

## Simple Assumptions of Finance

- Investors prefer more to less
- Money Paid in the future is worth less than same amount today
- Investors are risk-averse
- Financial Markets are competitive

### Discount Rate ( $r$ )

The discount rate ( $r$ ) is the number such that you're indifferent between

$$\$ \text{ today} \quad \square \quad \$ (1+r) \text{ next period}$$

### Present Value

$$PV(CF_T) = \frac{CF_T}{(1+r)^t}$$

### Net Present Value (NPV)

To value a stream of cash flows  $\rightarrow$  sum overall Present Values

$$NPV = \sum \frac{CF_t}{(1+r)^t}$$

### Properties of NPV

#### 1. NPV scales under Multiplication

$$\hookrightarrow NPV(a \cdot CF_1, \dots, a \cdot CF_T) = a * NPV(\dots)$$

#### 2. NPV is additive

$$\hookrightarrow NPV(x_T + Y_T) = NPV(x_T) + NPV(Y_T)$$

#### Conditions to use NPV

##### Cash flows are Known

$\hookrightarrow$  (in amount & time)

##### Discount rate is known & constant

when  $r$  uncertain or unknown

Use average rate of return for similar asset

$$\hookrightarrow R_f + \text{Risk premium}$$

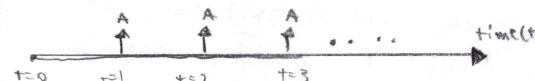
### Future Value

$$FV_T = CF_0 * (1+r)^T$$

## 15.401 Exam #1

### Present value of perpetuity

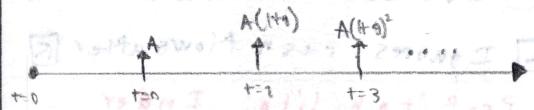
- Value of same cashflow, every period, forever.



$$PV(\text{Perpetuity}) = \frac{A}{r}$$

### Perpetuity w/ constant growth

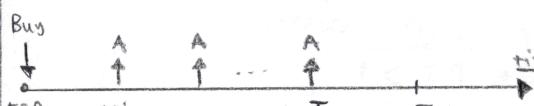
- value of a sequence of cash flows that grow at constant rate.



$$PV(\text{Perpetuity w/growth}) = \frac{A}{r-g}$$

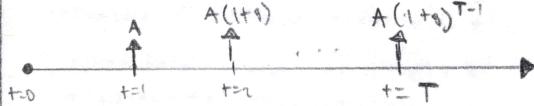
### Present value of Annuity

- A constant cash flow for T periods



$$PV(\text{annuity}) = \frac{A}{r} \left( 1 - \frac{1}{(1+r)^T} \right)$$

### Annuity w/ growth



$$PV(\text{w/ Growth}) = \frac{A}{r-g} \left( 1 - \frac{(1+g)^T}{(1+r)^T} \right)$$

### Compounding

- Interest may be charged more frequently than annually

#### Interest Rates quoted as

##### Annual Percentage Rate (APR)

$$APR = \left( \frac{\text{per Period Interest}}{\text{interest rate}} \right) * \left( \frac{\# \text{ of Periods}}{\text{in year}} \right)$$

### APR vs. EAR

- For per-period rate ( $r$ )

$$\hookrightarrow N = \# \text{ of periods per year}$$

$$APR = N * r$$

$$(1+EAR) = (1+r)^N$$

$$\hookrightarrow EAR = \text{Effective annual rate of Annual Percentage Yield (APY)}$$

## \* Conversion Formulas

$$EAR = \left( 1 + \frac{APR}{N} \right)^N - 1$$

$$APR = N \cdot \left[ \left( 1 + EAR \right)^{1/N} - 1 \right]$$

### Compounding for 1 year

$$e^{r_{\text{cont}}} \Leftrightarrow 1 + EAR \Rightarrow \left( 1 + r_{\text{monthly}} \right)^{12}$$

$$\left( 1 + r_{\text{semi}} \right)^2 \Leftrightarrow \left( 1 + \frac{APR}{365} \right)^{365}$$

$$e^{APR} \Leftrightarrow 1 + EAR \Rightarrow \left( 1 + \frac{APR}{12} \right)^{12}$$

