# B.Tech Sem I Date 7/1/2021, Tutorial-1 MATRICES

## Section-A

1. Define the following terms also give an example of each:
   (i) Lower Triangular and Upper Triangular matrix
   (ii) Transpose of a matrix (iii) Symmetric matrix and Skew Symmetric matrix
   (iv) Hermitian and Skew Hermitian (v) Idempotent matrix
   (vi) Orthogonal Matrix (vii) Nilpotent matrix (viii) Involutory matrix
   (ix) Unitary matrix (x) Adjoint of a matrix (xi) Rank of a matrix
   (xii) ECHELON FORM OF A MATRIX
   (xiii)Normal form of Matrix or Canonical form
   (xiv) Characteristic Matrix (xv) Characteristic Polynomial
   (xvi) Characteristic Equation (xvii) Characteristic Values(or roots)
   (xviii) Characteristic Vector
2. Prove the following

i. $Adj(kA) = k^{n-1}(\ AdjA)$ for any k,

ii. $|AdjA| = |A|^{n-1}$ , for any square matrix of order $n$.

iii. If $|A| \neq 0$ , then $Adj(AdjA) = |A|^{n-2}A$ , for any square matrix of order $n$.

iv. $|Adj(AdjA)| = |A|^{(n-1)^2}$, for any square matrix of order $n$.

3. Prove the following assertions

i. Eigen values of A and $A^T$ are same.

ii. If $A$ and $B$ are two square matrices and P be a $non\text{-}\ singular$ matrix, then $P^{-1}BP\ and\ A$ have same eigen values. In other words, the similar matrices have same eigen values .

iii. If $A$ and $B$ are two $non\text{-}\ singular$ (or invertible )matrices, then$AB\ and\ BA$ have same eigen values.

iv. If $A$ and $B$ are two square matrices and A be a $non\text{-}\ singular$ matrix, then$A^{-1}Band\ BA^{-1}$ have same eigen values.

v. Eigen values of a triangular matrix are just diagonal elements.

vi. Sum of the eigen values of a matrix is equal to its trace and product of the eigen values of a matrix is equal to its determinant.

vii. Eigen values of a Hermitian matrix are real.

viii. Eigen values of a real symmetric matrix are real.

ix. The eigen values of a of a $unitary$ matrix are of modulus unity.

x. Eigen values of a skew Hermitian matrix are either purely imaginary number or zero.

4. $If\ \lambda\ i$s an eigen value of a matrix A, then find the eigen values of the following matrices:

| Matrix | Ans |
|---|---|
| kA | k$\lambda$ |
| A$\pm kI$ | $\lambda \pm k.$ |
| $A^n$ | $\lambda^n$ |
| $A^{-1}$ | $\frac{1}{\lambda}$ |

| | |
|---|---|
| Adj A | $\frac{\|A\|}{\lambda}$ |

5. If A=$\begin{bmatrix}2&0&0\\0&2&0\\0&0&0\end{bmatrix}$ and B=$\begin{bmatrix}1&2&3\\0&1&3\\0&0&2\end{bmatrix}$, the value of |AB| is (a) 4, (b)8,(c*)16 ,(d)32

6. The determinate of $\begin{bmatrix}x+a&b&c&d\\a&x+b&c&d\\a&b&x+c&d\\a&b&c&x+d\end{bmatrix}$ is (a*) $x^3(x+a+b+c+d)$,

   (b)(x+a)(x+b)(x+c)(x+d),(c)$x^4+ax^3+bx^2+cx+d$ (d) None of the above.

7. If$\begin{vmatrix}x&x^2&1+x^3\\y&y^2&1+y^3\\z&z^2&1+z^3\end{vmatrix}$=0, then $xyz$ =(a*) -1, (b)1, (c) 0, (d) 3

8. If the system of equations $x+4ay+az=0, x+3by+bz=0, x+2cy+cz=0$ has a non-trivial solution, then $a, b, c,$ are in (a) A.P, (b)G.P, (c*) H.P., (d) None of these.

