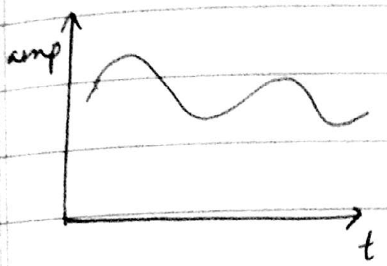
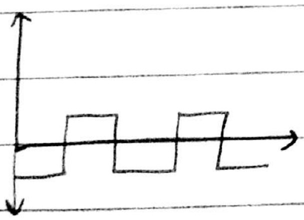
2.1.18

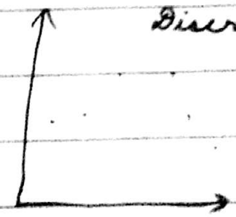
SIGNALS

Analog signal - Continuous in time and continuous in amplitude.



Digital signal - ~~Continuous~~ ~~Digital~~ Discrete in time and discrete in amplitude





Analog $\longrightarrow$ ~~digital~~ Discrete Sampling

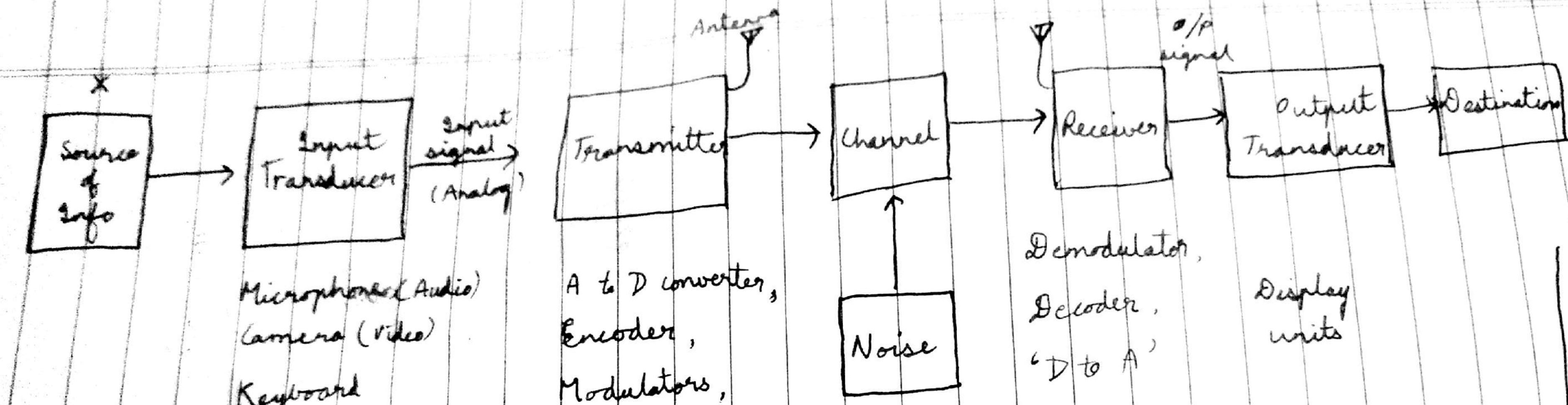
~~Digital~~ Discrete $\longrightarrow$ ~~Analog~~ Digital Quantisation

Analog - real time signal, low frequency more noise

Analog signals are converted to digital signals for processing.

1·18

COMMUNICATION SYSTEMS



Microphones (Audio)
Camera (Video)
Keyboard
(Text)

A to D converter,
Encoder,
Modulators,
Antenna

Demodulator,
Decoder,
'D to A'

Display
units

Audio signal 1D
Video " 2D
2D signal is converted to 1D before transmission. Video is broken down into images (frames)

ATTENUATION - Reduction in power of the signal.

DISTORTION - Change in wave form of the signal.

Satellite - Amplifies signal, filters noise.

$$\text{Transmitter} \xrightarrow{2x + \Delta x} \text{Channel} \xrightarrow[2x]{2.5x + \Delta x} \text{Receiver}$$

Noise is added | (Noise is filtered in satellite communication) | Noise is added

Internal noise & external noise

↓ Internal noise: Electronic components, diffusion, random motion of carriers

↓ External noise:
- Interference, radiation from mobile ignition of automobiles
- Can be eliminated completely
- Greater effect (Δx)

$I \propto f^2 A^2$ (Intensity)

Base band signal - low frequency signals

(few Hz to kHz) Audio

(MHz) Video

(100 MHz) FM radio

In transmitters, carrier signals have a high frequency. This process is known as modulation.

Carrier signals : GHz

Modulated signal → Base Band signal + Carrier signal

Low Pass Filter (LPF) should be used after removing noise.

Remove noise $\xrightarrow{\text{then}}$ Amplify signal

8.1.18

Freq. Range	Wavelength	Name	Tr. media	Application
3k - 30kHz	100 - 10km	VLF (Very low freq)	Wire line (copper wires)	Telephony, telegraph
30 - 300kHz	10 - 1km	LF		Aeronautical appl
300 - 3 MHz	1km - 100m	MF	Ground wave	AM broadcast
3M - 30MHz	100 - 10m	HF	Sky wave	CB radio (walkie-talkie)

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30 - 300 MHz	10 - 1 m	VHF	Co Axial	VHF, TV, FM, Mobile
300M - 3G	1m - 10 cm	UHF		
3G - 30G	10 cm - 10 mm	SHF	Wave guide	TV, SAT, mobile, LAN
10^{14} 10^{15}	$\sim 10^{-6}$ m	IR, VIS, UV	Optical fibre	Broadband

Band width - Range of frequency transmitted without any loss.

Freq. Band	Carrier - Freq	B.W.
LF	100 kHz	~ 2 kHz
HF	5 MHz	100 kHz
VHF	100 MHz	5 MHz
Micro wave	5 GHz	100 MHz
Optical	5×10^{14} Hz	10 G - 10 THz

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Tr. Medium	Typical Freq	Power (dB/km) (Loss)
Twisted Wire	1 kHz - 100 kHz	0.05 - 3
Coaxial Cable	100 kHz - 3 MHz	1 - 4
Wave guide	10 GHz	1.5
Optical fibre	$10^{14} - 10^{15}$	< 0.5

12.1

NEED OF MODULATION

- Height of antenna $\propto$ ~~Freq~~ Wavelength $\propto \dfrac{1}{\text{Frequency of the signal}}$

Modulation is used to reduce the height of antenna.

- To Reduce noise and distortion.

Signal to noise ratio (SNR) : $\dfrac{\text{Power/strength of signal}}{\text{Noise of signal}}$

It should be high for an efficient communication system.

Mostly, noise is added in the channel and sometimes at the transmitter end.

As the distance bet the ends of the comm. system increases, SNR decreases.

Transreceiver - Antenna ~~few~~ for transmitting as well as receiving signals.

6.1.18

SIGNALS

Signal is defined as a set of information.

Signal
- Deterministic - can be represented mathematically
- Random - ~~up~~ unpredictable, uncertain
  eg human speech

Deterministic signal

$$A\cos(\omega_0 t + \theta) = R_e\{Ae^{j(\omega_0 t + \theta)}\}$$

$$= \frac{1}{2}Ae^{j(\omega_0 t + \theta)} + \frac{1}{2}Ae^{-j(\omega_0 t + \theta)}$$

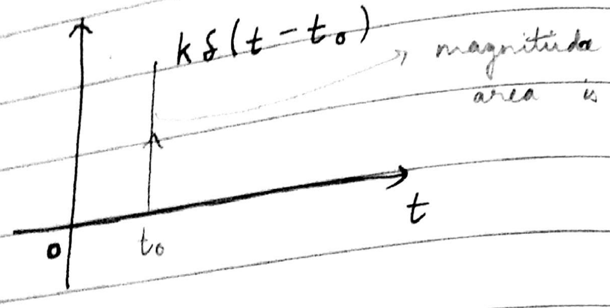
Ex. $x(t) = 2\sin\left(10\pi t - \frac{\pi}{6}\right)$

$$= e^{j\left(10\pi t - \frac{2\pi}{3}\right)} + e^{-j(10\pi t - 2\pi/3)}$$

Impulse function

$$\int_{-\infty}^{\infty} \delta(t)\,dt = 1$$

$$\delta(t) = 0 \quad , t \neq 0$$



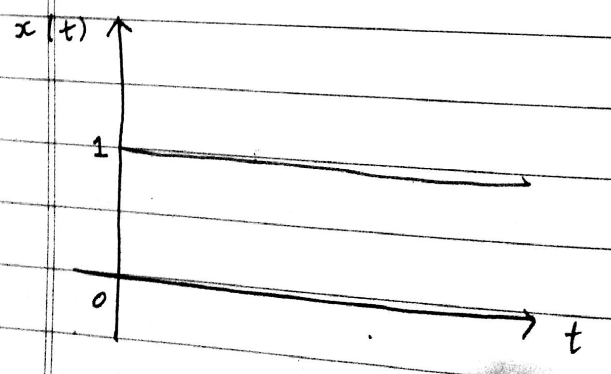
1. Shifting property $\int_{-\infty}^{\infty} x(t)\,\delta(t-t_0)\,dt = x(t_0)$

Proof also in course

2. Scaling property $\delta(at) = \frac{1}{|a|}\delta(t)$

3. $x(t)\,\delta(t-t_0) = x(t_0)\,\delta(t-t_0)$

Unit step function



~~u(t~~

$$u(t) = 1 \quad , t > 0$$
$$= 0 \quad , t < 0$$
$$= \int_{-\infty}^{t} \delta(x)\,dx$$

Fourier series

If $x(t + T_0) = x(t)$ $\forall\ t$

$$x(t) = \sum_{n=-\infty}^{\infty} X_n e^{j2\pi n f t} \qquad \omega_0 = \frac{2\pi}{T_0}$$

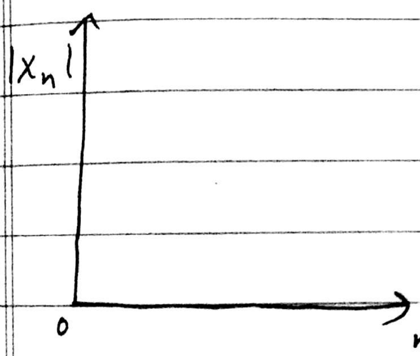
X_n = Coefficient of fourier series

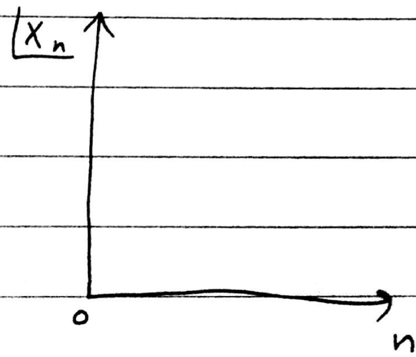
$$X_n = \frac{1}{T_0} \int_{t_0}^{t_0 + T_0} x(t) e^{-jn\omega_0 t}\, dt$$

(Applied for periodic signals)

Magnitude spectrum $|X_n|$

Phase " $\angle X_n = \theta_n$





Properties

1. For real $x(t)$; $|X_n| = |X_{-n}|$
   $\angle X_n = -\angle X_{-n}$

Mag. sp. is symmetric, phase sp. is asymmetric.

2. $X_n^* = X_{-n}$ (complex conjugate of X_n) defined for real signal

Energy signal

$x(t)$

$$E_x = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

$$x(t) = \begin{cases} e^{-t} & , t \geq 0 \\ 0 & , t < 0 \end{cases}$$

$$= e^{-t} u(t)$$

$$E_x = \int_{0}^{\infty} |e^{-t}|^2 dt$$

$$= \left[\frac{e^{-2t}}{-2}\right]_0^{\infty} = \frac{1}{2}$$

$$\boxed{E_x = 1/2}$$

$x(t)$ is the energy signal.

Power signal

$$P_x = \lim_{T \to \infty} \frac{1}{T} \underbrace{\int_{-T/2}^{T/2} |x(t)|^2 dt}_{\text{energy of the signal}}$$

$$= \lim_{T \to \infty} \frac{\text{energy in window of size } T}{T}$$

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If $P_x < \infty$ (Power is finite), then the signal is a power signal.

→ $$P_x = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 \, dt$$

$$= \lim_{T \to \infty} \frac{E_x}{T} = 0$$ ($x(t)$ is energy signal)

→ $x(t)$ is power signal :

$$E = P_x T$$
$$= \infty$$

Window - duration or size

PARSEVAL'S THEOREM

$$P = \frac{1}{T} \int |x(t)|^2 \, dt$$

$$= \sum_{n=-\infty}^{\infty} |x_n|^2$$

FOURIER TRANSFORM

Fourier series is defined for ~~infinite~~ periodic signals

" transform " " " any type of signal.

