3.032 Exam #1 Cheatsheet

Review: Forces

Magnitude of $\vec{R}$: $|\vec{R}| = \sqrt{(F_{1x}+F_{2x})^2 + (F_{1y}+F_{2y})^2}$

Direction: $\tan\theta = \frac{F_{1y}+F_{2y}}{F_{1x}+F_{2x}}$

External / Internal

R&F: External Forces; other bodies act on members

$F_{internal}$ = Force within member

Review: Moments

- Tendency of Force to cause Rotation about a point

$\bar{M}(\text{moment}) = |F| \cdot |d|$

Thumb out: (+) | Thumb in: (−)

Review: Equilibrium

$\sum F = 0$ $\quad$ $\sum M = 0$

Stress & Strain

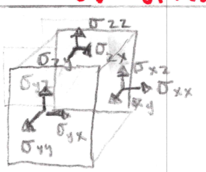
Normal Stress $(\sigma) = \frac{\text{Force} \perp \text{to Area}}{\text{Area}}$

Sign Convention: (+) Tension or Pulling; (−) Pushing or compression

Shear Stress $(\sigma_{xy}) = \frac{\text{Force} \parallel \text{to Area}}{\text{Area}}$

Sign Convention: (+) acts in Positive direction on positive face or Negative Direction on Negative Face

3D Stress-State



(3 Normal stresses)
(6 shear stresses)

$$\begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{xy} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{xz} & \sigma_{yz} & \sigma_{zz} \end{bmatrix}$$

* Matrix is symmetric

Normal & Shear strain

Normal Strain (ε) → amount of deformation δ per unit original length l_0

$\varepsilon = \frac{\delta}{l_0}$

Shear Strain (γ) → lengths of sides DONT CHANGE, but Rotate

$\tan(\gamma) = \frac{\delta}{l} \Rightarrow \gamma = \frac{\delta}{l}$

Hooke's law for Isotropic material

$\sigma = E\varepsilon$ — * Stress/strain related by Elastic Modulus, E

Joints & Supports

[1] Roller → can resist Force in y-direction; cant resist Moment

[2] Pin → can resist in x-direction & y-direction; cant resist moment

Shear Modulus

$\sigma_{xy} = G\gamma_{xy}$ — • Shear stress & strain related by G

Poisson's Ratio (ν)

Poisson's Ratio $(\nu_{yx}) = -\frac{\text{lateral strain } (x)}{\text{strain in Direction of Applied Uniaxial stress } (y)}$

- If Isotropic, All ν Equal
- No Equivalent for Shear Stress

$\varepsilon_x = \frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E} - \nu\frac{\sigma_z}{E}$

Hooke's Law for Isotropic solid

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{yz} \\ \gamma_{xz} \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} \frac{1}{E} & -\frac{\nu}{E} & -\frac{\nu}{E} & 0 & 0 & 0 \\ -\frac{\nu}{E} & \frac{1}{E} & -\frac{\nu}{E} & 0 & 0 & 0 \\ -\frac{\nu}{E} & -\frac{\nu}{E} & \frac{1}{E} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G} \end{bmatrix} \begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \sigma_{yz} \\ \sigma_{xz} \\ \sigma_{xy} \end{bmatrix}$$

- Elastic Modulus
- Shear Modulus
- Poisson's Ratio

Elastic Moduli — • 2 independent elastic constants for Isotropic

Relations between Elastic moduli

$G = \frac{E}{2(1+\nu)}$ $\quad$ $K = \frac{E}{3(1-2\nu)}$

Both necessitate $-1 < \nu < 0.5$

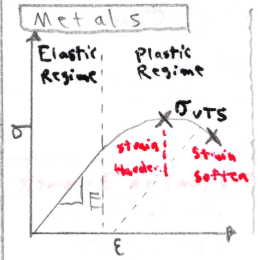
Elastic Strain Energy

- Total Elastic Strain Energy (Normal Stress): $U = \frac{1}{2}\sigma_x \varepsilon_x$
- Total Elastic Strain Energy: $U = \frac{1}{2}\sigma_{xy}\gamma_{xy}$

Total $U = U_{shear} + U_{Normal}$

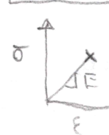
Stress-Strain Curves

Metals



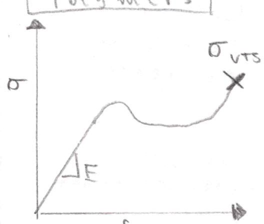
- Initial linear Elasticity
- 0.2% offset yield strength
- very Ductile

