# Basics of Economic Growth

Growth Rate: $\frac{y_{t+1}-y_t}{y_t} = g_t$

↳ given constant growth rate $y_t = y_0(1+g)^t$

• Rapid Econ growth is recent phenomenon

## The Neo classical Production Function

$$Y = F(K,L)$$ K=Capital, L=Labor

Properties:

1. constant returns to scale
2. Positive but Diminishing Marginal Returns

## Intensive Production & Capital

$y_t = \frac{Y_t}{L_t}$ | $k_t = \frac{K_t}{L_t}$

## Cobb Douglas & Facts of Model

$$F(K,L) = A(K)^{\alpha}(L)^{1-\alpha}$$

↳ A = Productivity

↳ α = Total Factor Productivity

$\frac{\partial F}{\partial K} = R$ (rental rate), $\frac{\partial F}{\partial L} = W$

## Equilibrium in Production Model

1. Households supply $\bar{R}$ & $\bar{K}$
2. Firms optimize, that is $(K^*, L^*)$ solves given $R^*$ & $w^*$
3. Markets Clear, $K^* = \bar{K}$, $R^* = \bar{R}$

• In this case profits are zero

• in cobb douglas
α - share given to Capital
1-α - share given to Labor

$Y^* = R^*\bar{K} + w^* L$
$= \alpha Y^* + (1-\alpha) Y^*$

## Solow Model

• Labor Grows at Rate $\boxed{n}$

Law of Capital Motion: $K_{t+1} = (1-\delta)K_t + sY_t$

# 14.05 Summary Sheet (Exam 1)

Consumption: $C_t = (1-s)Y_t$

Solving for law of motion for capital labor Ratio:

$K_{T+1} = (1-\delta)K_t + s f(k_t)$

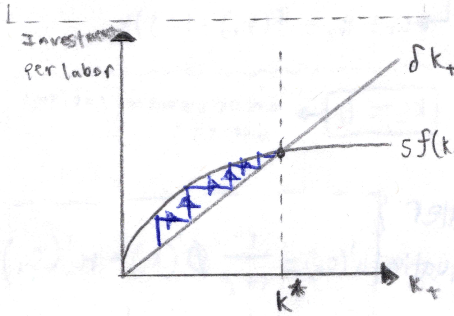
$\left(\frac{L_t}{L_{t+1}}\right)\frac{K_{t+1}}{L_t} = \left[(1-\delta)k_t + s f(k_t)\right] \cdot \frac{L_t}{(1+n)(L_t)}$

$$k_{t+1} = \frac{1}{1+n}\left[(1-\delta)k_t + s f(k_t)\right]$$

## Solving for Steady State

• when capital doesn't change year to year

$$sf(k^*) = \delta k^*$$




• Dynamics of Motion
↳ moves bounded by curves to new $k^*$

## Testable Implications of Solow Model

1. Conditional Convergence
↳ 2 countries with similar Economic Profiles, irregardless of starting, will converge to same steady state
2. Poverty traps
↳ countries under certain capital threshold have different savings rate

## Statics vs. Dynamics

• Statics → Final State

• Dynamics → motion in arriving to Final state

## Solow Model with Human capital

↳ Attributes that augment Labor to make it more productive

$$Y_t = A K_t^{\alpha} H_t^{\beta} L_t^{1-\alpha-\beta}$$

↳ $y_t = A k_t^{\alpha} h_t^{\beta}$

$k_{t+1} = s_k A k_t^{\alpha} h_t^{\beta} + (1-\delta)k_t$

$h_{t+1} = s_h A k_t^{\alpha} h_t^{\beta} + (1-\delta)h_t$

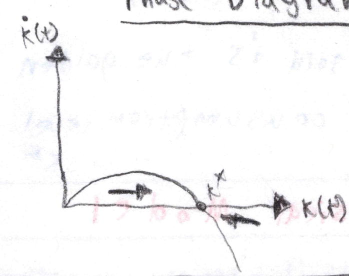
$k^* = \left(\frac{A s_k^{1-\beta} s_h^{\beta}}{\delta}\right)^{1/(1-\alpha-\beta)}$ $h^* = \left(\frac{A s_k^{\alpha} s_h^{1-\alpha}}{\delta}\right)^{1/(1-\alpha-\beta)}$

## Discrete → Continuous Time

Pop Growth: $\dot{L}(t) = nL(t)$

Law of Capital Motion: $\dot{K}(t) = sf(k(t)) - (\delta+n)k(t)$

Phase Diagram:




## Solow Model with Technological Progress

• we treat A(t) as labor augmenting Technological Progress

$$Y(t) = F(K(t), A(t)L(t))$$

↳ where $\frac{\dot{A}(t)}{A(t)} = g > 0$

Growth rate of Aggregate Output & Capital = $\boxed{n+g}$

Growth rate of per labor output & capital = $\boxed{n}$

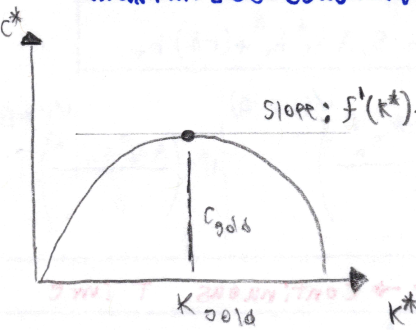
## Growth Accounting Formula

$$g_Y(t) = \alpha_K(t)\, g_K + \alpha_L\, g_L + TFP$$

## Consumption in Solow Model

$$c_t = (1-s)\, f(k_t)$$

↳ There should exist a level of savings that maximizes consumption




remember

$$c^* = f(k^*) - \underbrace{s\, f(k^*)}_{(\delta k^*)}$$

• Level $k_{gold}$ is the golden ~~steady~~ consumption level $k^*$

## Neo-Classical Model

A) output split between consumption and investment

$$c_t + i_t = f(k_t)$$

&

$$k_{t+1} = (1-\delta)k_t + i_t$$

combine

$$c_t + k_{t+1} = f(k_t) + (1-\delta)k_t$$

## Household utility from Consumption

$$V_0 = \sum_0^{\infty} \left(\frac{1}{1+\rho}\right)^t u(c_t)$$

↳ $\left(\frac{1}{1+\rho}\right)$ = Discount Factor

• Captures how much less people value one unit of consumption going forward

## Usual Utility Function:

$$u(c) = \frac{c^{1-\theta}-1}{1-\theta}$$

Marginal Utility: $u'(c) = c^{-\theta}$

- As $\theta \to 0$, utility is linear
- As $\theta \to 1$, $u(c) = \log(c)$
- as $\theta > 1$, Function becomes concave

* Parameter $\boxed{\frac{1}{\theta}}$ is Elasticity of Substitution

## Optimal consumption Problem

Ex. 2 Period case

1. Maximize $u(c_0) + \frac{1}{1+\rho} u(c_1)$

↳ $c_0 + k_1 = f(k_0) + (1-\delta)k_0$

↳ $c_1 + k_2 = f(k_1) + (1-\delta)k_0$

2. $\boxed{k_2 = 0}$ → unconsumed capital useless

Euler Equation: $$u'(c_0) = \frac{1}{1+\rho}\,\phi'(k_1)\cdot u'(c_1)$$

↓ taking $\phi(k) = f(k) + (1-\delta)k$

$$u'(c_0) = \frac{1}{1+\rho}\left(f'(k_1)+1-\delta\right) u'(c_1)$$

↓

$$\frac{c_1}{c_0} = \left(\frac{1}{1+\rho}\left(f'(k_1)+1-\delta\right)\right)^{1/\theta}$$

consumption grows if:

$$f'(k_1) - \delta > \rho$$

## Steady State Solution

$c_t = c^*$ | $k_t = k^*$

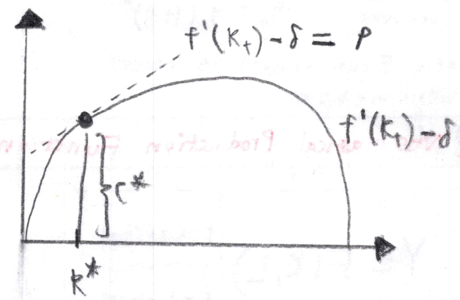
Note: this requires that $k_0 = k^*$

Recall:

$$\frac{c_{t+1}}{c_t} = \left(\frac{1}{1+\rho}\left(f'(k_{t+1})+1-\delta\right)\right)^{1/\theta}$$

## At Steady State:

$$\boxed{f'(k^*) - \delta = \rho}$$




## Understanding Steady State

• Remember all of this was to better understand the effects of savings Rate

$$s^* = \frac{i_t}{y_t} = \frac{\delta k^*}{f(k^*)}$$

↳ For each $k^*$ there is a corresponding $s^*$

$k^* < k^{gold}$ & $c^* < c^{gold}$

## Comparitive Statics of Steady State

ex. Cobb Douglas Function

$$f(k) = A k^{\alpha}$$

↓

$$\alpha A (k^*)^{\alpha-1} = \delta + \rho$$

| | | |
|---|---|---|
| $k^* = \left(\frac{\alpha A}{\delta+\rho}\right)^{\left(\frac{1}{1-\alpha}\right)}$ | $y^* = A(k^*)^{\alpha}$ | $s^* = \frac{\delta k^*}{y^*} = \frac{\delta\alpha}{\delta+\rho}$ |

## De-centralizing the Model

• $R_t = F_K(k_t,1)$
• $w_t = F_L(k_t,1)$
} we assume they're paid their marginal contribution to output

• Introduce Banks who promise $(1+r_t)$ return on funds

• Firms earn $(R_t+1-\delta)$ for renting capital

maximize $(R_t+1-\delta)k_t - (1+r_t)k_t$

so,

$$\boxed{(1+r_t) = (1+R_t-\delta)}$$

## Household Euler Equation

$$u'(c_t) = \frac{1}{1+\rho_t}(1+r_{t+1})\,u'(c_{t+1})$$

↓ Rewrite

$$\boxed{\frac{c_{t+1}}{c_t} = \left(\frac{1}{1+\rho}(1+r_{t+1})\right)^{1/\theta}}$$

$$r_{t+1} = R_{t+1} - \delta = \boxed{f'(k_{t+1}) - \delta}$$

* The fact that all expressions are equivalent is an example of a welfare theorem

