Coase Theorem

- If property rights are clearly specified and there are no transaction costs, bargaining will lead to an efficient outcome

Collective Decision Making

↳ There are a set of Alternatives, & each person has Preferences over the alternatives

The Model:

$\boxed{I}$ → set of individuals w/ element i

$\boxed{A}$ → Set of alternatives w/ element a

↳ Each individual has preference ordering $\boxed{\succ_i}$

Social welfare Function

↳ Takes input of Preferences & Produces Societal Preference

Plurality Rule

- Alternative w/ most "top choice votes" placed 1st
- Remove this alternative, and evaluate the rest the same until all alternatives ranked

Borda criterion

- Assign Points to alternatives based on each individuals ranking

ex. #1 = 1 points, #2 = 2 point

- Rank alternatives according to points

Rawlsian SWF

- Find individual that ranks a the lowest
- Do same for all alternatives & rank in Decreasing order

Requirements for an SWF

$\boxed{1.}$ Universal Domain

↳ Every Possible preference list input generates A well-defined Social Ranking

14.19 Summary Sheet

$\boxed{2.}$ Unanimity

↳ If $A \succ B$, for all individuals, then $A \succ B$ in societal preference

$\boxed{3.}$ Independence of Irrelevant Alternatives

- The Social ranking of A vs. B should only depend on individuals ranking of only those two

Dictatorship

- Some individual $\boxed{i}$ Determines social $\succ$ based on his $\boxed{\succ_i}$

(Satisfies all 3 SWF conditions)

Arrow's Impossibility Theorem

- The only SWF to satisfy all 3 conditions is Dictatorship

Social Choice Functions

- A central Planner chooses an alternative in $\boxed{A}$
- Each agent has preferences over types $t_i \in T$

* A SCF maps a type space to an alternative

Mechanisms & Implementation

- Message Space $\boxed{M_i}$ is what a Player can submit

$\boxed{\text{Direct Mechanisms}}$

- If Message Space = Type Space

↳ we call $\boxed{\varphi}$ a Direct Mechanism

↳ is incentive compatible if for all states $t \in T$, t is an Equilibrium

Revelation Principle

- For every mechanism Ψ and dominant strategy s^* there exists an incentive compatible Direct Mechanism

$$\boxed{\varphi(t) = \Psi(s^*(t))}$$

Properties of Mechanism

- A Direct Mechanism is strategy Proof if it is Dominant strategy (incentive) compatible
- Direct Mechanism is Pareto Efficient, if no other social outcome makes everyone better off
- Direct Mechanism is Dictatorial, if dictator's favored is chosen

Gibbard/Satterthwaite Theorem

- If $|A| \geq 3$ is finite, any Pareto-Efficient, Strategy-Proof Mechanism is Dictatorial

Properties:

$\boxed{1.}$ Monotonicity

$\boxed{2.}$ Re-ordering

$\boxed{3.}$ Swap

Single Peaked Preference

- Alternatives are indexed such that each agent's utility peaks at a given alternative, and decreases farther the social choice is from it

Median Voter Rule

- All agents have single Peaked Preferences

↳ Median Voters peak chosen as social alternative

Quasi-Linear Preferences

↳ utility of the agent for his type

$$\underbrace{H_i(t_i,(\delta,X))}_{\text{utility}} = \underbrace{v_i(t_i,\delta)}_{\text{value of decision}} + \underbrace{X_i}_{\text{Nonmetonic transfer}}$$

Mechanism $\varphi(d,y)$ is efficient for all $t \in T$ if

$$\sum_{i \in N} v(t_i, d(t)) = \max_{\delta \in D} \sum_{i \in N} (t, \delta)$$

* value maximized through decision φ

Vickrey Auction

- k identical good auctioned amongst n bidders
- Auctioneer gets values on all k units
- Ranks the bids, **winners are top k bidders**
- Each winner(i) pays total bids of player s who could win new goods if winner (i) not present