Ceramics



- Linear Elastic 'til failure
- Higher ↑ E than Metals

Polymers



- can drawn out
- As its stretched Polymers chains aligns strengthen allowing drawing out balancing necking

True Stress & strain

$\sigma_{True} = \sigma_{Eng}(1+\varepsilon_{Eng})$ $\quad$ $\varepsilon_{True} = \ln(1+\varepsilon_{Eng})$

Anisotropic Hooke's Law: New Notation

$\sigma_{xx} = \sigma_1$ $\quad$ $\sigma_{yz} = \sigma_4$

$\sigma_{yy} = 2$ $\quad$ $\sigma_{xz} = \sigma_5$

$\sigma_{zz} = 3$ $\quad$ $\sigma_{xy} = \sigma_6$

The Matrix:

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \\ \varepsilon_5 \\ \varepsilon_6 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & 0 & 0 & 0 \\ & S_{22} & S_{23} & 0 & 0 & 0 \\ & & S_{33} & 0 & 0 & 0 \\ & & & S_{44} & 0 & 0 \\ & & & & S_{55} & 0 \\ & & & & & S_{66} \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \\ \sigma_5 \\ \sigma_6 \end{bmatrix}$$

↳ 21 Independent Elastic constants

orthotropic:

- 9 independent Elastic Constants

Transversely

z-y Plane: $S_{12} = S_{13}$, $S_{22} = S_{33}$

x-z Plane: $S_{11} = S_{33}$, $S_{12} = S_{23}$

x-y Plane: $S_{11} = S_{22}$, $S_{13} = S_{23}$

↳ 5 Elastic constants

- $S_{66} = 2(S_{11} - S_{12})$

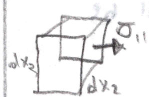
Composite Materials

Loaded Parallel: $E_{composite} = E_1V_1 + E_2V_2 \ldots$

(upper bound)

Loaded Perpendicular: $E_c = \frac{E_1 * E_2}{E_1V_2 + E_2V_1}$

Atomic Basis of Elastic Properties



dV = change in Internal Energy
dW = work done on Material
dQ = heat absorbed by material

dW: = Force * Displacement

$= \underbrace{\sigma_{11}(dx_2)(dx_3)}_{\text{Force}} * \underbrace{d\varepsilon_1\, dx_1}_{\text{displacement}}$

$= \boxed{\sigma_{11}\varepsilon_{11}V}$ — V = volume of solid

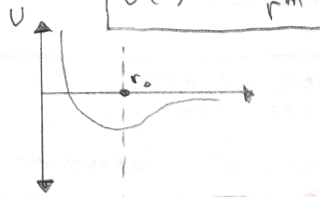
dQ $= \boxed{T\,dS}$ — T = Temp; dS = change in Entropy

$$\sigma = \underbrace{\frac{1}{V}\frac{dU}{d\varepsilon_1}}_{\text{Bond-Stretching (dominates crystalline)}} - \underbrace{\frac{T}{V}\frac{dS}{d\varepsilon_1}}_{\text{Entropy Term (dominates Amorphous)}}$$

crystalline + Amorphous Material ($T < T_g$)

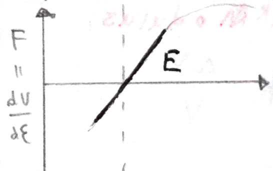
$\sigma = \frac{1}{\Omega}\frac{dU}{d\varepsilon}$ $\quad$ $E_{youngs} = \frac{1}{\Omega}\frac{d^2U}{d\varepsilon^2}$

where, $U(r) = -\frac{A}{r^m} + \frac{B}{r^n}$



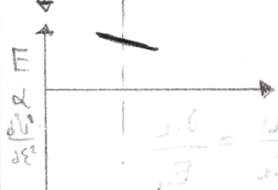
r_0 = Equilibrium spacing

$\varepsilon = \frac{r - r_0}{r_0}$



- F & r, for small displacement around r_0

(Proof & basis of Elasticity)



- E decreases as r increases

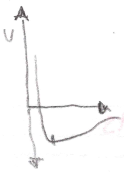
↳ As temp increase ↑r, ↓E

Young's Modulus of Crystalline

Given: $U(r) = -\frac{A}{r^m} + \frac{B}{r^n}$

$$\boxed{E = \frac{A\,m(n-m)}{r_0^{m+3}}}$$

Coefficient of Thermal Expansion



- At T = 0K, Equilibrium Spacing
- As ↑T, atoms oscillate in potential well

* b/c energy well is asymmetric we get $\boxed{\alpha}$

Thermal Strain

$$\varepsilon_{Thermal} = \alpha \Delta T$$

Thermal stress

* If constrained there'll be stress

$$\sigma_{thermal} = E\,\varepsilon_{Thermal}$$

Elastic Modulus: Rubber

- Entropy increases w/ decreased order

In Rubbers:

$K \gg E$ b/c Bond stretching >> Entropy

↳ Under Hydro-static stress

For Isotropic Materials:

$$\nu = \frac{1}{2} - \frac{1}{6}\frac{E}{K}$$

*why Rubber $\nu \approx 0.5$

Calculation of E:

Distance between cross-links: $\boxed{\bar{r} = \sqrt{\frac{2}{3}N}\,\lambda}$ N-steps between; λ = bond length

$$\boxed{E = 3 n_v k T}$$

$\Omega = N/\lambda^3$

$n_v = 1/\Omega$

Notes:

1. E increases w/ density of chains per volume
2. Increases w/ Temperature

Notes: Bulk Modulus

$$\sigma_{Hydrostatic} = K \frac{\Delta V}{V_0}$$

↳ $K = \frac{E}{3(1-2\nu)}$

Reciprocal Relation $\left\{ \frac{\nu_{21}}{E_2} = \frac{\nu_{12}}{E_1} \right.$

Cylindrical Pressure Vessel

radius [r] thickness [t]

length [L] $r \gg t$ • pressure [p]

1. Longitudinal Stress σ_ℓ

$\sigma_\ell(2\pi r t) = p(\pi r^2)$

$$\sigma_\ell = \frac{pr}{2t}$$

2. Hoop Stress

$\sigma_H(2\pi r t) = p(2Lr)$

$$\sigma_H = \frac{pr}{t}$$

3. Radial Stress

$$\boxed{\sigma_r = 0}$$

Spherical pressure vessel

radius [r] thickness [t]

pressure [p] $r \gg t$

1. Longitudinal Stress (σ_ℓ)

$\sigma_\ell(2\pi r t) = p(\pi r^2)$

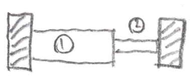
$$\sigma_\ell = \frac{pr}{2t}$$

2. Hoop Stress (σ_{Hoop})

$\sigma_{Hoop}(2\pi r t) = p(\pi r^2)$

$$\sigma_H = \frac{pr}{2t}$$

2 cylinder Thermal Expansion Example



- 2 cylindrical Bars
- $\Delta T = 500°C$
- $L_0 = 30$ cm

info:

$E_1 = 200$ GPa $E_2 = 70$ GPa

$\nu_1 = 0.28$ $\nu_2 = 0.33$

$L_1 = 18$ cm $L_2 = 12$ cm

$r_1 = 4$ cm $r_2 = 12$ cm

$\alpha_1 = 10 * 10^{-6}$ $\alpha_2 = 20 * 10^{-6}$

Questions:

1. What's σ in each Bar
2. What is ε in each bar
3. What is final length of each bar
4. Radius of each bar

SOLUTION

If unconstrained:

$\delta_{1T} = (10*10^{-6})(500°C)(0.18\,m) = \boxed{9*10^{-4}}$

$\delta_{2T} = (20*10^{-6})(500°C)(0.12\,m) = \boxed{1.2*10^{-3}}$

Total Displacement $\Rightarrow \boxed{2.1*10^{-3}}$

* recognize internal force in bar is the same

$\frac{FL_1}{A_1E_1} + \frac{FL_2}{A_2E_2} = 2.1*10^{-3} \Rightarrow$ Solve for F

$\sigma_1 = \frac{F}{A_1}$ $\sigma_2 = \frac{F}{A_2}$

$\varepsilon_1 = -\frac{\sigma_1}{E_1} + \alpha_1 \Delta T$

$\varepsilon_2 = -\frac{\sigma_2}{E_2} + \alpha_2 \Delta T$

$\Delta L_1 = \varepsilon_1 L_1$

$\Delta L_2 = \varepsilon_2 L_2$

Formula for Displacement

$$\delta = \frac{PL}{AE}$$

3.032 Exam #2 cheat sheet

Stress Transformations

$$\sigma_{x_1} = \frac{\sigma_x+\sigma_y}{2} + \frac{(\sigma_x-\sigma_y)}{2}\cos(2\theta) + \sigma_{xy}\sin(2\theta)$$

$$\sigma_{x_1y_1} = -\left(\frac{\sigma_x-\sigma_y}{2}\right)\sin(2\theta) + \sigma_{xy}\cos(2\theta)$$

$$\sigma_{y_1} = \left(\frac{\sigma_x+\sigma_y}{2}\right) - \left(\frac{\sigma_x-\sigma_y}{2}\right)\cos(2\theta) - \sigma_{xy}\sin(2\theta)$$

