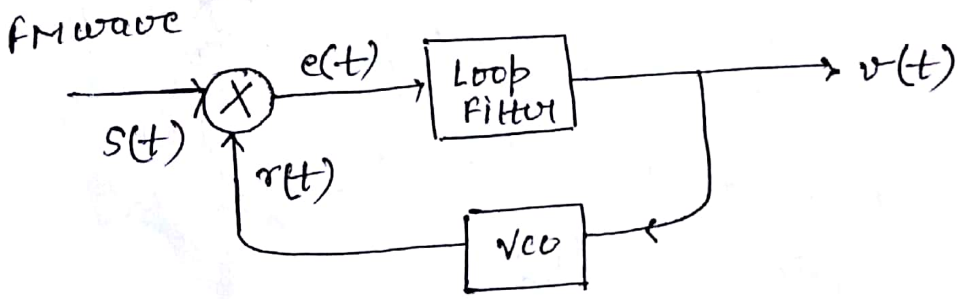
FM demodulation using Phase-Locked-Loop (PLL)

The phase-locked loop is (PLL) is primarily used in tracking phase and frequency of the carrier component of an incoming signal. PLL is useful for demodulating FM signals also in addition to synchronous demodulation of AM-SC signals or signals with few cycles of pilot carrier. PLL is particularly useful in demodulating FM signals in presence of large ~~tao~~ noise and low signal power (low signal-to-noise ratio). It is most suited for use in space vehicle-to-earth data links or where loss along the txn path is very large. It is popularly used in commercial FM receivers.



A PLL is basically a negative feedback system.

Fig 1: Phase Locked Loop (PLL)

In a typical feedback system, the signal fed back tends to follow the input signal. If the signal fed back is not equal to the input signal, the difference signal (known as the error signal) will change the value of the signal fed back until it is equal to the input signal. A PLL operates on a similar ~~principal~~ principle for the fact that the quantity fed back is not the amplitude, but generalized phase $\phi(t)$. The error signal is utilized to adjust the VCO freqn in such a way that the instantaneous phase angle comes close to the angle of the incoming signal. At this point, the two signals (the incoming and the VCO output) are in synchronism and the PLL is locked to the incoming signal. FM demodulation can be achieved by using PLL according to following steps.

Assuming that initially we have adjusted the VCO so that when the control voltage is zero, the following two conditions are satisfied

i) The freqn of the VCO is precisely set at the unmodulated carrier freqn f_c and

ii) The VCO o/p has a 90° phase shift with respect to unmodulated carrier wave.

Let the input signal applied to the PLL is an FM wave defined by

$$s(t) = A_c \sin(2\pi f_c t + \phi(t)) \quad \text{------(i)}$$

where A_c is unmodulated carrier amplitude

and $$\phi_1(t) = 2\pi k_f \int_0^t e_m(t)\,dt \quad \text{------(ii)}$$

$e_m(t) \rightarrow$ message signal

$k_f \rightarrow$ freqn sensitivity of the FM modulator

Let the VCO output be defined by

$$r(t) = A_v \cos\left(2\pi f_c t + \phi_2(t)\right) \quad \text{------ (iii)}$$

$A_v \rightarrow$ Amp of VCO o/p, when the control voltage applied to VCO is $v(t)$, then

$$\phi_2(t) = 2\pi k_v \int_0^t v(t)\,dt \quad \text{-------- (iv)}$$

$k_v \rightarrow$ frequency sensitivity of the VCO in Hz/V.

From eqn (i) & (iii), it is clear that VCO o/p and incoming signal are 90° out of phase while the VCO freqn in absence of $v(t)$ is precisely equal to the unmodulated freqn of the FM signal. The incoming FM wave $s(t)$ and the VCO o/p $r(t)$ are applied to a multiplier. The o/p of the multiplier has

(i) a high freqn component represented by

$$k_m A_c A_v \sin\left(2\pi f_c t + \phi_1(t) + \phi_2(t)\right)$$

(ii) a low freqn component represented by

$$k_m A_c A_v \sin\left(\phi_1(t) - \phi_2(t)\right)$$

wher k_m is the multiplier gain in volt^{-1}.

The high freqn component is eliminated by the filter. Thus, discarding the high freqn component, the effective input to the low-pass filter can be written as

$$e(t) = k_m A_c A_v \sin\left(\phi_1(t) - \phi_2(t)\right)$$

$$= k_m A_c A_v \sin \phi_e(t) \quad \text{------- (v)}$$

where, $\phi_e(t)$ is the phase error given by

$$\phi_e(t) = \phi_1(t) - \phi_2(t) \Rightarrow$$

$$\Rightarrow \phi_e(t) = \phi_1(t) - 2\pi k_v \int_0^t v(t)\,dt \quad \ldots\ldots (vi)$$

The loop filter can create on e(t) to produce the o/p v(t) given by

$$v(t) = \int_{-\infty}^{\infty} e(\tau)\, h(t-\tau)\, d\tau \quad \ldots\ldots (vii)$$

where h(t) is the impulse response of the filter.

