

An Improved WiFi Indoor Localization Method Combining Channel State Information and Received Signal Strength

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Abstract: WiFi indoor localization has attracted much attention owing to the pervasive penetration of wireless local area networks (WLANs) and WiFi enabled mobile devices. Traditional WiFi indoor localization systems rely on received signal strength (RSS), which is instability and low space distinguish ability. Recently, channel state information (CSI) has been adopted instead of RSS and proven to be an efficient method. However, CSI raw data is sensitive to environment change. In this paper, we propose an improved WiFi indoor localization system combining CSI and RSS. Firstly, this study applies a time domain filtering method to reduce CSI data dynamic range. Then, we use the coherence bandwidth for CSI dimensional reduction. Afterwards, we extract a robust positioning feature from CSI and RSS. Finally, an improved weighted k-nearest neighbor algorithm based on kernel methods is used to estimate the location. The experiments demonstrate the effectiveness of the proposed system.

Key Words: WiFi fingerprint, indoor localization, channel state information, kernel methods

1 Introduction

The indoor location-based service (ILBS) has spurred the development of many indoor localization techniques over the last several years [1]. As GPS signal cannot penetrate well in indoor scenarios, alternative methods such as WiFi [2], Bluetooth [3], RFID [4], UWB [5] and magnetic field [6] have been proposed. Among those methods, WiFi based indoor localization has attracted much attention due to the massive deployment of WiFi access points (APs) and WiFi enabled mobile devices [1]. Indoor environment is usually complex and dynamic. Many existing Wi-Fi based indoor positioning systems adopt fingerprinting technique for increased accuracy and robustness. Fingerprinting technique is usually conducted in two basic phases: offline phase and online phase [7]. During the offline phase, a site survey is performed in an environment. The location coordinates and respective localization feature from all the detected AP are collected and pre-processed. During the online phase, a localization algorithm uses the currently measured location feature and previously collected information to infer the location.

Traditional WiFi fingerprint-based localization systems use a vector of RSS from different APs as fingerprints due to easily measured and low hardware requirements. For instance, the Horus system [8]. However, the RSS is instability varying with time at a fixed position even in a static environment. Besides, RSS is coarse information, which may lead to a confusion among adjacent locations, and eventually degrade the performance of the localization system. With the development of new generation WiFi Network Interface Card (NIC) supporting IEEE 802.11n, used OFDM (orthogonal frequency division multiplexing) and MIMO (multiple input multiple output) techniques, it is now possible to extract channel state information (CSI) from some NICs such as Intel WiFi Wireless Link 5300 [9]. Compared

with RSS, CSI is able to provide richer multipath information from physical layer, which can be used as fingerprints to improve the performance of indoor localization. For example, the DeepFi system [10]. But in some cases, CSI raw data is more sensitive to influence of dynamic environment than RSS value. Most of existing WiFi indoor localization systems consider a single type feature of WiFi signal. However, they are not fully exploiting the feature of WiFi signal.

In this study, we propose an enhanced WiFi fingerprinting system based on CSI and RSS. The main idea involves the exploitation of complementary advantages of WiFi signal features, while simultaneously avoiding their disadvantages. We are not using CSI instead of RSS like the DeepFi system [10], but rather the combination of RSS and CSI in an intelligent manner to improved location performance. Firstly, we propose a time domain filtering method to pre-processing CSI raw data, in order to reduce the dynamic variation of CSI due to multipath effects and noise. Then, the channel responses of subcarriers within the same sub-channel is averaged to reduce the redundancy. Afterward, we extract a robust fingerprint from CSI and RSS. Finally, an improved weighted k-nearest neighbor (WKNN) algorithm based on kernel methods is used to estimate the location. We implemented the system and experimentally evaluate its performance in the typical laboratory scenario with single AP.

