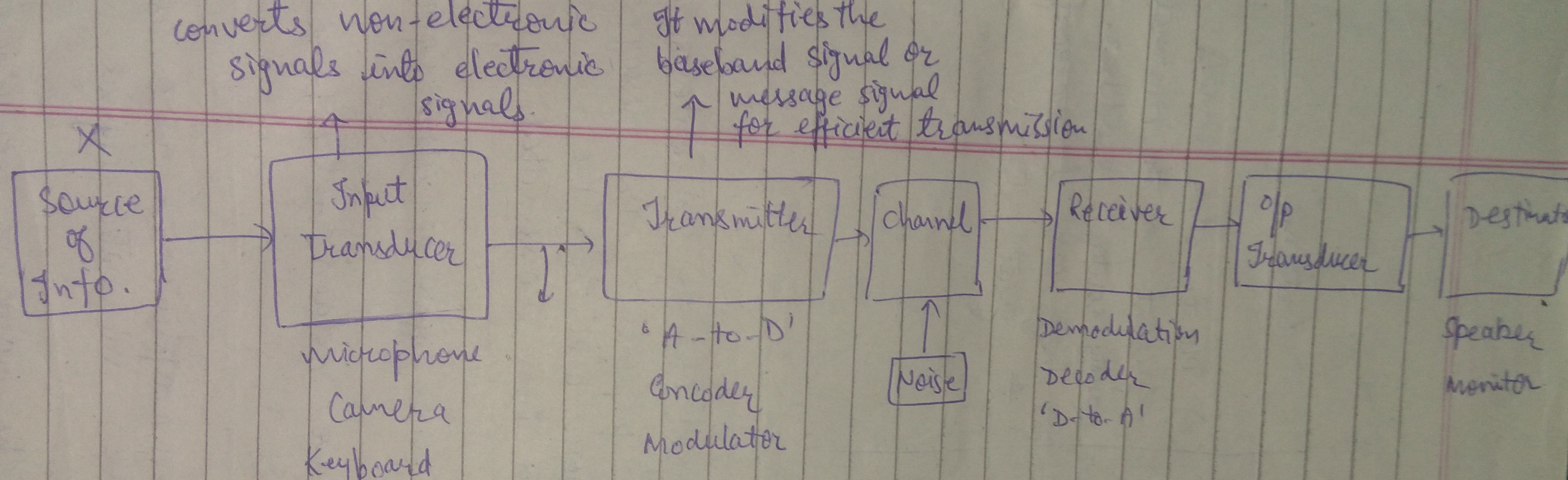

Message/Baseband signal → electrical form of message

Reduction in the power of the signal is called 'Attenuation'

Change in the shape of the waveform is called 'distortion'

Sources of Noise

Internal	External
a) Functioning of the electrical components	a) Due to other signals present in the channel faulty contact switches.
(b) These noises cannot be completely eliminated.	(b) They can be reduced to great extent.

$$I \propto A^2 f^2$$

** We use carrier signal (having high intensity) in transmitter to propagate our message signal by modulation and then the modulated signal is then transmitted.

=> Modulated signal: carrier signal + message signal

At receiver, we have certain filters (like low pass filters, etc.)

Before filtering and amplification noise is removed from the modulated signal.

* The bandwidth is the range of frequencies over which the ~~me~~ signal can be transmitted with fidelity (trustability)/not distortion.

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	Frequency	Wavelength	Name	Tr. media	Application
①	3K - 30 KHz	100km - 10km	Very low frequency	Wire lines (copper wires)	Telephony, Telegram
②	30 KHz - 300Kz	10km - 1km	Low freq.	Ground wave	
③	300 KHz - 3MHz	1km - 100m	Medium freq.		AM Broadcast
④	3MHz - 30MHz	100m - 10m	High freq.	Sky wave	CB Radio
⑤	30MHz - 300MHz	10m - 1m	Very high freq.	Coaxial transmission radio	VHF, TV, FM radios mobiles.
	300MHz - 3GHz	1m - 10cm	Ultra high freq.		TV, SAT. mobile
	3GHz - 30GHz	10cm - 1cm	Super high freq.	Wave guides	

Frequency Range am.

