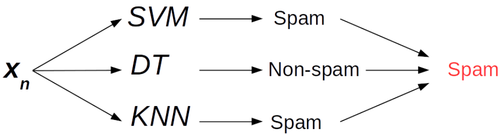
Some Simple Ensembles

- Voting or Averaging of predictions of multiple pre-trained models



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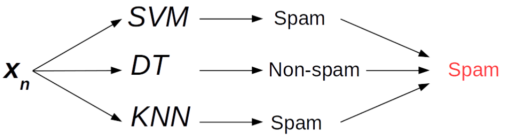
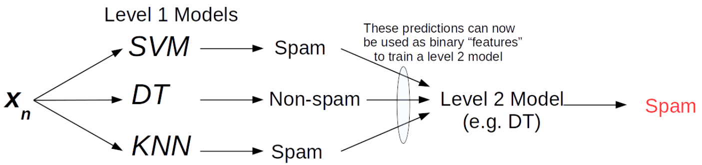


- "Stacking": Use predictions of multiple models as "features" to train a new model and use the new model to make predictions on test data



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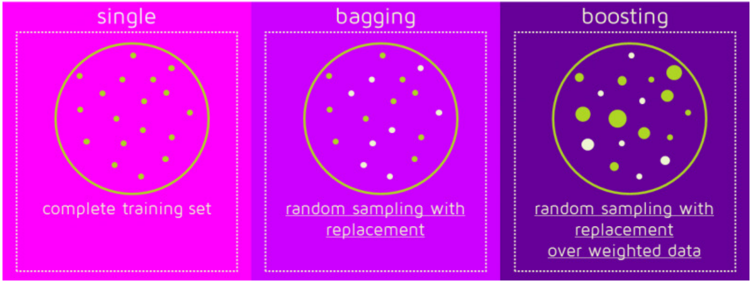

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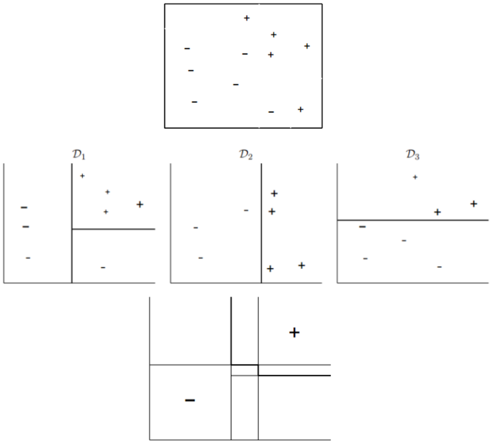
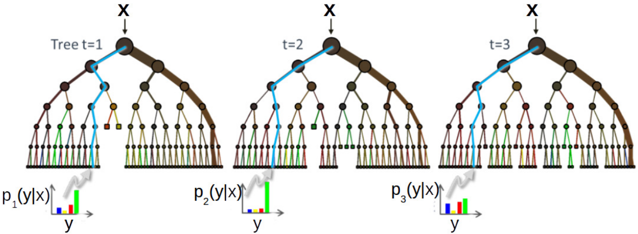
- Useful for models with high variance and noisy data

Bagging: illustration

Top: Original data, Middle: 3 models (from some model class) learned using three data sets chosen via bootstrapping, Bottom: averaged model



Random Forests



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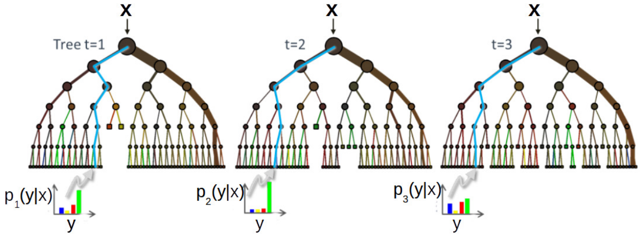

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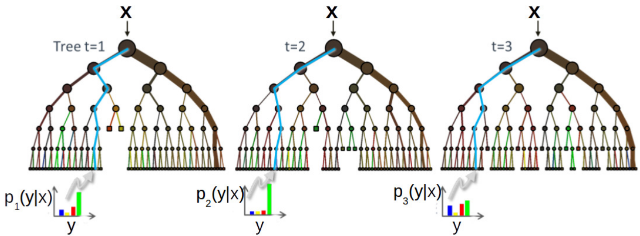