9. If $x, y, z \in \mathbb{R}^+$, then $\begin{vmatrix}1&\log_x y&\log_x z\\\log_y x&1&\log_y z\\\log_z x&\log_z y&1\end{vmatrix}$ is(a) 1, (b*)0,(c)-1 ,(D)$log2$

10. If$\begin{vmatrix}a_1-x&a_2&a_3\\b_1&b_2-x&b_3\\c_1&c_2&c_3-x\end{vmatrix}$=0, for $x=-1,2,3$ then the value of $\begin{vmatrix}a_1&a_2&a_3\\b_1&b_2&b_3\\c_1&c_2&c_3\end{vmatrix}$ (a) -6,

    (b*)6, (c) -4, (d) 4

11. If A=$\begin{bmatrix}1&2&3\\1&3&4\\1&4&3\end{bmatrix}$ then |AdjA| is (a) -4, (b*)4,(c)-2 ,(d)2

12. If A be a square matrix such that A,$A^2$, $A^3$are non-zero matrices but $A^4$is a zero matrix.Then$(I-A)^{-1}$ is (a) A+ $A^2+A^3$(b)I+A+ $A^2$ (c) I+ $A^2+A^3$,(D*)I+A+ $A^2+A^3$

13. The rank of the matrix $\begin{bmatrix}1&2&3\\2&4&7\\3&6&10\end{bmatrix}$ is (a) 1, (b*)2,(c)3 ,(d) None of these.

14. If $\rho(A_{3\times3})$=2 and $\rho(B_{3\times3})$=3,then ρ(AB)= (a) 5, (b)3,(c*)2 ,(d) 1

15. If the system of equations$3x-y+\lambda z=1, 2x+y+z=2, x+2y-\lambda z=-1$ has a unique solution if λ=(a) any value, (b)$\frac{-7}{2}$,(c*)≠$\frac{-7}{2}$,(d) ≠$\frac{7}{2}$

16. If A and B are square matrices of the same order, which of the following is true

(a)$(A+B)^2=A^2+2AB+B^2$,$(b)(A+B)(A-B)=A^2-B^2$,$(c)\ (A-B)\ (A+B)\ =A^2-B^2$,$(d*)\ (A+B)(A-B)+\ (A-B)\ (A+B)\ =2A^2-2B^2$

17. If A is a square matrix, prove that(i) $A+A'$ is symmetric,(ii) $A-A'$ is skew-symmetric.

18. Prove that

    (i) If $A, B$ are symmetric, then so is $A+B$

(ii) If $A,\ B$ are skew-symmetric, then so is $A+B$

(iii) If $A$ is a square matrix, then $AA'$ and $A'A$ are both symmetric.

(iv) If $A$ is symmetric, then $B'$ AB is symmetric.

19. If $A=\begin{bmatrix}1&2&1\\a&0&4\\1&1&1\end{bmatrix}$ and adj. (adj. A)=A, find a. Ans 3

20. If $A=\begin{bmatrix}1&2&-1\\3&0&2\\4&5&0\end{bmatrix}$ and $B=\begin{bmatrix}1&0&0\\2&1&0\\0&1&3\end{bmatrix}$, verify that $(AB)'=B'A'$.

21. Prove that matrices(i) $\frac{1}{3}\begin{bmatrix}-2&1&2\\2&2&1\\1&-2&2\end{bmatrix}$ and (ii) $\frac{1}{3}\begin{bmatrix}-8&4&1\\1&4&-8\\4&7&4\end{bmatrix}$ are orthogonal.

22. Express each of the following matrices as the sum of a symmetric and a skew- symmetric matrix. (i) $\begin{bmatrix}4&2&-3\\1&3&-6\\-5&0&-7\end{bmatrix}$ (ii) $\begin{bmatrix}a&a&b\\c&b&b\\c&a&c\end{bmatrix}$

23. *Employing elementary row transformations, find the inverse of .* $A=\begin{bmatrix}0&1&2\\1&2&3\\3&1&1\end{bmatrix}$

**Ans** $A^{-1}=\begin{bmatrix}\frac{1}{2}&-\frac{1}{2}&\frac{1}{2}\\-4&3&-1\\5/2&-3/2&1/2\end{bmatrix}$

## Section B

1. Reduce the matrix A to triangular form, $A=\begin{bmatrix}1&2&3\\2&5&7\\3&1&2\end{bmatrix}$.

2. If X, Y are non-singular matrices and $B=\begin{bmatrix}X&O\\O&Y\end{bmatrix}$, show that $B^{-1}=\begin{bmatrix}X^{-1}&O\\O&Y^{-1}\end{bmatrix}$ where O is a null matrix.

