USN

06CS42

Fourth Semester B.E. Degree Examination, Dec.09/Jan.10
Graph Theory and Combinatorics

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions, selecting at least TWO questions from each part.***

PART – A

1 a. Determine $|V|$ for the following graphs.
   i) G has nine edges and all vertices have degree 3.
   ii) G is regular with 15 edges. **(06 Marks)**

b. Define isomorphism of graphs. Show that no two of the following three graphs as shown in Fig.1(b) are isomorphic. **(07 Marks)**

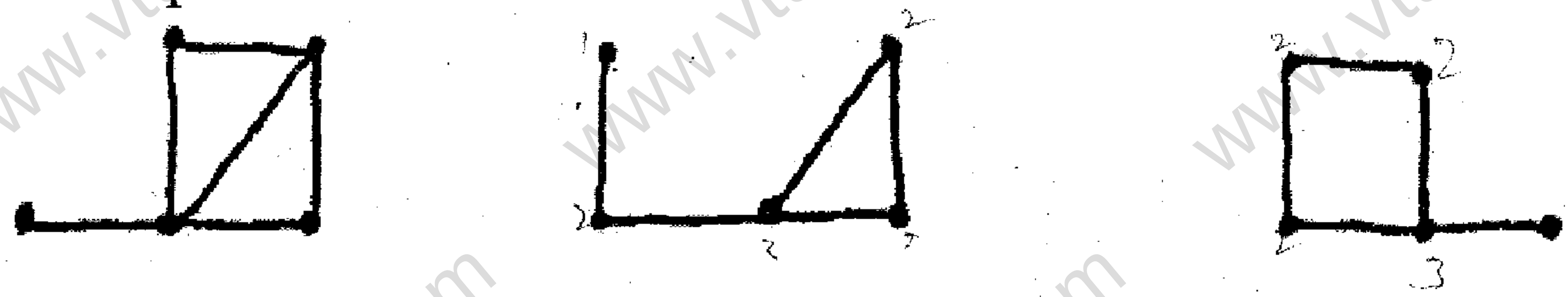

Fig.1(b).

c. Define Euler circuit. Discuss Konigsberg bridge problem. **(07 Marks)**

2 a. Define: i) Bipartite graph ; ii) Hamilton cycle and iii) Planar graph. Give one example for each. **(06 Marks)**

b. If $G = (V, E)$ is a loop-free connected planar graph with $|V| = n$, $|E| = e > 2$, and r regions, then prove that : i) $e \geq \frac{3r}{2}$; ii) $e \leq 3n - 6$. Further, if G is triangle free, then iii) $e \leq 2n - 4$. **(07 Marks)**

c. Define chromatic number. Find the chromatic polynomial for the cycle of length 4. Hence find its chromatic number. **(07 Marks)**

3 a. Define a tree. Prove that in every tree $T = (V, E)$, $|V| = |E| + 1$. **(06 Marks)**

b. Define: i) Rooted tree ; ii) Complete binary tree and iii) Spanning tree. Give an example for each. **(07 Marks)**

c. Obtain an optimal prefix code for the message ROAD IS GOOD using labelled binary tree. Indicate the code. **(07 Marks)**

4 a. State Max-flow and Min-cut theorem. For the network as shown in Fi.4(a), determine the maximum flow between the vertices A and D by identifying the cut-set of minimum capacity. **(06 Marks)**

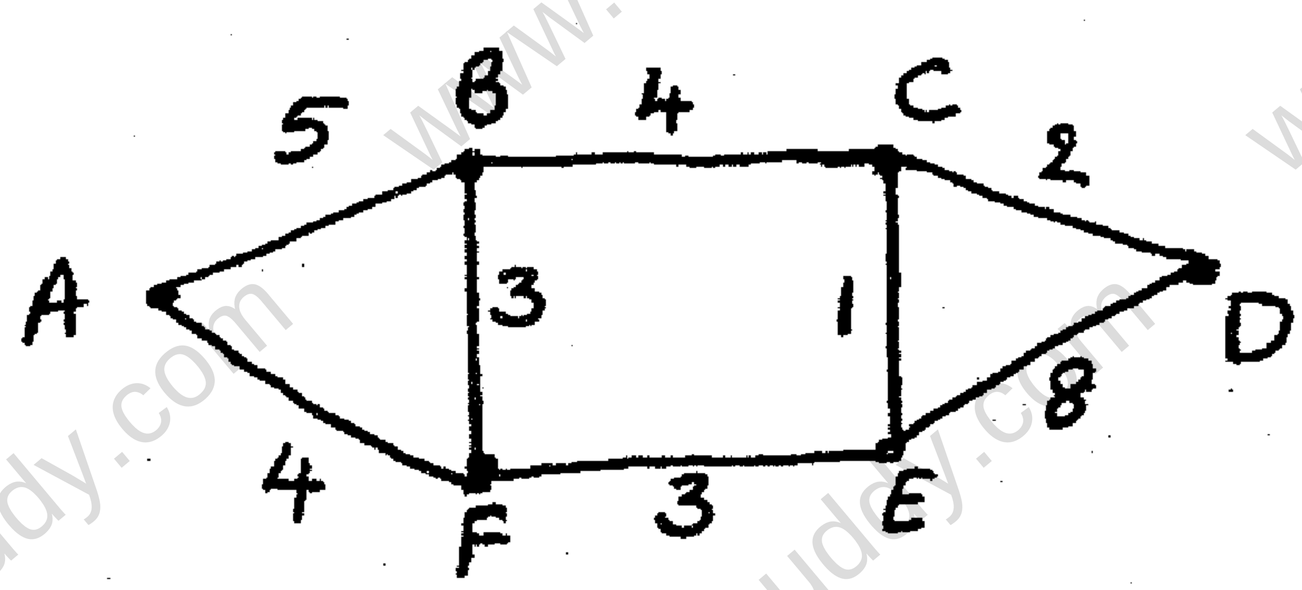

Fig.4(a).

