For the rising exponential,

$g(t) = \exp(at)u(-t)$, where $u(-t) = 1$, for $t<0$

$= 0$ for $t>0$

The fourier transform for this function is,

$$G(f) = \int_{-\infty}^{0} \exp(at)\exp(-j2\pi f t)\,dt$$

$$= \int_{-\infty}^{0} \exp((a-j2\pi f)t)\,dt$$

$$= \frac{1}{a-j2\pi f}$$

$$= \frac{1}{\sqrt{a^2+(j2\pi f)^2}} \exp\left(j\tan^{-1}\left(2\pi\left(\frac{f}{a}\right)\right)\right)$$

Thus. for the decaying and rising exponential functions the amplitude spectra are the same. and phase of one is negative while the phase spectrum of the other.

Gate function :-

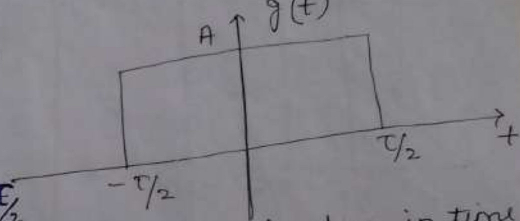


Fig: The gate function in time domain

The gate function is defined as:

$\Pi\left(\frac{t}{\tau}\right) = 1$; for $-\tau/2 < t < \tau/2$

$= 0$; for $|t| > \tau/2$

In terms of gate function the given rectangular pulse can be represented as

$$g(t) = A\Pi(t/\tau)$$

So, fourier transform of this function is given as - $G(f) = A\int_{-\tau/2}^{\tau/2} \exp(-j2\pi f t)\,dt$

$$= \frac{A}{j2\pi f}\left[\exp(j\pi f\tau) - \exp(-j\pi f\tau)\right]$$

$$= A\tau \frac{\sin(\pi f \tau)}{\pi f \tau}$$

$$= A\tau \operatorname{sinc}(f\tau)$$

Thus $A\Pi(t/\tau) \longleftrightarrow A\tau \operatorname{sinc}(f\tau)$

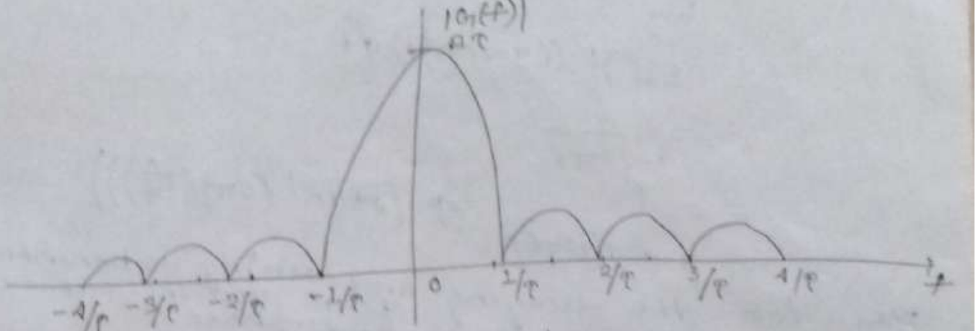


Fig: Amplitude Spectrum

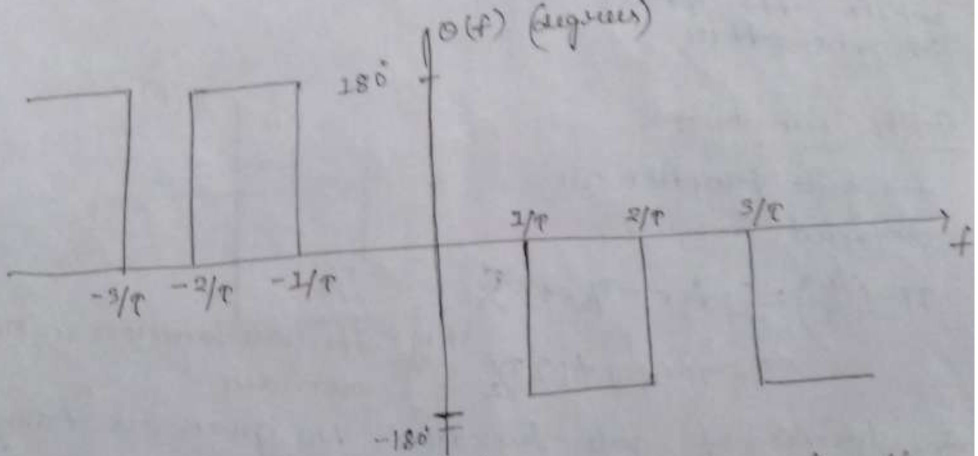


Fig: Phase Spectrum of the gate function

Triangular Pulse

The triangular pulse may is defined as

$$g(t) = 1 - \frac{|t|}{\tau}, \quad |t| \leq \tau$$
$$= 0, \quad |t| > \tau$$

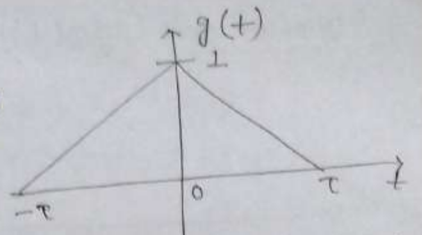


Fig: Triangular pulse in time domain

The Fourier transform of g(t) is given by

$$G(f) = \int_{-\tau}^{0} (1 + t/\tau) \exp(-j2\pi f t)\, dt + \int_{0}^{\tau} (1 - 1/t) \exp(-j2\pi f t)\, dt$$

$$= (1 + 1/\tau) \frac{\exp(-j2\pi f t)}{-j2\pi f} \Bigg|_{-\tau}^{0} + \frac{1}{\tau} \int_{-\tau}^{0} \frac{\exp(-j2\pi f t)}{-j2\pi f}\, dt$$

$$= \left(1 - \frac{t}{\tau}\right) \exp\left(\frac{-j2\pi f}{-j2\pi f}\right) \Bigg|_{0}^{\tau} - \frac{1}{\tau} \int_{0}^{\tau} \frac{\exp(-j2\pi f)}{-j2\pi f}\, dt$$

$$= \frac{1}{\tau} \frac{\exp(-j2\pi f t)}{(-j2\pi f)^2} \Bigg|_{-\tau}^{0} - \frac{1}{\tau} \frac{\exp(-j2\pi f t)}{(-j2\pi f)^2} \Bigg|_{0}^{\tau}$$

$$= \frac{2}{-4\pi^2 f^2 \tau} [1 - \cos 2\pi f \tau]$$

$$= \tau \frac{\sin^2 \pi f \tau}{\pi^2 f^2 \tau^2}$$

$$= \tau \frac{\sin^2 \pi f \tau}{\pi^2 f^2 \tau^2} \Rightarrow G(f) = \tau \operatorname{sinc}^2(f\tau)$$