3. Find the inverse of the matrix $\begin{bmatrix}1&0&k\\0&1&0\\0&0&1\end{bmatrix}\begin{bmatrix}1&0&0\\0&1&s\\0&0&1\end{bmatrix}\begin{bmatrix}1&0&0\\0&p&0\\0&0&1\end{bmatrix}$ without first evaluating the product.

4. Find the inverse of the following matrices by using elementary row operations:

(i) $\begin{bmatrix}2&0&-1\\5&1&0\\0&1&3\end{bmatrix}$ (ii) $\begin{bmatrix}2&1&-1\\0&2&1\\0&0&1\end{bmatrix}$ (iii) $\begin{bmatrix}8&4&3\\2&1&1\\1&2&1\end{bmatrix}$

(iv) $\begin{bmatrix}3&-1&1\\-15&6&-5\\5&-2&2\end{bmatrix}$ (v) $\begin{bmatrix}1&2&1\\3&2&3\\1&1&2\end{bmatrix}$ (vi) $\begin{bmatrix}1&0&-1\\3&4&5\\0&-6&-7\end{bmatrix}$

5. Employing elementary row transformations, find the inverse of the following non-singular matrix.

(i) $\begin{bmatrix} 1 & 2 & 3 & 1 \\ 1 & 3 & 3 & 2 \\ 2 & 4 & 3 & 3 \\ 1 & 11 & & 1 \end{bmatrix}$ (ii) $\begin{bmatrix} 2 & 1 & -1 & 2 \\ 1 & 3 & 2 & -3 \\ -1 & 2 & 1 & -1 \\ 2 & -3-1 & & 4 \end{bmatrix}$

**Answers**: 1. $\begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & -2 \end{bmatrix}$ 2. $\begin{bmatrix} 1 & 0 & -k \\ 0 & 1/p & -8/p \\ 0 & 0 & 1 \end{bmatrix}$

4. (i) $\begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$ (ii) $\begin{bmatrix} 8 & -1 & -3 \\ -5 & 1 & 2 \\ 10 & -1 & -4 \end{bmatrix}$ (iii) $\begin{bmatrix} 1 & -2 & -1 \\ 1 & -5 & 2 \\ -3 & 12 & 0 \end{bmatrix}$

(iv) $\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$ (v) $\begin{bmatrix} -1/4 & 3/4 & -1 \\ 3/4 & -1/4 & 0 \\ -1/4 & -1/4 & 1 \end{bmatrix}$ (vi) $\begin{bmatrix} 1/10 & 3/10 & 1/5 \\ 21/20 & -7/20 & -2/5 \\ -9/10 & -3/10 & 1/5 \end{bmatrix}$

5. (i) $\begin{bmatrix} 1 & -2 & 1 & 0 \\ 1 & -2 & 2 & -3. \\ 0 & 1 & -1 & 1 \\ -2 & 3-2 & & 3 \end{bmatrix}$ (i) $\begin{bmatrix} 2 & 5 & -7 & 1 \\ 5 & -1 & 5 & -2 \\ -7 & 5 & 11 & 10 \\ 1 & -210 & & 5 \end{bmatrix}$

## Section C

1. Find the rank of the matrices

(i) $\begin{bmatrix} 1 & 34 & 5 \\ 1 & 26 & 7 \\ 1 & 50 & 10 \end{bmatrix}$ (ii) $\begin{bmatrix} 1 & 23 & 0 \\ 2 & 43 & 2 \\ 3 & 21 & 3 \\ 6 & 87 & 5 \end{bmatrix}$

2. $\begin{bmatrix} 1^2 & 2^2 3^2 & 4^2 \\ 2^2 & 3^2 4^2 & 5^2 \\ 3^2 & 4^2 5^2 & 6^2 \\ 4^2 & 5^2 6^2 & 7^2 \end{bmatrix}$

3. (i) Reduce the matrix $A = \begin{bmatrix} 1 & 1-1 & 1 \\ -1 & 1-3 & -3 \\ 1 & 0\ 1 & 2 \\ 1 & -13 & 3 \end{bmatrix}$ to column echelon form and find its rank.