House Allocation Problems

↳ a 3-tuple Problem $\langle I, H, \succ \rangle$

$\boxed{I}$ = Set of Agents

$\boxed{H}$ = Set of Houses

$\boxed{\succ}$ = list of Preferences over houses

Assume | $|H| = |I|$, & Strict preferences

A matching: H

- A (house) Matching: $I \rightarrow H$ is a One to One Function
  - ↳ each agent assigned one house
  - ↳ No house assigned to more than one agent
- A matching $\boxed{H}$ Pareto Dominates another matching if

$$H(i) \succeq_i v(i) \quad \text{for all } i \in I$$
$$H(i) \succ_i v(i) \quad \text{for some } i \in I$$

- Pareto Efficient if can do no better

Housing Market Problems

- Housing Market is a 4-Tuple

$\boxed{I}$ Set of agents $\boxed{\succ}$ list of strict Preferences

$\boxed{H}$ set of Houses $\boxed{H}$ initial Endowment Matching

- Let $h_i = H(i)$ denote initial endowment
- A matching $\boxed{\eta}$ is individually rational if

$$\eta(i) \geq h(i) \quad \text{for all } i \in N$$

- A matching is in the core of housing market if there is no coalition $T \subseteq I$
  $\boxed{1.}$ $v(i) \in h_i$

Gale's Top Trading Cycle

- Each agent Points to his favorite house, and through a finite # of cycles all agents receive a house

Proofs of TTC

- a matching v is in the core
  - ↳ Let $C_1, C_2, C_3, \ldots C_n$ be the order of agents as they're removed from cycle
  - ↳ No agent C_1 wants to block give n that he gets 1st choice
  - ↳ No C_2 wants to block and so on

* A matching and price vector are in **competitive Equilibrium** if matching H(a) provides the best house within affordability

Mechanism For House Allocation

- Simple Serial Dictatorship
  - ↳ Agent induced by an ordering gets top choice and so and so forth

Lottery Mechanisms

- Random Serial Dictatorship
  - ↳ randomly constructed ordering
- Core from Random Endowment
  - ↳ **assign random properties and run TTC**

- For each PS H there exists a random SD or initial endowment that will result in it

Lotteries

- $\mathcal{L} = \sum \alpha_H \cdot H$ $\{ \alpha_H$ = probability of matching H
- Lottery is ex-Post efficient if positive weight is assigned to Pareto Efficient assignments
- Each lottery induces random assignment $\boxed{P}$
- P_i Stochastically Dominates Q_i
  - ↳ first choice as just as Probable or more than Q

Ordinal Efficient Mechanism: Probabilistic Serial

- Each house is 1 divisible commodity
- Each agent eats from their favorite house at same speed
- Once person's top choice is fully consumed, they'll resume eating from next best
- Ends when each unit consumes one unit

- **PS Not Strategy Proof except in Large Markets**

House Allocation w/ existing Tenants

- Is a 5-tuple

I_E - set of existing tenants

I_n - set of newcomers

H_o - set of occupied homes

H_v - finite set of vacant houses

RSD w/ Squatting Rights

- Each agent can choose to keep their house, or enter the lottery

Deficiency: Individually Rational nor Pareto Efficient

MIT NH4 Mechanism

- start with ordering f
- Run Top Trading Cycles until squatting conflict
  ↳ give house to owner and start from affected

Alternative: top trading cycles

- Fix an ordering f

$\boxed{\text{Step 1:}}$

Define the set of available houses for this step to be the set of vacant houses

- Each agent points to their favorite house and all houses point at their

Example

$I_E = \{i_1, i_2, i_3, i_4\}$, $I_N = \{i_5\}$

$H_O = \{h_1, h_2, h_3, h_4\}$, $H_V = \{h_5, h_6, h_7\}$

$\succ_1$	$\succ_2$	$\succ_3$	$\succ_4$	$\succ_5$
h_2	h_7	h_2	h_2	h_4
h_6	h_1	h_1	h_4	h_3
h_5	h_6	h_4	h_3	h_7
h_1	h_5	h_7	h_6	h_1
h_4	h_4	h_3	h_1	h_2
h_3	h_3	h_6	h_7	h_5
h_7	h_2	h_5	h_5	h_6
h_0	h_0	h_0	h_0	h_0