&

$$\boxed{\sigma_{x_1}+\sigma_{y_1} = \sigma_x+\sigma_y}$$

(sum of 2 $\perp$ Forces constant)

Principal Stresses

$$\sigma_1 = \frac{\sigma_x+\sigma_y}{2} + \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \sigma_{xy}^2}$$

$$\sigma_2 = \frac{\sigma_x+\sigma_y}{2} - \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \sigma_{xy}^2}$$

$$\tan(2\theta_p) = \frac{2\sigma_{xy}}{\sigma_x-\sigma_y} \qquad \sigma_{x_1y_1} = 0$$

Max Shear Stress

Max shear: $\sigma_{xy,max} = \frac{\sigma_1-\sigma_2}{2}$

Normal stress at max shear: $\frac{\sigma_x+\sigma_y}{2}$

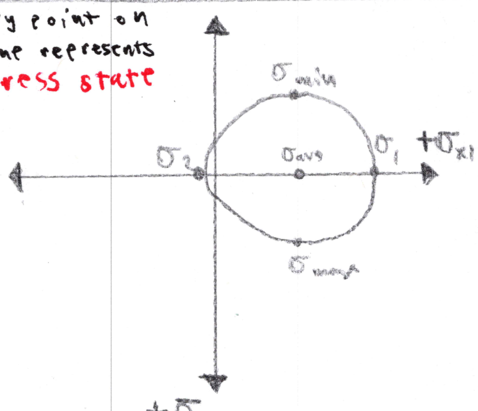
$$\tan(2\theta_s) = -\left(\frac{\sigma_x-\sigma_y}{2\sigma_{xy}}\right) \qquad \theta_s = \theta_p \pm 45^\circ$$

Mohr's circles

Circle Radius $R = \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \sigma_{xy}^2}$

Avg. stress $\sigma_{avg} = \frac{\sigma_x+\sigma_y}{2}$

- every point on plane represents stress state



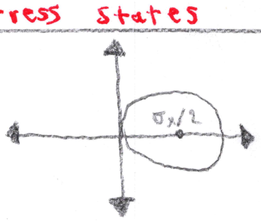
* If you rotate by $\boxed{\theta}$ degrees, move 2θ on Mohr's Circle

Special Stress States

a) Uni-axial

$\sigma_{avg} = \sigma_x/2$

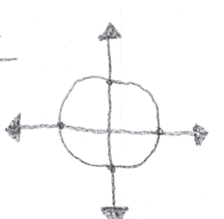
$R = \sigma_x/2$



b) Pure Shear

$\sigma_{avg} = 0$

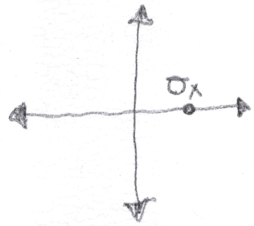
$R = \frac{\sigma_{xy}}{2}$



c) Equal Biaxial Stress

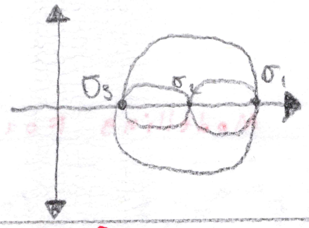
$\sigma_{avg} = \sigma$

$R = 0$



d)

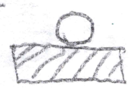
$\sigma_1 > \sigma_2 > \sigma_3$



Beams

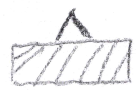
End Supports:

1. Roller



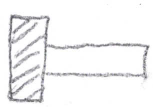
- can only resist w/ support reax in y-direction

2. Pin



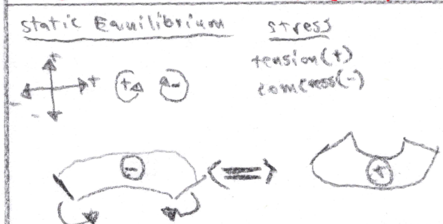
- can resist w/ support reaction in X & Y Direction