(ii) Reduce A to Echelon form and then to its row canonical form where

$$A = \begin{bmatrix} 1 & 3\ -1 & 1 \\ 0 & 11-5 & 3 \\ 2 & -53 & 1 \\ 4 & 1\ 1 & 5 \end{bmatrix}$$

Hence find the rank of A.

4. Using elementary transformation, reduce the following matrices to the canonical form.

(i) $\begin{bmatrix} 0 & 0 & 00 & 0 \\ 0 & 1 & 23 & 4 \\ 0 & 2 & 34 & 1 \\ 0 & 3 & 41 & 2 \end{bmatrix}$ (ii) $\begin{bmatrix} 0 & 4 & -128 & 9 \\ 0 & 2 & -6\ 2 & 5 \\ 0 & 1 & -36 & 4 \\ 0 & -8 & 243 & 1 \end{bmatrix}$ (iii) $\begin{bmatrix} & 1 & 2 & -14 \\ & 2 & 3 & 3\ 4 \\ & 1 & 2 & 3\ 4 \\ -1 & -2 & 6 & -7 \end{bmatrix}$

5. (i) Find non-singular matrices P and Q such that PAQ is in the normal form for the matrix

$$A = \begin{bmatrix} 1 & -1 & -1 \\ 1 & 1 & 1 \\ 3 & 1 & 1 \end{bmatrix}$$

(ii) Find non- singular matrix P and Q such that PAQ is in normal form of the matrix and hence find the rank of matrix $A = \begin{bmatrix} 1 & 2 & 3-2 \\ 2 & -2 & 1\ 3 \\ 3 & 0 & 4\ 1 \end{bmatrix}$

6. Find the rank of the following matrices using elementary transformations.

(i) $\begin{bmatrix} 1 & 2 & 3\ 4 & 5 \\ 2 & 3 & 4\ 5 & 1 \\ 5 & 8 & 1114 & 7 \end{bmatrix}$ (ii) $\begin{bmatrix} 3 & 4 & 11 \\ 2 & 4 & 36 \\ -1 & -2 & 6 & 4 \\ 1 & -1 & 4 & -3 \end{bmatrix}$

(iii) $\begin{bmatrix} 1 & 2 & 31 \\ 2 & 4 & 62 \\ 1 & 2 & 32 \end{bmatrix}$ (iv) $\begin{bmatrix} -2 & -1\ 3 & -1 \\ 1 & 2\ -3 & -1 \\ 1 & 01 & 1 \\ 0 & 11 & -1 \end{bmatrix}$

7. Find the rank of the following matrices by reducing it to normal form (or canonical form)

(i) $\begin{bmatrix} 1 & 2-1 & 3 \\ 4 & 1\ 2 & 1 \\ 3 & -11 & 2 \\ 1 & 2\,0 & 1 \end{bmatrix}$ (ii) $\begin{bmatrix} 1 & 2 & 32 \\ 2 & 3 & 51 \\ 1 & 3 & 45 \end{bmatrix}$ (iii) $\begin{bmatrix} 1 & 3 & 2\ 5 & 1 \\ 2 & 2 & -16 & 3 \\ 1 & 1 & 2\ 3 & -1 \\ 0 & 2 & 52 & -3 \end{bmatrix}$

8. Find the rank of the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{bmatrix}$; $a, b, c$ being all real.

9. Show that the rank of a skew-symmetric matrix cannot be unity.

10. (i) Find all values of $\mu$ for which rank of matrix

$$A = \begin{bmatrix} \mu & -1 & 0 & 0 \\ 0 & \mu & -1 & 0 \\ 0 & 0 & \mu & -1 \\ -6 & 11-6 & & 1 \end{bmatrix}$$

