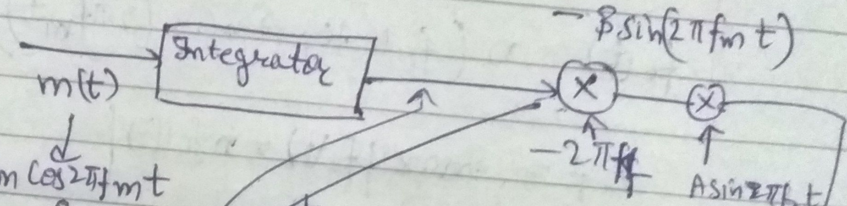
$$\cos(\beta \sin(2\pi f_m t)) \approx 1$$
$$\sin(\beta \sin(2\pi f_m t)) \approx \beta \sin(2\pi f_m t)$$

$$x(t)_{FM} = A\cos(2\pi f_c t) - A\beta\sin(2\pi f_c t)\cdot \sin(2\pi f_m t)$$



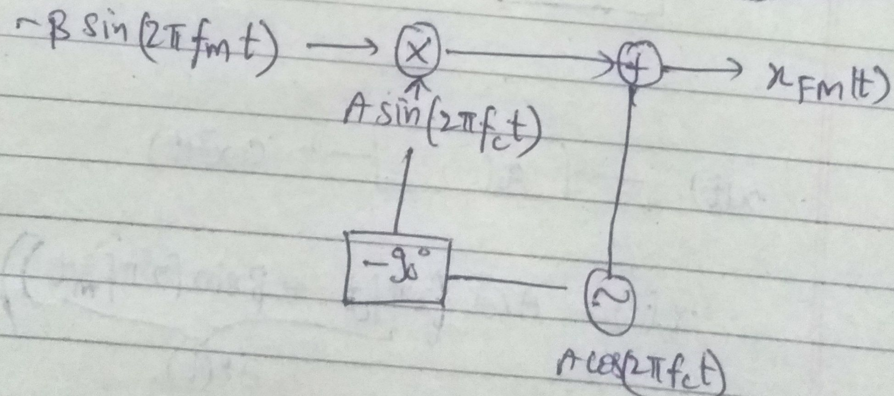
$m(t)$
$\downarrow$
$A_m \cos 2\pi f_m t$

$$\int_0^t A_m \cos(2\pi f_m t)\, dt$$

$$= \frac{A_m}{2\pi f_m} \sin(2\pi f_m t)$$

$$\left[\because \beta = \frac{K_f A_m}{f_m}\right]$$

(conti-)



<u>Wide Band FM :-</u>

It is done with the help of narrowband FM and its circuit is known as <u>frequency multiplier</u>.

$$x(t)_{NBFM} \longrightarrow \boxed{\text{Freq. Multiplier 'n'}} \longrightarrow x(t)_{WBFM}$$

$$\Downarrow$$

$$f_o(t) = n\, f_i(t)$$

$$f_i(t) = f_c + K_f A_m \cos(2\pi f_m t)$$

$$f_o(t) = n\left(f_c + K_f A_m \cos(2\pi f_m t)\right)$$

$$\Delta f = \max|f_o(t) - n f_c(t)|$$
$$= \underline{n f_m A_m}$$

$$\beta_{(NBFM)} = \frac{\Delta f}{f_m} = \frac{K_f A_m}{f_m}$$

$$\beta_{o\,(WBFM)} = \cancel{n \times} \frac{n K_f A_m}{f_m} = n\beta.$$

$$\boxed{\beta_o = n\beta}$$

$$x(t) \longrightarrow \boxed{a(\ )^3} \longrightarrow a x^3(t)$$

$$x(t) = A\cos\Big(\underbrace{2\pi f_c t + \beta \sin(2\pi f_m t)}_{\theta_i(t)}\Big)$$

$$\text{output} = a\left(A\cos(\theta_i(t))\right)^3$$
$$= a A^3 \cos^3(\theta_i(t))$$
$$= a\frac{A^3\left(\cos 3\theta_i(t) + 3\cos(\theta_i(t))\right)}{4}$$
$$= \frac{aA^3}{4}\cos\big(\overbrace{6\pi f_c t}^{3f_c} + 3\beta \sin(2\pi f_m t)\big)$$
$$+ \Big(\frac{3aA^3}{4}\cos(\underbrace{2\pi f_c t}_{f_c} + \beta \sin(2\pi f$$

using filter of '$3f_c$'

$$\text{output} = \frac{aA^3}{4}\cos\left(6\pi f_c t + 3\beta \sin(2\pi f_m t)\right)$$

$$\boxed{\beta_0 = 3\beta}$$

generalising $\boxed{\beta \longrightarrow \boxed{a(\,)^3} \longrightarrow 3^n \beta}$

$$x(t) = A\cos\left(2\pi f_c t + \beta \sin(2\pi f_m t)\right)$$

$$= \mathrm{Re}\left\{ A e^{j(2\pi f_c t + \beta \sin 2\pi f_m t)} \right\}$$

$$= \mathrm{Re}\left\{ \underbrace{A e^{j\beta \sin(2\pi f_m t)}}_{x_b(t)} * e^{j2\pi f_c t} \right\}$$

$$x_b(t) = A e^{j\beta \sin(2\pi f_m t)}$$

$$= A e^{j\beta \sin\left(2\pi f_m\left(t + \frac{K_t}{f_m}\right)\right)} \qquad T = \frac{1}{f_m}$$