A signal can be successfully transmitted if channel Bandwidth > message bandwidth.

P.T.O.

freq.	wavelength	Name	Transmedia	Application
8 ⑧ 10^{14} 10^{15}	$\sim 10^{-6}$ m	IR, visible, UV	optical fibre	Broadband

Carrier Signals

	Freq. Band	Carrier Freq.	Bandwidth.
1	LF	100 KHZ	~2 KHz
2	HF	5 MHZ	100 KHz
3	VHF	100 MHz	5 MHz
4	Microwave	5 GHz	100 MHz
5	Optical	5×10^{14} Hz	10 GHz – 10 Tera Hz

⇒ Bandwidth is the freq range over which message can be transmitted without much loses

Trans. Medium	Typical freq	Power (dB/km) (loss)
Twisted wire	1KHz – 100 KHz	0.05 3
Coaxial cable	100HZ – 30MHz	1 4
wave guide	10GHz –	1.5dB/km
Optical fibres	10^{14} – 10^{15}	<0.5

Height of Antenna $\propto$ wavelength of wave.

Signal to noise ratio (SNR)

Ratio of power of signal to ~~no~~ power of noise. If this ratio is high then very low losses.

* If distance increases, then SNR ~~also~~ decreases.

~~SR~~ $\boxed{SNR \propto \frac{1}{distance}}$

Signals:-

It carries the set of information.

① **Deterministic Signal:-**

They can be expressed in mathematical forms(expressions).

② **Random Signal:-**

These are probability based signals. (Mean square values) eg:- Noise.

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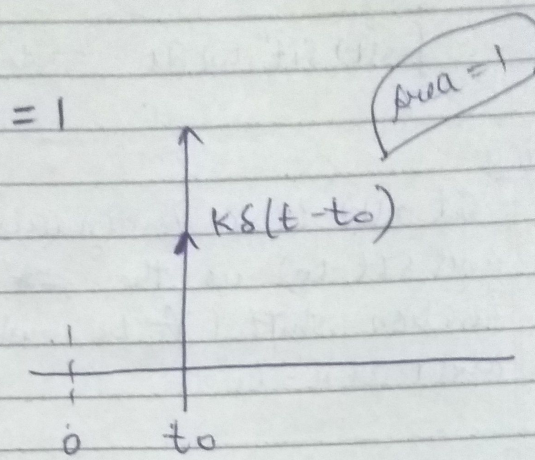
Impulse function (Dirac-Delta function)

$$\int_{-\infty}^{\infty} \delta(t)\, dt = 1$$



unit step function is defined as :-

$$u(t) = \begin{cases} 0, & t<0 \\ 1, & t>0 \end{cases}$$

(discontinuous at $t=0$)

The Dirac-Delta function $\delta(t)$ is defined as having zero amplitude everywhere except at $t=0$, where it is infinitely large, such that area under the curve is :-

$$\delta(t) = 0, \quad t \neq 0$$

$$\boxed{\int_{-\infty}^{\infty} \delta(t)\, dt = 1} \qquad -①$$

P.T.O.

~~Two~~ important properties of $\delta(t)$

① $\int_{-\infty}^{\infty} x(t)\,\delta(t-t_0)\,dt = x(t_0)$ (sifting property) / (sampling property)

Derivation:-

Let $x(t)$ be a continuous function of time. and $\delta(t-t_0)$ is the unit-impulse function shifted ~~to~~ to t_0, which is 0 at $x \neq t_0$ and 1 at $x = t_0$.

$$\delta(t-t_0) = \begin{cases} 0, & x \neq t_0 \\ 1, & x = t_0 \end{cases}$$

Now taking product of $x(t)$ and $\delta(t-t_0)$ and further integrating their product over $-\infty$ to ∞, we get,

$$= \int_{-\infty}^{\infty} x(t)\,\delta(t-t_0)\,dt \quad -①$$

Since $x(t)$ is continuous over time, ~~to~~ also $\delta(t-t_0)$ is 0 for $x \neq t_0$ and 1 ~~for~~ $x = t_0$. ~~for~~ so ~~writing~~ definitⁿ the eq. ① at very small ~~interval~~ neighbourhood of t_0, we write it as.