2 Related Work

WiFi-based fingerprinting has attracted continuous attention because WiFi signal is ubiquitous in the indoor environments. The first WiFi fingerprint-based positioning system is RADAR [11], this system builds fingerprint of RSS and uses a deterministic method for position estimation. Youssef et al. [8] proposed Horus system, which is an RSS based scheme utilizing probabilistic techniques to improve localization accuracy, where the RSS from an detected AP is modeled as a random variable in both time and spatial domains.

CSI is an emerging technique to replace RSS, and demonstrates an improved accuracy over RSS for indoor location estimation [13]-[14]. For instance, Jiang Xiao et al. [12] proposed FIFS system, which uses the average CSI values

This work is supported by China Scholarship Council, National Natural Science Foundation of China (Grant No. 61375087), Key Program of Nature Science Foundation of Tianjin (Grant No.14ZCDZGX00798) and Key Program of Natural Science Foundation of Tianjin (Grant No.15JCZDJC31200).

of all channels and antennas, but it does not consider the variation within channels. Yogita Chapre [16] proposed MIMO system, this approach incorporates MIMO information, and extracts the relatively robust fingerprints by a subtraction process that amplitude and phase value are subtracted for subsequent sub-carriers. But the relative amplitude and phase may lose some location information. Xuyu Wang et al. [10] proposed DeepFi system, which utilizes deep network on 90 CSI values to fully explore the feature of wireless channel data and obtain the optimal weights as fingerprint, but the algorithm is of high computational complexity. In the study [17], Shih-Hau Fang et al. reconstruct CSI through inverse multilevel discrete wavelet transform with histogram-equalization normalized wavelet coefficients, for extracting the robust positioning features and achieving enhanced location estimation accuracy. However, the algorithm failed to exploit the RSS feature information.

3 PRELIMINARY

In wireless communications, channel state information describes how a signal propagates from the transmitter to the receiver, which is able to reveal the combined effect of channel status, for example, fading, scattering, and power decay with distance. For WiFi network system, it is worth noting that only OFDM-based systems can demonstrate the frequency diversity in CSI since they use multiple subcarriers for data transmission [15]. In a narrow band flat-fading channel with MIMO, the OFDM system is modeled as

$$Y = H \cdot X + N \quad (1)$$

where X and Y are transmitted and received signal vector, respectively, matrix N represents additive white Gaussian noise, and matrix H represents the channel state information, which can be estimated from X and Y .

Thanks to NICs, such as Intel WiFi Link 5300 NIC, it is now easily to conduct CSI from laptop via the device driver. In proposed system, 5300 NIC implements an OFDM system at 2.4GHz band involving 56 subcarriers, and 30 out of which can be read for CSI. The channel frequency response H_i of subcarrier i is a complex value, which is defined by

$$H_i = |H_i| \exp \{j \angle H_i\} \quad (2)$$

where $|H_i|$ and $\angle H_i$ are the amplitude response and the phase response of subcarrier i , respectively. An example of CSI collection is shown in Figure 1, using 2 receive antennas and 2 transmit antennas. It manifests the possibility of determining the location using CSI.

4 Problem Statement

In this paper, we propose an improved WiFi fingerprinting system based on CSI and RSS. It requires only single WLAN AP to estimate the unknown location of mobile devices. Figure 2 gives an overview of the proposed localization system, which is divided into two phases: an offline phase, CSI raw data and RSS of all the reference location points are collected, whose precise location is known as a label. This is a challenging task in indoors due to radio interference, multipath effects, shadowing, and non-line-of-sight(NLOS) caused by complex environment. After data pre-processing, an radio

