BIBO → Bounded Input Bounded output → BI stability

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Causality:-

$$h(t) = 0 \quad \text{for} \quad t < 0$$

$$|x(t)| < M$$
$$|y(t)| < K$$

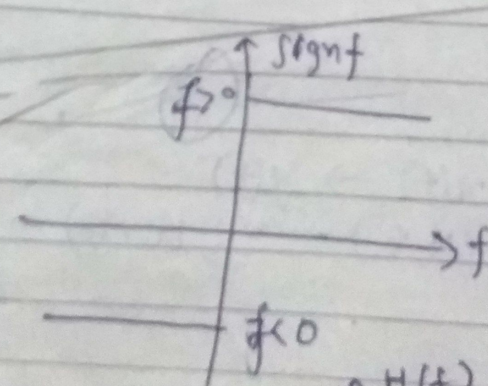
** A system is causal if it depends on the present and past system only at any poi

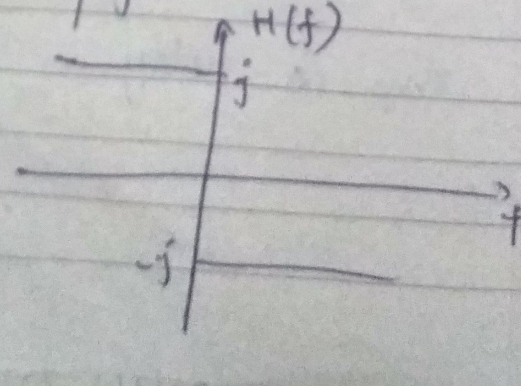
Hilbert Transform

Consider LTI filter,

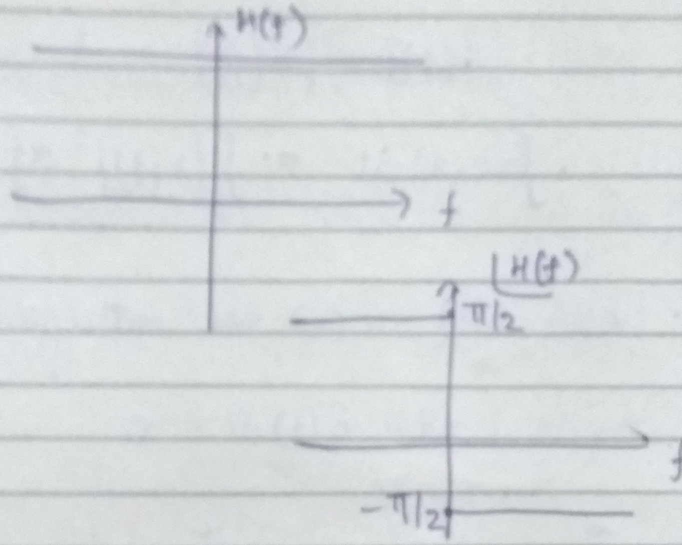
$$H(f) = -j \, \text{sgn} f$$

$$\text{sgn} f = \begin{cases} 1 & f > 0 \\ 0 & f = 0 \\ -1 & f < 0 \end{cases}$$ (signum function)





In hilbert transform, phase shift of all frequency components of the input or input signal by $-\pi/2$ for +ve frequency compo and $+\pi/2$ for -ve freq. components.



H.T.
Hilbert Trans is used for single side band modulation.

① $\frac{j}{\pi t} \xleftrightarrow{H.T.} \text{sgn} f$

Properties :-

1) $\hat{\hat{x}}(t) = -x(t)$ — $\hat{x}(t)$ = Hilbert transform

② ① $x(t) = \cos 2\pi f_0 t \Rightarrow \hat{x}(t) \Rightarrow \sin 2\pi f_0 t$ of $x(t)$.

② $x(t) = \sin 2\pi f_0 t \Rightarrow \hat{x}(t) = -\cos 2\pi f_0 t$

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③ $x(t) = e^{j2\pi f_0 t} \Rightarrow \hat{x}(t) = -j\,sgn(2\pi f_0)\, e^{j2\pi f_0 t}$

③ $\hat{X}(f) = (-j\,sgn f)\, X(f)$

$$|\hat{X}(f)| = |X(f)|$$

$$\int x^2(t)\,dt = \int |X(f)|^2\,df$$

④ $x(t)$ and $\hat{x}(t)$ are orthogonal.

$$\int_{-\infty}^{\infty} x(t)\cdot \hat{x}(t)\,dt = 0$$

$$\int_{-\infty}^{\infty} X(f)\,\hat{X}^*(f)\,df = \int_{-\infty}^{\infty} X(f)\left(j\,sgn f\; X^*(f)\right) df$$

$$= \int_{-\infty}^{\infty} \underbrace{(j\,sgn\,f)}_{odd}\, \underbrace{|X(f)|^2}_{even}\,df \quad [\because z\bar{z} = |z|^2]$$

$$= 0$$

⑤ Let $c(t)$ and $m(t)$ have none overlap spectrum

$c(t)$ = carrier (high frequency)
$m(t)$ = message (low frequency)

$$\widehat{m(t)\cdot c(t)} = m(t)\cdot \hat{c}(t)$$

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$\widehat{m(t)\cdot\cos\omega_0 t} = m(t)\cdot\sin\omega_0 t$

