SEMESTER EXAMINATION, (U.P.) 2018
Applied Mathematics-I
Code : 2041
1st Semester

Time : $2\frac{1}{2}$ **Hrs.** **Max. Marks : 50**

Note : Attempt All questions.

1. Answer any ten parts of the following from parts a to e select the correct choice in the following : **[10 × 1 = 10]**

(a) The sum of 10 terms of the series $4+8+12+\ldots$:

(i) 110 (ii) 220

(iii) 330 (iv) None

(b) If $f(x) = \tan x$ then the value of $f(60^\circ)$ is :

(i) $\sqrt{3}$ (ii) $\frac{1}{\sqrt{3}}$

(iii) ∞ (iv) 1

(c) If $\bar{a} = 2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + \hat{\mathbf{k}}$ then $|\bar{a}|$ is :

(i) $\sqrt{12}$ (ii) $\sqrt{14}$

(iii) $\sqrt{11}$ (iv) None

(d) The value of $\sin^2\alpha + \cos^2\alpha$ is :

(i) 0 (ii) 1

(iii) -1 (iv) None

(e) The value of $\frac{d^2y}{dx^2}$ is :

(i) $\frac{1}{x}$ (ii) $\frac{1}{x^2}$

(iii) $-\frac{1}{x^2}$ If $y = \log x$ (iv) (None) If $y = \log x$

(f) Evaluate $\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 7 \\ 7 & 8 & 9 \end{vmatrix}$

(g) Evaluate $\lim\limits_{x \to 3} \frac{x^2-9}{x-3}$

(h) Find the differential coefficient of $x \log x$.

(i) Change into Polar form $-1+i$.

(j) Find the equation of tangent to ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ at point (1, 2).

(k) If $\bar{a} = \hat{\mathbf{i}} + \hat{\mathbf{j}} - 2\hat{\mathbf{k}}$ and $\bar{b} = 3\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - \hat{\mathbf{k}}$ then find the value of $\bar{a}.\,\bar{b}$

(l) If $y = x^3 + 8x^2 - 7x + 9$ then find $\frac{d^3y}{dx^3}$.

2. Answer any five parts of the following : [5 × 2 = 10]

(a) Which term of series $1, \sqrt{3}, 3, \ldots$ is 81 ?

(b) Find the sum of series $101 + 99 + 97 + \ldots + 47$.

(c) $\frac{(2+3i)^2}{5-i}$ change into $a + ib$ form.

(d) Find the differential coefficient of $\cos(\tan x^2)$.

(e) Prove that in ΔABC

$$a\,(b\cos C - c\cos B) = (b^2 - c^2)$$

(f) If $y = \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x}}} + \ldots \infty$ then prove that $\frac{dy}{dx} = \frac{\cos x}{2y - 1}$

(g) Prove that $(1+i)^4 \cdot \left(1 + \frac{1}{i}\right)^4 = 16$

3. Answer any two parts of the following : [2 × 5 = 10]

(a) Find the independent term from x in the expansion of $\left(2x^4 - \frac{1}{3x^7}\right)^{11}$.

(b) Solve the equation $x^3 - 1 = 0$ using Demoivre's theorem.

(c) Show that the angle between vectors $\bar{a} = 3\hat{\mathbf{i}} + \hat{\mathbf{j}} + 2\hat{\mathbf{k}}$ and $\bar{b} = 2\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 4\hat{\mathbf{k}}$ is $\theta = \sin^{-1}\left(\frac{2}{\sqrt{7}}\right)$.

4. Answer any two parts of the following : [2 × 5 = 10]

(a) Solve the equations using Cramer's rule $6x + y - 3z = 5, x + 3y - 2z = 5$ and $2x + y + 4z = 8$.

(b) Find the differential coefficient of $\sqrt{\tan x}$ from the first principle.

(c) If $y = a\cos(\log x) + b\sin(\log x)$ then prove that $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$.

5. Answer any two parts of the following : [2 × 5 = 10]

(a) Find the differential coefficient $\left(\frac{x + \cos x}{\tan x}\right)$

(b) If vectors $\bar{a}, \bar{b}, \bar{c}$ are such that $\bar{a} + \bar{b} + \bar{c} = 0$ prove that $\bar{a} \times \bar{b} = \bar{b} \times \bar{c} = \bar{c} \times \bar{a}$,

(c) Find the equation of Normal to the curve $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at (x_1, y_1).

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ODD SEMESTER EXAMINATION (U.P.), DECEMBER-2019

Applied Mathematics-I

Code : 2041 (C)

First Semester

Time : 2.30 Hours] **[Maximum Marks : 50**

Notes :

(i) Attempt all questions.

(ii) Students are advised to specially check the Numerical Data of question paper in both versions. If there is any difference in Hindi translation of any question, the students should answer the question according to the English version.

(iii) Use of Pager and Mobile Phone by the students is not allowed.

