Digital Image Processing
Image Enhancement

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Contents

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Why image Enhancement?

- The principal objective of image enhancement is to process a given image so that the ***result is more suitable than the original image*** for a specific application.
- It accentuates or ***sharpens image*** features such as edges, boundaries, or contrast to make a graphic display more helpful for display and analysis.
- The enhancement doesn't increase the inherent information content of the data, but ***it increases the dynamic range of the chosen features*** so that they can be detected easily.

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Basics

As we have a function, we can apply operators to this function, e.g.

$$T(f(x,y)) = f(x,y) / 2$$

Operator

Image (= function !)

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BASICS CONT.....

T transforms the given image f(x,y) into another image g(x,y)

The operator T can be defined over

1. The set of pixels (x,y) of the image
2. The set of 'neighborhoods' N(x,y) of each pixel
3. A set of images f1,f2,f3,…

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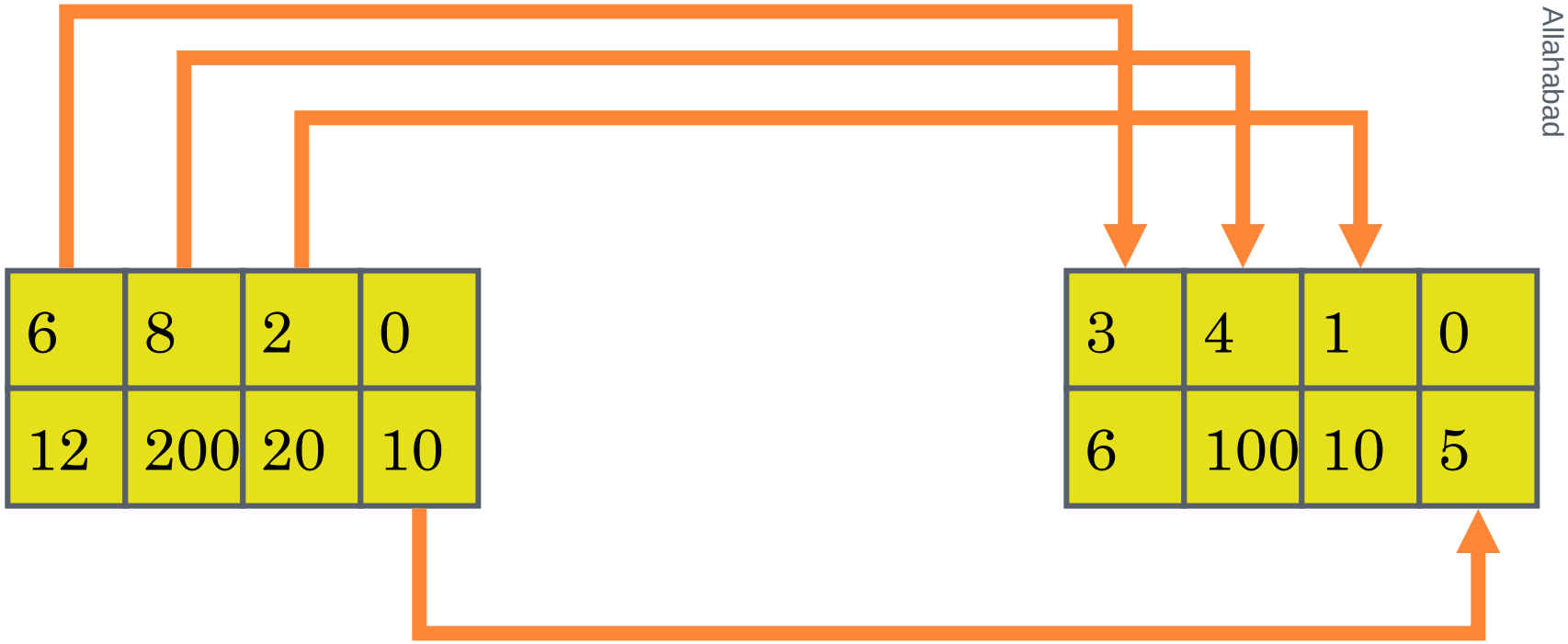
BASICS CONT.....

Operation on the set of image-pixels



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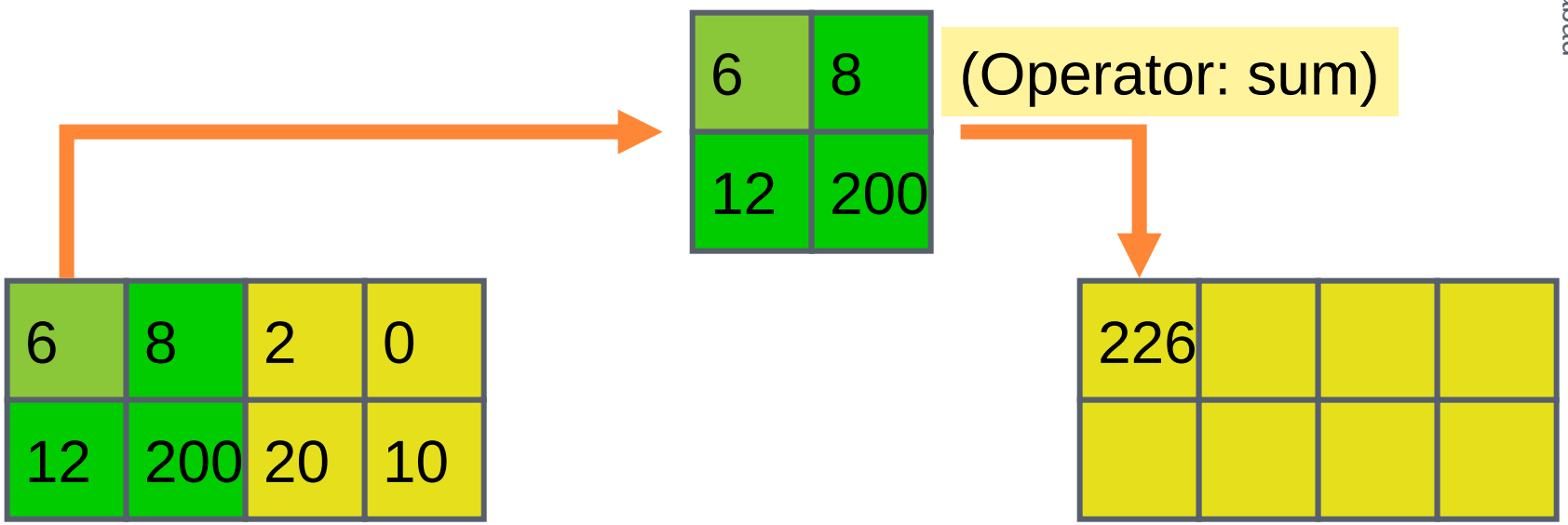
BASICS CONT.....

Operation on the set of 'neighborhoods' N(x,y) of each pixel



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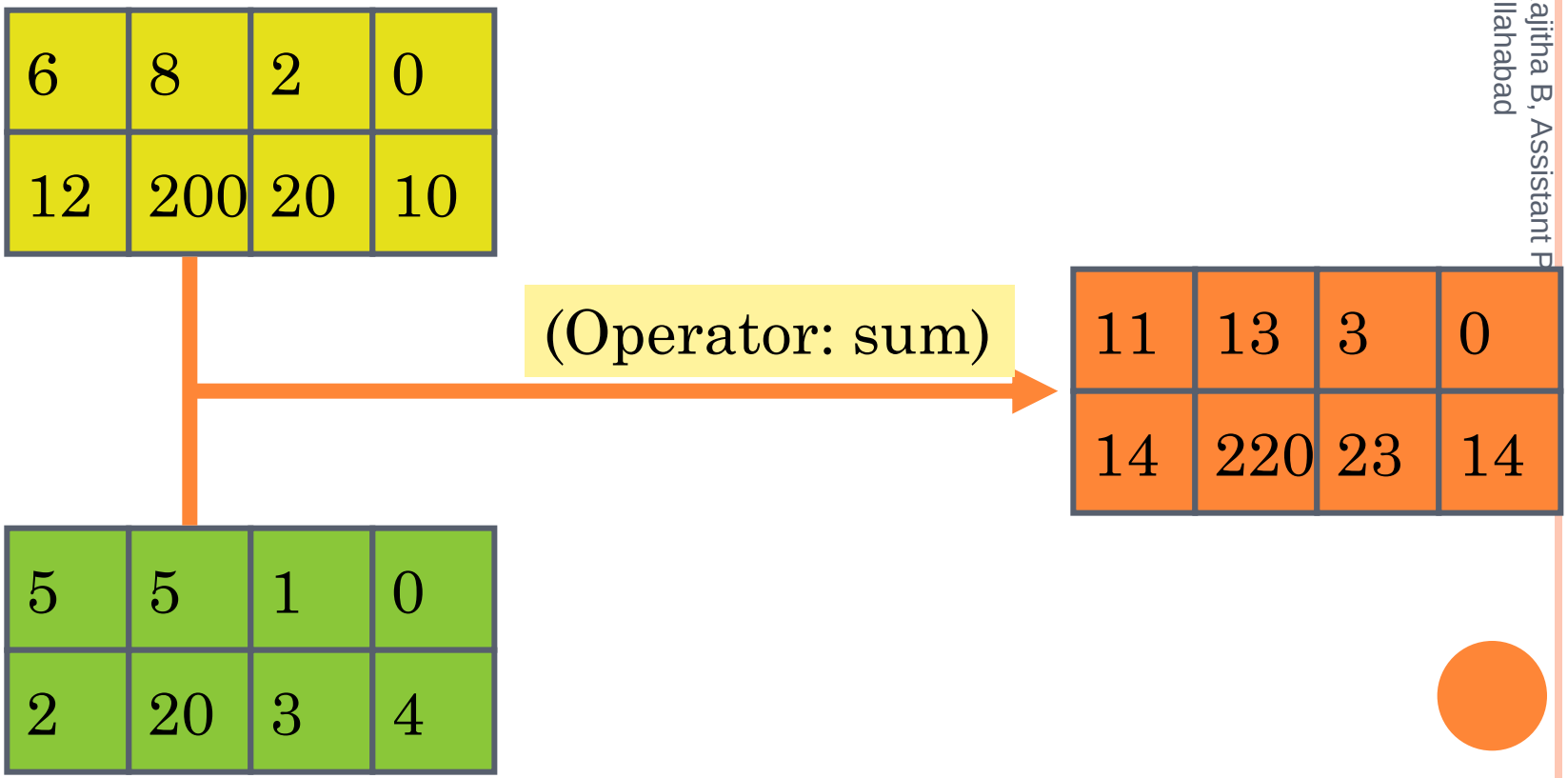
Basics cont……

Operation on a set of images f1,f2,…



SPATIAL DOMAIN TECHNIQUES

Spatial domain techniques are ***performed*** to the ***image plane itself*** and they are based on ***direct manipulation of pixels in an image.***

The operation can be formulated as

g(x,y) = T[f(x,y)]

where g is the output

f is the input image and

T is an operation on f defined over some neighborhood of (x,y).

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Spatial domain techniques (cont…)

According to the operations on the image pixels, it can be further divided into 2 categories:

1) ***Point operations***
2) ***spatial operations***

(*including linear* and non-linear operations).

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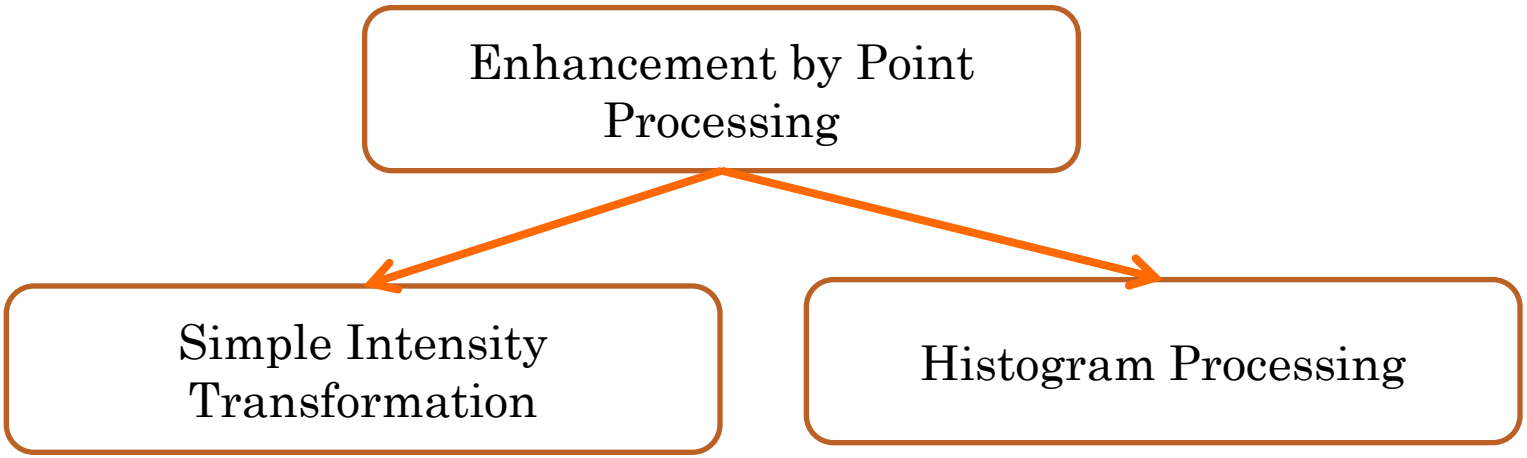
Enhancement by point processing

These processing methods are based only on the intensity of single pixels.



1. Image Negatives
2. Contrast stretching
3. Clipping
3. Compression of dynamic range
4. Gray-level slicing

1. Histogram Equalization
2. Histogram Specification
3. Local Enhancement

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Intensity Transformation: Image negatives

Transform function

$$T : g(x,y)=L-f(x,y)$$

where L is the max. intensity.

Original

Negative

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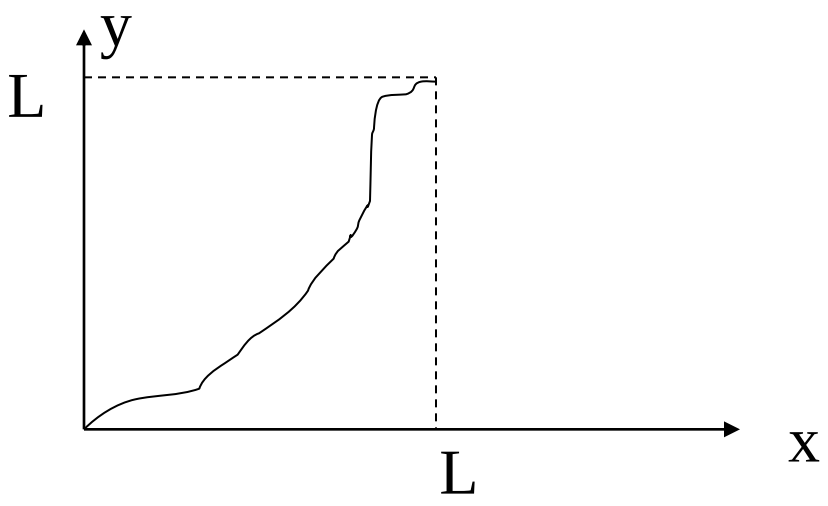
Intensity Transformation: Image negatives

Point operations are **zero-memory** operations where a given gray level $x \in [0,L]$ is mapped to another gray level $y \in [0,L]$ according to a transformation

$$y = f(x)$$



L=255: for grayscale images

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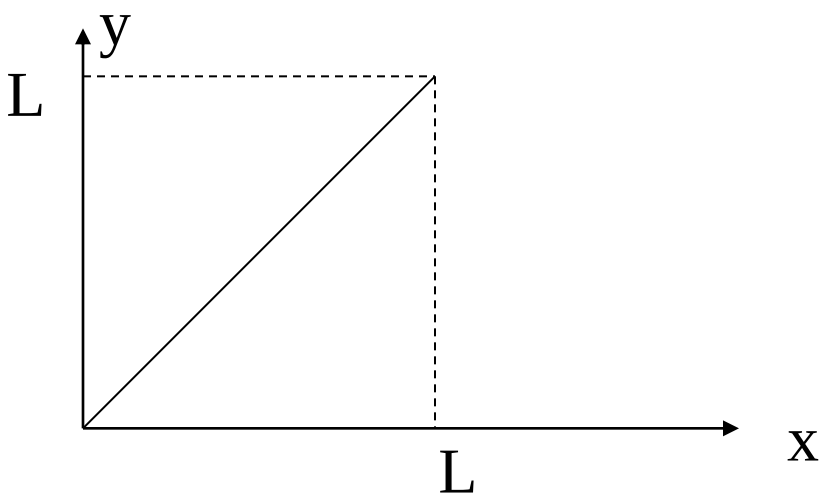
Intensity Transformation:
Image negatives

$$y = x$$



No influence on visual quality at all

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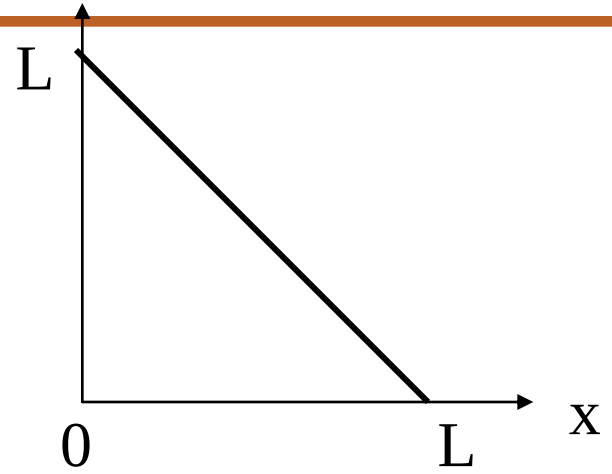
Intensity Transformation: Image negatives

$$y = L - x$$



Original

Negative

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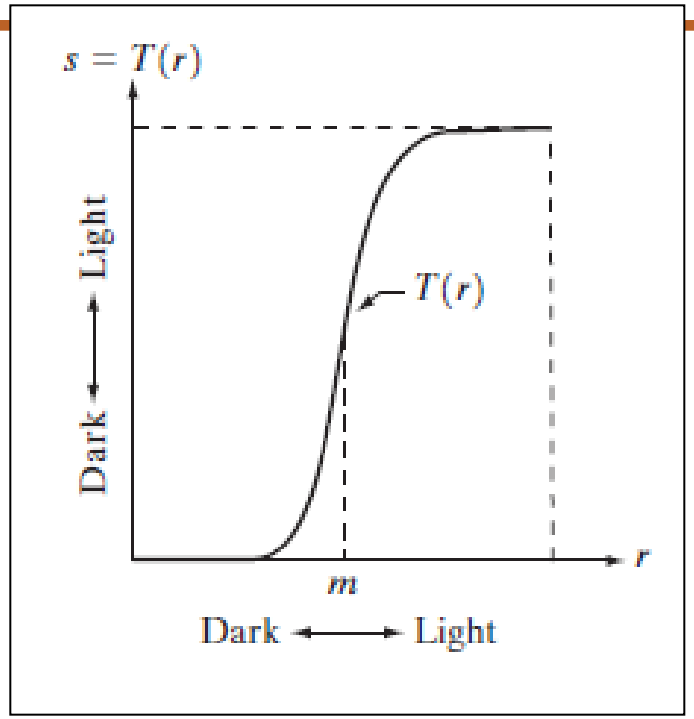
Intensity Transformation: Contrast Stretching

The effect of applying the transformation T to every pixel of **f** to generate the corresponding pixels in **g** would:

Produce higher contrast than the original image, by:

- ***Darkening the levels below m in the original image***
- ***Brightening the levels above m in the original image***

Typically, it uses a linear function



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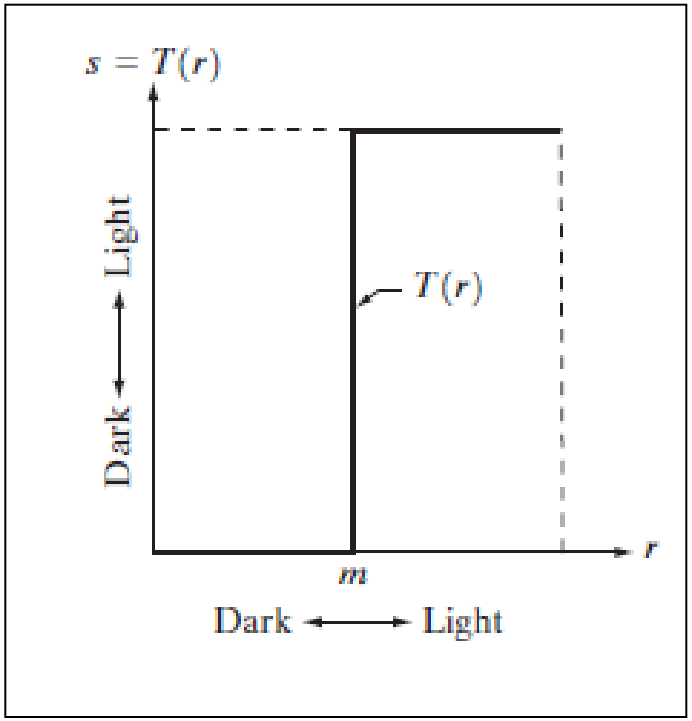
Intensity Transformation: Contrast Stretching

Thresholding

Is a limited case of contrast stretching, it produces a two-level (binary) image.



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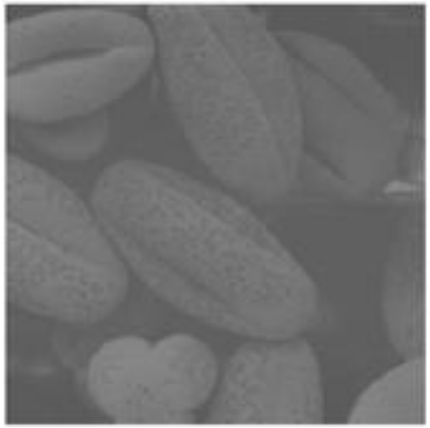
Intensity Transformation: Contrast Stretching & Thresholding example



Original Image

Contrast Image

Thresholding Image

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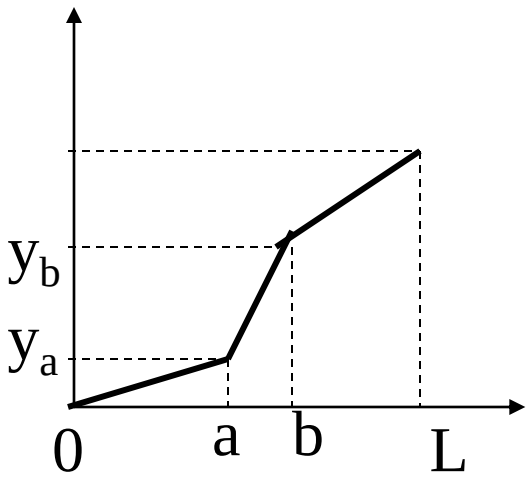
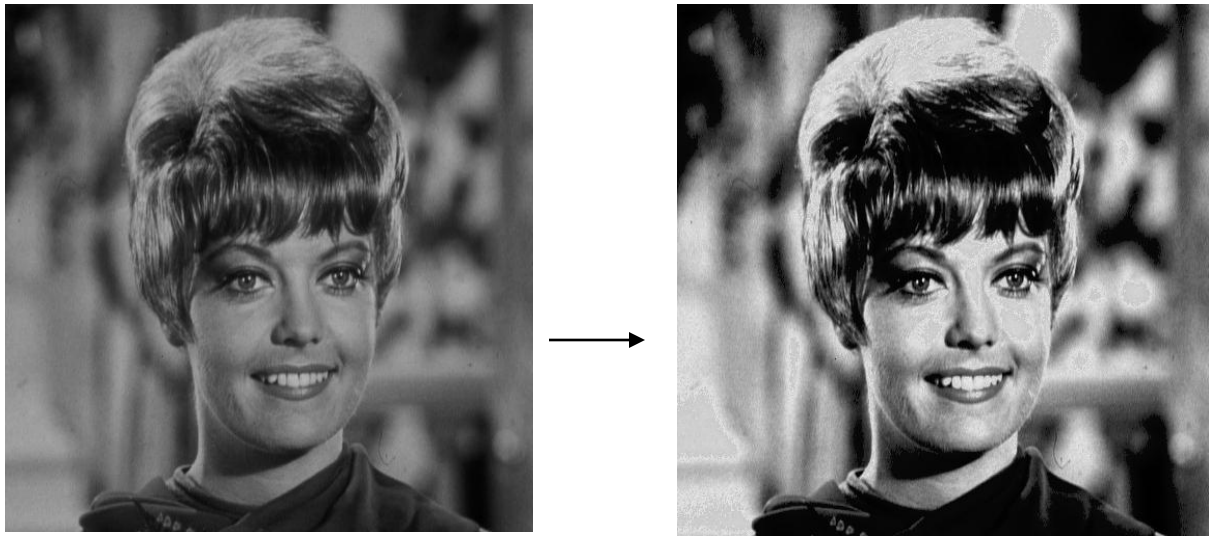
Intensity Transformation: Contrast Stretching

$$y = \begin{cases} \alpha x & 0 \le x < a \\ \beta(x-a) + y_a & a \le x < b \\ \gamma(x-b) + y_b & b \le x < L \end{cases}$$





$a = 50, b = 150, \alpha = 0.2, \beta = 2, \gamma = 1, y_a = 30, y_b = 200$

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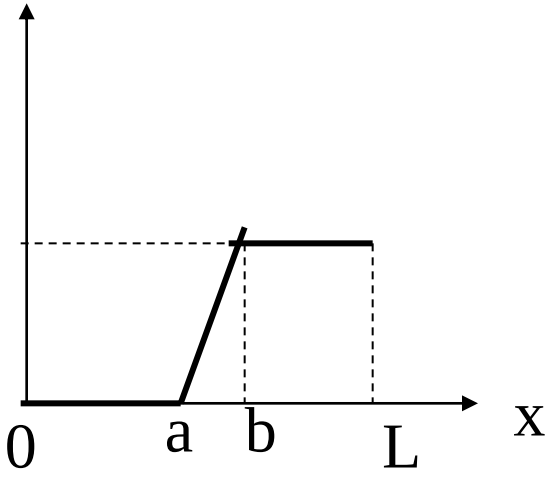
Intensity Transformation: Clipping

$$y = \begin{cases} 0 & 0 \le x < a \\ \beta(x-a) & a \le x < b \\ \beta(b-a) & b \le x < L \end{cases}$$



$a = 50, b = 150, \beta = 2$

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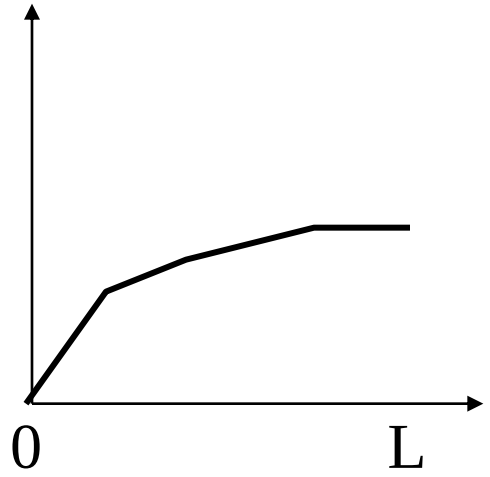
Intensity Transformation: Range Compression

$$y = c\log_{10}(1+x)$$



c=100

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Intensity Transformation: Gray level scaling

Highlighting a specific range of gray levels in an image often is desired. Applications include enhancing features such as masses of water in ***satellite imagery*** and enhancing flaws in ***x-ray images.***

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Intensity Transformation: Gray level scaling

- Grey-level transformation functions (also called, intensity functions), are considered the simplest of all image enhancement techniques.
- The value of pixels, before an after processing, will be ***denoted by r and s***, respectively. These values are related by the expression of the form:

$$s = T(r)$$

where T is a transformation ***that maps a pixel value r into a pixel value s.***

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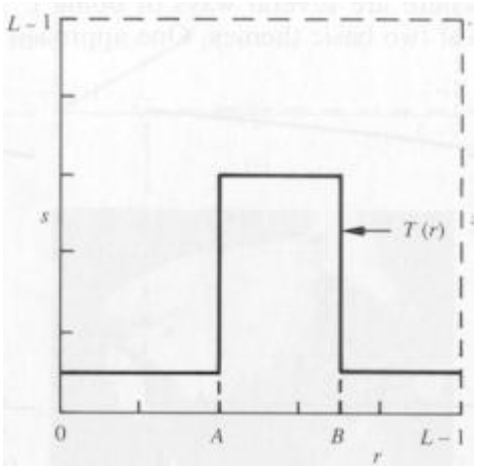
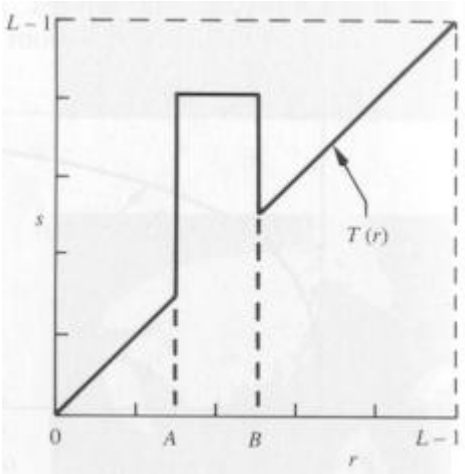
Intensity Transformation: Gray level scaling





A transformation function that highlights a range [A,B] of intensities while diminishing all others to a constant.

A transformation function that highlights a range [A,B] of intensities but preserves all others.

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Types of functions in enhancement

The three basic types of functions used frequently for image enhancement:

- Linear Functions:
  - Negative Transformation
  - Identity Transformation
- Logarithmic Functions:
  - Log Transformation
  - Inverse-log Transformation
- Power-Law Functions:
  - n^{th} power transformation

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Types of functions in enhancement: Linear functions

Identity Function

- Output intensities are identical to input intensities
- This function doesn't have an effect on an image, it was included in the graph only for completeness
- Its expression:

$$s = r$$

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Types of functions in enhancement: Logarithmic functions

Log Transformation

The general form of the log transformation:

$$s = c \log (1+r)$$

Where c is a constant, and $r \geq 0$

- Log curve maps a ***narrow range*** of ***low gray-level*** values in the input image into a ***wider range of the output levels.***
- Used ***to expand*** the values of ***dark pixels*** in an image while compressing the higher-level values.
- It compresses the dynamic range of images with large variations in pixel values.

