Deterministic Algorithms: Those algorithms where input is finite

Search space = ∞ (infinite no. of points)

So, we need to divide the search space into some fixed no. of points which the computer can handle.

Such problems are called NP (Non Polynomial) Hard Problems.

It is hard as the search space is infinite.

To generate fixed no. of points, we divide the domain in equispaced intervals which is determined by a formula — Binary genetic algorithm.

Calculate max of $f(x) = \sin x$ $\quad 0 \leq x \leq 3$.

$0 \leq x \leq 3$ can be divided in 4 parts or 6 parts.

But if we divide it in 2^n points then we can use binary representation

4 parts $\longrightarrow$ 00, 01, 10, 11

higher limit, lower limit

$$X_{actual(b)} = X_L + \frac{X_H - X_L}{2^l - 1} (Dec(B))$$

No. of points

Decimal value of binary string

We need to find where 00 lies in 0 to 3:

$$\therefore X_{actual(b)} = 0 + \frac{3-0}{4-1}(0) = 0$$

00 will lie at 0.

Maximise $f(x) = \frac{x^2}{1+x^2}$ $\quad 1 \leq x \leq 7$.

considering 2^6 points between 1 and 7, where does 110011 lie ?

$$X_{actual(6)} = 1 + \frac{7-1}{2^6-1}(51)$$

$$= 1 + \frac{6}{63} \times 51 = 1 + \frac{34}{7}$$

$$= 5.857$$

This is known as encoding of binary in decimal

Complexity count is no. of evaluations.

For eg. maximize $\sin x$ $\quad 0 \leq x \leq 3$

calculating points 0 1 2 3

→ 4+4 = 8 evaluations.

cal. their values .2 .3 .5 .6

(+4 comparison)

For finding the best point in these 4, we need 12 evaluations (need to compare).

If 2^{40} points : Total evaluation $= 3 \times 2^{40}$.

So, such problems are NP hard due to high complexity (2^n)

So, to solve such problems, we need to minimize the no. of comparisons (2^{40})

We can create all the points but we won't compare all of them — Randomized algorithms.

Called randomized because randomness is there.

Knowledge, Intelligence

Suppose max $f(x) = \sin x \quad 0 \le x \le 3$

Dividing in 16 points, from the graph we can conclude that maximum will be at peaks only.

$\therefore$ No. of points reduced $= 8$.

So, this is applying intelligence

No. of comparisons reduced

Such programs are called artificial intelligence

We knew the graph, it was knowledge.

Knowledge specific to a domain: Domain knowledge

Suppose if the graph was not known:

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$$f(x) < f(x') < f(x'')$$

GA $\rightarrow$ Darwin's principal of Natural Selection & Evaluation

$$\max f(x) = \frac{x}{1+x^2} \qquad 0 \le x \le 3$$

$$\downarrow$$

$$2^{10}$$

2^5 solutions $\leftarrow 0 \le x \le 3$

Binary genetic algorithm

First step is to initialize in GA.

$\therefore$ Initialization

Binary representation of any solution is called chromosome.

We will use a randomizer to generate 5 random values between (0 - 31). These 5 values must be distinct.

Suppose the initial 5 values are 1, 4, 7, 11, 21 — these are parents.

In binary, these are
00001
00100
00111
01001
10101

These must be such that all the values (search space) must be covered.

This was for single value problem.

(Selection, Crossover, Mutation)

We use selection to get better solutions.
We need to select good solutions and create copies of that, and remove worst solutions.

Convert every binary solution into actual solution and find its fitness.