Step 1

$h_1 \to i_1 \to h_2 \to i_2 \to h_7$

* h_1 added to market

Step 2

$i_3 \to h_1$, $i_4 \to h_4$

Remaining matching

$$\begin{pmatrix} i_1 & i_2 & i_3 & i_4 & i_5 \\ h_2 & h_7 & h_1 & h_4 & h_3 \end{pmatrix}$$

Proofs

- For any ordering f Top Trading cycles is
  - i) IR
  - ii) PE
  - iii) SP
- Consider case where $|H_V| = |I_N|$

YRMH - IGYT

- Move tenant whose property is demanded to the top of priority list

One-to-One: Marriage Problems

- The Marriage Problem is a triple $\langle M, W, R \rangle$ where

$M = \{m_1, \dots\}$ = set of men

$W = \{w_1, \dots, w_n\}$ = set of women

$R = \{R_{m_1}, \dots R_{m_n}, R_{w_1}, \dots, R_{w_n}\}$ = list of preferences

- R_m preference over $W \cup \{m\}$

R_w preference over $M \cup \{w\}$

Notation:

$w P_m w'$ – man prefers w over w'

$w P_m m$ – man prefers w to m

$m P_w w$ – woman w is unacceptable

- A matching μ is a function such that

$\boxed{1.}$ $\mu(m) \notin W \implies \mu(m) = m$

$\boxed{2.}$ $\mu(w) \notin M \implies \mu(w) = w$

$\boxed{3.}$ $\mu(m) = w \iff \mu(w) = m$

Properties of Matchings

- A matching $\boxed{\mu}$ is Pareto Efficient if no other matching is weakly or strictly better for every $M \cup W$ pair
- A matching μ is blocked by an individual if $i \, P_i \, \mu(i)$
- A matching is Individually Rational if not blocked by individual
- a $\boxed{\mu}$ is blocked by a pair if they prefer eachother to their own partners
- A matching is Stable if it is not Blocked by any individual or a pair

Man-Proposing deferred acceptance algorithm

- Each man proposes to his 1st choice
  ↳ Each one accepts and tentatively keeps best offer
- Keeps cycling through until no more rejections

Core & Stability:

- $\boxed{\mu}$ is the Core if there exists no matching ν such that $\nu(i) P_i \mu(i)$

3 Stable Match Results

1. Side-Optimality → there exists a man-optimal stable matching

2. Opposing Interests → The man-optimal matching is the worst stable matching for women

3. Rural Hospitals → The set of agents matched is the same for all stable matchings

Join & Meet

- let μ & μ' be stable Matchings

Join $\mu \vee_M \mu'$ → assigns each man the more preffered of his 2 assignments

Meet $\mu \wedge^M \mu'$ → assigns each man the least preffered of his 2 assignments

Lattice Theorem: if μ & μ' are stable Matchings then both $\mu \vee^M \mu'$ & $\mu \wedge^M \mu'$ are both stable Matchings

Mechanism

- A mechanism is Stable if $\varphi(R)$ is stable for any $R \in \mathcal{R}$
- A mechanism is Pareto-Efficient if it always selects a PE outcome
- A mechanism is Individually Rational if it always selects an IR outcome

* Each Mechanism induces a preference Revelation game

Roth's Impossibility Theorem

- There exists no mechanism that is both stable and strategy proof
- both women and men can manipulate or truncate preferences to change the stable matching outcome

Many-to-One Matching

- College Admissions is a 4 Tuple
  ↳ $\langle C, S, q, R \rangle$

1. Colleges: $C = \{c_1, c_2, \dots c_n\}$
2. Students: $S = \{s_1, s_2, \dots s_n\}$
3. college capacities: $q = \{q_{c_1}, \dots q_{c_m}\}$
4. Preferences: $R = \{R_{c_1}, \dots R_{c_m}, R_{s_1}, \dots R_{s_n}\}$

- R_s → Preference relation over $C \cup \{\emptyset\}$
- R_c → Preference relation of students

Responsiveness

Assume (No peer effects)

- A student's acceptability is not contingent on other students in her class
- Relative Desireability not dependent on composition of class

So, college Preferences are Responsive if

1. if for any $T \subset S$ with $|T| < q_c$ and any $s \in S$: $(T \cup s) P_c T$ & $\{s\} P_c \emptyset$
2. For any $T \subset S$ with $|T| < q_c$: $(T \cup \{s\}) P_c (T \cup \{s'\}) \Rightarrow \{s\} P_c \{s'\}$