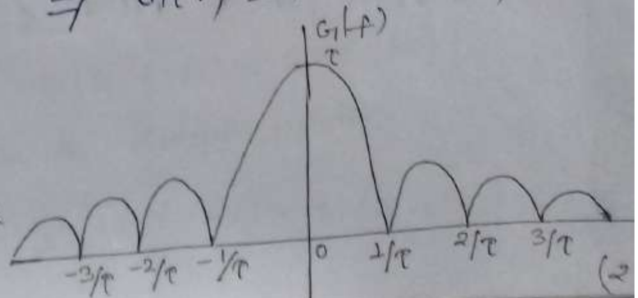
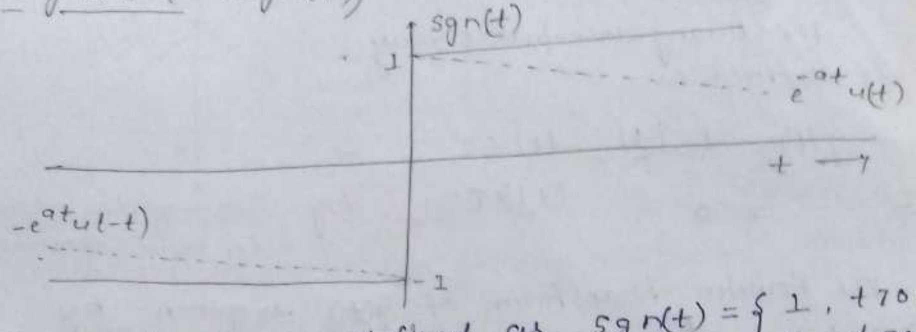


Fig: Spectrum of the triangular pulse

signum f (sgn(t))



signum function is defined as $sgn(t) = \begin{cases} 1, & t > 0 \\ 0, & t = 0 \\ -1, & t < 0 \end{cases}$

sgn(t) is not ~~proper~~ absolutely integrable to find its fourier transform because sgn(t) violates the Dirichlet condition. So, its transform can be obtained by considering sgn(t) as a sum of two exponentials, in the limit as a→0,

$$sgn(t) = \lim_{a \to 0} \left[e^{-at} u(t) - e^{at} u(-t) \right]$$

Therefore $F[sgn(t)] = \lim_{a \to 0} \left[F[e^{-at} u(t)] - F[e^{at} u(-t)] \right]$

$$= \lim_{a \to 0} \left(\frac{1}{a + j2\pi f} - \frac{1}{a - j2\pi f} \right)$$

$$= \lim_{a \to 0} \left(\frac{-j4\pi f}{a^2 + 4\pi^2 f^2} \right) = \frac{1}{j\pi f}$$

Using frequency shifting (frequency translation) property $g(t)\exp(j2\pi f_0 t) \longleftrightarrow G(f-f_0)$

similarly $g(t)\exp(-j2\pi f_0 t) \longleftrightarrow G(f+f_0)$

This theorem states that multiplication of a function in the time domain by $\exp(j2\pi f_0 t)$ is equivalent to shifting the spectrum in the frequency domain by f_0. Using this property spectra of low frequency signal is translated to higher frequency range. This is achieved using scheme, known as modulation.

Let $g(t)$ be the low frequency signal and $\cos(2\pi f_0 t)$ be the high frequency sinusoidal signal. Then we may write. the product of two signals. as-

$$g(t)(\cos 2\pi f_0 t) = \frac{1}{2}\left[g(t)\exp(j2\pi f_0 t) + g(t)\exp(-j2\pi f_0 t)\right]$$

using the linearity property we get,

$$F[g(t)\cos(2\pi f_0 t)] = \frac{1}{2}\{F[g(t)\exp(j2\pi f_0 t)] + \frac{1}{2}F[g(t)\exp(-j2\pi f_0 t)]$$

Using the frequency shifting property, we get

$$F[g(t)\cos 2\pi f_0 t] = \frac{1}{2}\left[G(f-f_0) + G(f+f_0)\right]$$

similarly, it can be shown that

$$F[g(t)\sin 2\pi f_0 t] = \frac{j}{2}\left[G(f+f_0) - G(f-f_0)\right]$$

Thus, the modulation process translates the frequency spectra of $g(t)$ by $\pm f_0$. The result is known as modulation theorem.

Ex

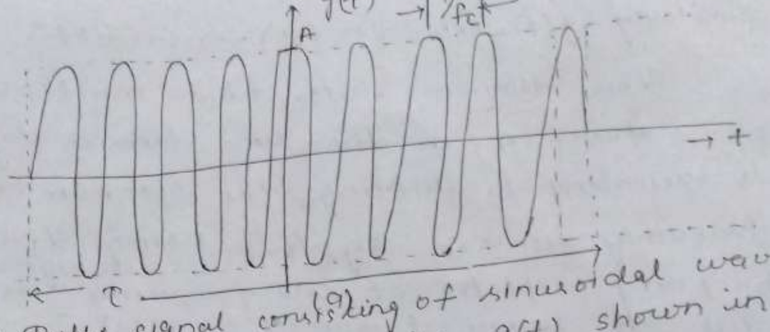


Fig: Pulse signal consisting of sinusoidal wave (a)

Consider the signal pulse $g(t)$ shown in above fig which consists of a sinusoidal wave of amplitude A & frequency f_c extending from $t = -\tau/2$ to $t = \tau/2$. Find the fourier transform of the signal using frequency shifting theorem.

Soln we can write above function mathematically as-

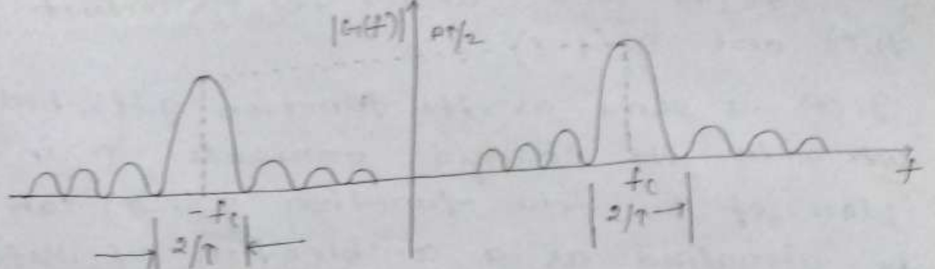
$$g(t) = A\Pi\left(\frac{t}{\tau}\right)\cos 2\pi f_c t$$

To, find the fourier transform of $g(t)$. we ~~get~~ have

$$A\Pi\left(\frac{t}{\tau}\right) \leftrightarrow A\tau\,\mathrm{sinc}(f/\tau)$$

Using the modulation theorem we get,

$$G(f) = \frac{A\tau}{2}\left[\mathrm{sinc}[(f-f_c)\tau] + \mathrm{sinc}[(f+f_c)\tau]\right]$$



(b)

Fig: Spectrum of the pulse signal consisting of sinusoidal wave.

Multiplication in time and frequency domain

Multiplication in time domain & frequency domain can be understood with the help of convolution. Suppose given two functions $g_1(t)$ and $g_2(t)$, we form the integral

$$g(t) = \int_{-\infty}^{\infty} g_1(\tau)\, g_2(t-\tau)\, d\tau.$$

The above integral is known as the convolution of the functions $g_1(t)$ and $g_2(t)$ and can be expressed symbolicall as

$$g(t) = g_1(t) * g_2(t)$$

Evaluation and interpretation of convolution integrals

From the above equation of convolutional integral, the value of $g(t)$ at any particular time t is seen to be area under the product $g_1(\tau)$ and $g_2(t-\tau)$.