00001. $lb = 0$, $ub = 3$.
$2^l - 1 = 32 - 1 = 31$.

$$x_{a(00001)} = lb + \frac{ub - lb}{2^l - 1}\,(\mathrm{Dec}(00001))$$

$$x_{a(00001)} = 0 + \frac{3-0}{31}(1) = \frac{3}{31}$$

$$x_{a(00100)} = 0 + \frac{3-0}{31}(4) = \frac{12}{31}$$

Fitness function $f(x) = \frac{x}{1+x^2}$.

$$f(x) = \frac{3/31}{1+(3/31)^2} = \frac{3/31}{1+9/31\times31} = \frac{31\times3}{(31^2+9)}$$

$$= \frac{93}{970}$$

Q. Volume of a cylinder is $\theta = \pi r^2 h$

Maximize $\theta = \pi r^2 h$

$0 \le r \le 3$, $1 \le h \le 4$

Let us consider 2^{10} solutions.

(Solution is a combination of r and h both)

We need to tell how many bits are for r and for h and the order also.

So, let r is of 4 bits and h is of 6 bits.

(These are identified by programmer.)

First 4 bits of r and last 6 for h.

∴ for 1011011000

1011 : $lb = 0$, $ub = 3$.

$$X_{a(1011)} = 0 + \frac{3-0}{2^4-1}(11) = \frac{33}{15} = 2.2$$

011000 : $lb = 1$, $ub = 4$.

$$X_{a(011000)} = 1 + \frac{4-1}{2^6-1}(24) = \frac{72}{63} + 1 = \frac{135}{63}$$

$= 2.14$

$f(x) = \frac{x}{1+x^2}$ $0 \le x \le 3$. 2^4 solutions

0100 $\rightarrow$ $X_{or(0100)} = 0 + \frac{3}{2^4 - 1}(4) = \frac{4}{5} = 0.8$

1000 $\rightarrow$ 1.6

1100 $\rightarrow$ 2.4

1111 $\rightarrow$ 3

Fitness values $= \frac{.8}{1+(.8)^2} = 0.4878$

$= \frac{1.6}{1+(1.6)^2} = 0.449$

$= \frac{2.4}{1+(2.4)^2} = .355$

$\frac{3}{1+3^2} = 0.3$

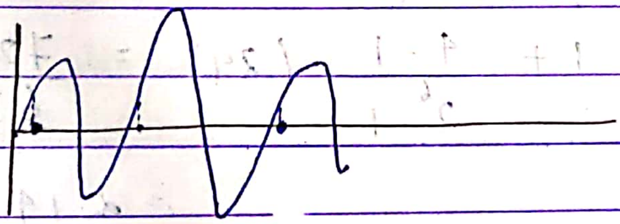
We can say that the best probability of getting best solution is around 0100.

Neighbourhood is in range (−1, +1) of any val

Searching in the neighbourhood is called exploitation. (where we have idea)

Exploration – randomly searching in the entire search space

Its not necessary that we get the solution always by exploitation (not guaranteed).



Here the fitness will be best of 1st point but that is local optimum, not global optimum

Here exploitation won't work, we need to do exploration

Selection operator : Binary tournament selection
Ranking based selection

Why are we selecting non-worst solutions?
Multi
It might be representing an unrepresented solution which may provide global optimum.

Now, we need to apply crossover and mutation. There is a probability associated with crossover always because we are mapping natural phenomena and it is never 100%.

Suppose probability = 0.7
$\therefore$ $4 \times 0.7 = 2.8 \Rightarrow 3$ (round off).
$\therefore$ We need to apply crossover 3 times.
For this, we again need a randomizer and pick any 2.

0100
0100 $\longrightarrow$ 0100
1100 1000
1000

3 2 1 0
0 1 0 0
1 1 1 0

$\therefore$ 0 1 0 1
1 1 1 0.

Select any position between 0 and 3 and make a cut and then ~~expb~~ append the values

We can see that it is exploitation (+1, -1 to values)

0 1 1 0 0 → 0 1 1 1
1 1 1 1 1 → 1 1 0 0

This can be said as exploitation.

0000 → 1st and 2 cut will be exploitation
1111 → 3rd cut will cause exploration.