3. Fixed End

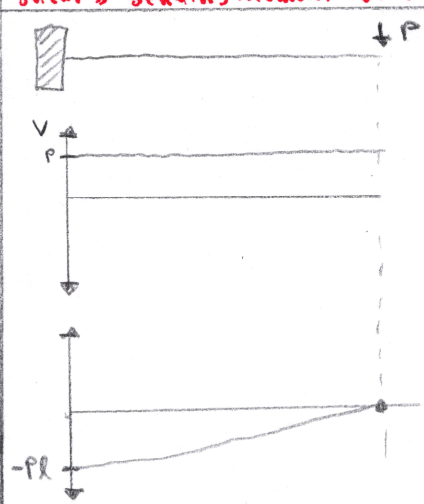


- can resist in X & Y Direction
- can have Moment

Beam Sign Conventions



Shear & Bending Moment Diagrams



$\sum F_y = 0$

$P - V = 0$

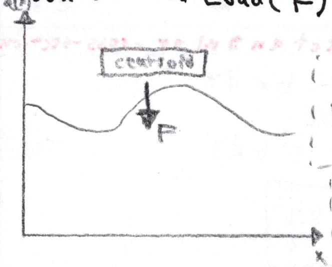
$\boxed{V = P}$

$+Pl - Px + M = 0$

$\boxed{M = -P(l-x)}$

Distributed Loads on Beams

- we derive an Equivalent Concentrated Load (F)



$$F = \int_0^L q(x)\,dx$$

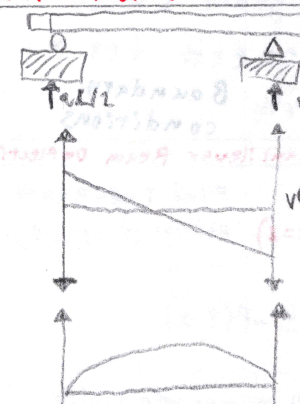
$$\text{centroid} = \frac{\int_0^L x\,q(x)}{\int_0^L q(x)\,dx}$$

- Equivalent Force $\boxed{F}$ acts through the centroid of Area of Distributed Load

Relations between q, V, M

$q = -\frac{dV}{dx}$	$V_B - V_A = -\int_A^B q\,dx$
$V = \frac{dM}{dx}$	$M_B - M_A = \int_A^B V\,dx$

Example: Distributed Load on Beam



$$V(x) = V_A - \int_0^x q\,dx$$

Stresses in Beams

Bending: one face in Tension, one face in compression

Neutral axis: Plane that sees no stress

$$\frac{1}{\rho} = \frac{d\theta}{dx} = K \qquad \varepsilon_x = -Ky$$

Normal stress: $\boxed{\sigma_x = -KyE}$

$$* \boxed{\sigma_x = \frac{-My}{I}}$$

- M (+) & y (+) $\Rightarrow \sigma_x < 0$, compression
- M (+) & y (−) $\Rightarrow \sigma_x > 0$, tension

- Max occurs at greatest M & y

Review + 2nd Moment of Area (I)

$I = \int_A y^2 \, dA$ | $\sigma = \frac{My}{I}$ | $K = \frac{M}{EI}$

I of Rectangular cross-section

$dA = b * dy$

$$I_z = \int_{-h/2}^{h/2} y^2 b \, dy = \boxed{\frac{bh^3}{12}}$$

Circular Cross-Section (I)

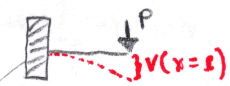
solid	thick-walled	thin walled
$\frac{\pi r^4}{4}$	$\frac{\pi(r_{out}^4 - r_{in}^4)}{4}$	$\pi r^3 t$

Deflection of Beam

$$\boxed{\frac{d\theta}{dx} = \frac{d^2 v}{dx^2} = \frac{M}{EI}}$$

↳ Integrate twice w/ respect for x & Apply Boundary conditions

Example: Cantilever Beam Deflection



Find End-Beam Deflection $v(x=\ell)$

Given $\quad M = -P(\ell - x)$

⇓ Integrate Twice

$$v = \frac{-P}{EI}\left(\frac{\ell x^2}{2} - \frac{x^3}{6}\right) + C_1 x + C_2$$

Boundary Conditions:

1. at $x=0, v=0$ → So $\boxed{C_2 = 0}$
2. at $x=0, \theta=0 \Rightarrow \frac{dv}{dx}$ → $\boxed{C_1 = 0}$

↳ at L: $\boxed{v(x=L) = \frac{-PL^3}{3EI}}$

Euler Buckling Load

$$P_{critical} = \frac{n^2 \pi^2 E I}{L^2}$$ (Euler Buckling Load)