### Real vs Nominal Returns

$$r_{\text{real}} = \frac{1 + r_{\text{nominal}}}{1 + i} - 1$$

$$\hookrightarrow (\text{For small}) \quad r_{\text{real}} = r_{\text{nominal}} - i$$

### Inflation & NPV

- Discount Nominal Cash Flow by Nominal rates

- Discount Real Cash Flow by Real rates

### Capital Budgeting: NPV

\* Companies should only take projects w/ positive NPV

① Use cash flows

② Use After-Tax Cash Flows

③ Use cash flows attributable to project

↳ i. Ignore sunk costs!

↳ ii. Include investment in working capital & capital expenditure

↳ iii. Include opportunity cost for existing facilities

## Cash Flow Estimation

↳ Must undo effects of accounting

### 1. Cash Flows

$$CF = \text{Operating Revenues} - \text{Non-capital costs} - (\text{Capital Expenditure}) - (\text{Income Taxes})$$

↳ Non-capital costs: COGS + OPEX

### 2. Operating Profit

$$\text{Operating Profit} = \text{Operating Rev} - \text{Non-capital cost} - \text{Dep}$$

### 3. Income & Taxes

$$\text{Income} - \text{Taxes} = T(\text{Operating Profit})$$

### 4. Working Capital

- a. Inventory
- b. Accounts receivable
- c. Accounts Payable

$$\text{Working Capital} = \text{Inv} + \text{AR} - \text{AP}$$

## \* PUTTING IT ALL TOGETHER \*

$$(1-T)(\text{Operating Profit}) - \text{CAPEX}$$

$$CF = +\text{Depreciation} - \text{change in working capital}$$

### Internal Rate of Return

↳ in words, IRR is the discount rate that makes project's NPV = 0

Accept Project:

### 1. Project IRR > Fixed IRR

#### Shortcomings of IRR

### 1. IRR sometimes doesn't exist

### 2. Multiple IRR's can exist

### 3. IRR ignores Project Magnitude

↳ IRR ignores difference in time patterns

### Pay back Period

• Minimum Value of periods such that cash flows of project become positive accept Project;

• If payback period less than some threshold value

### shortcoming of Payback Period

1. Ignores Time Value of Money

2. Ignores cash flows after

### Profitability Index

• Ratio of future NPV of Project to initial cost

$$PI = \frac{NPV}{I_0}$$

Accept:

if  $PI > 1$

+ ignores project size tho

### Stock Markets

#### 1. Common Stock

↳ ownership positions in companies

↳ rights to cash dividend, stock dividend, share repurchase

#### Legal characteristics:

1. Residual claimant

2. Limited liability

3. Voting rights

### Discounted Dividend Model

↳ Price of Stock

$$P_0 = \sum_{t=1}^{\infty} \frac{D_t}{(1+r_t)^t}$$

$P_t$  = price of stock at time  $t$   
 $D_t$  = expected cash dividend  
 $r_t$  = discount rate

### Serial Case: Gordon Growth Model

• Dividends expected to grow @ rate  $g$

$$P_0 = \frac{D_1}{r-g}$$

implied cost of capital:

$$\text{cost of capital} = \left( \frac{D_1}{P_0} \right) + g$$

## Forecasting Dividends: Key Variables

### 1. Earnings or EPS

↳ Total Profit or  $\frac{\text{Total Profit}}{\# \text{ of shares}}$

### 2. Payout Ratio

$$\frac{\text{Dividends}}{\text{Earnings}} = \frac{EPS}{EPS}$$

### 3. Retained Earnings

$$RE = \text{Earnings} - \text{Dividends}$$

### 4. Plowback Ratio

$$\frac{\text{retained Earnings}}{\text{Earnings}}$$

### 5. Book Value

↳ cumulative RE

### 6. Return on Equity

$$\frac{\text{Total Earnings}}{\text{Book Value}}$$

### Mini-Case: World Bakery

#### Data

- Company gets 18% ROE
- Book Value = \$150 mil
- Expands at 12% growth rate
- cost of capital = 18%