Matching

- A matching is a correspondance

$$\mu: C \cup S \Rightarrow C \cup S \cup \emptyset$$

where

1. $\mu(c) \subseteq S$ such that $|\mu(c)| \leq q_c$
2. $\mu(s) \subseteq C$ such that $|\mu_s| \leq 1$
3. $s \in \mu(c)$ if & only if $\mu(s) = \{c\}$

Stability

- Match is Blocked by a college if $s \in \mu(c)$ but $\emptyset P_c \{s\}$
- A matching is Blocked by a student $s \in S$ if $\emptyset P_s \mu(s)$
- A matching is individually rational if not blocked by college or student
- A matching is Blocked by a pair if $c P_s \mu(s)$

College Proposing Deferred Acceptance Algorithm

Step 1: Each college c proposes to its top q_c students and each student holds its most preferred

Step k: college applies to top q_c students who havent yet rejected

* Useful extension of Marriage Problem

- Divide each college c into q_c pieces with a capacity of one each piece with identical preferences to college c
- students rank seats at same college according to index

Stability Lemma

- A college admissions matching is stable if & only if its marriage Problem is stable

Implications:

1. Existence of student-optimal stable Matching
2. Existence of college-stable matching

Old Results:

- student-optimal matching worst for colleges
- set of students and positions filled is the same for all stable matches
- Join & Meet are also stable Matches

New Results

- Rural Hospitals → Any college that does not fill all its positions is assigned precisely the same students in every stable match

* There can be a case where
* there is an IR Matching strictly better for each college

ex. $R_{s_1}: \{c_1\}, \{c_2\}$ $\quad R_{c_1}: \{s_1, s_2\}, \{s_2\}, \{s_1\}$

$R_{s_2}: \{c_2\}, \{c_1\}$ $\quad R_{c_2}: \{s_1\}, \{s_2\}$

$$M_c = \begin{pmatrix} c_1 & c_2 \\ s_1 & s_2 \end{pmatrix}, \quad V = \begin{pmatrix} c_1 & c_2 \\ s_2 & s_1 \end{pmatrix}$$

Incentives

- Truth telling is favored for students
- TruthTelling is not always dominant strategy for colleges

* Student-Optimal is more manipulable than college optimal

Boston School System mechanism

- Mechanism used by Boston Public School system until '05

$\boxed{1.}$ For each school a priority ordering is established as such

- 1st: Sibling & walking zone
- 2nd: Sibling
- 3rd: walking zone
- 4th: other Students

$\boxed{2.}$ Each student submits Preference Ranking over schools

$\boxed{3.}$ Student assignment based on Preferences & priorities

Procedure:

- 1st round only 1st choices considered schools assign seats to students whole ranked it first and seats are filled on hierarchial system until no more 1st choice or

- in round K only k^{th} choices considered

* Was easy to Manipulate

↳ Boston Mechanism only incentivized ranking schools where they have highest priority

Cambridge Mechanism

Goals:
- Treat students fairly
- Empower choice
- Improve diversity
- Promote competition

The Mechanism:

$\boxed{1.}$ Assign all Pre-Assigned to the schools where they are gaurenteed

$\boxed{2.}$ students submit preferences of up to Three ranked schools

$\boxed{3.}$ Every student receives random Number

- All students assigned to 1st choice up to capacity
- Iterate to 2nd choice & so on

Boston Performance and Fact

- 80% get first choice
- 8% second choice
- 5-9% unassigned

Unsophisticated Play

↳ Rank 2 overdemanded choices as 1st & 2nd

- 1/3 of unassigned students could've gotten 3rd choice if they ranked second

Alternative Mechanisms

$P_{i_1}: s_2 > s_1 > s_3$

$P_{i_2}: s_1 > s_2 > s_3$

$P_{i_3}: s_1 > s_2 > s_3$

$\boxed{\text{Top Trading cycles}}$

$$M^{TTC}\begin{pmatrix} i_1 & i_2 & i_3 \\ s_2 & s_1 & s_3 \end{pmatrix}$$

$\pi_{s_1}: i_1 > i_3 > i_2$

$\pi_{s_2}: i_2 > i_1 > i_3$

$\pi_{s_3}: i_2 > i_1 > i_3$

$$M^{DA}\begin{pmatrix} i_1 & i_2 & i_3 \\ s_1 & s_2 & s_3 \end{pmatrix}$$

Strategy Proofness & Policy

- A strategy proof Algorithm levels the playing field for those who report truthfully

