Genetic Algorithm David Goldberg

Deterministic algorithm values are finite & we know the final output and we apply algorithm & can also verify it.

$L = \{3, 1, 2, 5\}$

$sort = \{1, 2, 3, 5\}$.

Now if $f(x) = \sin x$

$0 \leq x \leq 3$ find $\max(f(x))$

These are the N-P hard problems because input are not limited (non-polynomial)

so we divide the input in fixed no. of points.
these fixed no. of points will be calculated according to the computation limit of computer.

Binary genetic algorithm

$0 \leq x \leq 3$

divide in 4 points 0, 1, 2, 3 00, 01, 10, 11

if we divide in 2^n points

↳ we can assign these n points in binary encoding

$$X_{actual}(E_h) = X_{lower} + \frac{X_{higher} - X_{lower}}{2^l - 1}\left(Dec(B)\right)$$

X_L X_H
0 – 3

$$X_{ac} = 0 + \frac{3-0}{4-1}\left(Dec(B)\right)$$

$$= 0 + 1 \times 00 = 0$$

max $f(x) = \frac{x^2}{1+x^2}$ $\Rightarrow 110011$

$1 \le x \le 7$

2^6 points are considered

$$X_{ac} = 1 + \frac{7-1}{64-1}\left(\cancel{Dec(110011)}\ Dec(110011)\right)$$

$$= 1 + \frac{6 \times 51}{63} \boxed{\frac{(110011)}{(51)}}$$

$$= 1 + \frac{2 \times 17}{7} = 1 + \frac{34}{7} = 5.8571$$

$0 \le x \le 3$

0 1 2 3 $\quad 4+4 \Rightarrow 8$

2 3 5 6 $\quad 8+4 \Rightarrow 12$

calculate points 4

calculate $f(x)$ 4

find max. 4

Total 12 calculations

for 2^{40} points 3×2^{40} calculations are needed

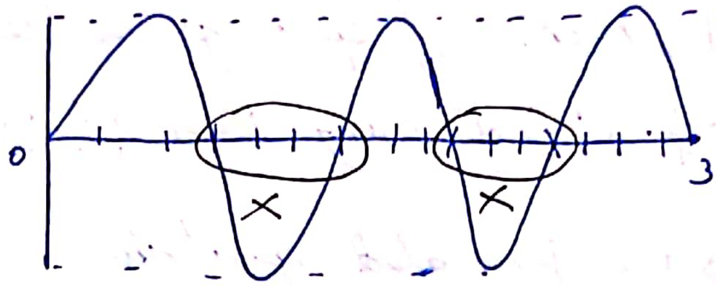
we don't go for the best one but for the good one

choosing path from Design center to SBIS via CSED not via ujjwal..

finding max. CPI in class

asking everyone O(N)

all > 9 complexity reduced



16 points

cut the one which occurs in area which is decreasing so points are reduced to 8 points.

This is intelligence

6 Sept' 2019

Binary GA

$$f(x) = \frac{x}{1+x^2} \qquad 0 \le x \le 3$$

find max.

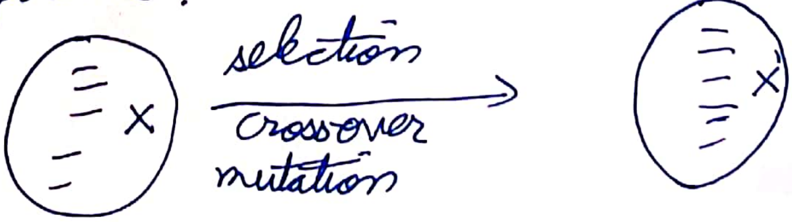
if dividing search space in 2^{10} points

first step is

Why GA?

process of GA is based on Darwin's principal of natural selection of evolution

if you have some set of species at a particular time 'T' then there will be some ~~crossovers~~, selection and some crossover & mutation will be done and some newly generated off springs are generated & these off springs are considered better than their parents.



how you will check off springs are better?

if x is parent & x' is off spring then $f(x') > f(x)$

if off springs are generated repeatedly

$f(x''') > f(x'') > f(x') > f(x)$ & so on.

so how many points to be considered for search space?

we choose atleast 40 bits but here let say we choose 5 bits.

so 2^5 solutions will be there

00000 to 11111 0 to 31

00000

11111 → initialization

binary representation of any solution is called chromosome.