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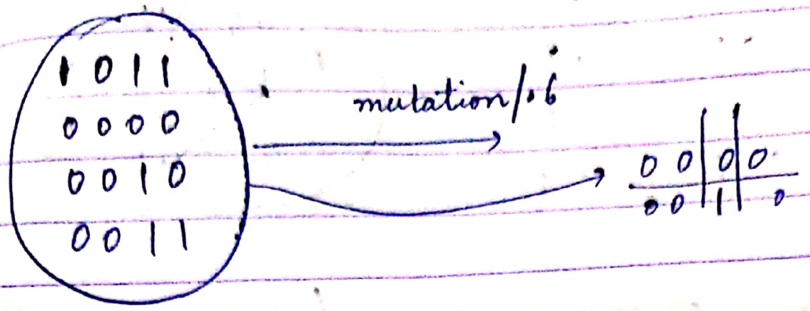
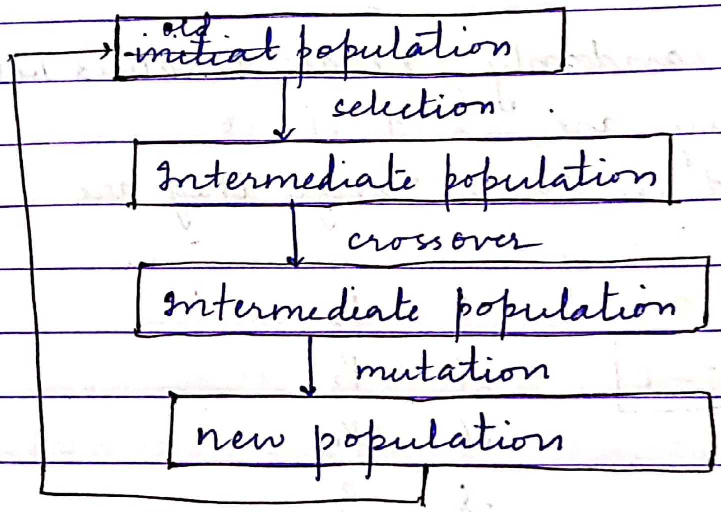


4×.6 = 2.4

Cutpoint is very far → exploration
near → exploitation

In genetic algorithm, we generate

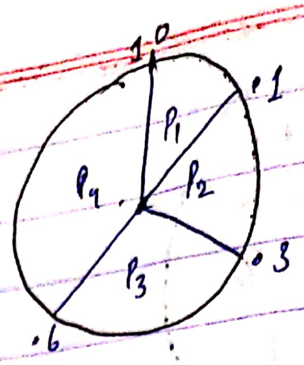


This process is iterated until we get a good solution or the no. which is fixed (given).

Roulette wheel selection.
Different from binary tournament selection.

We take out the fitness of each solution.

		fitness	$f_i / \sum_{i=0}^{3} f_i$
0	1010	1	.1
1	1011	2	.2
2	0000	3	.3
3	0010	4	.4



If the problem is of maximizing, (which is generally) we need to find summation of all fitness $\sum_{i=0}^{3} f_i = 10$
Thereafter, divide each fitness value by this. This is for normalizing each value between 0 to 1.
We make a fitness wheel which starts from 0 and ends at 1. We allot space based on fitness.

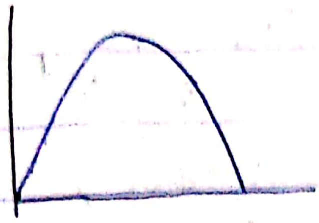
We need to randomly generate 4 values between 0 and 1
Suppose they are .2, .4, .6, .8
Now we find in whose range they are.