(ii) Find the value of P for which the matrix

$$A = \begin{bmatrix} 3 & P & P \\ P & 3 & P \\ P & P & 3 \end{bmatrix} \text{ is of rank 1.}$$

11. Under what condition, the rank of the following matrix A is 3?

Is it possible for the rank to be 1? Why?

$$A = \begin{bmatrix} 2 & 4 & 2 \\ 3 & 1 & 2 \\ 1 & 0 & x \end{bmatrix}$$

12. If $A = \begin{bmatrix} 1 & 5 & 4 \\ 0 & 3 & 2 \\ 2 & 3 & 10 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \end{bmatrix}$, find rank of A, rank of B, rank of A + B and of AB.

13. Show that if A is a non-zero column matrix and B is a non-zero row matrix then $\rho(AB) = 1$.

14. (i) Show that the rank of the transpose of a matrix is the same as that of the original matrix.

(ii) Prove that for a $m \times n$ matrix, whose every element is 1, the rank is one.

## ANSWERS

1. (i) 3 (ii) 3
2. (i) 4
3. (i) 2 (ii) 2
4. (i) $\begin{bmatrix} I_3 & \vdots & O_{3\times 2} \\ \hdashline O_{1\times 3} & \vdots & O_{1\times 2} \end{bmatrix}$ (ii) $\begin{bmatrix} I_3 & \vdots & O_{3\times 2} \\ O_{1\times 3} & \vdots & O_{1\times 2} \end{bmatrix}$ (iii) ) $\begin{bmatrix} I_3 & \vdots & O_{3\times 2} \\ O_{1\times 3} & \vdots & O_{1\times 1} \end{bmatrix}$
5. (i) $P = \begin{bmatrix} 1 & 0 & 0 \\ -\frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{2} & 1 & -\frac{1}{2} \end{bmatrix}, Q = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$

(ii) $P = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -1 & -1 & 1 \end{bmatrix}, Q = \begin{bmatrix} 1 & \frac{1}{3} & \frac{4}{15} - \frac{1}{21} \\ 0 & -\frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ 0 & 0 & -\frac{1}{5} & 0 \\ 0 & 00 & \frac{1}{7} \end{bmatrix}$; $\rho(A) = 2$

6. (i) 2 (ii) 4 (iii) 2 (iv) 3
7. (i) 3 (ii) 2 (iii) 3
8. $\rho(A) = 3$ if $a \neq b \neq c$ and $a + b + c \neq 0$;

$\rho(A) = 2$ if $a \neq b \neq c$ and $a + b + c = 0$

Also $\rho(A) = 2$ if $a = b \neq c$ while $\rho(A) = 1$ if $a = b = c$

10. (i) $\mu = 1, 2, 3$ (ii) $P = 3$
11. $x = \frac{3}{5}; No$
12 $\rho(A) = 3, \rho(B) = 1, \rho(A + B) = 3, \quad \rho(AB) = 1.$

# Section D

1. Fine the eigen values and corresponding eigen vectors of the following matrices:

   (i) $\begin{bmatrix} -5 & 2 \\ 2 & -2 \end{bmatrix}$ (ii) $\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$ (iii) $\begin{bmatrix} 1 & 2 & 2 \\ 0 & 2 & 1 \\ -1 & 2 & 2 \end{bmatrix}$ (iv) $\begin{bmatrix} a & h & g \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$

2. (i) Find the characteristic equation of the matrix $A = \begin{bmatrix} 1 & 2 & 2 \\ 0 & 2 & 1 \\ -1 & 2 & 2 \end{bmatrix}$

   (ii) Find the eigen values of the matrix $A = \begin{bmatrix} 2 & -3 & 1 \\ 3 & 1 & 3 \\ -5 & 2 & -4 \end{bmatrix}$

   (iii) Find the eigen vectors for the matrix $A = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}$

3. Find the eigen values of $3A^3 + 5A^2 - 6A + 2I$ where $a = \begin{bmatrix} 1 & 2 & -3 \\ 0 & 3 & 2 \\ 0 & 0 & -2 \end{bmatrix}$

4. Find the eigen value of matrix $\begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix}$ corresponding to the eigen vector $\begin{bmatrix} 101 \\ 101 \end{bmatrix}$.