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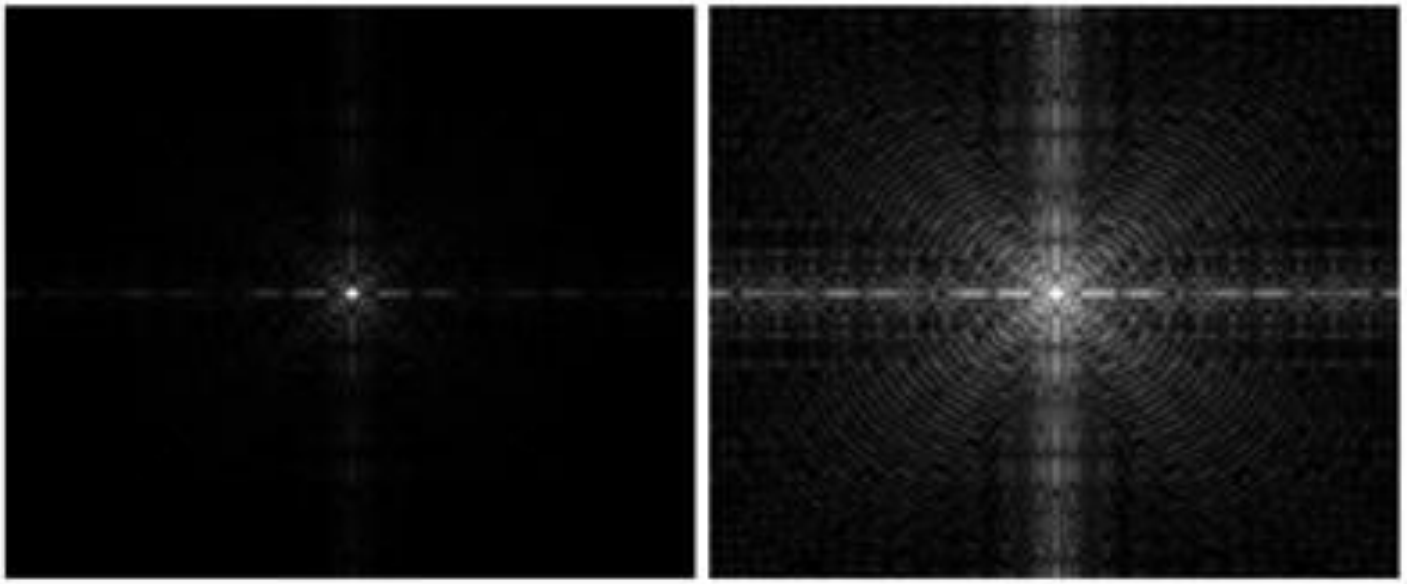
Types of functions in enhancement: Logarithmic functions example



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Types of functions in enhancement: Inverse Logarithmic functions

Inverse Logarithm Transformation

- Do opposite to the log transformations
- Used to expand the values of high pixels in an image while compressing the darker-level values.

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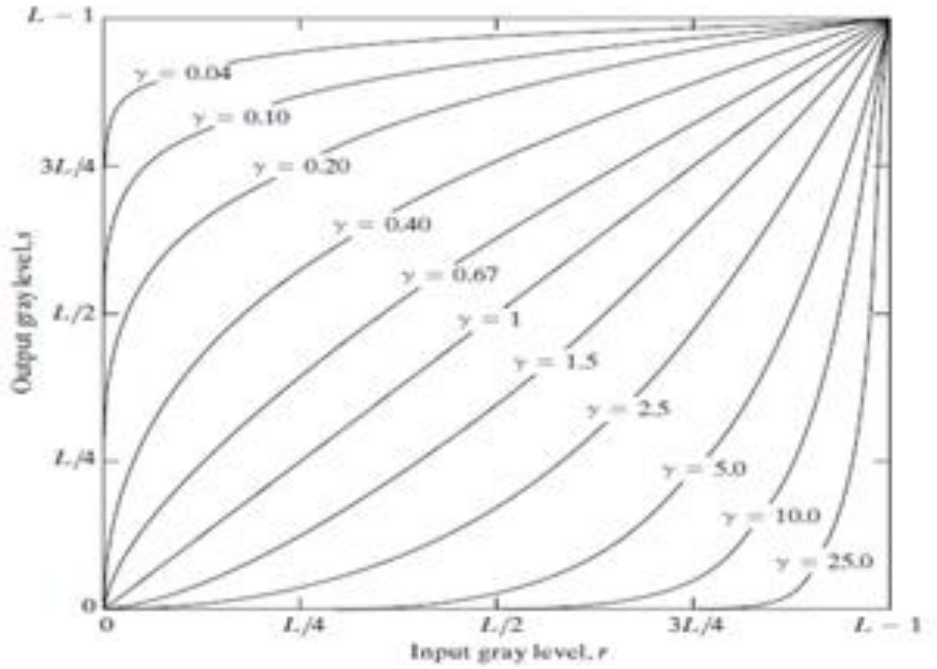
Types of functions in enhancement: Power law functions

Power-law transformations have the basic form of:

$$s = c.r^{\gamma}$$

Where **c and** γ are positive constants



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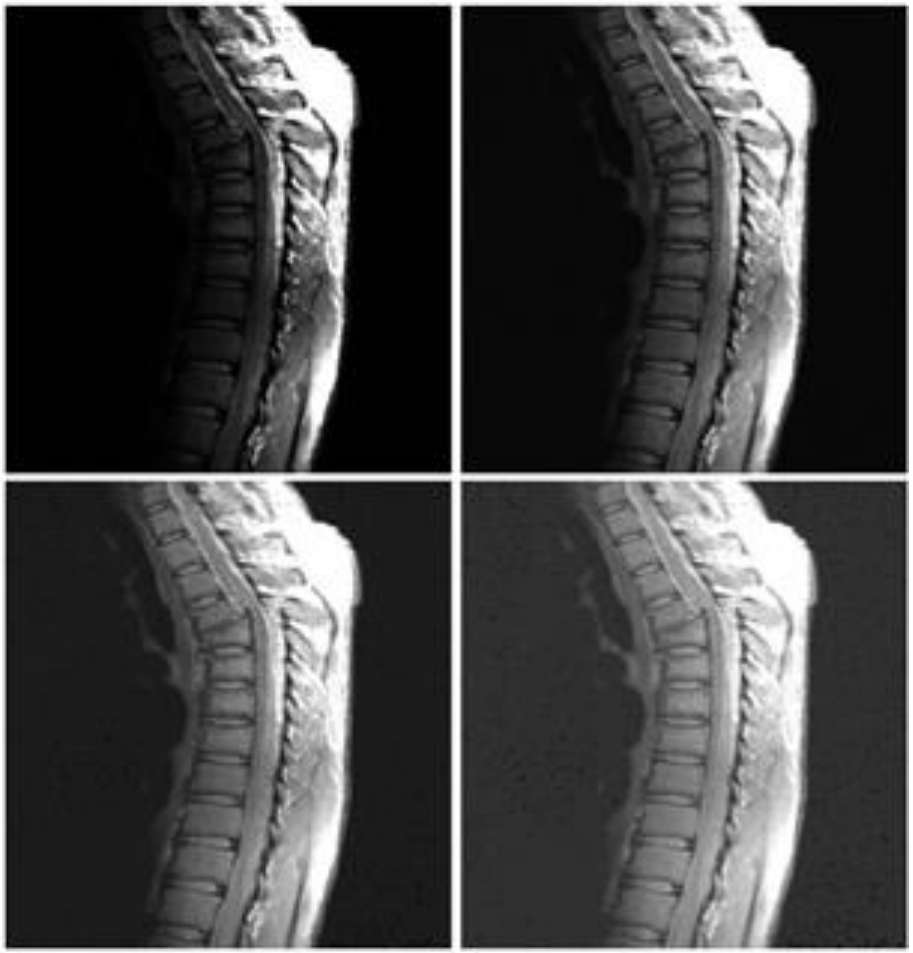
Types of functions in enhancement: Power law functions examples



a) MRI image of fractures human spine
b) C=1 and gamma = 0.6

c) C=1 and gamma = 0.4
d) C=1 and gamma = 0.3

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Types of functions in enhancement: Power law functions examples

a) MRI image of fractures human spine
b) C=1 and gamma = 3.0

c) C=1 and gamma = 4.0
d) C=1 and gamma = 5.0

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Histogram Processing

- ***Histogram of an image represents the relative frequency of occurrence of various gray levels in the image.***
- The histogram of a digital image with gray levels in the range [0,L-1] is a discrete function p(rk)=nk/n, where rk is the kth gray level, nk is the number of pixels in the image with that gray level, n is the total number of pixels in the image, and k=0,1..L-1.
- P(rk) gives an estimate of the probability of occurrence of gray level rk.

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Histogram Processing

- The shape of the histogram of an image gives us useful information about the possibility for contrast enhancement.
- A histogram of a narrow shape indicates little dynamic range and thus corresponds to an image having low contrast.

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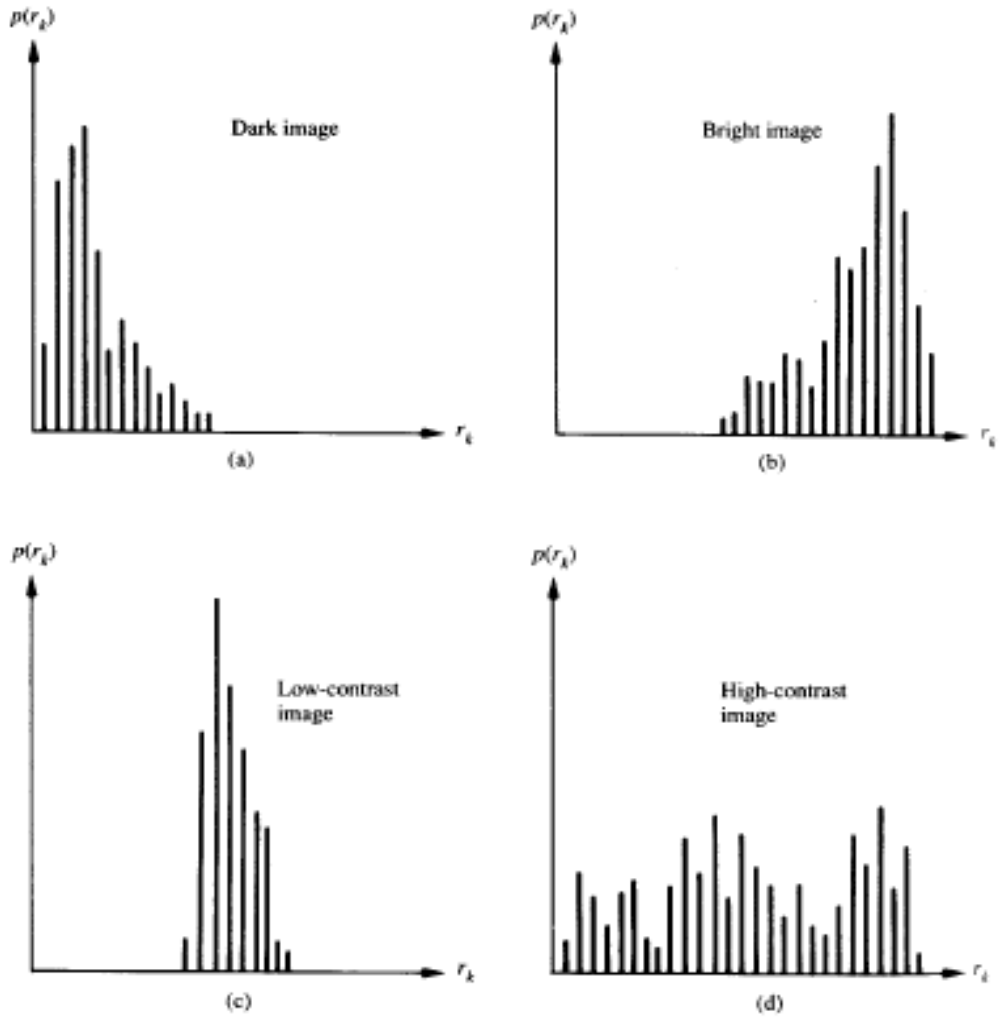
Histogram Processing



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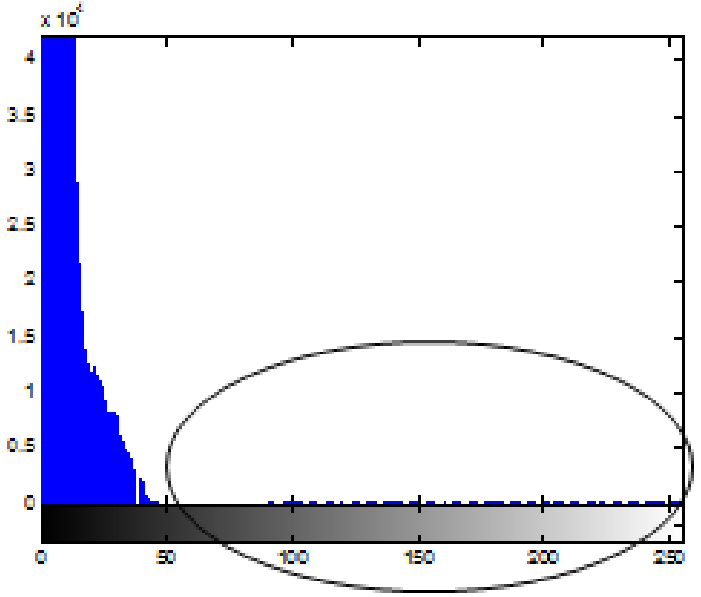
Histogram Need.....

It is a baby in the cradle!



Histogram information reveals that image is under-exposed

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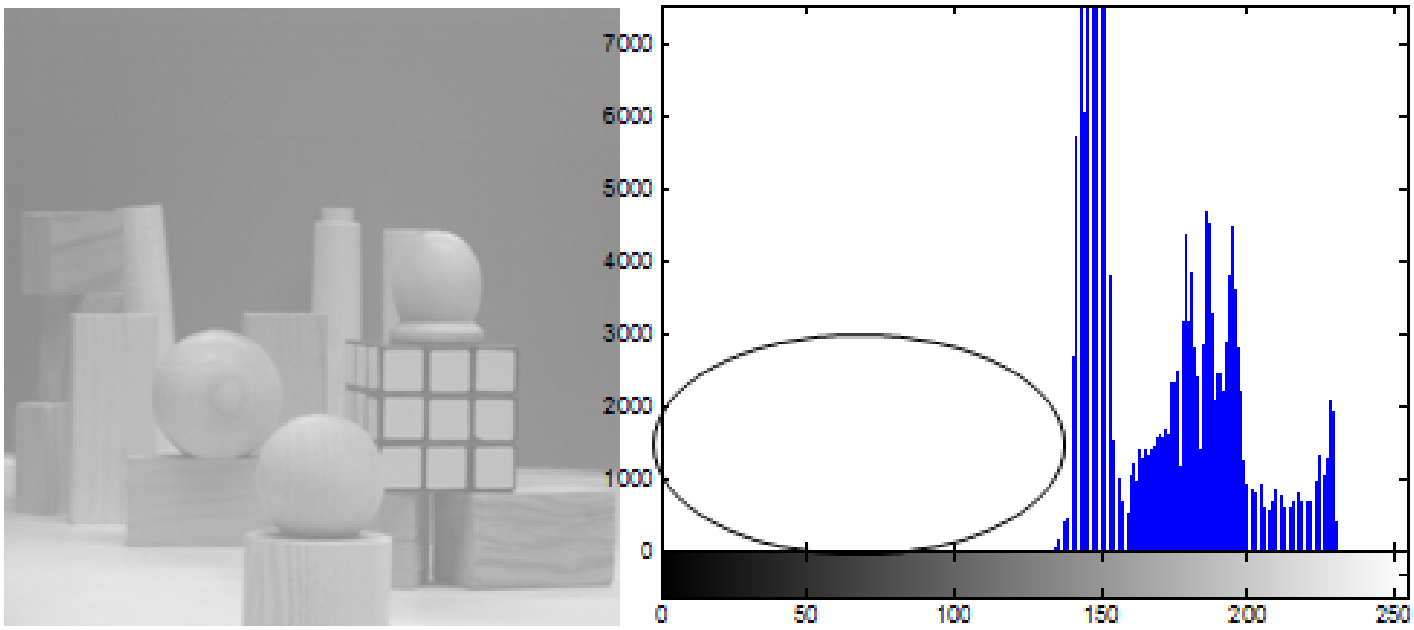
Histogram Need.....



Over-exposed image

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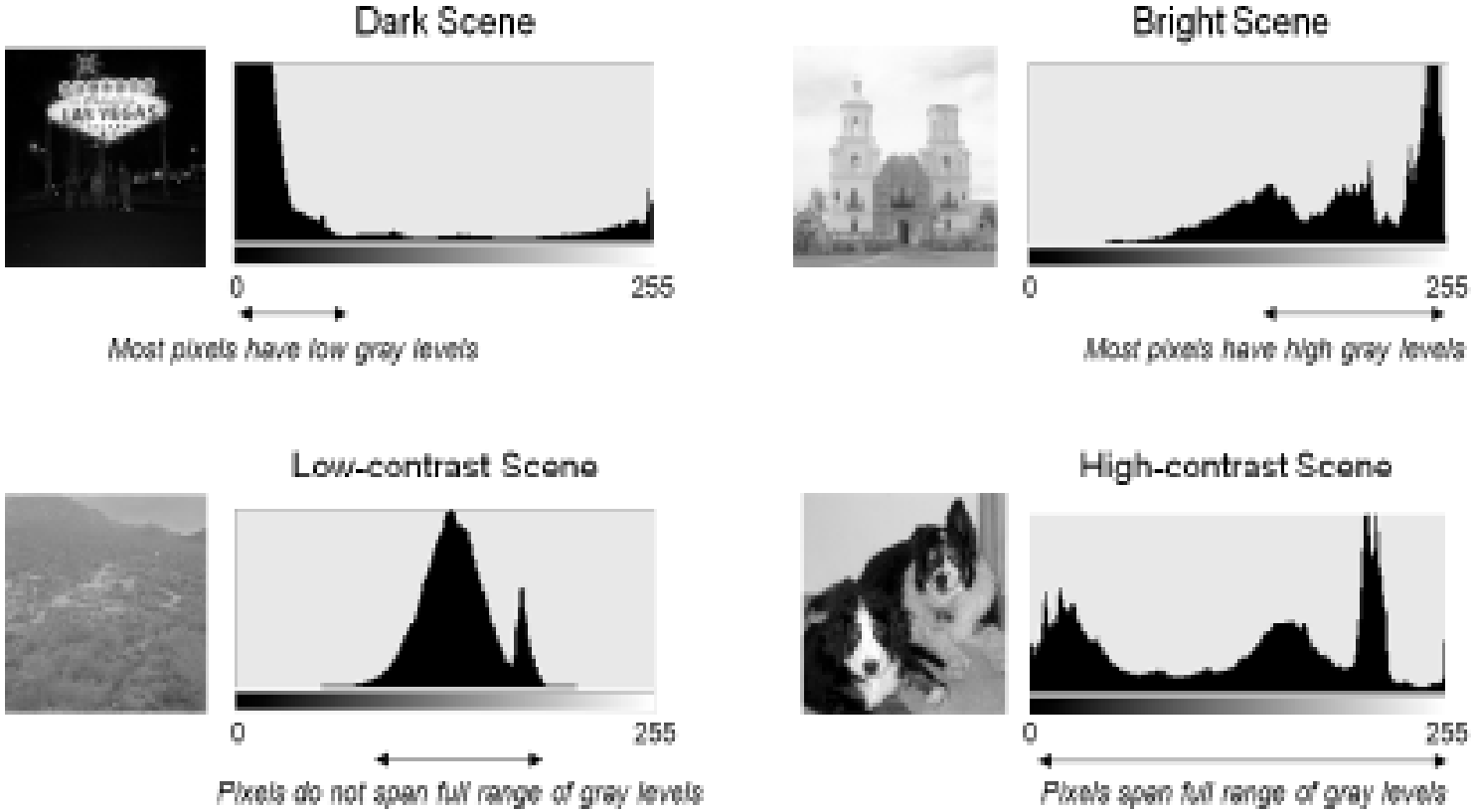
Histogram examples



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HISTOGRAM EQUALIZATION

- The objective is to map an input image to an output image such that its histogram is uniform after the mapping.
- Let *r represent the gray levels in the image to be* enhanced and *s is the enhanced output with a* transformation of the form ***s=T(r).***

Assumption:

- T(*r) is single-valued and monotonically* increasing in the interval [0,1], which preserves the order from black to white in the gray scale.
- 0 <= *T(r) <=1 for 0 <= r <=1, which guarantees the* mapping is consistent with the allowed range of pixel values

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HISTOGRAM EQUALIZATION

$$s_k = T(r_k) = \sum_{j=0}^{k} p_r(r_j)$$
$$= \sum_{j=0}^{k} \frac{n_j}{n} \qquad k = 0, 1, 2, \ldots, L-1.$$

Each pixel with level r_k is mapped into a corresponding pixel with level s_k in the output image called as histogram linearization.

T is the mapping function. It is the discrete cumulative distribution function.

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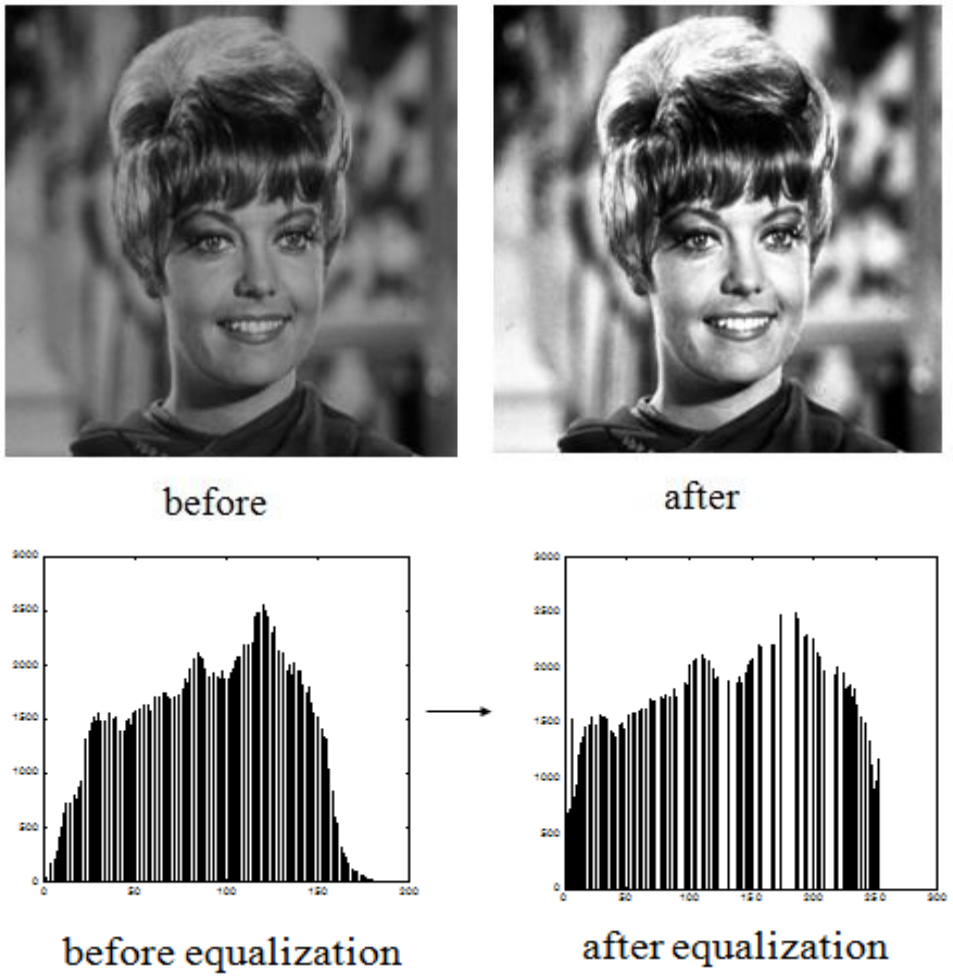
Histogram equalization example



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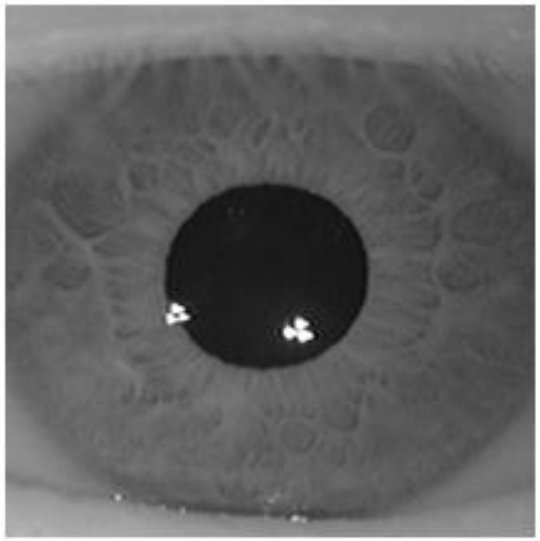
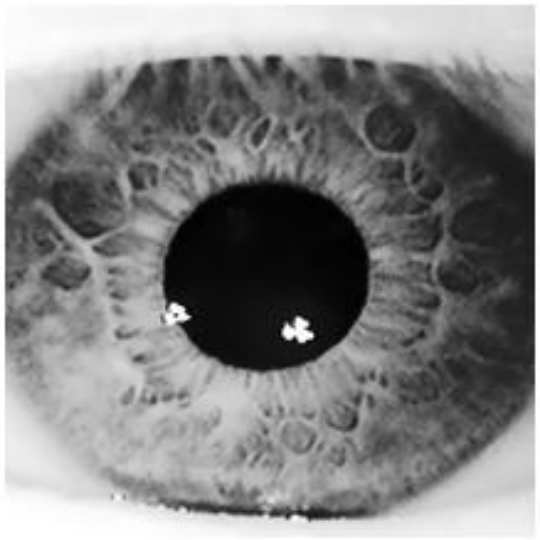
Histogram equalization example
Iris recognition



before



after

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Local enhancement

- It is often necessary to enhance details over small areas.
- The number of pixels in these areas may have negligible influence on the computation of a global transformation, so the use of global histogram specification does not necessarily guarantee the desired local enhancement.

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Local enhancement: Procedure

1. Define a square or rectangular neighborhood and move the center of this window from pixel to pixel.
2. Determine the histogram equalization or histogram specification transformation function with the histogram of the windowed image at each location.
3. Map the gray level centers in the window with the transformation function.
4. Move to an adjacent pixel location and the procedure is repeated.

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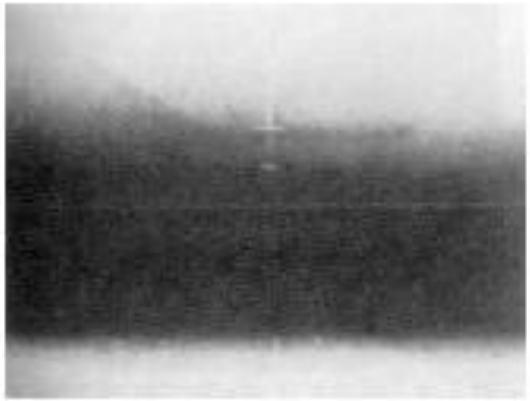
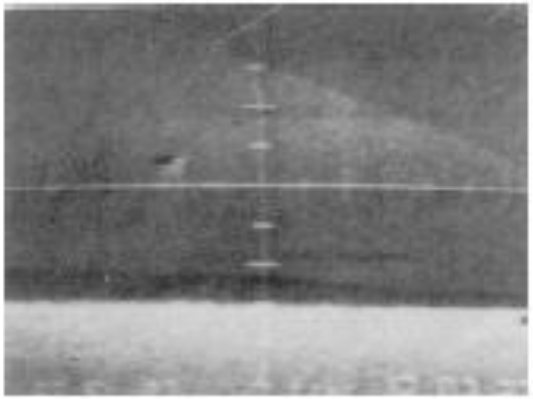
Local enhancement: Procedure





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Enhancement through the neighbour pixels

Simple neighbourhood operations example:

Min: Set the pixel value to the minimum in the neighbourhood

Max: Set the pixel value to the maximum in the neighbourhood

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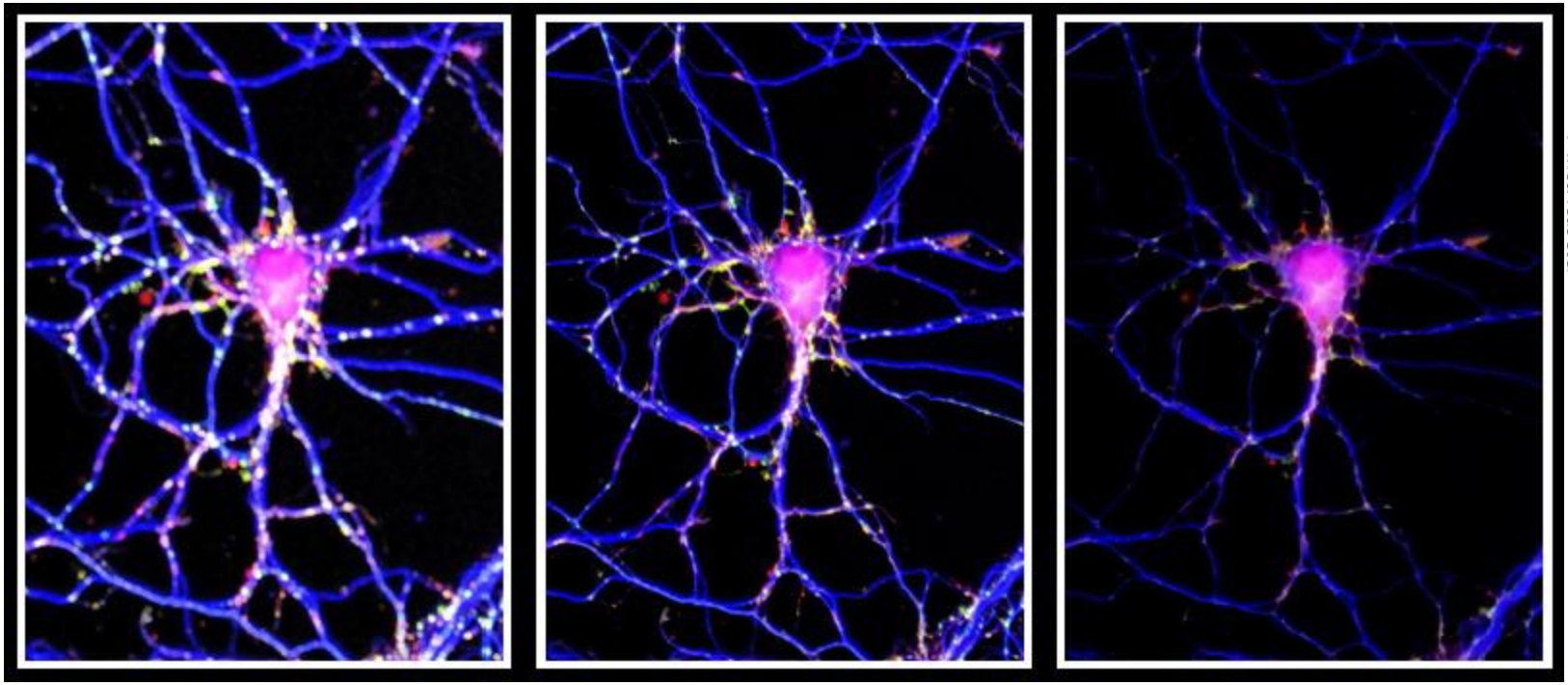
Enhancement through the neighbour pixels



Max original min

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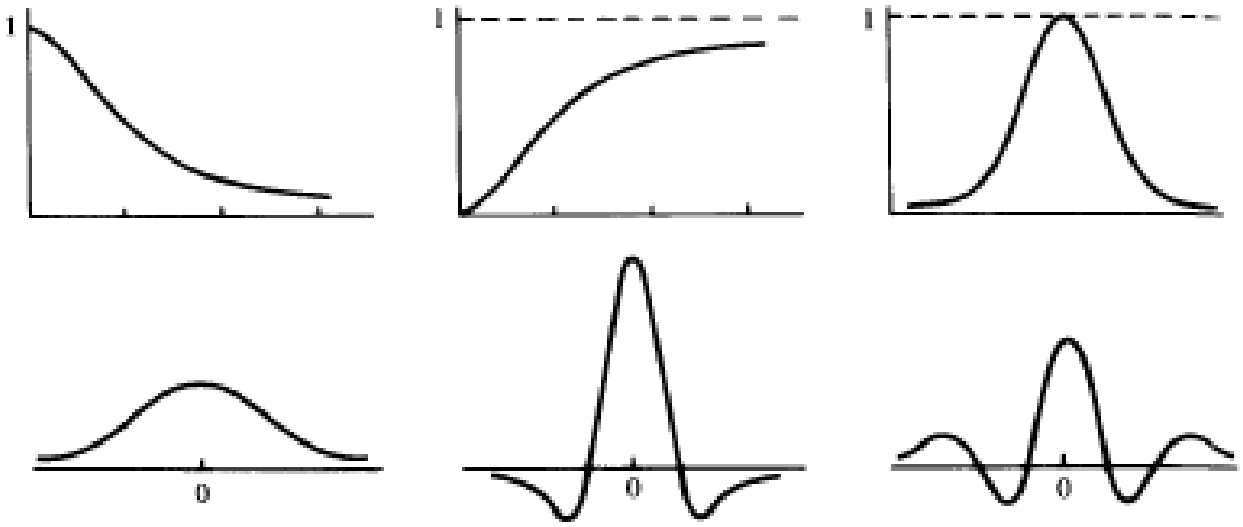
Spatial filtering

- The use of spatial masks for image processing is called *spatial filtering*.
- The masks used are called *spatial filters*.