~~x(t)~~ is the fourier transform of $x(t)$ is

$$x(f) = \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi f t}\, dt$$

Some analysis is easily done in time domain & some analysis in frequency domain. 15.1

Inverse fourier transform :

$$x(t) = \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi f t}\, df$$

$\underline{|x(f)|} =$ $\Big\}$ $x(f) = |x(f)|\, e^{j\theta(f)}$

Properties

1. For real signal $x(t)$, $X^*(f) = X(-f)$

2. $\int_{-\infty}^{\infty} |x(t)|\, dt = \int_{-\infty}^{\infty} |x(f)|\, df$. Parseval's theorem

3. Convulation property $x(t) = x_1(t) \circledast x_2(t)$

$$= \int_{-\infty}^{\infty} x_1(\lambda)\, x_2(t-\lambda)\, d\lambda$$

$x_1(t) \circledast x_2(t) \xleftrightarrow{F} X_1(f)\, X_2(f)$

Time delay :- $x(t) \longleftrightarrow x(f)$

$x(t-t_0) \longleftrightarrow x(f)e^{-j2\pi f t}$

15.1.18 PROOFS

1. Time delay property

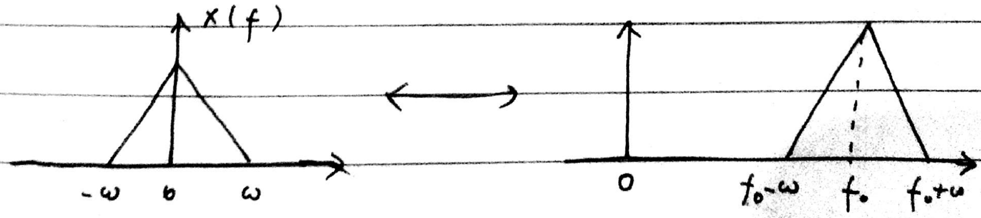
$$\delta(x) = \begin{cases} 1, & x=0 \\ 0, & x \neq 0 \end{cases}$$ } Impulse function

$$\delta(t-t_0) = \begin{cases} 1, & t = t_0 \\ 0, & t \neq t_0 \end{cases}$$

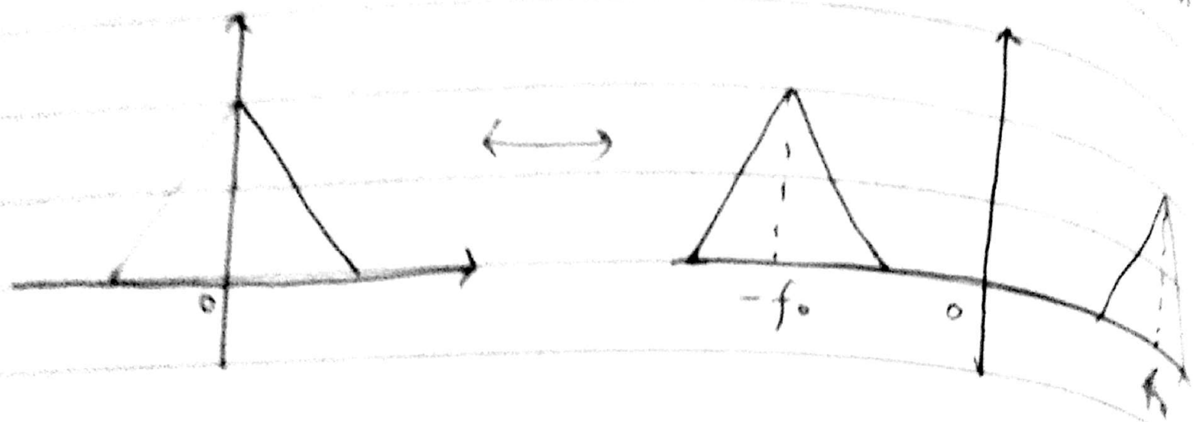
$$\underbrace{\delta(t)\,\delta(t-t_0)}_{t=t_0 \qquad t \neq t_0} = x(t)\,\delta(t-t_0)$$

FREQUENCY TRANSLATION

- $x(t) \cdot e^{j2\pi f_0 t} \longleftrightarrow X(f-f_0)$



- $x(t)\cos 2\pi f_0 t \longleftrightarrow \frac{1}{2}X(f-f_0)+\frac{1}{2}X(f+f_0)$



DIFFERENTIATION PROPERTY

- $\frac{d^n x(t)}{dt^n} \longleftrightarrow (j2\pi f)^n X(f)$

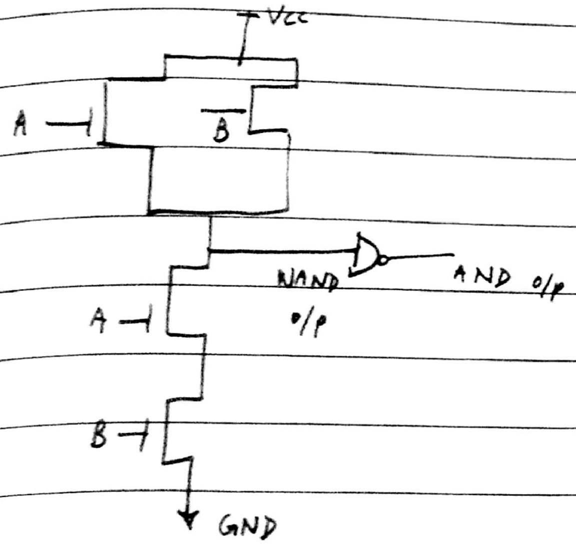
$$\int_{-\infty}^{t} x(\sigma)\, d\sigma \longleftrightarrow \frac{1}{j2\pi f} X(f) + kX(0)$$

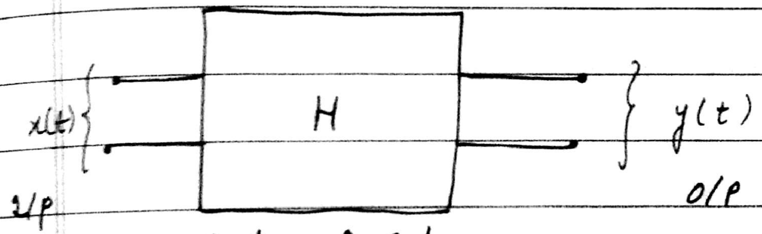
Date
Page

15·1·18

SYSTEMS

System - Interconnection of active and passive components to fulfill an application (Electronic and mechanical components).





System Operator

A system is designed for a specific application   System modelling

$$y(t) = H[x(t)] \quad , \; H \rightarrow \text{System operator}$$

LTI system - Linear time invariant
Linear time variant systems.

LTI system follows superposition property.

$$x_1(t) \longrightarrow y_1(t)$$

$x_2(t) \longrightarrow y_2(t)$

→ Linearity property :-

$\alpha_1 x_1(t) + \alpha_2 x_2(t) \longrightarrow \alpha_1 y_1(t) + \alpha_2 y_2(t)$

LTI system (superposition) →

→ Time invariant property

$x(t) \rightarrow y(t)$

$x(t-t_0) \longrightarrow y(t-t_0)$

Delay of the system is assumed to be zero.

Time taken by the system to process & propagate the ~~sti~~ input signal is known as propagation delay time.

t_0 - delay from input side

$$\text{Delay} \propto \frac{1}{\text{Speed of the system}}$$

→ CAUSALITY

$h(t) = 0$ for $t < 0$ (LTI + Causal system)

→ STABILITY

→ (LTI + stable system) follows ~~Big~~ BIBO property (Bounded input bounded output)

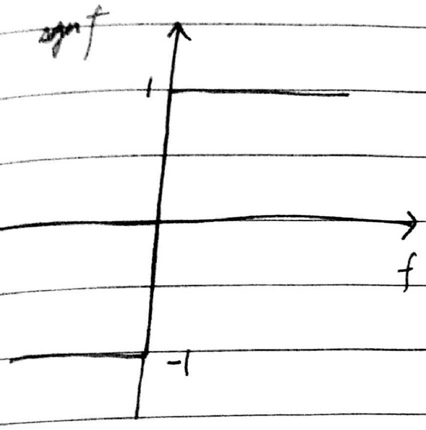
$|x(t)| < M$ (Finite I/P & O/P)

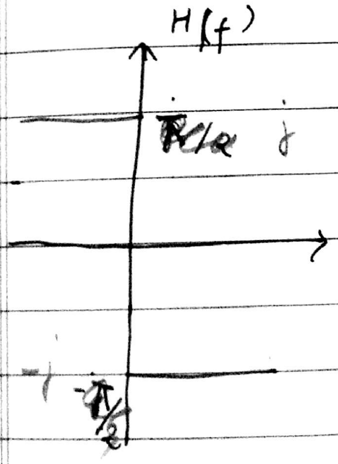
$|y(t)| < K$

→ Hilbert Transform

Single side band modulation technique
Consider LTI filter and its Hilbert transform, $H(f) = j \, sgn f$

Signum function : $sgn f = \begin{cases} 1 & f > 0 \\ 0 & f = 0 \\ -1 & f < 0 \end{cases}$

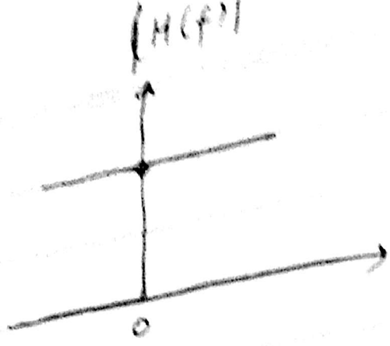


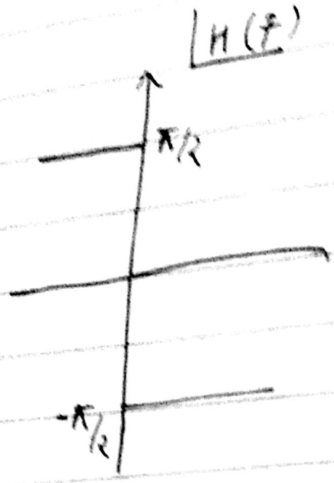


Phase plot

Frequency components are shifted.

Phase shift of all frequency components of input signal takes place by $-\pi/2$ for +ve component & $+\pi/2$ for -ve component.





Phase plot

$\frac{j}{\pi t} \longleftrightarrow \operatorname{sgn} f$ (Hilbert transform)

16.1.18 PROPERTIES

1. $\hat{\hat{x}}(t) = -x(t)$ Double Hilbert transf

$\hat{x}(t)$ - Hilbert transform of $x(t)$

2. $x(t) = \cos 2\pi f_0 t$
   $\hat{x}(t) = \sin 2\pi f_0 t$

- $x(t) = \sin 2\pi f_0 t$
  $\hat{x}(t) = -\cos 2\pi f_0 t$

- $x(t) = e^{j2\pi f_0 t}$
  $\hat{x}(t) = -j \operatorname{sgn}(2\pi f_0)\, e^{j2\pi f_0 t}$

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3. $\hat{X}(f) = (-j\,\mathrm{sgn} f)\, X(f)$ — $X(f)$ - Fourier transform of f

$\cdot$ $|\hat{X}(f)| = |X(f)|$

$\cdot$ $\int x^2(t)\,dt = \int |x(f)|\, df$

4. $x(t)$ and $\hat{x}(t)$ are orthogonal.

$$\int_{-\infty}^{\infty} x(t)\,\hat{x}(t)\,dt = 0$$

$$\Rightarrow \int_{-\infty}^{\infty} X(f)\,\hat{X}^*(f)\,df = \int_{-\infty}^{\infty} X(f)(j\,\mathrm{sgn} f\, X^*(f))\,df$$

$$= \int_{-\infty}^{\infty} \underbrace{j\,\mathrm{sgn} f}_{\text{asymmetric function}}\ \underbrace{|X(f)|^2}_{\text{even symmetric function}}\,df$$

5. Let $c(t)$ and $m(t)$ have non overlapping spectra (2 diff. signals whose spectra don't overlap)

$c(t)$: Carrier signal
$m(t)$: Message signal

$$\widehat{m(t)\,c(t)} = m(t)\,\hat{c}(t)$$

$$\widehat{m(t)\cos\omega_0 t} = m(t)\,.\sin\omega_0 t$$

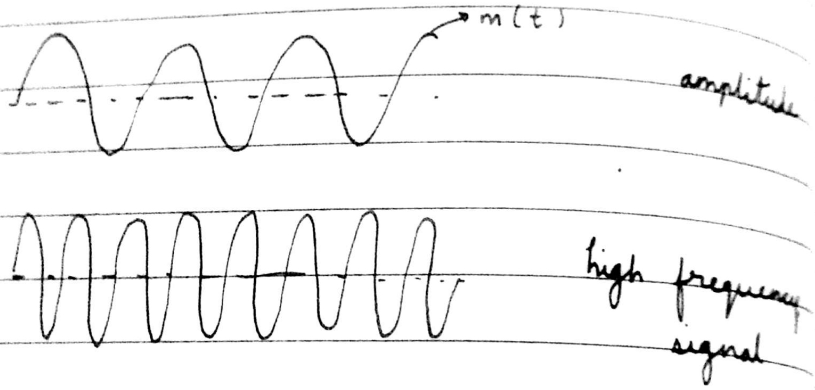
$$\widehat{m(t).\sin\omega_0 t} = -m(t)\cos\omega_0 t$$

