Permutation

K objects out of N $= \boxed{\dfrac{N!}{(N-k)!}}$

Combination

K objects out of N $= \boxed{\dfrac{N!}{k!(N-k)!}}$

Bernoulli's Principle

Probability $\boxed{A}$ happens x times out of N $= \boxed{\left(\dfrac{N!}{(N-x!)x!}\right) P(A)^x P(A^c)^{1-x}}$

Bayes theorem

$$P(A|B) = \frac{P(B|A)\,P(A)}{P(B|A)P(A) + P(B|A^c)P(A^c)}$$

Expectation

Continuous	* Discrete *
$E(x) = \int x f_x(x)$	$E(x) = \sum_{j=1}^{J} x_j f_x(x_j)$

Properties of Expectation

$\boxed{1.}$ $E(a) = a$

$\boxed{2.}$ $E(Y) = a + b\,E(x)$

$\boxed{3.}$ $E(Y) = E(x_1) + E(x_2)$

$\boxed{4}$ $E(xY) = E(x) * E(Y)$ if independent

Variance

Discrete:

$$Var(x) = \frac{1}{n}\sum_{i=1}^{n}\left(X_i - \frac{1}{n}\sum_{i=1}^{n} X_i\right)^2$$

Law of Large Numbers

$\lim_{n\to\infty} P(|\hat{\theta} - \theta_0| > \varepsilon) = 0$

- convergence in Probability

Properties of Variance

$\boxed{1.}$ $Var(a) = 0$

$\boxed{2.}$ $Var(Y) = b^2\,Var(x)$ | $Y = a + bX$

$\boxed{3.}$ $Var(Y) = Var(x_1) + Var(x_2) + \dots$

↳ $Y = x_1 + x_2 + x_3$ | Assuming Independence

$\boxed{4}$ $Var(x) = E[x^2] - (E[x])^2$

* Central Limit theorem *

$$\lim_{n\to\infty} P\left[\frac{\sqrt{n}\,(\bar{X}_n - \mu)}{\sigma} \le x\right] = \Phi(x)$$

↳ expression converges in Distribution

$\bar{X}_n$ = sample average

μ = Population mean

n = # of samples

Standard Error Estimator

$$\hat{SE}(\bar{x}) = \frac{\hat{\sigma}}{\sqrt{n}}$$

↳ $\hat{Var} = \hat{\sigma}_x^2 = \frac{1}{n}\sum_{i=1}^{n}(X_i - \bar{X})^2$

Deriving more Familiar CLT Term

$$P\left(\frac{\bar{X}}{\sqrt{n}}\right) < x\,) \to N(\mu, \sigma)$$

$\Downarrow$ $\boxed{\hat{\sigma}(\bar{x}) = \dfrac{\hat{\sigma}_x}{n}}$

$$P\left(\frac{\bar{x} - \mu}{\sqrt{n}/(\hat{\sigma}_x/n)}\right) \to N(0,1)$$

Sample Average Estimator

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} X_i$$

t-statistic

$$Z_n = \frac{\bar{x} - \mu}{\hat{SE}(\bar{x})}$$

- if $P_0 \in CI_{0.05}$ $\quad -1.96 \le Z_n \le 1.96$

Null Hypothesis: $\boxed{\mu = E[Y]}$

- if $|t| > 1.96$, Reject Null Hypothesis
- if $|t| < 1.96$, Dont reject Null

2 - sample test stat

$$\boxed{t = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{\hat{\sigma}_1}{n_1} + \frac{\hat{\sigma}_2}{n_2}}}}$$

Confidence Intervals

- corresponds to margin of error in results

$$\boxed{CI_{\alpha}^{\mu_x} = \left[\bar{x} - C_\alpha * \hat{s.e}(\bar{x}),\ \bar{x} + C_\alpha \hat{s.e}(\bar{x})\right]}$$

↳ C_α is Normal Distribution density Numbers

Covariance

$$cov(X,Y) = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})(Y_i - \bar{Y})$$

↳ if $Cov(x,Y) = 0$, x & Y independent

Law of Iterated Expectations

$$E[Y] = E[E[Y|x]]$$