### SEE Packet

#### But

$$\text{Dividends} = p * \text{ROE} * \text{BV}$$

\* b/c ~~ROE~~ & BV are constant

• Dividends & Book Value must grow at same rate

### Constant Growth Formula

w/ constant ROE & constant  $g$

$$EPS_1 = \text{ROE} * \text{BV}_0$$

$$RE_1 = g * \text{BV}_0$$

$$\text{Dividend per Share} = D_1 = (\text{ROE} - g) * \text{BV}_0$$

$$\text{Stock Price} = P_0 = \frac{(\text{ROE} - g) * \text{BV}_0}{r - g}$$

## Growth Opportunities

- Investment opportunities w/  $(ROE > r)$

### Indicators:

- Stock w/ EPS growing slower than cost of capital
- Stock w/ BPS growing slower than cost of capital

### Two Components of Stock Price

- Stock Price has 2 components

- Present value of earnings under No-Growth Policy

- Present Value of Growth (PVGO)

$$P_0 = \frac{EPS_1}{r} + PVGO$$

### Cash Flow of a Bond

- Cash Flow Depends on

- Maturity of Bond

- Principal (Full value of Bond to be paid)

- Coupons

### Discount Bonds

- Discount bond w/ maturity date  $\square$  pays \$1 at time  $t$  only

### Spot Interest Rates

- Prices of Discount Bonds offer information about spot rates of different horizons

$$B_+ = \frac{1.00}{(1+r_+)^t}$$

$\hookrightarrow B_+$  → price of Bond Maturing at time  $\square$

### Coupon Bonds

- Pays stream of Regular Payments  $\square$  Principal at Maturity

→ Price by treating as a portfolio of Discount Bonds

## Can Price Coupon bond Knowing

- Discount Bond Prices

- Spot rates of all relevant periods

### Yield to Maturity (YTM)

- YTM ( $y$ ) → rate that replace all spot rates & result in same bond Price

### Time Value of Money Across Horizons

- Markets implied time value of money described by Spot-Rates

### Term Structure of Interest Rate

- Spot interest rate  $r_t$  is annualized interest rate for horizon of  $t$  periods

- A set of spot rates for different Maturities is called a term structure  
(Also Yield Curve)

### Expectations Hypothesis

- States Long-term interest rates are determined fully by expectations of one period until maturity

