Volatility Modelling of INR-USD Conversion Rate using ARCH(Autoregressive Conditionally Heteroscedastic)/ GARCH(Generalised Autoregressive Conditionally Heteroscedastic) Models in the Time of COVID-19

Ankur Dutta

Masters Student, School of Mathematics and Statistics, University of Hyderabad

September 2021

Abstract

On March 11, 2020 the World Health Organisation (WHO) declared the COVID-19 as a pandemic, pointing to over 118,000 cases of the coronavirus illness in over 110 countries and territories and sustained risk of further global spread. At the end of March 2020, when India went into a sudden and unprecedented lockdown, the Indian currency felt some heat for a bit. In April, the rupee touched a record low of 76.92 against a dollar, the unprecedented policy response and the heightened uncertainty worldwide, large swings in prices of various asset classes such as currency, bond, equity and credit were observed. Emerging market currencies in particular, witnessed considerable volatility. Volatility is an entity that can not be observed directly, financial analysis are especially keen to obtain a precise estimate of this conditional variance process, and consequently, a number of models have been developed that are especially suited to estimate the conditional volatility of financial instruments, of which the most well known and frequently applied model for volatility is the conditional heteroscedastic models. The main motive of building these models is to make a good forecast of future volatility. Application of ARCH and GARCH models are widespread in situation where the volatility of the conversion rates is a central issue.

Introduction

Our objective is to look into how the information can be used to to forecast mean and variance of the conversion rates, conditional on past information

The usd-inr conversion rates are stationary and barely seem to be autocorrelated. This serial correlation in squared returns has been initially modelled by Engle (1982) with ARCH (Autoregressive Conditional Heteroscedasticity) model and by Bollerslev (1986) with the Generalised ARCH (GARCH). The data which have been considered here is from 11 March 2020 (start of the pandemic) to 24 August, 2021. We propose a set of ARCH and GARCH models for modelling the usd-inr conversion rates and find the most suitable model in this report.

Data

The Data has been sourced from https://www.kaggle.com/sheshngupta/usd-to-inr-conversion-rates-12-years from which the relevant dates has been taken. the dataset contains three columns namely Dates, USD and INR (Indian Rupees per 1 United States Dollar).

Methodology

A Descriptive Study:

First we look into a brief descriptive summary of the Indian Rupees per 1 United States Dollar over the period of time.

Summary Measure	INR/USD	Summary Measure	INR/USD
Minimum	72.35	Mean	74.20
1st Quartile	73.31	Range	4.5867
Median	74.14	Std Deviation	1.073218
3rd Quartile	74.97	Skewness	0.3534781
Maximum	76.94	Kurtosis	2.20456