Using eqn (V), (Vi) + (Vii) we get

$$\phi_e(t) = \phi_1(t) - 2\pi k_m k_v A_c A_v \int_0^t \int_{-\infty}^{\infty} \sin(\phi_e(\tau))\, h(t-\tau)\, dt\, d\tau$$

$$= \phi_1(t) - 2\pi k_o \int_0^t \int_{-\infty}^{\infty} \sin(\phi_e(\tau))\, h(t-\tau)\, d\tau\, dt \quad \ldots (viii)$$

where $k_o = k_m k_v A_c A_v$ .... (ix)

Differentiating both sides of eqn (viii), we get

$$\frac{d\phi_e(t)}{dt} = \frac{d\phi_1(t)}{dt} - 2\pi k_o \int_{-\infty}^{\infty} \sin(\phi_e(\tau))\, h(t-\tau)\, d\tau \quad \ldots (X)$$

In the above equation k_o has the dimension of frequency. On the basis of eqn (X) an equivalent model of PLL can be constructed as shown in figure below. In the model v(t) and e(t) are also included utilising the relationship between them as given in the equations (V) and (vii).

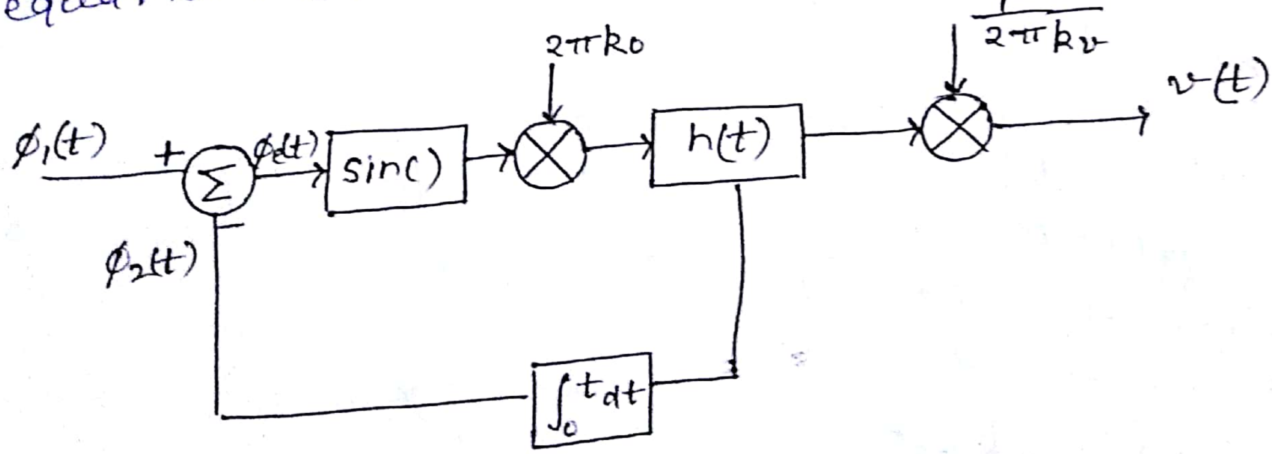


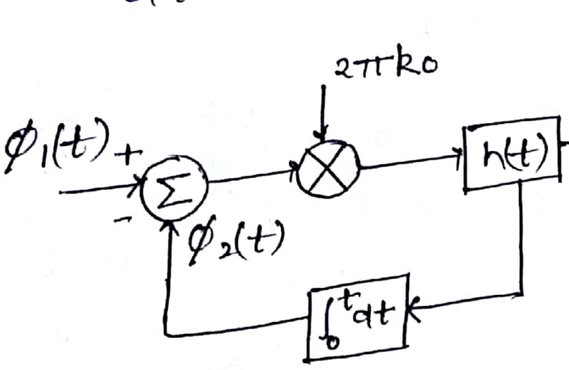
Fig2: Non-linear equivalent model of PLL

comparing ~~eqn~~ ~~i)~~ figures(1)+(2) it reveals that they are similar except for the fact the multiplier in the equivalent model has been replaced by a subtractor and a sinusoidal non-linearity and the VCO by an integrator when the phase error $\phi_e(t)$ is zero, the phase locked loop is said to be in phase-lock. when ~~the~~ $\phi_e(t)$ is at all times small compared to 1 radian, we may use the approximation