$$(1+r_{+,2-yr})^2 = (1+r_{+,1-yr})(1+E[r_{+1yr}])$$

so for  $\square$  maturity

$\square$  date

$$(1+r_{+,m})^m = (1+r_{+1})(1+E[r_{+1}]) \cdots (1+E[r_{+m}])$$

$\hookrightarrow$  Doesn't hold in data

### Liquidity Preference Hypothesis

- Same as above but add a Risk Premium Term

- Investors view long-term bonds as riskier

$$(1+r_{+,2})^2 = (1+r_{+1})(1+r_{+1}) + \text{Risk Premium}$$

### Implications:

- Term structure reflects Future one period Rates

$\square$

Risk premium demanded by investors

### Interest Rate Risk

- Change in interest rate could effect real value of your payment in future

### \* Duration \*

↳ Measure of interest-rate risk exposure

- consider a bond w/ YTM  $y$  & cash flows from coupons & principal

Duration → Weighted Average time to Maturity

$$D = \frac{1}{B} \sum_{t=1}^T \frac{CF_t}{(1+y)^t} * t$$

$B$  → Bond Price

### Modified Duration

- Relative Price change w/ respect to a unit change in yield

$$MD = \frac{D}{1+y}$$

### Duration of Perpetuity

$$\text{Duration of Perpetuity} = \frac{1}{(1+y)/y} = \frac{1}{y}$$

Using MD to calculate Bond Price Change

$$MD \times (-\Delta y) = \frac{\Delta B}{B}$$

Duration Matching for a firm	Book Value vs Market Value	Weighted Average Cost of Capital (WACC)
<ul style="list-style-type: none"> <li>Duration Matching makes duration of assets &amp; liabilities equal</li> </ul> $D_{Assets} = D_{Liab}$ <p>sensitivity to interest rate changes are as follows</p> $\Delta P = -\frac{D_{Liab}}{1+y} P_{(A)} + \frac{D_{Liab}}{1+y} P_{(M)} = 0$	<p>Book Value → what accountant says asset is worth</p> <p>Market Value → what you could sell asset for</p> <p>(MTY) <del>Expected or Last</del></p> <p>Book Leverage = <math>\frac{\text{Debt}}{(\text{Book Equity} + \text{Debt})}</math></p> <p>Market Leverage = <math>\frac{\text{Debt}}{\text{Market Equity} + \text{Debt}}</math></p> <p>Modigliani-Miller Theorem</p> <ul style="list-style-type: none"> <li>How should a firm finance itself ↳ M&amp;M says it's irrelevant!</li> </ul> <p><u>Assumptions:</u></p> <ol style="list-style-type: none"> <li>Markets are complete</li> <li>Markets are efficient</li> <li>No taxes</li> <li>Investment Decisions Fixed</li> </ol> <p><u>Intuition:</u></p> <ul style="list-style-type: none"> <li>Firm's cash flows don't matter on how it was financed</li> <li>Firm is a collection of projects, its value shouldn't matter on how it's financed</li> </ul>	$\lambda = \text{Leverage Ratio} = \frac{\text{Debt}}{\text{Debt} + \text{Equity}}$ $\lambda V = D$ (firm will make its value proportional to leverage ratio) <p>let <math>r_d</math> be interest rate on debt</p> <p>rate of return on firm (<math>r_v</math>)</p> $r_v = \lambda r_d + (1-\lambda) r_e$
<p>Dangers of Duration Mismatch</p> <ul style="list-style-type: none"> <li>when interest rates fall           <ul style="list-style-type: none"> <li>Value of Bonds increase</li> <li>But Liabilities increase more!</li> </ul> </li> </ul> <p>Example: <math>B_1 \cdot x + B_{30} \cdot z = \text{Portfolio Value}</math></p> $\frac{1}{B} ((B_1 \cdot x \cdot 1) + (B_{30} \cdot z \cdot 30)) = \text{Maturity of 15}$		<p>Weighted Average Cost of Capital (WACC)</p> $r_{WACC} = (1-\tau) r_d \left( \frac{D}{D+E} \right) + r_e \left( \frac{E}{D+E} \right)$ <p>↳ Takes into account tax advantages of debt</p> <p>Commuter <math>E = V - D</math></p> <p>* If Market Leverage Ratio is constant</p> $V_0 = \sum_{i=1}^T \frac{C_{F_i}}{(1+r_{WACC})^t}$
<p>Inflation Risk</p> <ul style="list-style-type: none"> <li>Real return of nominal payout will depend on inflation rate</li> </ul> <p>Default Risk</p> <ul style="list-style-type: none"> <li>Corporate bonds risk defaulting on their payments &amp; not making payments as promised</li> </ul> <p>How does a firm Finance Projects</p> <ul style="list-style-type: none"> <li>Internal Funds</li> <li>Equity</li> <li>Debt</li> </ul> <p>↳ which of these to use is the job of the CFO</p> <ul style="list-style-type: none"> <li>How firm is structured</li> <li>determines cash flow rights</li> </ul>	<p>M&amp;M Value of firm</p> $V = E + D$ <p><math>V</math> = value of firm <math>E</math> = Equity <math>D</math> = Debt</p> <p><u>Main result:</u> two identical companies financed differently, worth the same</p> <ul style="list-style-type: none"> <li>Doesn't strictly hold true but a good benchmark</li> </ul> <p>Debt Financing</p> <ul style="list-style-type: none"> <li>Key advantage of debt is tax-deductible</li> </ul> $CF_T = (1-\tau) EBITDA + T \left( \frac{\text{Debt}}{\text{Interest}} \right) - (\text{CAPEX} - \text{WC})$	<p>Illustration: In bond financing, firms can deduct interest payments as expenses, leading to lower tax bills.</p> <p>Excessive borrowing leads to excess fixed costs.</p> <p>Debt financing is a good benchmark for comparing different financing structures.</p>
<p>Important characteristics of Liabs</p> <ol style="list-style-type: none"> <li>Payoff structure</li> <li>Priority</li> <li>Maturity</li> <li>Voting rights</li> <li>Options</li> </ol>	<ul style="list-style-type: none"> <li>Define <math>C F^*</math> to be cash flows under no debt</li> </ul> $V = V^* + TD$	<p>as payment of cashflow + cash flow from changing a</p>