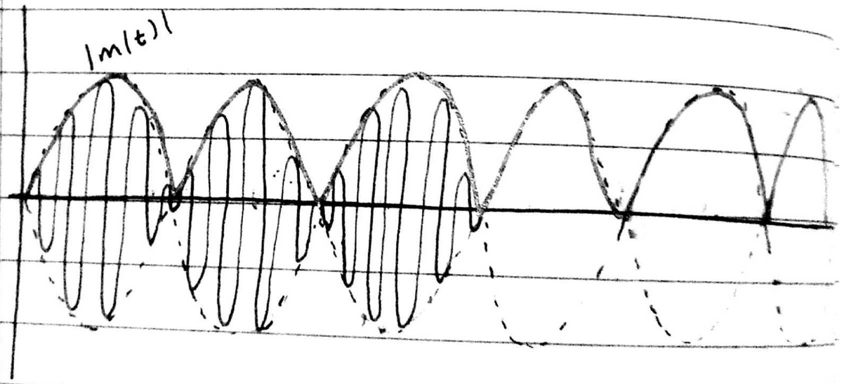
APPLICATIONS OF HILBERT TRANSFORM

$x(t)$ is a real valued function (signal)
$x_p(t)$: complex valued signal of $x(t)$

$$\boxed{x_p(t) = x(t) + j\hat{x}(t)}$$

$m(t).\cos(\omega_0 t + \theta)$





Modulating signal

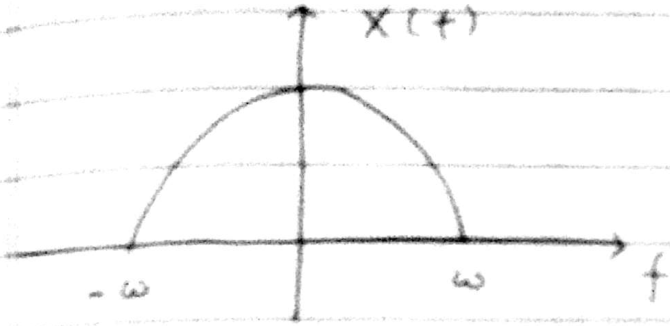
$$m(t)\cos(\omega_0 t+\theta) = m(t)\cos\omega_0 t\cos\theta - m(t)\sin\omega_0 t\sin\theta$$

$$\text{Magnitude} = \sqrt{m^2(t)\cos^2\theta + m^2(t)\sin^2\theta} = |m(t)|$$

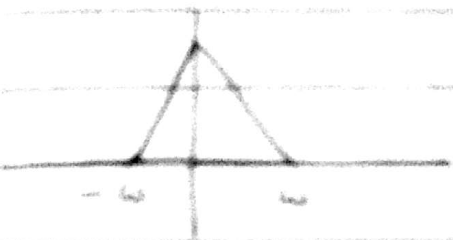
Complex envelope of $x(t)$

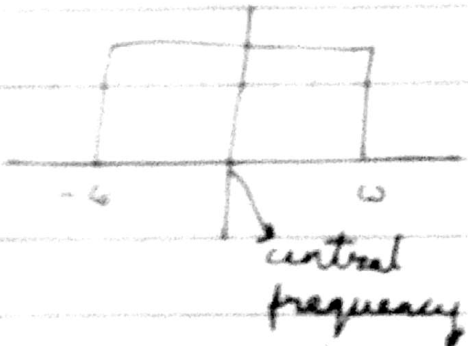
$x_p(t) = x(t) + j\hat{x}(t)$

$|x_p(t)|$



Parabolic





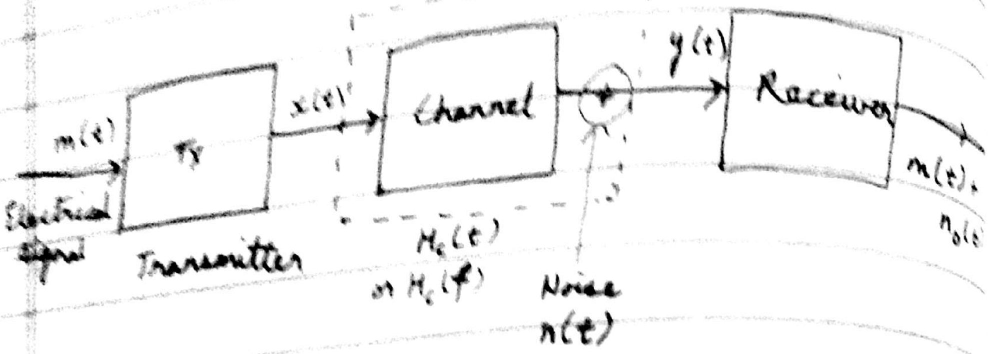
The shape may be any of the above but the frequency range remains the same

Complex Envelope representation Band Pass Signal

DSB & SSB modulation technique

ANALOG SIGNAL TRANSMISSION

19.1.15 ~~Analog~~



Channel is modelled as LTI system

y(t) = base band signal + noise

Receiver tries to reduce noise as much as possible.

$n_0(t)$: remnant noise (mostly internal)

<u>Ideal channel</u>

<u>Ideal filter</u> : Noise ~ 0

Distortion less transmission

Condition -

$$\boxed{y(t) = kx(t - t_0)}$$

k - Scaling factor

t_0 - Delay

$$\boxed{y(f) = k e^{j2\pi f t_0} x(f)}$$

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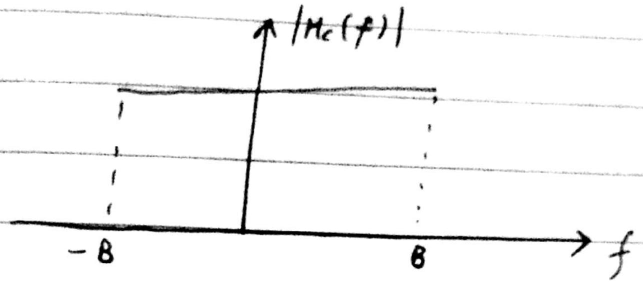
Ideal channel

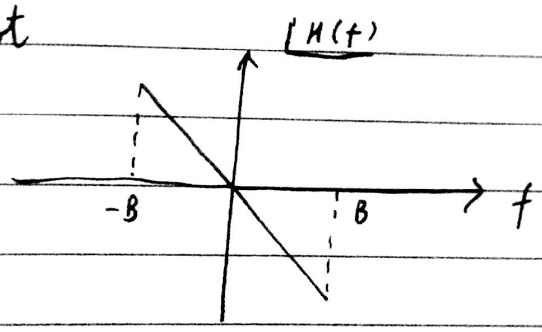
$$H_c(f) = K e^{-j2\pi f t_0}$$

t_0 - propagation delay

Magnitude plot of $H_c(f)$:



Phase plot



Slope $= -j2\pi t_0$

$K < 1$: Attenuation factor
$K > 1$: Amplification "

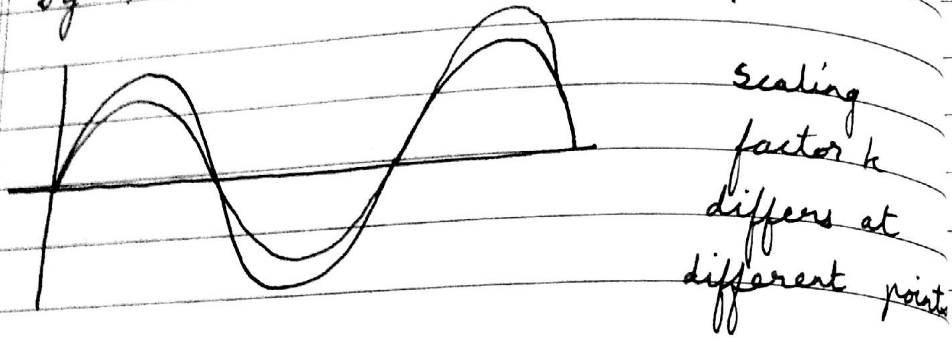
Amplitude should not be from $-\infty$ to ∞ (in the graph), it should be in the given range ($-B$ to B)

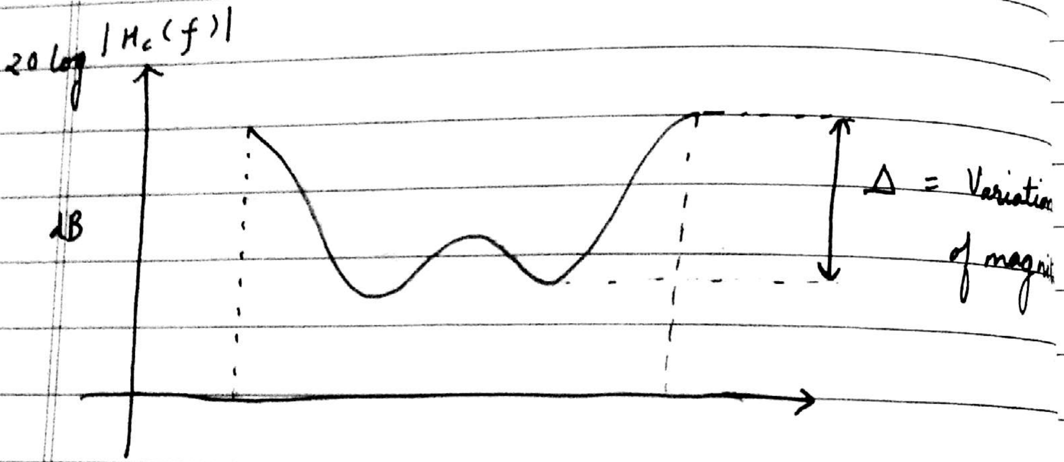
Distortion - If the range of f in mag. plot & phase plot are different.
(Linear distortion)

LINEAR AMPLITUDE DISTORTION

$|H_c(f)| \neq k$

All message signals are not modified by the same attenuation factor k.





If $\Delta > 1$, amplitude distortion should be eliminated.

$\Delta < 1$, distortion can be neglected.

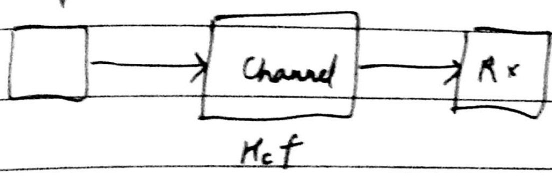
PHASE DISTORTION / DELAY DISTORTION

$$\angle H_c(f) \neq -2\pi f t_0 \pm m\pi \qquad |f| < B$$

If Phase plot is not linear
All frequency components are not delay by the same factor.

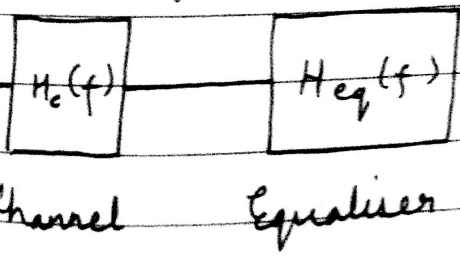
- Delay distortions are not important in speech transmission because ears are insensitive to delay distortion.

- Data transmission is highly sensitive to delay distortion.

- 

Receiver function is the inverse of that of channel to eliminate ~~noise~~ distortion.

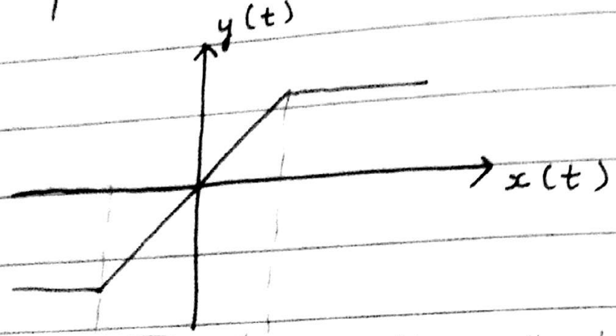
Equalisation - Amp and phase distortion eliminated by adding an equaliser at the receiver end, the function of which is the inverse of the transfer function of the channel.



$$\boxed{H_{eq}(f) = \frac{K e^{-j2\pi f t_0}}{H_c(f)}} \qquad |f| < B$$

NON LINEAR DISTORTIONS

Non linear distortion arises due to non linear transfer characteristics of amplifiers, mixers and other electronic components in the communication system.



linear characteristics followed after that saturation occurs

$$y(t) = A\,x(t)$$

(valid in the linear range)

$$y(t) = a_1 x(t) + a_2 x^2(t) + a_3 x^3(t) + \dots$$

q. $y(t) = a_1 x(t) + a_2 x^2(t)$

Evaluate a_1 and a_2.