$g_1(\tau)$ is same as the function $g_1(t)$ but ~~with~~ with the changed variable τ in place of t. The function $g_2(t-\tau)$ can be visualized as a combination of reflection and translation of original function $g_2(\tau)$. This process is called folding and sliding. ~~sam~~

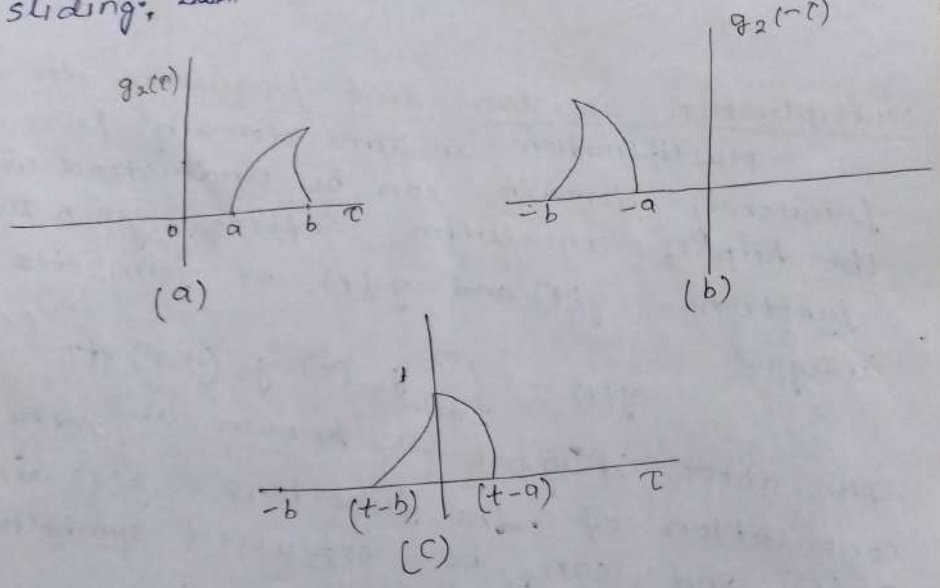
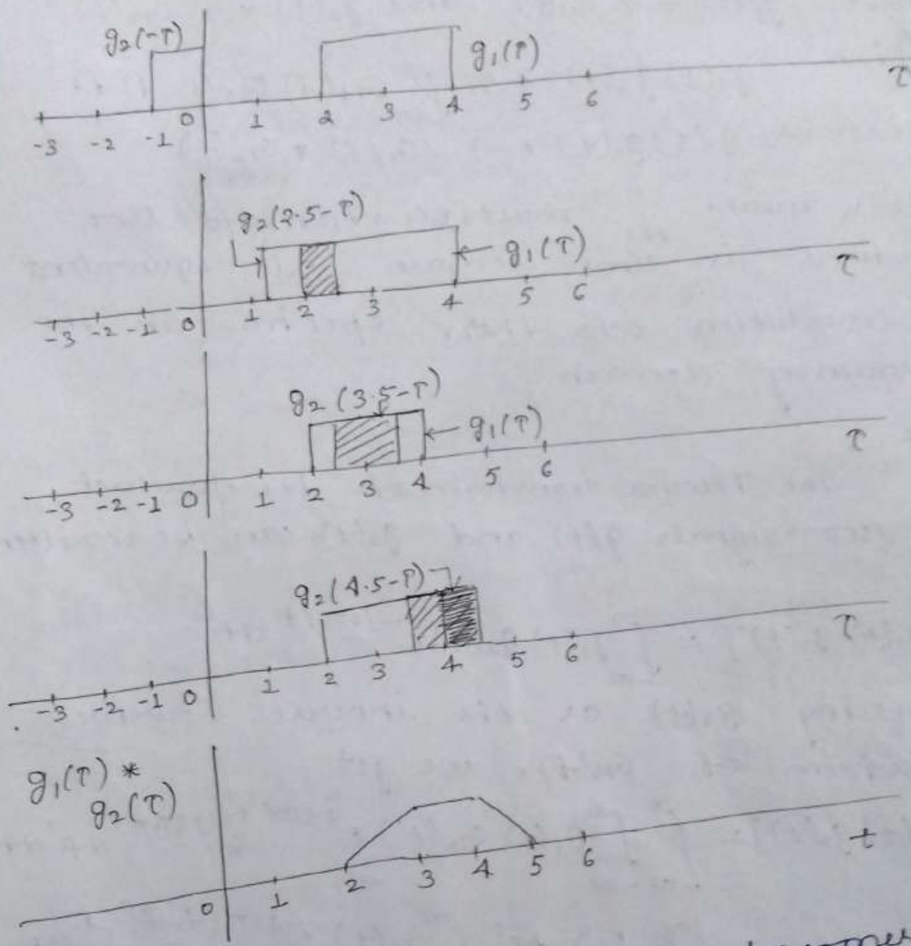


Fig: Folding and sliding the function

Convolution of two rectangular pulse:—



The convolvolved pulse becomes a trapezoidal pulse.

Multiplication in time domain :-

Let $g_1(t) \leftrightarrow G_1(f)$ and $g_2(t) \leftrightarrow G_2(f)$

then, $g_1(t)\,g_2(t) \leftrightarrow \int_{-\infty}^{\infty} G_1(\lambda)\,G_2(f-\lambda)\,d\lambda$,

that is, $g_1(t)\,g_2(t) \longleftrightarrow G_1(f) * G_2(f)$

This means the multiplication of two functions in the time domain is equivalent to convolution of their spectra in the frequency domain.

Proof :- The fourier transform of the product of two signals $g_1(t)$ and $g_2(t)$ can be written as-

$$F[g_1(t)\,g_2(t)] = \int_{-\infty}^{\infty} g_1(t)\,g_2(t)\,e^{-j2\pi ft}\,dt$$

Expressing $g_2(t)$ as the inverse fourier transform of $G_2(f)$, we get

$$F[g_1(t)\,g_2(t)] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g_1(t)\,G_2(f')\,e^{j2\pi f't}\,e^{-j2\pi ft}\,df'\,dt$$

$$= \int_{-\infty}^{\infty} G_2(f')\,df' \int_{-\infty}^{\infty} g_1(t)\,e^{-j2\pi(f-f')t}\,dt$$

putting $(f-f')$ as λ, we get

$$F[g_1(t)\,g_2(t)] = \int_{-\infty}^{\infty} G_2(f-\lambda)\,d\lambda \int_{-\infty}^{\infty} g_1(t)\,e^{-j2\pi\lambda t}\,dt$$

$$= \int_{-\infty}^{\infty} G_2(f-\lambda)\,G_1(\lambda)\,dt$$

$$= G_1(f) * G_2(f)$$

This property is known as convolution th.(3

Convolution in the time domain –

If $g_1(t) \leftrightarrow G_1(f)$ and $g_2(t) \leftrightarrow G_2(f)$

then,

$$F[g_1(t) * g_2(t)] = G_1(f)\, G_2(f) \;.\; \text{that is}$$

$$F[\int_{-\infty}^{\infty} g_1(\tau) g_2(t-\tau)\, d\tau] \leftrightarrow G_1(f)\, G_2(f)$$

Proof :-

$$F[g_1(t) * g_2(t)] = F[\int_{-\infty}^{\infty} g_1(\tau) g_2(t-\tau)\, d\tau]$$

$$= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g_1(\tau) g_2(t-\tau) \exp(-j2\pi f t)\, dt\, d\tau$$

$$= \int_{-\infty}^{\infty} d\tau\, g_1(\tau) \int_{-\infty}^{\infty} g_2(t-\tau) \exp(-j2\pi f t)\, dt \,.$$

utilizing Time shifting property of fourier transforms, above equation can be written as –

$$F[\int_{-\infty}^{\infty} g_1(\tau) g_2(t-\tau)\, d\tau] = \int_{-\infty}^{\infty} g_1(\tau)\, G_2(f)\, e^{(-j2\pi f\tau)}\, d\tau$$

$$= G_1(f)\, G_2(f)$$

This property is known as convolution theorem

Analog and Digital Modulation—

Sometimes digital transmission is not preffered.

