MRI Image Reconstruction from undersampled K-Space data

Prakar K. (13485) and Satyam Dwivedi (13629)

March 20, 2017

EE698K Project Update
Instructor: Prof. Tanaya Guha, IIT-Kanpur.

1 Introduction

MRI scans are collected using Magnetic-Gradient coils, which collect the image data in K-Space domain, which is basically just the Fourier Transform of the original image. Collecting these samples requires the patient to stay still for 15-90 minutes, which is often inconvenient. Hence, techniques have been developed for fair reconstruction of MRI image at sub-nyquist sampling rate. We explore these methods in our project.

2 Reconstruction as an Optimisation Problem

The idea is to reduce the collection-time by randomly collecting K-Space coefficients. This results in incoherent noisy interference in reconstructed image. Compressed Sensing (CS) suggests that if the image is sparse in any Transform Domain (other than K-Space domain), then this interference can be removed by enforcing sparsity in that domain.
Hence, let $\mathbf{m}$ be the image in pixel domain, $\mathbf{y}$ be the collected samples in Fourier domain, $F_u = AF$, where A is the sampling-mask, F is the Fourier-matrix, and let Ψ be the transform domain where m is sparse, then our optimisation problem to get reconstructed signal $\mathbf{m_r}$ is:

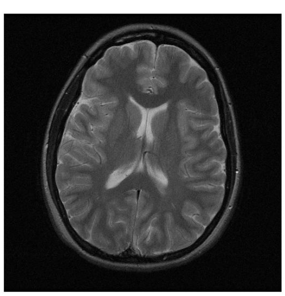
$$\mathbf{m_r} = ARGMIN_m \|\Psi\mathbf{m}\|_0 \text{ s.t. } F_u\mathbf{m} = \mathbf{y}$$

This problem is n.p. hard to solve so we relax the opmtimisation problem to be:

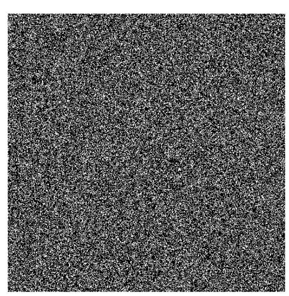
$$\mathbf{m_r} = ARGMIN_m\{\|F_u\mathbf{m} - \mathbf{y}\|_2^2 + \lambda\|\Psi\mathbf{m}\|_1\}$$

3 Setup

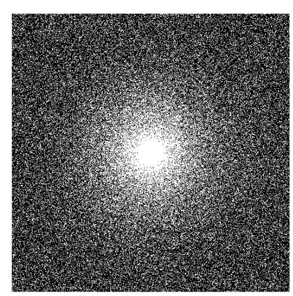
Michael Lustig has a very nice exercise on CS for MRI on his homepage [2]. We found the exercise very helpful in understanding the basic concepts of CS, and its application in medical imaging. We use fully sampled Brain Cross-Section image, from the same exercise. The subsampling in K-Space domain is simulated by undersampling this K-Space data randomly to retain just 33% coefficients. Since most of the energy is concentrated around origin so two undersampling masks (also provided in the exercise) are used: Uniform density mask, and Variable Density (Gaussian) mask.



Original Image

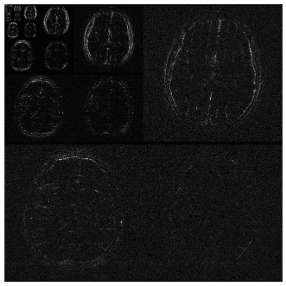


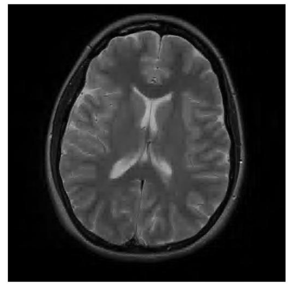
Uniform Density Mask



Variable Density Mask

The Sparsifying Transform is the Daubechies Wavelet Transform (DWT Toolbox given with the exercise). The following figures show the wavelet transform of image and the reconstruction using only top 2% coefficients:





Wavelet Transform of Image   Reconstruction using 2% coeffs

4 Some Methods:

4.1 Projection Onto Convex Sets (POCS)

The exercise suggested to use POCS method. Initialise $\mathbf{y_r} = \mathbf{y}$ and $\mathbf{m_r}$ and then repeat until convergence (more details in [2]):

- $\mathbf{m_r} = IFFT(\mathbf{y_r})$
- Take DWT of $\mathbf{m_r}$, soft-threshold all coefficients by λ, take IDWT and store in $\mathbf{m_r}$.
- $\mathbf{y_r} = FFT(\mathbf{m_r})$ and enforce data consistency (non-zero coefficients of $\mathbf{y}$ are forced-set into $\mathbf{y_r}$)

We implemented this algorithm ourselves, and the results (Structural Similarity Index (SSIM) and RMSE wrt original image of full K-Space data) are given in following table.