genetic structure is improving everytime.
genetic structure is binary number

for 0 to 31 choose 5 random numbers (distinct)

Let say 5 random values are 1, 4, 7, 11, 21

00001
00100
00111
01011
10101

we use sel
selection of
we create n
bad solution
size of initia
for selection
$X_{ac} =$
$lb = 0$
$ub = 3$
$2^l = 32$
for 1 ;
$X_{ac} = 0.096$
$\frac{3}{31}$
$f(x) = 0.0$
Question sol
$(=$
Let say
solution is c
1011011 0
we say r is

we use selection because we want better solution
selection operator is used

we create multiple copies of good solution & we discard bad solutions.
size of initial population is fixed

for selection we ~~use~~ need value of $f(x)$.

$$X_{ac} = X_{lower} + \frac{X_{higher} - X_{lower}}{2^{l} - 1} (Dec(B))$$

$lb = 0$
$up = 3$

$2^{l} = 32$

$$X_{ac} = 0 + \frac{3-0}{32-1} (Dec(B))$$

$$X_{ac} = \frac{3}{31} \times (Dec(B))$$

for 1, 4, 7, 11, 21

$X_{ac} = 0.09677, 0.38709, 0.67741, 1.0645, 2.0322$

$\frac{3}{31}$ $\frac{12}{31}$ $\frac{21}{31}$ $\frac{33}{31}$ $\frac{63}{31}$

$f(x) = 0.09512$

<u>Question</u> vol of cylinder is $\pi r^2 h$

$$0 \le r \le 3$$
$$1 \le h \le 4$$

Let say we consider 2^{10} solutions
solution is combination of r & h both

1011011000
we say r is represented by 4 bits therefore h is represented by 6 bits

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r by MSB 4 bits & h by ~~four~~ 6 LSB bits

$r_{ac} = 0 + \frac{3-0}{\cancel{1023}\ 2^4-1} \times Dec(1011)$ $\qquad h = 1 + \frac{4-1}{\cancel{1023}\ 2^6-1} Dec(011000)$

$r_{ac} = \frac{33}{\cancel{1023}\ 15}$ $\qquad = 1 + \frac{24\times 3}{\cancel{1023}\ 63}$

$r_{ac} = \cancel{0.0322}\ 2.2$ $\qquad h_{ac} = \cancel{[illegible]}\ 2.14285$

$vol = \cancel{[illegible]}\ \pi \times (2.2)^2 \times (2.1428)$

$= 32.58269$

solution here is $(2.2, 2.14) \to (32.5826)$

Ques

$\max f(x) = \frac{x}{1+x^2} \quad 0 \le x \le 3.$

2^4 solutions are considered.

0100
1000
1100
1111

$x = 0 + \frac{3-0}{15}(4)$

0.8	4/5
1.6	8/5
2.4	12/5
3	15/5

$x \begin{Bmatrix} 0.8 \\ 1.6 \\ 2.4 \\ 3 \end{Bmatrix} \xrightarrow{f(x)} \begin{Bmatrix} 0.487 \\ 0.449 \\ 0.355 \\ 0.3 \end{Bmatrix}$

we can say there is huge probability of getting optimal solution near 0001

so neighbour of 0001 are 0000, 0010

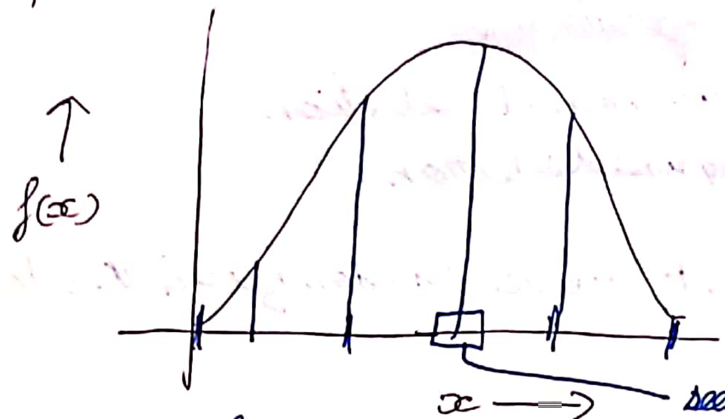
$x_{0001} = \frac{1}{5} = 0.2$

$x_0 = 0$

$x_2 = 0.4$

$x \begin{cases} 0.2 \\ 0 \\ 0.4 \end{cases} \xrightarrow{f(x)} \begin{matrix} 0.1923 \\ 0 \\ 0.3448 \end{matrix}$