$$= \int_{t_0-\varepsilon}^{t_0+\varepsilon} x(t)\,\delta(t-t_0)\,dt$$

Since ε is very small, so ~~f~~ $x(t)$ will attain its minimum (m) and maximum (M) values in the above interval, so, -

$$\int_{t_0-\varepsilon}^{t_0+\varepsilon} m\,\delta(t-t_0)\,dt \le \int_{t_0-\varepsilon}^{t_0+\varepsilon} x(t)\,\delta(t-t_0)\,dt \le \int_{t_0-\varepsilon}^{t_0+\varepsilon} \cancel{x(t)}\, M\,\delta(t-t_0)\,dt$$

$$\Rightarrow m \le \int_{t_0-\varepsilon}^{t_0+\varepsilon} x(t)\,\delta(t-t_0)\,dt \le M \quad -②$$

$[\because \delta(t-t_0) = 1 \text{ at } t = t_0]$

Now $\varepsilon \to 0$, so m and M will converge ~~fo~~ to $x(t_0)$. So writing eq. ② we get,

$$\Rightarrow \int_{t_0-\varepsilon}^{t_0+\varepsilon} x(t)\,\delta(t-t_0)\,dt \to x(t_0)$$

$\Rightarrow$ finally:-

$$\boxed{\int_{-\infty}^{\infty} x(t)\,\delta(t-t_0)\,dt \to x(t_0)}$$

② Scaling property

$$\delta(at) = \frac{1}{|a|}\,\delta(t)$$

Derivation.

We will derive this property using proof by cases.

P.T.O.

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Case1:- $c > 0$

- let $\Rightarrow y = cx$
$\Rightarrow dx = \frac{dy}{c}$

Now using the sifting property-

$$\Rightarrow \int_{-\infty}^{\infty} f(x) \cancel{\delta(x-0)\,dx}\ \delta(cx)\,dx = f(0) \quad -(1)$$

Substituting $x = \frac{y}{c}$, and $dx = \frac{dy}{c}$

$$\Rightarrow \int_{-\infty}^{\infty} f\left(\frac{y}{c}\right) \delta(y) \frac{dy}{c}$$

$$= \frac{1}{c} \int_{-\infty}^{\infty} f(y/c)\,\delta(y)\,dy = \frac{1}{c} f\left(\frac{0}{c}\right) = \frac{1}{c} f(0)$$

$$= \cancel{\frac{1}{c}\int_{-\infty}^{\infty} f(x)\,\delta(x-0)\,dx} = \frac{f(0)}{c} \quad -(2)$$

Now comparing eq. (1) and (2) we get

$$\delta(cx) = \frac{1}{c}\delta(x) \quad - (A)$$

Case2:- $c < 0$ $c = -b,\ b > 0$

let $y = -bx$

$dx = -\frac{dy}{b}$

using sifting property;

$$\int_{-\infty}^{\infty} f(x)\,\delta(bx)\,dx = f(0) \quad - (3)$$

Substituting $x = -\frac{y}{b}$, $dx = -\frac{dy}{b}$

$$\Rightarrow \int_{\infty}^{-\infty} f(-y/b)\, \delta(y)\, \frac{-dy}{b}$$

$$= -\frac{1}{b} \int_{\infty}^{-\infty} f(-y/b)\, \delta(y)\, dy$$

$$= +\frac{1}{b} \int_{-\infty}^{\infty} f(-y/b)\, \delta(y)\, dy = \frac{1}{b} f\left(\frac{0}{b}\right)$$

$$= \frac{1}{b} f(0)$$

$$= \frac{1}{|c|} f(0), \text{ since } c = -b,\ b > 0 \quad - ④$$

Now comparing eq. ③ and ④; we get

$$\cancel{\delta(t)}\ \delta(cx) = \frac{1}{|c|} \delta(x) \quad - (B)$$

So using (A) and (B),

$$\boxed{\delta(cx) = \frac{1}{|c|} \delta(x)} \quad \text{(Scaling property)}$$

③ $x(t)\,\delta(t-t_0) = x(t_0)\,\delta(t-t_0)$

$x(t)$ is a continuous function over time and $\delta(t-t_0)$ is a impulse function defined at $t = t_0$. So, their product will also we defined at $t = t_0$, or we can write the equation $x(t)\,\delta(t-t_0)$ as $= x(t_0)\,\delta(t-t_0)$. $x(t)$ is only valid at $t = t_0$.