The Model:

N → Sincere Students

M → Sophisticated Students

- we focus on the Nash Equilibria of the preference revelation game

Stability:

- M is stable if no (student, school) pair where

$\boxed{1.}$ student i prefers s to $M(i)$, $\boxed{\text{and}}$

$\boxed{2.}$ s has a vacant seat or lower priority student

Nash Equilibrium Example

- There are $\boxed{3}$ schools $\{a, b, c\}$, $\boxed{2}$ strategic students $\{i_1, i_2\}$ and $\boxed{1}$ sincere student $\{i_3\}$

utilities:

	a	b	c
M_{i_1}	1	2	0
M_{i_2}	0	2	1
M_{i_3}	2	1	0

Priorities:

$\pi_a: i_2 > i_1 > i_3$

$\pi_b: i_3 > i_2 > i_1$

$\pi_c: i_1 > i_3 > i_2$

Game Payoff

Player i_2

Player i_1	abc	acb	bac	bca	cab	cba
abc						
acb						
bac						
bca						
cab						
cba						

Nash Equilibrium:

$$\begin{pmatrix} i_1 & i_2 & i_3 \\ a & b & c \end{pmatrix}$$

Notes:

- Truthful Revelation is not Nash Equilibrium
- sincere player receives utility of 0
- This of course is not stable (i, b) is a blocking pair

Augmented Priorities

• Given an economy (P, π) and a school s, partition set $\boxed{I}$ of students into m sets:

I_1: Sophisticated and sincere student who rank s as 1st choice

I_2: sincere → 2nd

I_3: sincere → 3rd

↓

I_m: sincere who rank last

Constructing Augmented ordering

↳ $\tilde{\pi}_s$ • Hierchial
• up to agent within I_k who ne preferences

Augmented Economy { $(P, \tilde{\pi}_s)$

Former Example revisited

$\pi_a: i_2, i_1, i_3 \rightarrow \tilde{\pi}_a: i_2, i_1, i_3$

$\pi_B: i_3, i_2, i_1 \rightarrow \tilde{\pi}_B: i_2, i_1, i_3$

$\pi_c: i_1, i_3, i_2 \rightarrow \tilde{\pi}_c: i_1, i_2, i_3$

* in this case the Nash Equilibrium is equivalent to Boston Mechanism a unique stable Matching

Characterizing Nash Equilibrium Outcome

$\boxed{\text{Proposition 1}}$ → Nash Equilibrium from (P, π) is equivalent to $(P, \tilde{\pi})$

↳ sincere students lose Priority

↳ These Nash Equilibriums inherit all properties of stable matchings

$\boxed{\text{Proposition 2}}$ → Fix economy (P, π) sincere student i gets same assignment at each Nash Equilibrium

• Multiple Stable Matchings created by "conflict" between school priorities and student preferences

• sincere students never in conflict

• Truth-Telling is dominant strategy for sophisticated and only for sincere

$\boxed{\text{Proposition 3}}$ → Assignment of sophisticated student i under PD Nash equilibrium is at least as good as SOSM dominant strategy equilibrium

$\boxed{\text{Proposition 4}}$

↳ A sincere student weakly benefits from becoming sophisticated in PD Nash Equilibrium of Boston game

Chicago Public School

Round 1: Only 1st choice students considered & filled by descending test score until no more spaces or 1st choices

.... so on so forth 'till all placed

* Both old + new manipulable

Comparing Mechanisms

• Mechanism $\boxed{A}$ is more manipulable than Mechanism $\boxed{B}$ if for any problem where $\boxed{B}$ is, A must be

constrained school choice

let $\ell > k > 0$ and suppose there are ℓ schools. The student optimal stable mechanism where students can rank k schools is more manipulable than if ℓ schools could be ranked

