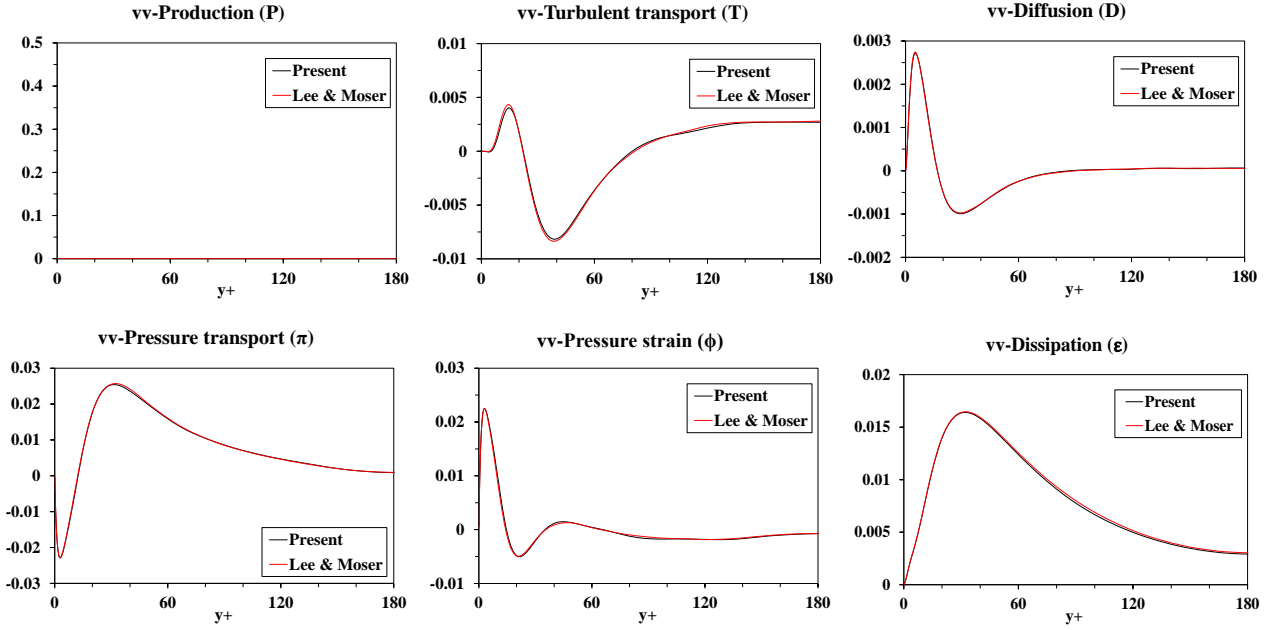


Fig.8. Budget profiles for  $\langle v'v' \rangle$

### 7.5 Budget profiles for $\langle w'w' \rangle$

Fig.9 shows the budget profiles for  $\langle w'w' \rangle$ .