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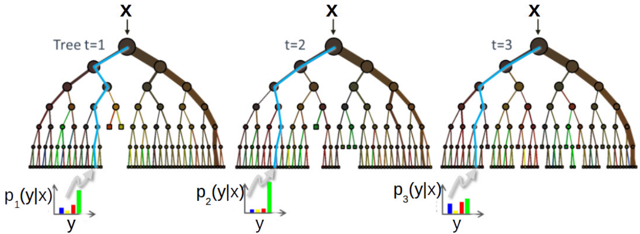

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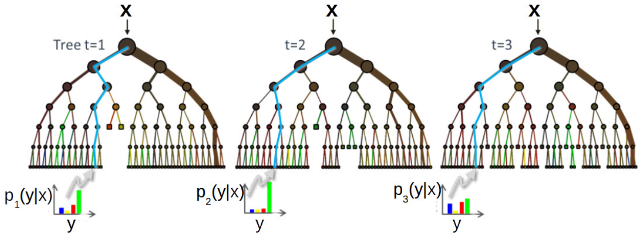

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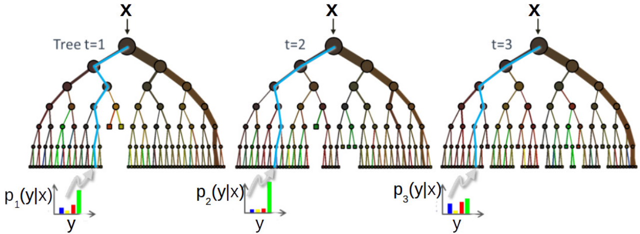

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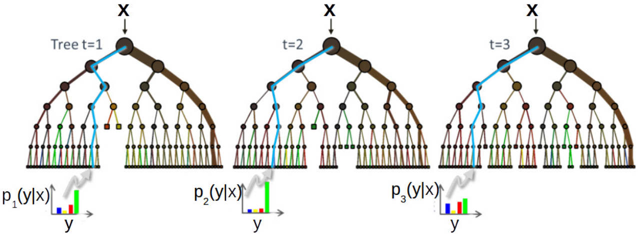

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- Prediction for a test example votes on/averages predictions from all the DTs

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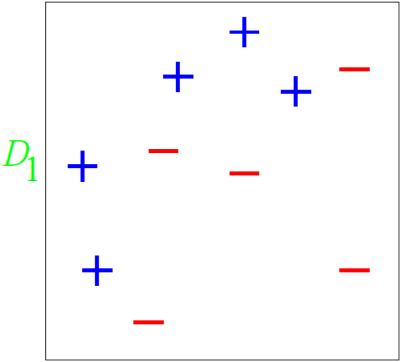
  - Normalize D_{t+1} so that it sums to 1: $D_{t+1}(n) = \frac{D_{t+1}(n)}{\sum_{m=1}^{N} D_{t+1}(m)}$
- Output the "boosted" final hypothesis $H(\boldsymbol{x}) = \text{sign}(\sum_{t=1}^{T} \alpha_t h_t(\boldsymbol{x}))$

AdaBoost: Example

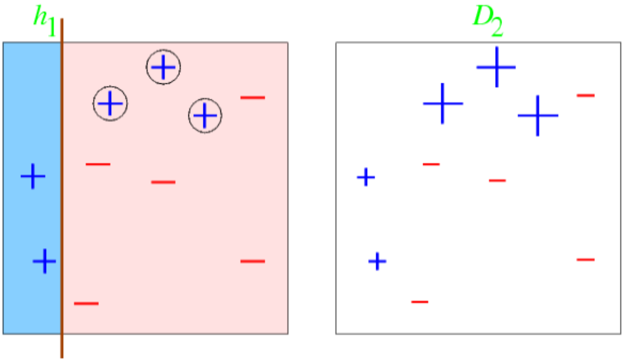
Consider binary classification with 10 training examples

Initial weight distribution D_1 is uniform (each point has equal weight $= 1/10$)



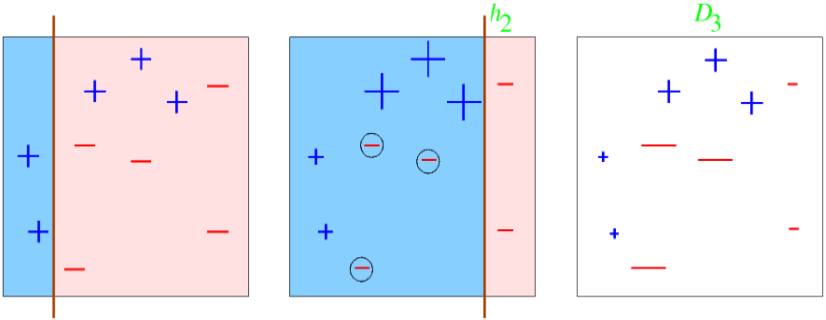
Each of our weak classifers will be an **axis-parallel linear classifier**