Key Eqns

$r_{t+1} = R_{t+1} - \delta$ & $R_{t+1} = f'(k_{t+1})$

## Equilibrium w/ Distortions

↳ Government Applies linear Tax ($\tau$)

$$\boxed{r_t = (R_t - \delta)(1-\tau)}$$

Distortions cause lower $k^*, s^*, \& \ c^*$

## Dynamics of Neo-Classical Model

• What if now $K_0 < K^*$ or $K_0 > K^*$

LOG of Euler's Equation

$$\log c_{t+1} - \log c_t = \frac{1}{\theta}[\quad]$$

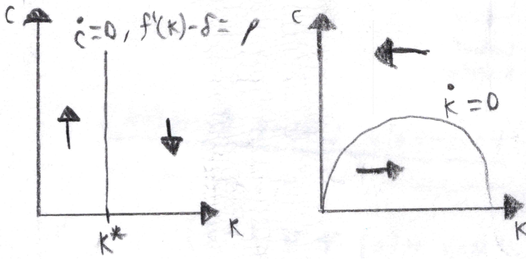
↓

$$\frac{\dot c(t)}{c(t)} = \frac{1}{\theta}\left(f'(k(t)) - \delta - \rho\right)$$

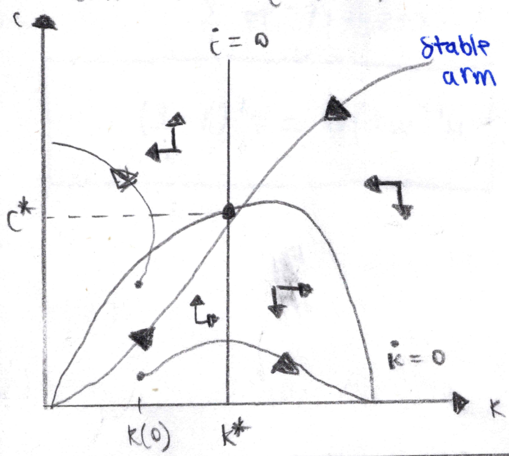
## Phase Diagrams & Consumption

$$\dot c = c\left(\frac{1}{\theta}\right)[f'(k) - \delta - \rho]$$

$$\dot k = f(k) - \delta k - c$$




* For Each $k(0)$ there is a $c(0)$ that will converge to $(k^*, c^*)$




## How $\theta$ affects speed to steady state

• Higher Substitution Elasticity
↑ $1/\theta$ → steeper stable arm

• Lower Substitution Elasticity
↓ $1/\theta$ → shallower stable arm

## Steady State Response

1. Increase in Patience

implies ↓ in $\rho$ (Discount Rate)

$\dot c$ loci shifts to the right

2. Increase in Productivity

↳ shifts $\dot c$ & $\dot k$ loci to the right & up

3. Foreign Aid

↳ Attributed to $\dot k$ loci Equation

↑ increases $\dot k$ loci ($c^*$ is higher)

4. Discretionary Tax

• can Decrease $\dot k$ loci if applied as lumpsum

• can decrease $\dot c$ loci if applied as lumpsum

## Endogenous Growth w/ research

Take Labor to be:

$$L = L_y + L_A \begin{cases} L_y - (\text{Production Labor}) \\ L_A - (\text{Research Labor}) \end{cases}$$

Assume Labor Augmenting Technology

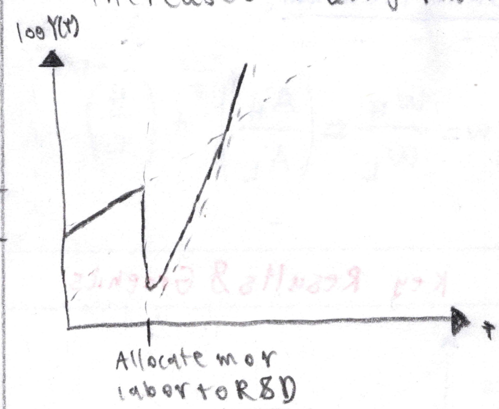
$$Y(t) = K(t)^{\alpha}(A(t)L_y)^{1-\alpha}$$

$$\dot A(t) = z \cdot L_A \cdot A(t)$$

• Model is identical to Solow except

$$\boxed{g = zL_a}$$

* Re-allocating Labor from Production to Research Decreases Short term output but increases in Long Run




• More realistic Model

$$A^j(t) = z * L_A^j * A(t)$$

## Intellectual Property Rights

1. Appropriability (+)
2. Escape Competition (−)
3. Availability (−)

IPR's effect on innovation

other Trends:

- Population Density

## Inequality and Biased Tech Change

- Suppose

$H(t)$ = College Educated Labor Force

$L(t)$ = Unskilled labor Force

$A_H(t)$ = Skill augmenting tech

$A_L(t)$ = Un-skilled Augmenting Tech

Production Function:

$$Y = \left((A_H H)^{(\sigma-1)/\sigma} + (A_L L)^{(\sigma-1)/\sigma}\right)^{\sigma/(\sigma-1)}$$

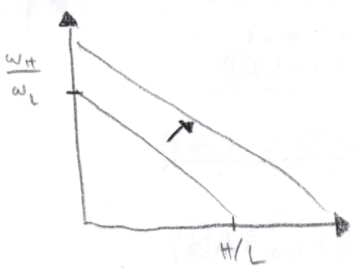
$\sigma$ = Elasticity of substitution

if $\sigma = 1$ (Cobb-Douglas case)

Skill Premium:

$$w = \frac{w_H}{w_L} = \left(\frac{A_H}{A_L}\right)^{\frac{\sigma-1}{\sigma}} * \left(\frac{H}{L}\right)^{-1/\sigma}$$