Single Unit Auctions

• single unit sold

• n bidders with value v_i

utility of winning agent: $v_i - p$

• ** Aside from Vickrey's 2nd Price Auction no other nth Price auctions are strategy-proof

For any $\ell > k \geq 2$, the ℓth price auction is strongly more manipulable than kth price

Multi-Unit Auctions

• k identical items for n-bidders

Utility of agent who gets ℓ objects $\left\{ M_i = v_i^1 + \dots v_i^\ell - p \right.$

Auction Formats:

1. Discriminatory Auction

- Each agent i bids $b_1^i \geq b_2^i \geq .. b_k^i$
- highest k bidders receive objects and pay their bids

2. Uniform Auction

- Each agent i bids b_i for the 1st where
- The highest k-bidders receive objects and pay the $(k+1)$th highest bid for each unit

* Discrimanatory Price Auction strongly more manipulable than Uniform Price Auction

DA vs. TTC

• Say we have $i_1: s_2 > s_1$
$i_2: s_1 > s_2$
$i_3: s_2 > s_1$

• 2 schools $\left\{ \begin{matrix} q_{s_1} = 2 \\ q_{s_2} = 1 \end{matrix} \right\}$ $s_1: i_1 > i_2 > i_3$
$s_2: i_2 > i_3 > i_1$

TTC

$s_1 \rightarrow i_1 \rightarrow s_2 \rightarrow i_2$

TTC:

i_1	i_2	i_3
s_2	s_1	s_1

↓ compare w/ justified and envy free outcome

i_1	i_2	i_3
s_1	s_1	s_2

Other Strategy Proof + Efficient mechanism

1. Serial Dictatorship

2. Clinch & Trade

Inspiration

- TTC w/ counters allows i to trade her priority at other objects even when i is among the highest q_s student at s
- Since i is among the highest ranked, its possible this trade creates envy

Method:

- Reduce q_s for every relevant s until no agent i is contained in his top school's q_s

Main Result: Envy

- Let φ be a PE, SP, mechanism

if φ has less envy than TTC at $\succ$, then
$$\varphi(\succ) = TTC(\succ)$$

* TTC minimizes envy in the class of PESP mechanisms

New York City

- Students who don't get into specialized HS can rank up to 12 schools they desire
  1. Main Round
  2. Supplementary Round
  3. Appeals Round

Main Round

School types

Audition, Screened, Unscreened, Ed-opt	Active rankers:	Audition, screened, ½ Ed-opt
	Passive rankers:	unscreened, ½ Ed-opt

* Half of all seats based on Active ranking

Design Question: how to order students within priority class?
1. Single lottery
2. Multiple school specific lotteries

Example

- For every preference profile (P_I, Π_s) and its student optimal matching M, there exists a single ordering $\boxed{I}$ of students where $DA^r(P_I, \Pi_s) = M$

* A student-optimal mechanism would dominate DA-single

- two tie breaking rules are single & multiple school lotteries

Theorem: For any tie breaking rule there exists no strategy-proof mechanism that dominates DA^τ

Back to Boston

- Walk-zone priority
  ↳ 1/2 of seats would be subject to this priority

Question: How does 50-50 compromise compare to 0 or 100

Fraction Assigned to walkzone:

0	61.3%
50-50	62.5%
100	76.5%

→ why?

The Model & Choice Function

- I: students
- School a, with slots S^a
- each slot has priority order Π^s over students
- slots ordered according to a linear order of precedence

Priorities

- Π^0 → Master priority order
- $I_a \subset I$ → subset of reserve eligible students

2 types of students

1. Open Slots → $\Pi^s = \Pi^0$
2. Reserve Slots → Any priority student takes precedence over non-priority

Priority & Precedence

- Suppose slot s^* is open under Π, but reserve under $\tilde{\Pi}$

Proposition 1

- Suppose that D is the choice function for a school a obtained from C by changing open slot → reserve slot

Result

- All reserve eligible children that are chosen in $\boxed{C}$ are chosen in $\boxed{D}$
- All reserve ineligible students at school a that are chosen from I under $\boxed{D}$ are chosen under $\boxed{C}$