$\widehat{m(t)\cdot\sin\omega_0 t} = -m(t)\cdot\cos\omega_0 t$

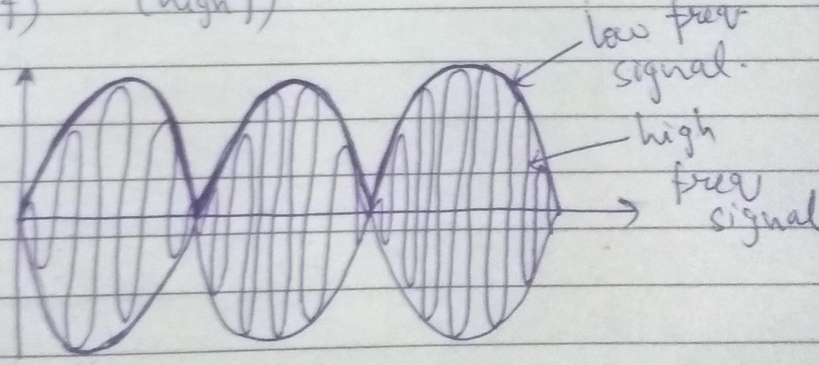
Applications of hilbert transform

Let $x(t)$ = real valued function (signal)

$x_p(t) = x(t) + j\cdot\hat{x}(t)$

$x_p(t)$ = complex valued function

$\Rightarrow$ $\underset{(\text{low } f)}{m(t)}\cdot\underset{(\text{high } f)}{\cos(\omega_0 t+\theta)}$



$\{m(t)\cos(\omega_0 t+\theta)\} = m(t)\cos\omega_0 t\cos\theta - m(t)\sin\omega_0 t\sin\theta$

$mag = \sqrt{m^2(t)\cos^2\theta + m^2(t)\sin^2\theta}$

$= |m(t)|$ = envelope of the signal

$x_p(t) = x(t) + j\,\hat{x}(t)$

$|x_p(t)|$ envelop are same

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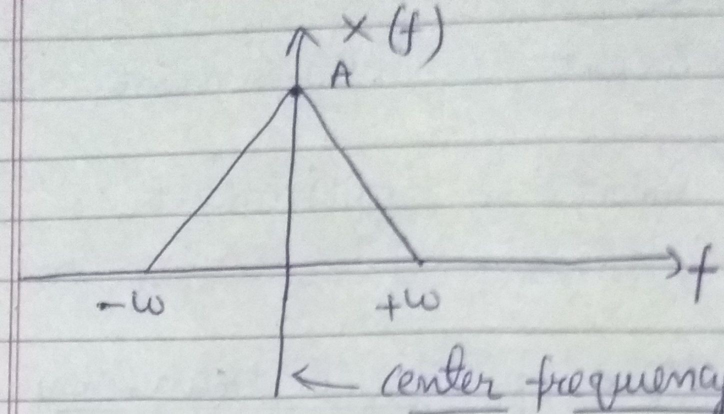
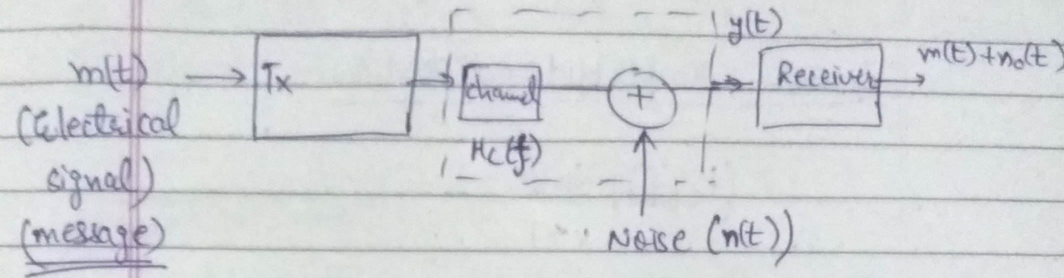


(not necessarily at the center).

→ <u>Complex envelop representation of band pass signal</u>

Analog Signal transmission (Tx)

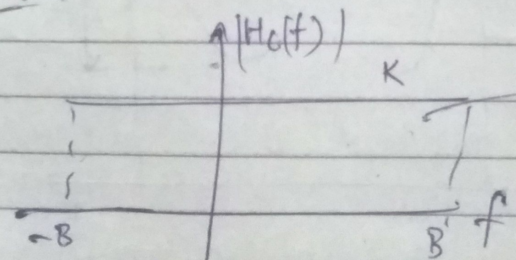


Ideal filter → noise is ~0.
distortionless transmission.