5. *Show that for a square matrix,*

   (i) *There are infinitely many eigen vectors corresponding to a single eigen value*

   (ii) *Every eigen vector corresponds to a unique eigen value*

6. Verify that the matrices $X = \begin{bmatrix} 0 & h & g \\ h & 0 & f \\ g & f & 0 \end{bmatrix}, Y = \begin{bmatrix} 0 & f & h \\ f & 0 & g \\ h & g & 0 \end{bmatrix}, Z + \begin{bmatrix} 0 & g & f \\ g & 0 & h \\ f & h & 0 \end{bmatrix}$ have same characteristic equation.

7. If $a + b + c = 0$, Find the characteristic roots of the matrix $A = \begin{bmatrix} a & c & b \\ c & b & a \\ b & a & c \end{bmatrix}$

8. Prove that for matrix $A = \begin{bmatrix} 2 & 1 & 1 \\ 2 & 3 & 4 \\ -1 & -1 & -2 \end{bmatrix}$, all its eigen values are distinct and real. Hence find the corresponding eigen vectors.

9. Show that the matrix $\begin{bmatrix} 3 & 10 & 5 \\ -2 & -3 & -4 \\ 3 & 5 & 7 \end{bmatrix}$ has leas than three linearly independent eigen vectors. Also find them.

10. If $M = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ and $P = \frac{1}{2}\begin{bmatrix} b+c & c-a & b-a \\ c-b & c+a & a-b \\ b-c & a-c & a+b \end{bmatrix}$, Show that the transform of $P$ byM $i.e., MPM^{-1}$ is a diagonal matrix and hence find the eigen values of $P$.

11. Find characteristic equation and eigen values of the matrix $A = \begin{bmatrix} 3 & 2 & 2{-}4 \\ 2 & 3 & 2{-}1 \\ 1 & 1 & 2{-}1 \\ 2 & 2 & 2 \quad -1 \end{bmatrix}$

12. If $A \begin{bmatrix} 4 & -1 & -1 \\ -1 & 4 & -1 \\ -1 & -1 & 4 \end{bmatrix}$ and $B = 1 - \frac{1}{4}\, A$, then show that $\mu_i = 1 - \frac{1}{4}\lambda_i$, Where $\lambda_i$ and $\mu_i$ are the eigen values of $A$ and $B$ respectively.

ANSWERS

1. (i) $-1, -6;\ k_1 \begin{bmatrix} 1 \\ 2 \end{bmatrix}, k_2 \begin{bmatrix} 2 \\ -1 \end{bmatrix}$ (ii) $1, 6;\ k_1 \begin{bmatrix} 1 \\ -1 \end{bmatrix}, k_2 \begin{bmatrix} 4 \\ 1 \end{bmatrix}$

(iii) $1, 2, 2\ ;\ k_1 \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}, k_2 \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$ (iv) $a, b, c\ ;\ k_1 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, k_2 \begin{bmatrix} h \\ b-a \\ 0 \end{bmatrix}, k_3 \begin{bmatrix} g \\ 0 \\ c-a \end{bmatrix}$

2. (i) $\lambda^3 - 5\lambda^2 + 8\lambda - 4 = 4$ (ii) $0, 1, -2$ (iii) $k_1 \begin{bmatrix} 1 \\ 1-i \end{bmatrix}, k_2 \begin{bmatrix} 1 \\ 1+i \end{bmatrix}$

3. $4, 110, 10$

4. $6$

7. $\lambda = 0, = \left[\frac{3}{2}\,(a^2 + b^2 + c^2)\right]^{1/2}$

8. $3, -1, 1;\ k_1 \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix}, k_2 \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}, k_3 \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$

9. $2, 2, 3;\ k_1 \begin{bmatrix} 5 \\ 2 \\ -5 \end{bmatrix}, k_2 \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}$

10. $a, b, c$

11. $\lambda^4 - 7\lambda^3 + 17\lambda^2 - 17\lambda + 6 = 0; \lambda = 1, 1, 2, 3$