(swapping precedence of seat has same effect as swapping ordering of seat)

Proposition 2 (Under DA)

- Replace an open slot w/ a reserve slot weakly increases # of Reserve eligible students
- Replacing a reserve precedence over position of open slot increases reserve students
- Big Result: can move closer to home simply by changing walkzone preference

Problems w/ BPS 50-50

1. Processing Order Bias

↳ The earlier the walk-zone slots are processed the fewer the # of walk-zone applicants are able to compete

2. Randomization Bias

↳ Unintended Implication of using same Random tie-break for both student groups

- Since walk-zone spots have precedence 1st, any that remain must have unfavorable Position

Affirmative Action

- Pro-Arguments include Diversity, student role models

Chicago's AA Plan

- Each Chicago census is given a score that determines if its in one of 4 tiers

40% by merit, 60% by tiers

What is the "objective" Function

Continuum Model

I: set of students with mass n

$T = \{1, 2, t_i\}$ → set of tiers

S = set of finite slots with unit mass capacity

$\triangleright$: Linear order of precedence of slots, which specifies the order in which seats are processed

$S = \{s_1, s_2, s_3\}$

$\boxed{C(S, \gamma, \triangleright, J)}$

- Slots are processed one-by-one according to $\triangleright$
- when Merit slot fill w/ highest test score
- when tier slot, fill w/ highest scoring student of said tier

* Many possibilities depending on In what order Tier & Merit slots Processed

$\triangleright$ is equivalent to $\tilde{\triangleright}$ if $\tilde{\triangleright}$ can be achieved by a sequence of swaes by adjascent tier slots

Maximal/Minimal Preference ordering

- Maximal Precedence ordering for tier t^*

m	m	m	t^*	t^*

↳ merit slots come before t^* slots

- Minimal Precedence order for tier t^* (+ weaks competition for merit slots to detriment of t^*)

* Preferential treatment across tiers only eliminated when tiers play no role

- A precedence $\triangleright$ is tier-blind if relabeling tier slots doesn't change outcome
- A precedence is balanced if it has equal slots for all tiers before, in between or after merit slots

*The best tier blind policy for the lowest tier is the worst for highest tier

Main Conclusions:

- Tier 1 students would benefit from tier-blind precedence
- Higher merit share of spots worsens the disparity

Auction Theory

Simple Model:

- N Bidders
- Each bidder i has value v_i

Ascending Auction

- Price starts at 0
- Buyers bid higher until only one remains
- Final Bidder pays price at which 2nd highest drops out

Optimal Stategy: continue bidding until price equals your value

- on average highest bid will be $N/(N+1)$
- on average 2nd highest will be $(N-1)/(N+1)$
- expected revenue: $(N-1)/(N+1)$

2nd Price Auction

- Bitters submit sealed bids
- Seller opens and highest bidder pays 2nd highest price

Vickrey Auction

- everyone submits bids
- Highest bidder wins but pays externality to those who didn't get the good

First Price Auction

- Highest Bidder pays his price

Deffered Acceptance Algorithm

- One party proposes to the other & tentatively accepts unless someone better comes along

* optimal for the side that is proposing

↳ best for Men is worst for women

↳ Agents who are matched remain matched in all stable matches

- Stable
- IR
- Not SP
- Not PE

Lattice Theorem: there can exist more than 2 stable Matchings

Join: $\vee_M$ → Best for Men | Meet: $\wedge_M$ → Best for women

Medical Labor Market: Student vs. Hospital Proposing

* The Student-Optimal Stable Mechanism is Strongly More Manipulable than college Optimal Stable Mechanism (No couples)

↳ As more students & # hospitals kept constant, # of stable matchings decrease

Main Result: Proportion of Colleges that can manipulate go to 0 as # of colleges go to infinity, and stable matchings tend to one

Medical Labor Market: Couples

- Initially, one in the couple, would submit himself as leading member
- Given large # of hospitals it is likely that a couple will not displace another & a stable matching will be achieved

Boston Mechanism: Pre-2005

- In Round 1 only first choices considered
  ↳ In round k only k^{th} choice considered
- Very easy to Manipulate
- School Priorities based on walk zone & sibling