$r_1, r_2, r_3, r_4 \rightarrow \boxed{0-1}$

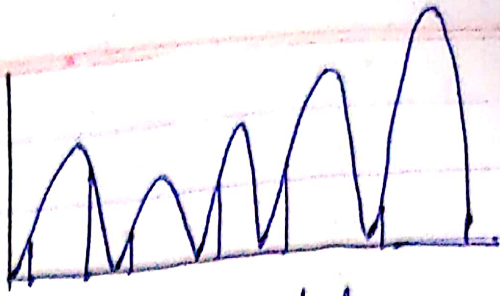
.2	P_2
.4	P_3
.6	P_4
.8	P_4

Thus, these are the selected solutions.
This is good because solutions which have high fitness have more chances.

Sharing fitness selection operator.
This method is used for solving multi modal problems.
Uni modal problem – Global optimum solution.
Multi modal



Unimodal.



Multimodal.

We will find how many solutions are representing the same region. Try to reduce the fitness of such solutions thereafter.

This is based on Euclidean distance. We fix what region will be considered close and far.

$\sigma_{share} = .2$

If $< .2 \Rightarrow$ close else far.

Sharing effect $sh(d_{ij}) = \begin{cases} 0 & d_{ij} \geq \sigma_{share} \\ 1 - \left(\dfrac{d_{ij}}{\sigma_{share}}\right)^{\alpha} \end{cases}$

α is a value which we have to choose based on the distribution of function. Generally = 1.

Advantage: We use this in solving multimodal problems because we need diversity.

Least count of a particular solution $^{n}C_1$.

1011 $\rightarrow$ 2.2 $\quad f(x)_{max} \quad 0 \leq x \leq 3$.

0000 $\rightarrow$ 0 $\quad \sigma_{share} = \cancel{2.5}$ 3.5

0011 $\rightarrow$ 0.6

1010 $\rightarrow$ 2

x_i		f_i
2.2	$\longrightarrow$	4
0	$\longrightarrow$	0
1.6	$\longrightarrow$	3
2	$\longrightarrow$	2

$sh(d_{11})$

$= 1 - \left(\frac{0}{\sigma_{share}}\right)'$

$= 1$

$\therefore sh(d_{11}) \rightarrow 1$

$sh(d_{12}) \rightarrow 0$

$sh(d_{13}) \rightarrow 0$

$sh(d_{14}) \rightarrow 0.6$

$\therefore$ Total effect $n_{C_1} = 1.6$

So, this solution has something in shared region

1st one is shared with 4.

$\therefore$ We divide by least count to reduce fitness

~~4/1.6~~ 4/1.6

$f(x,y) = x^2+y^2$ $\quad 0 \le x \le 3, \; 1 \le y \le 5$

$\sigma_{share} = 3.5$

x	y
1011	10111
0000	11111
0001	11000
1111	00000

1011 : $lb = 0$, $ub = 3$

$X_{1011} = 0 + \frac{3-0}{2^4-1}(11) = 2.2$

10111 : $lb = 1$, $ub = 5$

$X_{10111} = 1 + \frac{5-1}{2^5-1}(23) = 1 + \frac{4\times23}{31}$

$= 3.968$

$f_i = x^2+y^2 = \cancel{8^2+23^2} =$

$= (2.2)^2 + (3.968)^2 = 20.058$

$X_{0000} = 0 + \frac{3-0}{2^4-1}(0) = 0$, $X_{11111} = 1 + \frac{4}{31}(31) = 5.$

$f_i = 0^2+5^2 = 25$

$$X_{0001} = 0 + \frac{3-0}{15}(1) = 0.2, \quad X_{11000} = 1 + \frac{4}{31}(24) = 4.09$$

$$f_i = (0.2)^2 + (4.09)^2 = 16.8$$

$$X_{1111} = 0 + \frac{3}{15}(15) = 3, \quad X_{00000} = 1 + \frac{4}{31}(0) = 1.$$