$x(t) \rightarrow f_1$ $\qquad$ $y(t) = f_1$

$x(t) = \sin 2\pi f t$

$y(t) = a_1 \sin 2\pi f t + a_2 \sin^2 2\pi f t$

$2f_1$ (non linear dist.)

$x(t) \rightarrow f_1, f_2$

$y(t) = f_1, \quad f_1 \pm f_2$

$\qquad f_2, \quad 2f_2$

Interference /
Cross stock
- disturbing other frequency components

2.2.18

LINEAR MODULATION

Need of modulation

- SNR becomes high (efficient transmission).
- Reduce the size of antenna.
- Increase bandwidth of message signal.
- Frequency division multiplexing.
- ~~Ra~~ Noise reduction is better with modulation.
- Energy losses less for high frequency.

MODULATE - Change, tune or translate

Amp, freq, phase modulation
Modulating amp, freq, phase of carrier signal.

Baseband signal - data, audio, video
(Low frequency)

Advantages of Modulation

1. Ease of radiation
2. Multiplexing - multiple signals can be pass
3. Convenient signal processing
4. Noise reduction - depends on baseband signal & carrier frequency

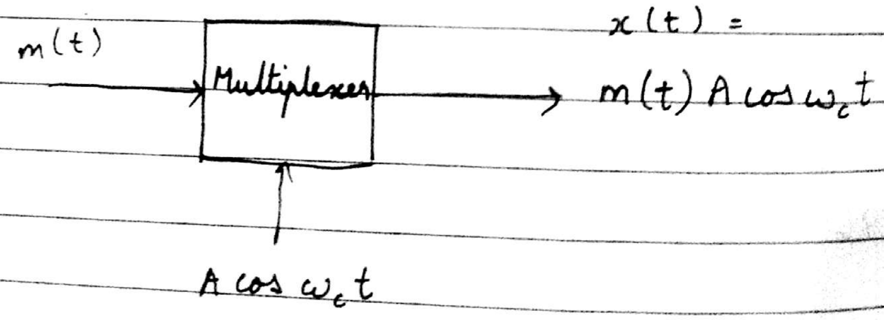
5 ~~Conver~~

Message signal : $m(t)$ (low freq)

~~m(t) = a~~ Message signal can be of arbitrary form. It need not be sinusoidal.

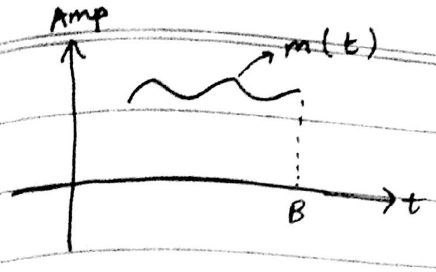
Carrier signal : $A \cos \omega_c t$ (high freq)

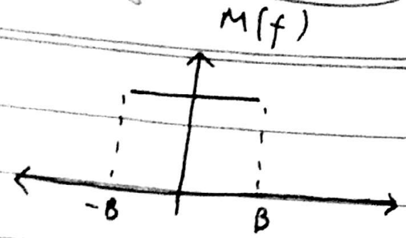
Frequency translation



DSB - SC : Double side band ~~with~~ suppressed carrier

$$X(f) = \frac{A}{2}\left[M(f+f_c) + M(f-f_c)\right]$$



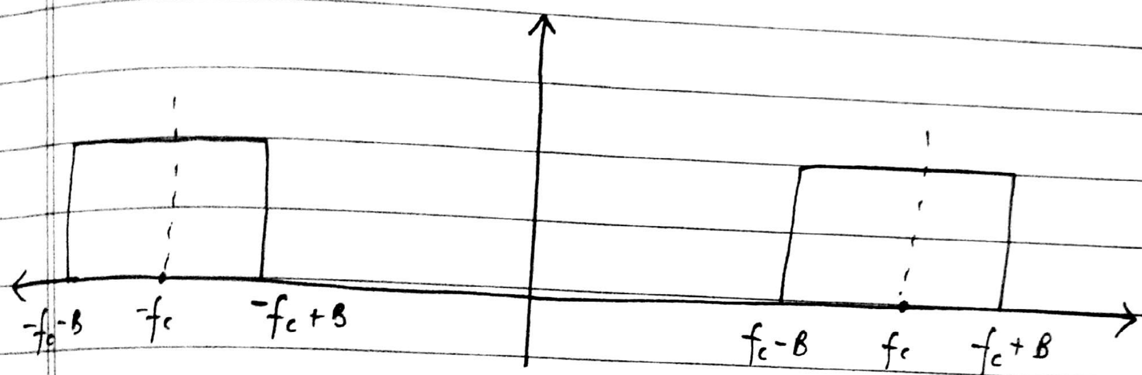


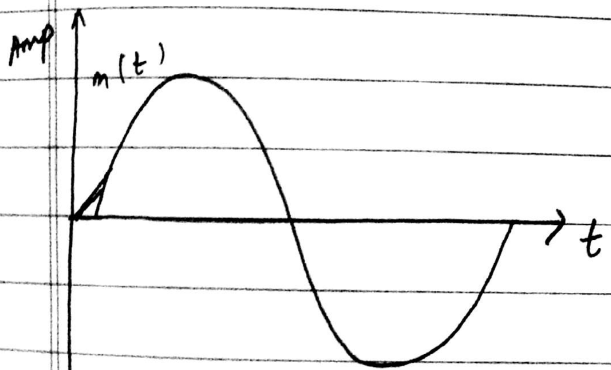
Bandwidth of message signal = B

Range [-B, B]

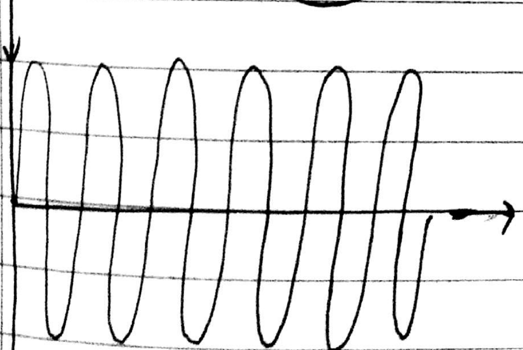
Message consists only [0, B]

We are representing ~~M(f)~~ m(t) in the frequency domain [-B, B].

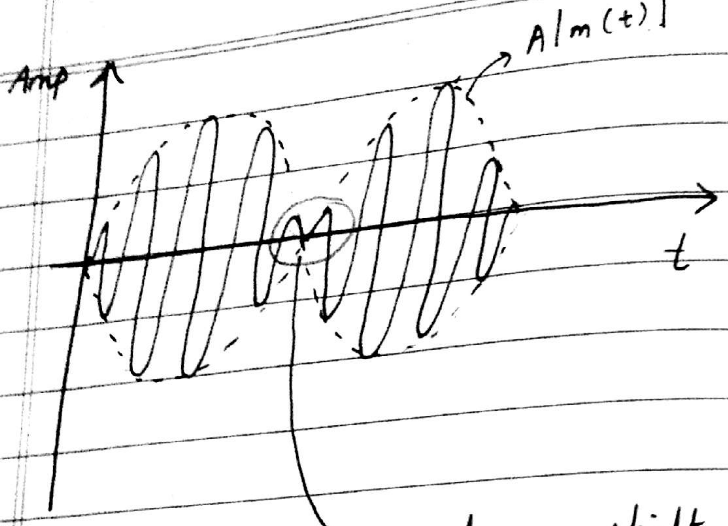




Message signal

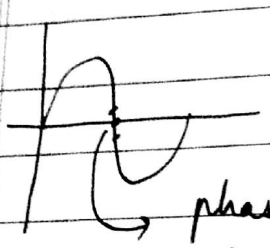


Carrier signal

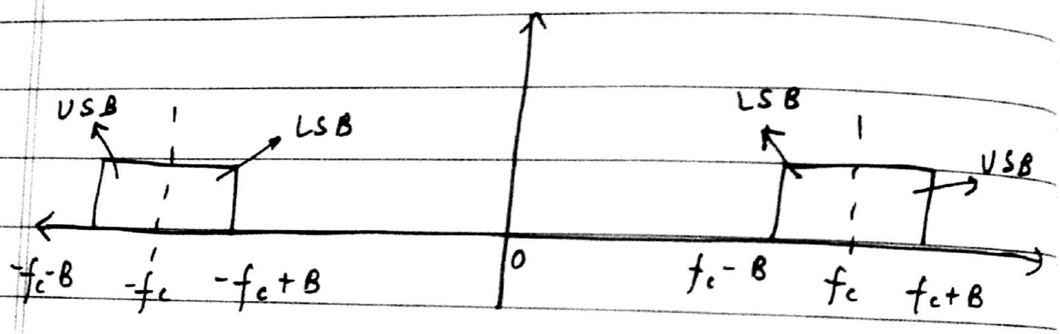


Modulated signal

phase shift of 180
not continuous



phase shift of 180
(+ve ~~the~~ 0 -ve)

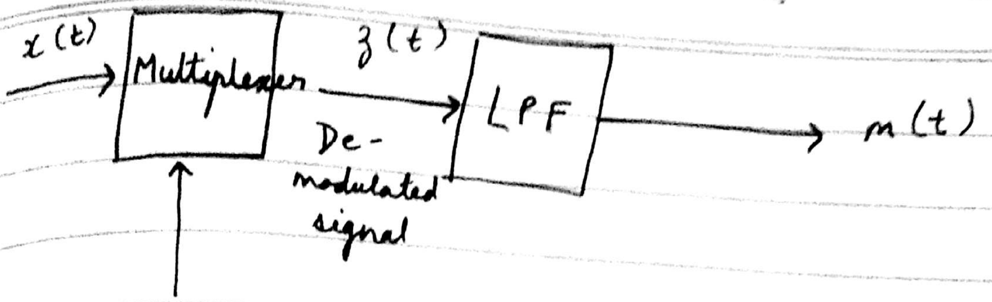


LSB - Lower side band
USB - Upper side band
(Double side band)

$x(t)$
$\rightarrow$

$m(t)$

Receiver end
(Modulated signal)



$2\cos\omega_c t$

(Carrier signal)

Quartz oscillator - used to produce constant frequency.

Capacitor does not generate const. freq.

LPF - Low pass filter
Filters high frequency and lets low freq. pass.

This multiplexing is known as demultiplexing.

DSB - SC

$x(t) = A\,m(t)\cos\omega_c t$

$2\cos\omega_c t$

$z(t) = k^*[m(t)\cos\omega_c t][2\cos\omega_c t]$

$= k\,[m(t)\,2\cos^2\omega_c t]$

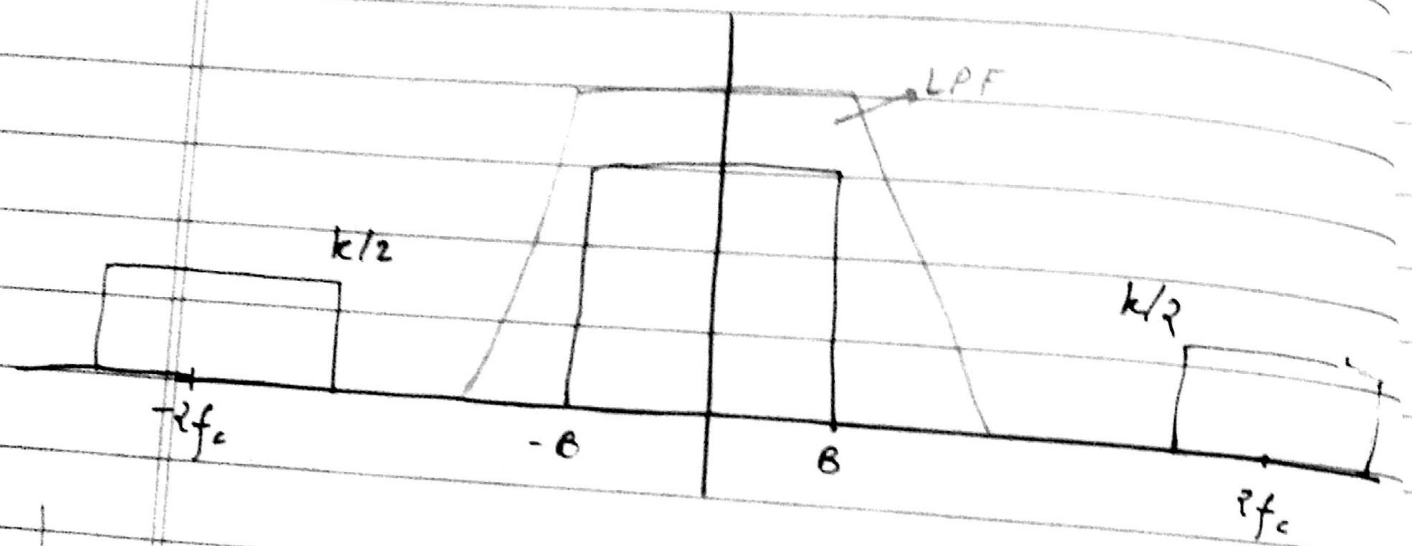
$= k\,m(t) + k\,m(t)\cos 2\omega_c t$

needed — LPF → passes

not needed — LPF → removed

$$z(f) = kM(f) + \frac{k}{2}\Big[\underbrace{M(f-2f_c) + M(f+2f_c)}_{\text{high freq (removed)}}\Big]$$

$M(f)$: Baseband signal



If modulated and demodulated freq. is not the same, accurate baseband signal will not be obtained.

