Diffraction

Note: Fer diffraction dimension of the obstacle must be same as the order of wavelength of light used.

Diffraction: Bending of light waves around the sharp edges of opaque obstacle or aperture encroachment of light within the geometrical shadow is called as diffraction.

Difference between diffraction & Interference

Diffraction	Interference
① In diffraction, diffraction occurs between the secondary wavelength of same wave starting from diff. points	① In interference, interference occurs between separate wave-fronts starting from two coherent sources.
② Several maxima's and minima's of equal magnitude are obtained.	② A central maxima & minima is obtained; rest all are secondary maxima's or minima's of different magnitudes.

Types of diffraction

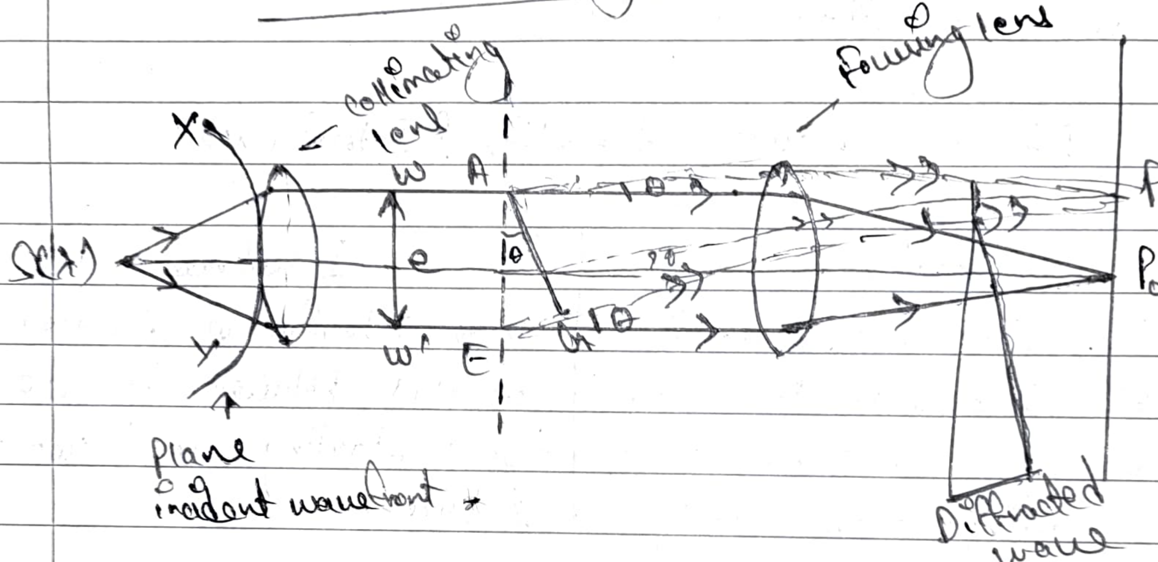
Two types of diffraction are:-

(i) Fresnel's class of diffraction
(ii) Fraunhofer diffraction

Fresnel's class of diffraction	Fraunhofer diffraction
① Source & Screen lies at finite distance from the diffracting agency.	① Source & Screen lies at infinite distance from the diffracting agency. This is possible with the help of polishing of focusing lens.

(2) No, use of lenses.	(2) lenses are used.
(3) They have only qualitative importance. ex. Straight line, Narrow wire, etc.	(3) They have only quantitative importance. Ex. Single slit, double slit

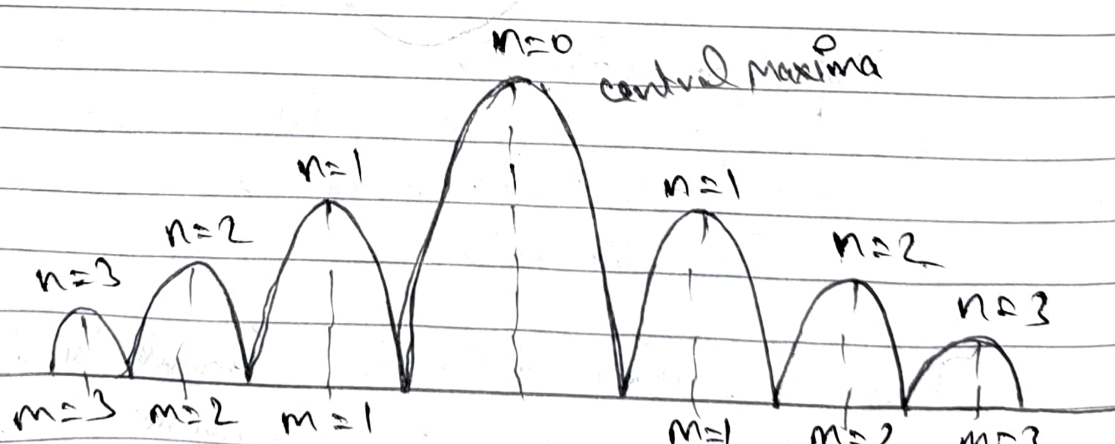
Diffraction at the Single slit double slit