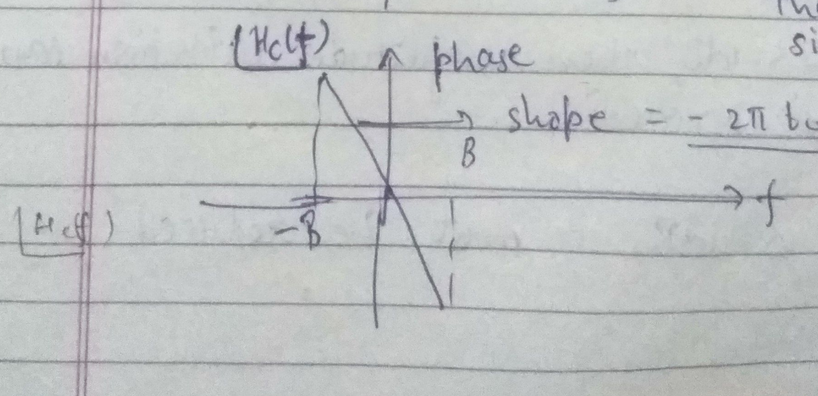
① $y(t) = K x(t-t_0)$ (propagation delay).
$y(f) = K \cdot e^{-2j\pi f t} X(f)$

Ideal channel

$H_c(f) = K e^{-j2\pi f_0 t_0}$



If within |B| amplitude remains constant, ~~[illegible]~~ then that is ideal signal.

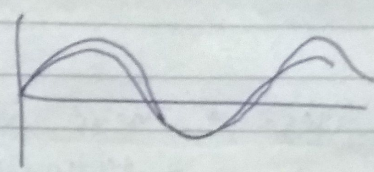


slope = $-2\pi t_0$

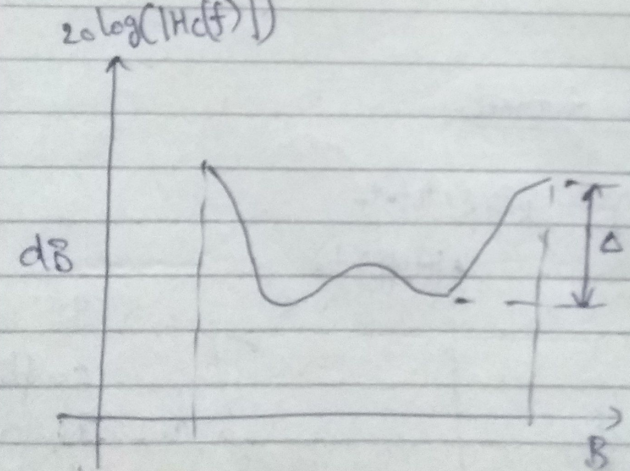
If $K<1$, attenuated signal

Linear Amplitude distortion

$$|H_c(f)| \neq K.$$
$$K \leq 1$$



If ~~the~~ all the amplitude of the signal is not multiplied with the scalar multiple K, then there is distortion.



⇒ $\Delta <= 1dB$, then amplitude distortion can be ignored.

→ if $\Delta > 1dB$, it must be reduced.

Phase distortion (delayed distortion)

~~It is~~ occurs becauz of non-linear ~~part~~ variation of phase.

$$\underline{|H_c(f)} = -2\pi f t_0 \pm m\pi$$

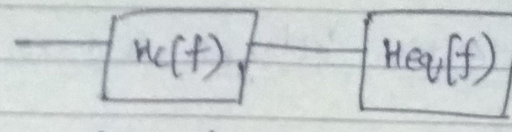
If $\underline{|H_c(f)}$ ~~is~~ within the band then there is no phase distortion otherwise there is distortion.

all frequency components are not delayed by same factor.

⇒ Delayed distortion are not important in speech transmission ~~becau~~ becaz ~~or~~ ears are insensitive to delayed distortion.

⇒ Data transmission is highly sensitive to delayed distortion.

To reduce the distortion we added the equalizer at the receiver end and its ~~~~ function ~~[illegible]~~ is exactly is equal to inverse of the message signal.



