

Km Mayawati Govt. Girls Polytechnic

Assignment (Unit-2)

Applied Mathematics-III

1-Identify the curve given by $2x^2 + 2y^2 - 6x + 5 = 0$.

2-If $u = e^{xyz}$, then show $\frac{\partial^3 u}{\partial x \partial y \partial z} = (1 + 3xyz + x^2 y^2 z^2) e^{xyz}$

3-If $u = \log(x^3 + y^3 + z^3 - 3xyz)$

$$\text{show that } \left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z} \right)^2 u = -\frac{9}{(x+y+z)^2}$$

4-If $u = \sin^{-1} \frac{\sqrt{x}-\sqrt{y}}{\sqrt{x}+\sqrt{y}}$, then show $\frac{\partial u}{\partial x} = -\frac{y}{x} \frac{\partial u}{\partial y}$

5-If $z = f(x, y)$, $x = e^u + e^{-v}$, $y = e^{-u} - e^v$, $z = e^{-u} - e^v$, then prove

$$\frac{\partial z}{\partial u} - \frac{\partial z}{\partial v} = x \frac{\partial z}{\partial x} - y \frac{\partial z}{\partial y}$$

6-If $u_1 = \frac{x_2 x_3}{x_1}$, $u_2 = \frac{x_3 x_1}{x_2}$, $u_3 = \frac{x_1 x_2}{x_3}$, then prove $J(u_1, u_2, u_3) = 4$

7-If $x = r \sin \theta \cos \phi$, $y = r \sin \theta \sin \phi$, $z = r \cos \theta$, then show $\frac{\partial(x, y, z)}{\partial(r, \theta, \phi)} = r^2 \sin \theta$

$$\text{And evaluate } \frac{\partial(r, \theta, \phi)}{\partial(x, y, z)}$$

8-If $x = r \cos \theta$, $y = r \sin \theta$, then find $\frac{\partial(x, y)}{\partial(r, \theta)}$ and $\frac{\partial(r, \theta)}{\partial(x, y)}$.

9- If $\vec{r} = t^2 \hat{i} - t \hat{j} + (2t + 1)\hat{k}$, find (i) $\frac{d\vec{r}}{dt}$ (ii) $\frac{d^2\vec{r}}{dt^2}$ (iii) $|\frac{d\vec{r}}{dt}|$ (iv) $|\frac{d^2\vec{r}}{dt^2}|$

10-If $\vec{f} = \text{grad}(x^3 + y^3 + z^3 - 3xyz)$, then find $\text{div } \vec{f}$, $\text{curl } \vec{f}$.

11- Prove (i) $\text{curl grad } \phi = 0$ (ii) $\text{div } \vec{r} = 3$

12- (i) $\int_1^2 \int_0^{y/2} y \, dy \, dx$ (ii) $\int_0^1 \int_0^1 \int_0^1 e^{x+y+z} \, dx \, dy \, dz$

13- If $\vec{r}(t) = 5t^2 \hat{i} + t \hat{j} - t^3 \hat{k}$ then find $\int_1^2 \vec{r} \times \frac{d^2\vec{r}}{dt^2} \, dt$

14- If $F = xy^3 - x^3y$ then find $\Delta^2 F$

15-Prove $\frac{\partial(u, v)}{\partial(x, y)} \times \frac{\partial(x, y)}{\partial(u, v)} = 1$