Chicago Selective Enrollment Mechanism

- Random Serial Dictatorship where they could rank k out of ℓ schools.
- New Chicago Mechanism is less manipulable than old one

* First-Pick-First Mechanisms more manipulable than

2012 Exam (Question 2)

[a] False, those are merely the 2 extreme cases, via the lattice theorem there can be more
men women

[b] we need to have Tie-Break Rule to ensure no one man rejected by a woman could complain & block

Prices vs. Non-Price Allocation (money vs. no money)

- Prices allow Expression of preferences
  - ↳ But if clearing price used. Income Dominates
- Rationing may lead to overdelivery of goods to those who don't really value them
  - ↳ But allows true need to be met (Equity)

Mechanism Properties

1. Pareto Efficiency
   - ↳ There exists no other outcome that makes everyone weakly better off
2. Strategy Proofness
   - ↳ Truthtelling is the Dominant Strategy of a mechanism
3. Core
   - ↳ if no agents want to make deal outside of mechanism
4. Individual Rationality
   - ↳ An individual prefers assignment to nothing
5. Stability → if not blocked by a pair or individual

Auctions

Generalized 1st Price

- everyone places bids simultaneously
- highest bidder receives 1st slot at their price

x click through rate

Generalized 2nd Price

- Each agent bids for a slot
- Highest bidder receives 1st slot at 2nd price

x click rate

Vickrey-Clark-Groves

- k identical goods will be auctioned to n bidders
- Auction elicits values on all goods
- Winners are top k Bidders

* Each winner pays
* total bids of those
* who'd win goods if
* wasn't present

Vickrey Clark Groves Mechanism (ex. 1)

- Apples $\boxed{A}$ & $\boxed{B}$ sold to 3 bidders
  - ↳ Bidder 1 wants one apple for \$2
  - ↳ Bidder 2 wants one apple for \$5
  - ↳ Bidder 3 wants both for \$6

Efficient Allocation

$$[\{A,2\},\{B,1\}]$$

↳ b/c combined \$7 of 1 & 2 > \$6 of bidder 3

transfer of payments:

$\boxed{1}$ = \$2 − \$6 (what bidder C would've gotten) = −\$4

$\boxed{2}$ = \$5 − \$6 = $\boxed{-\$1}$

$\boxed{3}$ = \$0

$\boxed{\text{seller Revenue} = \$5}$

- Produces PE & SP Allocation

Vickrey Clark Groves (2012 exam)

- Objects $\boxed{A}$ + $\boxed{B}$

Bidder 1: (A&B, 6)
Bidder 2: (A, 4)
Bidder 3: (B, 4)

a) Efficient Allocation

$$\begin{pmatrix} \text{Bidder 2} & \text{Bidder 3} \\ A & B \end{pmatrix}$$

b) Transfer of payments

$t_1 = 0$
$t_2 = 4 - 6 = \boxed{-2}$
$t_3 = 4 - 6 = \boxed{-2}$

c. Bidder cannot lie b/c Vickrey Mechanism is SP. Price he pays does not depend on his own report

d. Allocation is not in core b/c Bidder 1 & seller could leave and both be better off

e. Efficient Allocations

$$\begin{pmatrix} 2 & 3 \\ A & B \end{pmatrix}, \begin{pmatrix} 2 & 1 \\ A & B \end{pmatrix}$$

Top Trading Cycles (TTC)

- Each agent points to their favorite house & house points to owner, Run until done

Properties: SP, PE, IR

* With Indifference, core allocation doesn't always exist

1: a ~ b ~ c
2: a > b ~ c
3: a > b ~ c

Initial Allocation

$$\begin{pmatrix} 1 & 2 & 3 \\ a & b & c \end{pmatrix}$$

* Agents can Manipulate by swapping houses beforehand but not all strictly better

* series of Pareto improving trades does not always yield core

* Only running TTC with defined preferences delivers core allocation

* Random Serial Dictatorship is Equivalent to core from TTC w/ Random property rights

Lottery

Random Serial Dictatorship

- Probability of receiving choice dependent on your order of selection

x Not ordinally efficient

Probabilistic serial

- Constant eating Mechanism
- Is ordinally efficient but Not SP