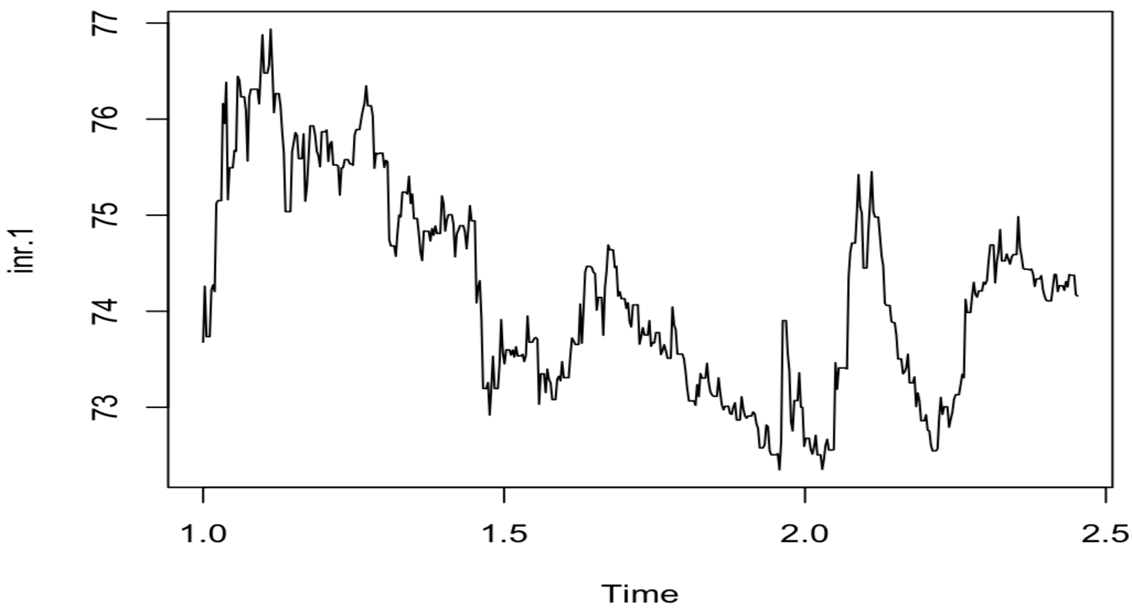


Figure 1. The trend graph of daily INR/USD conversion rates from March 11, 2020 to August 24, 2020

The ARCH Model:

The Autoregressive Conditional Heteroscedasticity (ARCH) model, originally introduced by Engle (1982) is a method that explicitly models the change in variance over time in a time series.
Specifically, an ARCH method models the variance at a time step as a function of the residual errors from a mean process (e.g. a zero mean).The approach expects the series is stationary, other than the change in variance, meaning it does not have a trend or seasonal component. An ARCH model is used to predict the variance at future time steps.

An ARCH(q) model is defined by

$$h_t = \omega + \sum_{i}^{q} \alpha_i e_{t-i}^2$$

where h_t is variance at time t, e_{t-i} is the model residual at time $t-i$

q: The number of lag squared residual errors to include in the ARCH model (A lag parameter must be specified to define the number of prior residual errors to include in the model.)

The GARCH Model:

The Generalised Autoregressive Conditional Heteroscedasticity, or GARCH was introduced Bollerslev (1986) is an extension of the ARCH model that incorporates a moving average component together with the autoregressive component.
Specifically, the model includes lag variance terms (e.g. the observations if modelling the white noise residual errors of another process), together with lag residual errors from a mean process.)

A GARCH(p,q) model is defined by

$$h_t = \omega + \sum_{i}^{q} \alpha_i e_{t-i}^2 + \sum_{j}^{p} \beta_j h_{t-j}$$

where h_t is variance at time t, e_{t-i} is the model residual at time $t-i$

p: The number of lag variances to include in the GARCH model. q: The number of lag residual errors to include in the GARCH model.

The introduction of a moving average component allows the model to both model the conditional change in variance over time as well as changes in the time-dependent variance. Examples include conditional increases and decreases in variance.

A GARCH model subsumes ARCH models, where a GARCH(0, q) is equivalent to an ARCH(q) model.

Testing Presence of any ARCH Effects:

Engle's (1982) ARCH LM Test provides a means of testing for serial dependence (auto-correlation) due to a conditional variance process by testing for auto-correlation within the squared residuals.

where, H_0 : no ARCH effects.
Executing the test we see that our p-value < 2.2e-16 which concludes that there is indeed presence of ARCH effects.

Suitable Mean Model and Model Selection:

We select our suitable mean model to be ARMA(4,3) and fit GARCH(1,1) , GARCH(1,2) , ARCH(1) and ARCH(2) to our data.

GARCH(p,q)	AIC	BIC
ARCH(1)	-0.2074558	-0.1270677
ARCH(2)	-0.2040900	-0.1156631
GARCH(1,1)	-0.2349660	-0.1465391
GARCH(1,2)	-0.2321871	-0.1357214

Based on the information criteria we see that the GARCH(1,1) has the lowest AIC and BIC of all the models and thus GARCH(1,1) was selected.

Having our model selected we make the following volatility plot.