## Key Results & Graphics




- All else equal

$\uparrow H/L = \downarrow \frac{w_H}{w_L}$

But if $A_H \uparrow$

$$\frac{w_H'}{w_L'}\left(\frac{H}{L}\right) > \frac{w_H}{w_L}\left(\frac{H}{L}\right)$$

## Labor Supply in Long Run

$$\underbrace{L}_{\text{Labor supply}} = E + U$$

Household Labor Decision:

$$\max_{c,\ell} H(c) + v(1-\ell)$$

↳ where $c = w\ell$

↳ Take Derivative w/ respect to $\ell$

$$\boxed{H'(w\ell)w = v'(1-\ell)}$$

# 14.05 Exam #2 Cheat Sheet

## Business Cycles

- Output seems to fluctuate about a trend

$$\tilde{y}_t = \frac{Y_t - \bar{Y}_t}{\bar{Y}_t}$$ where $\bar{Y}_t$ is • smooth trend

- Fluctuations known as Business Cycles

Recall:

$$C + I + G + NX_t = Y$$

- Investment is very volatile
- Summary of Comovement

↳ Output, consumption, investment, employment, interest rate all seem to move together

* Do movements in consumption/investment drive these cycles

## Two-Period Consumption

- We use two period model with constant interest Rate
- $\boxed{r_0 = r_1 = r}$
- she receives income $(y_0, y_1)$
- She starts with assets $a_0$
- Household Problem:

$$\max_{c_0, c_1 \geq 0, a_1} \; u(c_0) + \frac{1}{1+\rho} u(c_1)$$
$$c_0 + a_1 = y_0$$
$$c_1 = (1+r)a_1 + y_1$$

Lifetime Budget constraint:

$$c_0 + \frac{c_1}{1+r} = y_0 + \frac{y_1}{1+r}$$

Euler Equation:

$$u'(c_0) = \frac{1+r}{1+\rho} u'(c_1)$$

To Solve

$$\frac{c_1}{c_0} = \left(\frac{1+r}{1+\rho}\right)^{1/\theta}$$

$\boxed{8}$

$$c_0 + \frac{c_1}{1+r} = y_0 + \frac{y_1}{1+r}$$

## Permanent income Hypothesis

- consider $\boxed{r = \rho = 0}$

Euler Simplification:

$$\frac{c_1}{c_0} = 1^{1/\theta} \xrightarrow{\text{so}} \boxed{c_0 = c_1 = \frac{y_0 + y_1}{2}}$$

* local spending fluctuations don't effect consumptions. you only consider lifetime income

## implications

- if $y_0 > y_p$
  - ↳ $c_0 < y_0$
  - ↳ $a_1$ is positive
- if $y_0 < y_p$
  - ↳ $c_0 > y_0$
  - ↳ $a_1$ is negative
- consumption should not reflect income shift

## PIH & response to Tax Cuts

- Household subject to lumpsum tax $T$
- Economic agent anticipates all long-term cuts & taxes and their consumption won't change

## Consumption / Savings

Recall: 2 Period Consumption-savings

$$\max_{c_0, c_1 \geq 0, a_1} \; u(c_0) + \frac{1}{1+\rho} u(c_1)$$
$$c_0 + a_1 = a_0(D_0 + Q(D_1, r)) + y_0$$
$$c_1 = (1+r)a_1 + y_1$$

↳ add $a_0$ to mix

↳ $D_0$ (wealth that doesn't mature in value)

↳ $D_1$ (wealth that appreciates subject to $r$)

## Response of Consumption to interest

Substitution Effect:

↳ with higher $r$ higher consumption growth $(c_1/c_0)$ is implied

Current income Effect:

- suppose $y_1 = 0$, $D_0 = 1$, $D_1 = 0$
  - ↳ she saves $a_1 > 0$
- As $r$ increases the household feels richer due to its greater returns on $a_1$
- So $c_0$ is increased

Wealth Effect:

- if $a_1 < 0$ and $\uparrow r$
- Becomes effectively poorer and $c_0$ decreases

## Special case: log utility

* Substitution & income effects cancel
- so overall wealth effect seems to dominate as when $r\uparrow$, $c_0 \downarrow$

## Consumption w/ Borrowing Constraint

- Banks might not want to lend
  - ↳ Household faces Borrowing constraint

$$\boxed{a_1 \geq -\bar{b}_1}$$ • $\bar{b}_1$ = borrow limit

$b_1 = 0$ (No borrow)

$b_1 > 0$ (some borrow)

- Assuming $\boxed{r = \rho = 0}$ & $D_0 = 1, D_1 = 0$

$$\max \; u(c_0) + u(c_1)$$
$$c_0 + a_1 = a_0 + y_0$$
$$c_1 = a_1 + y_1$$
$$a_1 \geq -\bar{b}_1$$

- $y_p = \frac{a_0 + y_0 + y_1}{2}$
- Household desires $\boxed{c_0 = y_p}$

**Case 1:** $y_p \leq y_0 + a_0 + \bar{b}_1$

↳ No Borrowing occurs

**Case 2:** $y_p > y_0 + a_0 + \bar{b}_1$

↳ household wants to choose $a_1 < -b_1$ but cannot

so $a_1 = -\bar{b}_1$

$$c_0 = y_0 + a_0 + \bar{b}_1$$

## Investment

**Components of Investment:**

Residential: New Houses, renovation

Non-Res: Factories, Machines

Rental Rate

$$R_1 = \underbrace{A_1 f'(k_1)}_{\text{marginal product of capital}}$$

8

$$\underbrace{1 + r_1}_{\text{return on asset}} = \underbrace{1 + R_1 - \delta}_{\text{Return to capital}}$$