n th order maxima due to diffracted wave of $\angle \theta$

Central Maxima at diffraction angle $\theta = 0$

Observation :-



Assumptions: ① over the width e of the single slit there are infinite no. of coins which become the source of secondary wavelets at the incident plane wavefronts strike them. From these points diffracted waves reach at several points and make them maxima or minima.

② For a most general case, let us consider diffracted wave at an angle θ from these points.

③ Let us consider no. of these points be n. that in n diffracted wave at an angle θ meet at point P. Let the amplitude of each diffracted wave is a.

It has assume that light AE of the slit as one goes one point to next successive point, path diff. or phase diff. increase with Arithmetic Progression (A.P.) of common diff. (d). Finally the path diff. b/w first diffracted wave at A and last diffracted wave at E is given by

$$\Delta = EG = AE\sin\theta = e\sin\theta$$

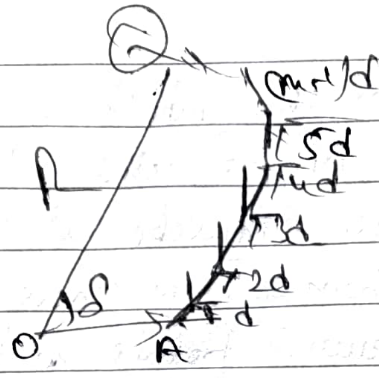
$$\because (AE = e)$$

and hence the corresponding phase diff. b/w first and last diff. wave is given by

$$\phi = \frac{2\pi}{\lambda}\Delta$$

$$= \frac{2\pi}{\lambda} e\sin\theta$$

Now let us calculate the resultant amplitude R and the resultant phase diff. of all these n diff. wave at diffraction angle θ to reach point P.



This can be calculated by using polygon law of vector addition as follows:-

$R\cos\delta = a + a\cos d + a\cos 2d + \ldots a\cos(n-1)d$ — (1)

$R\sin\delta = 0 + a\sin d + a\sin 2d + \ldots a\sin(n-1)d$ — (2)

Making some calculations with eq (1) & (2), finally we get n component as:

$$R\cos\delta = \frac{a\sin nd/2}{\sin d/2}\cos\frac{(n-1)d}{2} \quad \text{— (3)}$$

$$R\sin\delta = \frac{a\sin\frac{nd}{2}}{\sin\frac{d}{2}}\sin\frac{(n-1)d}{2} \quad \text{— (4)}$$

Squaring and adding eq (3) and (4), we get

$$R^2 = \frac{a^2\sin^2\frac{nd}{2}}{\sin^2\frac{d}{2}}$$

$$R = \pm\frac{a\sin\frac{nd}{2}}{\sin\frac{d}{2}}$$

dividing (3) by (4)

$$\tan\delta = \tan\frac{(n-1)d}{2}$$

$$\delta = \frac{(n-1)d}{2} \quad \text{— (6)}$$

study time

Subject

Date.: __/__/__

MON TUE WED THR FRI SAT SUN

Now take $n \to$ very very large (nearly infinite) and a & d are infinitesemaly small so that na & nd still remains finite. Hence, the phase difference between first and nth diffracted wave is given by

$$\phi = (n-1)d$$
$$\simeq nd$$
$$\simeq 2\alpha \text{ (say)}$$

$$\frac{2\pi}{\lambda} e \sin\theta = 2\alpha$$

$$\boxed{\alpha = \frac{\pi}{\lambda} e \sin\theta}$$

$$R = \pm \frac{a \sin\alpha}{\sin\frac{\alpha}{n}}$$

$$= \pm \frac{a \sin\alpha}{\frac{\alpha}{n}} \qquad \left(\because n \to \infty \text{ so } \sin\frac{\alpha}{n} \text{ is very small}\right)$$