4.2 Non-Linear Conjugate Gradient Descent (NLCGD) with Back-Tracking Line Search

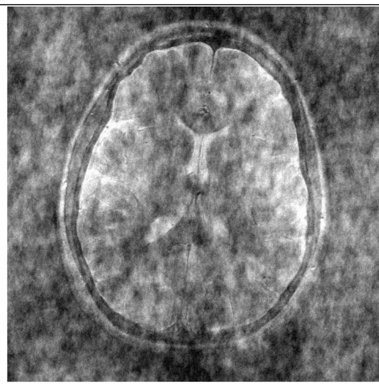
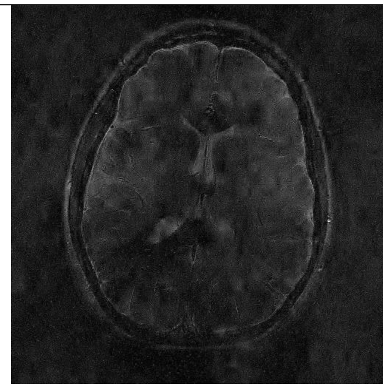
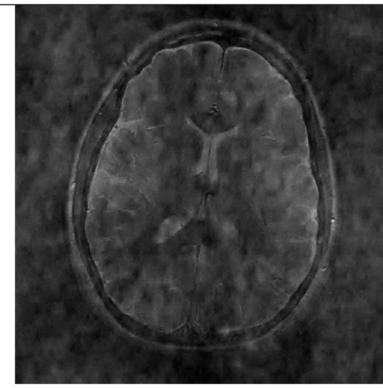
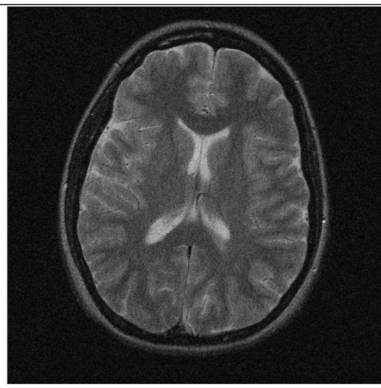
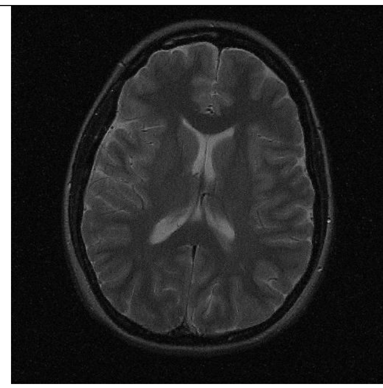
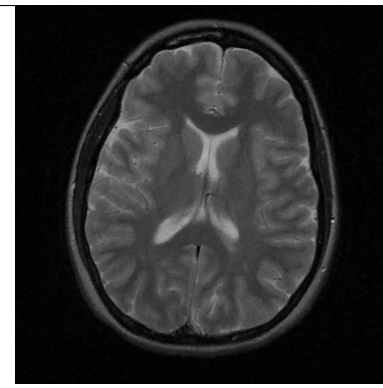
SparseMRI[1] modifies the original problem statement to include Finite Differences also. So basically they enforce sparsity in both DWT domain as well as in Finite Differences domain (FD):

$$\mathbf{m_r} = ARGMIN_m\{\|F_u\mathbf{m} - \mathbf{y}\|_2^2 + \lambda\|\Psi\mathbf{m}\|_1 + \alpha TV(\mathbf{m})\}$$

where Total Variation is $TV(\mathbf{m}) = \|FD(\mathbf{m})\|_1$. They solve this optimisation problem using NLCGD. We have used the author's implementation of NLCGD on our data. The SSIM and RMSE values wrt. original images are shown below. SparseMRI shows superior performance to POCS.

4.3 Adaptive Dictionary Learning

ADL[3] basically uses an overcomplete dictionary of image-patches as the sparse domain. The dictionary is adaptive because it is modified for each test-image separately. The paper claims that dictionary can be learnt on K-Space data of single MRI image as well i.e. no training data required. They follow an alternating approach of learning dictioanry and then computing K-Space coefficients (details in [3]).
Their code is not available so we are implementing it ourselves (implementation not completed yet).

	Inverse FFT	POCS	SparseMRI
Unif Sampled			
RMSE	0.0232	0.0187	0.0168
SSIM	0.2571	0.3498	0.4466
VarDens Sampled			
RMSE	0.0018	0.0062	0.0006
SSIM	0.5777	0.6785	0.7441

5 Future Plannings

We'll quickly complete our implementation of Adaptive Dictionary Learning, and then we'll try to come up with and test our own methods for reconstruction.

References

[1] Lustig, Michael, D. D., and Pauly., J. M. Sparse mri: The application of compressed sensing for rapid mr imaging. *Magnetic resonance in medicine 58.6 (2007): 1182-1195.*.

[2] Lustig., M. Cs exercise on mri, ee369c medical image reconstruction, autumn 2007, university of california, berkeley. *link: http://people.eecs.berkeley.edu/ mlustig/CS.html*.

[3] Ravishankar, S., and Bresler., Y. Mr image reconstruction from highly undersampled k-space data by dictionary learning. *IEEE transactions on medical imaging 30.5 (2011): 1028-1041.*.