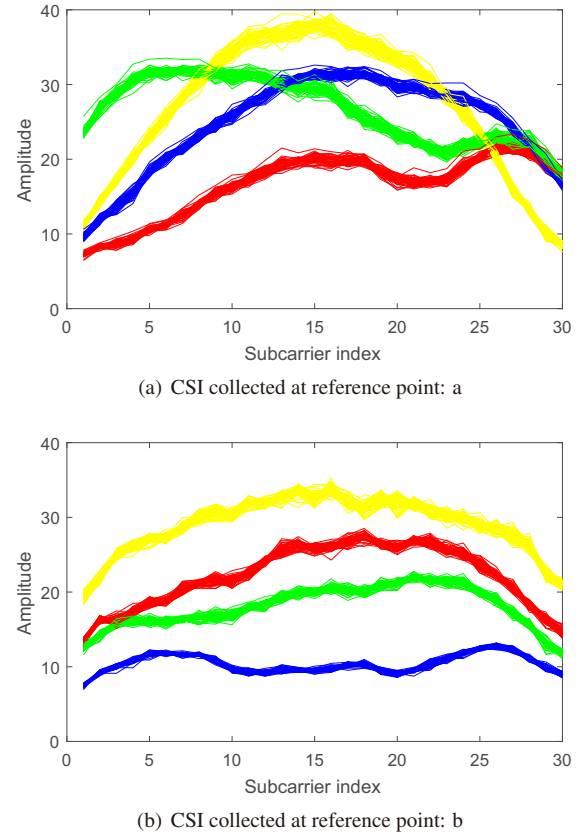


Fig. 1: Amplitudes of channel frequency response measured at two positions for 50 received packets (each link is plotted in a different color).

map is generated. In addition, the parameters are estimated by the radio map. In the online phase, CSI raw data and RSS collected by mobile devices are processed using the same algorithm of the offline phase. Moreover, an improved WKN-N algorithm based on kernel methods is used to estimate the location.

During the offline phase, the localization work as follows: Given a single base station AP, which transmitting radio signals in a field $A \subseteq \mathbb{R}^2$, we measured the RSS and CSI raw data of all the subcarriers at each reference point to build a raw fingerprint database that is a set $F' = \{p_{r_i}, \xi_{r_i}^j\}$, where $\xi_{r_i}^j$ is signal feature recorded in reference points, $i = 1, \dots, N_r$ is the index of reference points, $j = 1, \dots, N_s$ is the index of samples, N_r and N_s is total number of reference point and sample, respectively. After data pre-processing, we can generate radio map is $F = \{p_{r_i}, \xi_{r_i}\}$. where $p_{r_i} \in \mathbb{R}^2$ is reference point labeled by Cartesian coordinates $p_{r_i} = (x_{r_i}, y_{r_i})$, $\xi_{r_i} = \{csi_{r_i}, rss_{r_i}\}$ is signal feature information at p_{r_i} from the AP, and rss_{r_i} is averaged RSS, $csi_{r_i} = [csi_{11}, \dots, csi_{1L}, \dots, csi_{pq}, \dots, csi_{N1}, \dots, csi_{NL}]^T$, where N is the total number of links between each transmitter and receiver pair, and L is the total number of sub-channel are available for each link, csi_{pq} is the amplitude of the pq_{th} sub-channel at p_{r_i} .

During the online phase, a new CSI and RSS observation $\xi_{new} = \{csi_{new}, rss_{new}\}$ measured by mobile device. The location can be estimated as a weight average of selected

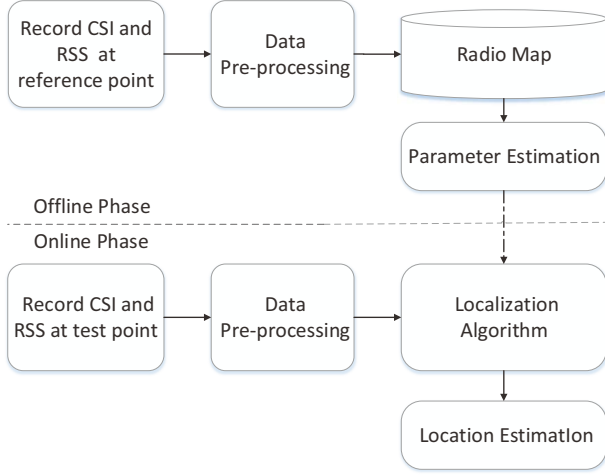


Fig. 2: Localization system architecture.

reference locations, as

$$\hat{p} = \sum_{k \in Q(p)} w_k p_k \quad (3)$$

where $Q(p)$ denotes the set of reference points whose fingerprint is most similar to the query fingerprint, the cardinality of $Q(p)$ be N_c , w_k is significance weight and is given as follows:

$$w_k = \frac{\phi(\xi_{new}, \xi_i)}{\sum_{i=1}^{N_c} \phi(\xi_{new}, \xi_i)} \quad (4)$$

where $\phi(\cdot)$ is kernel function to calculate similarity between reference fingerprint and query fingerprint, the function could be radius basis functions, polynomial functions, hybrid kernel function, etc.