Low Pass Filter High Pass Filter Band Pass Filter

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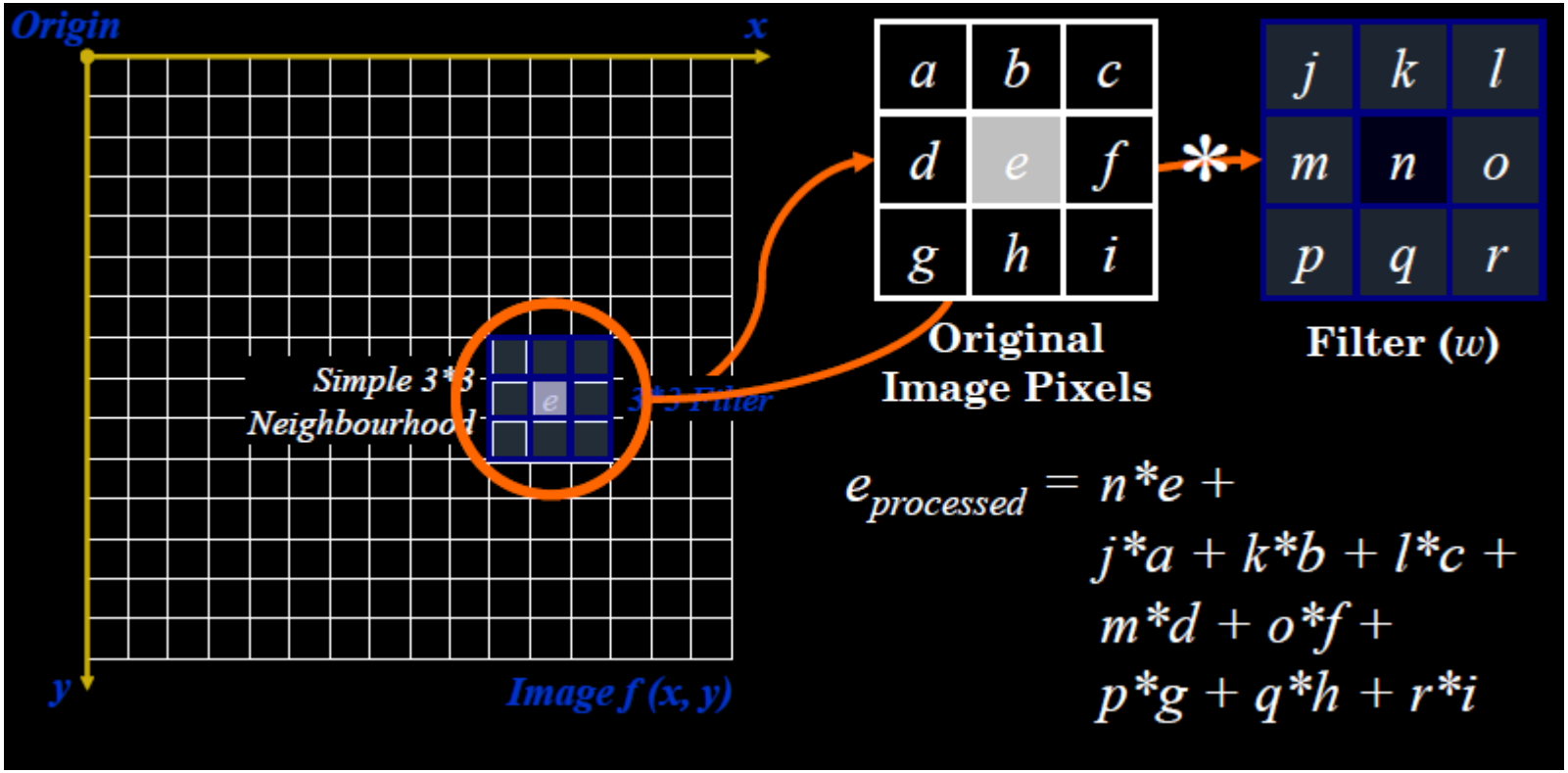
What is filtering?



Performed for each pixel in the original image to generate the filtered image

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Smoothing Filters

- Average all of the pixels in a neighbourhood around a central value according to selected mask.

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SMOOTHING FILTERS

- Smoothing filters are used for blurring and for noise reduction.
- Blurring is used in pre-processing steps, such as removal of small details from an image prior to object extraction, and bridging of small gaps in lines or curves.
- Noise reduction can be accomplishing by blurring with a linear filter and also by nonlinear filtering.

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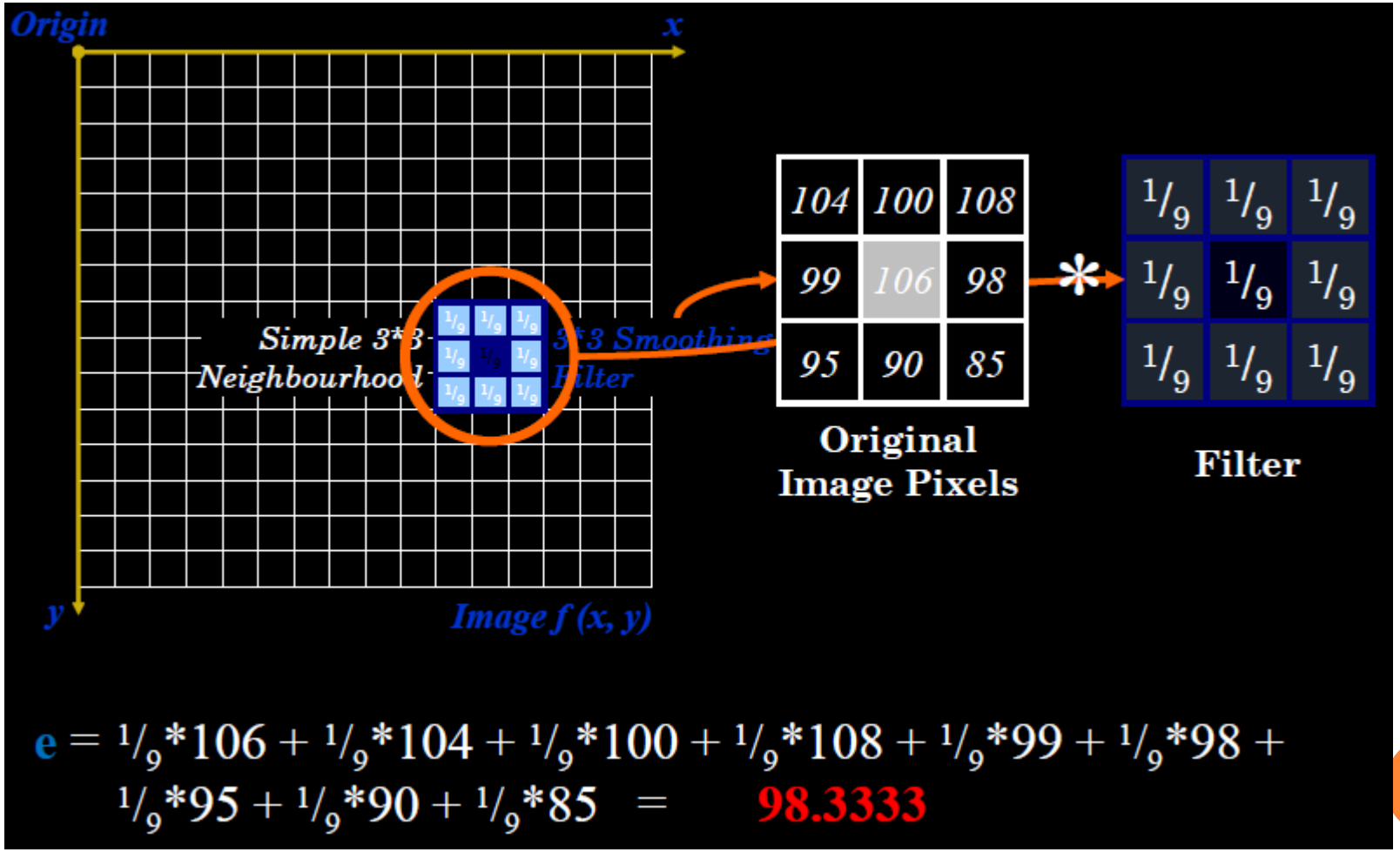
Smoothing Filtering Example



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Low Pass Filtering

- The key requirement is that all coefficients are positive.
- Neighborhood averaging is a special case of LPF where all coefficients are equal.
- It blurs edges and other sharp details in the image.

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Low Pass Filtering Example

Original with (a) spike noise (b) white noise

Low-pass filtering output

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Median Filtering

- If the objective is ***to achieve noise reduction instead of blurring***, this method should be used.
- This method is particularly ***effective*** when the noise pattern consists of ***strong, spike-like components*** and the characteristic to be reserved is edge sharpness.
- It is a nonlinear operation.
- For each input pixel *f(x,y), we sort the values of the* pixel and its neighbors to determine their median and assign its value to output pixel *g(x,y)*.

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Median Pass Filtering Example

Original with (a) spike noise (b) white noise

Median filtering output

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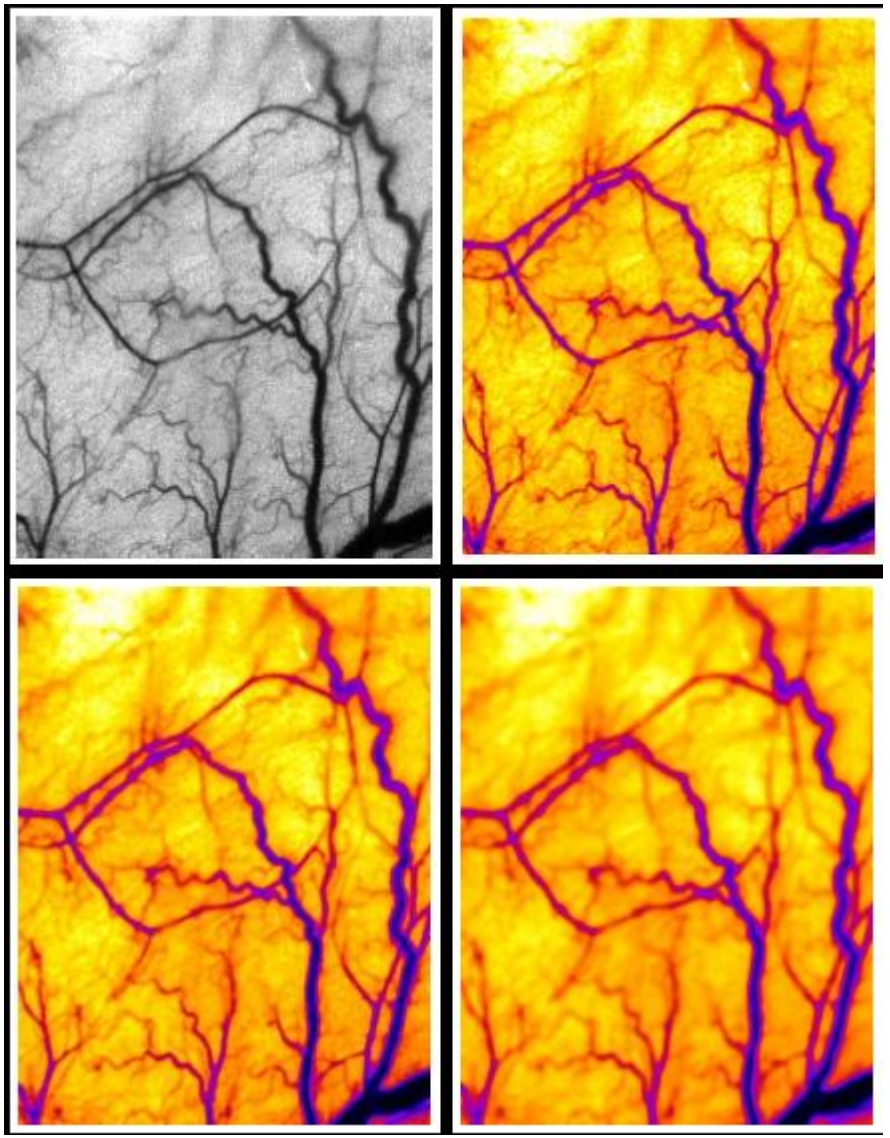
Median Filtering Example

As R grows
Fine details and noise vanish



Original, R =1, R=2, R=4

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Sharpening Filters

- ***To highlight*** fine detail in an image or ***to enhance*** detail ***that has been blurred***, either in error or as a natural effect of a particular method of image acquisition.
- Uses of image sharpening vary and include applications ranging from electronic printing and medical imaging to industrial inspection and autonomous target detection in smart weapons.

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High Pass Filter

- The shape of the impulse response needed to implement a highpass spatial filter indicates that the filter should have ***positive coefficients near its center, and negative coefficients in the outer periphery.***
- The results of highpass filtering involve some form of scaling and/or clipping to make sure that the gray levels of the final results are within [0,L-1].

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Derivative Filters

Differentiation can be expected to have the opposite effect of averaging, which tends to blur detail in an image, and thus sharpen an image and be able to detect edges.

The most common method of differentiation in image processing applications is the gradient.

For a function $f(x,y)$, the gradient of f at coordinates (x',y') is defined as the vector

$$\nabla f(x', y') = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix}_{(x', y')}$$

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Derivative Filters

- Its magnitude can be approximated in a number of ways, which result in a number of operators such as **Roberts, Prewitt and Sobel** operators for computing its value.


Sobel


Prewitt


Robert

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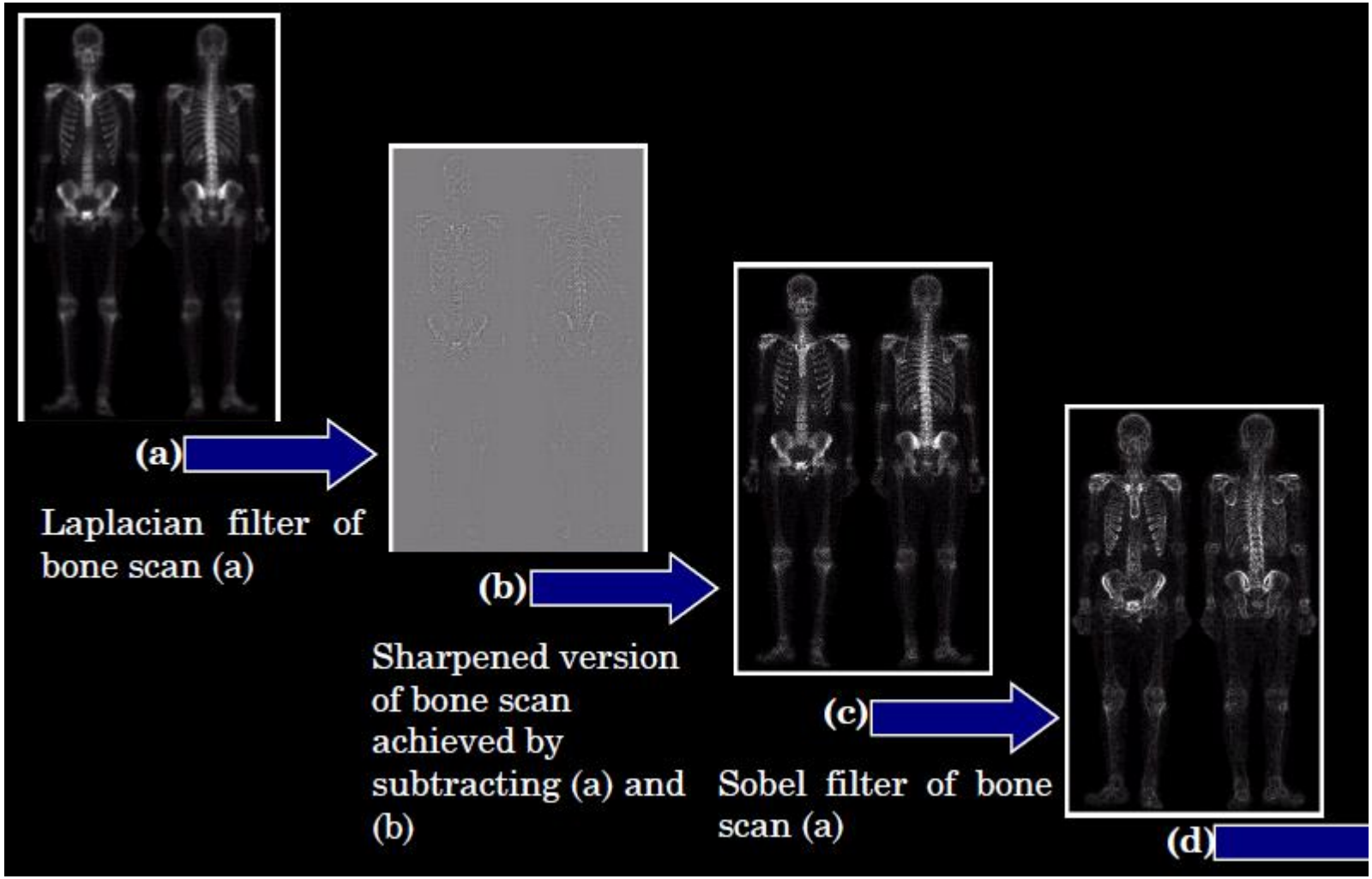
Mix of Different filters



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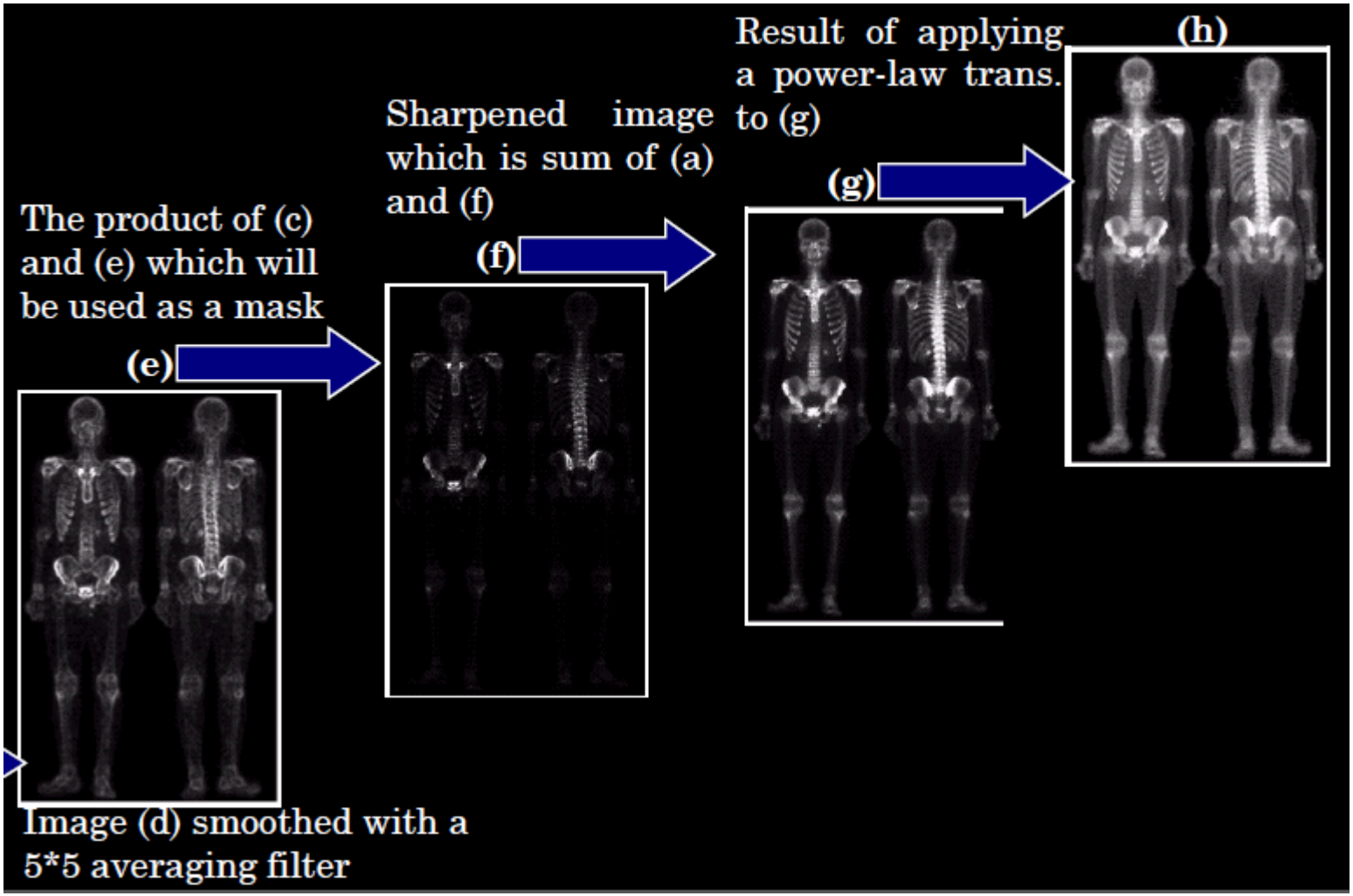
Mix of Different filters



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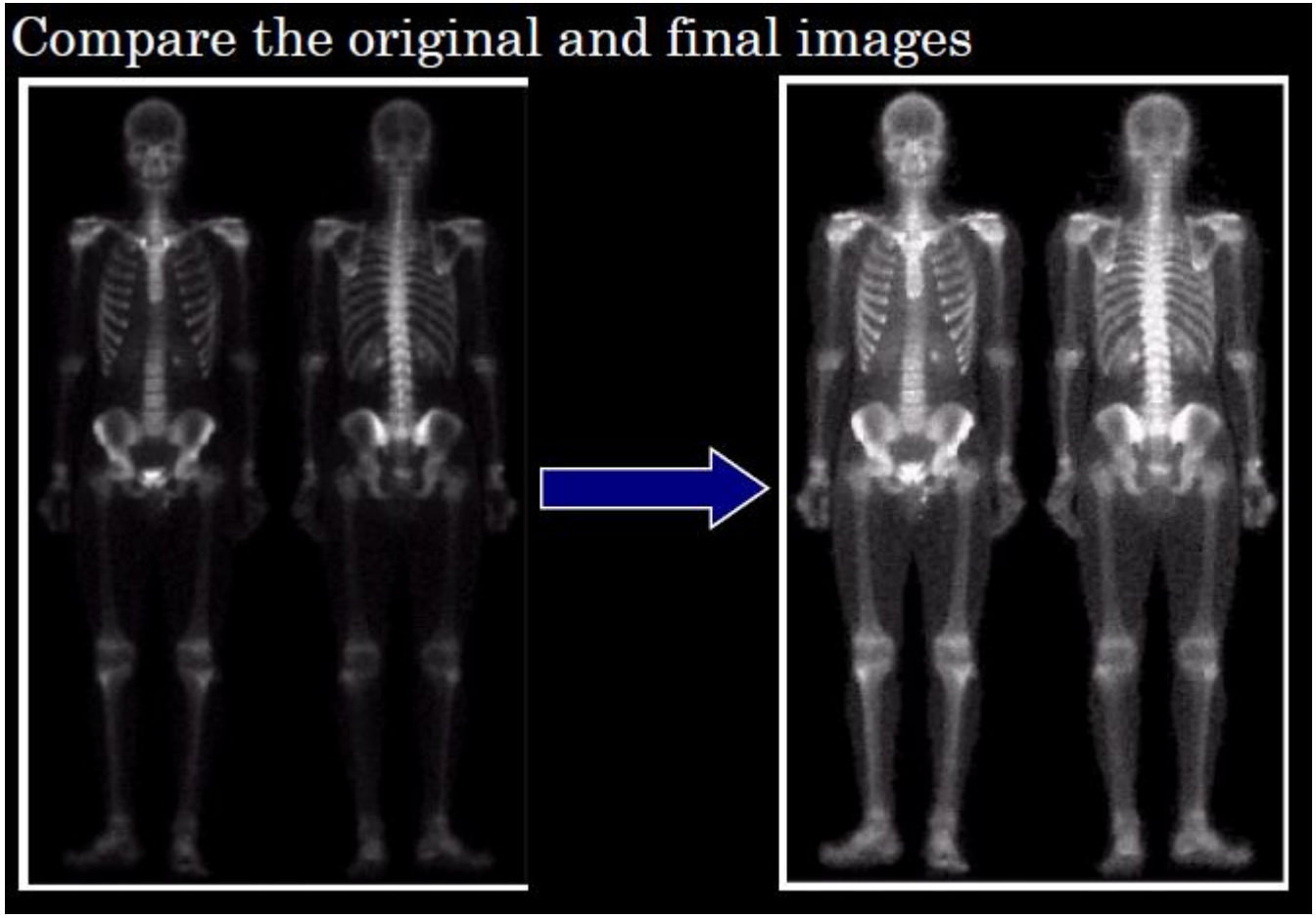
Mix of Different filters



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MATLAB CODE:

```
img = imread('img1.jpg');
img1 = imread('img2.jpg');
img2 = rgb2gray(img);
width = 210;
images = {img1,img2};
for k = 1:2
dim = size(images{k});
images{k} = imresize(images{k},[width*dim(1)/dim(2)
    width],'bicubic');
End
imgadj = imadjust(img2);
imghist = histeq(img2);
imghistequ = adapthisteq(img2);
subplot(2,3,1), imshow(img);
subplot(2,3,2), imshow(img2);
subplot(2,3,3), imshow(imgadj);
subplot(2,3,4), imshow(imghist);
subplot(2,3,5), imshow(imghistequ);
```

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An example on ENHANCEMENTS

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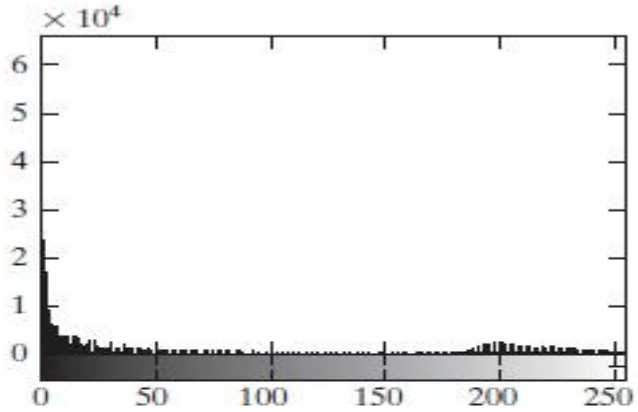
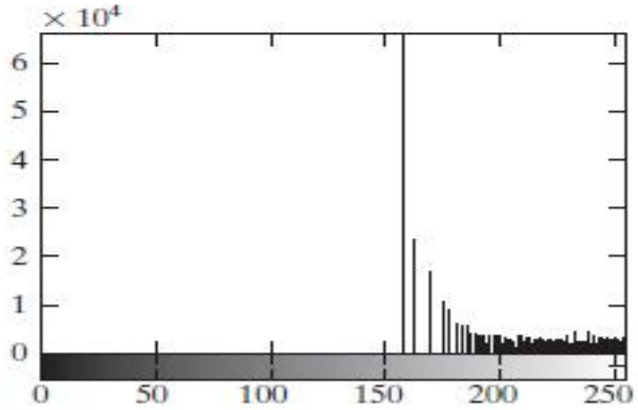
NEED FOR HISTOGRAM SPECIFICATION:





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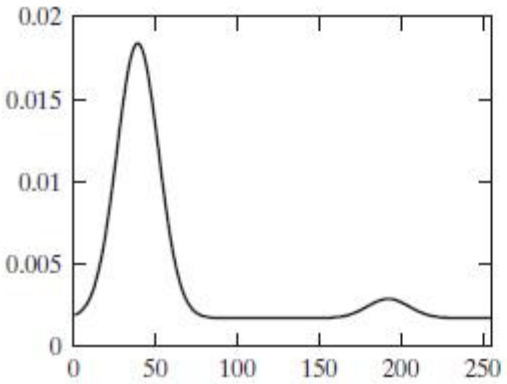
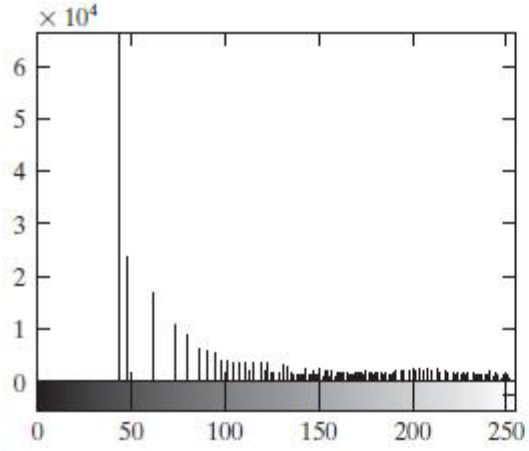
- Link to text file





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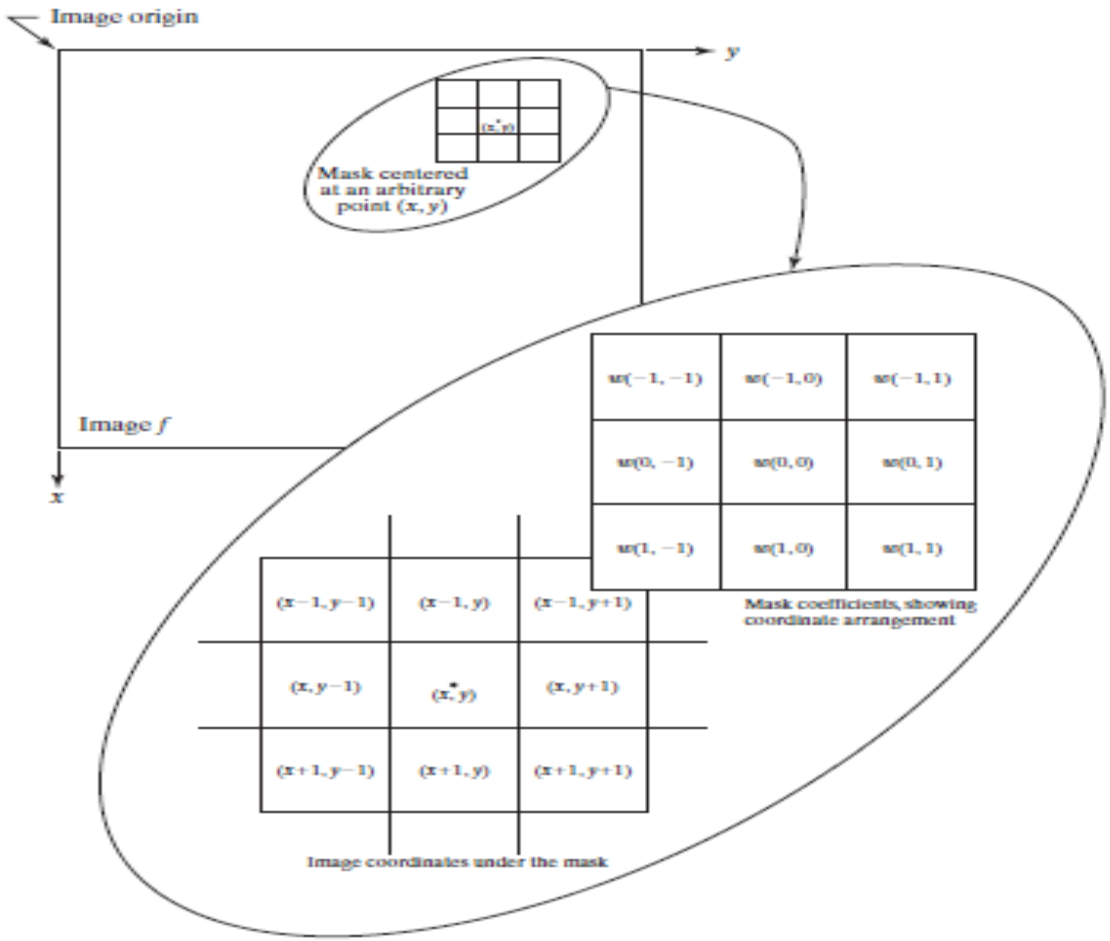
SPATIAL FILTERING IN MATLAB

These are divided in to two categories such as

1. Correlation
2. Convolution



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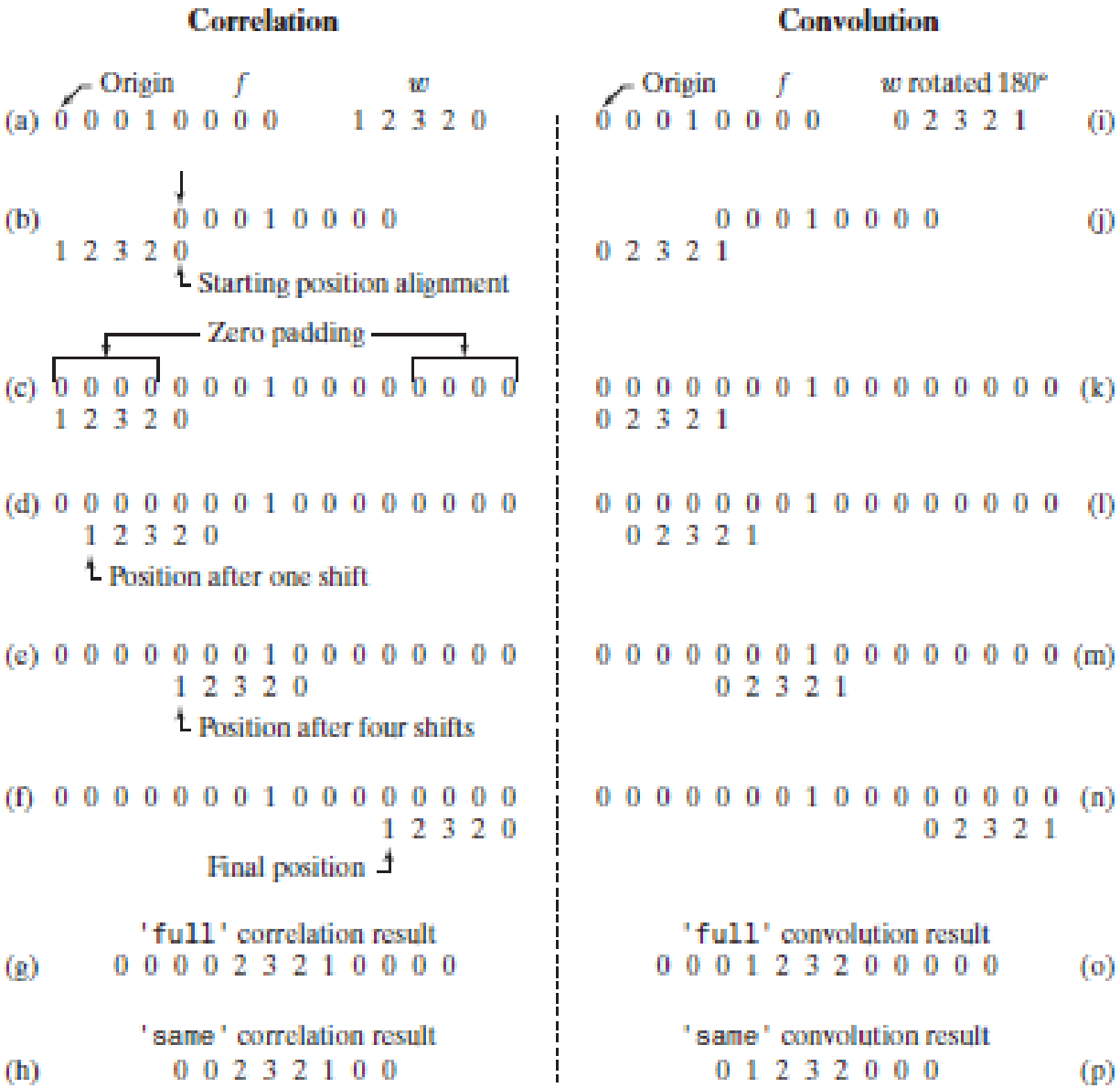

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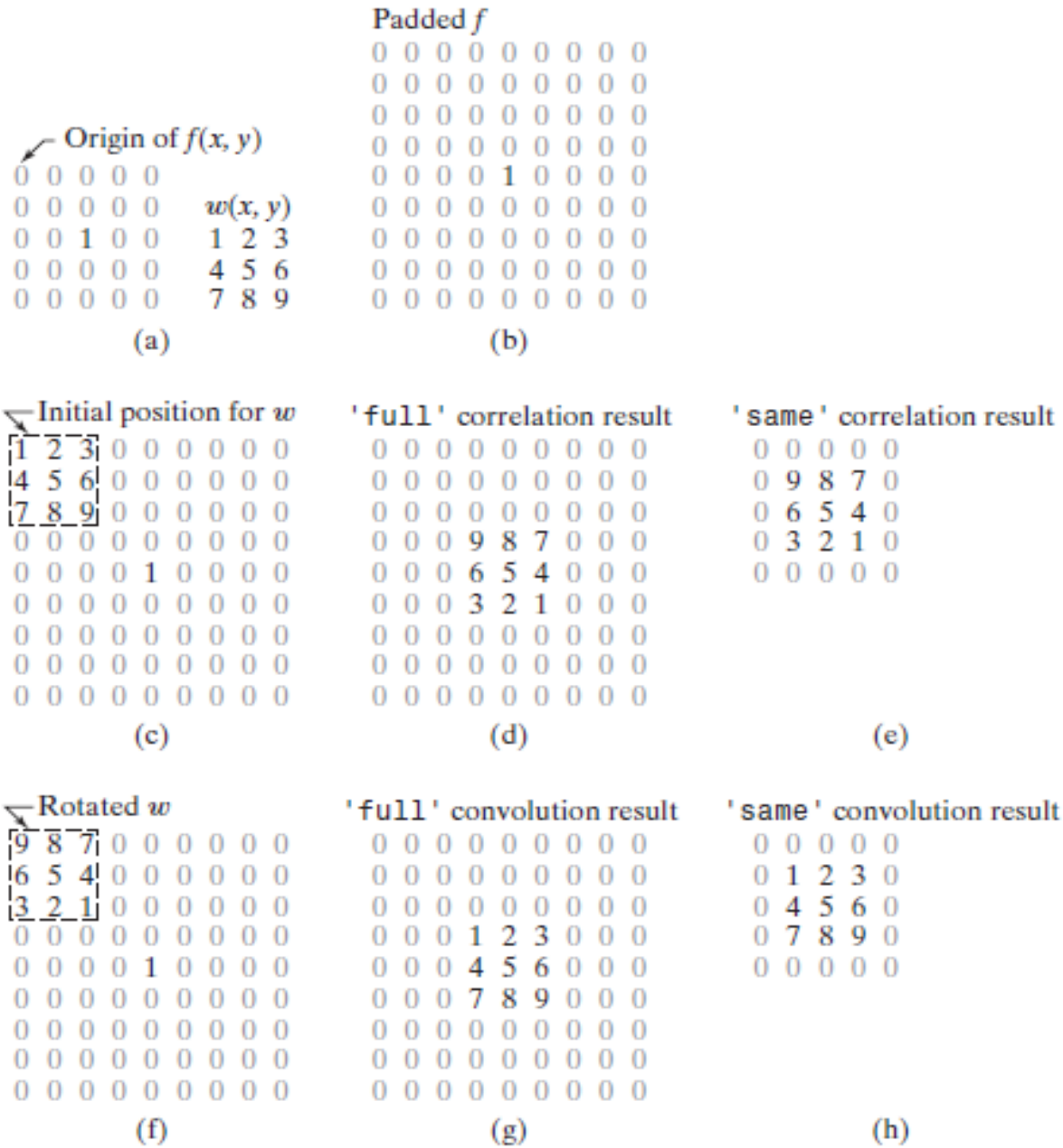

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SUPPORTING FUNCTIONS IN MATLAB

IMFILTER:

g = imfilter(f, w, filtering_mode,
boundary_options, size_options)

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Options	Description
Filtering Mode	
`'corr'`	Filtering is done using correlation (see Figs. 3.13 and 3.14). This is the default.
`'conv'`	Filtering is done using convolution (see Figs. 3.13 and 3.14).
Boundary Options	
`P`	The boundaries of the input image are extended by padding with a value, `P` (written without quotes). This is the default, with value 0.
`'replicate'`	The size of the image is extended by replicating the values in its outer border.
`'symmetric'`	The size of the image is extended by mirror-reflecting it across its border.
`'circular'`	The size of the image is extended by treating the image as one period a 2-D periodic function.
Size Options	
`'full'`	The output is of the same size as the extended (padded) image (see Figs. 3.13 and 3.14).
`'same'`	The output is of the same size as the input. This is achieved by limiting the excursions of the center of the filter mask to points contained in the original image (see Figs. 3.13 and 3.14). This is the default.

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Linear Spatial filters

SPECIAL FILTER:

w = fspecial(‘type’, parameters)

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Type	Syntax and Parameters
`'average'`	`fspecial('average', [r c])`. A rectangular averaging filter of size `r` × `c`. The default is 3 × 3. A single number instead of `[r c]` specifies a square filter.
`'disk'`	`fspecial('disk', r)`. A circular averaging filter (within a square of size `2r + 1`) with radius `r`. The default radius is 5.
`'gaussian'`	`fspecial('gaussian', [r c], sig)`. A Gaussian lowpass filter of size `r` × `c` and standard deviation `sig` (positive). The defaults are 3 × 3 and 0.5. A single number instead of `[r c]` specifies a square filter.
`'laplacian'`	`fspecial('laplacian', alpha)`. A 3 × 3 Laplacian filter whose shape is specified by `alpha`, a number in the range [0, 1]. The default value for `alpha` is 0.5.

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`'log'`	`fspecial('log', [r c], sig)`. Laplacian of a Gaussian (LoG) filter of size `r × c` and standard deviation `sig` (positive). The defaults are 5 × 5 and 0.5. A single number instead of `[r c]` specifies a square filter.
`'motion'`	`fspecial('motion', len, theta)`. Outputs a filter that, when convolved with an image, approximates linear motion (of a camera with respect to the image) of `len` pixels. The direction of motion is `theta`, measured in degrees, counterclockwise from the horizontal. The defaults are 9 and 0, which represents a motion of 9 pixels in the horizontal direction.
`'prewitt'`	`fspecial('prewitt')`. Outputs a 3 × 3 Prewitt mask, `wv`, that approximates a vertical gradient. A mask for the horizontal gradient is obtained by transposing the result: `wh = wv'`.
`'sobel'`	`fspecial('sobel')`. Outputs a 3 × 3 Sobel mask, `sv`, that approximates a vertical gradient. A mask for the horizontal gradient is obtained by transposing the result: `sh = sv'`.
`'unsharp'`	`fspecial('unsharp', alpha)`. Outputs a 3 × 3 unsharp filter. Parameter `alpha` controls the shape; it must be greater than 0 and less than or equal to 1.0; the default is 0.2.

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NON LINEAR FILTERS

Order statistic filters/ Rank Filters

Min filters:

```
g = ordfilt2(f, 1, ones(m, n))
```

Max Filters:

```
g = ordfilt2(f, m*n, ones(m, n))
```

Median Filters

```
g = ordfilt2(f, median(1:m*n), ones(m, n))
```

```
g = medfilt2(f)
```

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Frequency domain enhancement

- These methods enhance an image f(x,y) by convoluting the image with a linear, position invariant operator.
- The 2D convolution is performed in frequency domain with DFT.

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Objective

Basic understanding of the frequency domain, Fourier series, and Fourier transform.

Image enhancement in the frequency domain.

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Fourier Series

Any function that periodically repeats itself can be expressed as the sum of sines and/or cosines of different frequencies, each multiplied by a different coefficients. This sum is called a Fourier series.

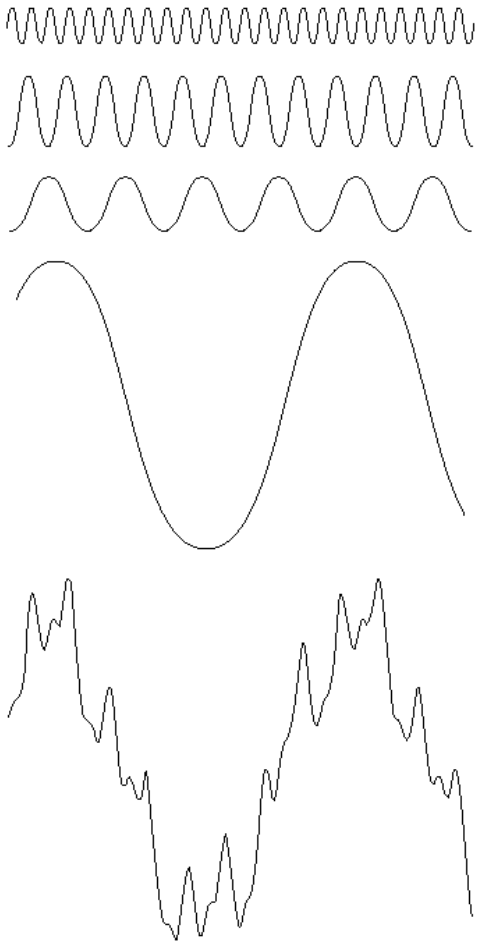

FIGURE 4.1 The function at the bottom is the sum of the four functions above it. Fourier's idea in 1807 that periodic functions could be represented as a weighted sum of sines and cosines was met with skepticism.

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Fourier Series

$$g(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos nw_0 t + b_n \sin nw_0 t$$

$$a_0 = \frac{1}{T_0} \int_{t_0}^{t_0+T_0} g(t)dt$$

$$a_n = \frac{2}{T_0} \int_{t_0}^{t_0+T_0} g(t) \cos nw_0 t \; dt$$

$$b_n = \frac{2}{T_0} \int_{t_0}^{t_0+T_0} g(t) \sin nw_0 t \; dt$$

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Fourier Transform

A function that is not periodic but the area under its curve is finite can be expressed as the integral of sines and/or cosines multiplied by a weighing function. The formulation in this case is Fourier transform.

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Continuous One-Dimensional Fourier Transform and Its Inverse

$$F(u) = \int_{-\infty}^{\infty} f(x) e^{-j2\pi ux} dx$$

$$f(x) = \int_{-\infty}^{\infty} F(u) e^{j2\pi ux} du$$

Where $j = \sqrt{-1}$

(*u*) is the frequency variable.

***F*(*u*)** is composed of an infinite sum of sine and cosine terms and…

Each value of u determines the frequency of its corresponding sine-cosine pair.

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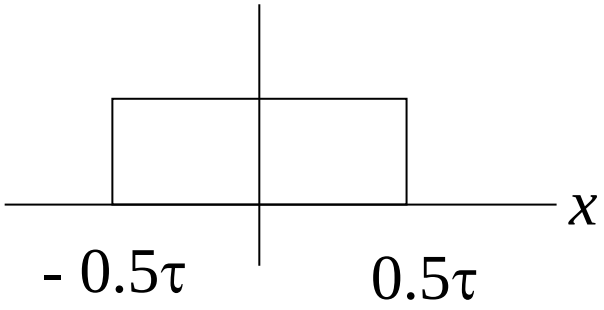
Continuous One-Dimensional Fourier Transform and Its Inverse

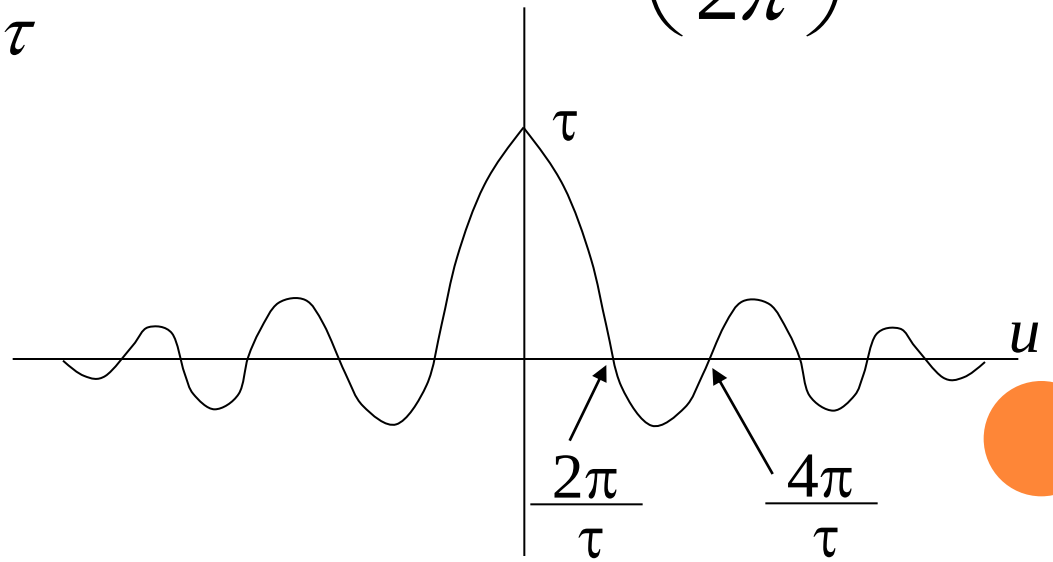
Example

Find the Fourier transform of a gate function Π(t) defined by

$$\Pi(x) = \begin{cases} 1 & |x| < \frac{1}{2}\tau \\ 0 & |x| < \frac{1}{2}\tau \end{cases}$$

$$F(u) = \tau \,\mathrm{sinc}\left(\frac{u\tau}{2\pi}\right)$$





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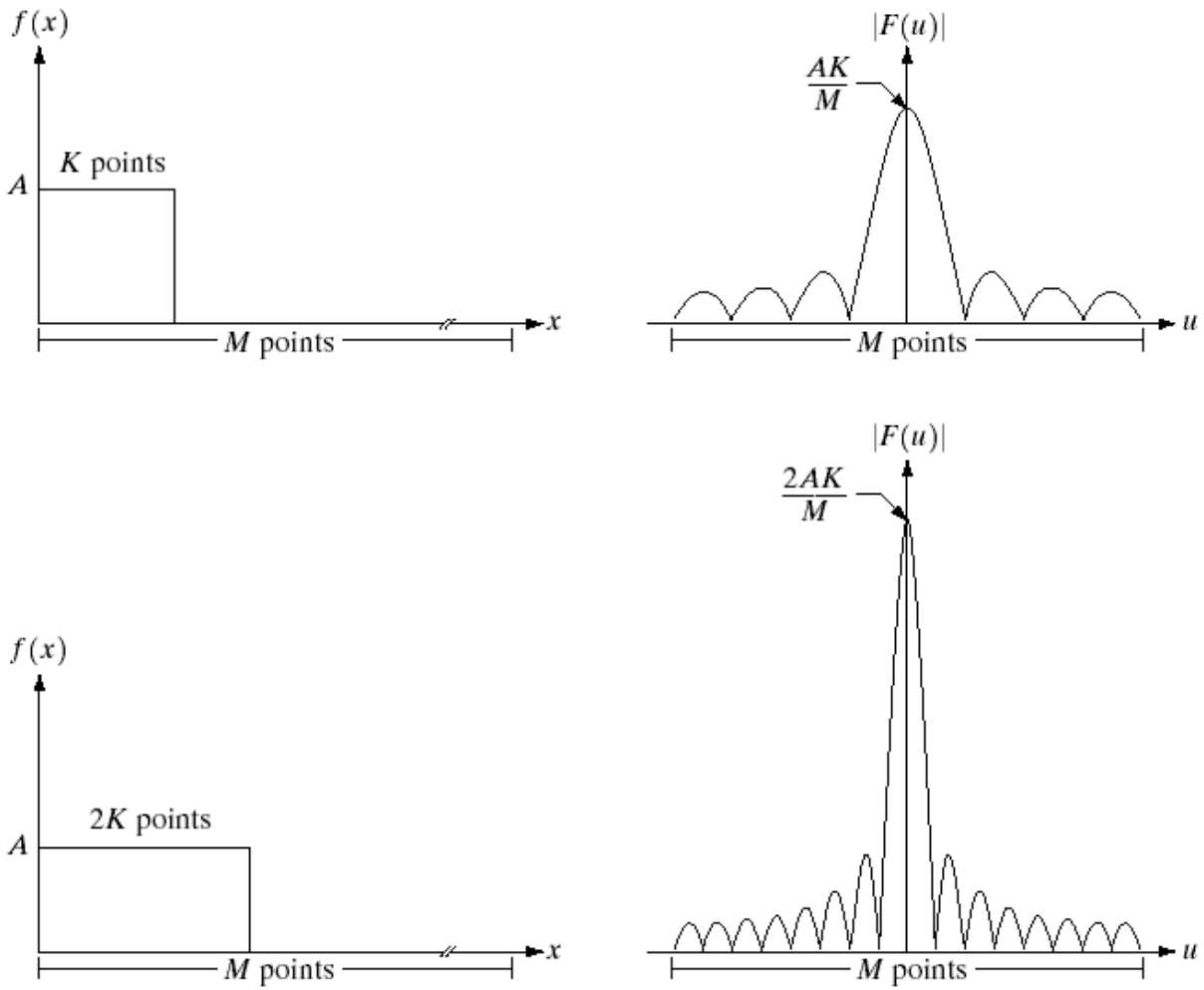
Continuous One-Dimensional Fourier Transform and Its Inverse



a b
c d

FIGURE 4.2 (a) A discrete function of M points, and (b) its Fourier spectrum. (c) A discrete function with twice the number of nonzero points, and (d) its Fourier spectrum.

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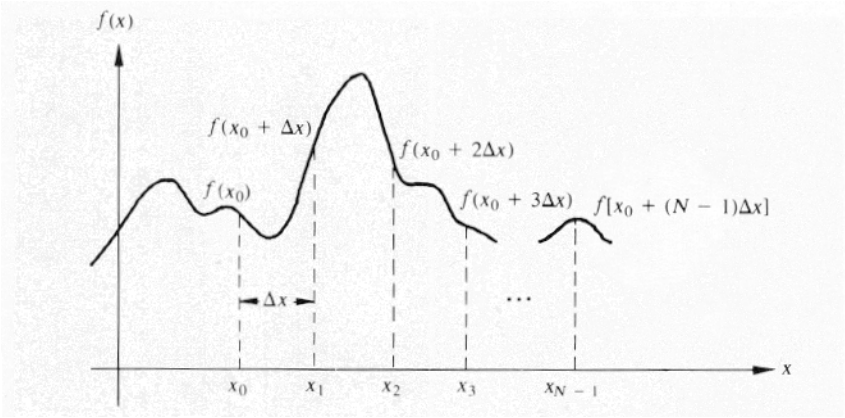
Discrete One-Dimensional Fourier Transform and Its Inverse

- **A continuous function f(x) is discretized into a sequence:**

$$\{f(x_0), f(x_0+\Delta x), f(x_0+2\Delta x),..., f(x_0+[N-1]\Delta x)\}$$

by taking N or M samples Δx units apart.



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Discrete One-Dimensional Fourier Transform and Its Inverse

- **Where x assumes the discrete values (0,1,2,3,…,M-1) then**

$$f(x) = f(x_0 + x\Delta x)$$

- **The sequence {f(0),f(1),f(2),…f(M-1)} denotes any M uniformly spaced samples from a corresponding continuous function.**

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Discrete One-Dimensional Fourier Transform and Its Inverse

$$F(u) = \frac{1}{M} \sum_{x=0}^{M-1} f(x) e^{-j2\pi u \frac{x}{M}} \qquad u = [0,1,2, \ldots, M\text{-}1]$$

$$F(u) = \frac{1}{M} \sum_{x=0}^{M-1} f(x) \left[\cos 2\pi u \frac{x}{M} - j \sin 2\pi u \frac{x}{M} \right]$$

$$f(x) = \sum_{u=0}^{M-1} F(u) e^{j2\pi \frac{u}{M} x}$$

$$x = [0,1,2, \ldots, M\text{-}1]$$

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Discrete One-Dimensional Fourier Transform and Its Inverse

- **The values u = 0, 1, 2, …, M-1 correspond to samples of the continuous transform at values 0, Δu, 2Δu, …, (M-1)Δu.**

  i.e. F(u) represents F(uΔu), where:

$$\Delta u = \frac{1}{M\Delta x}$$

- **Each term of the FT (F(u) for every u) is composed of the sum of all values of f(x)**

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Discrete One-Dimensional Fourier Transform and Its Inverse

- **The Fourier transform of a real function is generally complex and we use polar coordinates:**

$$F(u) = R(u) + jI(u)$$

$$F(u) = |F(u)|e^{j\phi(u)}$$

$$|F(u)| = [R^2(u) + I^2(u)]^{1/2}$$

$$\phi(u) = \tan^{-1}\left[\frac{I(u)}{R(u)}\right]$$

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Discrete One-Dimensional Fourier Transform and Its Inverse

- **|F(u)| (magnitude function) is the Fourier spectrum of f(x) and ϕ(u) its phase angle.**

- **The square of the spectrum**

$$P(u) = \left|F(u)\right|^2 = R^2(u) + I^2(u)$$

is referred to as the Power Spectrum of f(x) (spectral density).

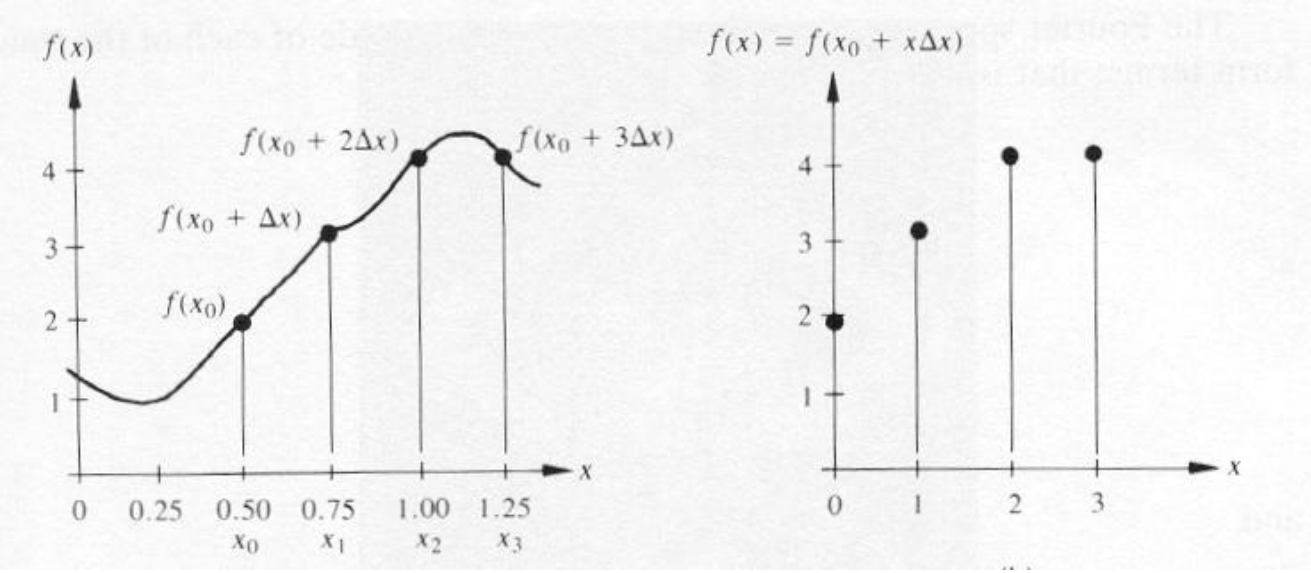
Discrete 2-Dimensional Fourier Transform

- **Fourier spectrum:** $|F(u,v)| = \left[R^2(u,v) + I^2(u,v)\right]^{1/2}$

- **Phase:** $\phi(u,v) = \tan^{-1}\left[\frac{I(u,v)}{R(u,v)}\right]$

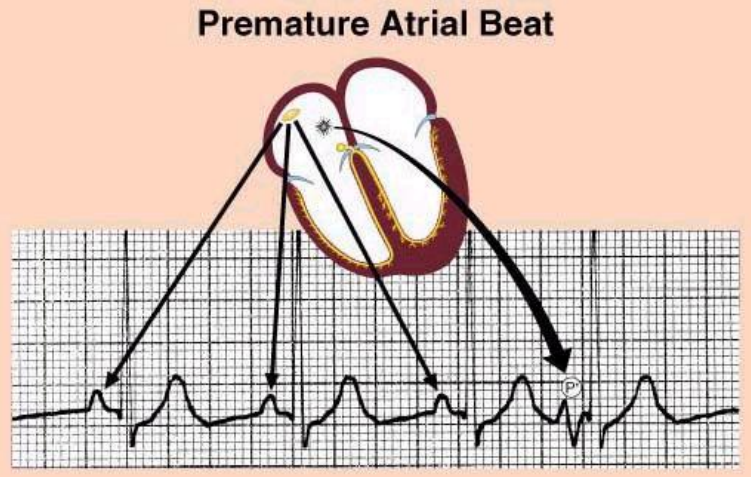
- **Power spectrum:** $P(u,v) = |F(u,v)|^2 = R^2(u,v) + I^2(u,v)$

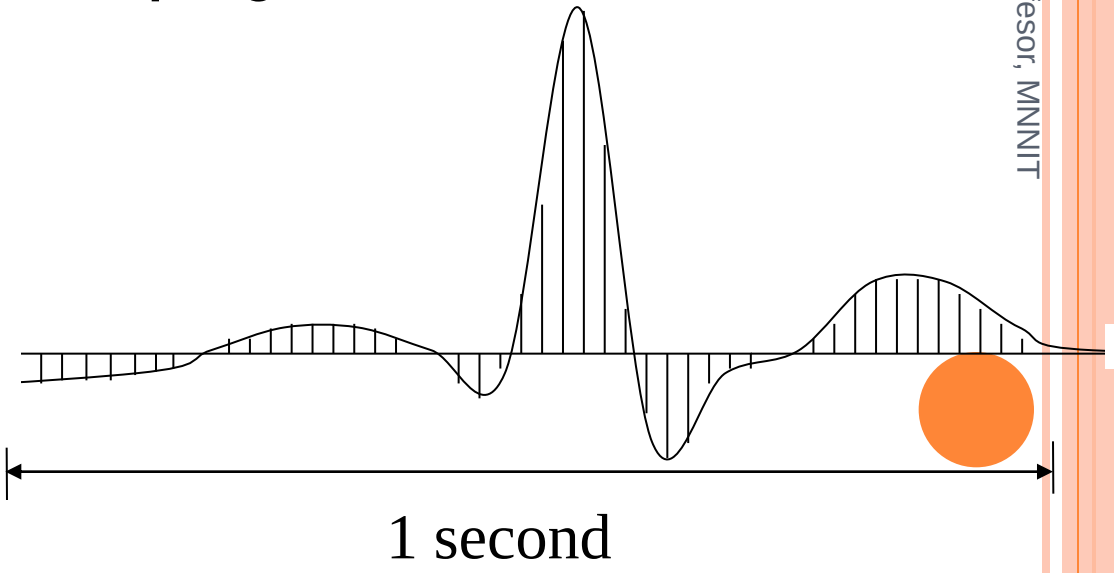
Discrete One-Dimensional Fourier Transform and Its Inverse



Time and Frequency Resolution and Sampling

F_{max} = 100 Hz
What is the sampling rate (Nyquist)?
What is the time resolution?
What is the frequency resolution?
What if the sampling rate is higher than the Nyquist sampling rate?
What if we take samples for two seconds with the Nyquist sampling rate?





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Discrete Two-Dimensional Fourier Transform and Its Inverse

$$F(u,v) = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y) e^{-j2\pi(u\frac{x}{M}+v\frac{y}{N})}$$

$$f(x,y) = \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} F(u,v) e^{j2\pi(\frac{u}{M}x+\frac{v}{N}y)}$$

$$|F(u,v)| = \left[R^2(u,v) + I^2(u,v)\right]^{1/2}$$

Fourier Spectrum

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Discrete Two-Dimensional Fourier Transform and Its Inverse

$$F(0,0) = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x, y)$$

$F(0,0)$ is the average intensity of an image

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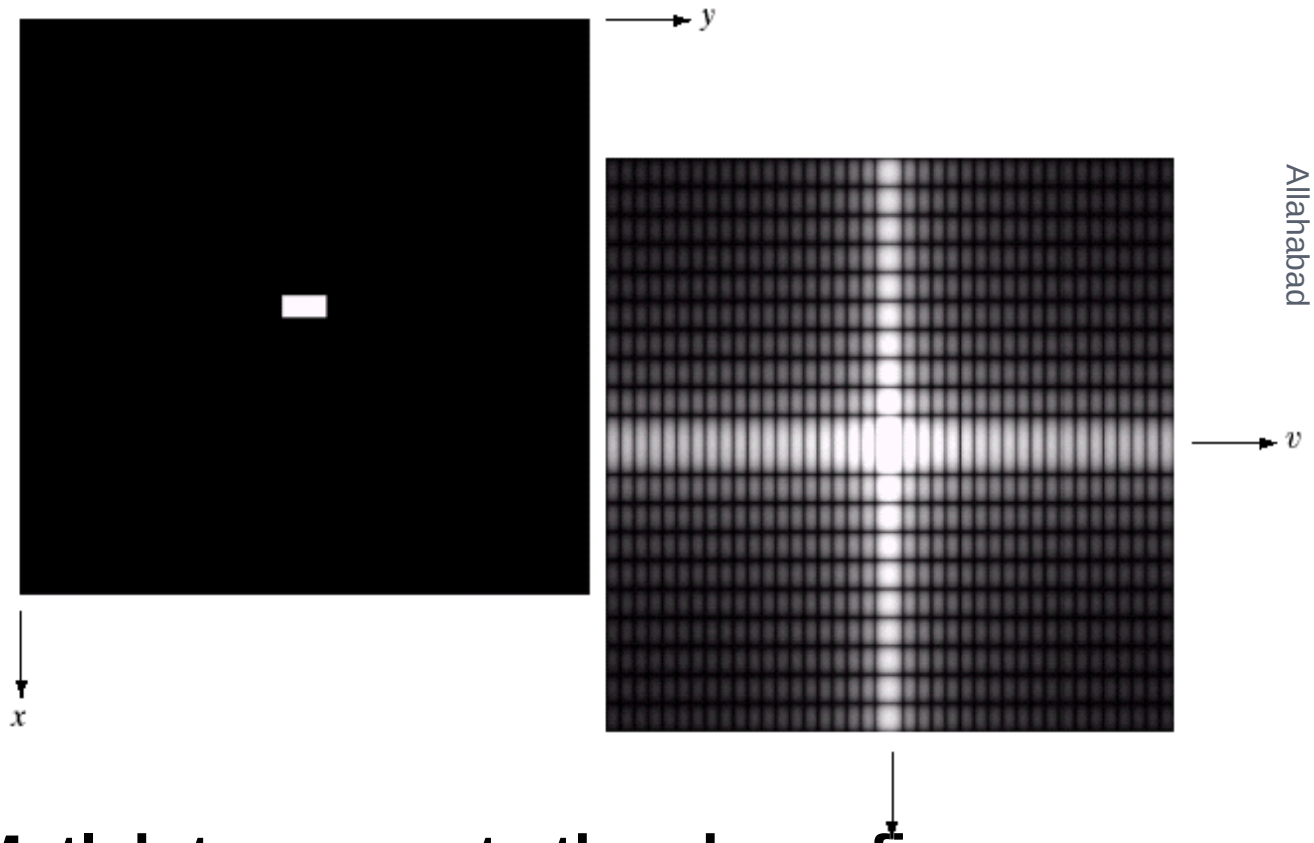
Discrete Two-Dimensional Fourier Transform and Its Inverse

a b

FIGURE 4.3
(a) Image of a 20 × 40 white rectangle on a black background of size 512 × 512 pixels.
(b) Centered Fourier spectrum shown after application of the log transformation given in Eq. (3.2-2). Compare with Fig. 4.2.



Use Matlab to generate the above figures

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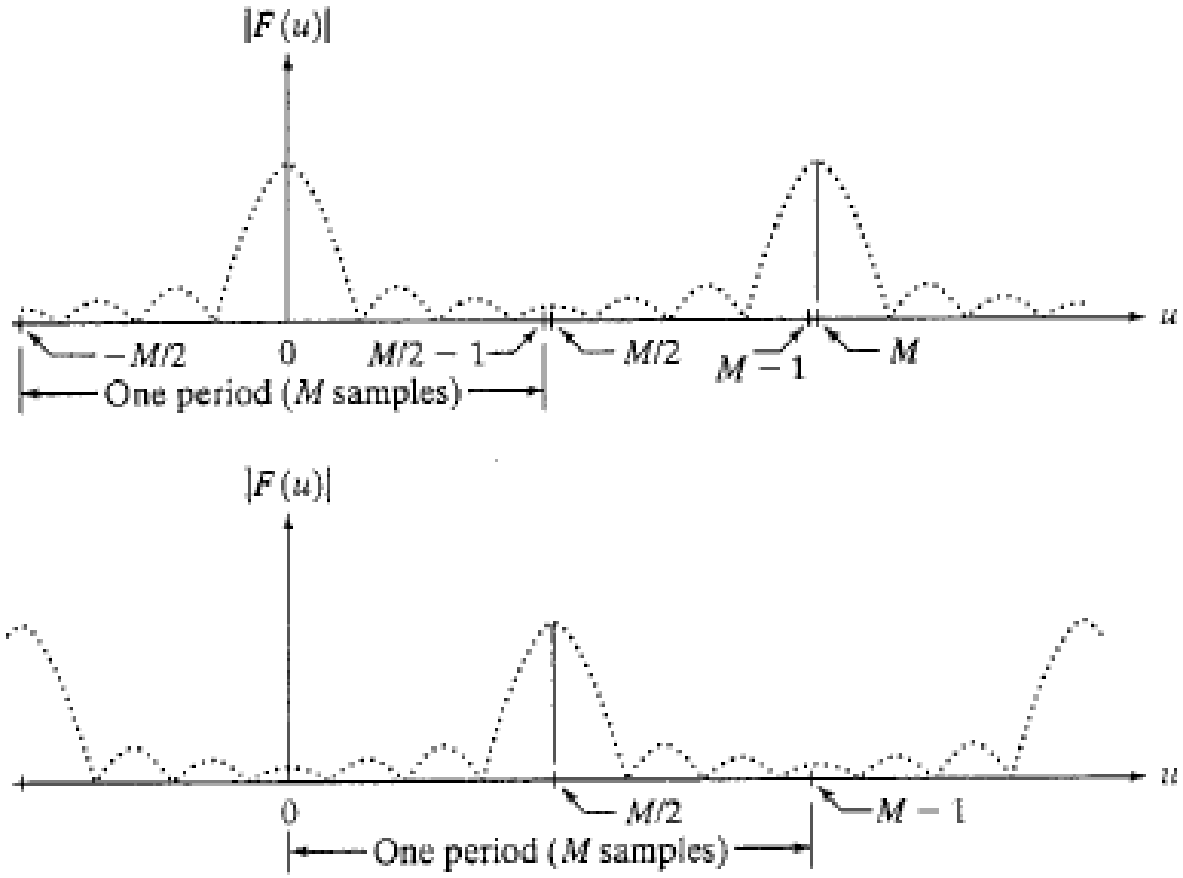

(a) Fourier spectrum showing back-to-back half periods in the interval $[0, M-1]$. (b) Centered spectrum in the same interval, obtained by multiplying $f(x)$ by $(-1)^x$ prior to computing the Fourier transform.

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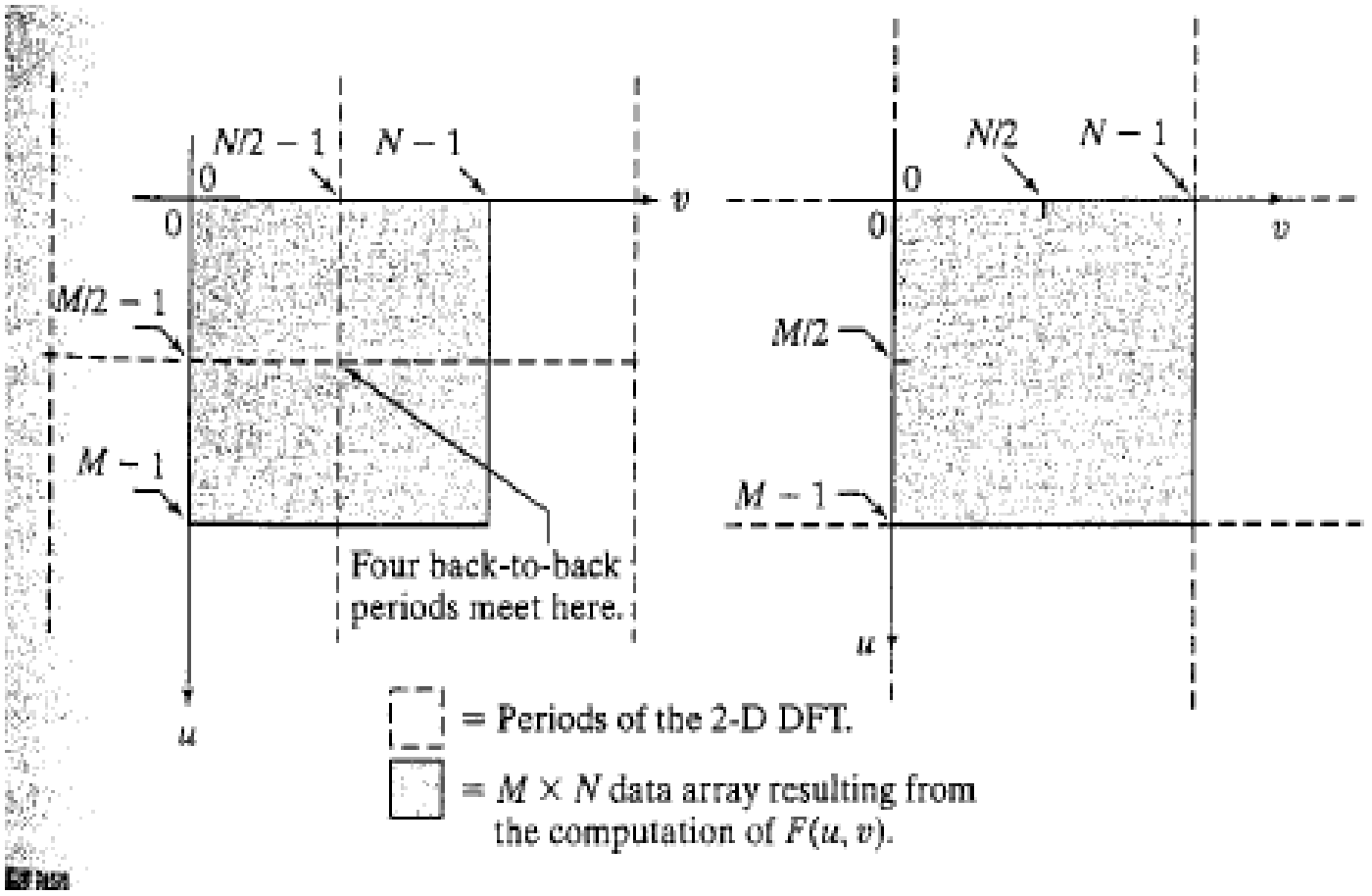

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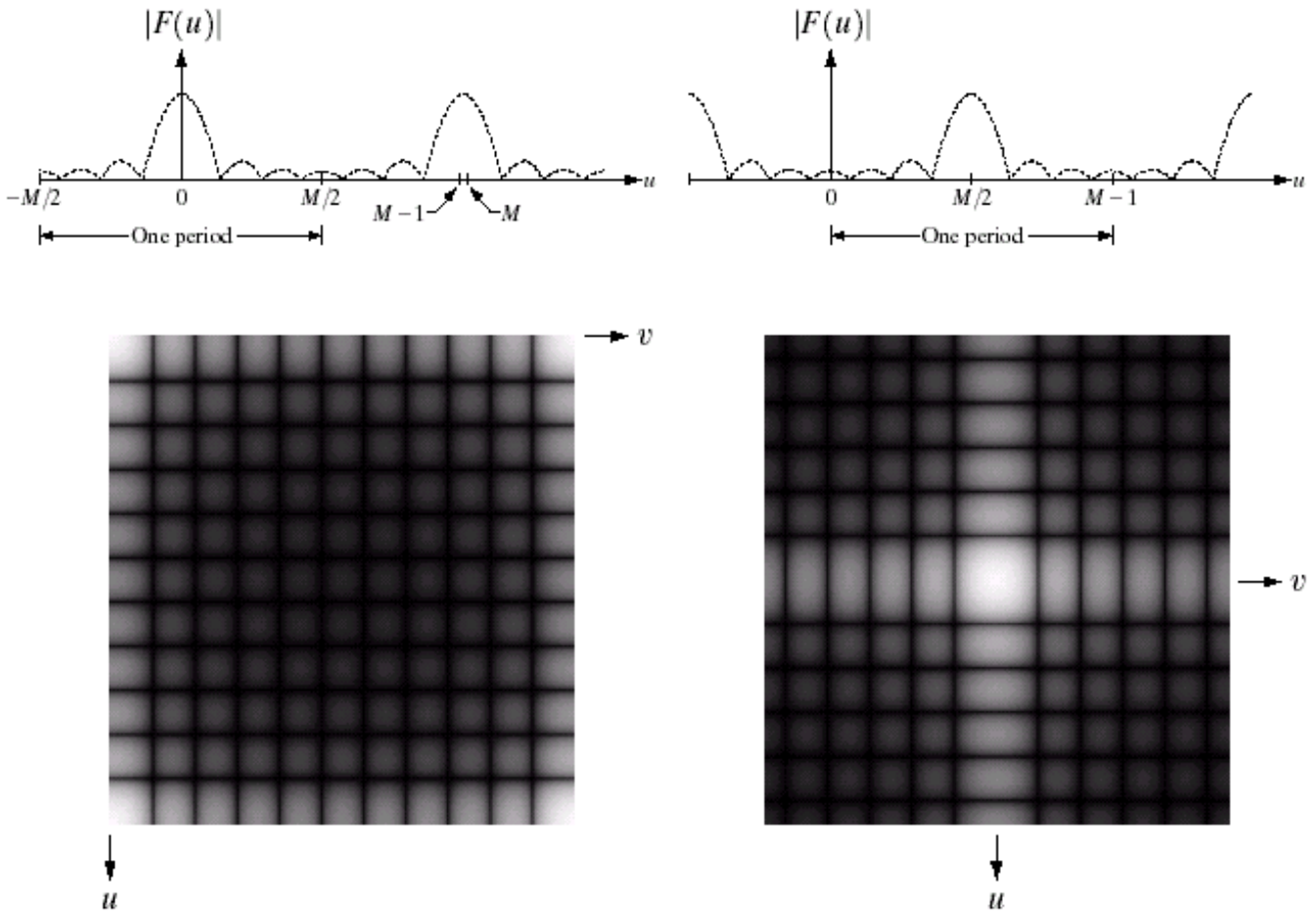
Frequency Shifting Property of the Fourier Transform

a b
c d

FIGURE 4.34
(a) Fourier spectrum showing back-to-back half periods in the interval $[0, M-1]$.
(b) Shifted spectrum showing a full period in the same interval.
(c) Fourier spectrum of an image, showing the same back-to-back properties as (a), but in two dimensions.
(d) Centered Fourier spectrum.



Basic Filtering in the Frequency Domain using Matlab

```
function Normalized_DFT = Img_DFT(img)
img=double(img);    % So mathematical operations can be conducted on
                    %  the image pixels.
[R,C]=size(img);
for r = 1:R             % To phase shift the image so the DFT will be
  for c=1:C             %    centered on the display monitor
      phased_img(r,c)=(img(r,c))*(-1)^((r-1)+(c-1));
   end
end
fourier_img = fft2(phased_img);          %Discrete Fourier Transform
mag_fourier_img = abs(fourier_img );    % Magnitude of DFT
Log_mag_fourier_img = log10(mag_fourier_img +1);
Max = max(max(Log_mag_fourier_img ));
Normalized_DFT = (Log_ mag_fourier_img )*(255/Max);
imshow(uint8(Normalized_DFT))
```

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Basic Filtering in the Frequency Domain

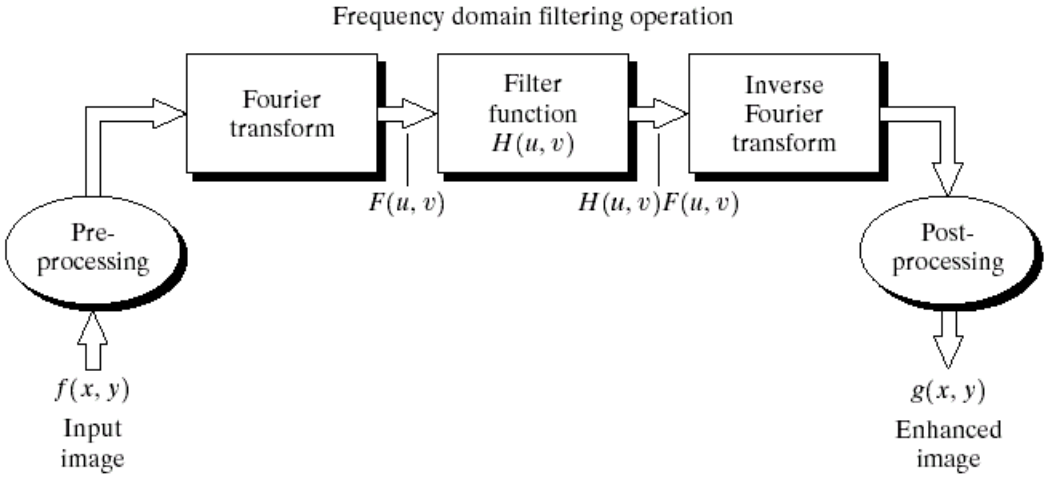

FIGURE 4.5 Basic steps for filtering in the frequency domain.

1. Multiply the input image by $(-1)^{x+y}$ to center the transform
2. Compute F(u,v), the DFT of the image from (1)
3. Multiply F(u,v) by a filter function H(u,v)
4. Compute the inverse DFT of the result in (3)
5. Obtain the real part of the result in (4)
6. Multiply the result in (5) by $(-1)^{x+y}$

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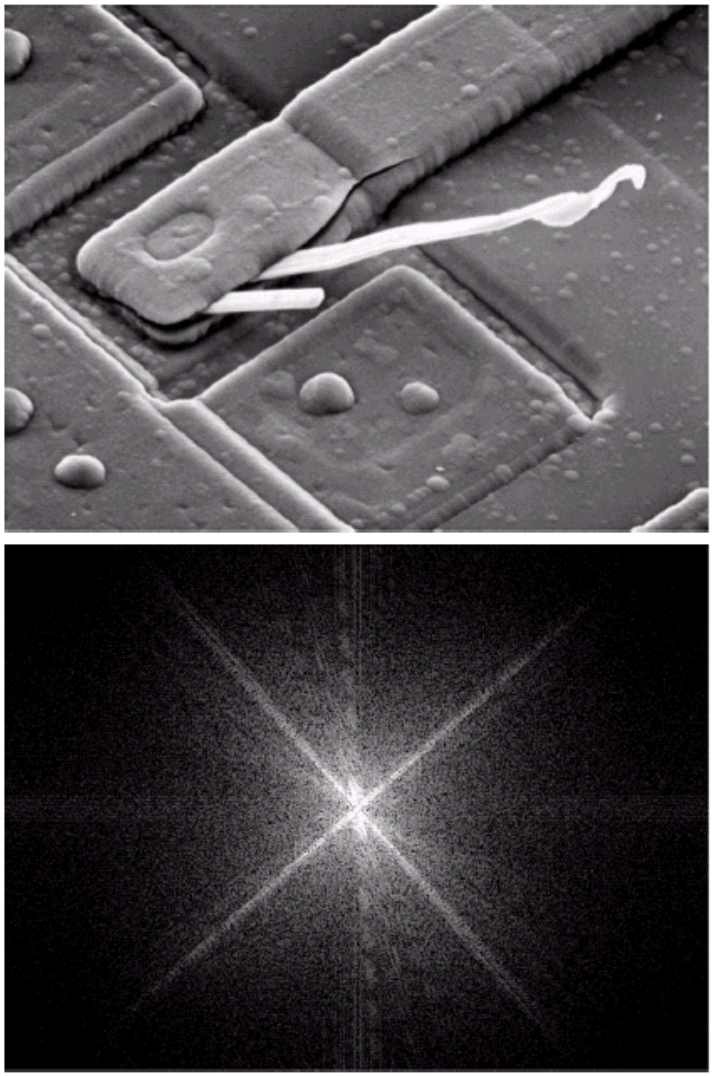
Filtering out the DC Frequency Component



a
b

FIGURE 4.4
(a) SEM image of a damaged integrated circuit. (b) Fourier spectrum of (a). (Original image courtesy of Dr. J. M. Hudak, Brockhouse Institute for Materials Research, McMaster University, Hamilton, Ontario, Canada.)

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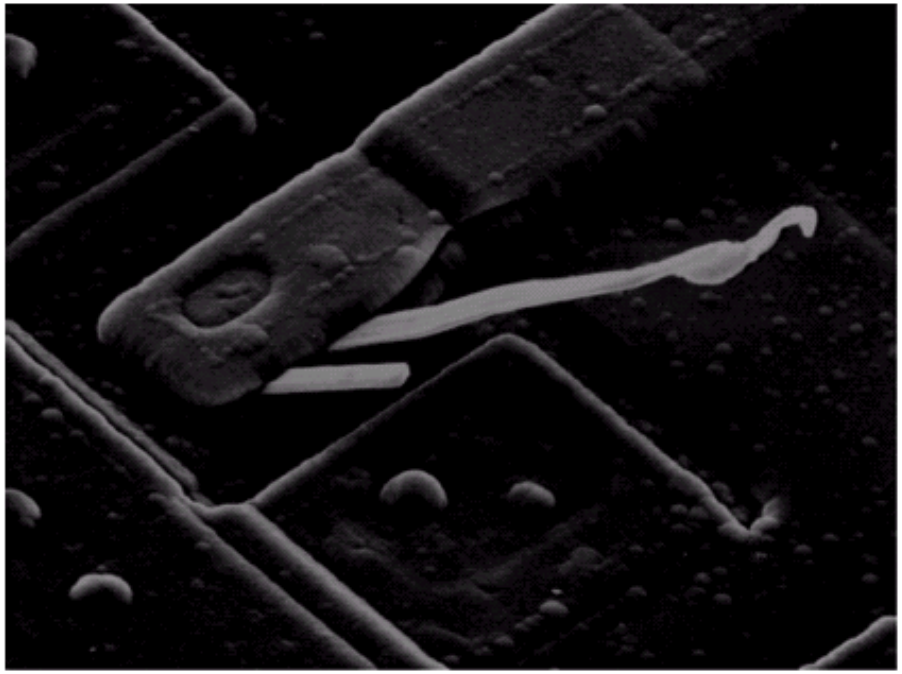
Filtering out the DC Frequency Component

FIGURE 4.6
Result of filtering the image in Fig. 4.4(a) with a notch filter that set to 0 the $F(0, 0)$ term in the Fourier transform.



Notch Filter

$$H(u,v) = \begin{cases} 0 & \text{if } (u,v) = (M/2, N/2) \\ 1 & \text{otherwise} \end{cases}$$

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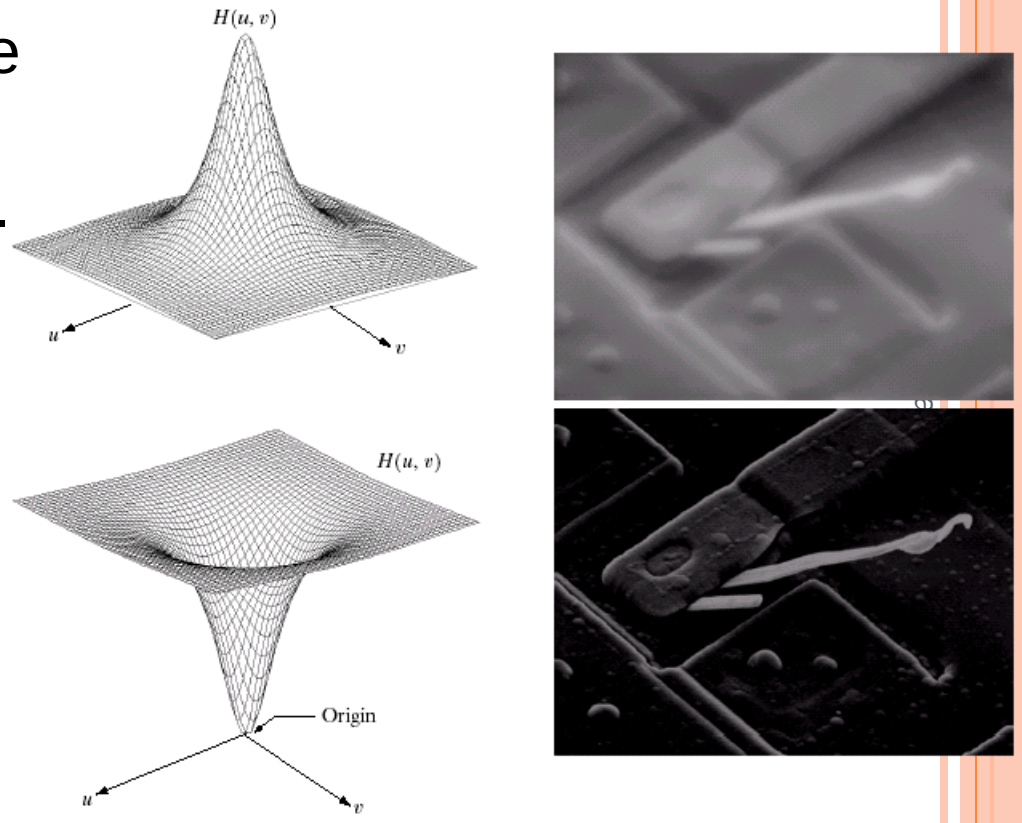
Low-pass and High-pass Filters

Low Pass Filter attenuate high frequencies while "passing" low frequencies.

High Pass Filter attenuate low frequencies while "passing" high frequencies.

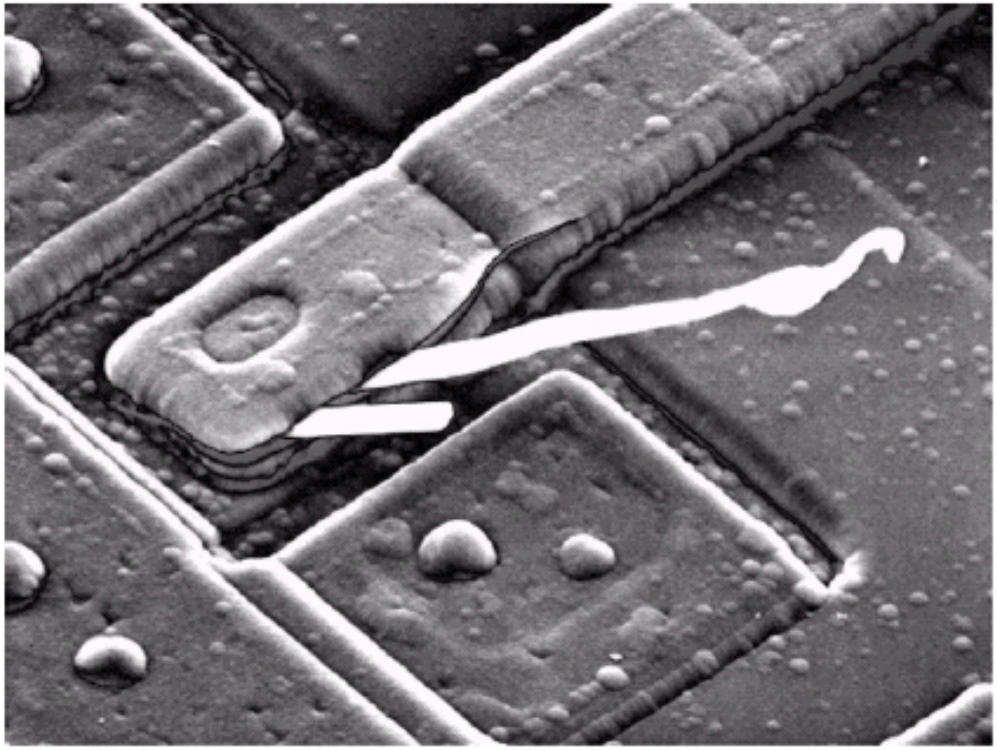


a b
c d

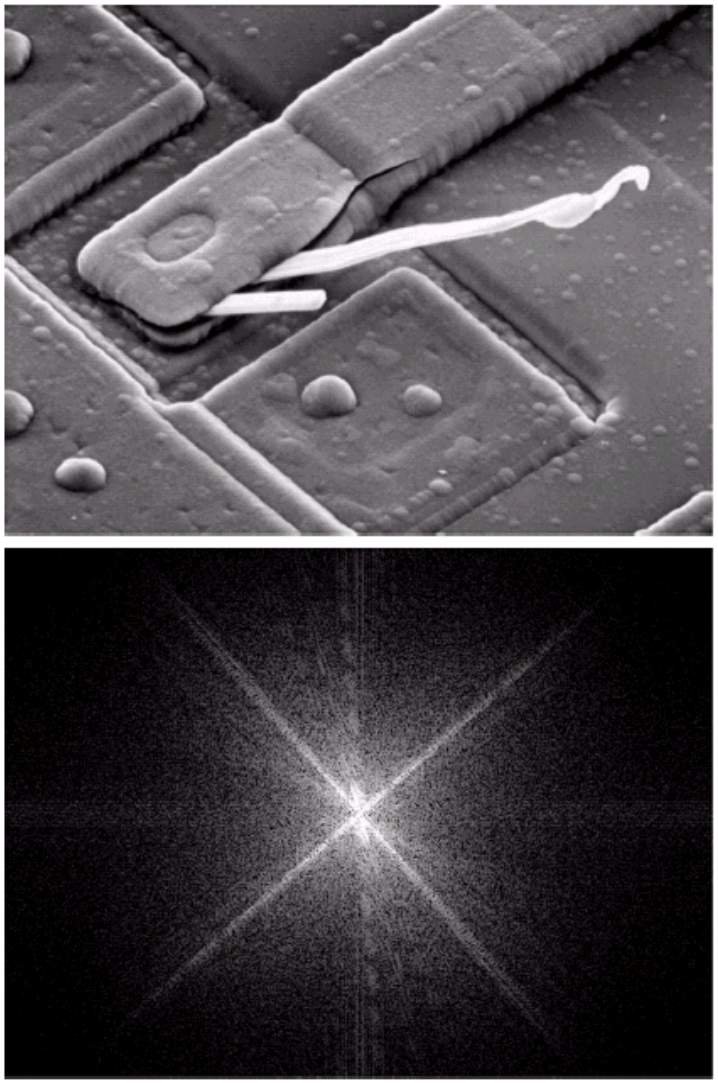
FIGURE 4.7 (a) A two-dimensional lowpass filter function. (b) Result of lowpass filtering the image in Fig. 4.4(a). (c) A two-dimensional highpass filter function. (d) Result of highpass filtering the image in Fig. 4.4(a).

Low-pass and High-pass Filters

FIGURE 4.8
Result of highpass filtering the image in Fig. 4.4(a) with the filter in Fig. 4.7(c), modified by adding a constant of one-half the filter height to the filter function. Compare with Fig. 4.4(a).



Low-pass and High-pass Filters



a
b

FIGURE 4.4
(a) SEM image of a damaged integrated circuit. (b) Fourier spectrum of (a). (Original image courtesy of Dr. J. M. Hudak, Brockhouse Institute for Materials Research, McMaster University, Hamilton, Ontario, Canada.)

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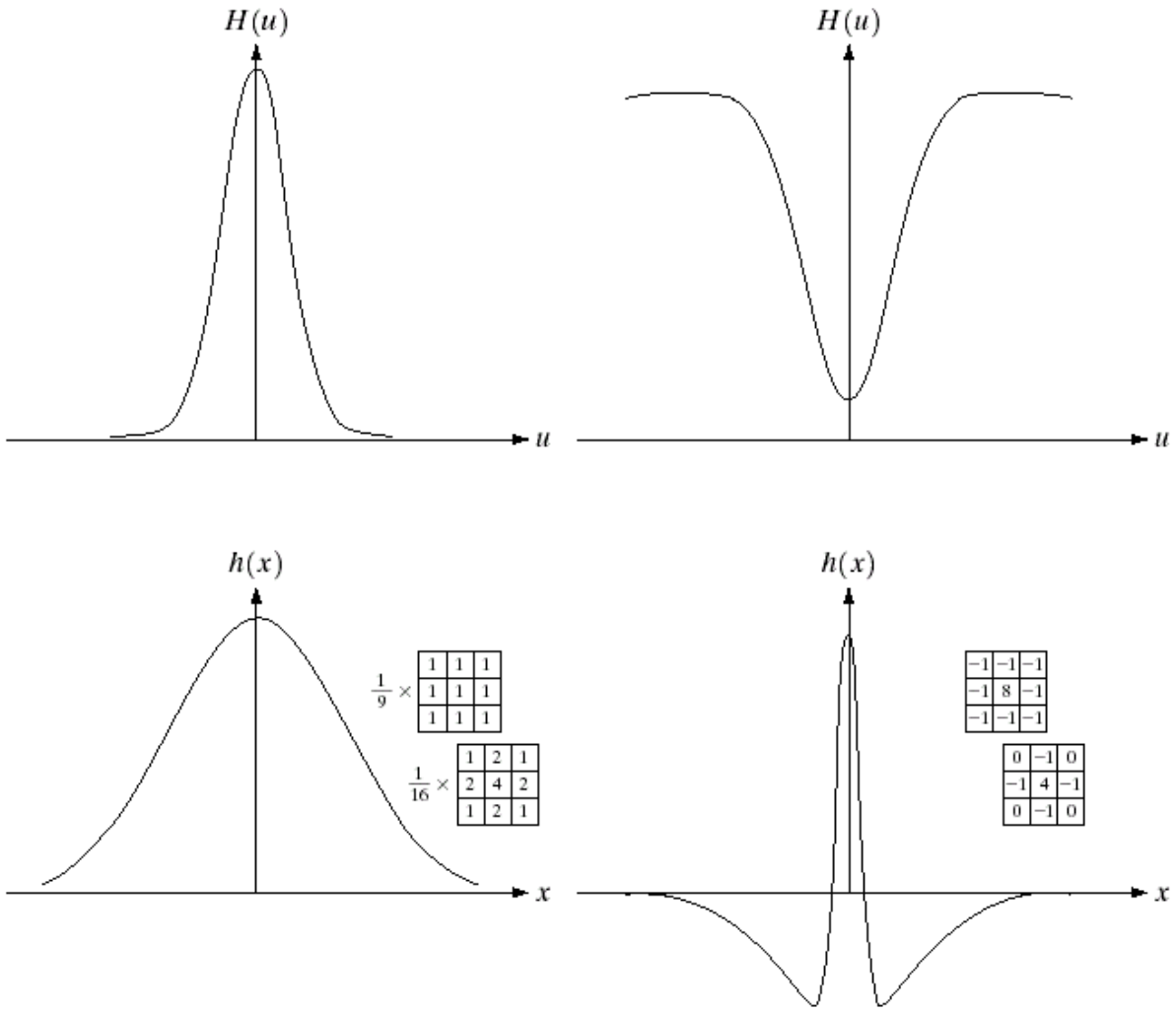
Low-pass and High-pass Filters



a b
c d

FIGURE 4.9
(a) Gaussian frequency domain lowpass filter. (b) Gaussian frequency domain highpass filter. (c) Corresponding lowpass spatial filter. (d) Corresponding highpass spatial filter. The masks shown are used in Chapter 3 for lowpass and highpass filtering.

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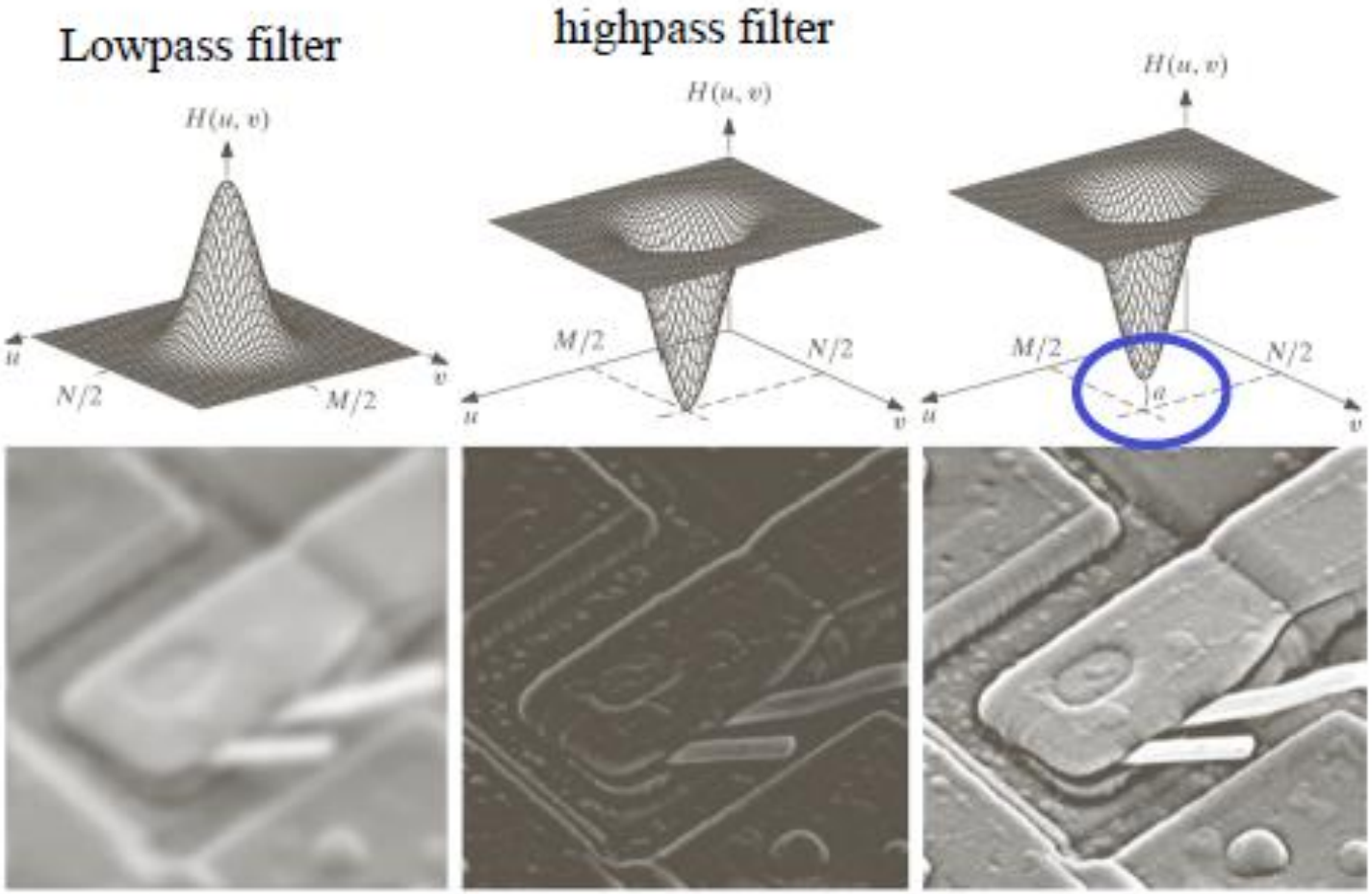

FIGURE 4.31 Top row: frequency domain filters. Bottom row: corresponding filtered images obtained using Eq. (4.7-1). We used $a = 0.85$ in (c) to obtain (f) (the height of the filter itself is 1). Compare (f) with Fig. 4.29(a).

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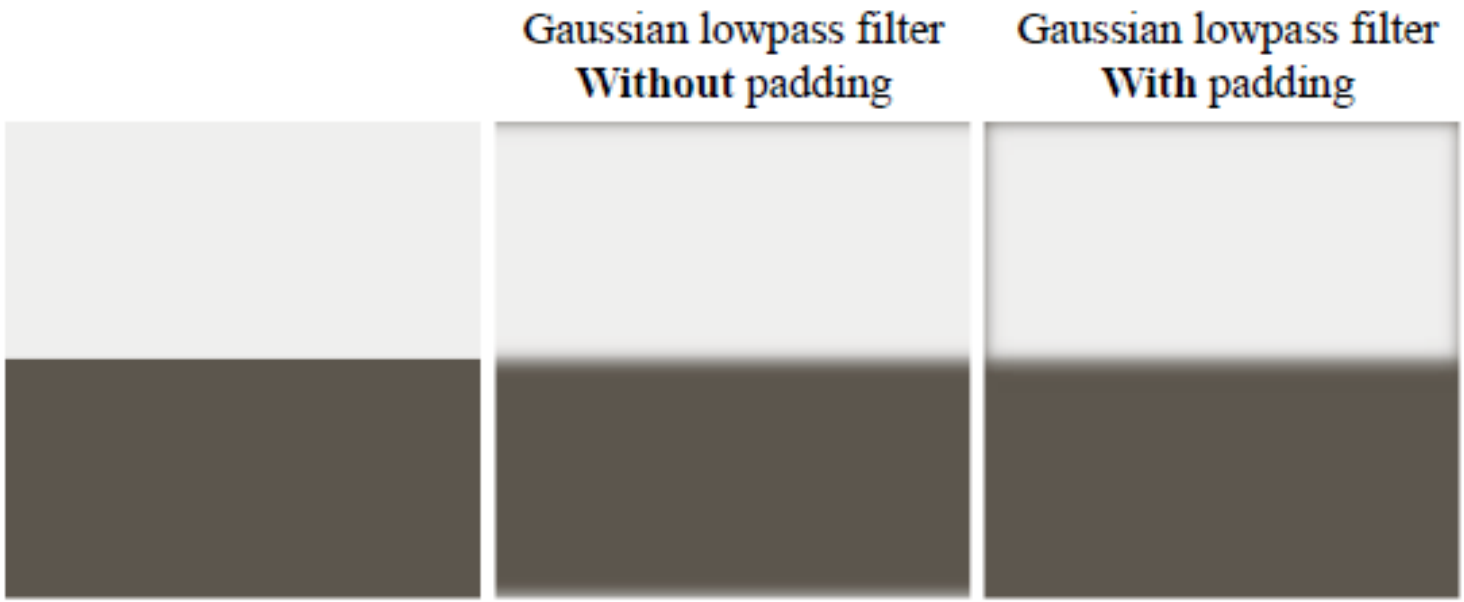

a b c

FIGURE 4.32 (a) A simple image. (b) Result of blurring with a Gaussian lowpass filter without padding. (c) Result of lowpass filtering with padding. Compare the light area of the vertical edges in (b) and (c).

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4.7.3 Steps for Filtering in the Frequency Domain

1. Given an input image $f(x,y)$ of size MxN, obtain padding parameters P and Q. Typically, $P=2M$ and $Q=2N$.
2. Form a padded image $f_p(x,y)$ of size PxQ by appending the necessary number of zeros to f(x,y).
3. Multiply $f_p(x,y)$ by $(-1)^{x+y}$ to centre its transform.
4. Compute the DFT, $F(u,v)$, of the image from step 3.
5. Generate a real, symmetric filter function, $H(u,v)$, of size PxQ with centre at coordinates $(P/2,Q/2)$. Form the product $G(u,v)=H(u,v)F(u,v)$ using array multiplication.
6. Obtain the processed image: $g_p(x,y) = \text{real}\left[IDFT\left[G(u,v)\right]\right](-1)^{x+y}$
7. Obtain the final processed result, $g(x,y)$, by extracting the MxN region from the top, left quadrant of $g_p(x,y)$

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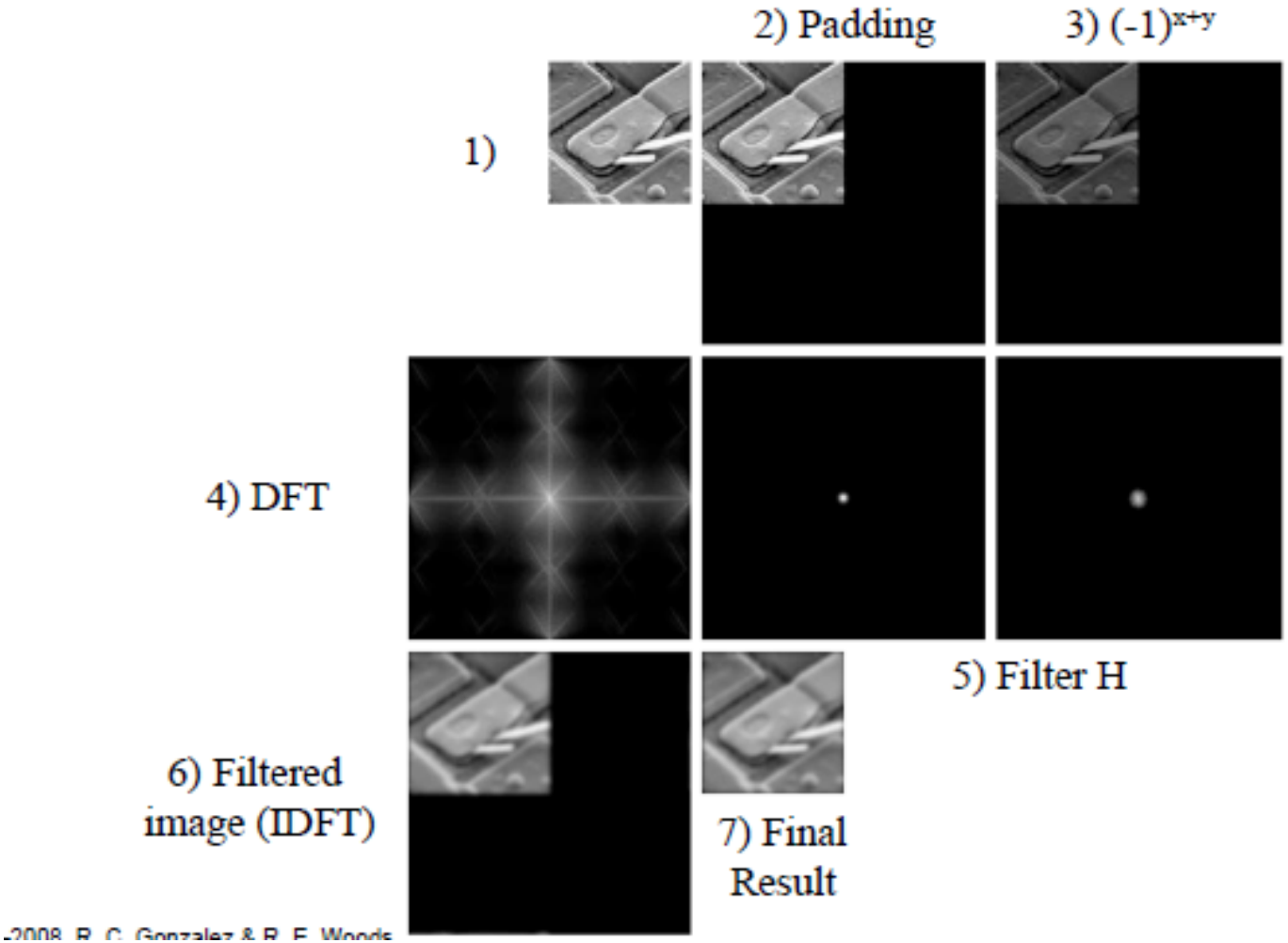
4.7.3 Steps for Filtering in the Frequency Domain



a b c
d e f
g h

FIGURE 4.36
(a) An $M \times N$ image, f.
(b) Padded image, f_p of size $P \times Q$.
(c) Result of multiplying f_p by $(-1)^{x+y}$.
(d) Spectrum of F_p. (e) Centered Gaussian lowpass filter, H, of size $P \times Q$.
(f) Spectrum of the product HF_p.
(g) g_p, the product of $(-1)^{x+y}$ and the real part of the IDFT of HF_p.
(h) Final result, g, obtained by cropping the first M rows and N columns of g_p.

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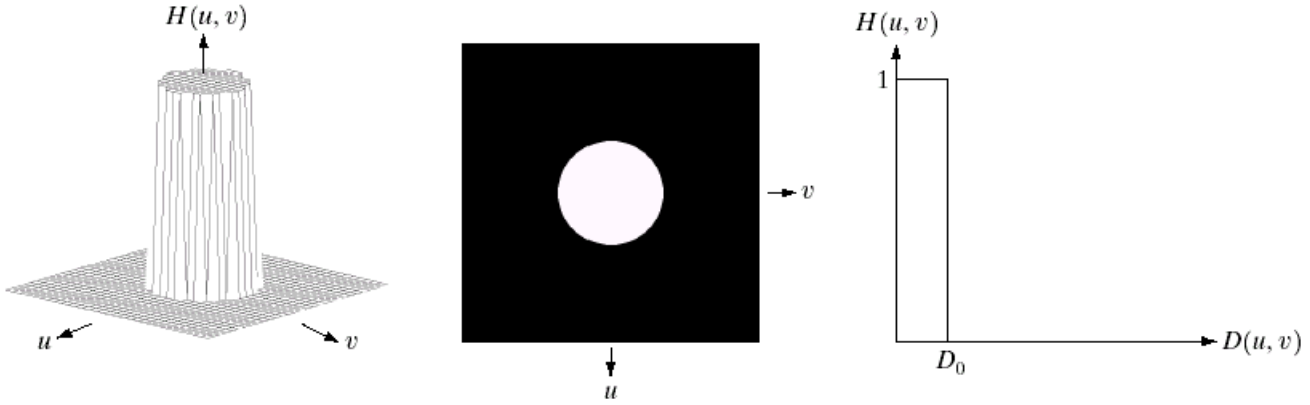
Smoothing Frequency Domain, Ideal Low-pass Filters



a b c

FIGURE 4.10 (a) Perspective plot of an ideal lowpass filter transfer function. (b) Filter displayed as an image. (c) Filter radial cross section.

$$H(u,v) = \begin{cases} 1 & \text{if } D(u,v) \le D_0 \\ 0 & \text{if } D(u,v) > D_0 \end{cases}$$

$$D(u,v) = \left[(u - M/2)^2 + (v - N/2)^2\right]^{1/2}$$

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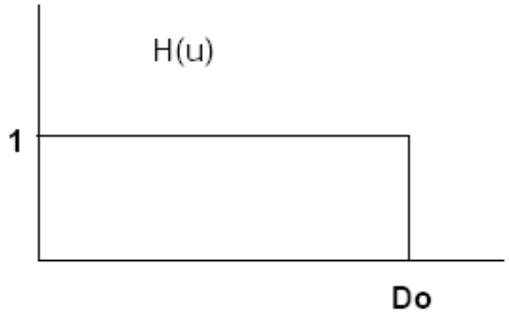
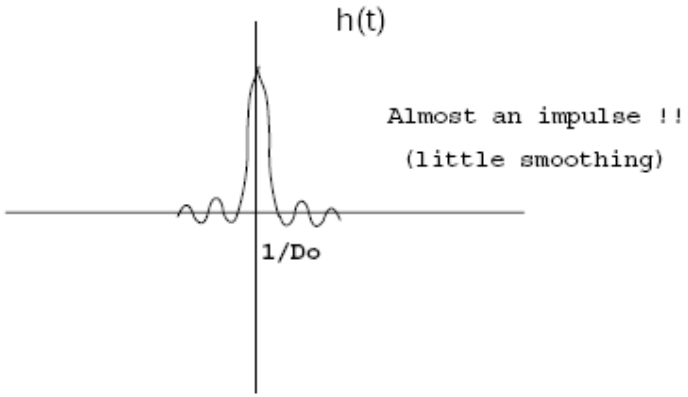
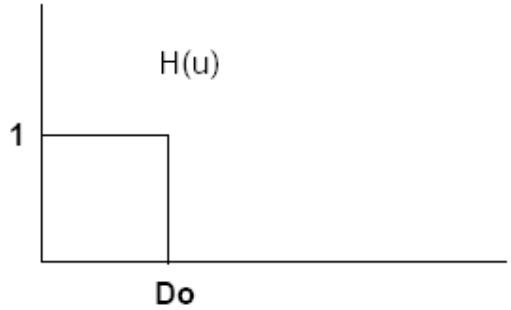
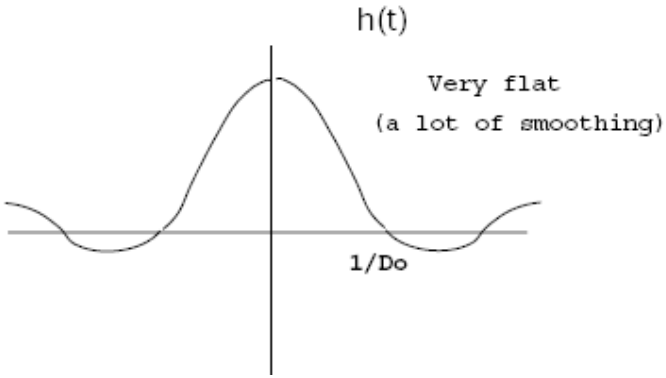
- D_0 controls the amount of blurring





r=78 (99%)





r=8 (90%)

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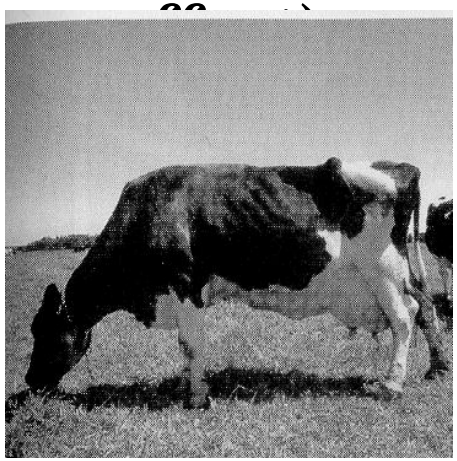
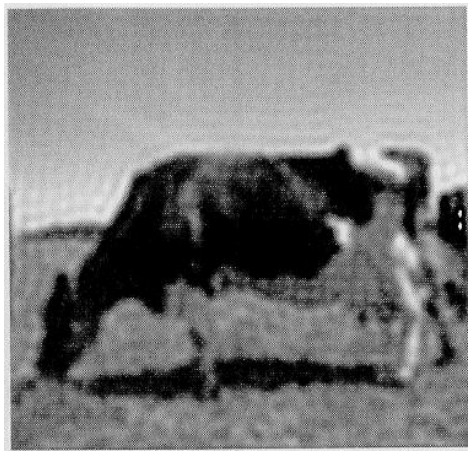
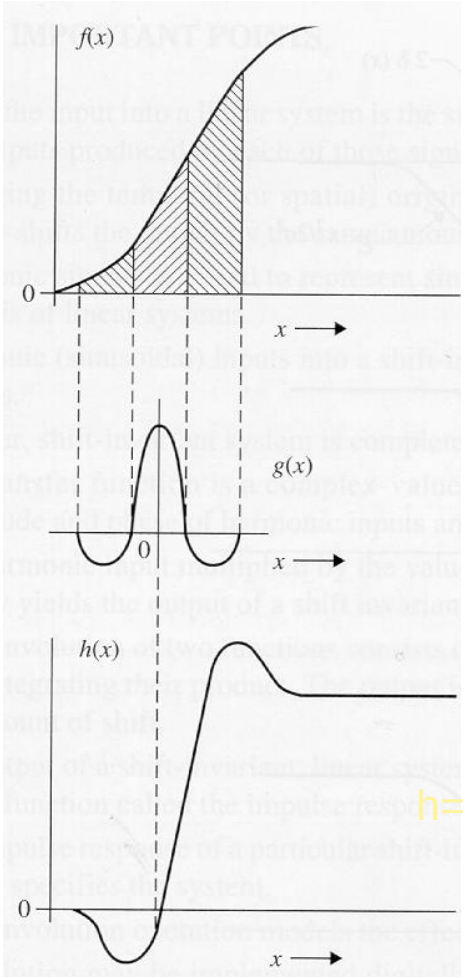
Ringing Effect

- Sharp cutoff frequencies produce an overshoot of image features whose frequency is close to the cutoff frequencies **(ringing**







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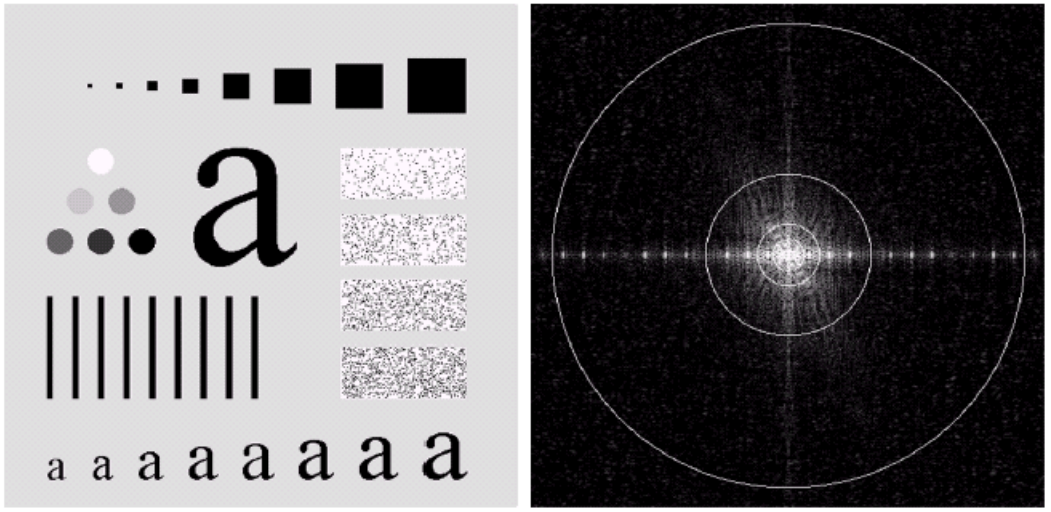
Smoothing Frequency Domain, Ideal Low-pass Filters



a b

FIGURE 4.11 (a) An image of size 500 × 500 pixels and (b) its Fourier spectrum. The superimposed circles have radii values of 5, 15, 30, 80, and 230, which enclose 92.0, 94.6, 96.4, 98.0, and 99.5% of the image power, respectively.

Total Power $\longrightarrow$ $$P_T = \sum_{u=0}^{M-1}\sum_{v=0}^{N-1}\left|F(u,v)\right|^2$$

The remained percentage power after filtration $\longrightarrow$ $$\alpha = 100 \times \left[\sum_{u}\sum_{v}\left|F(u,v)\right| / P_T\right]$$

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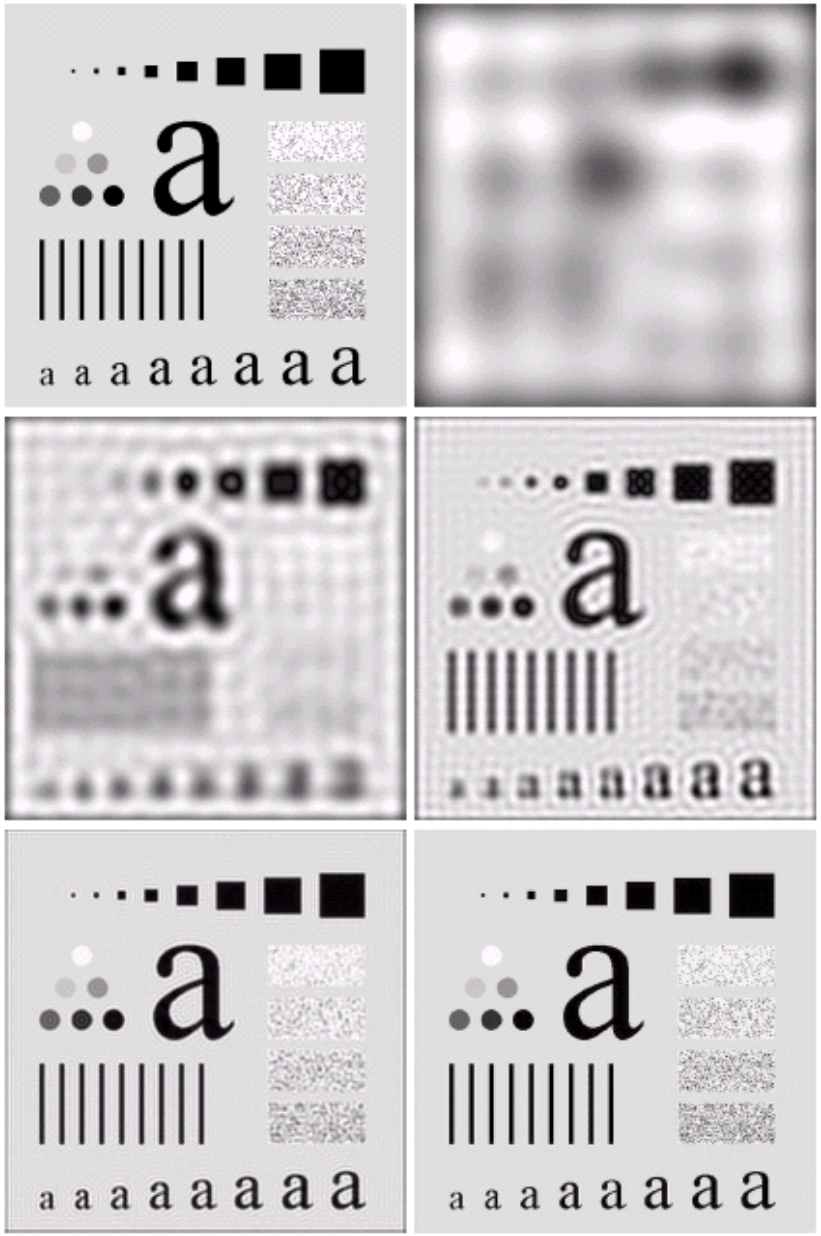
Smoothing Frequency Domain, Ideal Low-pass Filters



f_c =5
α = 92%

f_c =15
α = 94.6%

f_c =30
α = 96.4%

f_c =80
α = 98%

f_c =230
α = 99.5%

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Project #5, Ideal Low-pass Filter

Implement in Matlab the Ideal low-pass filter in the following equation.

$$H(u,v) = \begin{cases} 1 & \text{if } D(u,v) \le D_0 \\ 0 & \text{if } D(u,v) > D_0 \end{cases}$$

$$D(u,v) = \left[(u - M/2)^2 + (v - N/2)^2\right]^{1/2}$$

1. You must give the user the ability to specify the cutoff frequency D_0
2. Calculate the The remained percentage power after filtration.
3. Display the image after filtration
4. Use Elaine image to test your program

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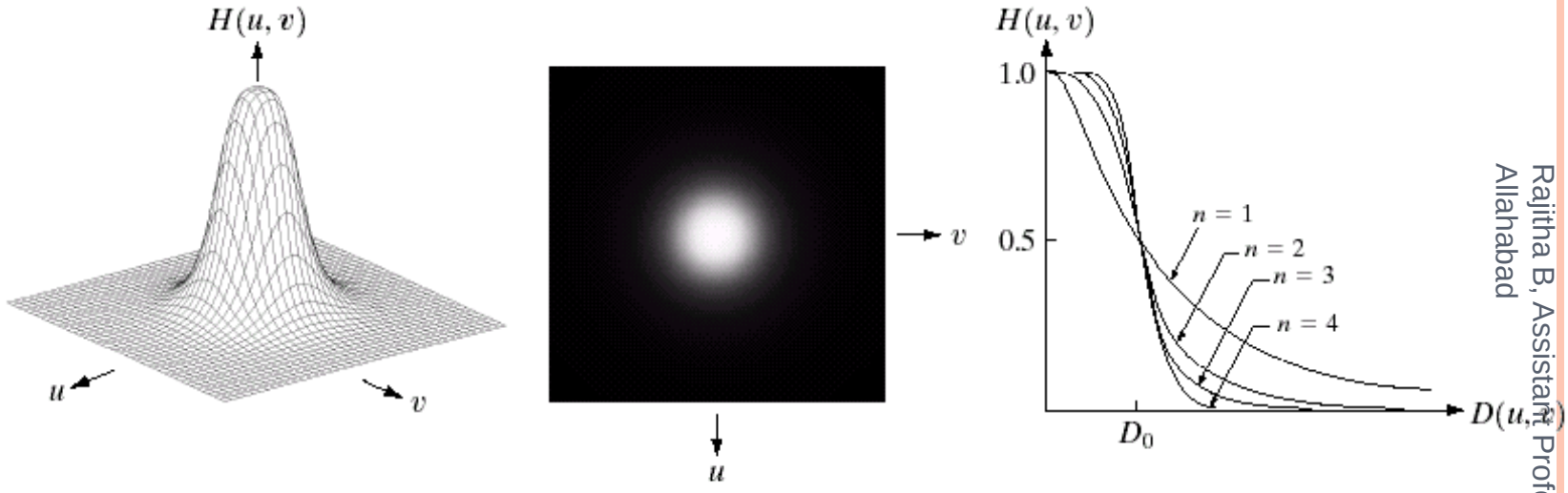
Smoothing Frequency Domain, Butterworth Low-pass Filters

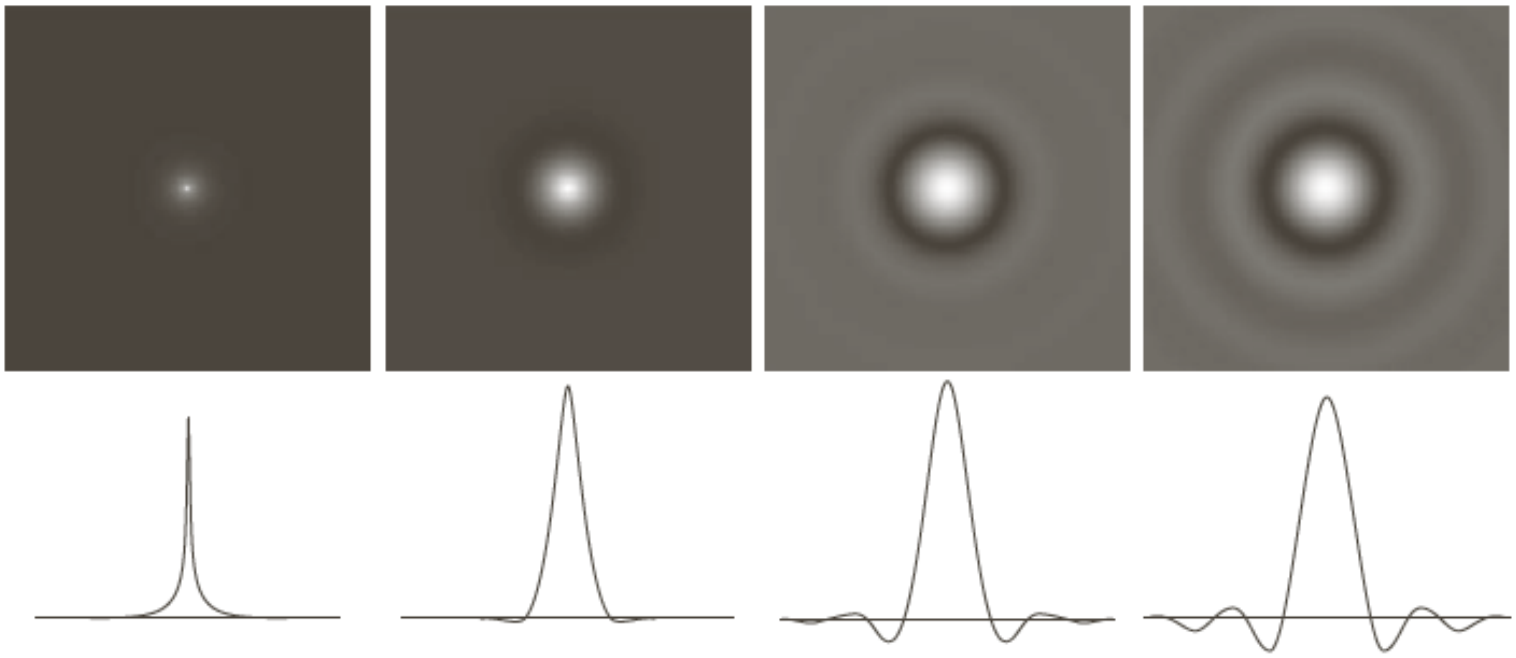


a b c

FIGURE 4.14 (a) Perspective plot of a Butterworth lowpass filter transfer function. (b) Filter displayed as an image. (c) Filter radial cross sections of orders 1 through 4.

$$H(u,v) = \frac{1}{1+\left[D(u,v)/D_0\right]^{2n}}$$

Spatial Representation of BLPFs



a b c d

FIGURE 4.46 (a)–(d) Spatial representation of BLPFs of order 1, 2, 5, and 20, and corresponding intensity profiles through the center of the filters (the size in all cases is 1000 × 1000 and the cutoff frequency is 5). Observe how ringing increases as a function of filter order.

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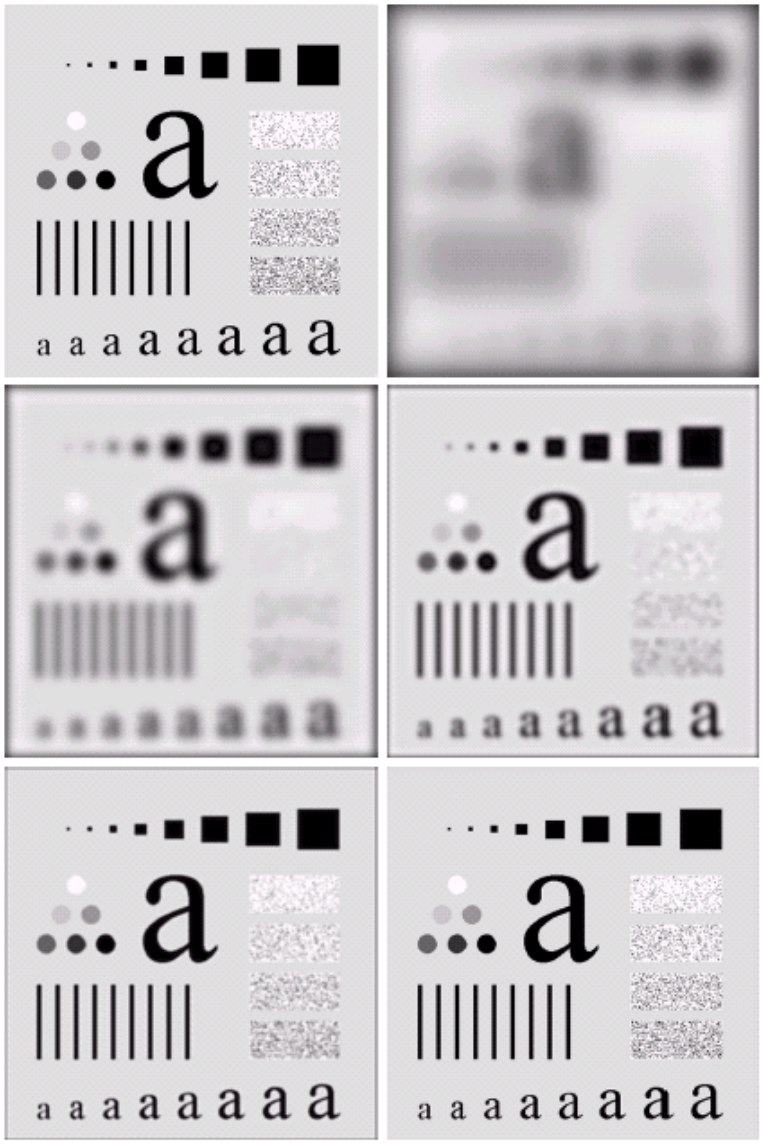
Smoothing Frequency Domain, Butterworth Low-pass Filters

Butterworth Low-pass Filter: *n*=2



Radii= 5

Radii= 15

Radii= 30

Radii= 80

Radii= 230

a b c d e f

FIGURE 4.15 (a) Original image. (b)–(f) Results of filtering with BLPFs of order 2, with cutoff frequencies at radii of 5, 15, 30, 80, and 230, as shown in Fig. 4.11(b). Compare with Fig. 4.12.

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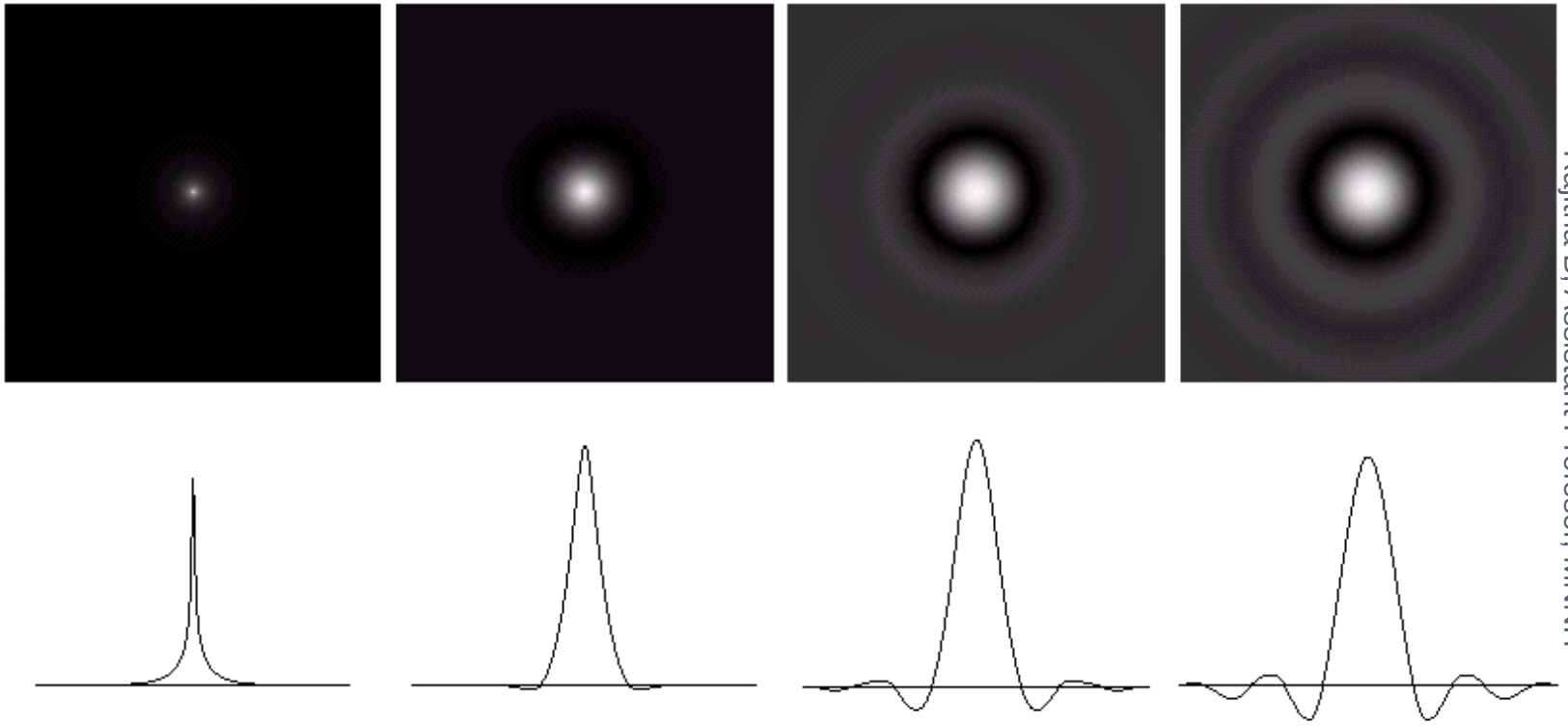
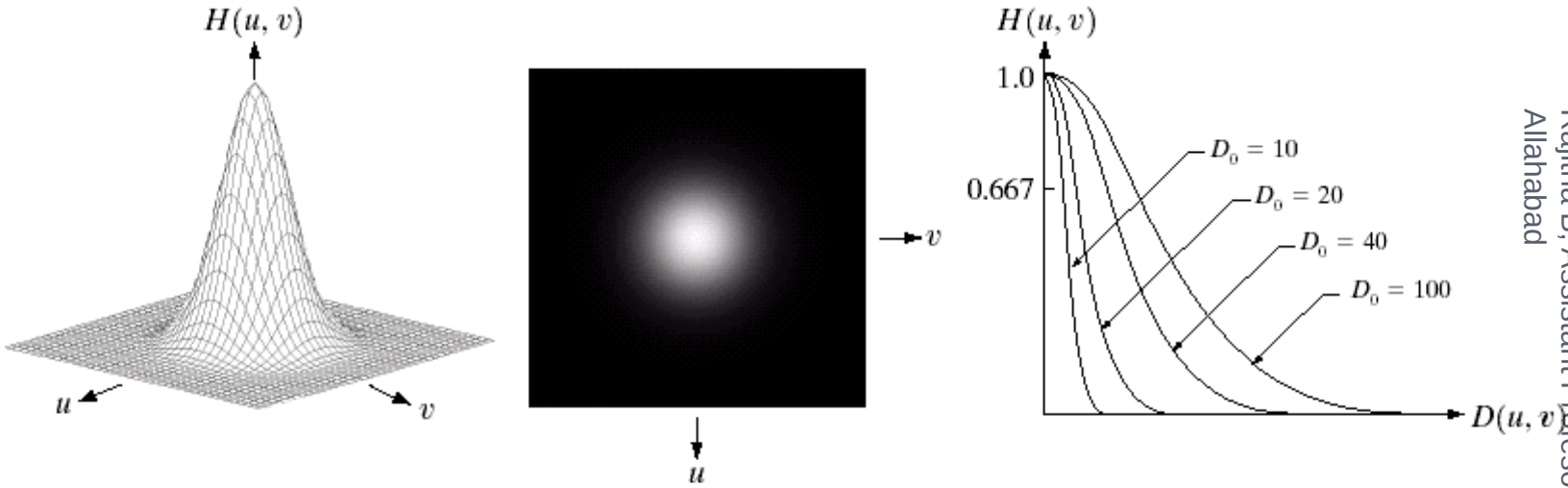
Smoothing Frequency Domain, Butterworth Low-pass Filters



a b c d

FIGURE 4.16 (a)–(d) Spatial representation of BLPFs of order 1, 2, 5, and 20, and corresponding gray-level profiles through the center of the filters (all filters have a cutoff frequency of 5). Note that ringing increases as a function of filter order.

Smoothing Frequency Domain, Gaussian Low-pass Filters



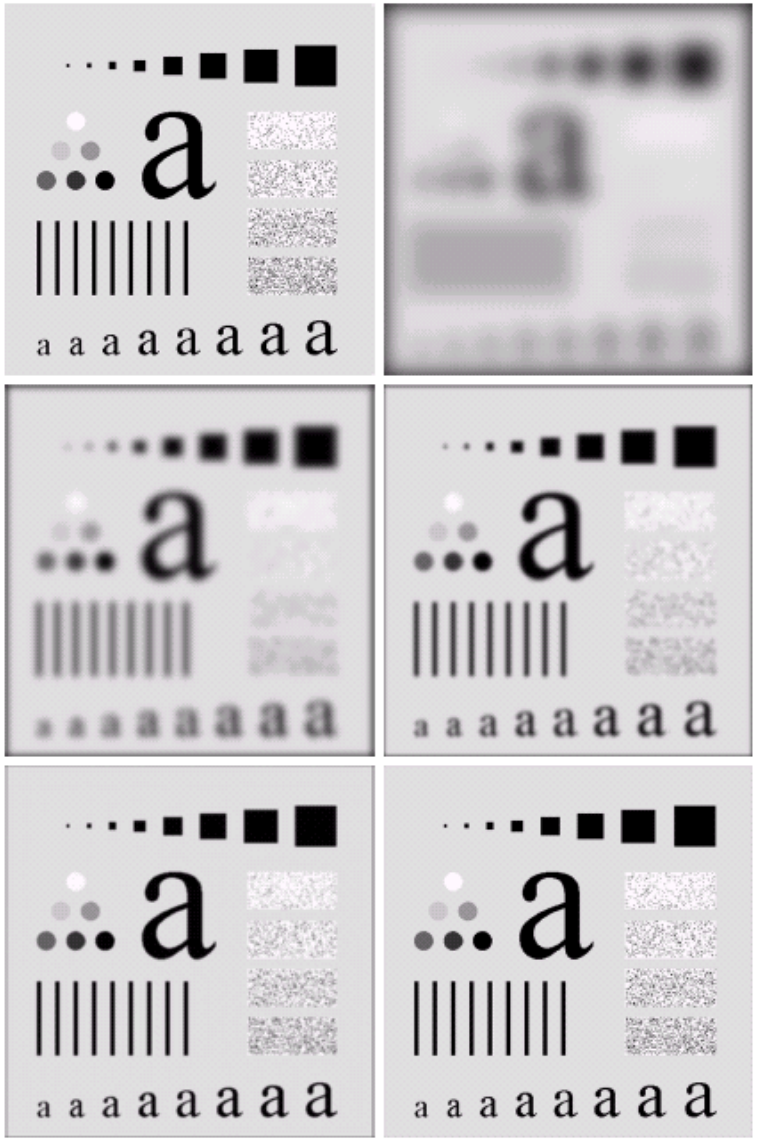
a b c

FIGURE 4.17 (a) Perspective plot of a GLPF transfer function. (b) Filter displayed as an image. (c) Filter radial cross sections for various values of D_0.

$$H(u,v) = e^{-D^2(u,v)/2D_0^2}$$

Smoothing Frequency Domain, Gaussian Low-pass Filters

Gaussian Low-pass



Radii= 5

Radii= 15

Radii= 30

Radii= 80

Radii= 230

FIGURE 4.18 (a) Original image. (b)–(f) Results of filtering with Gaussian lowpass filters with cutoff frequencies set at radii values of 5, 15, 30, 80, and 230, as shown in Fig. 4.11(b). Compare with Figs. 4.12 and 4.15.

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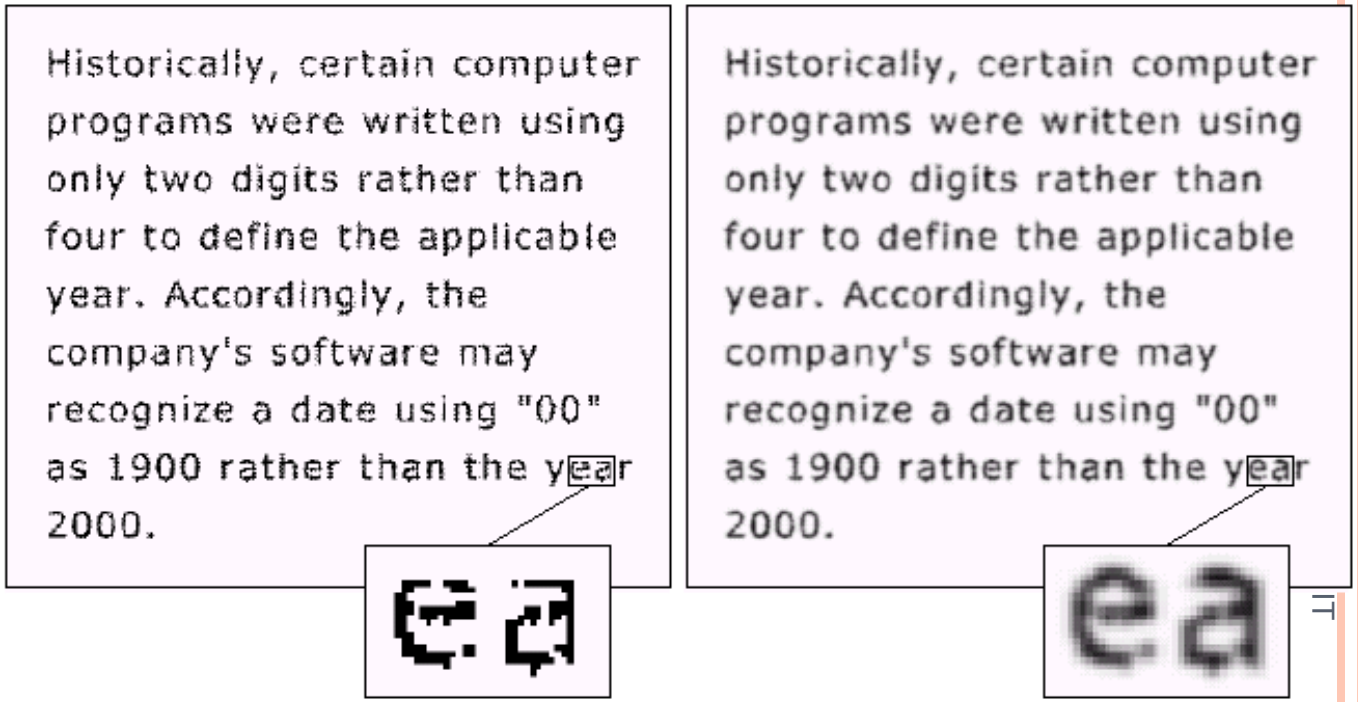
Smoothing Frequency Domain, Gaussian Low-pass Filters

a b

FIGURE 4.19
(a) Sample text of poor resolution (note broken characters in magnified view). (b) Result of filtering with a GLPF (broken character segments were joined).



Smoothing Frequency Domain, Gaussian Low-pass Filters

a b c

FIGURE 4.20 (a) Original image (1028 × 732 pixels). (b) Result of filtering with a GLPF with $D_0 = 100$. (c) Result of filtering with a GLPF with $D_0 = 80$. Note reduction in skin fine lines in the magnified sections of (b) and (c).

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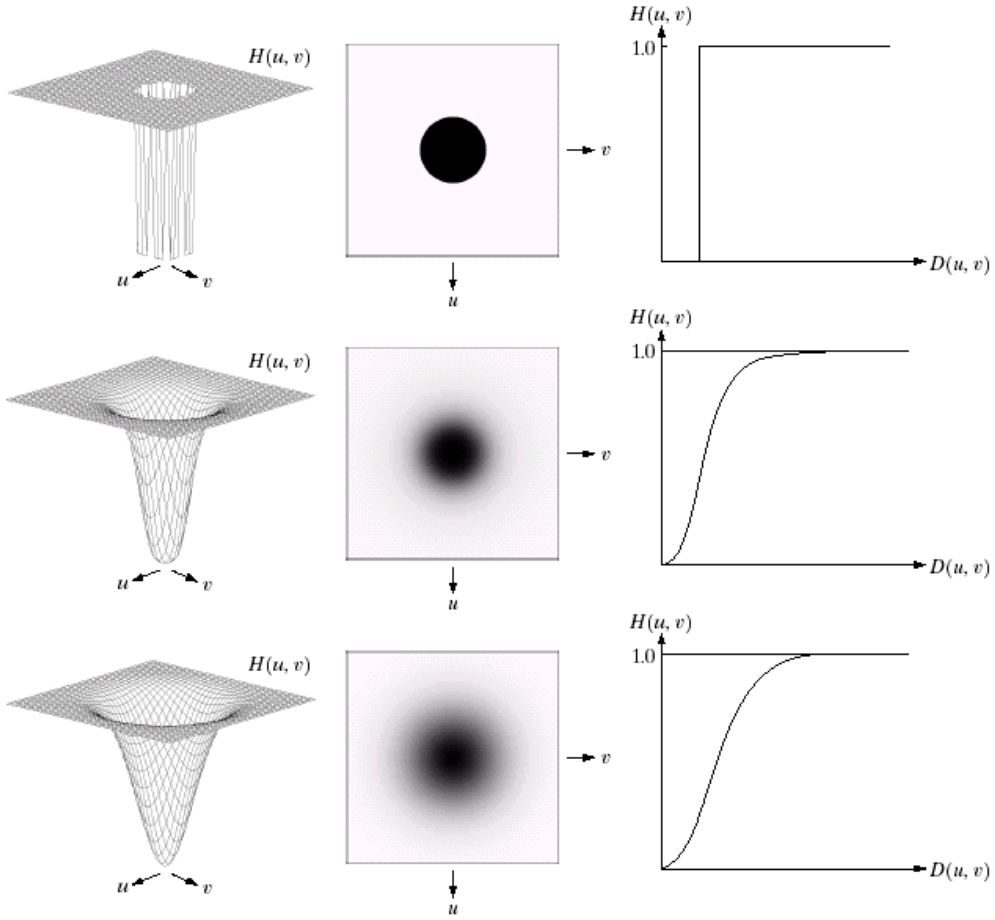
Smoothing Frequency Domain, Gaussian Low-pass Filters

a b c

FIGURE 4.21 (a) Image showing prominent scan lines. (b) Result of using a GLPF with $D_0 = 30$. (c) Result of using a GLPF with $D_0 = 10$. (Original image courtesy of NOAA.)

Sharpening Frequency Domain Filters



a b c
d e f
g h i

FIGURE 4.22 Top row: Perspective plot, image representation, and cross section of a typical ideal highpass filter. Middle and bottom rows: The same sequence for typical Butterworth and Gaussian highpass filters.

$$H_{hp}(u,v) = 1 - H_{lp}(u,v)$$

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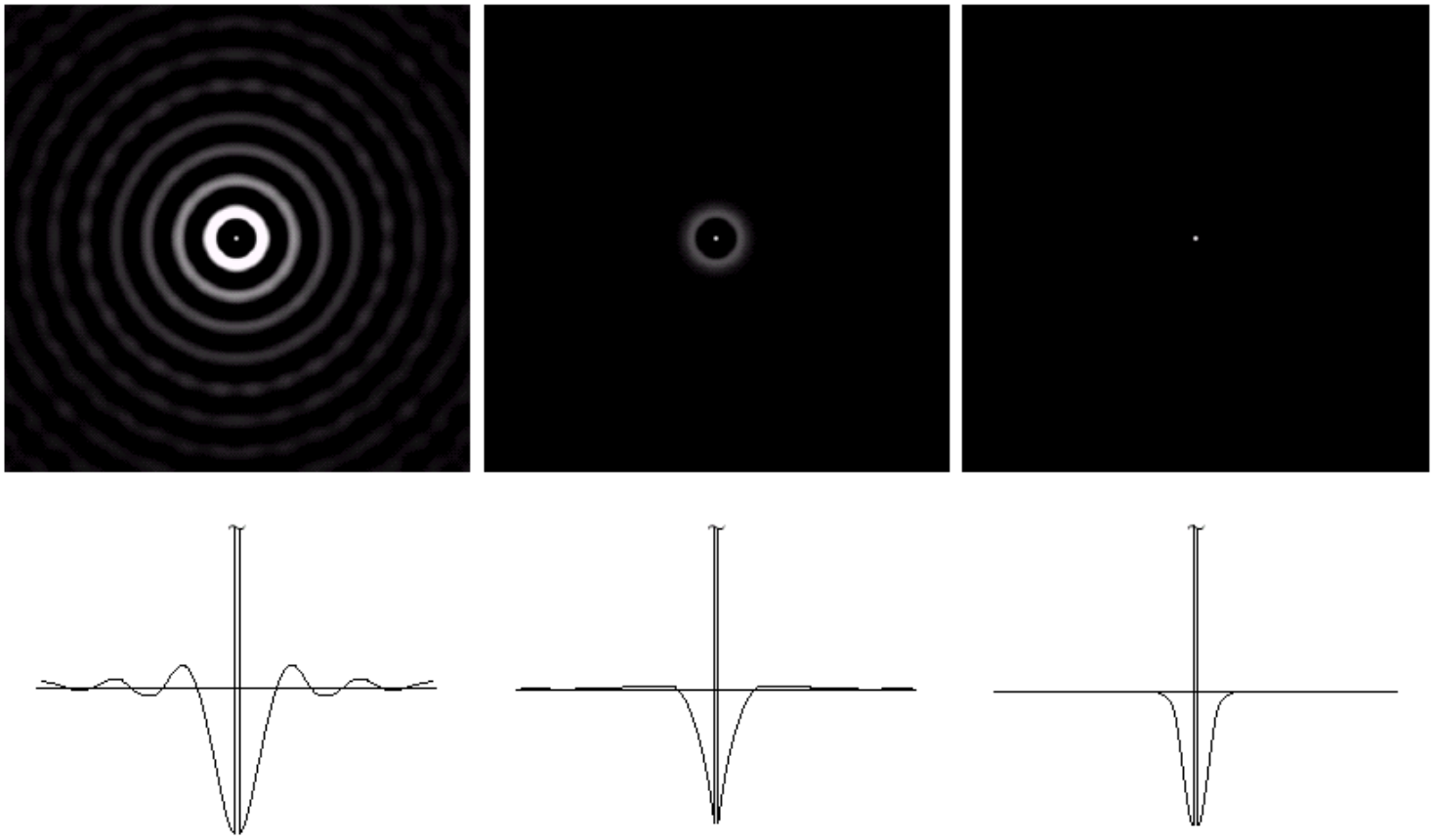
Sharpening Frequency Domain Filters



a b c

FIGURE 4.23 Spatial representations of typical (a) ideal, (b) Butterworth, and (c) Gaussian frequency domain highpass filters, and corresponding gray-level profiles.

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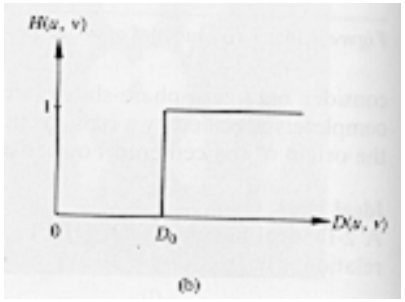
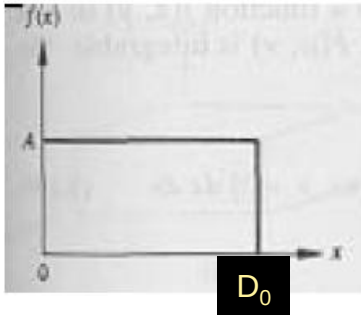
High-Pass filtering

A high-pass filter can be obtained from a low-pass filter using:

$$H_{HP}(u,v) = 1 - H_{LP}(u,v)$$



= 1 -



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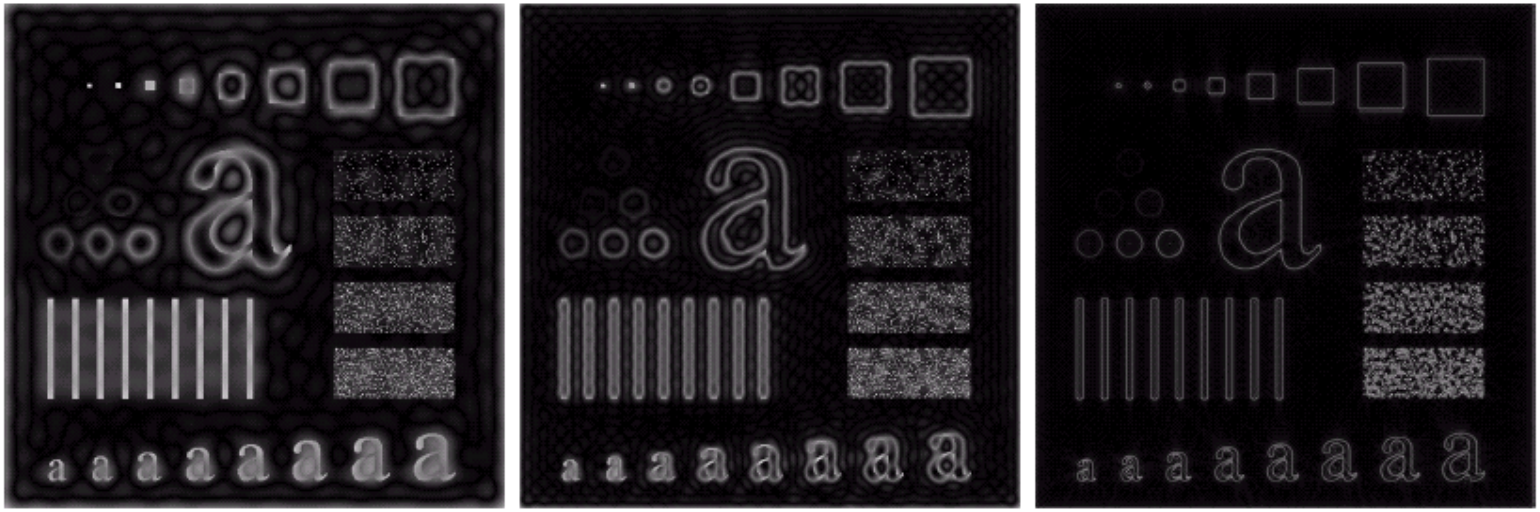
Sharpening Frequency Domain, Ideal High-pass Filters



a b c

FIGURE 4.24 Results of ideal highpass filtering the image in Fig. 4.11(a) with $D_0 = 15$, 30, and 80, respectively. Problems with ringing are quite evident in (a) and (b).

$$H(u,v) = \begin{cases} 0 & \text{if } D(u,v) \le D_0 \\ 1 & \text{if } D(u,v) > D_0 \end{cases}$$

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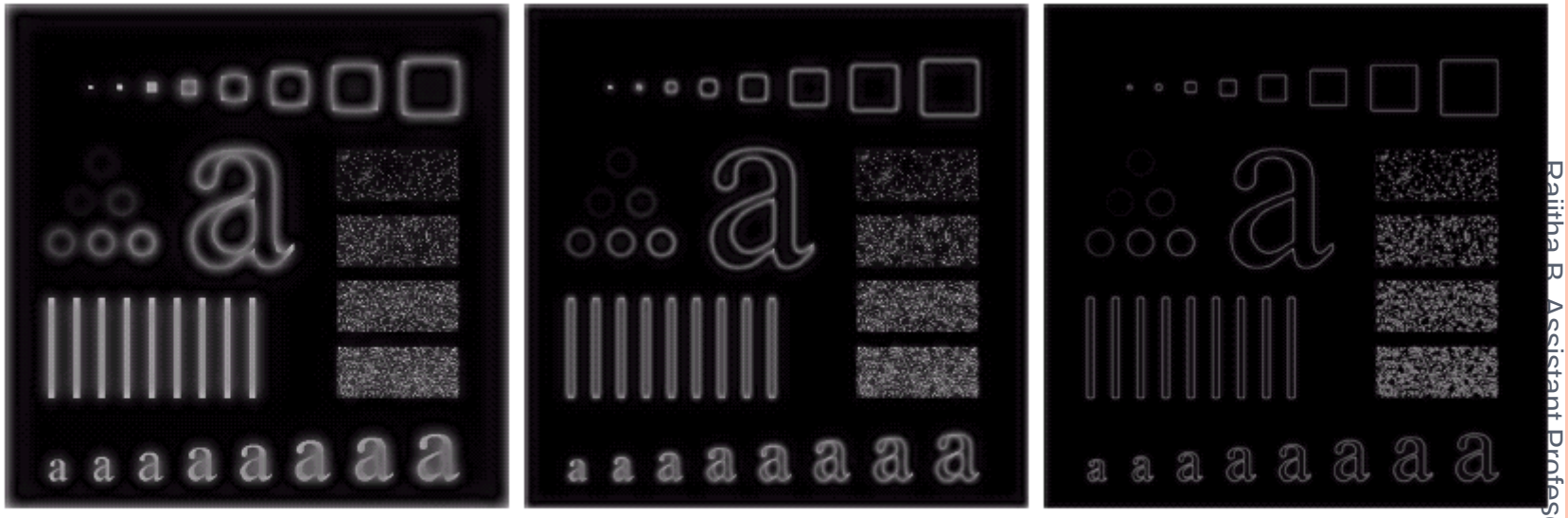
Sharpening Frequency Domain, Butterworth High-pass Filters



a b c

FIGURE 4.25 Results of highpass filtering the image in Fig. 4.11(a) using a BHPF of order 2 with $D_0 = 15$, 30, and 80, respectively. These results are much smoother than those obtained with an ILPF.

$$H(u,v) = \frac{1}{1+\left[D_0 / D(u,v)\right]^{2n}}$$

Sharpening Frequency Domain, Gaussian High-pass Filters

a b c

FIGURE 4.26 Results of highpass filtering the image of Fig. 4.11(a) using a GHPF of order 2 with $D_0 = 15$, 30, and 80, respectively. Compare with Figs. 4.24 and 4.25.

$$H(u,v) = 1 - e^{-D^2(u,v)/2D_0^2}$$

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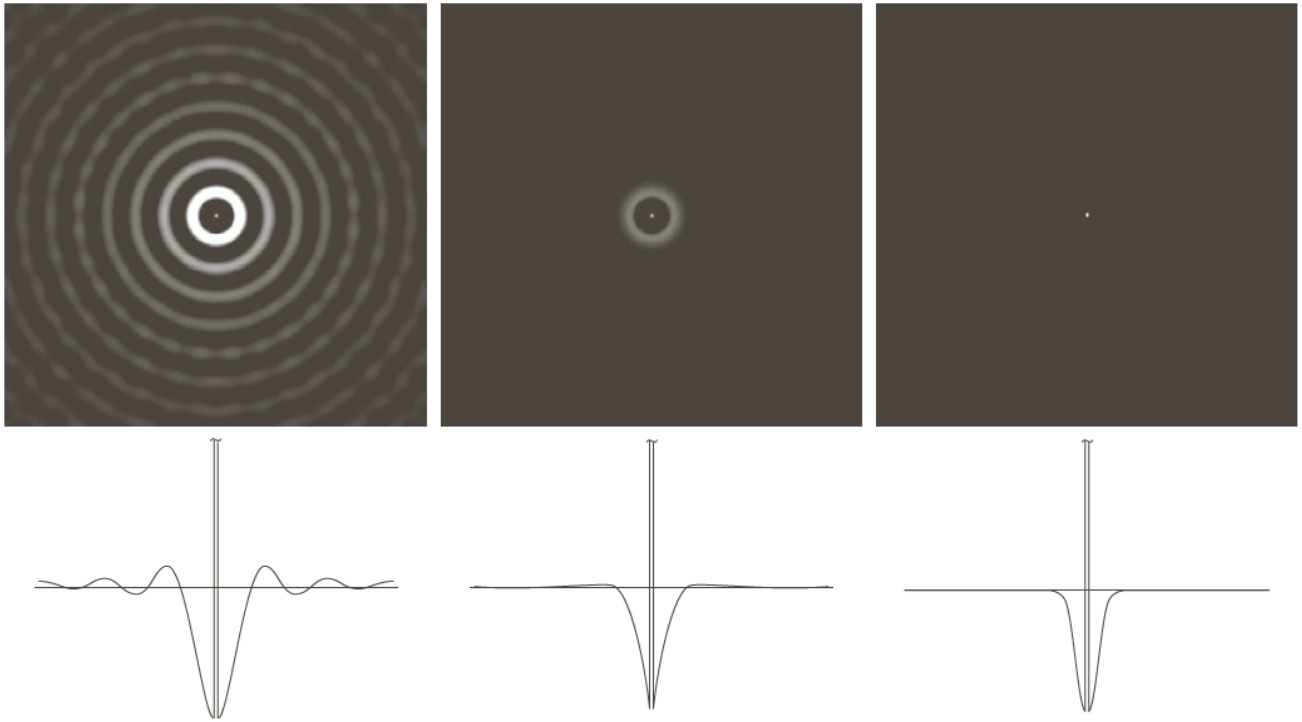
Spatial Representation of High-pass Filters



a b c

FIGURE 4.53 Spatial representation of typical (a) ideal, (b) Butterworth, and (c) Gaussian frequency domain highpass filters, and corresponding intensity profiles through their centers.

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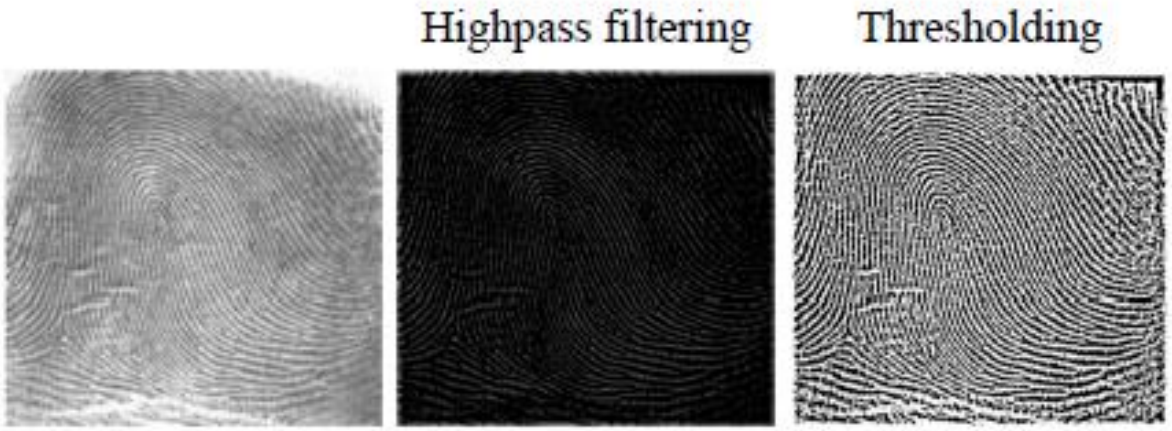
Example: using highpass filtering and thresholding for image enhancement



a b c

FIGURE 4.57 (a) Thumb print. (b) Result of highpass filtering (a). (c) Result of thresholding (b). (Original image courtesy of the U.S. National Institute of Standards and Technology.)

Homomorphic Filtering (cont'd)

- **Intensity Range Compression + Contrast Enhancement**
- Consider the following model of image formation:

$$f(x,y) = i(x,y)\, r(x,y)$$

i(x,y): illumination
r(x,y): reflection

- In general, the illumination component i(x,y) varies **slowly** and affects **low** frequencies mostly.
- In general, the reflection component r(x,y) varies **faster** and affects **high** frequencies mostly.

IDEA: separate low frequencies due to i(x,y) from high frequencies due to r(x,y)

How are frequencies mixed together?

- Low and high frequencies from **i(x,y)** and **r(x,y)** are mixed together.

$$f(x,y) = i(x,y)\, r(x,y) \quad \Rightarrow \quad F(u,v) = I(u,v) * R(u,v)$$

- When applying filtering, it is difficult to handle low/high frequencies separately.

$$F(u,v)H(u,v) = [I(u,v) * R(u,v)]H(u,v)$$

Can we separate them?

- Idea:

Take the ln( ) of $f(x,y) = i(x,y)\, r(x,y)$

$$ln(f(x,y)) = ln(i(x,y)) + ln(r(x,y))$$

Steps of Homomorphic Filtering

(1) Take $ln(f(x,y)) = ln(i(x,y)) + ln(r(x,y))$

(2) Apply FT: $F(ln(f(x,y))) = F(ln(i(x,y))) + F(ln(r(x,y)))$

or $Z(u,v) = Illum(u,v) + Refl(u,v)$

(3) Apply H(u,v)

$$Z(u,v)H(u,v) = Illum(u,v)H(u,v) + Refl(u,v)H(u,v)$$

Steps of Homomorphic Filtering (cont'd)

(4) Take Inverse FT:

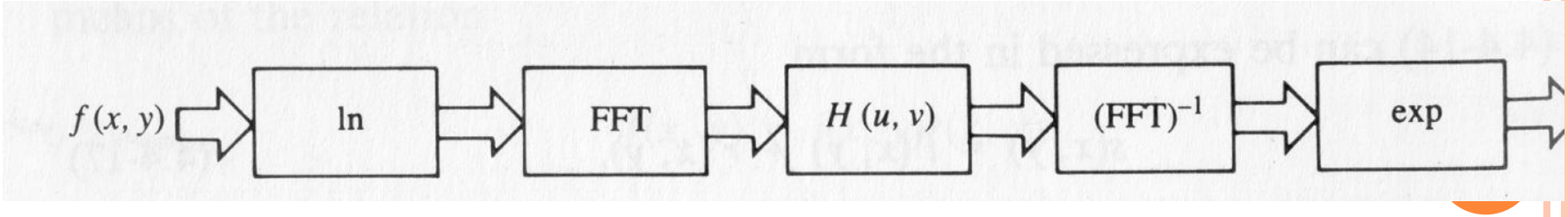
$$F^{-1}(Z(u,v)H(u,v)) = F^{-1}(Illum(u,v)H(u,v)) + F^{-1}(Refl(u,v)H(u,v))$$

or

$$s(x,y) = i'(x,y) + r'(x,y)$$

(5) Take exp( ) or

$$e^{s(x,y)} = e^{i'(x,y)} e^{r'(x,y)} \qquad g(x,y) = i_0(x,y) r_0(x,y)$$



Example using high-frequency emphasis

$$H(u,v) = (\gamma_H - \gamma_L)\left[1 - e^{-c\left[(u^2+v^2)/D_0^2\right]}\right] + \gamma_L$$

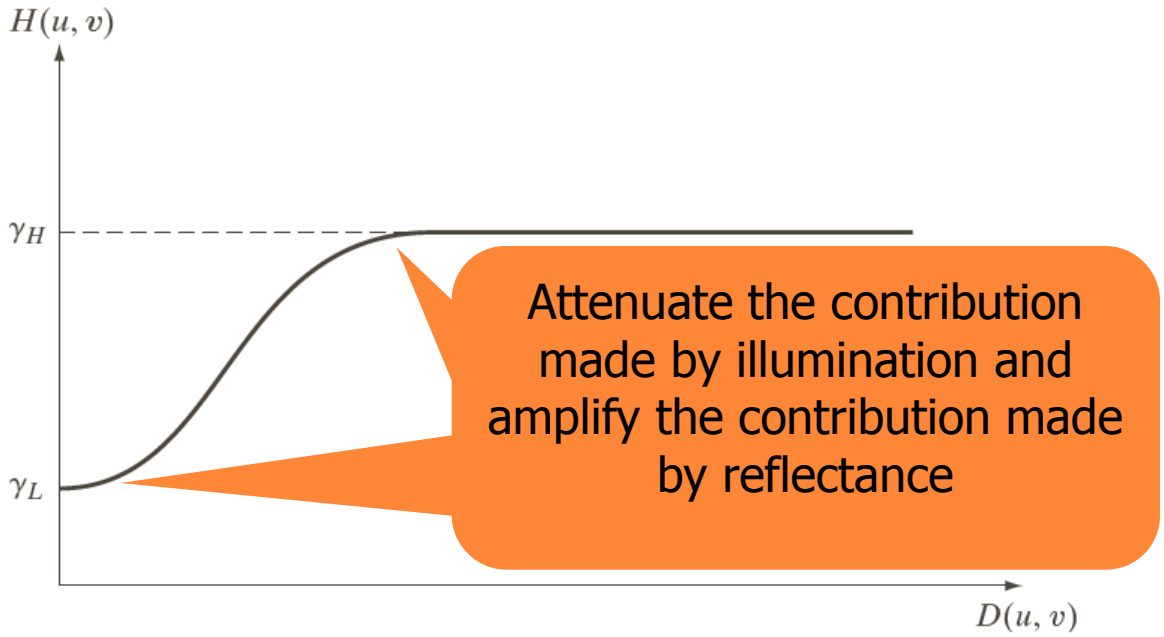
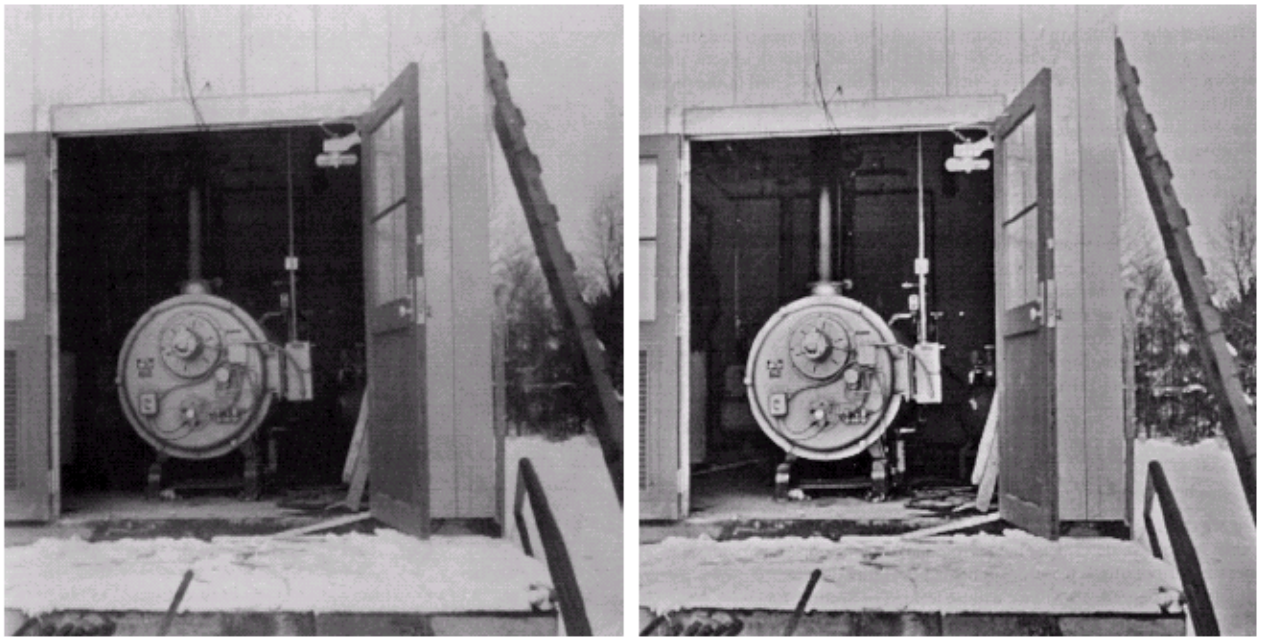


FIGURE 4.61 Radial cross section of a circularly symmetric homomorphic filter function. The vertical axis is at the center of the frequency rectangle and $D(u, v)$ is the distance from the center.

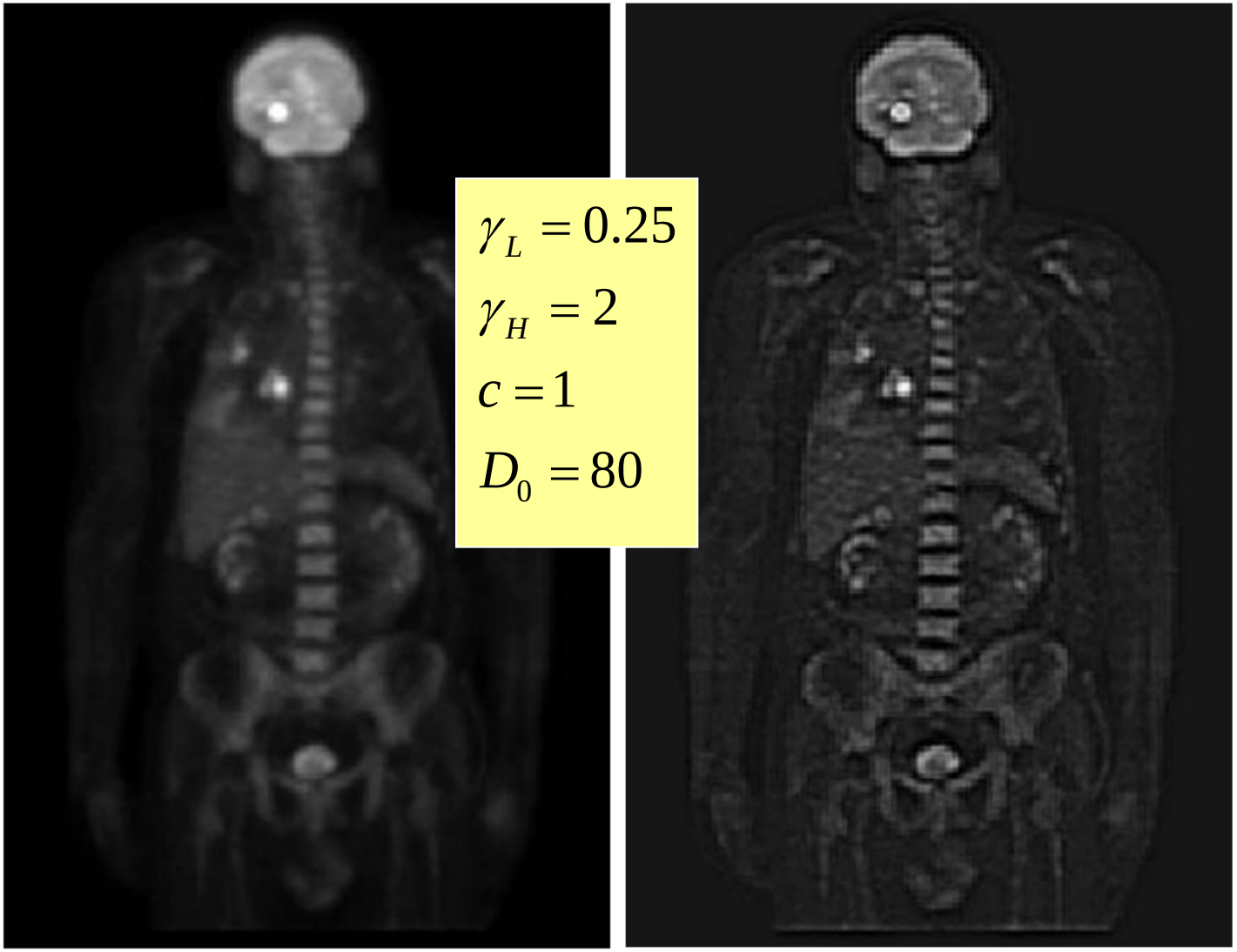
Homomorphic Filtering: Example

a b

FIGURE 4.33
(a) Original image. (b) Image processed by homomorphic filtering (note details inside shelter). (Stockham.)



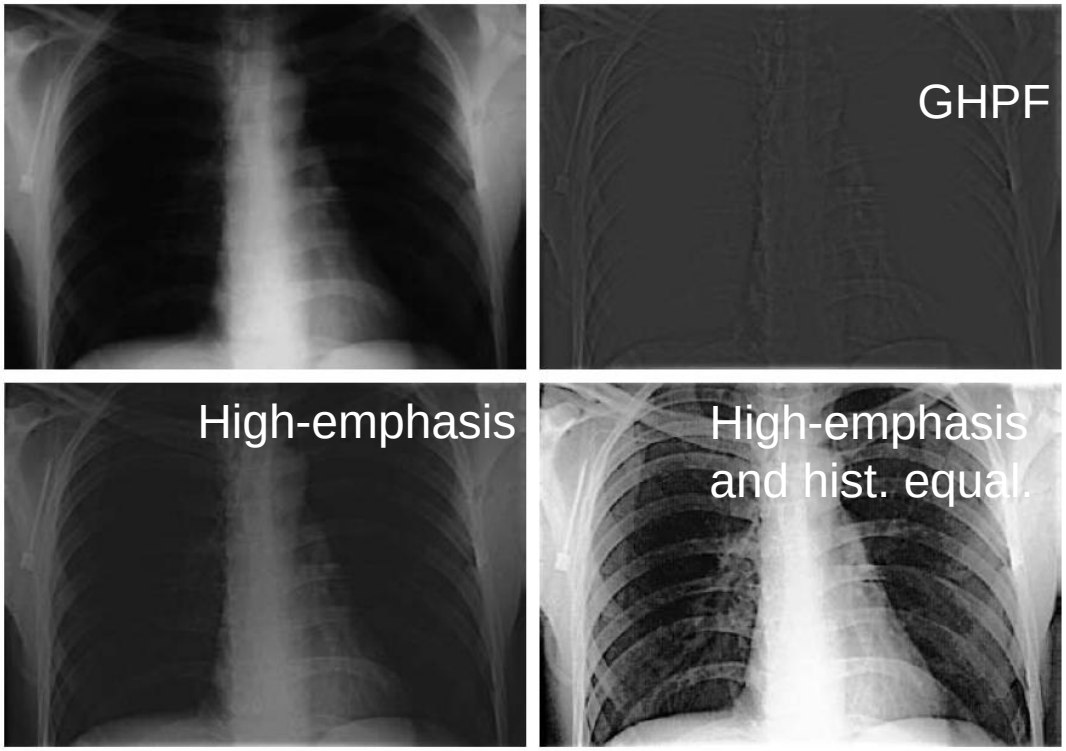
Homomorphic Filtering: Example



a b

FIGURE 4.62
(a) Full body PET scan. (b) Image enhanced using homomorphic filtering. (Original image courtesy of Dr. Michael E. Casey, CTI PET Systems.)

High-Frequency Emphasis filtering Using Gaussian filter
k_1=0.5, k_2=0.75



D_0=40

a b
c d

FIGURE 4.59 (a) A chest X-ray image. (b) Result of highpass filtering with a Gaussian filter. (c) Result of high-frequency-emphasis filtering using the same filter. (d) Result of performing histogram equalization on (c). (Original image courtesy of Dr. Thomas R. Gest, Division of Anatomical Sciences, University of Michigan Medical School.)

Selective Filtering

Bandreject and Banfpass Filters:

D(u,v) : The distance from center of frequency rectangle

D0: radial center of the band

W: width of the band

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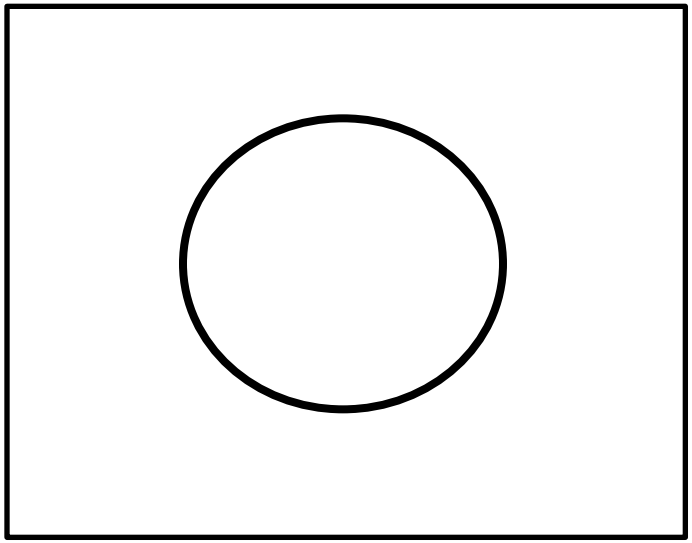
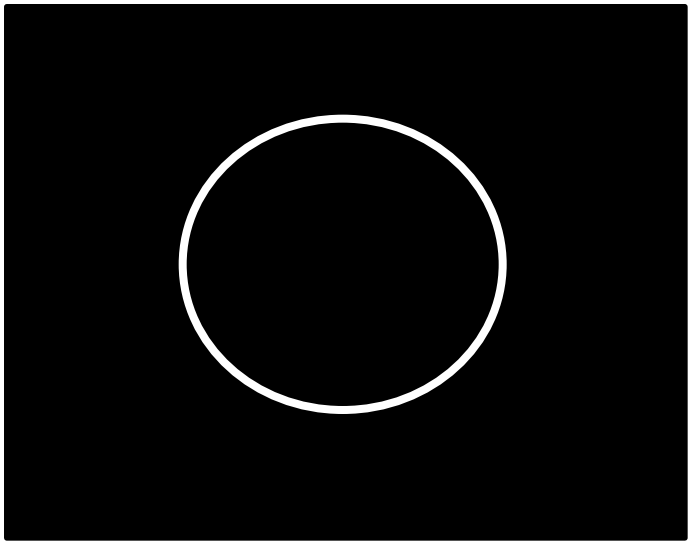
BANDREJECT



BANDPASS



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Three filters : Notch Filter

$$H(u,v) = \begin{cases} 0 & \text{if } D0 - W/2 \le D \le D0 + W/2 \\ 1 & \text{otherwise} \end{cases}$$

IDEAL

$$H(u,v) = \frac{1}{1 + \left[DW/(D^2 - D_0{}^2) \right]^{2n}}$$

Butterworth

$$H(u,v) = 1 - e^{-((D^2 - D_0^2)/DW)^2}$$

Gaussian

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