~~period~~

$$x_b(t) = A e^{j\beta \sin(2\pi f_m t + 2\pi K_t)}$$

integer

$$x_b(t) = \sum_{n=-\infty}^{n=\infty} C_n e^{j2\pi n f_m t}$$

$$C_n = \frac{1}{T}\int_0^T x_b(t)\cdot e^{-j2\pi n f_m t}\, dt$$

$$= \frac{1}{T}\int_0^T A e^{j(\beta \sin(2\pi f_m t) - 2\pi n f_m t)}\, dt$$

$$2\pi f_m t = x$$
$$dt = \frac{dx}{2\pi f_m}$$

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$$C_n = \frac{1}{T}\int_0^{2\pi} A e^{j(\beta \sin x - nx)} dx$$

$$C_n = \cancel{\frac{1}{T}} \frac{A}{2\pi}\int_0^{2\pi} e^{j(\beta \sin x - nx)} dx$$

(Bessel's function of nth order)

$$J_n(\beta) = \frac{1}{2\pi}\int_0^{2\pi} e^{j(\beta \sin x - nx)} dx$$

$$\boxed{C_n = A J_n(\beta)}$$ (C_n = n^{th} coefficient of fourier series)

$$x_b(t) = A e^{j\beta \sin(2\pi f_m t)}$$

$$A e^{j\beta \sin(2\pi f_m t)} = \sum_{n=-\infty}^{\infty} \underbrace{A J_n(\beta)}_{C_n} e^{j2\pi n f_m t}$$

(In terms of discrete fourier series)

$$x(t) = Re\{x_b(t)\, e^{j2\pi f_c t}\}$$

$$= Re\left\{\sum_{n=-\infty}^{\infty} A J_n(\beta)\, e^{j2\pi n f_m t} \cdot e^{j2\pi f_c t}\right\}$$

$$= Re\left\{\sum_{n=-\infty}^{\infty} A J_n(\beta)\, e^{j2\pi(n f_m + f_c)t}\right\}$$

modulated signal (FM) $x(t) = \sum_{n=-\infty}^{\infty} A J_n(\beta) \cos(2\pi(f_c + n f_m) t)$

$$X(f) = \sum_{n=-\infty}^{n=\infty} \frac{1}{2} A J_n(\beta)\left[\delta(f-(f_c+nf_m)) + \delta(f+(f_c+nf_m))\right]$$

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Bandwidth of FM

$$x(t) = A\cos\left(2\pi f_c t + \beta \sin(2\pi f_m t)\right)$$

$$x(t) = \frac{1}{2} A \sum_{n=-\infty}^{\infty} J_n(\beta) \left\{ \delta\left(f-(f_c+nf_m)\right) + \delta\left(f+(f_c+nf_m)\right) \right\}$$

Carson's Rule :-

~~Smaller~~ Larger $\beta \Rightarrow BW \rightarrow \Delta f$

Smaller $\beta \Rightarrow BW \rightarrow 2f_m$

$$BW \approx 2\left(1+\frac{1}{\beta}\right)\Delta f$$

$\Rightarrow$ larger $\beta \Rightarrow \frac{1}{\beta} \approx 0 \Rightarrow BW \approx 2\Delta f$

$\Rightarrow$ Smaller $\beta \Rightarrow \frac{1}{\beta} >> 1 \Rightarrow BW \approx 2\left(\frac{1}{\beta}\right)\Delta f$

$\Rightarrow BW \approx 2\left(\frac{f_m}{\cancel{\Delta f}}\right)\cancel{\Delta f}$

$\Rightarrow BW \approx 2f_m$

$$BW \approx 2\left(1+\frac{1}{\beta}\right)\Delta f = \left(2\Delta f + 2\frac{\Delta f}{\beta}\right)$$

Ex:-

$f_c = 10\,MHz$, $\Delta f = 50\,kHz$

Determine the bandwidth of fm.

a) $f_m = 0.5\,KHz$

b) $f_m = 50\,KHz$

c) $f_m = 250\,KHz$

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a) $f_m = 0.5$ KHz

$$\beta = \frac{\Delta f}{f_m} = \frac{50 \text{ KHz}}{0.5 \text{ KHz}} = 100 >> 1$$

$$BW \approx 2\Delta f$$

$$BW \approx 2(50) = 100 \text{ KHz}$$

(b) $f_m = 50$ KHz

$$\beta = \frac{\Delta f}{f_m} = \frac{50 \text{ KHz}}{50 \text{ KHz}} = 1$$

$$BW \approx \cancel{\left(2\Delta f + \frac{2\Delta f}{\beta}\right)} = \cancel{2\left(50 + \frac{50}{50}\right)}$$

$$BW \approx 2\left(1 + \frac{1}{\beta}\right)\Delta f = 2(1+1)\,50 = \underline{200 \text{ KHz}}$$

c) $f_m = 250$ KHz

$$\beta = \frac{\Delta f}{f_m} \approx \cancel{250}\ \frac{5\cancel{0}}{25\cancel{0}} = 0.2 < 1$$

~~Bω~~ $BW \approx 2 f_m$

$$BW \approx 2 \times 250$$

$$\boxed{BW = 500 \text{ KHz}}$$

Demodulation of FM

Since we have used integrator at the modulation side so we have to use differentiator at the ~~der~~ receiver side.