$$f_i = 3^2 + 1^2 = 10.$$

x_i	y_i	f_i
2.2	3.97	20.5
0	5	25
0.2	4.09	16.8
3	1	10

$sh(d_{11}) = 0.$

$sh(d_{12}) = (2.2, 3.97), (0, 5)$

$$\sqrt{(2.2-0)^2 + (3.97-5)^2} = 2.43.$$

$\therefore d_{ij} = 2.43 < \sigma_{share} = 3.5$

$$sh(d_{12}) = 1 - \left(\frac{2.43}{3.5}\right)^1 = 0.3$$

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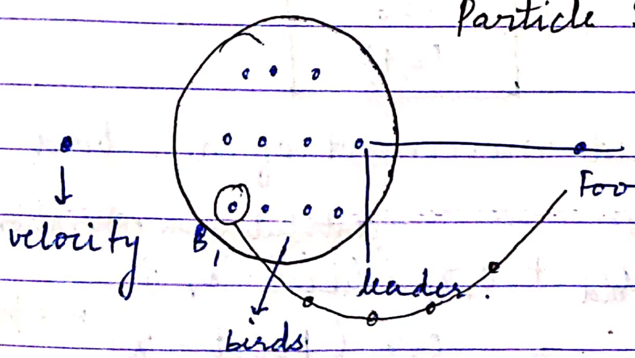
P.S.O $\longrightarrow$ Birds Flocking.

Particle Swarm Optimization.



Leader is the bird which has maximum information about food source (minimum distance of it).

Each bird will change its velocity based on two thin
Gbest. (used for best bird leader)
pbest.

- Bird will identify where is the leader. It will change its velocity in that direction.
- Best position of itself which it has occured during (It will keep history of its movement and always retain its best position) history of the flight. (Its own intelligence).

Thereafter, the updated velocity is added to position to get the current position.

velocity + position = current position.

Minimize

$f(x,y) = x^2 + y^2 \qquad 0 \le x \le 3, \; 1 \le y \le 4.$

Apply PSO and initial velocity

Initial positions of birds are assumed randomly by the programmer from the given search space.

x	v
(0,1)	(1,1)
(1,2)	(2,2)
(2,3)	(3,3)
(3,4)	(3,4)

(4)

Now, we have to decide movement of each bird

Formula of velocity

$$V_{new} = w \times V_{old} + \underbrace{c_1 r_1 (Gbest - x)}_{\text{contributing in exploration}} + \underbrace{c_2 r_2 (pbest - x)}_{\text{in exploitation}}$$

c_1, c_2 — constants; w = constant called inertia (=0.7)

r_1, r_2 — value between 0 and 1, random

Only positions are evaluated, velocities aren't.

Values are (0, 1) = 1
(1, 2) = 5.
(2, 3) = 13
(3, 4) = 25

Since it is a minimization problem, so (0, 1) has minimum value, thus it is the leader.

∴ (0, 1) is the current Gbest.

Other birds will adjust their position and try to achieve this.

All other birds will try to come in this area (0, 1). Thus, exploitation. They can't jump exactly on (0, 1).

(0,1)

$v_{new} = .7 \times [1,1] + 1\,([0,1] - [0,1])$

$v_{new} = [.7, .7]$

$x_{new} = x_{prev} + \cancel{x_{old}}\ v_{new}$

$= [0.7, 1.7]$

(1,2) $v_{new} = .7 \times [2,2] + 1\,([0,1] - [1,2])$

$= [1.4, 1.4] + [-1,-1] = [0.4, 0.4]$

$x_{new} = x_{prev} + v_{new} = [1.4, 2.4]$

(2,3) $v_{new} = .7 \times [3,3] + 1\,([0,1] - [2,3])$

$= [2.1, 2.1] + [-2,-2] = [0.1, 0.1]$

$x_{new} = [2.1, 3.1]$

(3,4) $v_{new} = .7 \times [3,4] + 1\,([0,1] - [3,4])$

$= [2.1, 2.8] + [-3,-3] = [-0.9, -0.2]$

$x_{new} = [3,4] - [-0.9, -0.2] = [3.9, 4.2]$

x_{new}

.7	2.7
1.4	2.4
2.1	3.1
3.93	4.2

.7	1.7
.4	1.4
.1	-1
-.9	-.2

v_{new}.