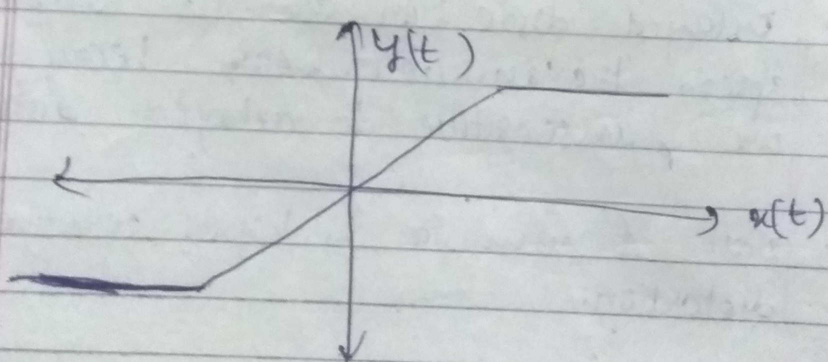
channel equalizer

$$H_{eq}(f) = \frac{K e^{-j2\pi f_0 t_d}}{H_c(f)}, \quad |f| < B.$$

<u>other type of distortions:-</u>

① <u>Non-linear distortions :-</u>

Non linear distortion arises due to non-linear transfer characteristics of amplifiers, mixers and other electronic components.



$$y(t) = A\,x(t)$$

$$y(t) = A_1 \cdot x(t) + A_2 x^2(t) + A_3 x^3(t) + \ldots$$

$|f_1 \pm f_2| < B$

$$y(t) = a_1 x(t) + a_2 x^2(t)$$

$x(t) = f_1$

$y(t) = 2f_1$ → due to this non linear behaviour occurs

$x(t) = f_1\ f_2$, $y(t) = f_1, (f_1 \pm f_2)$

$= f_2, 2f_2$

Linear Modulation:-

① Amplitude Modulation.
② Frequency Modulation.
③ Phase Modulation.

Advantages of modulation

(a) Ease of radiation.
(b) Multiplexing (multiple message signals).
(c) ~~Conveinien~~ Conveninent ~~message~~ signal processing.
(d) Noise reduction is better.
(e)

message signal: $m(t)$ [Low pass signal]

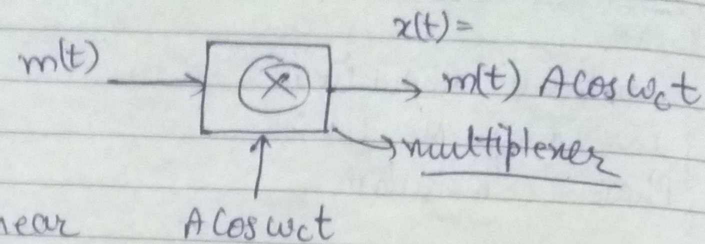
$$m(t) = a_m \cos 2\pi t$$

Carrier signal:- $A \cos \omega_c t$

ω_c = carrier frequency

$m(t)$ is low freq. signal. (upto few Megaherts)
$c(t)$ is high freq signal (~~tt~~ from megahertz to terahertz).

Frequency Translation



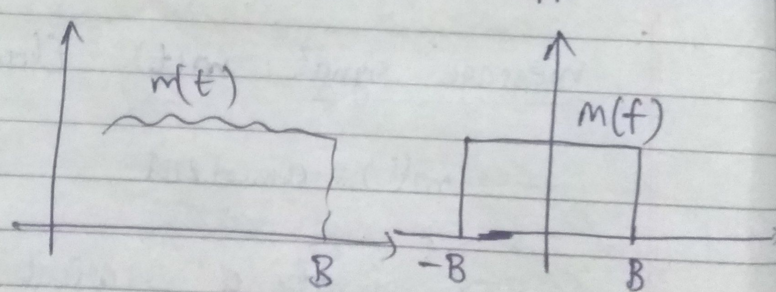
Here
A is the linear fu^n of the m(t)