Modulation is the systematic alteration of one wave-form called the carrier, according to the characteristics of another waveform, the modulating signal or message signal or baseband signal. The term baseband is used to designate the band of frequencies representing the original signal as defined by source of information. The fundamental goal of modulation is to produce an information bearing umodulated wave whose properties are best sutted for efficient utilization of communication channel.

Some important advantages of modulation are summarised below:

Frequency translation:- Modulation translates the signal from one region of frequency domain to another region. This helps to ~~translate~~ transmit the modulated signal with minimum attenuation through a particular medium.

Practical size of antenna:- Modulation translates baseband signal to higher frequency, which can be transmitted through a bandpass channel using an antenna of smaller sizes.

This has made communication practical.

Narrow-banding :— As modulation a signal from lower freq^n domain to higher frequency domain the ratio between highest to lowest freq of the signal becomes close to 1.

Multiplexing :— Different baseband signals originating from different sources can be translated to different frequency range. This allows transmission of different signals through the same medium using frequency division multiplexing (FDM).

Modulation schemes

↓ Linear modulation (superposition theorem holds)

↓ Non-linear modulation (exponential mod^n) (superposition theorem does not hold)

Modulation techniques (based on type of carrier)
→ continious wave (CW) mod^n (carrier is sinusoid)
→ Pulse modulation (carrier is pulse train

both linear and non-linear modulations are types of continious-wave modulation.

Linear mod^n ↓ Amplitued mod^n.

non-linear mod^n ↓ frequency and phase mod^n

Amplitude Modulation (AM) :— Amplitude modulation is defined as the process in which amplitude of the carrier wave is varied about a mean value, linearly with the baseband signal. The basic version of the amplitude modulation is also termed as double sideband full carrier (DSBFC) technique. Modulating freqn is lower than carrier freqn. AM is defined as a system of modulation in which the amplitude of the carrier is made proportional to the instantaneous amplitude of the modulating voltage.

Suppose the carrier voltage and the modulating voltage, v_c and v_m, respectively be represented by,

$$v_c = V_c \sin w_c t \quad \text{------} \quad (i)$$

$$v_m = V_m \sin w_m t \quad \text{------} \quad (ii)$$

Phase angle has been ignored, since it is unchanged by amplitude modulation.

For AM, the maximum amplitude V_c of the unmodulated carrier will have to made proportional to the instantaneous modulating voltage $V_m \sin w_m t$.

$m = V_m / V_c$ is defined as modulation index, which is a number lying between 0 & 1. and it is often expressed as a percentage and called the percentage modulation.

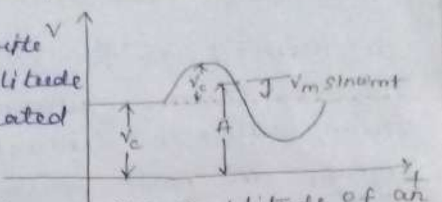


Fig1: Amplitude of an AM wave

from fig 1, we can write an equation for the amplitude of the amplitude modulated voltage, we have

$$A = V_c + v_m = V_c + V_m \sin\omega_m t$$
$$= V_c + mV_c \sin\omega_m t$$
$$= V_c (1 + m\sin\omega_m t)$$

The instantaneous voltage of the resulting amplitude modulated n wave is

$$v_{AM} = A\sin\omega_c t = V_c(1+m\sin\omega_m t)\sin\omega_c t$$

using trigonometric relation si

$$\sin x \sin y = \frac{1}{2}[\cos(x-y) - \cos(x+y)]$$. to expand above eqn of v_{AM}, we get,

$$v_{AM} = V_c \sin\omega_c t + \frac{mV_c}{2}\cos(\omega_c - \omega_m)t$$
$$- \frac{mV_c}{2}\cos(\omega_c + \omega_m)t$$

Thus the process of AM has the effect of adding to the unmodulated wave, rather the changing it. The additional terms produced are the two sidebands added. The frequency of lower sideband is (LSB) $(f_c - f_m)$ and uppersideband (USB) is $(f_c + f_m)$

So the bandwidth required for the AM is twice the freqn of the modulating signal. (4)

that is, $B_{AM} = (f_c+f_m) - (f_c-f_m) = 2f_m$

From fig2ii, we can say that AM contains three discrete frequencies. The central freqn, the carrier has the highest amplitude, and the other two are disposed symmetrically about it, having amplitudes equal to each other, but which can never exceed half the carrier amplitude. m cannot be more than unity.

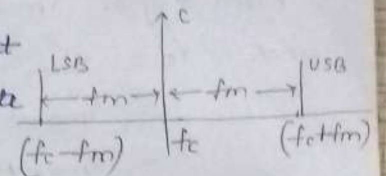


Fig2ii: Frequency spectrum of an AM wave.

Time Domain Representation of the AM Wave

Fig 3. shows one cycle of sinusoidal modulating wave,. Now the envelope of the AM wave may be given as-

$$A = V_c + V_m \sin\omega_m t.$$

The maximum negative amplitude or bottom envelope, is given by $-A = -(V_c+V_m \sin\omega_m t)$ The modulated wave extends between these two limiting envelops and has a repetition rate equal to

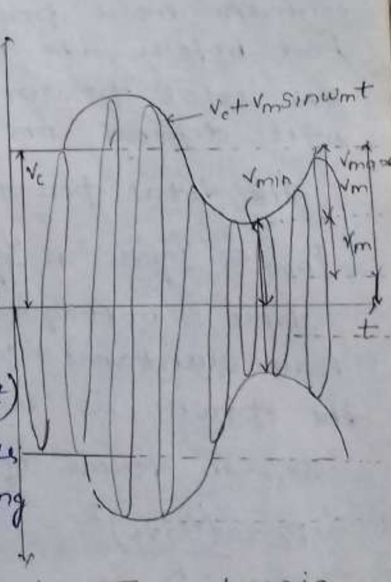


Fig 3: Time domain representation of AM wave

the unmodulated carrier freq^n.

From fig 3, $V_m = \frac{V_{max}\,V_{min}}{2}$

and, $V_c = V_{max} - V_m$

$$= V_{max} - \frac{(V_{max} - V_{min})}{2}$$

$$= \frac{V_{max} + V_{min}}{2}$$

Dividing the eq^n of V_m, by the eq^n V_c, we have

$$m = \frac{V_m}{V_c} = \frac{V_{max} - V_{min}}{V_{max} + V_{min}}$$

Power relations in the AM wave — The modulated wave contains extra energy in the two sideband components. Therefore, the modulated wave contains more power than the carrier had before the modulation took place. The total power in the modulated wave will depend on the modulation index also.

The total power of the modulated wave—

$$P_{AM} = \frac{V_{carr}^2}{R} + \frac{V_{LSB}^2}{R} + \frac{V_{USB}^2}{R}$$

where voltages are in rms value. (antenna resistance), in which R is resistance the power is dissipated.

so, in terms of peak value of voltages,

$$P_c = \frac{(V_c/\sqrt{2})^2}{R} = \frac{V_c^2}{2R}$$

$$P_{LSB} = P_{USB} = \frac{V_{SB}^2}{2} = \left(\frac{mV_c/2}{\sqrt{2}}\right)^2 \div R = \frac{m^2V_c^2}{8R}$$
$$= \frac{m^2}{4} \cdot \frac{V_c^2}{2R}$$

So, total power of the AM modulated waveform is

$$P_{AM} = \frac{V_c^2}{2R} + \frac{m^2}{4}\frac{V_c^2}{2R} + \frac{m^2}{4}\frac{V_c^2}{2R}$$

$$\Rightarrow \frac{P_{AM}}{P_c} = \left(1 + \frac{m^2}{2}\right)$$

The maximum power in the AM wave is,
$P_{AM} = 1.5P_c$, when $m = 1$.