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$$x(t) \longrightarrow \boxed{\frac{d}{dt}(\,)} \xrightarrow{\frac{d(x(t))}{dt}} \boxed{E.D.} \longrightarrow$$

LTI system

$$x(t) = A\cos\left(2\pi f_c t + 2\pi K_f \int_0^t m(\tau)d\tau\right)$$

$$\frac{dx(t)}{dt} = -A\sin\left(2\pi f_c t + 2\pi K_f \int_0^t m(\tau)d\tau\right) \times \frac{d}{dt}\left(2\pi K_f \int_0^t m(\tau)d\tau + 2\pi f_c t\right)$$

$$\frac{dx(t)}{dt} = -A\sin\left(2\pi f_c t + 2\pi K_f \int_0^t m(\tau)d\tau\right) \times \left(2\pi K_f m(t) + 2\pi f_c\right)$$

$$o/p \text{ 'D'} = \underbrace{-A\left(2\pi f_c + 2\pi K_f m(t)\right)}_{\text{envelope}} \left(\sin\left(2\pi f_c t + 2\pi K_f \int_0^t x(\tau)d\tau\right)\right)$$

$\Downarrow$

E.D.

Output of E.D. $= -A\left(2\pi f_c + 2\pi K_f m(t)\right)$

$$= -2\pi A\left(f_c + K_f m(t)\right)$$

$$= -2\pi f_c A\left(1 + \frac{K_f}{f_c} m(t)\right)$$

Condition to get the distortionless output is:-

$$(f_c > \Delta f)$$

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Ex: $f_c = 100\,MHz$, $f_m = 10\,KHz$, $\beta = 10$

Sol^n: $\beta = \frac{\Delta f}{f_m} \Rightarrow 10 = \frac{\Delta f}{10}$

$\Delta f = 100\,KHz = 0.1\,MHz$

$f_c > \Delta f$

$100 > 0.1$

∴ The output is distortionless.

Advantage of Digital signal

① Less effect of noise.
② Modulation and Demod^n is easier.

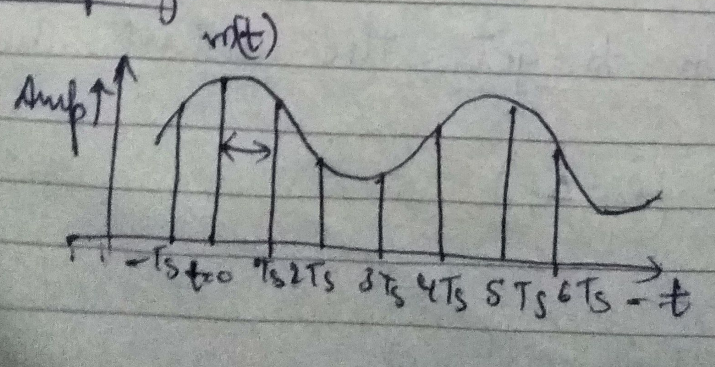
Conversion of A to D

~~Sam~~

Analog → discrete in amplitude and continuous in time.

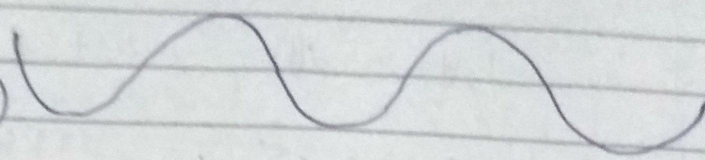
Digital → discrete in ampt and discrete in time.

Sampling:-



$s(t)$ $\delta(t-T_s)$

sine wave (analog)



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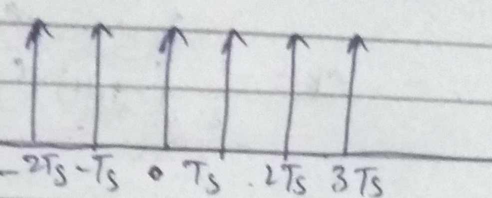
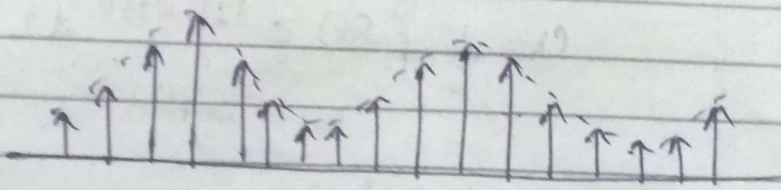
continuous (pulse train)





Pulse train :- $\delta(T+2T_s) + \delta(T+T_s) + \delta(t) + \delta(t-T_s) + \delta(t-2T_s) + \cdots$

$$g_\delta(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s) \qquad T_s = \frac{1}{f_s}$$