$$X(f) = \frac{A}{2}\left[M(f+f_c) + M(f-f_c)\right]$$

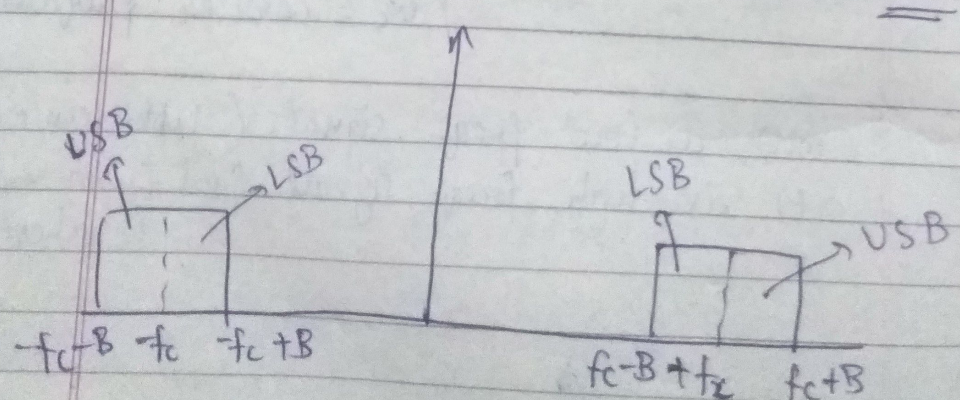
~~DSS~~ DSB-SC:-

① Amplitude modulation:-

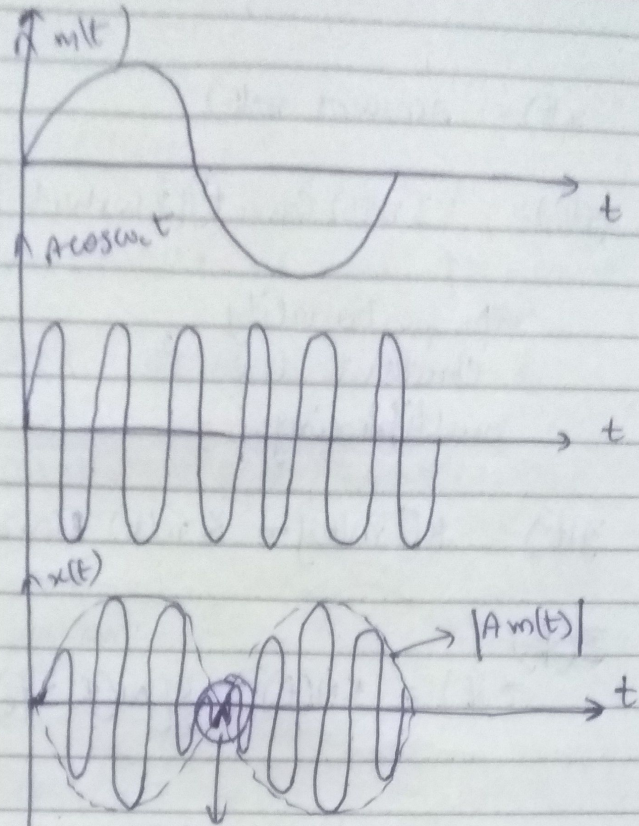
(a) DSB-SC :- Double Side Band Suppressed Carri



Bandwidth = B.

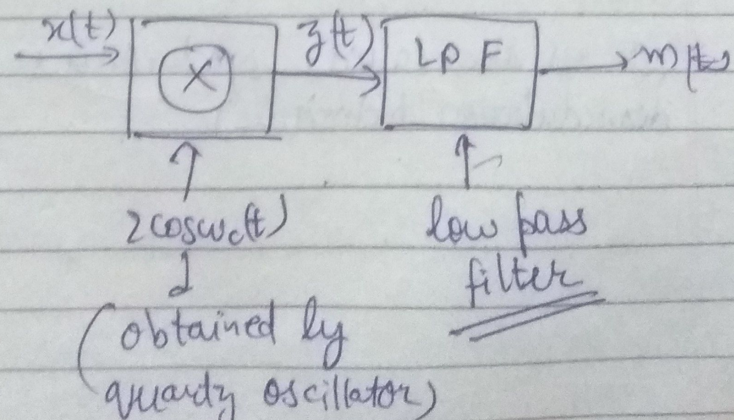


The frequency of carrier fc is suppresed in t
that's why DSB-SC.



This shows the
180° phase shift
due to negative amplitude.

At the ~~de~~ receiver end, we use again multiplexer ~~to~~



$x(t) = A\cos\omega_c t \, m(t)$

$z(t) = K[m(t)\cos\omega_c t][2\cos\omega_c t]$

$\uparrow$

proportionality constant due to multiplexing.

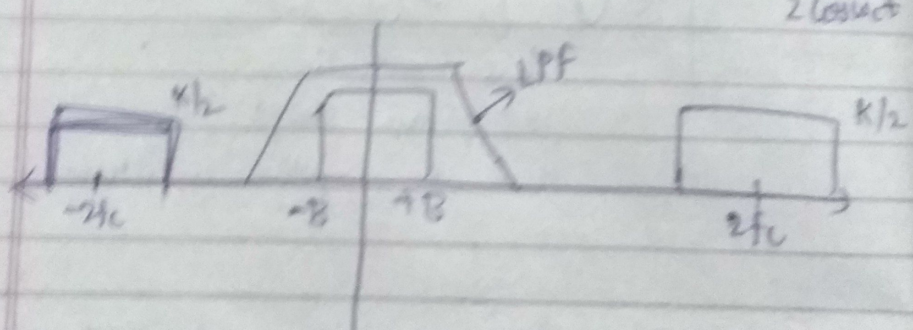
$z(t) = K[m(t)] + K\,m(t)\cos 2\omega_c t$

$\cancel{z(f)}$

$Z(f) = KM(f) + \frac{K}{2}[M(f-2f_c) + M(f+2f_c)]$

$\downarrow$

$2\cos\omega_c t$



Synchronisation is needed in DSB-SC

(It is also called synchronous modulation and demodulation technique).

~~Cho~~

Coherent Effect

When carrier is not of same frequency

expected carrier = $2\cos(\omega_c t)$

if carrier = $2\cos((\omega_c + \delta\omega)t + \theta)$

(i) $\delta\omega = 0$

carrier = $2\cos(\omega_c t + \theta)$

(a) if $\theta = +\frac{\pi}{2}$

carrier = $-2\sin\omega_c t$

(b) if $\theta = 0$

carrier = $2\cos\omega_c t$

(ii) $\delta\omega \neq 0$ and $\theta = 0$

carrier = $2\cos(\omega_c t + \delta\omega t)$

warbling effect occurs → demodulated signal is going to be distorted.