- $1 + r_1$ = cost of funds (rate households require to invest)

## Theory of Investment

$$\boxed{1 + r_1 = 1 + A_1 f'(k_1) - \delta}$$

- Rewards to capital increase in productivity, $f$, and decrease in Depreciation

$$k_1 = k_0(1-\delta) + i_0$$

↳ Target level of capital ($k_1$) satisfies Theory of investment

**Long Run Implications**

1. $\uparrow A_1$, $\uparrow i_0$
2. $\uparrow \tau$, $\downarrow i_0$
3. $\uparrow r_1$, $\downarrow i_0$
4. $\uparrow k_0$, $\downarrow i_0$

## Q Theory of Investment

- A small firm operates capital $\boxed{K_t}$
- Has Earnings $D_t K_t$, which it'll distribute as Dividends

capital evolution

$$\boxed{K_{t+1} = K_t(1-\delta) + i_t}$$

↳ Goal: understand $i_t$

**2 costs of investment:**

1. Direct Costs
2. Adjustment Costs

$$\boxed{i_t + \frac{1}{2}\phi_t(i_t)^2}$$

↳ $\phi_t$ = severity of adjustment costs

**Firm's Net Profits:**

$$\Pi_t = D_t K_t - i_t - \frac{1}{2}\phi_t(i_t)^2$$

* Firm is trading current costs for future benefits

- But future Profits discounted at rate $r_d$

## Discount rate ($r_d$)

- Key determinant is riskless interest rate $r$

$$r^d = r + \text{appropriate risk premium}$$

**Firm Optimization**

$$\max \ \Pi_0 + \frac{\Pi_1}{(1+r_d)} + \frac{\Pi_2}{(1+r_d)^2} + \frac{\Pi_3}{(1+r_d)^3}$$

where $\Pi_t = D_t K_t - i_t - \frac{1}{2}\phi_t(i_t)^2$

$$K_t = k_t(1-\delta) + i_t$$

**Simplifications:**

1. Depreciation Rate is Small
2. Firm can invest at $t=0$, but not $t \geq 1$
3. Future earnings are constant ($D$)

## Simplified Optimization Problem

$$\max_{i_0} \ D_0 K_0 - i_0 - \frac{1}{2}\phi_0(i_0)^2 + \sum_{n=1}^{\infty} \frac{D}{(1+r_d)^n} K_1$$

↳ $K_1 = K_0 + i_0$

**Value of Future profits:**

$$\boxed{Q = \frac{D}{r^d}}$$

- Increase in $D$ b/c greater amount every perid
- Decrease w/ $r_d$ b/c he values future profit less

- $Q$ denotes Tobin's Q

⇓ substitution in Tobin's Q

1st order condition

$$\underbrace{1 + \phi_0 i_0}_{\text{Marginal cost}} = \underbrace{Q}_{\text{Marginal Benefit}}$$

$$\boxed{i_0 = \frac{Q-1}{\phi_0}}$$

* Optimal investment increases in $Q$

* $i_0$ positive if $Q > 1$
* $i_0$ negative if $Q < 1$

## Main Result Implications

- Adjustment cost is short-run consideration
- Only in short run can $Q$ deviate from 1

## Empirical Q theory evidence

1. Is aggregate $Q$ correlated w/ aggregate investment
2. Is the relation tight?

## Investment w/ Borrowing constraints

- Firms, like consumers, might be constrained

**Budget constraint:**

$$\Pi_0 + i_0 + \frac{1}{2}\phi_0(i_0)^2 = D_0 K_0$$

- if current earnings > current spending

↳ $\Pi_0$ is positive

- but if $\Pi_0 < 0$, company has a cash shortage

* The natural assumption is the company will borrow to pay investors

↳ Investors look to $Q$

## Firm's Problem w/ Borrowing

$$\max_{i_0} \; \pi_0 + QK_1$$

where, $\pi_0 + i_0 + \frac{1}{2}\phi_0 (i_0)^2 = D_0 K_0$

$K_1 = K_0 + i$

$\pi_0 \geq -b_1$

**1.** unconstrained case

$$i_0^{unconstrain} = \frac{Q-1}{\phi_0}$$

Firm spending:

$$i_0^{uncontrained} + \frac{1}{2}\phi_0 (i_0^{unconstrained})^2$$

But

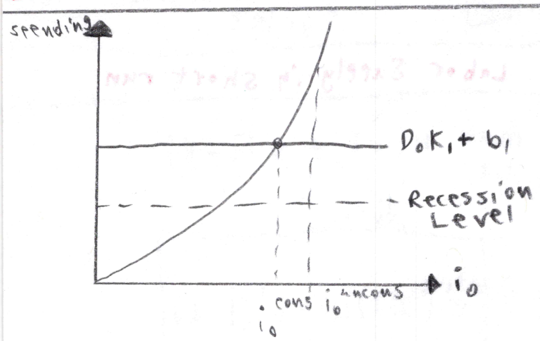
Firm spending $< D_0 K_0 + \bar{b}_1$

**2.** Constrained case

$$i_0^{cons} + \frac{1}{2}\phi_0 (i_0^{cons})^2 = D_0 K_0 + \bar{b}_1$$

→ Has single positive solution

## Investment w/ insufficient funds




$$i_0^{cons} < i_0^{uncons} = \frac{Q-1}{\phi_0}$$

* Financial crises lower borrowing threshold

## Summary

- Q is a sufficient static for investing
- If unconstrained Q dictates
- If constrained $D_0$ & $b_1$ dictates