Case-1



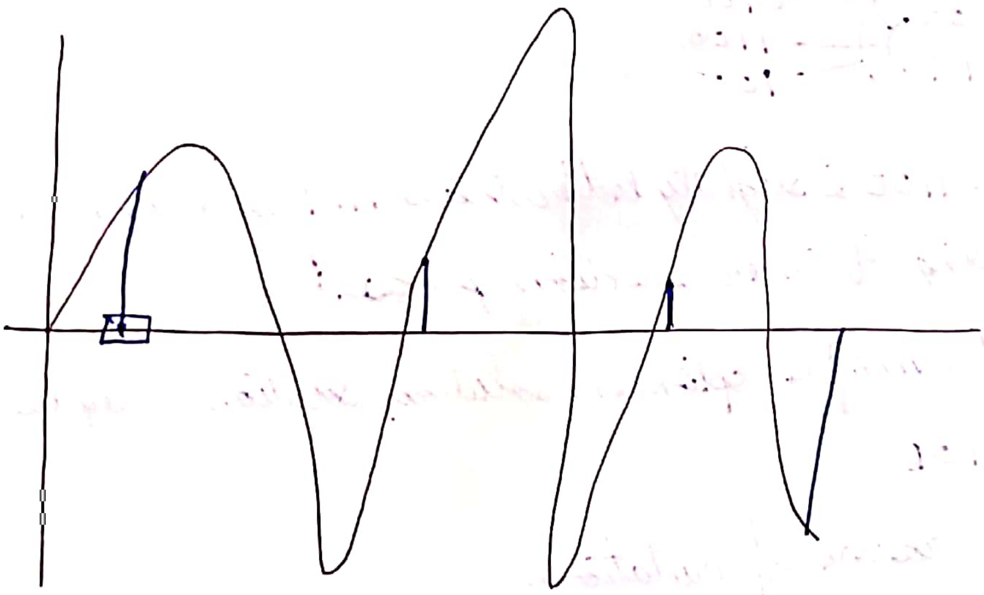
seeing in this region will give us maximum

maximise fitness of these x are f(x)

we consider values increases smoothly in function

in this graph we can get maximum by exploiting that region

Case-2



here we will not get best solution by exploiting that region because it is local maxima

so exploitation is done & then exploration.

without proper exploitation & proper exploration best solution cannot be achieved.

② Ranking based selection.

in binary tournament, we use randomizer & then do a selection tour

pick 0100 & then check which point has better
1100
fitness so we transfer the solution in selected pool

(0100, 1100) $\xrightarrow{\text{0100 is better}}$ (0100, 1111) $\longrightarrow$ 0100

(1100, 1111) $\longrightarrow$ 1100

0100 → 0100
1000 → 0100
1100 → 1100
1111 → 1000

but here 1100 is slightly better than 1111 then why we are choosing it in our selection process?
in case of multiple optimum solution, solution may be near 1100

we apply crossover & mutation.
There is always a probability associated with crossover
here the prob. of crossover is let say 0.7.
so $\underbrace{4}_{\text{4 values}} \times 0.7 = 2.8 \approx 3$ time we need to apply the crossovers

0100
1000
1100
1111

This operator is used for both exploitation & exploration

example

3 2 1 0
0 1 0 0
1 1 1 1

one-point crossover

choose a point & make a cut over there & append the points

0 1 0 | 0
1 1 1 | 1

0101
1110

exploitation

0 1 | 0 0
1 1 | 1 1

0111
1100

it is exploitation

0 | 1 0 0
1 | 1 1 1

0111
1100

it is exploitation

but there are certain cases where both can be performed

crossover is used to generate new points around the given point.

after selection, let the points we have are

1100
0011
1001
0101

if randomizer pick 1100 & 0101 out of them.