- ↑ E, Harder to buckle
- ↑ I, harder to Buckle
- ↑ ℓ, easier to buckle

Factor: n=1, n=2

Local Buckling Stress

$$\sigma_{local\ buckling} = \frac{Et}{R\sqrt{3(1-\nu^2)}}$$

Column Buckling Example

Modelling Foams

$$E^{Foam} = C_1 E_{solid} * \left(\frac{\rho_{foam}}{\rho_{solid}}\right)^2$$

$$\sigma^*_{elastic} = C_2 E_{solid} * \left(\frac{\rho_{foam}}{\rho_{solid}}\right)^2$$

Linear Visco Elasticity

- Deformation dependent on temperature & time

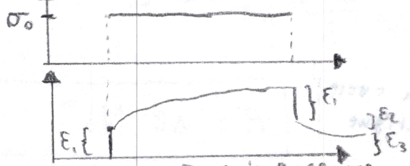
Review: Fluids

$$\tau = \mu \frac{dv_x}{dy}$$

↳ where $\boxed{\mu}$ = viscosity

Creep test

- Apply constant stress $\boxed{\sigma}$, measure strain as a function of time $\boxed{\varepsilon(t)}$



ε_1 = immediate Elastic Response

ε_2 = unrecoverable Deformation

Creep Compliance: $J(t) = \dfrac{\varepsilon(t)}{\sigma}$

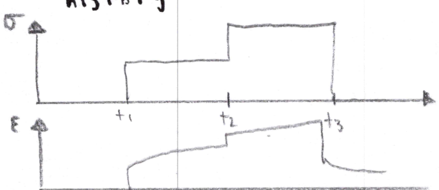
Stress-Relaxation test

- Apply a constant strain $\boxed{\varepsilon}$ & measure $\boxed{\sigma}$

Relaxation modulus: $\dfrac{\sigma(t)}{\varepsilon}$

Boltzmann Superposition Principle

- Creep & Stress Relaxation is a function of **entire loading history**

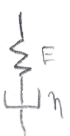


$$\varepsilon(t=t_n) = \Delta\sigma_1 J(t_n - t_1) + \Delta\sigma_2 J(t_n - t_2) + \ldots$$

Spring-Dashpot system for Linear Viscoelastic

- Spring: $E = \sigma/\varepsilon$
- Dashpot: viscosity $(\eta) = \dfrac{\sigma}{\dot{\varepsilon}}$

Maxwell Element

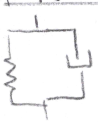


$\sigma = \sigma_{spring} = \sigma_{dashpot}$

$\varepsilon = \varepsilon_s + \varepsilon_d$

$$\frac{d\varepsilon}{dt} = \frac{1}{E}\frac{d\sigma}{dt} + \frac{\sigma}{\eta}$$

Voight Element



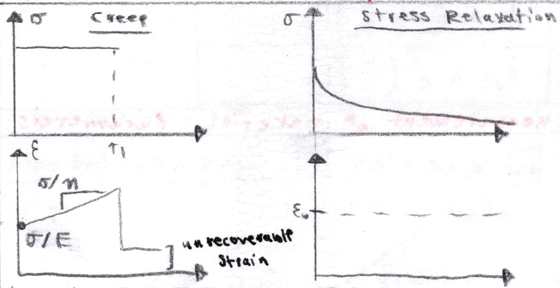
$\sigma = \sigma_s + \sigma_d$

$\varepsilon = \varepsilon_s = \varepsilon_d$

$$\sigma = E\varepsilon + \eta\dot{\varepsilon}$$

3.032 Cheat Sheet # 3

Maxwell: Creep & Stress Relaxation

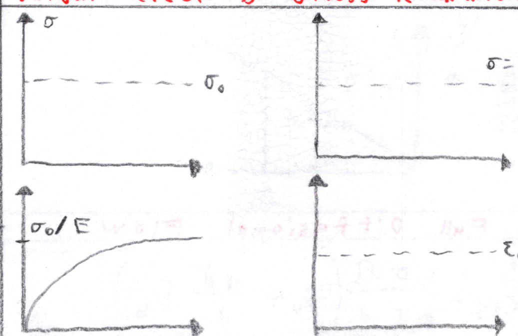


$t<t_1$: $\varepsilon = \frac{\sigma}{E} + \frac{\sigma}{\eta}t$

$t>t_1$: $\varepsilon = \frac{\sigma}{\eta}t_1$

$$\sigma = E\varepsilon_0 e^{\wedge}\left(-\frac{E}{\eta}t\right)$$

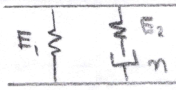
Voight: Creep & Stress Relaxation



$t<t_1$: $\varepsilon = \frac{\sigma_0}{E}\left(1 - e^{\wedge}\left(-\frac{Et}{\eta}\right)\right)$