## Gordon Growth Model & US stock market

- we now allow for earnings growth

$$D_t = D(1+g)^{t-1}$$

New Firm Maximization:

$$\max_{i_0} \; DK_0 - i_0 - \frac{1}{2}\phi_0 (i_0)^2 + \left(\frac{P}{1+r_d} + \frac{D(1+g)}{(1+r_d)^2}\right)(K_0+i)$$

→ $i_0 = \frac{Q-1}{\phi_0}$ Same result as before

But

Asset Price $= \boxed{\frac{D}{r^d - g}}$

(Gordon Growth Model)

* Provides benchmark for Price-Dividend Ratio

$$\frac{D}{P} = r^d - g$$

## Why stock prices are so high

- Higher steady state g could result from higher productivity
- lower $r^d$ could be due to market participation

## Bubbles

- leading cause is optimism
  - → optimism about high g or low $r^d$

Short Selling:

- Selling an asset you do not own
- This asset originally belongs to another institution
- Make Money if Price goes down
  - → Feasible & Easy w/ stock
  - → Infeasible w/ houses

- For houses D = rent
- Rent growing @ rate g

if $r^d = 7\%$ & $g = 2\%$

$$P = 20 \cdot D \xrightarrow{D=10} 200$$

## Price Benchmark

- 3 units up for sale
- 5 buyers each with $440
- Investors cannot short asset
- All agree $D = 10$

Result:

* 3 investors buy the asset @ 200

Different Beliefs

| investor | D |
|---|---|
| 1 | 12 |
| 2 | 11 |
| 3 | 10 |
| 4 | 9 |
| 5 | 8 |

Result:

* investor 1 buys 2 shares and investor 2 buys 1 share for $P = 220$

overvaluation → Price is 10% higher than average valuation. There exists bubble even if rational on average

intuition:

Marginal Investor: Price determined by her evaluation

Buying on Margin:

* investors pay only 20% of Price as Downpayment

Most optimistic buys all three

## Trading Volume & Speculation

- People more often buy asset to sell in Bubble Episodes
- This generates Trading Volume

w/ speculation

$$P_0 = \frac{D_1}{1+r^d} + \frac{P_1}{1+r^d}$$

↳ $P_1 = \frac{D_2}{1+r^d} + \frac{D_3}{(1+r^d)^2} + \ldots$

- Difference lies in investor's beliefs

## The Model

- at $t=0$, each has their own belief about $D_i$
- Now she also believes that others will come to share the same beliefs

* we apply the formula taking into account Dividends + price that will be obtained

$$\boxed{P_0 = P_0^{optimist} = \frac{D_1^{opt} + P_1^{opt}}{1+R}}$$

- Price is not the same as market holding value due to speculation
- Optimists are willing to pay more b/c they don't plan to hold asset

## Equilibrium Implications

- Higher Price level than buy-and-hold valuation (Greater Fool)
- Large trading volume

## Asset Price & Investment bubbles

- Absent Borrowing constraints

↳ $i_0 = \frac{Q-1}{\phi_0}$ | $Q = \frac{D_1}{(1+r^d)} + \frac{D_2}{(1+r^d)^2}$

- High Prices induce high Investment

↳ Price Bubbles Beget Investment Bubble

Investment overhang Mechanism

## Banks & Financial Crises

- Banks are Source of Investment
- Has power to raise or lower consumer borrowing
- influence $i_0$ & $C_0$

## Modeling Bank Problem

- 2 Period Model
- Each Bank starts with cash $\boxed{N}$
- Invests at Period 0 $\boxed{I}$

Bank Budget Constraint:

$$I = N + \rho I$$

$\rho$ = percentage of investment that can be borrowed

$$I = \frac{1}{1-\rho} N$$

| Assets | Liability |
|---|---|
| $I$ | $\rho I$ |
| | $N$ |

$\boxed{\frac{1}{1-\rho}}$ → Leverage Ratio

- However Banks are always constrained by some value $\boxed{\bar{\rho}}$

## Limited Pledgeability

- Imagine Bank Project Value comes in 2 parts

$$Q = \underbrace{\bar{\rho}}_{\text{less risky}} + \overbrace{Q - \bar{\rho}}^{\text{risky}}$$

$\boxed{\rho}$ → Loan-to-Value Ratio

$\boxed{1-\rho}$ → haircut

Model:

$$I = \frac{1}{1-\rho} N, \quad \rho \leq \bar{\rho}$$

Bank's Return: $\boxed{\left(1 + \frac{Q-1}{1-\rho}\right) N}$

- So bank wants to invest at max Pledgeability

## Net Worth Channel

- A banks Net worth $\boxed{N}$ determines Amount its able to invest

$$\boxed{I = \frac{1}{1-\rho} N} \quad \downarrow N, \downarrow I$$

## The Credit Crunch

- How Banks effect firms

Assumptions:

- Firm has no internal funds
- Firm can only borrow from banks
- Firms can't switch banks

## Implications

- A financial shock can lower a banks $\boxed{N}$, lowering Borrowing for firms
- How Problems on Wall St can effect Main St

## Leverage as Amplification Mechanism

- Initial Financial shock can cause much larger Damage

↳ $\boxed{\text{Leverage}}$

- All else variable, the debt the bank takes on is fixed
- Drop in a bank's net worth reduces its future investment by means of lower $\boxed{N}$ & $\boxed{\rho}$

## Short-Term Equilibrium in Goods Market

Market Clearing:

$$C_0(Y_0, r_1) + I_0(Y_0, r_1) + G_0 = A K_0^{\alpha} L_0^{1-\alpha}$$

1. Real Business Cycles
2. Keynesian Business Cycles

## Labor Supply in short run

$$\max_{c,\ell} \; H(w\ell) + v(1-\ell)$$

⇓

$$\boxed{H'(w\ell) w = v'(1-\ell)}$$

2 Forces at Play:

1. Substitution Effect

↳ wage increase makes you want to work to earn more

2. Income Effect

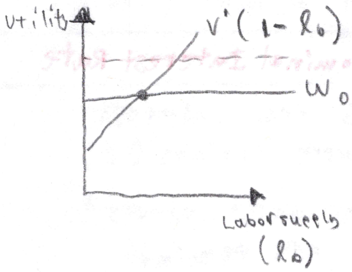
↳ wage increase makes you feel richer & want to work less

## Special case: Linear utility

$\boxed{H(C_0) = C_0}$ → weakens Income Effect

optimality condition:

$$w_0 = v'(1-\ell_0)$$




Substitution effect only present

## Short Run Labor Demand

1. Real Business Cycles → Labor Demand determined by Marginal productivity

2. Keynesian Business cycles

↳ Determined by Demand for Goods & Services

## RBC Model

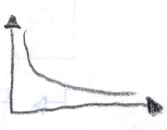
Firm Optimization:

$$\max_{L_0} A_0 K_0^\alpha L_0^{1-\alpha} - w_0 L_0$$

Dependent on production side

⇓

$$A_0(1-\alpha) K_0^\alpha L_0^{-\alpha} = w_0$$



RCB: Employment changes due to Productivity Shocks

Keynesian: Fluctuates due to Demand shocks

## Keynesian Model

• Firms must meet level of output demanded $\boxed{Y_0^d}$

• Now firms have no choice & must hire enough to produce $Y_0^d$

$$L_0^d = \left(\frac{Y_0^d}{A_0 K^\alpha}\right)^{1/(1-\alpha)}$$

## Real Business Cycles

• Measured TFP is volatile & procyclical

Main Result: These productivity shocks explain comovement of $C_0, I_0, Y_0, L_0, w_0$

**Simple RBC Model**

Production Function: $\boxed{Y_t = A_t K_t^\alpha L_t^{1-\alpha}}$

Investment Technology: $\boxed{K_{t+1} = (1-\delta)K_t + i_t}$

Equilibrium described by Euler relation: $\boxed{\frac{C_{t+1}}{C_t} = \left(\frac{1}{1+\rho}\left(A_{t+1} f'(k_{t+1}) + 1-\delta\right)\right)^{1/\theta}}$

w/ Resource constraint

$$C_t + K_{t+1} = A_t f(K_t) + (1-\delta) K_t$$

## Modeling Productivity Shock

Productivity Shock:

$$\log(A_t/\bar{A}) = \rho_A \log(A_{t-1}/\bar{A}) + \varepsilon_{A,t}$$

↳ $\bar{A}$ → steady state Productivity

↳ $\varepsilon_{A,t}$ → shocks uncorrelated to anything

↳ $\boxed{\rho_A}$ = captures persistence of shock over time

## Special case: Permanent Shock

• Equilibrium After Shock is known as Impulse Response

• Suppose $\boxed{\rho_a = 1}$

↳ This is equivalent to permanently changing productivity level $A_{old} \to A_{new}$

• So we can use same conclusions Derived before

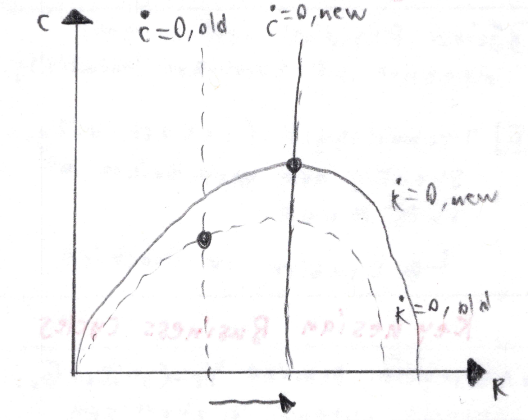
• $Y_0$ output will increase immediately