$m_\delta(t)$ = Sampled signal $= m(t) \times g_\delta(t)$

$$= m(t) \times \sum_{n=-\infty}^{\infty} \delta(t-nT_s)$$

$$m_\delta(t) = \sum_{n=-\infty}^{\infty} m(t)\,\delta(t-nT_s)$$

$m(t)\,\delta(t-t_0) = m(t_0)\,\delta(t-t_0)$ (Sampling property)

$$m_\delta(t) = \sum_{n=-\infty}^{\infty} m(nT_s)\,\delta(t-nT_s)$$

$$m_\delta(t) \longleftrightarrow M_\delta(f)$$

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$$g_s(t) = \delta(t) \quad , \quad 0 \le t \le T_s$$

$$g_s(t) = \sum_{K=-\infty}^{\infty} c_K \, e^{j2\pi K f_s t}$$

$$c_K = \frac{1}{T_s} \int_0^{T_s} g_s(t) \, e^{-j2\pi K f_s t} \, dt$$

$$c_K = \frac{1}{T_s} \int_0^{T_s} \underbrace{\delta(t) \, e^{-j2\pi K f_s t}}_{\text{using sampling}} \, dt$$

$$c_K = \frac{1}{T_s}(1) = \frac{1}{T_s}$$

$$\boxed{g_s(t) = \frac{1}{T_s} \sum_{K=-\infty}^{\infty} e^{j2\pi K f_s t}}$$

$$e^{j2\pi K f_s t} \longleftrightarrow \delta(f - K f_s)$$

$$g_s(t) \longleftrightarrow G_s(f) = \frac{1}{T_s} \sum_{K=-\infty}^{\infty} \delta\left(f - \frac{K}{T_s}\right)$$

To avoid Aliasing $\boxed{f_s \ge 2 f_m}$ (Nyquist criterion)

<u>Reconstruction (D-to-A):-</u>

$$m_s(t) = m(t) \times g_s(t)$$

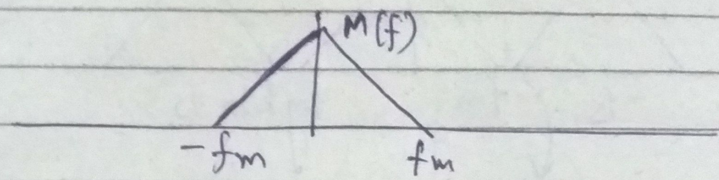
$$M_s(t) = M(f) * G_s(f)$$

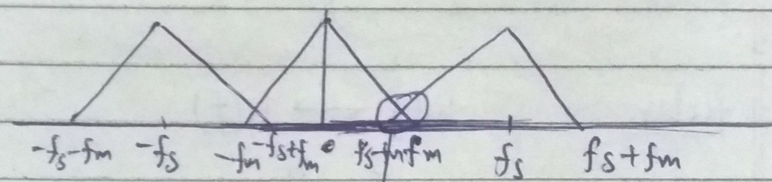
$$= M(f) * f_s \sum_{K=-\infty}^{\infty} \delta(f - K f_s)$$

$$\Rightarrow f_s \sum_{K=-\infty}^{\infty} M(f) * \delta(f - K f_s)$$

$$M(f) * \delta(f - f_0) = M(f - f_0)$$

$$\boxed{M_\delta(f) = f_s \sum_{k=-\infty}^{\infty} M(f - kf_s)}$$





This overlapping causes distortion.

To avoid Aliasing

$\boxed{f_s \geq 2f_m}$

(Nyquist criterion)

$f_s - f_m < f_m$

$\boxed{f_s < 2f_m}$ * (Aliasing effect)

This causes reconstruction difficult.

$$\cancel{M_\delta(t)} = m_\delta(t) = \sum_{k=-\infty}^{\infty} m(nT_s)\,\delta(t - nT_s)$$

$$M_\delta(f) = f_s \sum_{k=-\infty}^{\infty} M(f - kf_s)$$

$$+ \cdots f_s M(f + f_s) + f_s M(f) + f_s M(f - f_s) + \cdots$$

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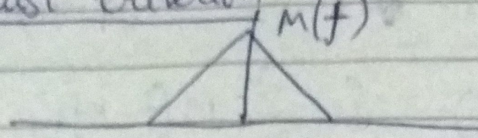
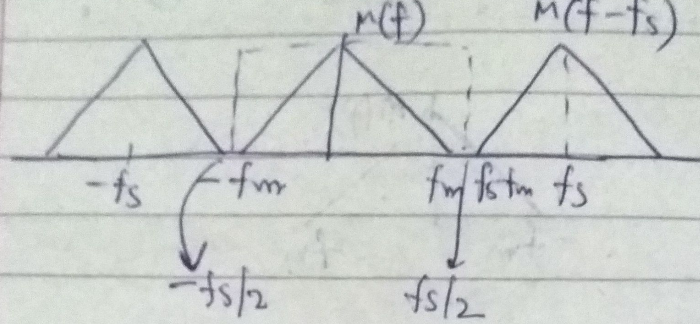
Nyquist criterion

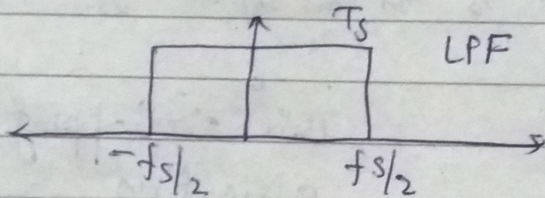


$\boxed{f_s \geq 2f_m}$



sampelled signal

filter :- $h(t) \longleftrightarrow H(f)$



$$H(f) = \begin{cases} T_s & , \text{ if } |f| \leq f_s/2 \\ 0 & , \text{otherwise} \end{cases}$$