Synchronised carrier is required for this process.

3-2-18

COHERENCE

Carrier (at receiver end) : $2\cos[(\omega_c + \delta\omega)t + \theta]$ (very small value)

Carrier should be the same at receiver end and transmitter end.

1. $\delta\omega = 0$ $\qquad 2\cos(\omega_c t + \theta)$
   If $\theta = \pi/2$ $\qquad -2\sin\omega_c t$

2. $\delta\omega \neq 0$, $\theta = 0$

~~Wobbling of~~ Warbling effect - Extra frequency ($\delta\omega$) component is introduced at the receiver end.
Demodulated signal is distorted.

m(t) message, c(t) carrier $\Big\} \rightarrow x(t)$

$x(t) \rightarrow \bigcirc \rightarrow \boxed{LRF} \rightarrow m(t)$

$\uparrow$ $c'(t)$ (receiver end carrier)

If $c'(t)$ & $c(t)$ are not the same then Warbling effect occurs for linear amplitude modulation $\theta = 0$

DSB Carrier is not suppressed.
Also referred to as DSB-AM

DSB-AM
DSB-SC

$c(t) = A_c \cos(2\pi f_c t)$

A_c = amp

f_c = freq

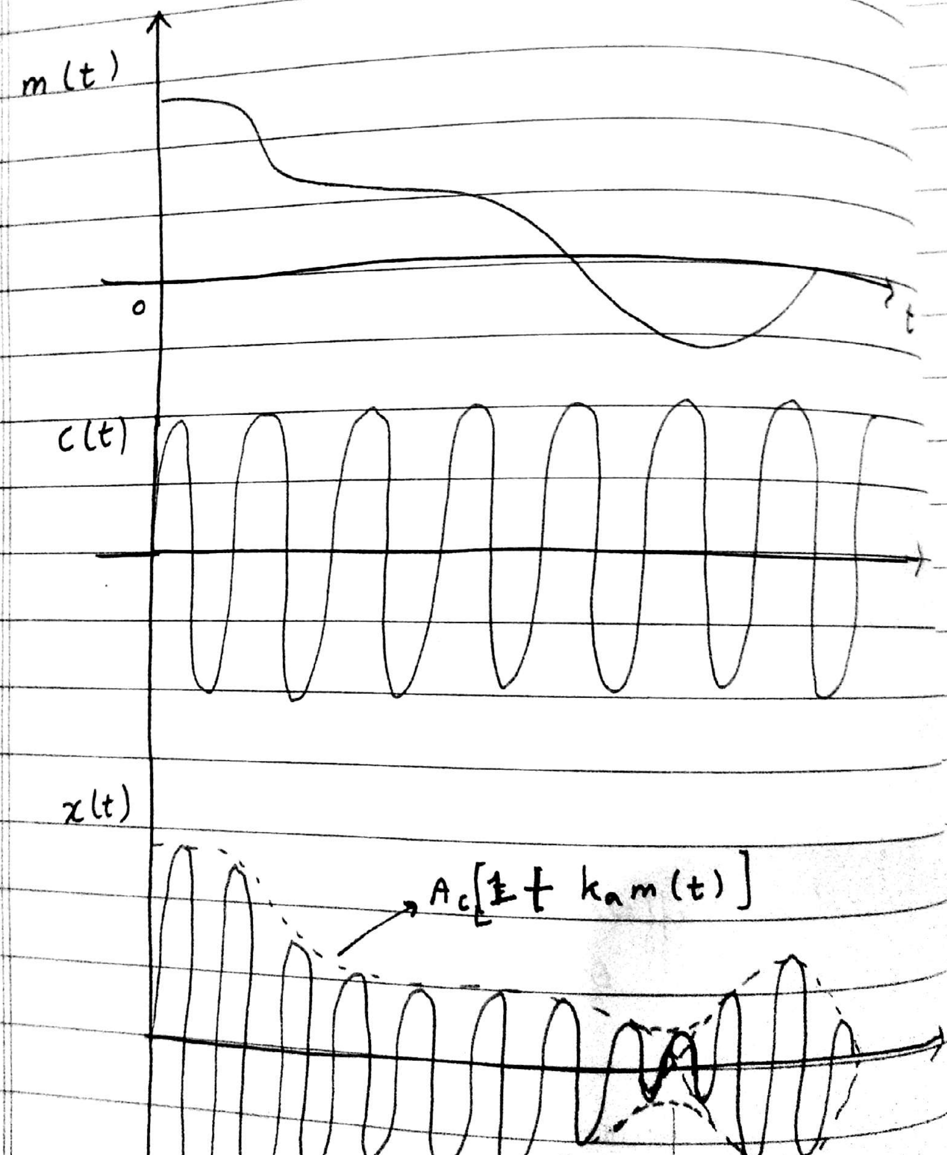
$x(t) = A_c [1 + k_a m(t)] \cos(2\pi f_c t)$

DSB-AM

k_a = amp. sensitivity factor

(not present in DSB-SC)

Volt^{-1}



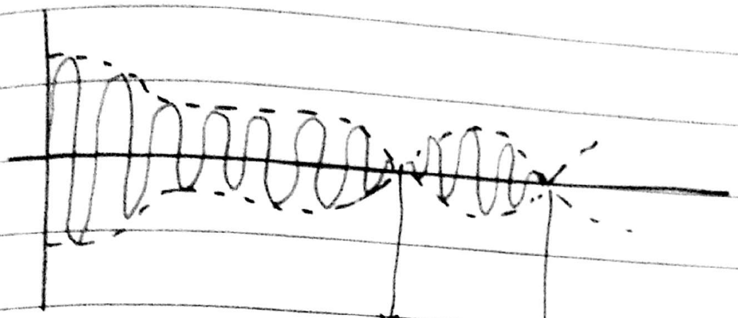
$k_a m(t) < 1$

$1 + k_a m(t) > 1$

$\therefore$ x(t) does not meet zero crossing

Envelope of $x(t)$ = $\cancel{|m(t)|}$

- The waveform $x(t)$ occurs when $k_a m(t) < 1$
- The waveform goes till zero crossing if $k_a m(t) \nless 1$ and modulated signal envelope is distorted.



Distorted envelope

Phase reversal at 0 crossing

Assumption $f_c >> B$
(BW of message signal)

Fourier transform of $x(t)$

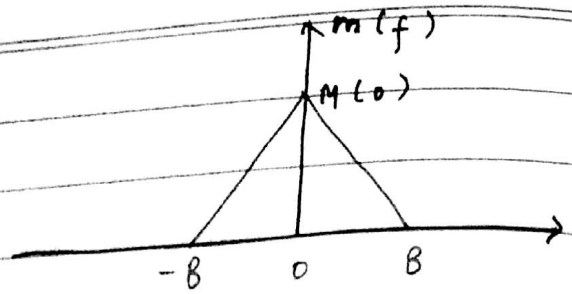
$$X(f) = \frac{A_c}{2}\left[\delta(f-f_c) + \delta(f+f_c)\right]$$

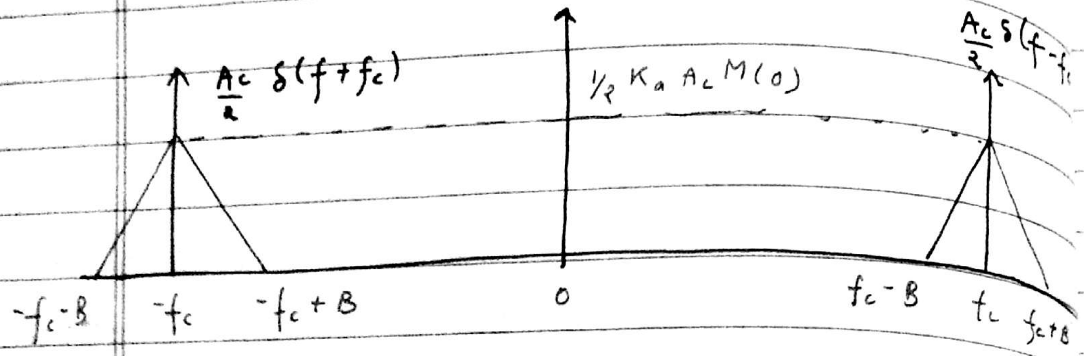
because of this carrier freq is present

DSB-AM

$$+ K_a \frac{A_c}{2}\left[M(f-f_c) + M(f+f_c)\right]$$

Consider message signal $m(t)$ is band limited.





BW of message signal $= B$

$$B_T = 2B$$

BW of transmitted signal

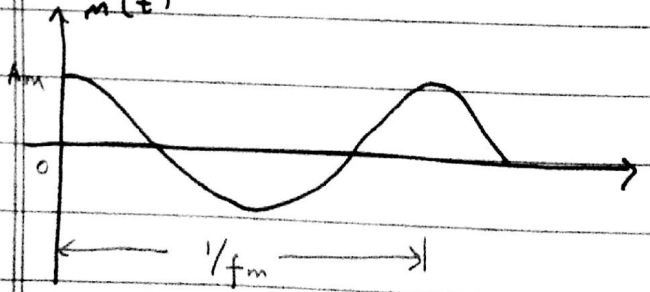
→ $m(t) = A_m \cos(2\pi f_m t)$

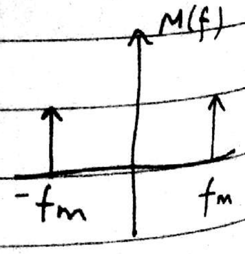
Single tone message signal - consists of only one frequency

$$x(t) = A_c[1 + k_a A_m \cos(2\pi f_m t)] \cos 2\pi f_c t$$

$\mu = k_a A_m$ modulation factor

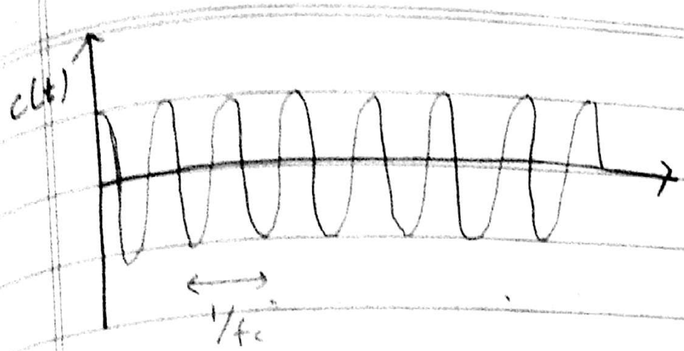
$\mu < 1$ to avoid envelope distortion or overmodulation.