cut point

11|00
01|01

1101
0100

if probability of crossover is .7

4 → initial population

$.7 \times 4 = 2.8 \approx 3$

so 3 crossovers are needed

Let after crossover, we have

1011
0000
~~0100~~
0010
0011

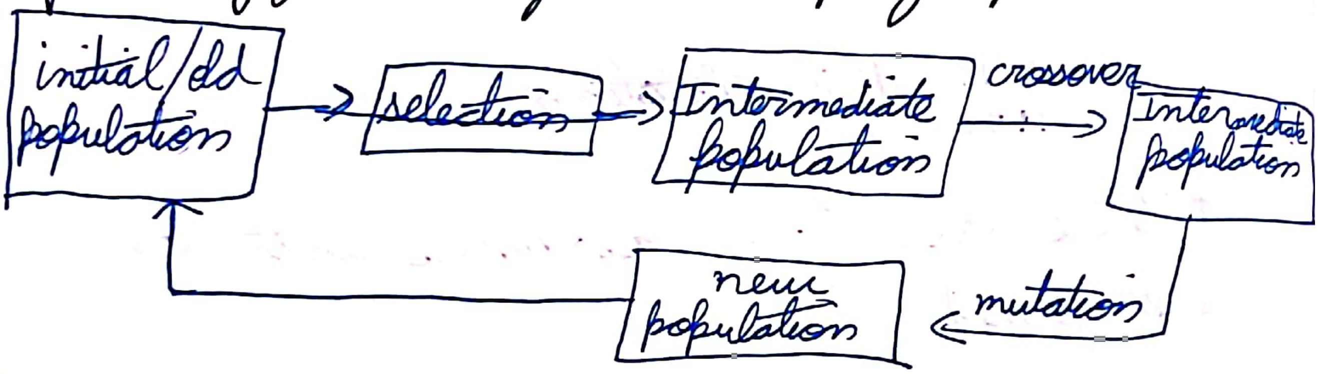
$\xrightarrow{\text{mutation}}$

probability of mutation is 0.6

$4 \times 0.6 = 2.4 \approx 2$

pick 00|1|0 → 0010.
↳ reverse this bit

process of genetic algorithm step by step is :-



$40 \times \underbrace{100}_{\text{no of times this cycle will repeat}} = 4000$

Types of selection operator :-

① roulette wheel selection operator :-

0 → 1010
1 → 1011
2 → 0000
3 → 0010

we calculate fitness of initial population

Let fitness be

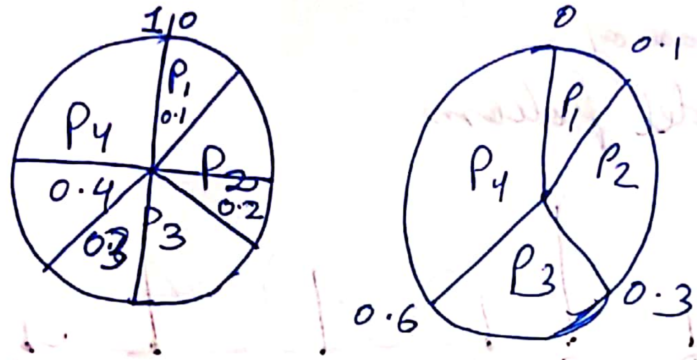
i		f_i	$f_i/\Sigma f_i$
0	1010 →	1	0.1
1	1011 →	2	0.2
2	0000 →	3	0.3
3	0010 →	4	0.4

if problem is of maximization

$$\sum_{i=0}^{3} f_i = 1+2+3+4 = 10$$

$$\frac{f_i}{\sum_{i=0}^{3} f_i}$$ Normalisation

create a wheel starting from 0 ending at 1



r_1		0.2	P_2
r_2		0.4	P_3
r_3	$\rightarrow \boxed{0 \rightarrow 1}$	0.6	P_3
r_4		0.8	P_4

check this no comes in which range from 0.1

0.2 is P_2 0.4 in P_3 0.6 in P_3/P_4 (boundary condition)   0.8 in P_4

higher the fitness more the chances of having random number in that area.