$$= \pm \frac{n a \sin\alpha}{\alpha}$$

$$= \pm R_0 \frac{\sin\alpha}{\alpha} \qquad (\because na = R_0)$$

Now the resultant phase angle is given by (6)

$$\delta = \frac{(n-1)d}{2}$$

$$\delta = \frac{nd}{2} \qquad (\because n \to \infty \text{ so } n-1 = n)$$

Hence we reach the following important results :-

study time

Subject ____________ Date.: __/__/__

MON TUE WED THR FRI SAT SUN

Resultant amplitude and I is given by

$$R = R_0 \frac{\sin\alpha}{\alpha}$$

where $R_0 = na$ = max. amplitude due to undiffracted wave

$$\beta = \alpha = \frac{\pi}{\lambda} e\sin\theta$$ and the resultant intensity is given by

$$I_p = R^2$$

$$= R_0^2 \frac{\sin^2\alpha}{\alpha^2}$$

$$= I_0 \frac{\sin^2\alpha}{\alpha^2}$$

where $I_0 \to$ max. resultant intensity due to non-diffracted wave.

Explanation

$$R = R_0 \frac{\sin\alpha}{\alpha}$$

$$= \frac{R_0}{\alpha}\left[\alpha - \frac{\alpha^3}{3!} + \frac{\alpha^5}{5!} - \frac{\alpha^7}{7!} + \dots\right]$$

$$= \frac{R_0}{\cancel{\alpha}} \cdot \cancel{\alpha}\left[1 - \frac{\alpha^2}{3!} + \frac{\alpha^5}{5!} - \frac{\alpha^6}{7!} + \dots\right]$$

For central Maxima

$$R = R_0$$

hence $\alpha = 0$

It correspond to the non-diffracted wave which meet at P_0 and arise central max. intensity at central Maxima will be R_0^2.

Condition for various order of Minima on both side of central Maxima

Resultant amplitude at P is given by

$$R = R_0 \frac{\sin\alpha}{\alpha}$$

~~For Minima to take place~~

$R \to 0$ at P for this in above eq^n

$$\sin\alpha = 0$$

But $\alpha \neq 0$ because it correspond to central Maxima

so $\sin\alpha = 0$ OR $\alpha = \pm m\pi$

where $m = 1, 2, 3, \ldots$

give the order of Minima.

$$\alpha = \frac{\cancel{\pi}}{\lambda} e\sin\theta = \pm m\cancel{\pi}$$

$$[\alpha = e\sin\theta = \pm m\lambda]$$

Formation of Primary Maxima on both side of Central Maxima

Intensity at P is given by

$$I = I_0 \frac{\sin^2\alpha}{\alpha^2}$$

diff. the above eq^n w.r.t. α, we get

$$\frac{dI}{d\alpha} = I_0 \left[\frac{2\sin\alpha\cos\alpha\,\alpha^2 - 2\alpha\sin^2\alpha}{\alpha^4} \right]$$

study time

Subject

Date.: __/__/__

MON TUE WED THR FRI SAT SUN

$$= I_0 \left[\frac{\alpha\cos\alpha - \sin\alpha}{\alpha^4}\right] 2\sin\alpha\ \alpha$$

$$= I_0 \left[\frac{\alpha\cos\alpha - \sin\alpha}{\alpha^2}\right] \frac{2\sin\alpha}{\alpha}$$

if $\frac{dI}{d\alpha} = 0$

$\alpha \neq 0$ (central Maxima)

$\alpha = \tan\alpha$

therefore only remaining condition is $\alpha\cos\alpha - \sin\alpha = 0$ this eqn can be solve graphically by plotting the curve $y = \alpha$, $x = \tan\alpha$. This eqn is St. line passing through origin & making an angle of 45° with axis.

The eqn $y = \tan\alpha$ represents a discontinous curve having a no. of branches with asymptotes at intervals $\alpha = \pi$ as shown in the figure the point of intersection of these two curves gives the value of α satisfying the eqn $\alpha = \tan\alpha$.