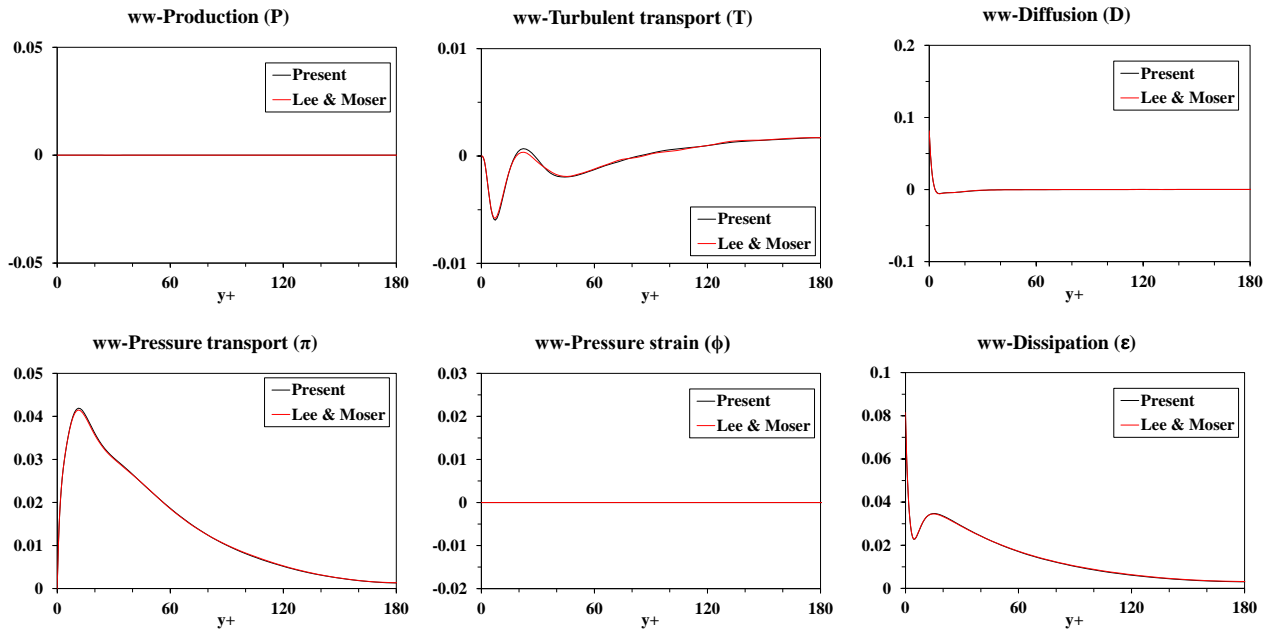


Fig.9. Budget profiles for  $\langle w'w' \rangle$

## 7.6 Budget profiles for $\langle u'v' \rangle$

Fig.10 shows the budget profiles for  $\langle u'v' \rangle$ .

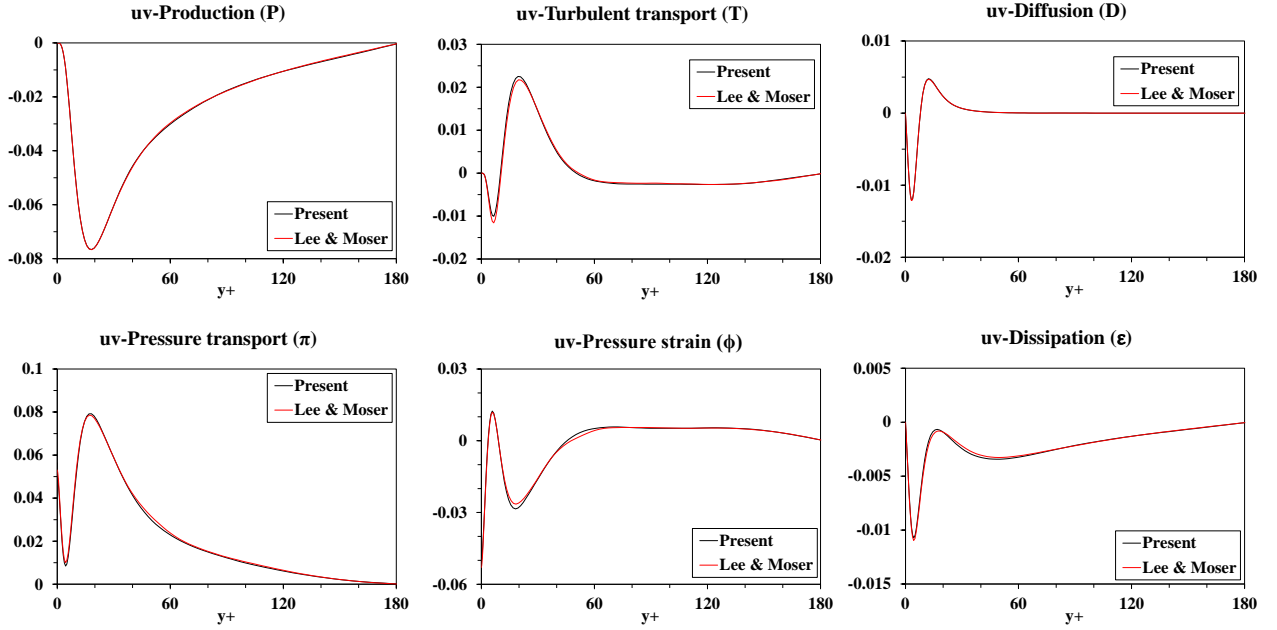


Fig.10. Budget profiles for  $\langle u'v' \rangle$

## 7.7 Energy spectra profiles

Fig.11 shows the energy spectra profiles at  $y^+ \approx 36.5$ .