Inverse fourier

$$h(t) = \text{sinc}(f_s t) = \frac{\sin(\pi f_s t)}{\pi f_s t}$$

$m_\delta(t) \longrightarrow \boxed{h(t)} \longrightarrow m(t)$

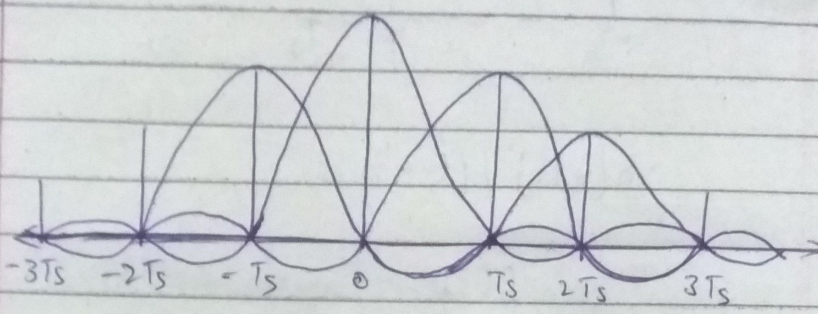
$$m_\delta(t) = \sum_{n=-\infty}^{\infty} \cancel{m(t)\,\delta(t-nT} \; m(nT_s)\,\delta(t-nT_s)$$

(sampelled signal)

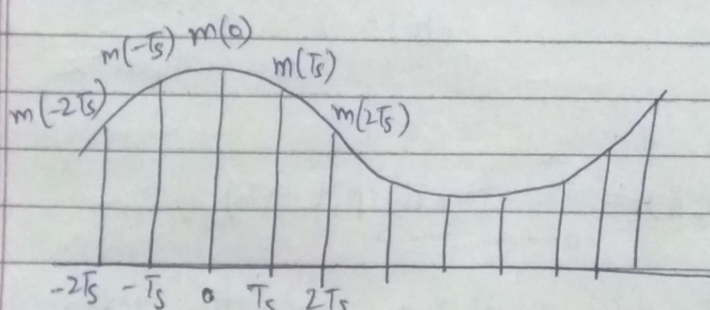
$$m(t) = m_\delta(t) * h(t)$$
$$= m_\delta(t) * \text{sinc}(f_s t)$$
$$= \text{sinc}(f_s t) * \sum_{n=-\infty}^{\infty} m(nT_s)\,\delta(t-nT_s)$$

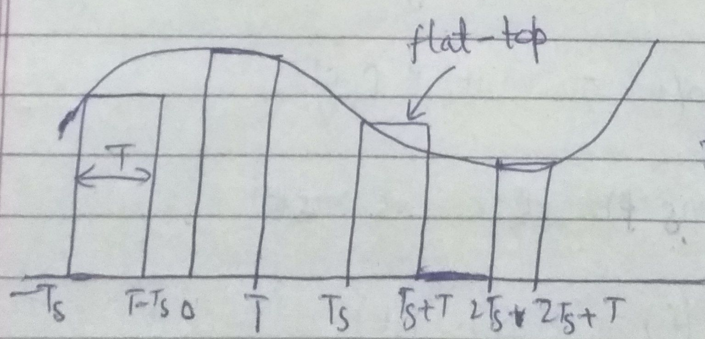
$$m(t) = \sum_{n=-\infty}^{\infty} m(nT_s)\ \mathrm{sinc}(f_s(t - nT_s))$$

$\mathrm{sinc}(f_s(t - nT_s))$



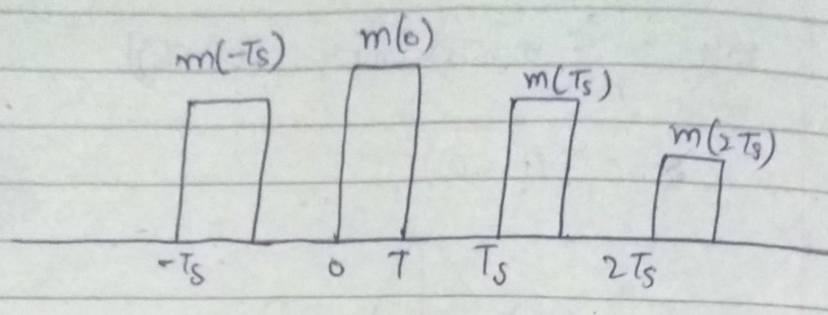
PAM :-



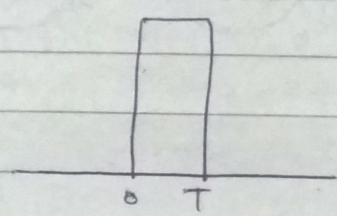


flat-top sampling

$T << T_s$ (T_s: time interval
T: width of the pulse).