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Fourier Series

Fourier series is defined for the periodic functions only.

Let ~~f~~ $x(t)$ be a periodic signal.

$$x(t+T_0) = x(t).$$

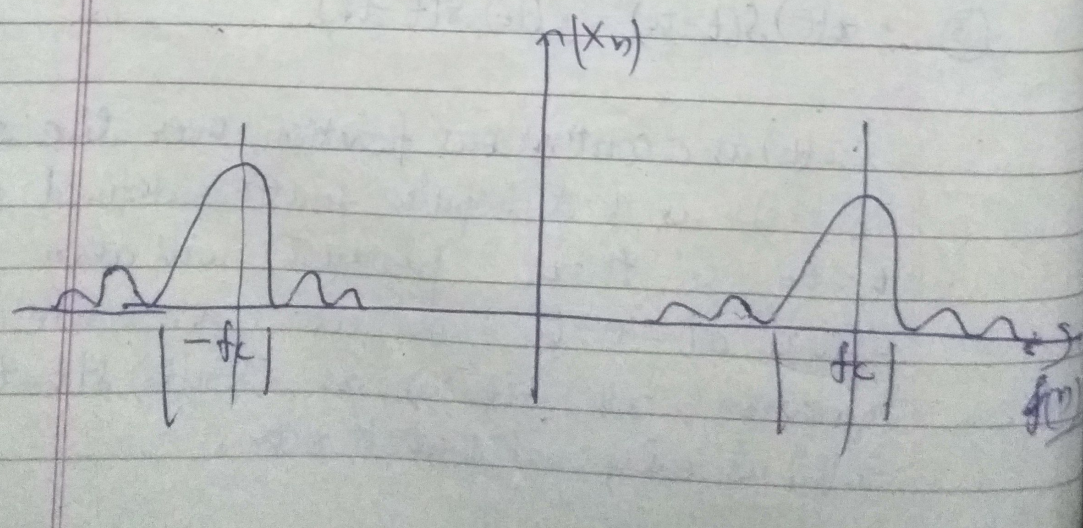
So the periodic function $x(t)$ can be written as fourier series as.

$$x(t) = \sum_{n=-\infty}^{n=+\infty} X_n \, e^{j2\pi n f t} \quad , \omega_0 = \frac{2\pi}{t_0}$$

where X_n = coeff. of fourier series.

$$\Rightarrow \quad X_n = \frac{1}{T_0} \int_{t+0}^{t+T_0} x(t) \, e^{-jn\omega_0 t}$$

$|x_n|$ = magnitude Spectrum , $\angle X_n$ = face spectrum. $= \theta_n$



$|X_n| = |X_{-n}|$ ... (symmetric about y axis)

$\angle X_n = -\angle X_{-n}$ (assymmetric)

Energy Signal

$$E_x = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

$$x(t) = \begin{cases} e^{-t}, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

$$x(t) = e^{-t} u(t)$$

$$E_x = \int_0^{\infty} |e^{-t}|^2 dt = \int_0^{\infty} e^{-2t} dt$$

$$= -\frac{1}{2}\left(e^{-2t}\right)_0^{\infty} = \underline{\underline{\frac{1}{2}}}$$

If energy of a signal is finite then it is called energy signal and its power is 0.

Power Signal

$$P_x = \lim_{T \to \infty} \frac{1}{T} \left| \int_{-T/2}^{T/2} |x(t)|^2 dt \right.$$

$$P_x = \lim_{T \to \infty} \frac{\text{energy in window of size } T}{T}$$

if $P_x < \infty$ or finite then it is called power signal. and its energy is ∞.