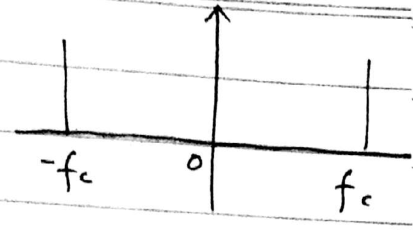




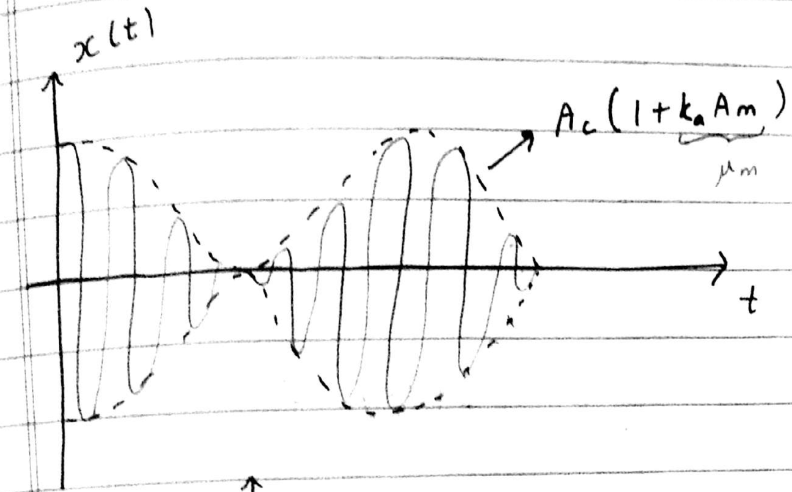
Low freq. signal

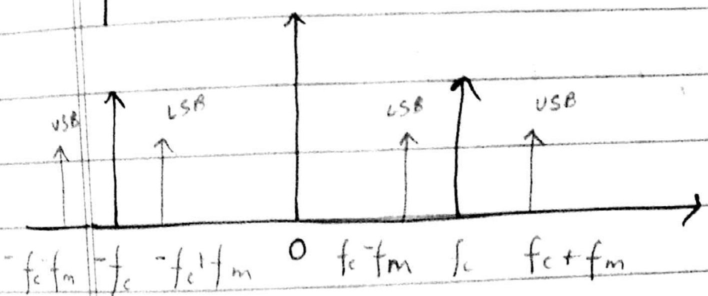
f_m close to 0





High freq signal
fc far from 0





$$\frac{A_{max}}{A_{min}} = \frac{A_c[1+\mu]}{A_c[1-\mu]}$$

$$\boxed{\mu = \frac{A_{max} - A_{min}}{A_{max} + A_{min}}}$$

Modulation factor

Power of carrier signal $P_c = \frac{1}{2} A_c$

$P_{USB} = \frac{1}{8} A_c^2 \mu^2$; $P_{LSB} = \frac{1}{8} A_c^2 \mu^2$

Date
Page

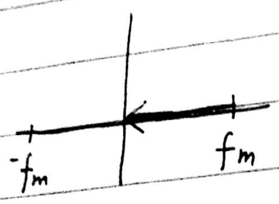
single tone
only 1 value of f_m

6.2.18:

$$x(t)_{DSB-AM} = A_c[1+\mu \cos(2\pi f_m t)] \cos(2\pi f_c t)$$

$$m(t) = A_m \cos(2\pi f_m t)$$

$$x(t)_{DSB-AM} = A_c[1+k_a m(t)] \cos 2\pi f_c t$$



imp for ques

Fourier transform

$$X(f) = \frac{1}{2} A_c[\delta(f-f_c) + \delta(f+f_c)]$$

$$+ \frac{1}{4}\mu A_c[\delta(f-f_c-f_m) + \delta(f+f_c+f_m)]$$

$$+ \frac{1}{4}\mu A_c[\delta(f-f_c+f_m) + \delta(f+f_c-f_m)]$$

→ rms of this → USB

Case I : Voltage is given

→ Carrier power $(P_c) = \frac{A_c^2}{2}$

$P = VI$

$P = \frac{V^2}{R}$ $V^2 \to V_{rms}$, $R \to 1\,\Omega$

→ Upper side band Power $(P_{USB}) = \frac{1}{8}\mu^2 A_c^2$

→ Lower " " " $(P_{LSB}) = \frac{1}{8}\mu^2 A_c^2$

Total power $P_t = P_c + P_{USB} + P_{LSB}$

$$= \frac{A_c^2}{2}\left[1 + \frac{\mu^2}{2}\right]$$

$$P_t = P_c[1+\mu^2/2]$$

$$\boxed{\frac{P_t}{P_c} = \frac{2+\mu^2}{2}}$$

$$\frac{P_t}{P_{SB}} = \frac{A_c^2}{2}\,\frac{4}{\mu^2 A_c^2}\left[1+\mu^2/2\right]$$

$P_{SB} \leftarrow P_{USB} + P_{LSB}$

$$\boxed{\frac{P_t}{P_{SB}} = \frac{2+\mu^2}{\mu^2}}$$

To avoid distortion $\mu < 1$

$\rightarrow$ $P_c = \dfrac{\left(\dfrac{A_c}{\sqrt{2}}\right)^2}{R} \xrightarrow{rms} = \dfrac{A_c^2}{2}$ $(R = 1\,\Omega)$

$\rightarrow$ $\mu = 1$ Perfect modulation

$$\frac{P_t}{P_{SB}} = 3$$

Case II : Current is given

$$\frac{P_{AM}}{P_c} = \frac{P_t}{P_c} = \left(1+\frac{\mu^2}{2}\right) = \frac{(I_t^2)R}{(I_c^2)R}$$

$\{ P = VI = I^2 R \}$

$$\Rightarrow \left(\frac{I_t}{I_c}\right)^2 = 1+\frac{\mu^2}{2}$$

$$\boxed{I_t = I_c\sqrt{1+\frac{\mu^2}{2}}}$$

MULTI FREQUENCY COMPONENTS

$f_{m1}, f_{m2}, f_{m3}, \ldots$

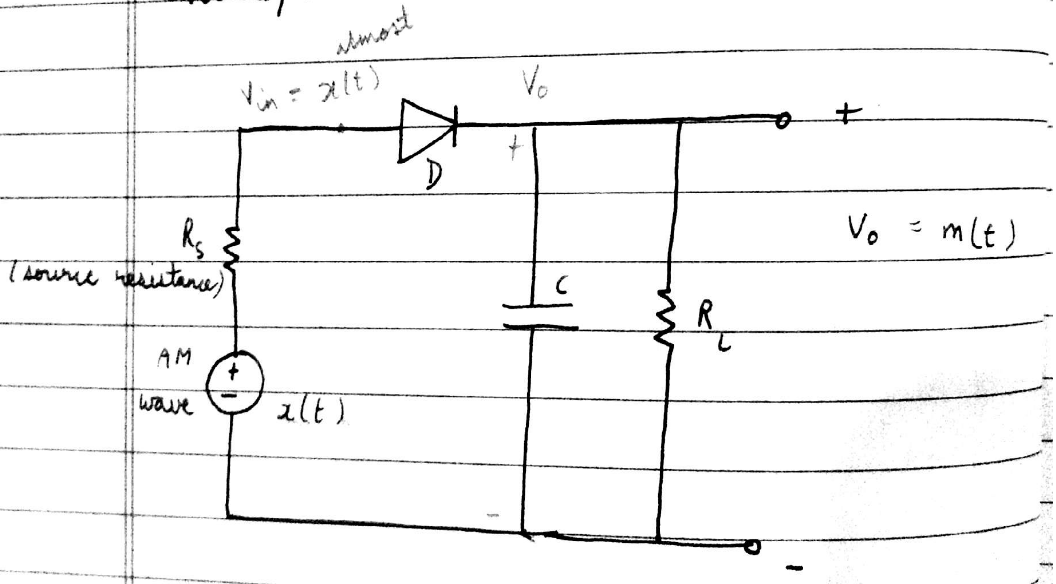
Multiple modulation factors are obtained

$\mu_1, \mu_2, \mu_3, \ldots$

$$\boxed{\mu_t = \sqrt{\mu_1^2 + \mu_2^2 + \mu_3^2 + \ldots}}$$ Total modulation index

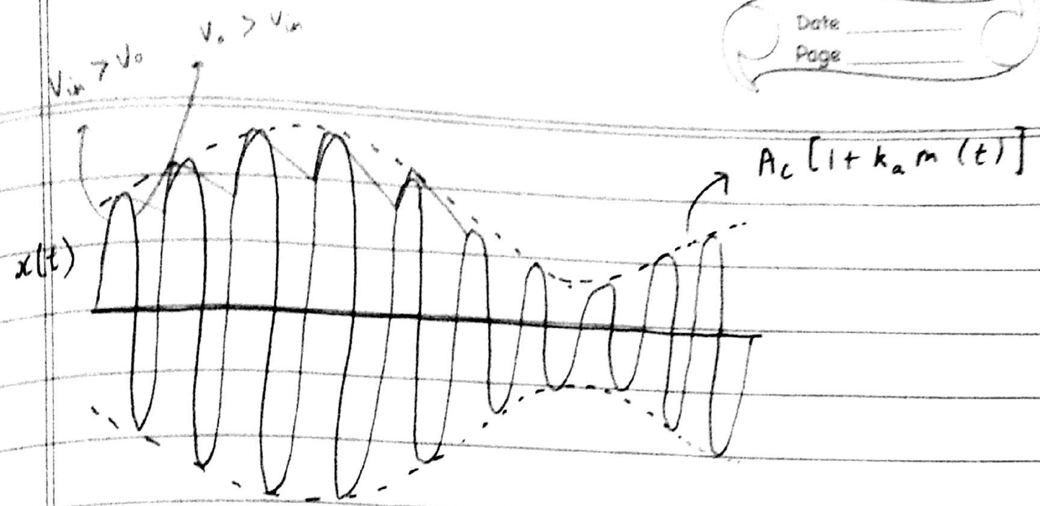
Retrieving m(t) from x(t) at receiver end

Envelope detector is used.



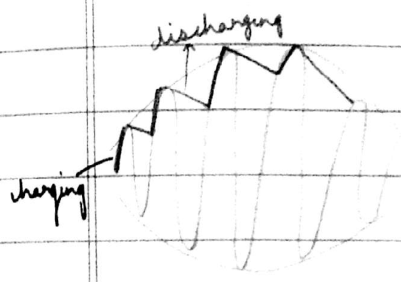
$R_s \rightarrow$ to avoid overloading hazards

V_o traces the envelope of the modulated signal.



$V_{in} > V_o$ D is forward biased
Capacitor charges

$V_o > V_{in}$ D is reversed biased
Capacitor discharges



C and R_L values should be chosen appropriately to avoid too fast or too slow charging and discharging

The exact envelope is not tracked, path close to the envelope is tracked.

Envelope detector -: TV receivers & radio receivers use this to make tuning easy

Frequency band of FM -: 98 - 108 MHz

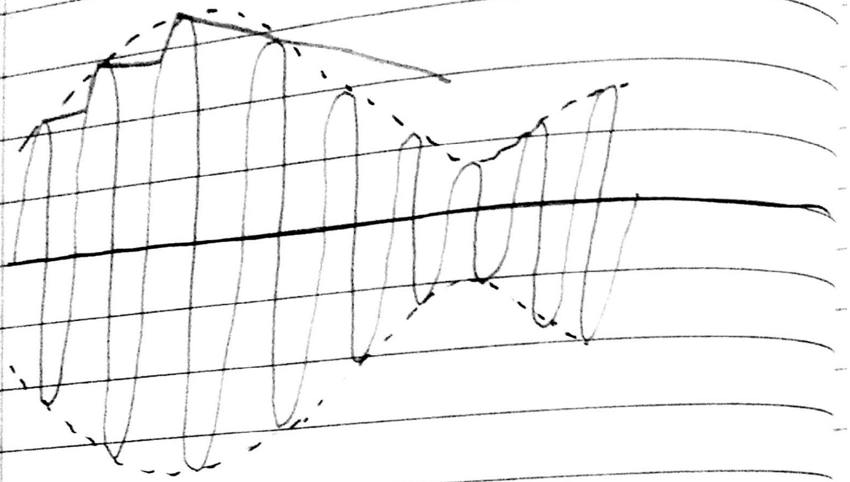
Knob present on TV - for tuning diff. frequency bands.

Time constant

$\tau = R_L C$

- **If τ is very large** R_L, C not chosen properly

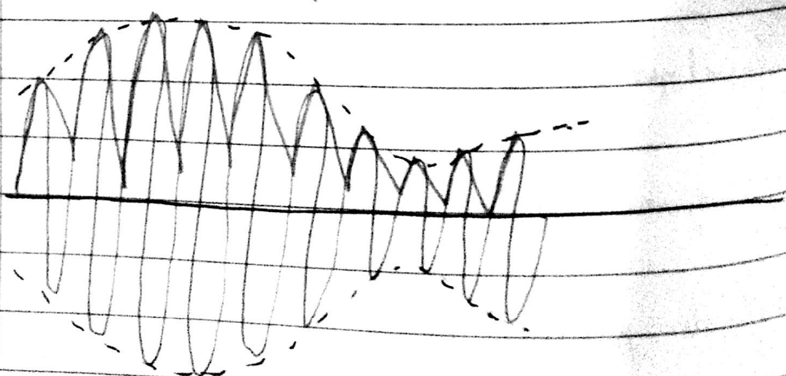
Capacitor discharges very slowly



Peaks are not tracked due to slow discharging ; message signal is lost.

- **If τ is very small**

Capacitor discharges very fast



Instead of tracing the envelope , it traces the carrier signal

Date
Page

• Selection of τ

$$\boxed{\frac{1}{f_c} << \underset{(R_L C)}{\tau} << \frac{1}{f_m}}$$

to avoid small τ

to avoid large τ

message signal

** Frequency is not utilized properly. 2B is transmitted but freq. band of signal is B. 2 freq bands on left & right of origin (L & USB)

2 signals : $B_1 < B_2$
left side → right side

Single side band modulation — complete removal of 1 band
Vestigial " " " — partial removal of 1 band

One side band has to be transmitted & the other has to be eliminated.