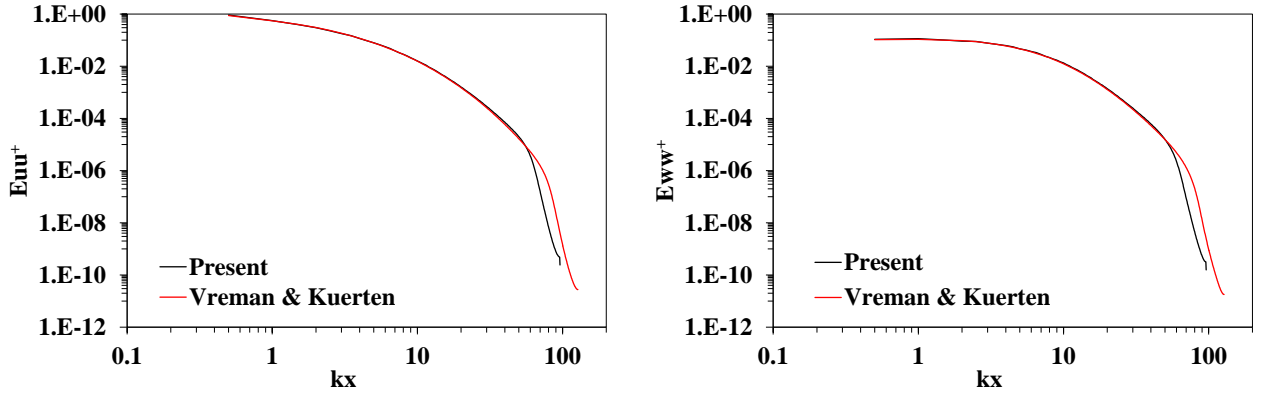


Fig.11. Energy spectra profiles at  $y^+ \approx 36.5$

## Reference:

- [1] Lee M, Moser RD. Direct numerical simulation of turbulent channel flow up to  $Re_\tau \approx 5200$ . J Fluid Mech. 2015;774:395-415. doi:[10.1017/jfm.2015.268](https://doi.org/10.1017/jfm.2015.268).
- [2] Vreman AW, Kuerten JGM. Comparison of direct numerical simulation databases of turbulent channel flow at  $Re_\tau = 180$ . Phys Fluids. 2014;26(1):1-21. doi:[10.1063/1.4861064](https://doi.org/10.1063/1.4861064).



## Appendix A. Definition of turbulent budget terms for wall-bounded turbulence

The general Reynolds stress transport equation reads:

$$\frac{\partial \langle u'_i u'_j \rangle}{\partial t} + \langle u_k \rangle \frac{\partial \langle u'_i u'_j \rangle}{\partial x_k} = P_{i,j} + T_{i,j} + \phi_{i,j} + \pi_{i,j} + D_{i,j} - \varepsilon_{i,j} \quad (\text{A.1})$$

where  $P_{i,j}$ ,  $T_{i,j}$ ,  $\phi_{i,j}$ ,  $\pi_{i,j}$ ,  $D_{i,j}$ , and  $\varepsilon_{i,j}$  stand for production term, turbulent transport term, pressure transport term, pressure strain term, viscous transport (diffusion) term, and viscous dissipation term respectively:

$$P_{i,j} = -\langle u'_i u'_k \rangle \frac{\partial \langle u_j \rangle}{\partial x_k} - \langle u'_j u'_k \rangle \frac{\partial \langle u_i \rangle}{\partial x_k} \quad (\text{A.2a})$$

$$T_{i,j} = -\frac{\partial \langle u'_i u'_j u'_k \rangle}{\partial x_k} \quad (\text{A.2b})$$

$$\phi_{i,j} = -\frac{\partial \langle p' u'_i \rangle}{\partial x_j} - \frac{\partial \langle p' u'_j \rangle}{\partial x_i} \quad (\text{A.2c})$$

$$\pi_{i,j} = \left\langle p' \left( \frac{\partial u'_i}{\partial x_j} + \frac{\partial u'_j}{\partial x_i} \right) \right\rangle \quad (\text{A.2d})$$

$$D_{i,j} = \nu \frac{\partial^2 \langle u'_i u'_j \rangle}{\partial x_k \partial x_k} \quad (\text{A.2e})$$

$$\varepsilon_{i,j} = 2\nu \left\langle \frac{\partial u'_i}{\partial x_k} \frac{\partial u'_j}{\partial x_k} \right\rangle \quad (\text{A.2f})$$