$m(nT_s)\, p(t - nT_s)$



$$p(t) = \begin{cases} 1 & , 0 \le t \le T \\ 0 & , \text{otherwise} \end{cases}$$

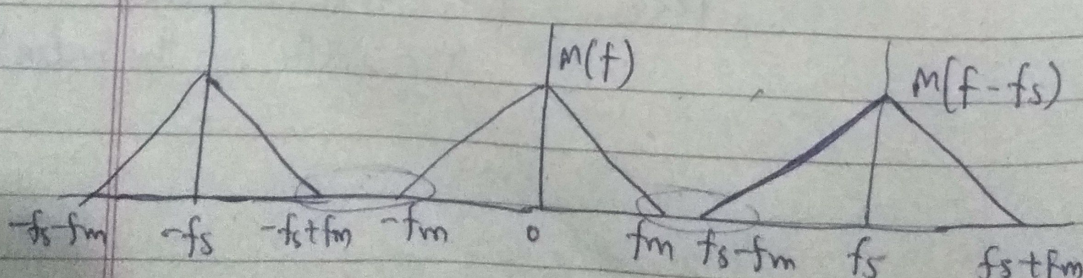
$$m_p(t) = \sum_{n=-\infty}^{\infty} m(nT_s)\, p(t - nT_s)$$

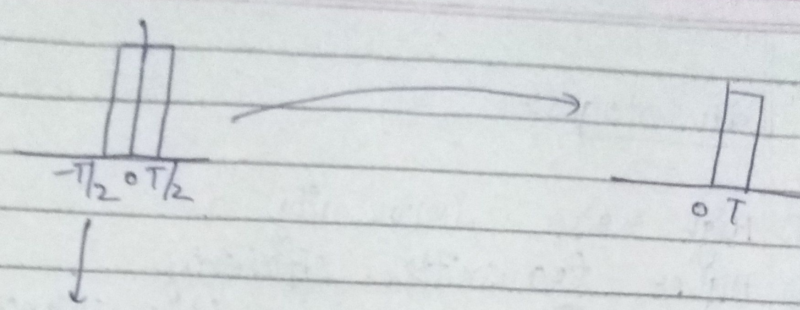
flat-top sampling is also referred as PAM.

$M_p(f) = m_\delta(t) \times P(f)$

$m_\delta(f)$ = F.T. of $m_\delta(t)$

$P(f) = M_p(f) \cdot P(t)$





$$P_{T/2} = \begin{cases} 1 & , |t| \le T/2 \\ 0 & , \text{otherwise} \end{cases}$$

$$P(t) = P_{T/2}(t - T/2)$$
$$P(f) = T\,\text{sinc}(fT)\, e^{-j2\pi f T/2}$$

$$P(f) = T\,\text{sinc}(fT)\, e^{-j\pi f T}$$

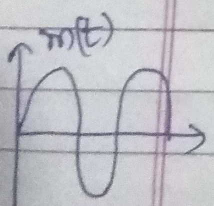
$$|P(f)| = T\,\text{sinc}(fT)$$

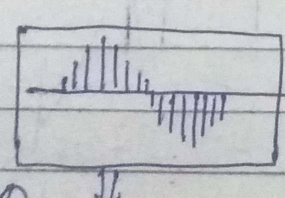
Note - $\boxed{\text{Study PPM, PWM theoretical from book}}$

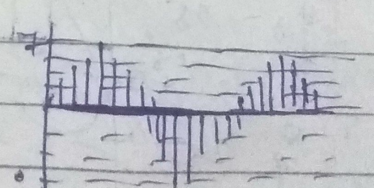
AM.

PCM (Pulse Code Modulation) :-

$m(t) \rightarrow$ [Sampling] $\rightarrow$ [Quantization] $\rightarrow$ [Encoding] $\rightarrow$ binary

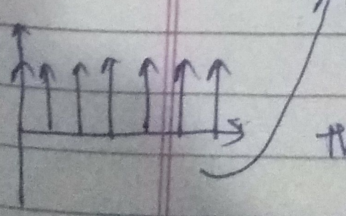






$L = 8 = 2^3$ (levels)

as $L \uparrow$, quantization noise $\uparrow$



By maintaining the Nyquist criteria

binary $\rightarrow$ [Quantizer] $\rightarrow$ [Decoder] $\rightarrow$ [Filter] $\rightarrow m(t)$.

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Advantages:-

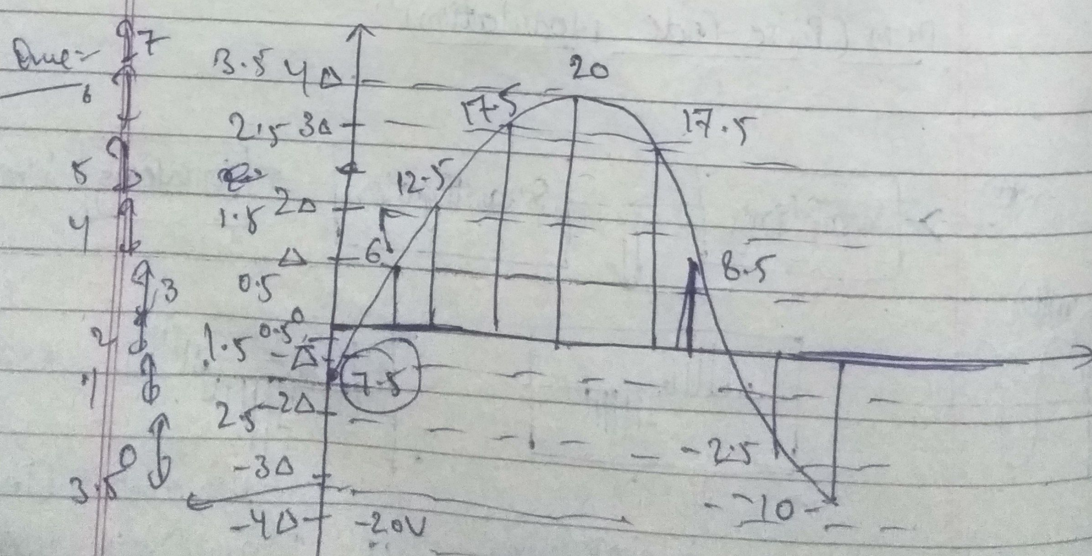
① High noise immunity.
② Higher transmitter efficiency.
③ ~~Conve~~ convenient for long distance communication