~~Ch~~

Synchronous detection / Coherent detection

At the detector side when same frequency carrier is used at

$c(t) = A_c \cos(2\pi f_c t)$ — ①

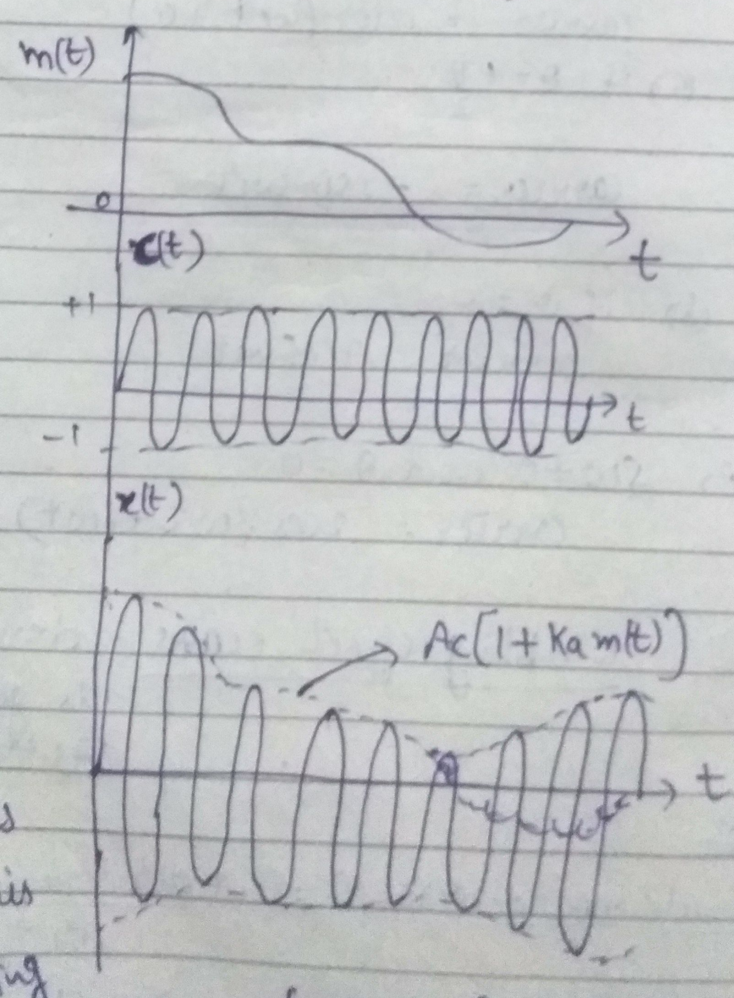
A_c = amp.

f_c = freq.

$x(t) = A_c[1 + K_a m(t)] \cos(2\pi f_c t)$
(DSB-AM)

In $x(t)_{\text{DSB-SC}}$ K_a is not there.

K_a = amplitude sensitivity factor
unit of K_a = $(\text{voltage})^{-1}$.



$\boxed{|K_a m(t)| < 1}$

due to this
zero crossing is
not occuring

~~fcc~~ → $f_c \gg B$ (bandwidth of the signal)

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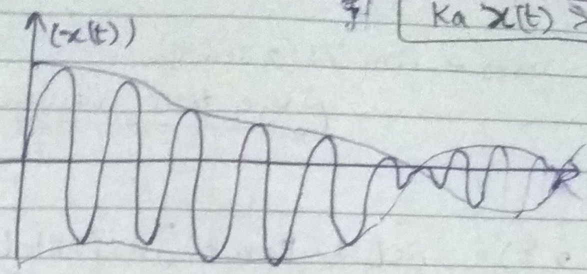
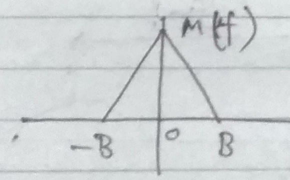
If $|K_a\, x(t)| > 1$

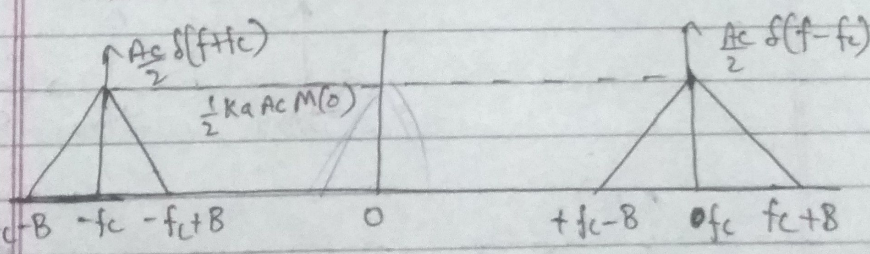


F.T. of $x(t)$

$$X(f)_{DSBAM} = \frac{A_c}{2}\left[\delta(f-f_c) + \delta(f+f_c)\right] + \frac{K_a A_c}{2}\left[M(f-f_c) + M(f+f_c)\right]$$



<u>representation of X(f)</u>



Here f_c is present and not suppressed, hence it is DSB-AM.