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$f(x, y) = x^2 + y^2$

$0 \le x \le 3$

$1 \le y \le 5$

Question

$\sigma\ share = 3.5$

initial population

x	y
1011	10111
0000	11111
0001	11000
1111	00000

x_i	y_i	f_i	y_i	f_i	new f_i
2.2	~~3.225~~	~~9.29~~	3.96	20.52	14.65
0	~~3.5~~	~~25~~	5	25	
0.2	~~2.322~~	~~5.4224~~	4.096	16.817	
3	~~1~~	~~10~~	1	10	

$r_x = 0 + \frac{3}{15}(D(x))$

$= \frac{D(x)}{5} \Rightarrow$

$\frac{11}{5} = 2.2$

$\frac{0}{5} = 0$

$\frac{1}{5} = 0.2$

$\frac{15}{5} = 3$

$r_{x_y} = \cancel{1} + \frac{5-1}{31}(D(y))$

$= 1 + \frac{4}{31}(D(y))$

$\cancel{1 + \frac{23 \times 4}{31} = 3.225}$

$\cancel{1 + \frac{31 \times 4}{31} = 3.5}$

$\cancel{1 + \frac{24 \times 4}{31} = 3.322}$

$\cancel{1 + \frac{0 \times 4}{31} = 1}$

$1 + \frac{23}{31} \times 4 = 3.96$

$1 + \frac{4 \times 31}{31} = 5$

$1 + \frac{4 \times 24}{31} = 4.096$

$1 + \frac{4 \times 0}{31} = 1$

$sh(d_{11}) = 1$

$sh(d_{12}) = 1 - \left(\frac{2.43}{3.5}\right)^1 = 0.30$

$sh(d_{13}) = 1 - \frac{2.00}{3.5} = 0.42$

$sh(d_{14}) =$

$n_{c_1} =$

$\frac{20.52}{1.84}$

17 Sept'19

DE -> Diffe

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values.

in DE an

GA operato

P.S.O. (part

PSO is base

a group of

of food. Th

birds cha

Birds

The bird h

is called lea

for changing

velocity

↓

G best

P best

$sh(d_{14}) = 1 - \frac{3.06}{3.5} = 0.125$

$n_{C_1} = 1.84$

$\frac{20.52}{1.84}$

17 Sept '19

DE -> Differential Evolution

DE is always used with real problems involving real values.

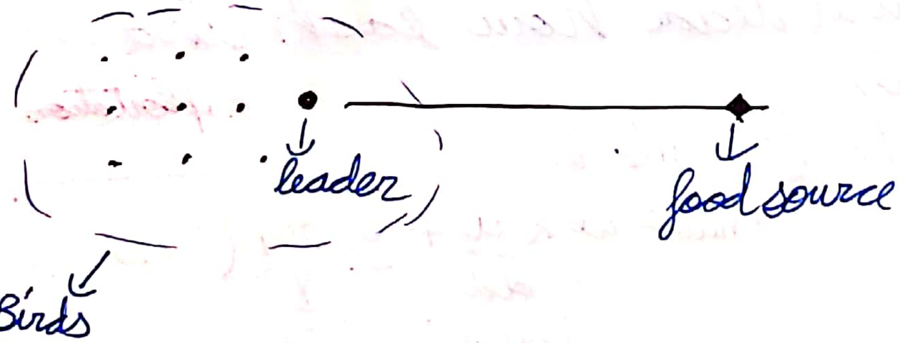
in DE any operator can be used any time. while in GA operators are used step by step.

P.S.O. (particle swarm optimization) swarm intelligence

PSO is based on birds flocking.

a group of birds moving in same direction in search of food. This phenomenon is called bird flocking.

birds changes the position according to their leader



The bird having max. information of the food source is called leader bird

for changing the position velocity is to be changed

velocity is changed based on the 2 factors given below.

velocity
↓
G best
P best

Gbest → ~~velocity~~ position of the best bird / leader.

Pbest → best position of itself during flight taken till now.

velocity position

current position

Q minimise

$f(x,y) = x^2 + y^2$ PSO

$0 \leq x \leq 3$ $1 \leq y \leq 4$

birds initial positions will be random & initial velocity will also be random

X	v	f(x)/f(x,y)
(0,1)	(1,1)	1
(1,2)	(2,2)	5
(2,3)	(3,3)	13
(3,4)	(3,4)	25

4 values (random)

Now we will decide how each bird will change its position.

for this formula is

exploitation explorat

$$v_{new} = w \times \underset{\text{old initial}}{u} + c_1 \underset{0-1}{r_1} (G_{best} - x) + c_2 r_2 (P_{best} -$$

$c_2 = 2$ generally

random value like 0 & 1

only positions can be evaluated.

velocity can never be evaluated.

(0,1) has the minimum value so it will be the leader bird.