Disadvantages:-

① Requires larger bandwidth.
② Encoding, decoding and quantizing circuits of PCM is very complex.

Applications:-

① Used in satellite transmission system.
② Used in telephony.
③ It is also used in CD's.



$\Delta m = \frac{mp}{}$ (peak of magnitude)

normalized out
close to which level.

quantisation noise power

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Step Size : $\Delta v = \frac{Am-(-Am)}{L}$

$$\Delta v = \frac{2Am}{L}$$

$Am = mp$ (peak of magnitude)

$$L = 8 \rightarrow \Delta v = \frac{2 \times 20}{8} = 5V$$

Actual amp. $\rightarrow -7.5$

Normalized value $\left(\frac{m(t)}{\Delta}\right) \Rightarrow \frac{-7.5}{5} = -1.5$

Quantized avalue $= -1.5$

actual amp = 6 17.5 20

Normal val = 1.2 3.5 4

normalized Quantized value = 1.5 3.5 3.5

diff = 0.3 0 0.5

close to which level.

Signal to noise ratio (SNR) = $\frac{\text{Signal Power}}{\text{noise power}} = \left(\frac{S}{N}\right)$

quantisation noise power

$$\text{Q.N.P.} \ (N_q) = \tilde{q}^2 = \frac{1}{\Delta v} \int_{-\frac{\Delta v}{2}}^{\frac{\Delta v}{2}} q^2 \, dq$$

$$N_q = \frac{1}{\Delta v} \left[\frac{q^3}{3} \right]_{-\frac{\Delta v}{2}}^{\frac{\Delta v}{2}} = \left(\frac{\Delta v^2}{12}\right)$$

$$= \frac{\left(\frac{2Am}{L}\right)^2}{12} = \frac{Am^2}{3L^2}$$

$m(t) \Rightarrow S_0 = \widetilde{m^2(t)}$ mean square

$$\frac{S}{N} = \frac{\widetilde{m^2(t)}}{\frac{Am^2}{3L^2}} \Rightarrow \frac{S}{N} = \frac{3L^2 \, \widetilde{m^2(t)}}{Am^2}$$

$$\frac{S}{N} = \frac{3L^2 \, \widetilde{m^2(t)}}{A_m^2} \quad (\text{for receiver}) \qquad \frac{3\widetilde{m^2(t)}}{A_m^2}$$

$10\log_{10} e$

$$\left(\frac{S}{N}\right)_{dB} = (\alpha + 6n)_{dB}$$

$$B_T = nB \qquad \boxed{B = 4K}$$

Digital Modulation

ASK → AM
PSK → PM
FSK → FM
Keying → Switching

Requirements for digital modulation

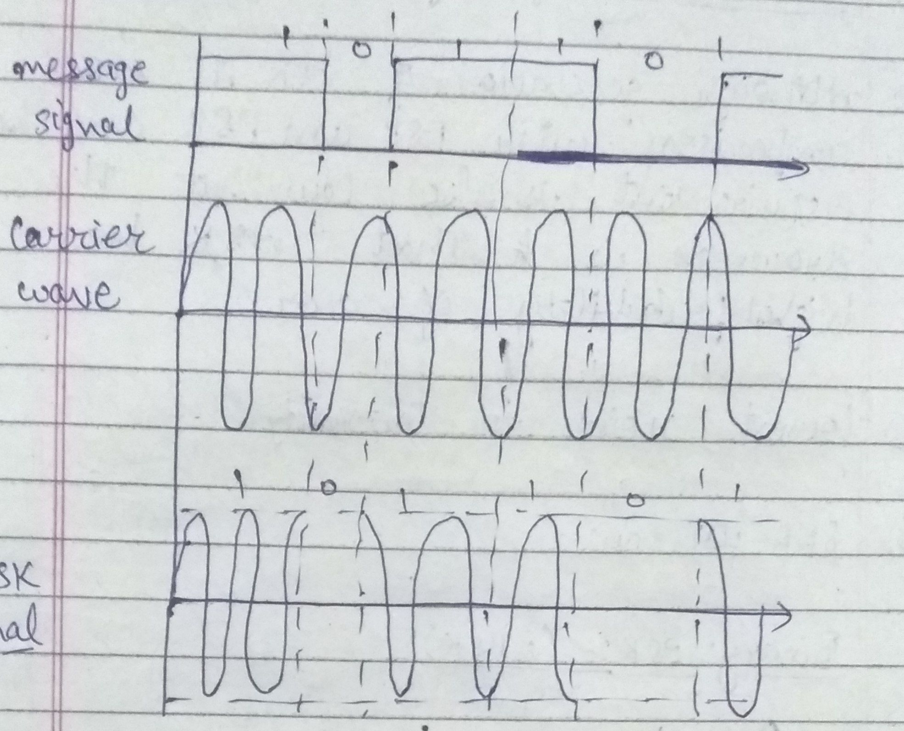
1. Low bitrate and Low SNR.
2. Perform well in multiple threading.
3. Occupies minimum of bandwidth.
4. Easy & cost-effective implementation.