B_T = bandwith of transmitter signal $= 2B$

$B_M = B$ (message bandwidth)

$m(t) = A_m \cos(2\pi f_m t)$

(Single tone message signal)

$$x(t) = A_c[1 + K_a m(t)] \cos(2\pi f_c t)$$

DSB-AM

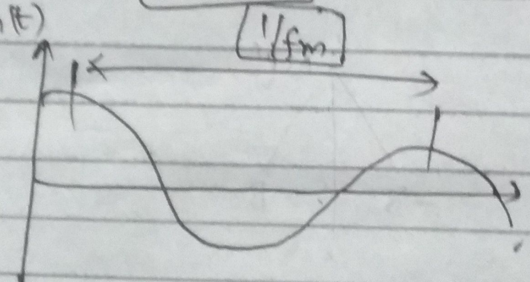
$$= A_c[1 + \underbrace{K_a A_m}_{\mu} \cos(2\pi f_m t)] \cos(2\pi f_c t)$$

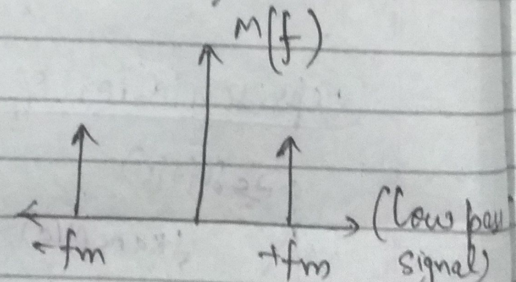
$\mu = K_a A_m$ = modulation factor

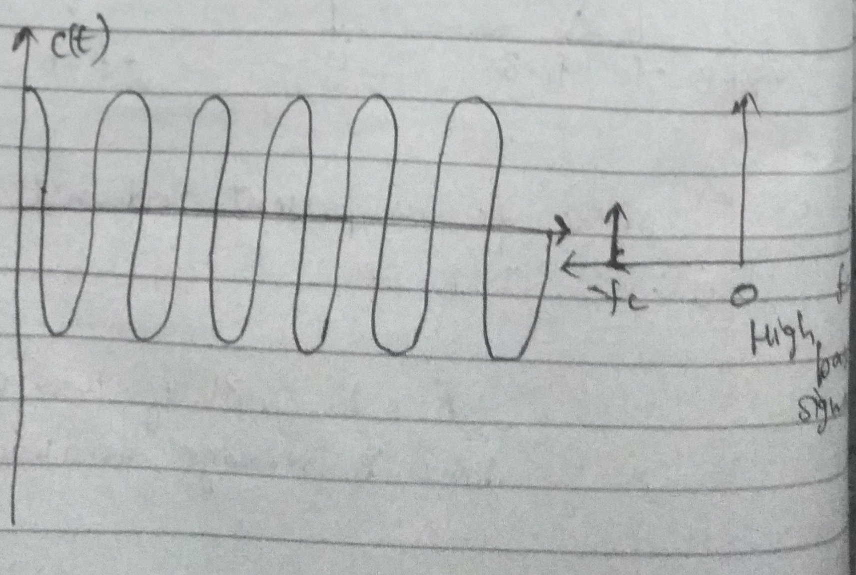
** to avoid the distortion

$\mu < 1$

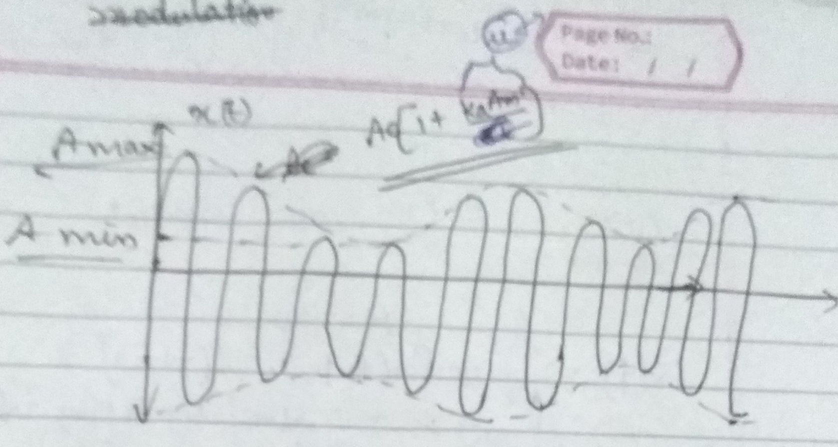
** $\boxed{K_a A_m < 1}$

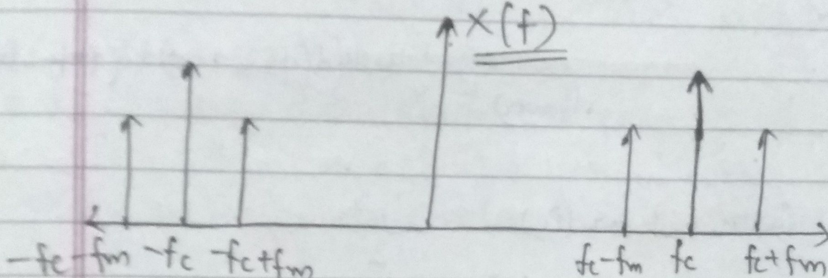






~~modulation~~





$$\frac{A_{max}}{A_{min}} = \frac{A_c[1+\mu]}{A_c[1-\mu]}$$

$$\boxed{\mu = \frac{A_{max}-A_{min}}{A_{max}+A_{min}} \quad \text{(modulation factor)}}$$

modulating signal = message signal.
modulated signal = x(t).

P_C = Power of carrier = $\frac{1}{2} A_c^2$

P_{USB} = Power of upper side band = $\frac{1}{8} A_c^2 \mu^2$

P_{LSB} = Power of lower side band = $\frac{1}{8} A_c^2 \mu^2$

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$\boxed{m(t) = A_m \cos 2\pi f_m t}$

$$\underset{DSB-AM}{x(t)} = A_c[1 + K_a m(t)] \cos(2\pi f_c t)$$

$$X(f) = \underbrace{\frac{1}{2} A_c [\delta(f-f_c) + \delta(f+f_c)]}_{\text{(carrier)}}$$

(upper) $+ \frac{1}{4} \mu A_c [\delta(f - f_c - f_m) + \delta(f + f_c + f_m)]$

(lower) $+ \frac{1}{4} \mu A_c [\delta(f - f_c + f_m) + \delta(f + f_c - f_m)]$

$\cancel{*}$ $m(f-f_c) + m(f+f_c)$

⇒ Carrier Power (P_c) $= \frac{A_c^2}{2}$

