**CSE 551 – Assignment 2**

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**Solution for Question (1)**

For finding using above definition and matrix chain multiplication method we use a 1x4 matrix with values as:

[] …………..**eq(i)**

= []

= []

Let . Therefore, above equation now becomes:

]

Now expanding with a similar approach like we used for eq(i), we get:

]

.

.

. (continuing this till the last step)

.

.

]

Substituting the given values for in the above equation, we get:

]

Above equation can be rewritten as:

], because

]…………….**eq(ii)**

Now,

Therefore,

Substituting this value in eq(ii):

Therefore,

[] **……..eq(iii)**

Now we know that, from the proof of using matrix chain multiplication technique, value of can be determined in time.

**Therefore, can also be computed in time** using eq(iii)**.**

**Solution for Question (2)**

2

3

4

5

7

* **Give a bound of the form O(f(n)) on the running time of this algorithm**

Outer for-loop (line 1) iterations =

Inner for-loop (line 2) iterates up to =

Number of ‘Add up array entries through ’ operations are up to =

Store the result in operation =

Total operations =

***Therefore, running time of this algorithm on an input of size n is***

* **Show that** **the running time of the algorithm on an input of size is also**

To prove,

To prove above equation, by the definition of - notation, we need to prove that:

where and are positive constants.

We can re-write the above equation as:

, where

We can see that if is a positive fraction less than 1 like 0.01 and , above equation holds true as will be less than

***Hence proved,***

* **Algorithm to solve this problem, with an asymptotically better running time.**

We replace adding all array entries to just adding to which stores sum of all the previous iterations of for a given value.

Since, can be done in , we are only left with two nested for-loop running and (.

***Therefore, running time of this modified algorithm on an input of size n is***

**Solution for Question (3)**

*Given*: Unsorted sequence of numbers

Below is the algorithm to find smallest number in an unordered sequence –

*High level description:*

***min­2***

***min2***

In step 1, comparisons are done in tournament approach as all elements need to be compared. Below is a diagram, explaining tournament approach for finding max element.

Shape, circle

Description automatically generated

In step 2, since the problem is being divided by 2 in every iteration, no of calls =

Therefore, max numbers of elements that can be in

Hence, max numbers of comparisons that can be done in

­

Total comparisons in step 1 and 2 =

*Therefore, above algorithm computes the 2nd smallest number in an unordered (unsorted) sequence of numbers in comparisons in the worst case.*

**Solution for Question (4)**

Given,

and, Multiplications in

Let first calculate for a 4x4 matrix where,

and

Then,

=

Values of are calculated using the vector product VW by the formula:

Generalizing above equation for a matrix we get:

= **…….eq(i)**

We are given that will need multiplications using the vector product formula.

Now, for we will only require multiplication as term is independent of and need not be calculated as its already done in

Similarly, for we will only require multiplication as term is independent of and need not be calculated as its already done in

Expanding on this thought process we can denote multiplications needed for calculating elements for matrix with dimensions in eq(i), we get:

Calculating total multiplications needed:

Substituting value

**Therefore, vector product VW formula for the multiplication of two matrices require multiplications.**

**Solution for Question (5)**

After iterations we will get:

……………(i)

This recurrence will end when

We are given that , therefore recurrence ends when in eq(i). Substituting values in eq(i) we get,

, where is given

Since,

=

Therefore,

**Solution for Question (6)**

=

This recurrence will end when

We are given that , therefore recurrence ends when in eq(i). Substituting value in eq(i) we get,

, where is given ………………eq(ii)

Now,

= =

We are given . Substituting value of in above equation we get:

Therefore,

Substituting this value in eq(ii), we get: