Preliminary results of the S2 redshift analysis within the RAR model

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Abstract

The recent fly by of the S2 star through its pericenter allowed to verify relativistic effects in the galactic center based on the robust detection of the combined gravitational redshift and relativistic transverse Doppler effect for S2 [1]. The detection of relativistic corrections for the S2 orbit are yet out of precision due to the large distance ($1400 r_{\mathrm{Sch}}$) of the pericenter to an assumend BH in the galactic center. Argüelles *et al.* [2] has shown that a quantum core, composed of degenerate fermions, provides a feasible alternative for the BH scenario. It is shown here that their semi-degenerate mass distributions of massive fermions are able to reproduce the relativistic redshift corrections.

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INTRODUCTION

The expected redshift $z(r)$ is a combination of the gravitational redshift $z_g(r)$ and the full relativistic Doppler shift z_D [3],

$$z(r) = z_g(r) + z_D(r) \tag{1}$$

Gravitational redshift

The gravitational redshift is given by

$$z_g(r) = \frac{g_{00}(R)}{g_{00}(r)} - 1 \tag{2}$$

where r is the position of the emitted photon (emitter or source) and R is the position of the receiver. Here, $R = R_\odot \approx 8\,\mathrm{kpc}$ is the distance of the sun to the galactic center. For a spherically symmetric metric with $g_{00}(r) = \mathrm{e}^{\nu(r)}$ eq. (2) becomes

$$z_g(r) = \mathrm{e}^{[\nu(R)-\nu(r)]/2} - 1 \tag{3}$$

In the special case of a BH the 00-component of the metric is given by $g_{00}(r) = (1 - r_{\mathrm{Sch}}/r)$ with the Schwarzschild radius $r_{\mathrm{Sch}} = 2GM/c^2$, the BH mass M, the gravitational constant G and the speed of light c. For an emitter close to the galactic center we have $r/R_\odot \ll 1$. The gravitational redshift simplifies then to

$$z_g(r) \approx \frac{1}{2}\frac{r_{\mathrm{Sch}}}{r} \tag{4}$$

Further, it is assumed that the emitter is not to close to the central compact object such that its orbit is Keplerian to a good approximation. Following the vis-viva equation yields

$$\beta(r)^2 \approx \frac{r_{\mathrm{Sch}}}{r} - \frac{r_{\mathrm{Sch}}}{2a} \tag{5}$$

with the semi-major axis a. Finally, eqs. (4) and (5) give

$$z_g(r) \approx \frac{r_{\mathrm{Sch}}}{4a} + \frac{1}{2}\beta(r)^2 \tag{6}$$

with the substitution $\beta(r) = v(r)/c$ and $v(r)$ the object's velocity.

Full relativistic Doppler shift

The full relativistic Doppler shift is given by

$$z_D(r) = \frac{1 + \beta(r)\cos\vartheta}{\sqrt{1 - \beta(r)^2}} - 1 \tag{7}$$

where ϑ is the angle between the velocity vector and the line of sight. Of interest here is only the transverse Doppler shift, given for $\theta = \pi/2$, what is a prediction of Special Relativity. For small velocities $\beta(r) \ll 1$ the transverse Doppler shift $z_{tD}(r)$ becomes

$$z_{tD} \approx \frac{1}{2}\beta(r)^2 \tag{8}$$

Redshift comparison at the S2 pericenter

In the following, the combined gravitational redshift and relativistic transverse Doppler effect at the pericenter of the S2 star will be compared with the RAR prediction.

The former redshift based on the S2 orbit parameters is calculated via

$$z(r_p) \approx \frac{r_{\rm Sch}}{4a} + \beta(r_p)^2 \tag{9}$$

where the velocity at the pericenter is given by

$$v_p^2 = \frac{GM}{a}\frac{1+e}{1-e} \tag{10}$$

The numerical values of the orbit parameters (e.g. M, a and e) are provided in Gravity Collaboration *et al.* [1, Table A.1]. Note that the semi-major axis a is given there only in *mas*. It is therefore convenient to calculate its value via

$$a^3 = \frac{GMT^2}{4\pi^2} \tag{11}$$

where T is the orbit period. The relative uncertainty of eq. (9) is given by

$$\Delta z(r_p) = \frac{\Delta M}{M} + \frac{\Delta a}{a} + \frac{4\Delta e}{(1-e)(3+e)} \approx 1.1\,\% \tag{12}$$

while the uncertainty of the pericenter is given by

$$\Delta r_p = \frac{\Delta a}{a} + \frac{\Delta e}{1-e} \approx 0.3\,\% \tag{13}$$

In the RAR case the redshift is calculated by eqs. (1), (2) and (7) with $\vartheta = \pi/2$ in order to take into account only the transverse Doppler shift. In sum

$$z(r_p) = \frac{1}{\sqrt{1-\beta(r_p)^2}} + \mathrm{e}^{[\nu(R)-\nu(r_p)]/2} - 2 \tag{14}$$

It is important to emphasize that the calculated redshifts, given by eqs. (9) and (14), consider only the relativistic corrections. Thus, a deviation of the angle $\theta = \pi/2$ would contribute the Newtonian Doppler shift to both redshifts in the same amount. Additionally, it is expected that the Roemer time delay would contribute in the same way.[1]

RESULTS

The inferred redshift corrections at the S2 pericenter from its orbit parameters and the predicted corrections of the RAR model are compared in fig. 1. Note that the considered RAR solutions consider a slightly larger central core mass ($M_c = 4.2 \times 10^6\,\mathrm{M}_\odot$) compared to $M_c \approx 4.1 \times 10^6\,\mathrm{M}_\odot$ as obtained by Gravity Collaboration *et al.* [1]. Nevertheless, the results

[1] To me it is not completely clear yet how to include the effect due to the Roemer time delay. And if it is necessary at all.

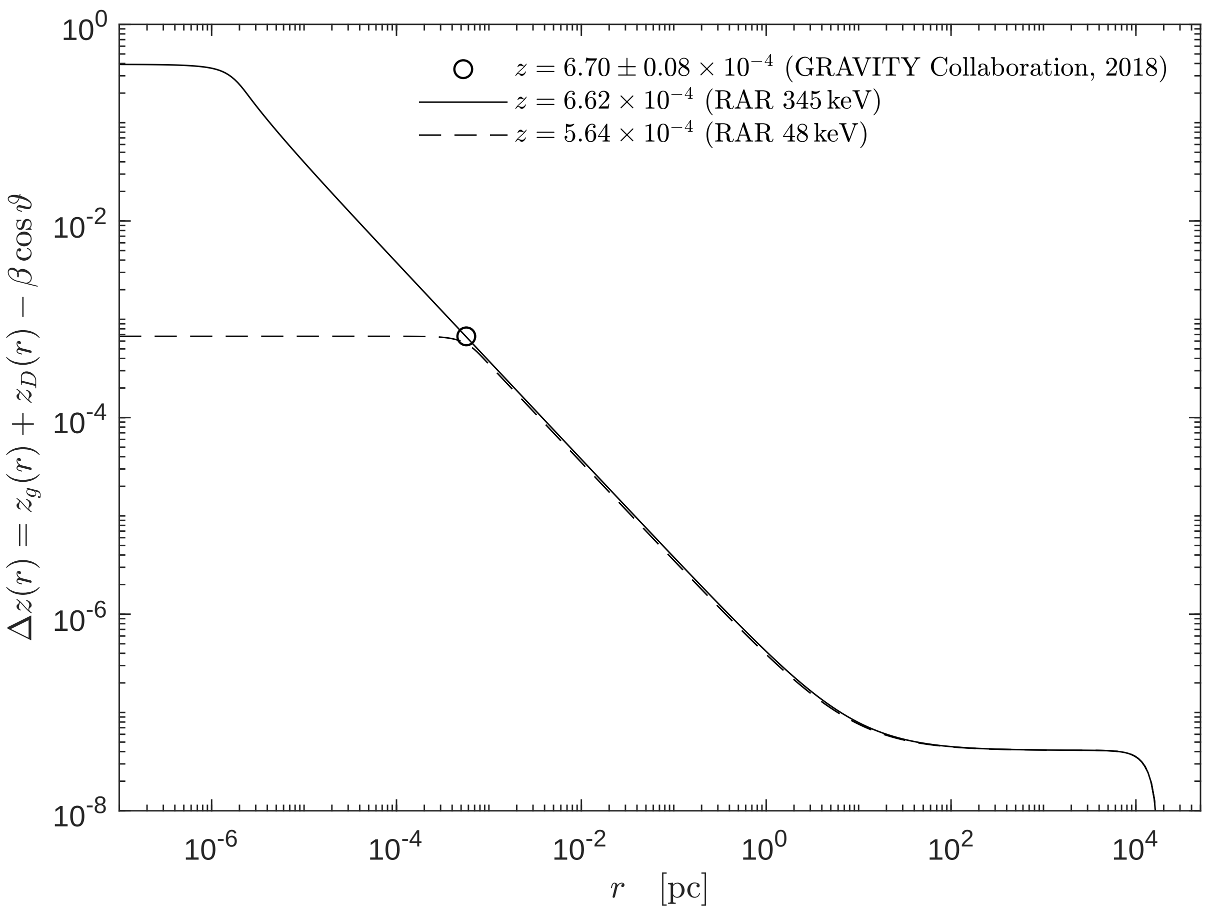

FIG. 1. Comparison of the inferred redshift correction at the S2 pericenter (circle) and the predicted curves of the RAR solutions as obtained in Argüelles *et al.* [2].

show that mass distributions of the RAR model are able to reproduce the relativistic redshift correction. However, note that for the solution corresponding to the minimal DM particle mass ($mc^2 = 48\,\mathrm{keV}$) the redshift prediction shows a little discrepancy with respect to the reference value due to the close proximity of the S2 pericenter to the *semi-surface*[2] of the quantum core.

[1] Gravity Collaboration, R. Abuter, A. Amorim, N. Anugu, M. Bauböck, M. Benisty, J. P. Berger, N. Blind, H. Bonnet, W. Brandner, A. Buron, C. Collin, F. Chapron, Y. Clénet, V. Coudé Du Foresto, P. T. de Zeeuw, C. Deen, F. Delplancke-Ströbele, R. Dembet, J. Dexter, G. Duvert, A. Eckart, F. Eisenhauer, G. Finger, N. M. Förster Schreiber, P. Fédou, P. Garcia, R. Garcia Lopez, F. Gao, E. Gendron, R. Genzel, S. Gillessen, P. Gordo, M. Habibi, X. Haubois, M. Haug, F. Haußmann, T. Henning, S. Hippler, M. Horrobin, Z. Hubert, N. Hubin, A. Jimenez Rosales, L. Jochum, K. Jocou, A. Kaufer, S. Kellner, S. Kendrew, P. Kervella, Y. Kok, M. Kulas, S. Lacour, V. Lapeyrère, B. Lazareff, J.-B. Le Bouquin, P. Léna, M. Lippa, R. Lenzen, A. Mérand,

[2] Is ok to call it a surface?

E. Müler, U. Neumann, T. Ott, L. Palanca, T. Paumard, L. Pasquini, K. Perraut, G. Perrin, O. Pfuhl, P. M. Plewa, S. Rabien, A. Ramírez, J. Ramos, C. Rau, G. Rodríguez-Coira, R.-R. Rohloff, G. Rousset, J. Sanchez-Bermudez, S. Scheithauer, M. Schöller, N. Schuler, J. Spyromilio, O. Straub, C. Straubmeier, E. Sturm, L. J. Tacconi, K. R. W. Tristram, F. Vincent, S. von Fellenberg, I. Wank, I. Waisberg, F. Widmann, E. Wieprecht, M. Wiest, E. Wiezorrek, J. Woillez, S. Yazici, D. Ziegler, and G. Zins, A&A **615**, L15 (2018), arXiv:1807.09409.

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[3] S. Zucker, T. Alexander, S. Gillessen, F. Eisenhauer, and R. Genzel, ApJ **639**, L21 (2006), astro-ph/0509105.