Digital modulation divided into two parts:-

1. Coherent
2. Non-coherent.

ASK → amplitude shift Key.

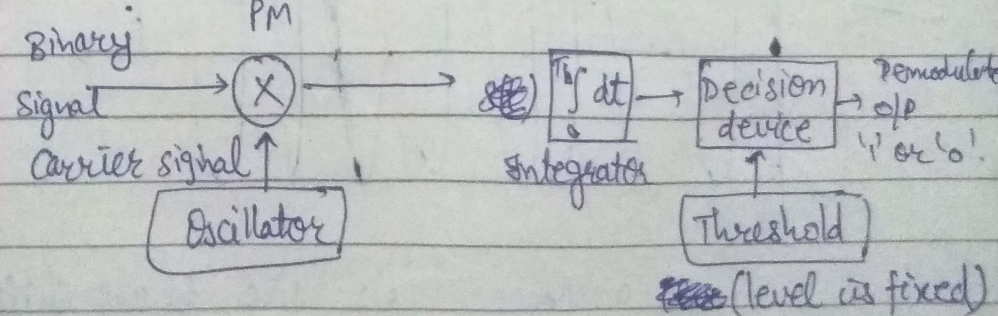
Coherent:-



⇒ . Through 0 carrier is not allowed to pass. Freq. and amplitude remains same.

$$s(t)_{ASK} = \text{modulated signal} = m(t)\cos(\omega_c t)$$

$$= \begin{cases} A.\cos(\omega_c t), & m(nT_b) = A(\text{"1"}) \\ 0, & m(nT_b) = A(\text{"0"}) \end{cases}$$



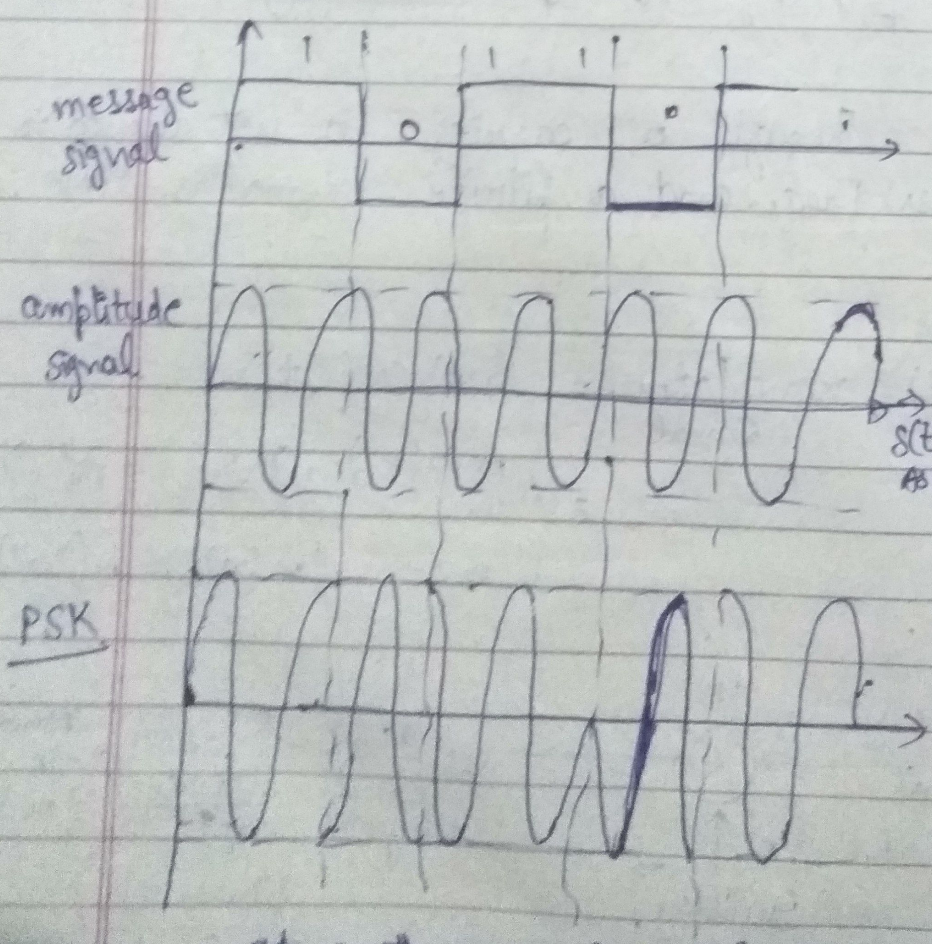
Few disadvantages of ASK :-

① Although, generation of ASK is easier in comparison with FSK and PSK and bandwidth requirement is also low but the first drawback is that, ASK has highest probability of error.

② lowest noise immunity.

③ SNR is low.

Binary PSK :- (BPSK) :-



at 0 there is 180° phase shift.