7.8.18

DSB - SC $\longrightarrow$ 67%

DSB - AM

$$\frac{P_{SB}}{P_t} = \frac{\mu^2}{2+\mu^2}$$

$$= \frac{1}{3} = 33\% \quad (\mu \leq 1)$$

$$P_t = P_c\left(1+\frac{\mu^2}{2}\right) = P_c\left[\frac{\mu^2}{2}\right]$$

To reduce power DSB-SC method

$$x_{DSB-SC}(t) = \frac{m(t) \times c(t)}{[1+k_a m(t)]\, c(t)}$$

QCM Quadrature curvature multip

2 carriers (90° phase diff.) & 2 mess sig

$\cos(2\pi f_c t)$, $\sin(2\pi f_c t)$

To utilize power $\rightarrow$ DSB-SC

$$x(t)_{DSB-SC} = m(t)\, c(t)$$

$$x(t) = A_c m_I(t) \cos(2\pi f_c t) - A_c m_Q(t) \sin(2\pi f_c t)$$

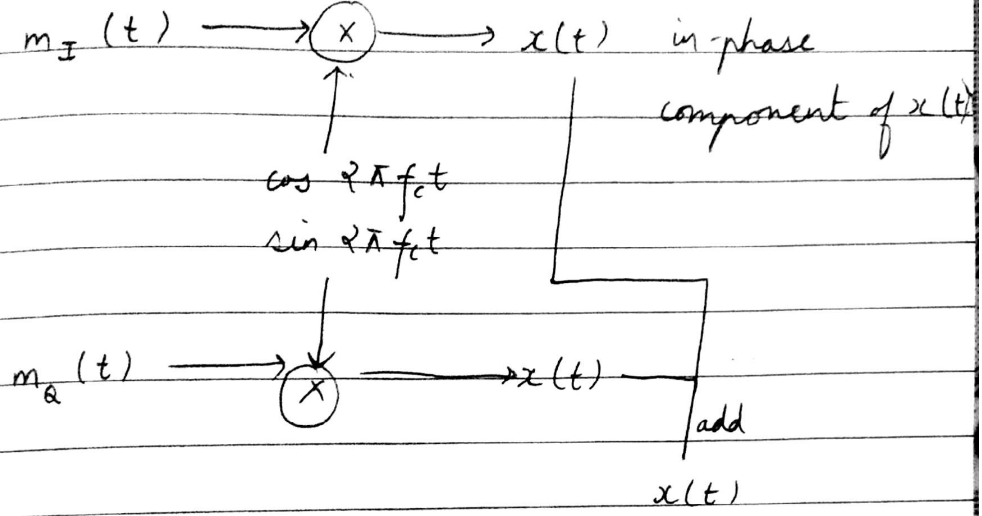
m_I modulated with cos( )

m_Q " " sin( )

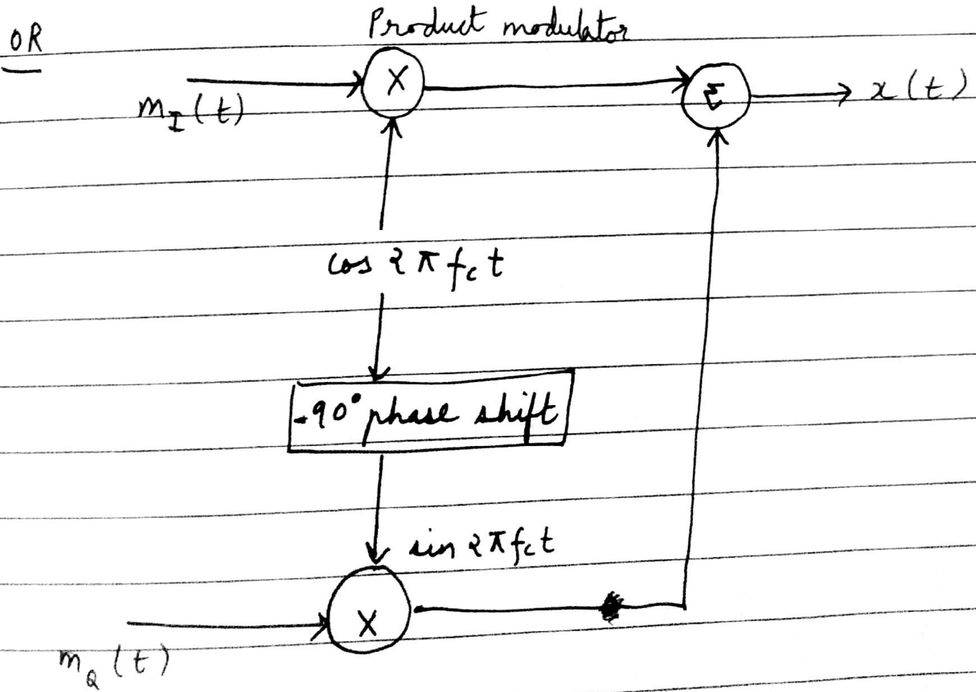
m_I - In phase component
m_Q - Quadrature "

At the receiver end -:

Carriers -: $\sin 2\pi f_c t$ $\cos 2\pi f_c t$



OR

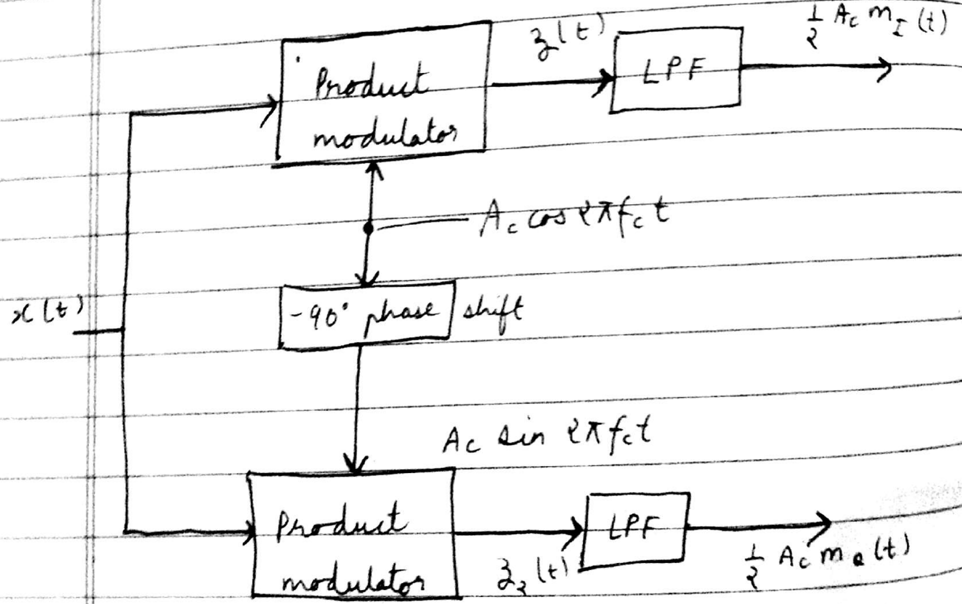


→ If A_c is multiplied with $\cos(2\pi f_c t)$ we get in-phase component at demodulation end

→ $A_c \times \sin 2\pi f_c t$ → quadrature comp.

Orthogonal signals (carrier) : $\cos(2\pi f_c t)$, $\sin(2\pi f_c t)$

$$\cos(2\pi f_c t) \times \sin(2\pi f_c t) = \frac{1}{2} \sin(4\pi f_c t$$



$$z(t) = [A_c m_I(t) \cos(2\pi f_c t) - A_c m_Q(t) \sin(2\pi f_c t)] \cos 2\pi f$$

$$= \frac{A_c m_I(t)}{2} (1 + \cos(4\pi f_c t)) - \frac{A_c}{2} m_Q(t) \sin(4\pi f_c t)$$

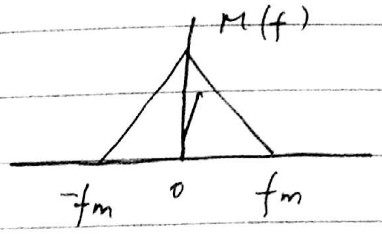
$$z(t) = \underbrace{\frac{A_c}{2} m_I(t)}_{LPF} + \overbrace{\frac{A_c}{2} m_I(t)\cos(4\pi f_c t)}^{2f_c}$$

$$\underbrace{- \frac{A_c}{2} m_Q(t)\sin(4\pi f_c t)}_{2f_c}$$

$$z_2(t) = \Big[A_c m_I(t)\cos(2\pi f_c t) - A_c m_Q(t)\sin(2\pi f_c t)\Big]\sin(2\pi f_c t)$$

SSB (Single side band) modulation

$$m(t) \longleftrightarrow M(f)$$

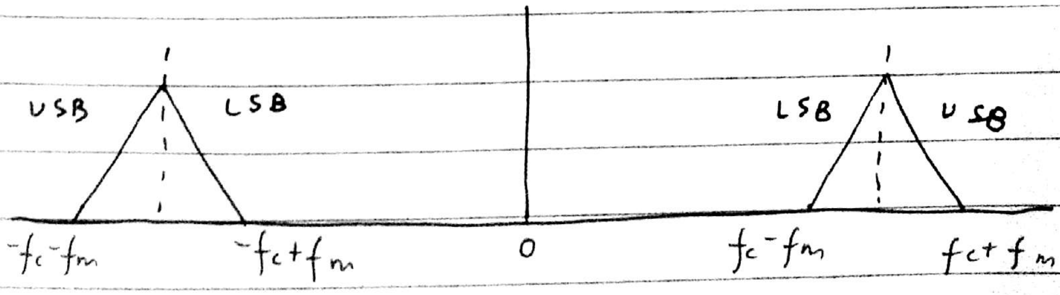


DSB signal with single tone modulation :-

$$x(t) = m(t)\cos(2\pi f_c t)$$

$$X(f) = M(f)\left[\frac{1}{2}\delta(f-f_c) + \frac{1}{2}\delta(f+f_c)\right]$$

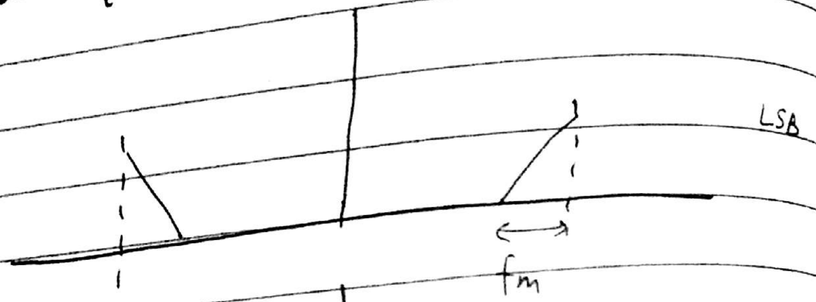
$$= \frac{1}{2}M(f-f_c) + \frac{1}{2}M(f+f_c)$$



Either LSB or USB has to be eliminated.

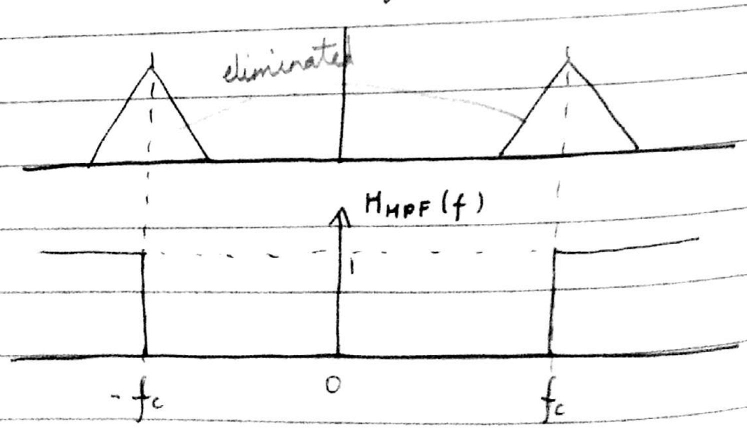
$USB = \{(-f_c - f_m, -f_c) \cup (f_c, f_c + f_m)\}$

$LSB = \{(-f_c, -f_c + f_m) \cup (f_c, f_c + f_m)\}$



BW is f_m in both cases.

