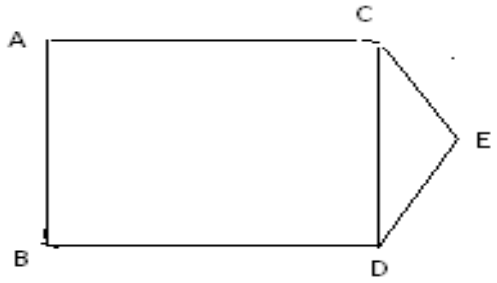



(v) Explain covering and portioning of a graph.

MODEL PAPER

Printed Page – 4 ECS 505

(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID: 2168 Roll No

B.Tech.

(SEM V) THEORY EXAMINATION 2011 -12

GRAPH THEORY

*Time: 2 Hours* *Total Marks: 50*

Note: (1) Attempt ALL questions.

(2) Make suitable assumptions whenever necessary.

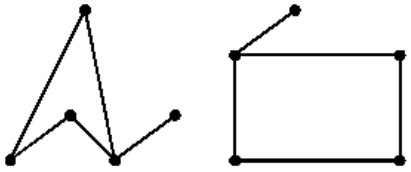
1. Attempt any four parts of the following: [3 x 4 = 12]

(i) Prove that a simple graph with n vertices and k components can have atmost $\frac{(n-k)(n-k+1)}{2}$ edges.

(ii) Define the following terms with examples:

(a)Bipartite Graph (b) Ring Sum of graphs (c) Walk
(d) Path

(iii) Define isomorphism between two graphs. Verify whether the following graphs are isomorphic to each other.



(iv) Prove that a graph is an Euler graph if and only if it can be decomposed into circuits

(v) Define a Hamiltonian path. Find an example of a non Hamiltonian graph with a Hamiltonian path.

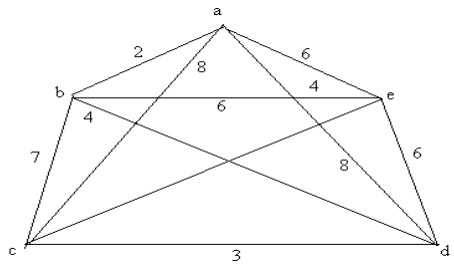
2. Attempt any two parts of the following: [6 x 2 =12]

(i) Show that

(a) If in a graph G, there is one and any one path between every pair of vertices, G is a tree.

(b) A connected graph with n vertices & n – 1 edges is a tree.

(ii) Find all the minimum spanning trees in the following graph using Prim's algorithm.




(iii)

(a) Define fundamental circuit and cut set. Prove that if v is a cut vertex of a graph G, then v is not a cut vertex of the complement $\overline{G}$ of G.

(b) Find the dimension of the circuit sub-space of the $K_4$

3. Attempt any two parts of the following: [6 x 2 =12]

(i) Define the edge connectivity, vertex connectivity of a graph. Prove that for any graph: $k(G) \leq l(G) \leq d(G)$ where k(G), $\lambda$(G), $\delta$(G) are connectivity number, edge connectivity number and minimum degree among the vertices in a graph respectively.

(ii) State and prove Euler's theorem for planarity of a graph.

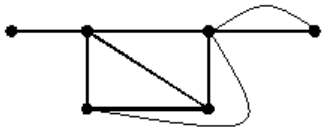
(iii) Define thickness and crossing number of a graph. Find the thickness and crossing number of the complete graph with n vertices, where n ≤ 8.

4. Attempt any four parts of the following [3.5 x 4 = 14]

(i) If the intersection of two paths in a graph is a disconnected graph, show that the union of the paths has atleast one circuit.

(ii) Define the adjacency matrix X(G) of a graph. Let X(G) be adjacency matrix of a simple graph G, then prove that $ij^{th}$ entry in $X^T$ is the number of different edge sequences of *r* edges between vertices $v_i$ and $v_j$.

(iii) Define cut-set vector and circuit vector of a graph. Find the set of all cut-set vectors and the set of all circuit vectors of the following graph:



(iv) What is the chromatic polynomial of the graph below