5 THE PROPOSED SYSTEM

In this section, we present details on the proposed localization system. Figure 2 shows the system architecture which conducted in two phases: an offline phase followed by an online phase. To enhance the performance of estimation, the CSI raw data is pre-processed to reduce CSI data dynamic range and redundancy. Then, we explore a new robust fingerprint by combining with CSI and RSS. In addition, an improved WKNN algorithm based on kernel methods is used to estimate the location.

5.1 Data Pre-processing

Indoor environment is often complex, characterized by NLOS of objects, presence of obstacles, environmental changes, wireless signal noise, etc. The CSI raw value is the dynamic range due to channel interference, noise, and multipath effects. Additionally, not all the features contribute equally to the system performance, and the high dimension of fingerprint vectors can result in a redundant computational cost. Therefore, before generating fingerprint, we need to extract the robust positioning feature by a time domain filtering method, and reduce the dimension of CSI feature. Besides, the RSS value varies significantly over different times. We collect lots of RSS at fix location. Take the average of the RSS, and then the average RSS is utilizing to generate fingerprint.

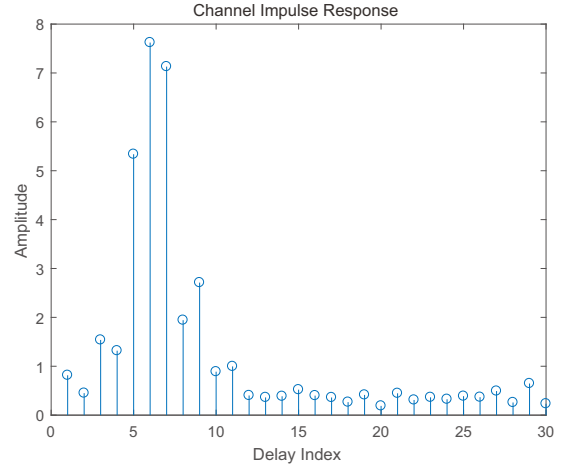


Fig. 3: Channel Impulse Response of NLOS dominant condition

5.1.1 Time Domain Filtering

For wireless communication, the radio propagation between a pair of transmitter and receiver can easily be disturbed by the severe multipath effects. Transmitted signal is pass through multiple paths and reflections to receiver in indoor scenarios. In general, the longer path of signal propagation is more likely to be interfered. To fully characterize the individual paths, the wireless propagation channel is modeled as a temporal linear filter, known as Channel Impulse Response (CIR) [13]. Under the time-invariant assumption, CIR is described as follow:

$$h(\tau) = \sum_{i=1}^n a_i e^{-j\theta_i} \delta(\tau - \tau_i) \quad (5)$$

where a_i , θ_i , and τ_i are the amplitude, phase, and time delay of the i_{th} path, respectively. n is the total number of multipath and $\delta(\tau)$ is the Dirac delta function.