• To analyze $i_0, C_0$ we need Phase Diagram

$$\frac{\dot{c}(t)}{c(t)} = \frac{1}{\theta}\left(A f'(k(t)) - \delta - \rho\right)$$

$$\dot{k}(t) = A f(k(t)) - \delta k(t) - c(t)$$

• Response to Productivity increase




## Effect on consumption & investment

$$\dot{k}(t) = i(t) - \delta k(t)$$

• Before the shock $\dot{k}(t) = 0$

• afterwards $\dot{k}(t) > 0$, so $\boxed{i_0 \uparrow}$

Productivity shock increases investment in short run

**Consumption**

• consumption increases

## Effect on wages:

$$w_t = \frac{dY_t}{dL_t} = (1-\alpha) A^{new} K_t^\alpha (L^*)^{-\alpha}$$

• So at $t=0$ we have $K_0 = K^*_{old}$

so $\boxed{w_0^{new} > w_0^{old}}$

↳ Wages increase in the short run

## Effect on interest Rate:

Recall: $\boxed{r_t = R_t - \delta}$

$$R_t = \frac{dY_t}{dK_t} = (1-\alpha) A_t K_t^{\alpha-1}$$

• So greater Productivity increases Rental Rate on Capital and in turn increases interest Rate

## What about Employment

• We've so far assumed $L_0 = L^*$

• Higher Short Term Wage would lead to higher Employment

## Problems with RBC

1. Solow Residual isn't a good measure of Short-Run productivity
2. Technology & Productivity shocks are main driver of this Model
   ↳ Elusive in Practice

## Keynesian Business Cycles

• Aggregate Demand $Y_0 = C_0 + I_0 + G_0$ affects output in short run

• consider lone firm $\boxed{f}$

• Assume the firm is a price setter $\boxed{P_0^f}$

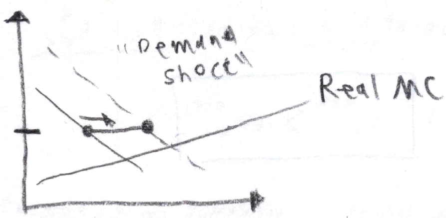
• Denote average firm price with $\boxed{P_0}$

↳ Firm's Real price = $\boxed{P_0^f / P_0}$

Key Assumption: Price $P_0^f$ is sticky over short horizons

(we assume firm can't always re-optimize its prices)

ex. Positive Demand shock




• firm locked in at short run to that price

• Real-price level is sub-optimal but still makes positive profit and will supply Quantity Demanded

Price Stickiness Summary:

• W/ sticky prices & Demand shock

1. Increases Output in Short Run
2. Eventually Generates Inflation

## Aggregate Demand & IS curve

• Suppose $\boxed{G_0 = T_0 = 0}$

Short Run:

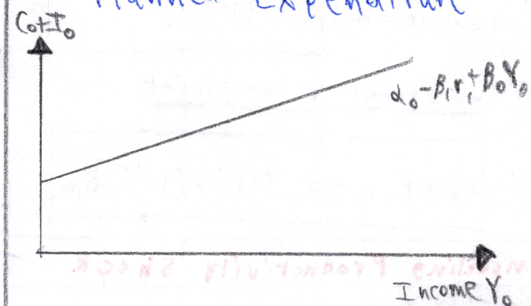
$$C_0(Y_0,r_1) = \alpha_0^c + \beta_0^c Y_0 - \beta_1^c r_1$$

$$I_0(Y_0,r_1) = \alpha_0^i + \beta_0^i Y_0 - \beta_1^i r_1$$

Spending Relation:

$$C_0(Y_0,r_1) + I_0(Y_0,r_1) = \alpha_0 + \beta_0 Y_0 - \beta_1 r_1$$

↳ Expression known as Planned Expenditure



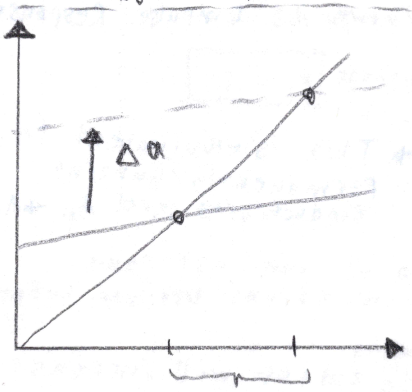

• In Equilibrium Planned Expenditure equals Actual Income

NOTE*: Income Appears on Both sides of Equation

$$\boxed{C_0(Y_0,r_1) + I_0(Y_0,r_1) = Y_0}$$

## Solving of Keynesian Cross

$$Y_0 = \frac{\alpha_0}{1-\beta_0} - \frac{\beta_1}{1-\beta_0} r_1$$




$$\frac{\Delta\alpha}{1-\beta_1} > \Delta a$$

• Though $C_0+I_0$ only increased by $\Delta a$, Demand increased more

• This is called Keynesian Multiplier

## Keynesian IS curve

$$Y_0 = \frac{\alpha_0}{1-\beta_0} - \frac{\beta_1}{1-\beta_0} r_1$$

• Higher Interest Decreases output

• Demand shock shifts curve right/left by Multiplier

## Real & Nominal Interest Rate

• $r_1$ denotes real interest rate between periods 0 & 1

↳ However most interest rates are nominal

Inflation:

• Let $P_0$ & $P_1$ denote Price levels

Inflation Rate

$$\boxed{\pi_1 = \frac{P_1}{P_0} - 1}$$

## Real & Nominal Interest Rate

$$(1+r_1) = \frac{(1+r_1^n)}{(1+\pi_1^e)}$$

or (for small interest Rates)

$$\boxed{r_1 = r_1^n - \pi_1^e}$$

↳ Fisher Equation

## Applications of IS-MP

• Liquidity trap when Real Interest Rate goes lower than $\boxed{0}$

• Any IS curve shift at liquidity trap will change output as Bank can't counter & chang interest Rate