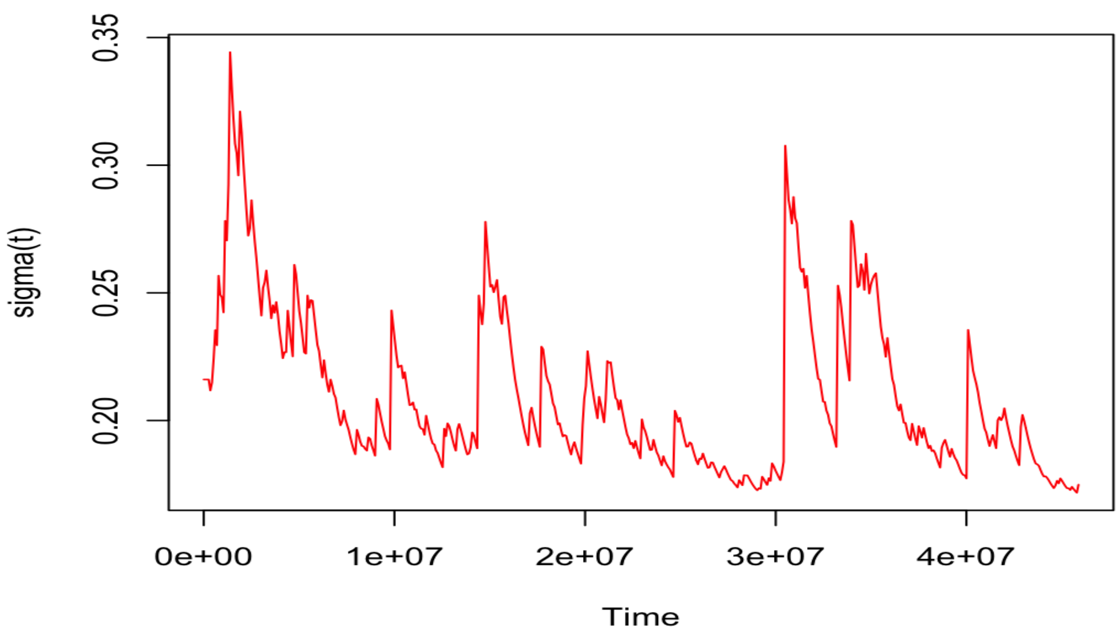


Figure 2 . The Volatility Plot

The presence of higher peaks suggests high instability in the conversion rates over the time period.

Model Adequacy Checking:

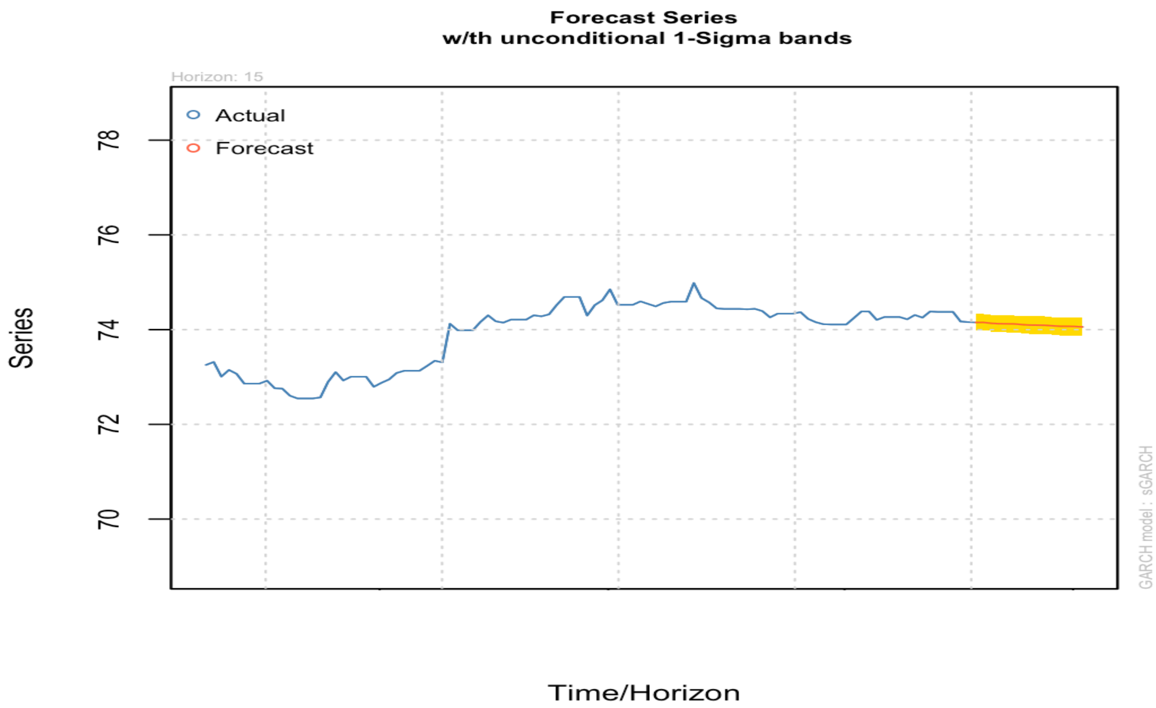
Standardised Residual Tests

Test	Residual	Statistic	p-value
Ljung-Box Test	R	$Q = 0.14583$	0.7026
Ljung-Box Test	R^2	$Q = 0.0082616$	0.9276
ARCH LM Test	R	$LM = 6.158$	0.9079
Jarque-Bera Test	R	$JB = 1229.1$	0
Shapiro-Wilk Test	R	$W = 0.89342$	0

The Ljung-Box Test suggest that the standardised residuals have no autocorrelation thus conclude that the model does not exhibit significant lack of fit and the residuals square and thus it does not exhibit presence of autoregressive conditional heteroscedasticity. The ARCH LM Test on the residuals suggest that there isn't any remaining significant ARCH effects in the residuals. However, the Jarque-Bera Test and the Shapiro-Wilk Test suggest that the distribution is non-normal.

Forecasting:

Now with the model fitted we forecast the next 15 days INR/USD conversion rates from August 25, 2021 to September 8, 2021.



Date	INR/USD	Date	INR/USD	Date	INR/USD	Date	INR/USD	Date	INR/USD
25 Aug	74.15	28 Aug	74.13	31 Aug	74.11	3 Sept	74.09	6 Sept	74.07
26 Aug	74.15	29 Aug	74.12	1 Sept	74.10	4 Sept	74.08	7 Sept	74.07
27 Aug	74.13	30 Aug	74.12	2 Sept	74.10	5 Sept	74.07	8 Sept	74.06

Conclusion

Volatility is considered as an important concept in many economic and financial applications. Here we used the GARCH(1,1) model based on the lowest AIC for modelling and forecasting the INR/USD conversion rates during the time of pandemic.

References

1. Robert Engle, “Autoregressive Conditional Heteroscedasticity with Estimates of the Variance of United Kingdom Inflation” Econometrica, Vol. 50, No. 4 (Jul., 1982).
2. Tim Bollerslev. “Generalized Autoregressive Conditional Heteroskedasticity” Journal of Econometrics 31 (1986): 307-327
3. Robert Engle. “GARCH 101: The Use of ARCH/GARCH Models in Applied Econometrics.” Journal of Economic Perspectives 15.4 (Fall 2011): 157-168.
4. Akaike, H. (1973). “Information theory and an extension of the maximum likelihood principle.” In B. N. Petrov & B. F. Csaki (Eds.),Second International Symposium on Information Theory, (pp. 267–281).
5. Bera A., Jarque C. (1981). “Effcient tests for normality, heteroskedasticity and serial independence of regression residuals: Monte Carlo evidence”, Economics Letter 7, 313 – 318.
6. Shapiro, S.S. and Wilk, M.B. (1965). “An analysis of variance test for normality (complete samples).”, Biometrika, 52, 591–611.
7. Ljung, G. M. and Box, G. E. P. (1978). “On a Measure of Lack of Fit in Time Series Models.” , Biometrika, vol. 65, 297-303.
8. Measuring Volatility Using the ARCH Model - Czar Yobero, 2018.
9. ARMA-GARCH modelling and white noise tests - James Proberts & Georgi N. Boshnakov, University of Manchester.
10. Risk, Volatility, and Factor Models - Robert L. Kissell, in Algorithmic Training Methods (Second Edition), 2021.
11. Price Volatility - Morton Glantz, Robert Kissell, in Multi-Asset Risk Modelling, 2014.
12. A Stationary Unbiased Finite Sample ARCH-LM Test Procedure - Pär Sjölander, 2011.