Specifically, for wall-bounded turbulent flow, where periodic boundary conditions are applied in streamwise and spanwise directions. So Eq.(A.2) can be further simplified as follows:

$$\begin{aligned} P_{i,j} &= -\langle u'_i v' \rangle \frac{\partial \langle u_j \rangle}{\partial y} - \langle u'_j v' \rangle \frac{\partial \langle u_i \rangle}{\partial y}, & T_{i,j} &= -\frac{\partial \langle u'_i u'_j v' \rangle}{\partial y} \\ \phi_{i,j} &= -\frac{\partial \langle p' u'_i \rangle}{\partial x_j} - \frac{\partial \langle p' u'_j \rangle}{\partial x_i}, & \pi_{i,j} &= \left\langle p' \left( \frac{\partial u'_i}{\partial x_j} + \frac{\partial u'_j}{\partial x_i} \right) \right\rangle \\ D_{i,j} &= \nu \frac{\partial^2 \langle u'_i u'_j \rangle}{\partial y^2}, & \varepsilon_{i,j} &= 2\nu \left\langle \frac{\partial u'_i}{\partial x_k} \frac{\partial u'_j}{\partial x_k} \right\rangle \end{aligned} \quad (\text{A.3})$$

And for  $\langle u'u' \rangle$ :

$$\begin{aligned} P_{1,1} &= -2\langle u'v' \rangle \frac{\partial \langle u \rangle}{\partial y}, & T_{1,1} &= -\frac{\partial \langle u'u'v' \rangle}{\partial y}, & \phi_{1,1} &= 0 \\ \pi_{1,1} &= 2\left\langle p' \frac{\partial u'}{\partial x} \right\rangle, & D_{1,1} &= \nu \frac{\partial^2 \langle u'u' \rangle}{\partial y^2}, & \varepsilon_{1,1} &= 2\nu \left\langle \frac{\partial u'}{\partial x_k} \frac{\partial u'}{\partial x_k} \right\rangle \end{aligned} \quad (\text{A.4})$$

For  $\langle v'v' \rangle$ :

$$\begin{aligned} P_{2,2} &= -2\langle v'v' \rangle \frac{\partial \langle v \rangle}{\partial y} = 0, \quad T_{2,2} = -\frac{\partial \langle v'v'v' \rangle}{\partial y}, \quad \phi_{2,2} = -2\frac{\partial \langle p'v' \rangle}{\partial y} \\ \pi_{2,2} &= 2\left\langle p' \frac{\partial v'}{\partial y} \right\rangle, \quad D_{2,2} = \nu \frac{\partial^2 \langle v'v' \rangle}{\partial y^2}, \quad \varepsilon_{2,2} = 2\nu \left\langle \frac{\partial v'}{\partial x_k} \frac{\partial v'}{\partial x_k} \right\rangle \end{aligned} \quad (\text{A.5})$$

For  $\langle w'w' \rangle$ :

$$\begin{aligned} P_{3,3} &= -2\langle v'w' \rangle \frac{\partial \langle w \rangle}{\partial y}, \quad T_{3,3} = -\frac{\partial \langle w'w'v' \rangle}{\partial y}, \quad \phi_{3,3} = 0 \\ \pi_{3,3} &= 2\left\langle p' \frac{\partial w'}{\partial z} \right\rangle, \quad D_{3,3} = \nu \frac{\partial^2 \langle w'w' \rangle}{\partial y^2}, \quad \varepsilon_{3,3} = 2\nu \left\langle \frac{\partial w'}{\partial x_k} \frac{\partial w'}{\partial x_k} \right\rangle \end{aligned} \quad (\text{A.6})$$

For  $\langle u'v' \rangle$ :

$$\begin{aligned} P_{1,2} &= -\langle v'v' \rangle \frac{\partial \langle u \rangle}{\partial y}, \quad T_{1,2} = -\frac{\partial \langle u'v'v' \rangle}{\partial y}, \quad \phi_{1,2} = -\frac{\partial \langle p'u' \rangle}{\partial y} \\ \pi_{1,2} &= \left\langle p' \left( \frac{\partial u'}{\partial y} + \frac{\partial v'}{\partial x} \right) \right\rangle, \quad D_{1,2} = \nu \frac{\partial^2 \langle u'v' \rangle}{\partial y^2}, \quad \varepsilon_{1,2} = 2\nu \left\langle \frac{\partial u'}{\partial x_k} \frac{\partial v'}{\partial x_k} \right\rangle \end{aligned} \quad (\text{A.7})$$