Double Sideband Suppressed Carrier (DSBSC) Technique

In this case message to be txed is present only in LSB and USB. So, the power relation becomes-

$$P_{AM} = P_c\left(1+\frac{m^2}{2}\right)$$

$$\Rightarrow P_c = \frac{P_{AM}}{\left(1+m^2/2\right)}$$

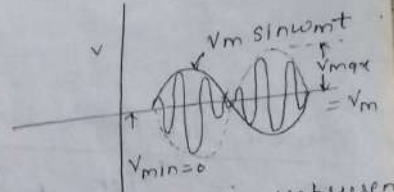


Fig 1: Time domain representation of DSBSC wave

Let $m=1$,
then, $P_c = \frac{2}{3}P_{AM}$

The the two-third of the total power is utilized by the carrier component. So, power which does not bear any message the carrier can be saved by suppressing before transmission. This variant of basic AM is termed as double sideband suppressed carrier technique. The instantaneous voltage of DSBSC may be written as-

(7)

$v_{DSBSC} = v_{AM} - V_c \sin w_c t$

So, $v_{DSBSC} = V_c \sin w_c t + \frac{mV_c}{2} \cos(\omega_c - \omega_m)t \mp \frac{mV_c}{2} \cos(\omega_c + \omega_m)t$

$\Rightarrow v_{DSBSC} = \frac{mV_c}{2} \cos(\omega_m - \omega_c)t \mp \frac{mV_c}{2} \cos(\omega_c + \omega_m)t$

So, the bandwidth required for the DSBSC is twice the frequency of the modulating signal.

$B_{DSBSC} = (f_c + f_m) - (f_c - f_m)$

$= 2f_m$

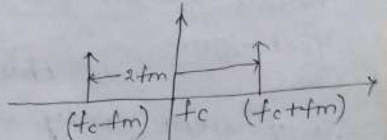


Fig5: Frequency spectrum of DSBSC wave.

Power Relations in the DSBSC wave:-

$P_{DSBSC} = \frac{V_{LSB}^2}{R} + \frac{V_{USB}^2}{R}$

where V_{LSB} & V_{USB} are in r.m.s value, and R is the resistance in which power is dissipated

$P_{LSB} + P_{USB} = \frac{V_{SB}^2}{R} = \left(\frac{mV_c/2}{\sqrt{2}}\right)^2 \div R + \left(\frac{mV_c/2}{\sqrt{2}}\right)^2 \div R$

$= \frac{m^2}{4} \frac{V_c^2}{2R} + \frac{m^2}{4} \frac{V_c^2}{2R}$

$= \frac{m^2}{2} \frac{V_c^2}{2R}$

$\Rightarrow P_{LSB} \mp P_{USB} = P_c (m^2/2)$

for $m=1$, $P_{LSB} \mp P_{VSB} = P_c/2$

Thus sidebands contain 50% the unmodulated carrier power.

~~carrier power.~~

Generation of AM signal:—

Using Analog Multiplier:—

The output of the analog multiplier is given by

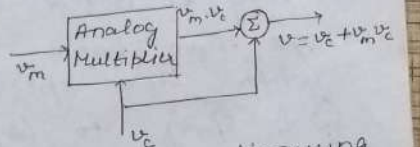


Fig 6: AM generation using analog multiplier

$$v' = v_m v_c = V_m \sin\omega_m t \sin\omega_c t$$

$$= \frac{mV_c}{2}\cos(\omega_c-\omega_m)t - \frac{mV_c}{2}\cos(\omega_c+\omega_m)t$$

At the output of the analog multiplier we have two sidebands. Now adding the unmodulated carrier component to this, the requisite AM signal and is given by.

$$v = v_c + v_m \cdot v_c = V_c \sin\omega_c t + \frac{mV_c}{2}\cos(\omega_c-\omega_m)t + \frac{mV_c}{2}\cos(\omega_c+\omega_m)t$$

Demodulation of AM signal:—

Envelope detector:—

At the receiving end the signal is being demodulated to get the original data.

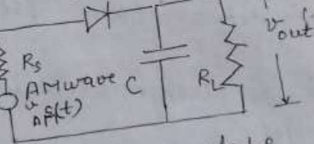


Fig 7: Envelope detector for AM wave

Let a baseband signal $v_m(t)$ is translated out by multiplication with the carrier signal $\cos\omega_c t$ to get $v_m(t)\cos\omega_c t$. By multiplying second time with the carrier, we get

$(v_m(t)\cos\omega_c t)\cos\omega_c t$

$= v_m(t)\cos^2\omega_c t \quad = v_m(t)\left\{\frac{1}{2}(1+\cos 2\omega_c t)\right\}$

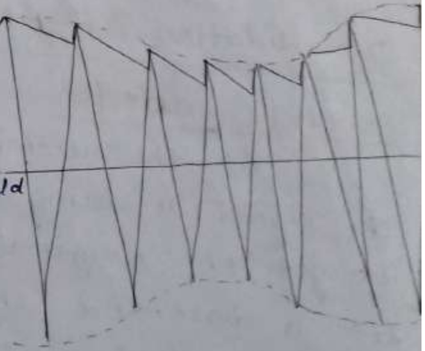
$= \frac{v_m(t)}{2} + \frac{v_m}{2}\cos 2\omega_c t$

It may be noted that

- The base band signal reappears along with two spectral components of freq^n $(2f_c - f_m)$ to $(2f_c + f_m)$
- The Spectral components $2f_c - f_m$ to $2f_c + f_m$ can be easily removed by a low-pass filter. This process is known as synchronous detection.

The synchronous detection approach has the disadvantage. that the carrier signal used in the second multiplication has to be precisely synchronous. Fig 7, shows, the accomplishment of the base band signal using envelope detection

The detector consists of a diode and a resistor/capacitor filter



the design criterion of RC should be.

$\frac{1}{\omega_c} << RC < 1/2\pi B.$

or

$2\pi B < \frac{1}{RC} << \omega_c$

Fig 8: Envelope detection

$\omega_c \rightarrow$ ripple of frequency

dc term can be blocked by a capacitor o a simple high-pass filter. The ripple may be reduced by another (low-pass) RC filter. (1

Exponential Modulation.

Phase and Frequency Modulation –

In the linear modulation scheme, the modulated spectrum is basically the translated message spectrum and the transmission bandwidth never exceeds twice the message BW. Also in linear mod^n schemes, the signal-to-noise ratio never at the Rx is no better than the base-band transmission and can be improved only by increasing the transmitted power.

In contrast to linear modulation, exponential mod^n is a non-linear process. As a consequence, the modulated spectrum is related in a complicated fashion to the message spectrum. Also the transmission BW of exponential modulation is usually much greater than twice the message BW. Because of this larger BW., exponential mod^n inherently provides increased signal-to-noise ratio without increasing txed power. In exponential mod^n the angle of the carrier is varied in accordance with the baseband signal, + so, it is also called angle mod^n. There are two forms of angle mod^n –

Frequency Modn (FM) and Phase Modn (PM).
The properties of one modn can be
derived from those of other.