## Randomness of Asset Returns

Most Asset Returns are random	
↳ $P_t$ Price today	
↳ $D_t$ Dividend next Period	
↳ $P_{t+1}$ Price next Period	
Return	Expected Return
$\tilde{r}_t = \frac{\tilde{P}_{t+1} - P_t}{P_t}$	$E[\tilde{r}_t]$
Excess Return	Risk premium
$\tilde{r}_t - r_f$	$E[\tilde{r}_t] - r_f$

## Simple Metrics for Risk & Return

1. Mean  $\bar{r} = E[\tilde{r}]$
2. Variance  $\text{Var}(\tilde{r}) = E[(\tilde{r} - \bar{r})^2]$
3. Standard Deviation  $\sigma = \sqrt{\text{Var}(\tilde{r})}$

## More Metrics to Analyze

Median: 50<sup>th</sup> percentile

Covariance & Beta

### 1. Covariance

$$\sigma_{ij} = E[(\tilde{r}_i - \bar{r}_i)(\tilde{r}_j - \bar{r}_j)]$$

### 2. Correlation

$$\rho_{ij} = \text{Cov}(\tilde{r}_i, \tilde{r}_j) / \sigma_i \sigma_j$$

### 3. Beta

$$\beta_{ij} = \text{Cov}(\tilde{r}_i, \tilde{r}_j) / \sigma_j^2$$

## Portfolios & Weights

- A combination of different Assets

$$\text{Value}_P = \sum_{i=1}^n w_i P_i$$

Portfolio weights

• (+) long positions

• (-) short positions

$$w_i = \frac{N_i P_i}{N_1 P_1 + N_2 P_2 + \dots + N_n P_n}$$

$$w_1 + w_2 + \dots + w_n = 1$$

## Portfolios vs. Assets

- Portfolios provide Diversification

↳ allows balance of Risk / Reward

## 15.401 Part 2 Notes

### What is "best" portfolio?

- Highest reward
- lowest Risk

### Micro Econ: Indifference Curve

- All combinations of two items that deliver same level of utility

### In Finance: Risk-Reward Tradeoff

### Mean-Variance Investors

### Mean-Variance Utility

$$U(\tilde{r}) = E[\tilde{r}] - \frac{1}{2} \lambda * \text{Var}(\tilde{r})$$

→  $\lambda$  is Risk-Aversion

→  $\lambda$  = "certainty Rate of Return"

### Selecting Desirable Portfolio

1. Investors like more to less  $E[\tilde{r}]$
2. Investors dislike volatility  $\sigma$
3. Investors care about performance of overall Portfolio

### Portfolio Properties

$$\text{Expected Portfolio Return} : E[\tilde{r}_P] = w_1 E[\tilde{r}_1] + \dots + w_n E[\tilde{r}_n]$$

$$\text{Variance of Portfolio Returns} : \sigma_P^2 = \sum_{i=1}^n w_i \sigma_i^2 + \sum_{i=1}^n \sum_{j \neq i} w_i w_j \sigma_{ij}$$

### Why Diversification Works

- Returns contain independent sources of risk, when uncoupled, cancel each other out

### Takeaways:

① Diversification Benefit has limit

② Remaining Risk is Systematic

### (2) constrained optimization Problem

Highest Return

$$\text{Max } E[r_p] = \sum_{i=1}^n w_i r_i$$

s.t.