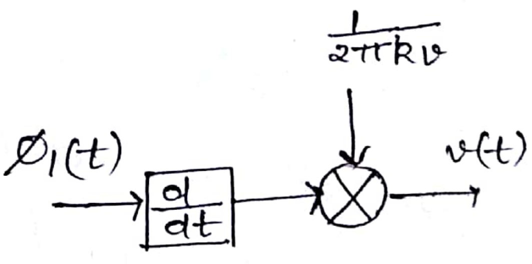
$$\sin(\phi_e(t)) \simeq \phi_e(t) \quad \text{.......(xi)}$$

which is fairly accurate as long as $\phi_e(t)$ is less than 0.5 radian. In this case the loop is said to be near-lock condition and the sinusoidal non-linearity by can be disregarded. The linearised model of PLL valid under this condition is shown in figure 3(a). In this model, phase error $\phi_e(t)$ is related to the input phase $\phi_1(t)$ by the integro-differential eqn obtained (eqn (viii)) as.

$$\frac{d\phi_e(t)}{dt} + 2\pi k_0 \int_{-\infty}^{\infty} \phi_e(\tau)\, h(t-\tau)\, d\tau = \frac{d\phi_1(t)}{dt} \quad \text{--(xii)}$$



(a)



(b)

Fig 3 : Equivalent model of PLL

Taking Fourier transform of both sides of ~~eqn(xi)~~ eqn (xii) we get.

$$\phi_e(f) = \frac{1}{1 + k_0 \frac{H(f)}{jf}} \phi_1(f) \quad \text{------} (xiii)$$

$\phi_e(f)$ & $\phi_1(f)$ → Fourier transforms of $\phi_e(t)$ & $\phi_1(t)$ respectively and $H(f)$ → transfer function of the loop filter.

$k_0 \frac{H(f)}{jf}$ is called the open-loop transfer function of the phase-locked loop.

Let $$L(f) = k_0 \frac{H(f)}{jf} \quad \text{-----} (xiv)$$

eqn (xiii) in terms of $L(f)$ can be written as –

$$\phi_e(f) = \frac{1}{1+L(f)} \phi_1(f) \quad \text{------} (xv)$$

Let for all values of f inside the base-band, we make ~~k f~~ magnitude of $L(f)$ very large compared to unity. So, from eqn (XV) we ~~get(t)~~ get

$$\phi_e(f) \to 0 \quad \text{as} \quad L(f) \gg 1$$

Under this condition the phase of the VCO becomes asymptotically equal to the phase of the incoming wave, and the phase lock is thereby established. From figure 3(a), we see that $V(f)$, the Fourier transform of $v(t)$ is related to $\phi_e(f)$ by

$$V(f) = \frac{K_0}{K_v} H(f) \phi_e(f) \quad \text{.........} (xvi)$$

This can also be obtained by taking Fourier transform on both sides of equation (viii) and using the approximation (xi). Substituting the value of $H(f)$ in terms of $L(f)$ from eqn (xiv) in eqn (xvi) we get

$$V(f) = \frac{jf}{K_v} L(f) \phi_e(f) \quad \text{---------} (xvii)$$

Substituting the value of $\phi_e(f)$ from eqn (xv) into eqn (xvii) we get

$$V(f) = \frac{\left(\frac{jf}{K_v}\right) L(f) \phi_1(f)}{1 + L(f)} \quad \text{------} (xviii)$$

when $|L(f)| >> 1$, eqn (xviii) can be approximated as

$$V(f) = \left(\frac{jf}{K_v}\right) \phi_1(f) \quad \text{---------} (xix)$$

The corresponding time-domain representation can be obtained by taking inverse Fourier transform on both sides of eqn (xix). Thus,

$$v(t) = \frac{1}{2\pi K_v} \frac{d\phi_1(t)}{dt} \quad \text{.......} (xx)$$

Thus provided the magnitude of Lt $L(f)$ is very large for all frequencies of interest, the PLL may be modelled as a differentiator with its o/p scaled by a factor $\frac{1}{2\pi K_v}$, as as shown in figure 3(b). The simplified model shown is figure 3(b) provides the basis of using PLL as an FM demodulatar. This can be easily verified by substituting the value of $\phi_1(t)$ from ~~eqn (6.2) into eqn (~~ eqn (ii) into eqn (xx). Thus,

$$v(t) = \frac{k_f}{K_v} e_m(t) \quad \text{.......} (xxi)$$

Therefore, the output $v(t)$ of phase locked loop is approximately same, except for a scale factor $\frac{k_f}{K_v}$, as the original baseband signal $e_m(t)$ and the frequency demodulation is accomplished.

It is to note that the incoming wave can have much wider BW than that of the loop filter characterised by $H(f)$ (which is restricted to baseband). Thus the control signal of the VCO has a BW of the baseband signal while the O/P of the VCO is a wideband freqn modulated wave whose instantaneous frequency tracks the incoming FM. The complexity of a PLL is determined by the transfer function $H(f)$ of the loop filter. The simplest PLL is one which has $H(f) = 1$, that is there is no loop filter, and the PLL is referred to as first order phase locked loop. The order of the PLL is determined by the order of the denominator polynomial of the closed loop transfer function, which determines the O/P transfer function $V(f)$ in terms of the input $\phi_1(f)$. as given by equation (xviii).