Mathematical Representation:-

An angle modulated sinusoidal
carrier can be written in the form-

$$s(t) = A_c \cos(\theta_i(t)) \qquad \text{.......(i)}$$

$\theta_i(t)$ → angle of the modulated sinusoidal
carrier and A_c is the amplitude, which
is same as that of the unmodulated
carrier.

$\theta_i(t)$ → 2π radians ~~represen~~ represents a
completed oscillation.

Assuming $\theta_i(t)$ to increase monotonically
with time, the avg freqn in Hz over
the interval t and $t+\Delta t$ can be written
as-

$$f_{\Delta t} = \frac{\theta_i(t+\Delta t) - \theta_i(t)}{2\pi\Delta t} \quad \text{(ii)}$$

→ angular vel ~~freqn~~
= rate of change of phase angle of the angle

The instantaneous freqn by
modulated wave $s(t)$,

$$f_i(t) = \lim_{\Delta t \to 0} f_{\Delta t}(t)$$

$$= \lim_{\Delta t \to 0} \frac{\theta_i(t+\Delta t) - \theta_i(t)}{2\pi\Delta t}$$

$$= \frac{1}{2\pi}\frac{d\,\theta_i(t)}{dt} \qquad \text{....(iii)}$$

Thus the angle modulated wave s(t) can be considered as a rotating phasor of length Ac and angle $\theta_i(t)$. The instantaneous angular velocity is given by

$$\omega_i = \frac{d\theta_i(t)}{dt} \quad \cdots (iv)$$

In case of an unmodulated carrier, the instantaneous phase angle is,

$$\theta_i(t) = 2\pi f_c t + \theta_0 \quad (v)$$

where θ_0 is the initial value of the phase angle i.e. $\theta_i(t)$ at $t=0$. An unmodulated carrier thus can be represented as

$$e_c(t) = A_c \cos(2\pi f_c t + \theta_c) \quad \cdots\cdots (vi)$$

The unmodulated carrier can be represented as ~~at~~ phasor of length Ac, which rotates with a constant angular velocity $2\pi f_c$.

Phase Modulation—(PM)

Phase mod^n is a form of angle mod^n in which the angle $\theta_i(t)$ is made to vary linearly with the baseband signal $e_m(t)$. Assuming the initial phase angle $\theta_0 = 0$, we get

$$\theta_i(t) = 2\pi f_c t + K_p e_m(t) \quad \cdots\cdots (vii)$$

where $(2\pi f_c t)$ represents the angle of the unmodulated carrier having a frequency f_c and the constt K_p represents the phase sensitivity of the modulator expressed in radians/volt.

Thus the phase modulated wave in the time domain can be expressed in the form –

$$s(t) = \cos\left(2\pi f_c t + k_p\, e_m(t)\right) \qquad \text{(viii)}$$

Frequency Modulation – (FM)

FM is the form of angle modn in which the instantaneous freqn $f_i(t)$ of the carrier is varied linearly with the baseband signal $e_m(t)$.

Thus the instantaneous freqn of an FM signal can be represented as

$$f_i(t) = f_c + K_f\, e_m(t) \qquad \text{(ix)}$$

f_c → freqn of unmodulated carrier

K_f → frequency sensitivity of the modulator expressed as in Hz/volt.

from eqn (iii) + (ix)

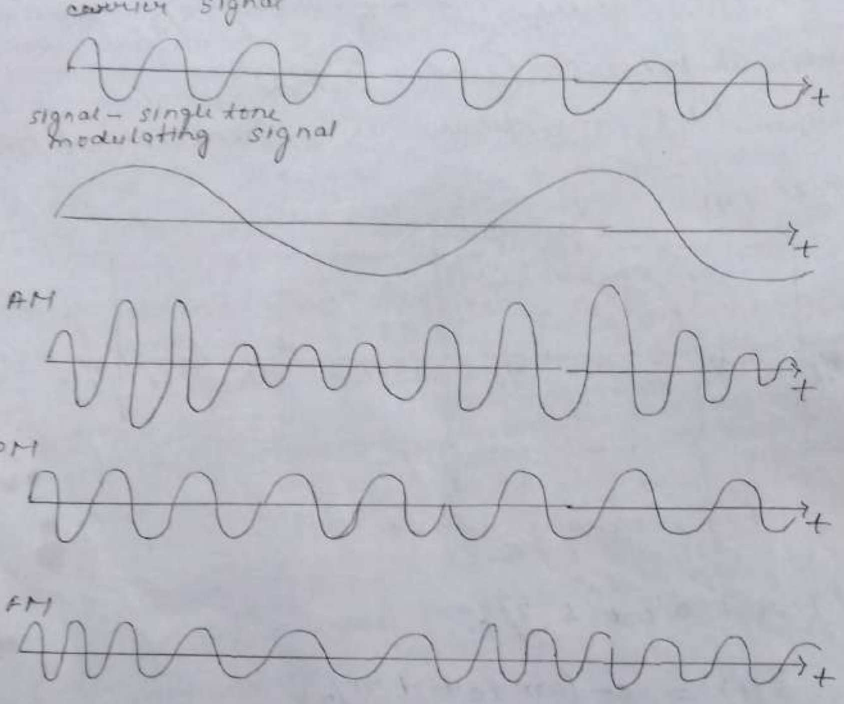
$$\frac{1}{2\pi}\frac{d}{dt}\theta_i(t) = f_c + K_f\, e_m(t)$$

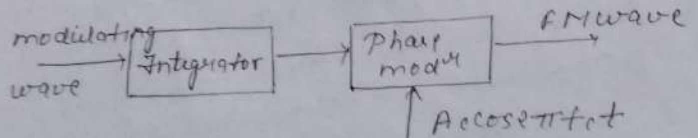
That is, $$\theta_i(t) = 2\pi f_c t + 2\pi K_f \int_0^t e_m(t)\,dt \qquad \text{(x)}$$

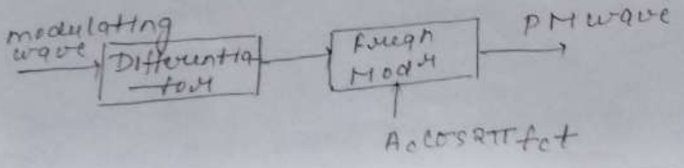
In the above eqn angle of un modulated carrier is assumed to zero. (1)

Thus the generalized form of angle modulated wave is given by

$$s(t) = A_c \cos\left(2\pi f_c t + 2\pi k_f \int_0^t m(t)\,dt\right) \qquad (xi)$$







Single - tone Frequency Modulation:-

Ex :- Consider a signal

$$s(t) = \cos(2\pi f_c t + \phi(t))$$

where $\phi(t)$ is a square wave taking on the values $\pm \pi/3$ every $2/f_c$ sec.

a) Sketch $\cos(2\pi f_c t + \phi(t))$

b) Plot the phase as a function of time.