$t>t_1$: $\varepsilon = \frac{\sigma_0}{E}\left(e^{\wedge}(-Et/\eta)\right)$

Standard Linear Solid



constitutive Eqn

$$\sigma + \tau\dot{\sigma} = E_1\varepsilon + (E_1 + E_2)\tau\dot{\varepsilon}$$

↳ $\tau = \dfrac{\eta}{E_2}$

Linear Viscoelasticity in Polymers

1. A low temperature, Glassy Behavior
2. At medium Temp, we get viscoelastic behavior at $\boxed{T_g}$, secondary bonds melt, easier sliding of polymer segments
3. At high temperature, get Rubbery behavior

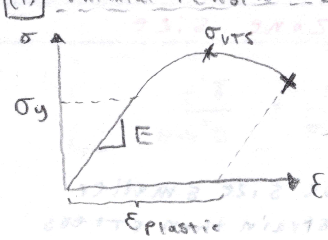
Plasticity

Plasticity → at/near room temperature materials have defined yield

$\sigma < \sigma_y$	Recoverable Elastic Deformation
$\sigma \geq \sigma_y$	Irrecoverable Plastic Deformation

Measuring Yield Strength

(1) Uniaxial Tensile test



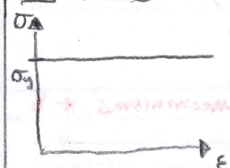
(2) Hardness Tests

- Indenter pushed into surface of solid specimen

* size of indent related to $\boxed{\sigma_y}$

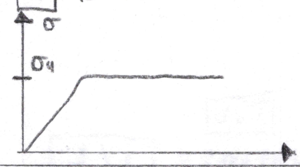
Idealization of Plastic Behavior

1. Rigid Perfectly Plastic



- "rigid" $E \to \infty$
- If $\varepsilon_{elastic} << \varepsilon_{plastic}$

2. Elastic-Perfectly Plastic



Plasticity: Assumptions

(1) Plastic-Flow occurs at constant Volume

$$* \quad \frac{\Delta V}{V_0} = \varepsilon_{11}^{plastic} + \varepsilon_{22}^{plastic} + \varepsilon_{33}^{plastic} = 0$$

- Plasticity associated w/ Dislocation **motion**

(2) Hydrostatic Stress doesn't cause yielding

(3) Neglect strain Hardening

Yield Criteria

1. Rankine Criterion
   - yield when $\boxed{\sigma_1 = \sigma_y}$
2. Tresca Criterion
   - yield when $\boxed{\sigma_1 - \sigma_3 = \sigma_y}$
3. Von Mises Criterion
   - yield when $\boxed{\sigma_{eq} = \sigma_y}$

$$\sigma_{eq} = \sqrt{\frac{1}{2}\left[(\sigma_{11}-\sigma_{22})^2 + (\sigma_{22}-\sigma_{33})^2 + (\sigma_{33}-\sigma_{11})^2\right] + 3(\sigma_{12}^2) + \ldots}$$

* max dif between Tresa & Von Mises when $\boxed{\sigma_1 = -\sigma_3}$

Dislocations

Edge Dislocation $\boxed{b \perp t}$

Screw Dislocation $\boxed{b \parallel t}$

Plastic shear Strain Rate: $\boxed{\dot{\gamma}^p = \rho b v}$

Elastic Strain energy of Dislocation

Elastic Energy of Screw: $\dfrac{U_{el}}{L} = \dfrac{Gb^2}{4\pi}\ln\left(\dfrac{R}{r_0}\right)$

Elastic Energy of Edge: $\dfrac{U_{el}}{L} = \dfrac{Gb^2}{4\pi(1-\nu)}\ln\left(\dfrac{R}{r_0}\right)$

where

$R = 1/\sqrt{\rho}$ - ρ = dislocation density

Line Tension

$$\boxed{T = \frac{G b^2}{2}}$$

Glide Force (f)

$$\boxed{f = \tau b}$$

** Alloy Strengthening Mechanisms **

1. Solid-State Solution Hardening

- solute atoms dissolved in primary metal

3 parameters

- Glide Force = $\boxed{\tau_0 b}$
- spacing of solute atoms = $\boxed{1/L}$
- concentration of solute atoms = $\boxed{1/L^2}$