$$(1) \sum_{i=1}^n w_i = 1$$

Lowest Risk

$$\text{Min } \sigma_p^2 = \sum_{i=1}^n \sum_{j \neq i} w_i w_j \sigma_{ij}$$

s.t.

$$(1) \sum_{i=1}^n w_i = 1$$

## Effect of Short-selling

- \* shorting allows positions past 100% in one asset
- \* adding more equities generally improves mean-variance frontier

### Tools to Select "best" portfolio

#### Tools:

- Risky Asset or combination of risky assets
- Max Return
- Min Risk
- Risk-free asset

\* Best Portfolio is a combination of Risk-Free Asset & Tangent Portfolio

### Sharpe Ratio

$$\text{Sharpe Ratio} : \frac{E[r_p] - r_f}{\sigma_p}$$

• Tangency Portfolio has highest Possible Sharpe Ratio

• Only portfolios along the Capital Market Line can be someone's best choice

Tangent Portfolio is Market Portfolio

① Efficient CML portfolios are combos of market portfolio

### Risk-free Asset

② Expected return on asset must satisfy

$$E[r_i] = r_f + \beta_i (E[r_m] - r_f)$$

↳  $\beta_i = \text{Cov}(r_i, r_m) / \text{Var}(r_m)$

③ gives us Benchmark CAPM Pricing Model

## CAPM & Implementation

For any arbitrary portfolio of stocks

$$E[r_p] = r_f + \beta_p \times (E[r_m] - r_f)$$

Gives expression to calculate

1. required rate of return

2. Opportunity cost of capital

3. Risk-Adjusted Discount Rate

Decomposing Asset to get CAPM

Return of asset i

Regression:

$$r_{i,t+1} - r_{f,t+1} = d_i + \beta_i(r_{m,t+1} - r_f)$$

$\beta_i$  is regression coefficient

$$\rightarrow E[\epsilon_i] = 0$$

$$\rightarrow \text{Cov}[\epsilon_i, r_{m,t+1}] = 0$$

$\beta_{im}$  → measures i's exposure to systematic risk

$SD(\epsilon_i)$  → measures i's idiosyncratic risk

$d_i$  → measures i's return beyond its reward

for a risk-free asset  
investors respect reward for systematic risk

$$SD[r_{i,t+1}] = \beta \cdot SD[r_{m,t+1}] + SD[\epsilon_{i,t+1}]$$

Security Market Line can compare Asset Performance

CAPM & Cost of Capital

• CAPM based cost-of-capital for a firm's assets; need weighted average of Debt & Equity Betas

$$\beta_A = \frac{D}{D+E} \beta_D + \frac{E}{D+E} \beta_E$$

• When Debt Beta is zero

$$\beta_E = \left( \frac{\text{Assets}}{\text{Equity}} \right) \beta_A$$

\* Capital structure matters for equity Betas

## How to Implement CAPM in Practice

### Option 1

- use T-Bill rates for  $r_f$
- Estimate  $\beta$  in linear regression
- Decide which index to use as proxy for market return
  - use long time horizon  
daily frequency

### Option 2 Estimate Beta

$$\beta = \frac{\text{Cov}[r_i - r_f, r_m - r_f]}{\text{Var}(r_m - r_f)}$$

### Fama & French 3-factor Model

$$E(r_i) - r_f = \beta_{im}[E(r_m) - r_f] + \beta_{is}(s) + \beta_{in}(r_{md})$$

### Arbitrage Pricing Theory (APT)

• Extension to CAPM Model

$$E(r_i) - r_f = d_i + \beta_{im}[E(r_m) - r_f] + \sum \beta_k (r_{k,t+1})$$

1. identify the factors

2. Estimate factor loadings of assets

3. Estimate factor premiums

### Options

• Gives owner the right to buy or sell an underlying asset at a pre-specified price, on or before specific date