Soln (a)

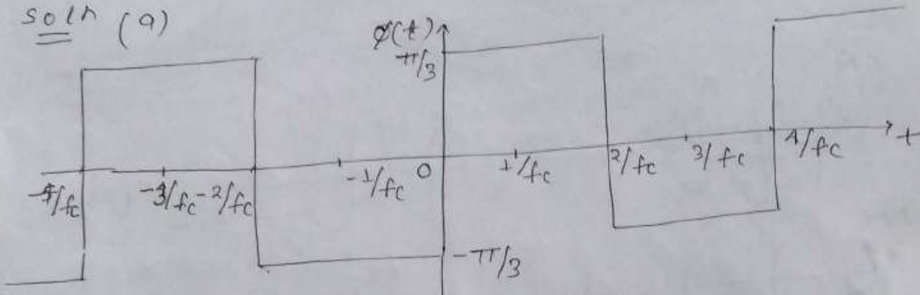


Fig: square wave $\phi(t)$

for $0 < t < 2/f$

$$s(t) = \cos(2\pi f_c t + \pi/3)$$

Single-tone Frequency Modulation :-

Eqn of freqn modulated wave is -

$$s(t) = A_c \cos\left(2\pi f_c t + 2\pi k_f \int_0^t e_m(t)\, dt\right) \quad \text{---- (11)}$$

From the above eqn it is clear that FM signal is non-linear function of the baseband signal. Thus freqn modulation is basically a non-linear scheme. The spectrum of ~~AF~~ FM wave is related in a complicated manner with the baseband signal.

Consider a simple case in which the modulating signal is considered as a single-tone (single freqn) sinusoidal given by

$$e_m(t) = A_m \cos(2\pi f_m t) \quad \text{---- (12)}$$

The instantaneous freqn from eqn (ix),

$$f_i(t) = f_c + \overset{(Hz/V)}{k_f} A_m \cos(2\pi f_m t) \quad \text{---- (13)}$$

$$= f_c + \Delta f \cos(2\pi f_m t)$$

where $\Delta f = k_f A_m$ denotes the deviation frequency from the unmodulated carrier frequency.

$$\text{---- 14(a)}$$

Therefore, $f_i|_{max} = f_c + \Delta f$ ---- 14(b)

$$f_i|_{min} = f_c - \Delta f$$

Thus Δf represents the maxm departure of the instantaneous freqn of the FM wave from the unmodulated carrier freqn f_c.

Δf is called freqn deviation.

Thus Δf represents the maxm departure of the instantaneous freqn of the FM wave from the unmodulated carrier freqn called freqn deviation.

Thus Δf is proportional to the amplitude of the base band (modulating) signal and is independent of the modulating frequency.

The instantaneous angle from of the FM wave can be obtained from eqn(x) as-

$$\theta_i(t) = 2\pi f_c t + 2\pi k_f \int_0^t A_m \cos(2\pi f_m t)\,dt$$

$$= 2\pi f_c t + \frac{k_f A_m}{f_m} \sin(2\pi f_m t) \quad \cdots (15)$$

$$= 2\pi f_c t + \frac{\Delta f}{f_m} \sin(2\pi f_m t)$$

$\frac{\Delta f}{f_m}$ is called as modulation index of FM wave.

Thus we may write

$$\beta = \frac{\Delta f}{f_m} \quad \cdots (16)$$

and $\theta_i(t) = 2\pi f_c t + \beta \sin(2\pi f_m t)$ --- (17)

so, β $\theta_i|_{max} = 2\pi f_c t + \beta$ ..... 18(a

$\theta_i|_{min} = 2\pi f_c t - \beta$ ..... 18(b)

Thus, the parameter β represents the phase deviation of FM which corresponds to the max^m departure of the instantaneous angle $\theta_i(t)$ from the unmodulated carrier angle $2\pi f_c t$.

Thus from eqn(11) FM wave can be written as $s(t) = A_c \cos(2\pi f_c t + \beta \sin(2\pi f_m t))$

So, depending the value of β. we may distinguish two types of FM.

(1) Narrow-band freqn modulation (NBFM)- for which β is small compared to 1 radian.

(2) Wide-band freqn modulation (WBFM)- for which β is large compared to 1 radian.

Narrow band FM ($\beta << 1$ rad) requires the same bandwidth
2fm as AM wave.

For larger value of $\beta >> 1$ radian, the FM
signal becomes wideband., Ideally the BW
is ~~wideband~~ infinite in such cases.
So, For single tone modulation the FM wave
can be written as -

$s(t) = A_c \cos(2\pi f_c t + \beta \sin(2\pi f_m t))$ ....(19)

The WBFM can be expressed as $\cos(\beta \sin(2\pi f_m t))$

$s(t) = A_c \cos(2\pi f_c t) \sin(\beta \sin(2\pi f_m t))$ --- (20)

$- A_c \sin(2\pi f_c t) \cos(\beta \sin(2\pi f_m t))$ and
From the terms in eqn (30) it reveals, that former
$\sin(\beta [\sin(2\pi f_m t))$, and later
is an even function of fm. So, eqn (30) may be
is an odd fn of fm. So, eqn (20) may be
written as right side of eqn fourier series
So, terms at the in terms of
written as in fundamental freqn of fm.
with coefficients of the terms appearing
The coefficients of the representation of
in the fourier series a function of β.
the two fn will be is an even fn, the
since $\cos(\beta \sin(2\pi f_m t))$ harmonics will be
coefficients of the odd is an odd
zero. $\sin(\beta \sin(2\pi f_m t))$ of the even
function, the coefficients
harmonics will be zero.
So, $\cos(\beta \sin(2\pi f_m t))$ and $\sin(\beta \sin(2\pi f_m t))$
can be expressed in fourier series in
following forms-

$$\cos(\beta \sin(2\pi f_m t)) = J_0(\beta) + 2J_2(\beta)\cos(2\pi 2 f_m t) + 2J_4(\beta)\cos(2\pi 4 f_m t) + \dots + 2J_{2n}(\beta)\cos(2\pi 2n f_m t) + \dots \quad (21)$$

and

$$\sin(\beta \sin(2\pi f_m t)) = 2J_1(\beta)\sin(2\pi f_m t) + 2J_3(\beta)\sin(2\pi 3 f_m t) + \dots + 2J_{2n-1}(\beta)\sin(2\pi(2n-1) f_m t) \quad (22)$$

The function $J_n(\beta)$ is known as 'Bessel function' of a first kind of order n, having an argument.

So, from eqns. (20),(21) & (22) we get.

$$s(t) = A_c \cos(2\pi f_c t)[J_0(\beta) + 2J_2(\beta)\cos(2\pi 2 f_m t) + 2J_4(\beta)\cos(2\pi 4 f_m t) + \dots] - A_c \sin(2\pi f_c t)[2J_1(\beta)\sin(2\pi f_m t) + 2J_3(\beta)\sin(2\pi 3 f_m t) + 2J_5(\beta)\sin(2\pi 5 f_m t) + \dots]$$

$$= A_c J_0(\beta)\cos(2\pi f_c t) - A_c J_1(\beta)[\cos(2\pi(f_c - f_m)t) - \cos(2\pi(f_c + f_m)t)] + A_c J_2(\beta)[\cos(2\pi(f_c - 2f_m)t) + \cos(2\pi(f_c + 2f_m)t)] - A_c J_3(\beta)[\cos(2\pi(f_c - 3f_m)t) - \cos(2\pi(f_c + 3f_m)t)] + A_c J_4(\beta)[\cos(2\pi(f_c - 4f_m)t) + \cos(2\pi[f_c + 4f_m)t] + \dots \quad (23)$$

Thus from the properties of ~~even~~ Bessel function, we can find that for even value of n, we have

$$J_n(\beta) = J_{-n}(\beta)$$

whereas for odd values of n, we have

$$J_n(\beta) = -J_{-n}(\beta)$$

that is, $J_n(\beta) = (-1)^n J_{-n}(\beta)$ ..... (24)

So, using properties of Bessel function eqn (23) becomes –

$$s(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cos(2\pi (f_c + n f_m) t) \quad \cdots \quad (25)$$

The above eqn (25) is desired form of for the fourier-series representation of the single-tone FM signal for an arbitrary value of β. The spectrum of s(t) can be found by taking fourier transform of both sides of eqn (25), we get,

$$S(f) = \frac{A_c}{2} \sum_{n=-\infty}^{\infty} J_n(\beta) \left[\delta(f - f_c - n f_m) + \delta(-f_f + f_c + n f_m)\right] \quad \cdots \quad (26)$$

The variation of the Bessel function $J_n(\beta)$, which determines the amplitude of various side freqn components of wideband FM, has been plotted against the modulation index β (the argument of the Bessel function), for different positive integer values of n in eqn (26).