→ Time shifting

(-) sign

** $\boxed{g(t-t_0) \Longleftrightarrow G(f)\,e^{-2\pi f t_0}}$ → Only phase changes but amplitude remains same

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Parseval's Theorem

① $P = \frac{1}{T}\int |x(t)|^2\,dt$

$= \sum_{n=-\infty}^{\infty} |x_n|^2$

Fourier Series and Transformation

Fourier series is for periodic signals and transform is for any kind of signal

$$x(f) = \int_{-\infty}^{\infty} x(t)\,e^{-j2\pi f t}\,dt$$

$$x(t) = \int_{-\infty}^{\infty} x(f)\,e^{j2\pi f t}\,df$$

$$\left.\begin{matrix} \underline{x(t)} = \\ \underline{x(f)} \end{matrix}\right\} \cancel{x}(f) = |x(f)|\,e^{j\theta(f)}$$

→ phase

↓ amplitude

for real signal $x(t)$

$\cancel{X^*(f) = X(-f)}$

Derivation:

$\cancel{X(f) = Af + jBf}$

$\cancel{X^*(f) = Af - jBf}$

$\cancel{X(-f) = +Af - jBf}$

~~u(t) =~~ **

$$u(t) \Longleftrightarrow \boxed{\frac{1}{j\omega} + \frac{\delta(f)}{2}}$$

① $X^*(f) = X(-f)$. (if $x(t)$ is a real function)

Now, $X(f) = \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi f t}\, dt$ — ①

$\Rightarrow X^*(f) = \int_{-\infty}^{\infty} x(-t)\, e^{j2\pi f t}\, dt$ —

substitute $t = -\lambda$.

$\Rightarrow X^*(f) = -\int_{\infty}^{-\infty} x(\lambda)\, e^{-j2\pi f \lambda}\, d\lambda$

$\Rightarrow X^*(f) = \int_{-\infty}^{\infty} x(\lambda)\, e^{-j2\pi f \lambda}\, d\lambda$ — ②

Now from eq. ① and ②, we get,

$$X^*(f) = \int_{-\infty}^{\infty} x^*(t)\, e^{+j2\pi f t} dt$$

$$x(t) \xleftrightarrow{Ft} \cancel{X(j\omega)}\ X(f).$$

$$X(f) = \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi f t}\, dt \quad — ①$$

using eq. ①.

$$X^*(f) = \int_{-\infty}^{\infty} x^*(t)\, e^{j2\pi f t}\, dt$$

f by $-f$.

$$X^*(-f) = \int_{-\infty}^{\infty} x^*(t)\, e^{-j2\pi f t} dt \quad — ②$$

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from eq.-② we get,

$$x^*(t) \xrightarrow{FT} X^*(-f) \quad -③$$

Also from eq.-①

$$X^*(f) = \int_{-\infty}^{\infty} x^*(t)\, e^{j2\pi f t}\, dt$$

replacing f with -f.

$$X^*(-f) = \int_{-\infty}^{\infty} x^*(t)\, e^{-2j\pi f t}\, dt \quad -④$$

Now if $x(t)$ is real, then ~~for~~

$$x^*(t) = x(t) \quad -⑤$$

Now from eq. ~~④~~, ~~②~~ ① and ⑤, we get;

~~$X^*(-f) = X$~~

$$X^*(f) = \int_{-\infty}^{\infty} x^*(t)\, e^{j2\pi f t}\, dt$$

$$X^*(f) = \int_{-\infty}^{\infty} x(t)\, e^{j2\pi f t}\, dt$$

$$\underline{X^*(f) = X(-f)}$$

Hence proved

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(2) $$\int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\infty}^{\infty} |X(f)|^2 df.$$

$$\int_{-\infty}^{\infty} |x(t)^2| dt = \int_{-\infty}^{\infty} x(t) \cdot x^*(t) dt$$

$$= \int_{-\infty}^{\infty} x(t) \left[\int_{-\infty}^{\infty} X^*(f) e^{-j2\pi ft} df \right] dt$$

Reverse the order of integration,

$$\int_{-\infty}^{\infty} |x(t)^2| dt = \int_{-\infty}^{\infty} X^*(f) \left[\int_{-\infty}^{\infty} x(t) e^{-j2\pi ft} dt \right] df$$

$$= \int_{-\infty}^{\infty} X^*(f) \cdot X(f) df$$

$$\int_{-\infty}^{\infty} |x(t)^2| dt = \int_{-\infty}^{\infty} |X(f)|^2 df$$

Hence proved

(3) $$x_1(t) \otimes x_2(t) = \int_{-\infty}^{\infty} x_1(d) \cdot x_2(t-d) \, dd$$

$$F(x(t)) = \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} x_1(d) \cdot x_2(t-d) \, dd \right] e^{-2\pi jft} dt$$

reversing the ordering

$$F(x(t)) = \int_{-\infty}^{\infty} x_1(d) \left[\int_{-\infty}^{\infty} x_2(t-d) e^{-2j\pi ft} dt \right] dd$$