In this study, the CSI raw data represent the channel response in frequency domain. We process frequency domain CSI into time domain CIR with inverse fast Fourier transform (IFFT), then an estimation of CIR with time resolution of $1/20\text{MHz}=50\text{ns}$ is exposed for 20MHz bandwidth of IEEE 802.11n WLAN. Due to the limitation of bandwidth, the path with length differences less than 15m might be mixed in one CIR sample. An example of CIR shown in Figure 3, We can observe that the different paths come with different time delay. Generally, the LOS path signal contains higher energy, and arrives at the receiver more quickly than all other reflected paths. Since the first maximal amplitude, record the corresponding index of delay as t , is chosen for LOS path or shortest path NLOS reflection. Considering that there is an uncertain time lag at the start of measured CIR samples, then we extract the main components of multiple paths by keeping $(|t-2|)_{th}$, $(|t-1|)_{th}$, t_{th} , $(t+1)_{th}$, $(t+2)_{th}$ amplitude, and filter out the residual amplitude. After the time domain filtering, the frequency domain CSI is reacquired using fast Fourier transform (FFT). Figure 4 shows the CSI results after time domain filtering.

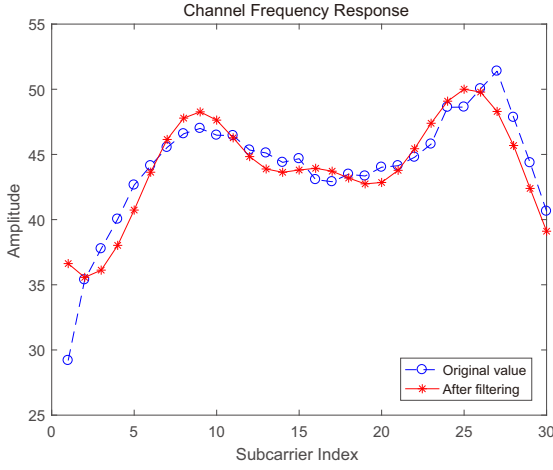


Fig. 4: The results after time-domain filtering

5.1.2 CSI Dimensional Reduction

The wideband channel of 802.11n can provide abundant diversity in the frequency domain due to the multipath effect. The metric to evaluate the frequency diversity is coherence bandwidth. Coherence bandwidth is a characteristic of the propagation channel and is considered as the minimum bandwidth, within which different frequencies of a signal are likely to experience comparable or correlated amplitude fading. In this study, we consider that coherence bandwidth is 5MHz [12], and then divided the whole 20MHz channel into 4 sub-channels ($L=4$). As a result, the channel responses of subcarriers within the different sub-channel can be viewed as fading independently. Finally, after averaging over each sub-channel, the feature of CSI is denoted by $[csi_{11}, \dots, csi_{14}, \dots, csi_{pq}, \dots, csi_{N1}, \dots, csi_{N4}]^T$. Therefore, the CSI dimensional is reduced to 13.3%.

5.2 Location Estimation based on Feature Fusion

Traditional fingerprint-based localization methods can be easily implement based on k-nearest neighbor (KNN) and the computational complexity is often low. However, KNN is obviously affected by multiple diverse feature. In order to overcome this problem, we propose an improved weighted k-nearest neighbor (WKNN) algorithm based on kernel methods. Kernel methods make use of kernel functions defining a measure of similarity between pairs of instances. Commonly used kernels are $\phi(\xi_{new}, \xi_{r_i}) = \xi_{new}^T \xi_{r_i}$ (linear kernel), $\phi(\xi_{new}, \xi_{r_i}) = (\xi_{new}^T \xi_{r_i} + 1)^d$ (polynomial kernel of degree d), $\phi(\xi_{new}, \xi_{r_i}) = \exp\left(-\frac{\|\xi_{new} - \xi_{r_i}\|^2}{2\sigma^2}\right)$ (Gaussian kernel, where σ is a smooth factor). Conventional kernel functions evaluate kernel value between two input vectors in a feature-to-feature manner. In the context of feature fusion, it is useful to associate a kernel to each WiFi signal feature. Therefore, We define a hybrid kernel function as follow:

$$\phi(\cdot) = \mu\phi_1(rss_{new}, rss_{r_i}) + (1 - \mu)\phi_2(csi_{new}, csi_{r_i}) \quad (6)$$