Using the following approximations of Bessel function $J_n(\beta)$, for small values of β.

$$J_0(\beta) \simeq 1 \quad \cdots \quad (27)$$
$$J_1(\beta) \simeq \frac{\beta}{2}$$
$$J_n(\beta) \simeq 0, \quad n > 1,$$

eqn (25) may be written as,

$$s(t) = A_c \cos(2\pi f_c t) + \frac{\beta A_c}{2} \left[\cos 2\pi (f_c + f_m) t - \cos(2\pi (f_c - f_m) t\right] \quad \cdots \quad (28)$$

which is same as the narrow band FM (NBFM) representation given in eqn (17).

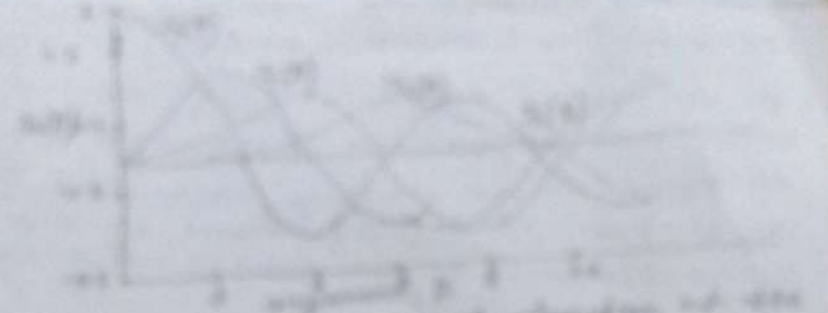


Fig: [illegible] of the Bessel function of the first kind with argument β.

Transmission bandwidth of frequency modulated wave:- In the case of single-tone frequency modulation, for large values of β, the bandwidth [illegible] only slightly greater than the total frequency deviation (2Δf). On the other hand, for small values of β, the [illegible] to carrier [illegible] which is [illegible] and a pair of side-frequencies at [illegible] so that bandwidth approaches [illegible] calculating [illegible] is [illegible] of the signals, known as the approximate [illegible] Carson's rule. (2x)

$B_T \simeq 2\Delta f + 2f_m$

$B_T \simeq 2\Delta f\left(1+\frac{1}{\beta}\right)$

Vestigial Side band Modulation— One of the side band is partially suppressed and vestige of other sideband is txed to compensate for that suppression.. Use frequency discrimination method.

DSB-SC- signal generated and then pass it through a band-pass filter

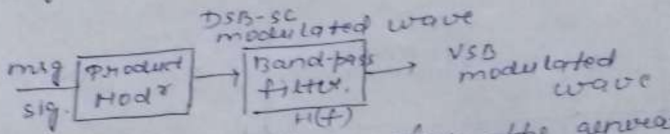


Fig: Filtering scheme for the generation of VSB modulated wave.

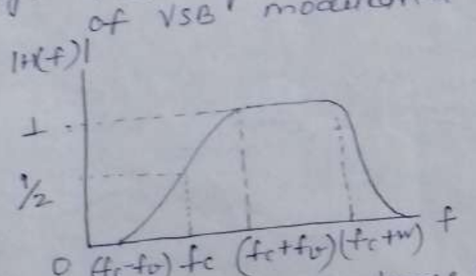


Fig: Magnitude response of VSB filter, only +ive frequency portion is shown.

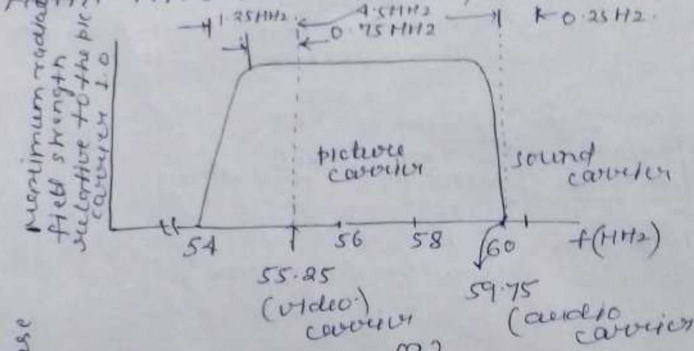
<u>Television signals</u> Use of vestigial sideband in commercial TV broadcasting.

Reason—

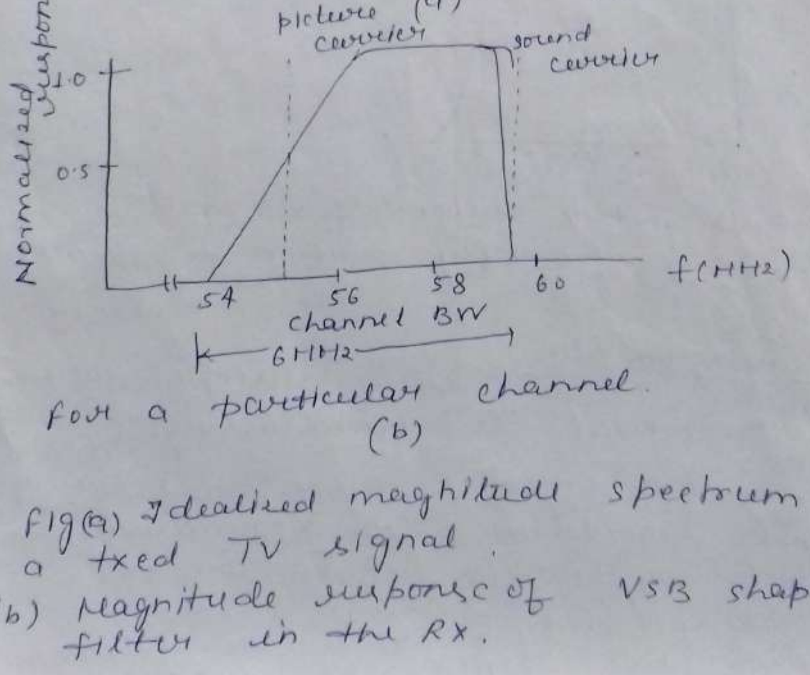
1. The video signal exhibits a large BW content and significant low frequency ~~compo~~ which suggest the use of VSB.
2. The circuitry used for demmod^n in the Rx should be simple and(1)

Therefore inexpensive. this suggests the use of envelope detection which require the addition of the carrier to the VSB-modulated wave.

Channel BW for TV Broadcasting in North America 6 MHz.,



(a)



For a particular channel.

(b)

Fig(a) Idealized magnitude spectrum of a txed TV signal.

(b) Magnitude response of VSB shaping filter in the RX.

(2)