$\tau_0 b L b = U_s$ | $\tau_g^0 = \frac{U_s \sqrt{\text{concentration}}}{b^2}$

2. Precipitation Hardening

L = spacing between precipitate
R = Radius of precipitate

work done by Dislocation: $\boxed{\tau_p^0 b L (2R)}$

surface Energy of Precipitate: $\boxed{\Gamma (2 r b) * 2}$

* Equate the 2 to find shear stress required to break through

$$\tau_p = \frac{2\Gamma}{L}$$

3. Dispersion Hardening

$\boxed{\tau_d^0 = \frac{Gb}{L}}$ set equal to τ_p^0 to find critical Γ

4. Grain Boundary Strengthening

$\tau_{gb}^0 = \tau^0 + \frac{K}{\sqrt{d}}$ | K = material property, d = Grain size

Creep

1. Diffusional Flow $\dot{\varepsilon}_{ss} = \sigma$

2. Power-Law Creep $\dot{\varepsilon}_{ss} = \sigma^n$

Creep: Diffusional flow

$$\dot{\varepsilon}_{ss} = \dot{\varepsilon}_0 \left(\frac{\sigma}{\sigma_0}\right)$$

$$\dot{\varepsilon}_0 = A e^{\wedge}\left(-\frac{Q}{RT}\right)$$

A = 1
Q = Activation energy
R = gas constant
T = Absolute Temp

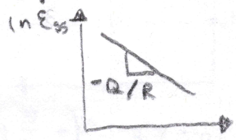
Creep: Power-Law Creep

$$\dot{\varepsilon}_{ss} = \dot{\varepsilon}_0 \left(\frac{\sigma}{\sigma_0}\right)^n$$

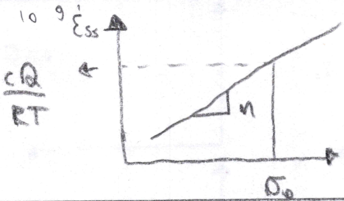
$$\dot{\varepsilon}_0 = A e^{\wedge}\left(-\frac{Q}{RT}\right)$$

Measurement of Material Parameters

1. Keeping $\boxed{\sigma}$ constant and vary T



2. Hold T constant & vary stress



Full Diffusional Flow

$$\dot{\varepsilon}_{ss} = \frac{2\sigma\Omega}{kTd}\left(D_v + \frac{\pi\delta}{d} D_b\right)$$

σ = stress
T = ab solute Temp
Ω = atomic volume
d = Grain size
k = Boltzmanns

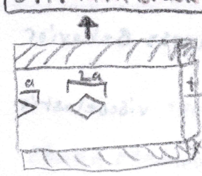
Creep Fracture

Larson-Miller Parameter

$$\frac{Q}{R} = T(\ln A + \ln(t_{Fail}))$$

Fracture Mechanics

Griffith Crack • only if $\frac{r_p}{a} < 0.02$



- plane thickness $\boxed{t}$
- crack length $\boxed{a}$

- Roughly change in energy as crack goes from 0 to a

$$U^{elastic} = \frac{\pi a^2}{2} +$$

$$\boxed{\sigma^2 \pi a = E G_c} \quad G_c = 2\Gamma$$

Stress Intensity Factor (K_I): $\boxed{Y\sigma\sqrt{\pi a}}$

Fracture Toughness: $\boxed{K_{IC}}$

Plastic Zone Size

$$r_p \approx \frac{1}{2\pi} * \frac{K_I}{\sigma^2_{yield}}$$

- Plastic zone size smaller for plane strain than stress

Ductile → Brittle Transition

$$\boxed{\sigma_{yield} = \sigma_y^0\left[1 - \frac{k_B T}{\sigma_b} \ln\left(\frac{\dot{\gamma}}{\dot{\gamma}_0}\right)\right]}$$

as $T\downarrow$, $\sigma_y\uparrow$, $r_p\downarrow$

Fatigue

$\sigma_{mean} = \frac{\sigma_{max}+\sigma_{min}}{2}$ | $\Delta\sigma = \sigma_{max} - \sigma_{min}$

High cycle Fatigue: $\boxed{B = \Delta\sigma N_f^{\alpha}}$

Low cycle Fatigue: $\boxed{\Delta\varepsilon^{plastic} N_f^{b} = C_2}$

Paris Law

$$\boxed{\frac{da}{dN} = A(\Delta K)^m}$$

$$N_f = \int_{a_0}^{a_f} \frac{da}{A(\Delta K)^m}$$