$t \to (t-d)$

$$= \int_{-\infty}^{\infty} x_1(d) \left[X_2(f) \, e^{-2j\pi fd} \, dd \right]$$

$$= X_2(f) \int_{-\infty}^{\infty} x_1(d) e^{-2j\pi fd} dd$$

$$= X_1(f) \cdot X_2(f)$$ H.P. //

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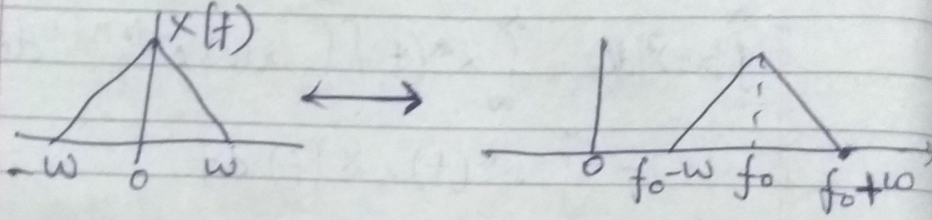
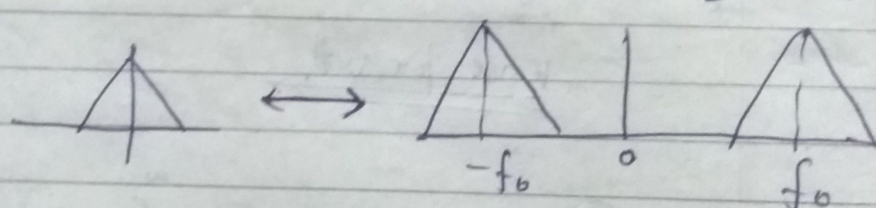
(4) $\boxed{\begin{array}{l} x(t) \leftrightarrow X(f) \\ x(t-t_0) \leftrightarrow X(f)\, e^{-2\pi j f t_0} \end{array}}$ (time delay)

Frequency Translation

$$x(t).\, e^{2j\pi f_0 t} \longleftrightarrow X(f-f_0)$$



$$x(t) \cos 2\pi f_0 t \longleftrightarrow \frac{1}{2}X(f-f_0)+\frac{1}{2}X(f+f_0)$$

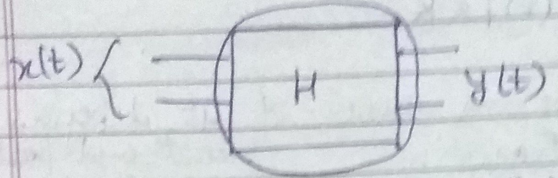


** $$\frac{d^n x(t)}{dt^n} \longleftrightarrow (j2\pi f)^n X(f)$$

** $$\int_{-\infty}^{t} x(\sigma)\, d\sigma \longleftrightarrow \frac{1}{j2\pi f} X(f) + k X(0)$$

** $$\int_{-\infty}^{t} g(\tau)\, d\tau \Leftrightarrow \frac{G(f)}{j2\pi f} + \frac{1}{2} G(0)\delta(f)$$

<u>Systems</u>:-
Interconnection of active and passive components.



(there will be some propagation delay)

$y(t) = H[x(t)]$

LTI → Linear Time Invariant systems

$$\begin{cases} x_1(t) \longrightarrow y_1(t) \\ x_2(t) \longrightarrow y_2(t) \\ \\ \alpha_1 x_1(t) + \alpha_2 x_2(t) \longrightarrow \alpha_1 y_1(t) + \alpha_2 y_2(t) \end{cases}$$

(Linearity Property)

$x(t) \longrightarrow y(t)$

$x(t-t_0) \longrightarrow y(t-t_0)$ ⇒ this delay is of propagation delay.

This ~~is~~ delay is from the input side.