Now other birds will try to reach the same position as of leader bird velocity of leader is assume $c_1 r_1 = c_2 r_2 = 1$ if $c_1 = 2$, $r_1 = \frac{1}{2}$

for (0,1)

$v_{new} = .7 \times (1,1) + 1(\{0,1\} - \{0,1\}) + 1(0)$

$v_{new} = [.7, .7]$ update $[0.7, 1]$

~~$X_{new} = X_{prev} + X\,dt +$~~

$X_{new} = X_{prev} + v_{new}$

$= \{0,1\} + \{.7, \cancel{.7}\,1\}$

$X_{new} = \{.7, \cancel{1.7}\,2\}$

damping

v_{new}	v_{new}'
0.7, 0.7	.7, 1
.4, 1	.4, 1
0.1, 0.1	0.1, 1
−0.9, −0.2	0, 1

Now for (1, 2)

$v_{new} = .7 \times (2,2) + 1(\{0,1\} - \{1,2\}) + 1(0)$

$= \{1.4, 1.4\} + \{-1, -1\}$

$v_{new} = \{+.4, \cancel{.4}\,1\}$

$X_{new} = \{.4, \cancel{.4}\,1\} + \{1,2\}$

$X_{new} = \{1.4, \cancel{2.4}\,3\}$

Now for (2, 3)

$v_{new} = 0.7 \times (3,3) + 1(\{0,1\} - \{2,3\}) + 1(0)$

$= \{2.1, 2.1\} + \{-2, -2\}$

$v_{new} = \{0.1, \cancel{0.1}\,1\}$

$X_{new} = \{2,3\} + \{0.1, \cancel{0.1}\,1\}$

$X_{new} = \{2.1, \cancel{3.1}\,4\}$

Now for (3, 4)

$v_{new} = 0.7 \times (3,4) + 1(\{0,1\} - \{3,4\}) + 0$

$= \{2.1, 2.8\} + \{-3, -3\}$

$= \{-\cancel{0.9}, -\cancel{0.2}\,1\}$

$X_{new} = \{3,4\} + \{-\cancel{0.9}, -\cancel{0.2}\} = \cancel{\{2.1, 3.8\}}$ $\{3, \cancel{5}\} = \{3, 4\}$

	x_{new}	x_{new}'	$f(x)$
(0,1)	(.7, 1.7)	(.7, 2)	(0,1) vs (0.7, 2)
(1,2)	(1.4, 2.4)	(1.4, 3)	(1,2) vs (1.4, 3)
(2, 3)	(2.1, 3.1)	(2.1, 4)	(2,3) vs (2.1, 4)
(3, 4)	(2.1, 3.8)	(3, 4)	(3,4) vs (3, 4)

P_{best}	v_{prev}
(0, 1)	(.7, 1)
(1, 2)	(.4, 1)
(2, 3)	(.1, 1)
(3, 4)	(0, 1)

g_{best} will be first of P_{best} = (0, 1)

$v_{new} = \cancel{0.7 \times (.7, 1) + 1 ((0,1) - (0,1)) + 1(}$

$= 0.7 \times (0.7, 1) + 1(\{0,1\} - \{0.7, 2\}) + 1(\{0,1\} - \{.7, 2\})$

$= (0.49, 0.7) + \{-0.7, -1\} + \{0.7, -1\}$

$v_{new} = \{-0.91, -1.3\}$

$v_{new}' = \{0, 1\}$

$x_{new} = (0, 1) + (.7, 2)$

$= (0.7, 3)$

x_{new}	v_{new}
(0.7, 3)	(0, 1)
(1.4, 3)	(0, 1)
(2.1, 4)	(0, 1)
(3, 4)	(0, 1)

$v_{new} = 0.7 \times (.7, 1) + 1((0,1) - (1.4, 3)) + ((1,2) - (1.4, 3))$

$= (0.49, 0.7) + (-1.4, -2) + (-.4, -1)$

$v_{new} \Rightarrow$

damping $v_{new} = 0, 1$

Limitation it is a greedy approach.
even though it has a very fast convergence rate but it may reach the local optimal solutions & trap there.

1 What is population?
2 How will you define convergence rate of an algorithm?
3 in what kind of problems we use sharing fitness approach?
4 What will happen if we remove (Pbest - x)
from the velocity equation
5 What is Roulette wheel selection operator in GA
state with an example.
6 What equations or operators are used in GA for exploiting & exploring the search space
7 What will happen if we apply sharing fitness approach in PSO?
8 What is a chromosome? genetic representation of it.