### Characterizing Options

#### 1. Option Type

Call → Right to Buy an asset for a given Price

Put → Right to Sell an asset for a given price

## Session 8: Option Pricing

### 2. Exercise Style

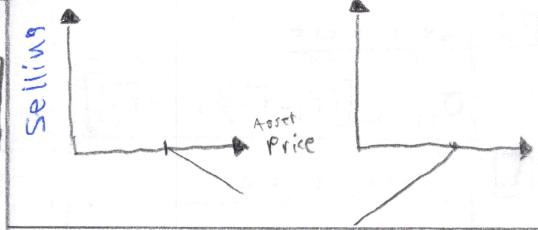
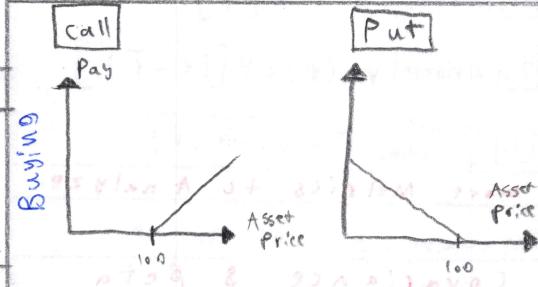
European → option can only be exercised on expiration date

American → can exercise at any time

### Cash flow of Call

$$CF_T(\text{Call}) = \max(S_T - K, 0)$$

### Duality of Option



### Net Payoff of Options

#### Net Payoff =

$$\max(S_T - K) - \text{Cost}(1+r)^T$$

Hence,

Break even is  $S_T$  such that

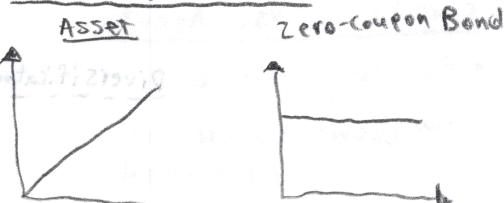
$$\text{Net payoff} = 0$$

CALL	$S < K$	$S = K$	$S > K$
Payoff	0	0	$S - K$
Profit	$-C(1+r)^T$	$-C(1+r)^T$	$S - K - C(1+r)^T$

PUT	$S < K$	$S = K$	$S > K$
Payoff	$K - S$	0	0
Profit	$K - S - C(1+r)^T$	$-C(1+r)^T$	$-C(1+r)^T$