where $\phi_1(rss_{new}, rss_{r_i}) = \exp\left(-\frac{\|rss_{new} - rss_{r_i}\|^2}{2\sigma_1^2}\right)$ and $\phi_2(csi_{new}, csi_{r_i}) = \exp\left(-\frac{\|csi_{new} - csi_{r_i}\|^2}{2\sigma_2^2}\right)$, σ_1, σ_2 is smooth factor estimated by empirical method (The space distinguish ability of RSS is lower than CSI, so σ_1 should smaller than σ_2), and μ is fusion-weight parameter estimated by double cross validation approach in the offline phase. Then, we calculate the similarity between online fingerprint measurement and offline fingerprint data by hybrid kernel function, and build a set $Q(p)$ of reference points whose fingerprints is most similar to the query fingerprint. Moreover, the significance weight of selected reference points calculated by equation (4). Finally, we estimate the coordinates by equation (3). We summarize the complete localization algorithm described in Algorithm 1.

Algorithm 1 Proposed localization algorithm

Require: Online fingerprint measurement $\xi_{new} = \{csi_{new}, rss_{new}\}$, offline fingerprint database F ;

Ensure: estimated location $\hat{p} = (\hat{x}, \hat{y})$;

- 1: **procedure** LOCALIZATION(ξ_{new}, F)
- 2: **for** $i = 1; i \leq N_r; i++$ **do**
- 3: $s_i = \mu\phi_1(rss_{new}, rss_{r_i}) + (1 - \mu)\phi_2(csi_{new}, csi_{r_i})$
- 4: Sort the offline fingerprint data in descending order by s_i ;
- 5: Build the set $Q(p)$, the cardinality of $Q(p)$ be N_c ;
- 6: **for** $k = 1; k \leq N_c; k++$ **do**
- 7: $w_k = \frac{\phi(\xi_{new}, \xi_i)}{\sum_{i=1}^{N_c} \phi(\xi_{new}, \xi_i)}$
- 8: Compute the estimated location
- 9: $\hat{p} = \sum_{k \in Q(p)} w_k p_k$

6 EXPERIMENT VALIDATION AND ANALYSIS

6.1 Experiment Methodology

The proposed system is tested through practical experiments. The experiment testbed consists of two major part, an access point and a mobile device, respectively. The access point, which is a TL-WR842N router manufactured by TP-Link company. The router has two transmit antennas and supports IEEE 802.11n standard. The mobile device is Acer laptop equipped with the Intel WiFi Link 5300 NIC. We installed the 64-bit Ubuntu LTS 12.04 system on the laptop, and modify the kernel of the wireless driver.

The experiments are conducted in room 13-212, The Robotics Virtual Simulation Lab of Nankai University. The area size is about $9 \times 9m^2$. Figure 5 shows the photo of the location where the experiments were performed, while Figure 6 shows the 138 reference points and 75 test points according to the layout of facilities. The average distance between adjacent reference points is $0.5m$. The access point was deployed at the corner, and the height of antennas is $1.20m$.

After the AP is connected by the mobile device, CSI raw data is available in the mobile device when 5300 NIC receives data packet. In our experiments, the 5300 NIC recorded CSIs using two receive antennas, and thus there were 4 links available as location features ($N = 4$). Each link reported CSI value contains 30 subcarriers. 1000 CSIs and 1000



Fig. 5: Photo of the experiment environment in Room 13-212

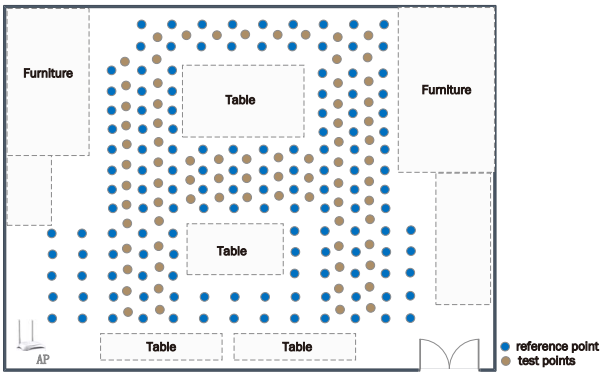


Fig. 6: Layout of the laboratory for reference/test positions

RSSs were measured over 20s at a reference point, but 50 CSIs and 50 RSSs were measured over 1s at a test point to reduce the operating cost in online phase.