After DSB modulation, add some component to remove USB (if LSB is required)



Ideal filter

High pass filter (with sharp cut off)

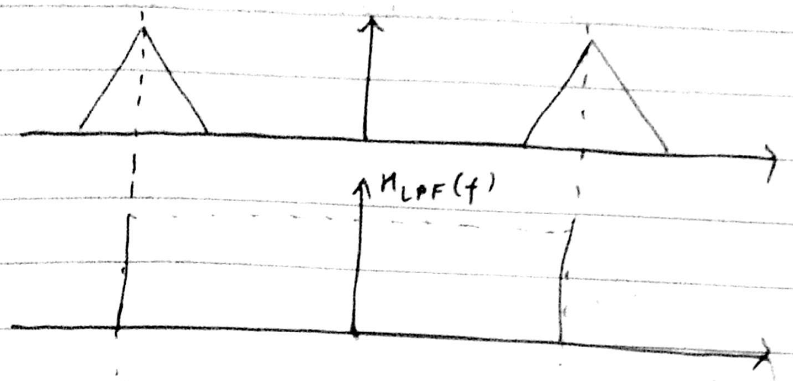
$$H_{HPP}(f) = \begin{cases} 1 & ; |f_c| \geq f_c \\ 0 & ; \text{otherwise} \end{cases}$$

Mid-sem

Fourier transform - Proof
Properties

Till SSB mod & demod

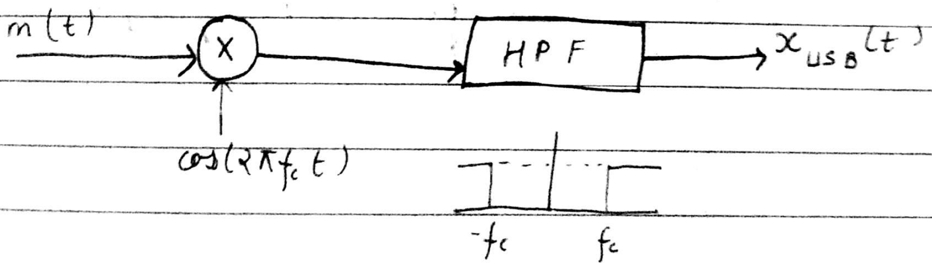
To cut-off LSB -:

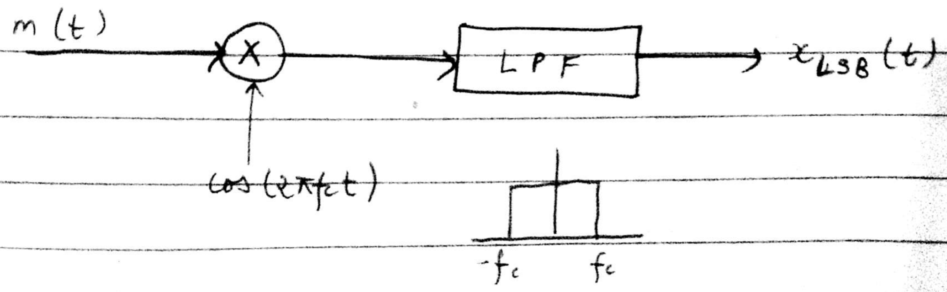


$$H_{LPF}(f) = \begin{cases} 1 & ; \ |f_c| \le f_c \\ 0 & ; \ \text{otherwise} \end{cases}$$

Frequency discrimination method
(using low pass or high pass filter after DSB - AM)

USB GENERATION





Problem related to DSB, SSB
Mathematical representation

12.2.18

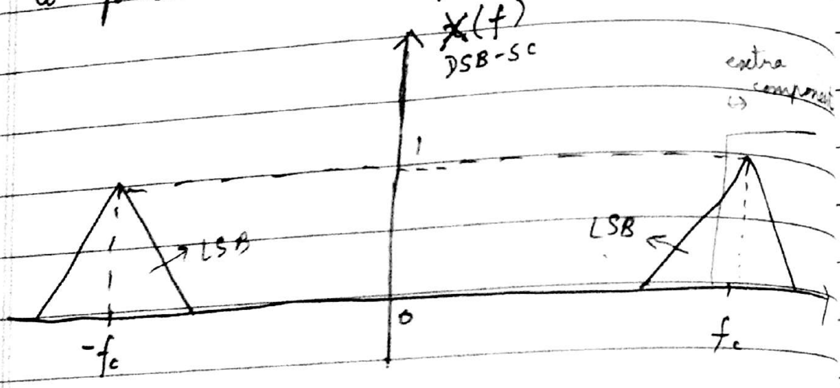
~~Io~~ To generate $x(t)$ -: balanced modulator & ring modulators are used.

diag; operation

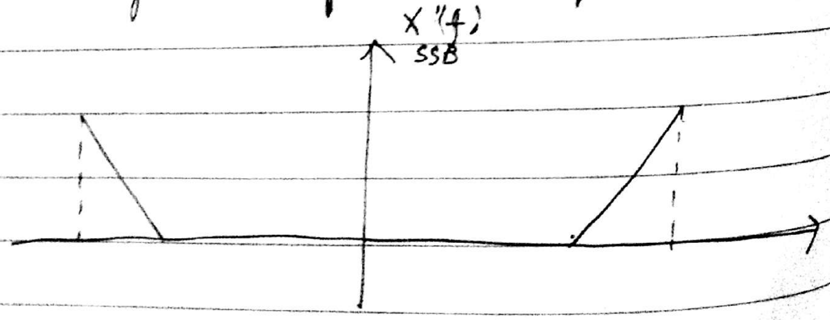
SSB modulation and demodulation

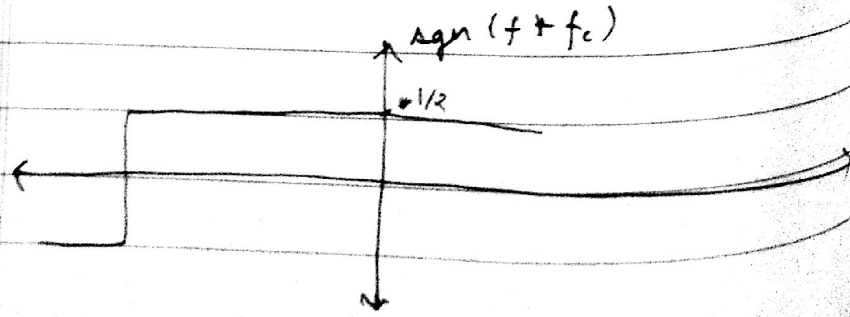
$x(t)_{DSB-SC}$

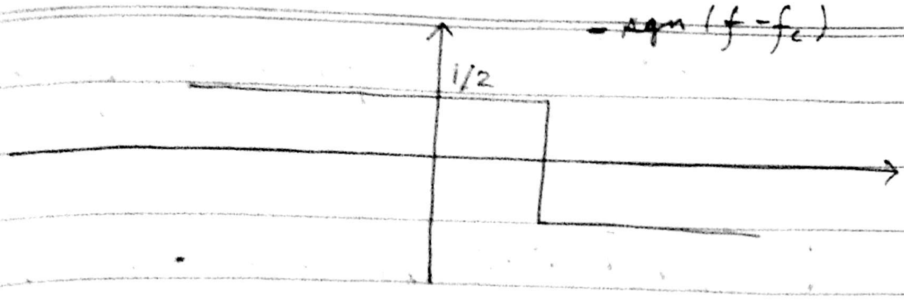
Ideal filter designing is impossible
From DSB-SC modulated signal is used to produce SSB signal.



High pass or low pass filters may not be ideal. Phase transformers can be suitably used for this purpose.







$$H_t(f) = \frac{1}{2}\left[\operatorname{sgn}(f+f_c) - \operatorname{sgn}(f-f_c)\right]$$

$$X_{DSB}(f) = \frac{1}{2} A_c \left[M(f+f_c) + M(f-f_c)\right]$$

Multiplying the above equations :-

$$X_{SSB}(f) = \frac{1}{4} A_c \left[\overset{A}{M(f+f_c)\operatorname{sgn}(f+f_c)} + \overset{B}{M(f-f_c)\operatorname{sgn}(f+f_c)} \right] - \frac{1}{4} A_c \left[\overset{C}{M(f+f_c)\operatorname{sgn}(f-f_c)} + \overset{D}{M(f-f_c)\operatorname{sgn}(f-f_c)} \right]$$

$$= \frac{A_c}{4}\left[M(f+f_c)\operatorname{sgn}(f+f_c) - M(f-f_c)\operatorname{sgn}(f-f_c)\right]$$

$$+ \frac{A_c}{4}\left[M(f-f_c)\operatorname{sgn}(f+f_c) - M(f+f_c)\operatorname{sgn}(f-f_c)\right]$$

$$= \frac{1}{4} A_c \left[M(f+f_c) + M(f-f_c)\right] +$$

$$\frac{1}{4} A_c \left[M(f+f_c)\operatorname{sgn}(f+f_c) - M(f-f_c)\operatorname{sgn}(f-f_c)\right]$$

$$* \ \frac{1}{2} A_c\, m(t) \cos \omega_c t \Leftrightarrow \frac{1}{4} A_c \left[M(f+f_c) + M(f-f_c)\right]$$

$$\hat{m}(t) \longleftrightarrow -j \operatorname{sgn} f \, M(f)$$

$$m(t)\, e^{\pm j 2\pi f_c t} \Leftrightarrow M(f \mp f_c)$$

$$\hat{m}(t)\, e^{\pm j 2\pi f_c t} \longleftrightarrow -j\, M(f \mp f_c)\, sgn(f \mp f_c)$$

$$\mathcal{F}^{-1}\left\{ \frac{1}{4} A_c \left[M(f+f_c)\, sgn(f+f_c) - M(f-f_c)\, sgn(f-f_c) \right. \right.$$

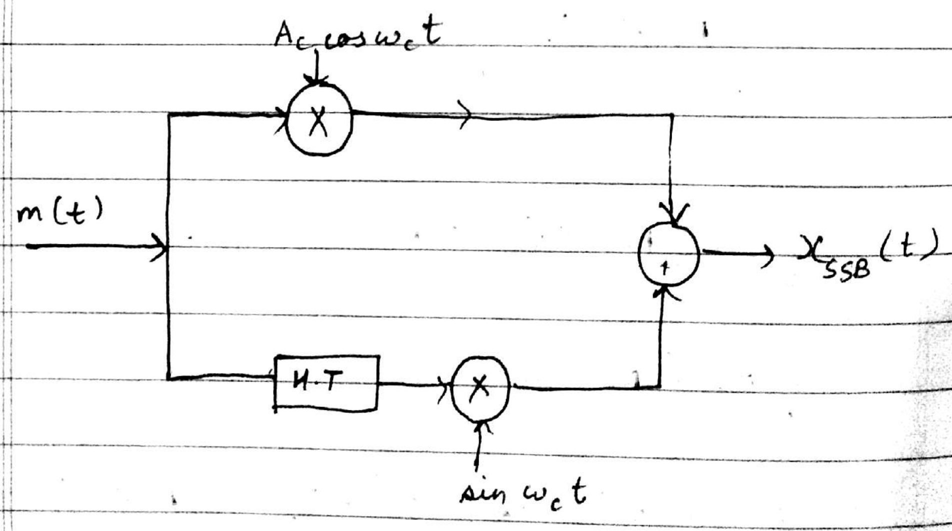
$$= -A_c \frac{1}{4j}\, \hat{m}(t)\, e^{-j 2\pi f_c t} \quad +$$

$$A_c \frac{1}{4j}\, \hat{m}(t)\, e^{j 2\pi f_c t}$$

$$= \frac{1}{2} A_c\, \hat{m}(t) \sin 2\pi f_c t$$

$$x_{SSB}(t) = \frac{1}{2} A_c\, m(t) \cos \omega_c t \quad +$$

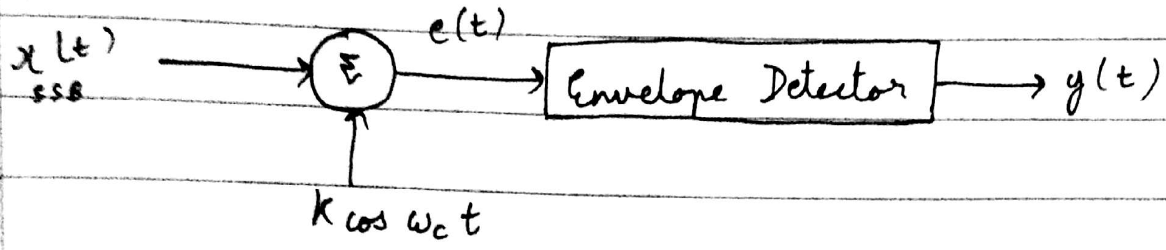
$$\frac{1}{2} A_c\, m(t) \sin \omega_c t$$



Phase component must be zero, otherwise message signal is not received properly

{ No warbling effect, $\theta = 0$

Synchronous modulation - carrier same at mod & demod end }



SSB → to utilize bandwidth

✓ Transmitting multiple signals Book

$m_1(t)$, $m_2(t)$

Interference effect