### Basic Option strategies

#### 2 Building Blocks:



Put-Call Parity	Five Key Determinants of Option Price	Black-Scholes Formula																
<ul style="list-style-type: none"> <li>2 ways one can have a protective put</li> </ul> <p>Recipe 1: \$100 strike call + Bond w/ par \$100</p> <p>Payoff of Portfolio</p> <p>Asset Price at T</p>	<ol style="list-style-type: none"> <li>Current Asset Price</li> <li>Volatility of underlying asset</li> <li>Time to Maturity</li> <li>Spot interest rate</li> <li>Strike Price</li> </ol> <p>Example: European call on stock</p> <ul style="list-style-type: none"> <li>Current stock Price = \$50</li> <li>One period till Maturity</li> <li>Stock price either will go up or down (75 or 25)</li> <li>call price (\$0)</li> </ul> <table border="0"> <tr> <td><math>t=0</math></td> <td><math>t=1</math></td> </tr> <tr> <td>Asset Price</td> <td><math>S_0 = \\$50</math></td> </tr> <tr> <td>Bond price</td> <td><math>B_0 = 1</math></td> </tr> <tr> <td>Call Price</td> <td><math>C_0 = ?</math></td> </tr> <tr> <td></td> <td><math>S_u = \\$75</math></td> </tr> <tr> <td></td> <td><math>S_d = \\$25</math></td> </tr> <tr> <td></td> <td><math>B_1 = \begin{cases} \\$1.1 \\ \\$1.1 \end{cases}</math></td> </tr> <tr> <td></td> <td><math>C_1 = \begin{cases} \\$25 \\ \\$0 \end{cases}</math></td> </tr> </table>	$t=0$	$t=1$	Asset Price	$S_0 = \$50$	Bond price	$B_0 = 1$	Call Price	$C_0 = ?$		$S_u = \$75$		$S_d = \$25$		$B_1 = \begin{cases} \$1.1 \\ \$1.1 \end{cases}$		$C_1 = \begin{cases} \$25 \\ \$0 \end{cases}$	<p>Motivations</p> <ul style="list-style-type: none"> <li>Time isn't discrete</li> <li>Price can take more than 2 possible values</li> </ul> $C(S, K, T, \sigma) = S \cdot N(X) - K \cdot (1+r)^{-T} \cdot N(X - \sigma \sqrt{T})$ $X = \ln\left(\frac{S}{K(1+r)^{-T}}\right) + \frac{1}{2}\sigma^2 T$ <p><math>T</math> = option maturity in years</p> <p><math>r</math> = risk-free interest rate annualized</p> <p><math>\sigma</math> = volatility of annual returns of underlying asset</p> <p><math>N(x)</math> → Standard Normal CDF interpretation:</p> <ol style="list-style-type: none"> <li>→ Call</li> <li>→ amount invested in stock of replicating portfolio</li> <li>+ ④ → dollar amount borrowed in replicating portfolio</li> </ol> <p>Black-Scholes Takeaway</p> <ul style="list-style-type: none"> <li>Volatility determines price of a call</li> </ul>
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<p>Key Takeaway: Payoffs are Identical</p> <p>→ must be same price</p> $C + \frac{K}{(1+r)^T} = P + S$	<p>Key Tool: Portfolio Replication</p> <p>Want to Know</p> <table border="0"> <tr> <td>Asset Price</td> <td>Price of Option</td> </tr> <tr> <td>Bond Price</td> <td>Price of Bond &amp; Stock</td> </tr> </table> <p>We Know</p> <table border="0"> <tr> <td><math>aS_u + bB_u = C_u</math></td> </tr> <tr> <td><math>aS_d + bB_d = C_d</math></td> </tr> </table> <p>Price of Option = <math>aS_0 + bB_0</math></p> <p>For Multi-Period Problems Work Backwards</p> <p>↳ can plug result into put-call parity to get results</p>	Asset Price	Price of Option	Bond Price	Price of Bond & Stock	$aS_u + bB_u = C_u$	$aS_d + bB_d = C_d$	<p>First Term:</p> <p><math>S \cdot N(X)</math> is discounted expected value of stock at strike time</p> <p>Second Term:</p> <p><math>K \cdot (1+r)^{-T}</math> → Discounted Strike Price</p> <p><math>N(X - \sigma \sqrt{T})</math> → Probability of exercising option</p>										
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<p>Starting Assumptions:</p> <ol style="list-style-type: none"> <li>Prices follow "no-arbitrage"</li> <li>*Price follows a binomial process (two possible values for price at current date)</li> </ol>																		

## Black-Scholes: volatility increases Value

- ↑ w/ stock Price  $S$
- ↓ w/ Exercise "Strike" Price
- ↑ Increases w/ Maturity
- ↑ Increases w/ Volatility

Equity & Debt as Options

- 2 firms, identical Assets, different capital structures

Firm A	Firm B
\$60(A) { \$60(Bond)	\$60(A) { \$60(Bond)
\$60(Equity)	\$10(E)

value of Firm B's stock mirrors call on its assets

Asset value	Firm B stock value	V value of Call
\$40	\$0	\$0
\$60	\$10	\$10

Equity ↑  
Firm B → Asset value

Replicating payoffs with options

1. Start w/ RF bond or asset
2. add puts or calls to get desired payoff structure

Options in Everyday life

- In capital investment decisions, we face strategic options
  - option to wait
  - option to make follow-on investments
  - option to abandon projects

option to wait → 100 - 100 = 0  
100 (final value) - 100 (initial value)

## Options Example

- 100 stock buying ↑
- probability of profit ↓
- standard deviation ↑
- stock price ↓
- stock price ↑

X 2018 AS HS MU3907A7 - 93008X

823-077 State lottery

Probability of success ↓

Standard deviation ↑

Stock price ↓

Stock price ↑

Stock price ↓