6.2 Performance Evaluation

We adopted the distance error as performance metric, which was evaluated using root-mean-square (RMSE) with the Euclidean distance between the true position and its estimate. Assume the estimated location of unknown mobile device i is (\hat{x}_i, \hat{y}_i) and the actual position of the user is (x_i, y_i) . For N_t test point, the RMSE is computed as

$$e = \sqrt{\frac{\sum_{i=1}^{N_t} ((\hat{x}_i - x_i)^2 + (\hat{y}_i - y_i)^2)}{N_t}} \quad (7)$$

Figure 7 demonstrates three example of cumulative distribution of positioning error with different fusion-weight μ , and we can conclude that appropriate parameters will improve accuracy.

For comparison purpose, we implemented three existing methods, including Horus, FIFS, and MIMO. In our experiment, these schemes use the same measured dataset in order to guarantee the reliability of the results. Figure 8 plots the cumulative localization errors of four different systems. With the proposed system, about 55% of the test positions have an error under 1.5m and 70% have an error under

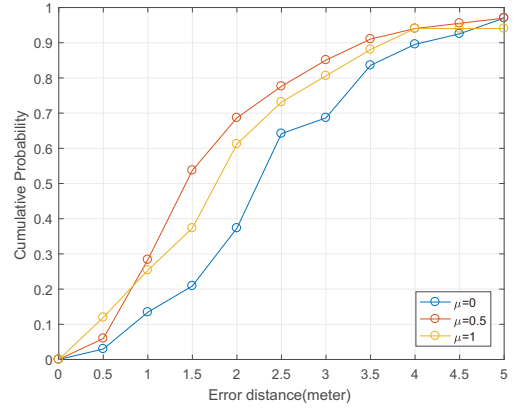


Fig. 7: Cumulative distribution of positioning error with different fusion-weight

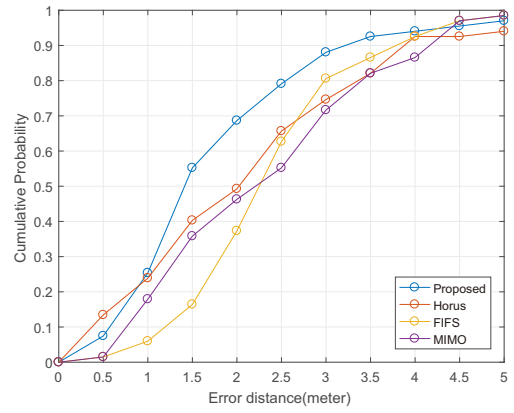


Fig. 8: Cumulative distribution of positioning error using four different localization systems

2m. Table 1 shows that the proposed system outperforms traditional methods, reducing the RMES by 8.7%, 12.9%, 11.5%, compared to Horus, FIFS, and MIMO, respectively. It shows that the proposed system can achieve improved localization performance.

Table 1: COMPARISON OF RMSE RESULTS

System	Error mean(m)	Error std (m)	RMSE(m)
Proposed system	1.7928	1.2550	1.3390
Horus	2.1514	1.5175	1.4668
FIFS	2.3641	0.9720	1.5376
MIMO	2.2899	1.2273	1.5133

7 Conclusion

In this paper, we propose an improved WiFi-based indoor localization system. The main contribution of this paper is not using CSI instead of RSS, but rather the combination of RSS and CSI in an intelligent manner to improve localization performance. A time domain filtering method is used to reduce CSI dynamic range. Then, we use the coherence bandwidth for CSI dimensional reduction. Moreover, an improved WKNN algorithm based on kernel methods is utilized to estimate the location. The proposed system was implemented in the typical laboratory scenario, and experimen-

tally evaluate its performance compared with several existing systems. Results indicate that the proposed system is efficient in improving the positioning performance.

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