b. State Kruskals algorithm. Apply Kruskal's algorithm to find a minimal spanning tree for the weighted graph as shown in Fig.4(b). **(08 Marks)**

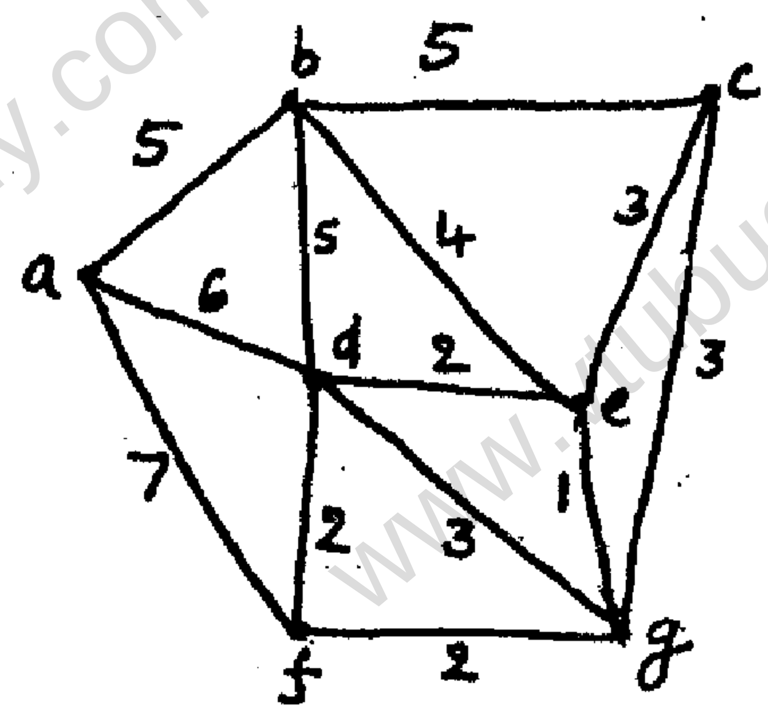

Fig.4(b).

c. Explain the steps in Dijikstra's shortest path algorithm. **(06 Marks)**

PART – B

5 a. i) How many distinct four digit integers can one make from the digits 1, 3, 3, 7, 7 and 8?
ii) Find the number of arrangements of the letters in TALLAHASSEE which have no adjacent A's **(06 Marks)**

b. In how many ways can 10 identical dime be distributed among five children if,
i) There are no restrictions
ii) Each child gets at least one dime
iii) The oldest child gets at least two dimes. **(07 Marks)**

c. Define Catalan number. In how many ways can one arrange three 1's and three – 1's so that all six partial sums (starting with the first summand) are nonnegative? List all the arrangements. **(07 Marks)**

6 a. Determine the number of positive integers n such that $1 \le n \le 100$ and n is not divisible by 2, 3 or 5. **(06 Marks)**

b. Define derangement. Find the number of derangements of 1, 2, 3, 4. List all the derangements. **(07 Marks)**

c. A girl student has sarees of 5 different colours: blue, green, red, white and yellow. On Mondays, she does not wear green; on Tuesdays, blue or red; on Wednesday, blue or green; on Thursdays red or yellow; on Fridays, red. In how many ways can she dress without repeating a colour during a week (from Monday to Friday)? **(07 Marks)**

7 a. Find a generating function for each of the following sequences:
i) $1^2, 2^2, 3^2, 4^2, \ldots\ldots\ldots$
ii) $8, 26, 54, 92, \ldots\ldots\ldots\ldots$ **(06 Marks)**

b. Using the generating function, find the number of ways of forming a committee of 9 students drawn from 3 different classes so that students form the same class do not have an absolute majority in the committee. **(07 Marks)**

c. If a leading digit 0 is permitted, using exponential generating function, find the number of r – digit binary sequences that can be formed using an even number of 0's and an odd number of 1's. **(07 Marks)**

8 a. The number of virus affected files in a system is 1000 (to start with) and this increases 250% every two hours. Use a recurrence relation to determine the number of virus affected files in the system after one day. **(06 Marks)**

b. Solve the following recurrence relations:
i) $a_n - 3\,a_{n-1} = 5\,(3^n), n \ge 1, a_0 = 2.$
ii) $a_{n+2} + 4\,a_{n+1} + 4\,a_n = 7, n \ge 0, a_0 = 1, a_1 = 2.$ **(08 Marks)**

c. Find the generating function for the recurrence relation:
$a_{n+2} - 2\,a_{n+1} + a_n = 2^n, n \ge 0$ with $a_0 = 1, a_1 = 2$. Hence solve it. **(06 Marks)**

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