Since the velocity is $-0.9, -0.2$ which is out of bounds, we do clamping i.e. if less than boundary, we take the boundary value.

$\therefore v_{new}$ for -0.9 we will take 0.

If value > max, we take max value.

This is called boundary clamping.

After clamping velocity, we will calculate ~~dis~~ x.

$\therefore v_{new} = [-0.9, -0.2] \longrightarrow [0, 1]$

$\therefore x_{new} = x_{prev} + v_{new} = [3, 4] + [0, 1]$

$= [3, 5]$.

$\therefore 0 \leq x \leq 3 \longrightarrow 3$ lies.

$0 \leq y \leq 4 \longrightarrow 5$ is out of range.

$\therefore x_{new} = [3, 4]$.

Now, for each bird we identify pbest.

Fitness value for bird 1 for $(.7, 2) = 4.49$.

for $(0, 1) = 1$.

$\therefore$ pbest $= [0, 1]$.

Gbest = best of pbest

$= [0, 1]$.

pbest

0, 1
1, 2
2, 3
3, 4

x_{prev}

.7, 2
1.4, 3
2.1, 4
3, 4

v_{prev}

.7, 1
.4, 1
.1, 1
0, 1

$[.7, 2]$

$v_{new} = .7 \times [.7, 1] + 1([0, 1] - [.7, 2]) + 1([0, 1] - [.7, 2])$

$= [.49, .7] + [-.7, -1] + [-.7, -1]$

$= [-.91, -1.3] = [0, 1]$.

$x_{new} = x_{prev} + v_{new} = [.7, 2] + [0, 1] = [.7, 3]$

$[1.4, 3]$ $\quad v_{new} = .7 \times [0.4, 1] + 1([0,1] - [1.4, 3]) + 1([1,2] - [1.4, 3])$

$= [0.28, .7] + [-1.4, -2] + [-0.4, -1]$

$= [-1.52, -2.3] = [0, 1]$

$x_{new} = x_{prev} + v_{new} = [1.4, 3] + [0, 1]$

$= [1.4, 4]$

$[2.1, 4]$

$v_{new} = .7 \times [.1, 1] + 1([0,1] - [2.1, 4]) + 1([2,3] - [2.1, 4])$

$= [.07, .7] + [-2.1, -3] + [-0.1, -1]$

$= [-2.13, -3.3] = [0, 1]$

$x_{new} = [2.1, 4] + [0, 1] = [2.1, 5] = [2.1, 4]$

$[3, 4]$

$v_{new} = .7 \times [0, 1] + 1([0,1] - [3,4]) + 1([3,4] - [3,4])$

$= [0, .7] + [-3, -3] + [0, 0]$

$= [-3, -2.3] = [0, 1]$

$x_{new} = [3, 4] + [0, 1] = [3, 5] = [3, 4]$

x_{new}	v_{new}
(.7, 3)	(0, 1)
(1.4, 4)	(0, 1)
(2.1, 4)	(0, 1)
(3, 4)	(0, 1)

How fast it is capturing the global optimum solution : convergence rate.

Here, it is not rapid. This is because the velocity becomes same in second round, so we need to change the bounds, allow negative values also for velocity.
So, we can have different bounds than position for velocity

It can give local optimum, can be stuck there because it has greedy approach although the convergence rate is very fast.

Genetic → Binary

Q. What is population.

Q. How will you define the convergence rate of an algorithm.

Q. In what kind of problems we use sharing fitness approach.

Q. What will happen if in PSO we remove (pbest − x) from the velocity equation?

Q. What is roullete wheel selection operator in GA. State with an example.

Q. What factors operation/equation are used in GA for exploiting and exploring the search space.

Q. What will happen if we apply sharing fitness in PSO? How will we apply?

Q. What is chromose?

Q. What is genetic representation?

Scanned by CamScanner