# Section E

1. Show that the matrix A is Hermitian and iA is Skew-Hermitian where A is.

(i) $\begin{bmatrix} 2 & 3-4i \\ 3+4i & 2 \end{bmatrix}$ (ii) $\begin{bmatrix} 3 & 5+2i & -3 \\ 5-2i & 7 & 4i \\ -3 & -4i & 5 \end{bmatrix}$

(iii) $\begin{bmatrix} -1 & 2+i & 5-3i \\ 2-i & 7 & 5i \\ 5+3i & -5i & 2 \end{bmatrix}$ (iv) $\begin{bmatrix} 2 & 3+2i & -4 \\ 3-2i & 5 & 6i \\ -4 & -6i & 3 \end{bmatrix}$

2. (i) Express the matrix $A = \begin{bmatrix} i & 2-3i & 4+5i \\ 6+i & 0 & 4-5i \\ -i & 2-i & 2+i \end{bmatrix}$. as a sum of Hermitian and Skew- Hermitian matrix.

(ii) Express the Hermitian matrix $A = 1\begin{bmatrix} 1 & -i & 1+i \\ i & 0 & 2-3i \\ 1-i & 2+3i & 2 \end{bmatrix}$ as $P + iQ$ where P is a real symmetric and Q is a real skew-symmetric matrix.

3. Show that $A = \begin{bmatrix} i & 0 & 0 \\ 0 & 0 & i \\ 0 & i & 0 \end{bmatrix}$ is skew- Hermitian and also unitary

4. (i) If A is any square matrix, prove that $A + A^*, AA^*, A^*A$ are all Hermitian and $A - A^*$ is Skew-Hermitian.

(ii) If A, B are Hermitian or Skew-Hermitian, then so is A + B.

(iii) Show that the matrix $B^*AB$ is Hermitian or Skew-Hermitian as A is Hermitian or Skew-Hermitian.

(iv) If A is a Hermitian matrix, then show that $iA$ is a Skew-Hermitian matrix.

5. (i) Define unitary matrix. Show that the following matrix is unitary.

$$\frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1+i \\ 1-i & -1 \end{bmatrix}$$

(ii) Prove that $\frac{1}{2}\begin{bmatrix} 1+i & -1+i \\ 1+i & 1-i \end{bmatrix}$ is a unitary matrix.

(iii) Show that $A = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{bmatrix}$ is a unitary matrix, where $\omega$ is complex cube root of unity.

6. Verify that the matrix $A = \begin{bmatrix} \frac{1+i}{2} & \frac{1-i}{2} \\ \frac{1-i}{2} & \frac{1+i}{2} \end{bmatrix}$ have eigen values with unit modulus.

## Section F

Q1. Suppose that T : $R^3 \to R^2$ and U : $R^2 \to R^3$ are linear transformations, and for all $v \in R^2$ , (T o U)(v) = v.

(a) Prove that T is onto.

(b) What is the nullity of T?

Ans: Nullity of T = 1

Q2. Let C be a 3×5 real-valued matrix. Assume that the span of the columns of C is all of $R^3$ . Determine the nullity (=dimension of the null space or kernel) of C?

Ans: 2

Q3. Find [bases for] the column space and the null space (or kernel) for the matrix A =

$$\begin{bmatrix} 1 & 2 & 2 & 3 \\ 1 & 2 & 1 & 2 \\ 2 & 4 & 3 & 5 \end{bmatrix}$$

Ans: Column space of A = Span{(1, 1, 2),(2, 1, 3)}

Ker(A) = Span{(−2, 1, 0, 0),(−1, 0, −1, 1)}.

Q4. TRUE or FALSE? Justify your answer.

a) There exists a $4 \times 5$ matrix of rank 3 and such that the dimension of the space spanned by its columns is 4.
   Ans: False. The dimension of the image is the rank of A.
b) If A is a $4 \times 5$ matrix, then it is possible for rank(A) to be 3 and dim(ker(A)) to be 3.
   Ans: No. dim ker + dim im = dim source = 5. So we would have 3 + 3 = 5, which is impossible.

c) There exists a $4 \times 5$ matrix A of rank 3 such that dim(ker(A)) is 2.

Ans: True. It is possible and in fact always true by Rank Nullity.