1. Answer any ten parts of the following, from parts a to e select correct choice. **[10 × 1 = 10]**

(a) If $\sin\theta = \cos\theta$, value of angle θ is :

(i) $\frac{\pi}{3}$ (ii) $\frac{\pi}{2}$ (iii) $\frac{\pi}{4}$ (iv) None

(b) If $f(n) = \frac{n}{1+n}$ then value of $f(n) + \frac{1}{f(n)}$ is :

(i) $\frac{1}{2n}$ (ii) 4 (iii) 1 (iv) None

(c) If vectors $2\hat{i} + m\hat{j} + 3\hat{k}$ and $4\hat{i} - 2\hat{j} - 2\hat{k}$ are perpendicular, then value of m is :

(i) 2 (ii) 1 (iii) 3 (iv) None

(d) The sum of infinite terms of the series $0.9 + 0.03 + 0.001 + \ldots\ldots\ldots\ldots\ \infty$ is :

(i) $\frac{29}{27}$ (ii) $\frac{27}{29}$ (iii) $\frac{25}{27}$ (iv) None

(e) If $y = e^x \sin x$, value of $\frac{dy}{dx}$ is :

(i) $e^x \cos x$ (ii) $e^x(\sin x + \cos x)$ (iii) $e^x \sec x$ (iv) None

(f) Evaluate $\lim\limits_{x\to\infty} \frac{4x^2 - 5x - 3}{7x^2 + 3x + 1}$

(g) Evaluate $\left(6i^{74} + \frac{4}{i^{174}}\right)$

(h) In ΔABC prove that $\frac{\sin A + \sin B}{\sin A - \sin B} = \frac{a+b}{a-b}$

(i) Evaluate $\begin{vmatrix} 2 & -1 & 3 \\ 1 & 3 & 11 \\ 3 & -2 & 1 \end{vmatrix}$

(j) Expand $(x+a)^4$ by Binomial theorem.

(k) If $y = x^5 + 7x^3 + 5x^2 + 7x + 9$, find y_4.

(l) Evaluate $(\cos\theta + i\sin\theta)^5 \times (\cos\theta - i\sin\theta)^4$

2. Answer any five parts of the following : **[5 × 2 = 10]**

(a) Evaluate $0.1\dot{2}\dot{3}$

(b) Find the cosine of the angle between the two vectors $\bar{a} = 2\hat{i} + \hat{j} - 3\hat{k}$ and $\bar{b} = 3\hat{i} - 2\hat{j} - \hat{k}$

(c) In ΔABC prove that $\dfrac{\cos A}{a} + \dfrac{\cos B}{b} + \dfrac{\cos C}{c} = \dfrac{a^2 + b^2 + c^2}{2\,abc}$.

(d) Differentiate with respect to x, $\dfrac{1 - \cos x}{1 + \cos x}$.

(e) Prove that $\sin^{-1}\dfrac{4}{5} + \sin^{-1}\dfrac{5}{13} + \sin^{-1}\dfrac{16}{65} = \dfrac{\pi}{2}$.

(f) If $y = \sec x \tan x$, find $\dfrac{dy}{dx}$.

(g) Find modulus and amplitude of $\sqrt{\dfrac{1+i}{1-i}}$

3. Answer any two parts of the following : **[2 × 5 = 10]**

(a) If A is the sum of odd terms, and B the sum of even terms in the expansion of $(x + a)^n$, then prove that $4AB = (x + a)^{2n} - (x - a)^{2n}$.

(b) Prove that :

$$(1 + \cos\theta + i\sin\theta)^n + (1 + \cos\theta - i\sin\theta)^n = 2^{n+1}\cos^n\frac{\theta}{2}\cos\frac{n\theta}{2}.$$

(c) If $y = x\tan^{-1}\dfrac{x}{y}$, prove that $\dfrac{dy}{dx} = \dfrac{y}{x}$.

4. Answer any two parts of the following : **[2 × 5 = 10]**

(a) If $x = r\cos\theta,\ y = r\sin\theta,\ r = a(1 + \cos\theta)$ then find $\dfrac{dy}{dx}$.

(b) If $y = e^{kx}(a\cos nx + b\sin nx)$ then prove that :

$$\frac{d^2y}{dx^2} - 2k\frac{dy}{dx} + (k^2 + n^2)\,y = 0.$$

(c) Find the middle term in the expansion of $\left(\dfrac{4x}{5a} - \dfrac{5a}{4x}\right)^{12}$.

5. Answer any two parts of the following : **[2 × 5 = 10]**

(a) Prove that if $(\bar{a} \times \bar{b}) \times \bar{b} = (\bar{b} \times \bar{a}) \times \bar{a}$, then $(\bar{a} \times \bar{b}) = \bar{0}$.

(b) Distance travelled by a particle in t seconds is given by the equation $s = 180\,t - 16t^2$, find velocity and acceleration of the particle. Calculate the time when velocity will be zero.

(c) Prove that $2\tan^{-1}\dfrac{1}{5} + \tan^{-1}\dfrac{1}{7} + 2\tan^{-1}\dfrac{1}{8} = \dfrac{\pi}{4}$.

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