$P = VI$

$P = \frac{V^2}{R} \rightarrow R = 1\Omega$

⇒ Upper side Band Power (P_{USB}) $= \frac{1}{8} \mu^2 A_c^2$

⇒ Lower Side Band Power (P_{LSB}) $= \frac{1}{8} \mu^2 A_c^2$

$$P_t = \frac{1}{2} A_c^2 + \frac{1}{8} \mu^2 A_c^2 + \frac{1}{8} \mu^2 A_c^2$$

$$P_t = \frac{A_c^2}{2}\left[1 + \frac{2}{4}\mu^2\right] = \frac{A_c^2}{2}\left[1 + \frac{\mu^2}{2}\right]$$

$$\boxed{P_t = P_c\left[1 + \frac{\mu^2}{2}\right]}$$

$$\boxed{\frac{P_t}{P_c} = \frac{2 + \mu^2}{2}}$$

P_t = total transmitted power.

$P_{USB}+P_{LSB}$

$$\frac{P_t}{P_{SB}} = \frac{\frac{A_c^2}{2}\left[1+\frac{\mu^2}{2}\right]}{\frac{1}{4}\mu^2 A_c^2} \Rightarrow \frac{\cancel{4}^{2}(2+\mu^2)}{\cancel{2}\,2\mu^2}$$

$$\frac{P_t}{P_{SB}} = \frac{2(2+\mu^2)}{\mu^2} \neq \cancel{\frac{2}{\mu^2}}$$

$$\eta = \boxed{\frac{P_{SB}}{P_t} = \frac{\mu^2}{2+\mu^2}}$$

$P_{SB} = LSB + USB$.

→ If $\mu = 1 \Rightarrow \frac{P_{SB}}{P_t} = \frac{1}{3} = 33\%$ utilized by P_{SB}

→ 67% by P_C

** μ = modulation index

when $\mu = 1$ perfect modulation.

** to avoid envelope distortion ($\mu \leq 1$)

$$\frac{P_{AM}}{P_c} = \frac{P_t}{P_c} = \left(1+\frac{\mu^2}{2}\right) = \frac{(I_t^2)R}{(I_c^2)R} = \left(\frac{I_t}{I_c}\right)^2$$

$$\Rightarrow \boxed{I_t = I_c\sqrt{1+\frac{\mu^2}{2}}}$$

To reduce this loss by P_c, we suppressed the carrier and used DSB-SC.

**

If there are multiple frequency components, then:-

$$\boxed{\mu_t = \sqrt{\mu_1^2+\mu_2^2+\mu_3^2+\ldots}}$$