After Round 1



- Error rate of h_1: $\epsilon_1 = 0.3$; weight of h_1: $\alpha_1 = \frac{1}{2}\ln((1-\epsilon_1)/\epsilon_1) = 0.42$
- Each misclassified point upweighted (weight multiplied by $\exp(\alpha_2)$)
- Each correctly classified point downweighted (weight multiplied by $\exp(-\alpha_2)$)

After Round 2



- Error rate of h_2: $\epsilon_2 = 0.21$; weight of h_2: $\alpha_2 = \frac{1}{2}\ln((1-\epsilon_2)/\epsilon_2) = 0.65$
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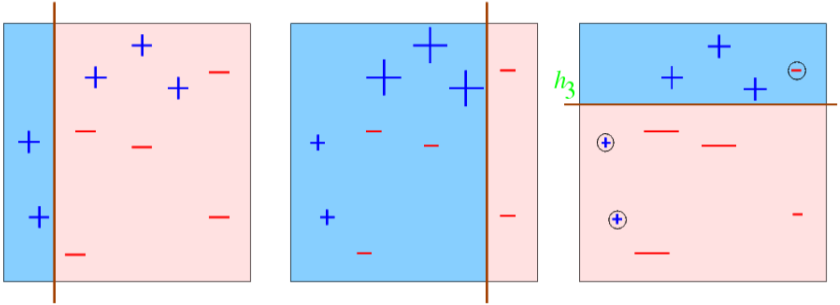
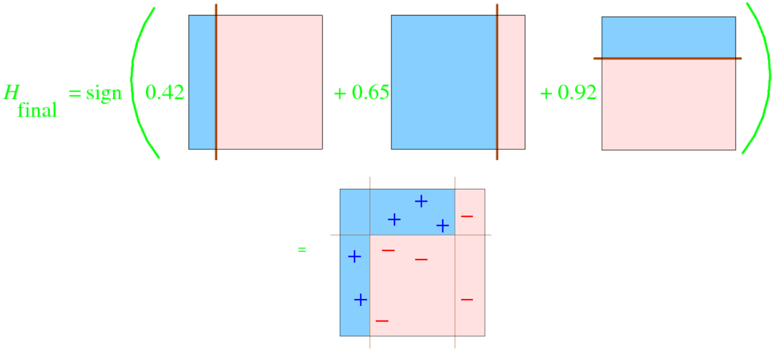
After Round 3



- Error rate of h_3: $\epsilon_3 = 0.14$; weight of h_3: $\alpha_3 = \frac{1}{2}\ln((1-\epsilon_3)/\epsilon_3) = 0.92$
- Suppose we decide to stop after round 3
- Our ensemble now consists of 3 classifiers: h_1, h_2, h_3

Final Classifier

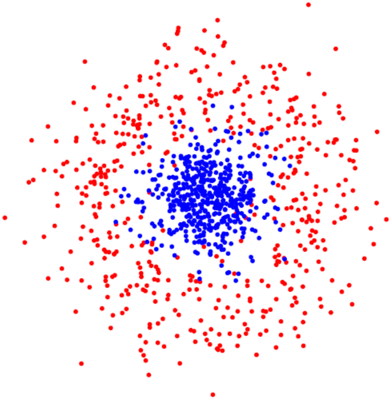
- Final classifier is a weighted linear combination of all the classifiers
- Classifier h_i gets a weight α_i



- Multiple weak, linear classifiers **combined** to give a strong, nonlinear classifier

Another Example

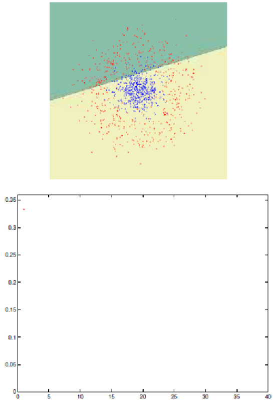
- Given: A nonlinearly separable dataset
- We want to use Perceptron (linear classifier) on this data



AdaBoost: Round 1

- After round 1, our ensemble has 1 linear classifier (Perceptron)
- Bottom figure: X axis is number of rounds, Y axis is training error

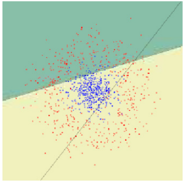
$t = 1$

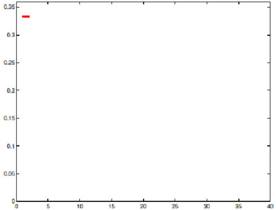


AdaBoost: Round 2

- After round 2, our ensemble has 2 linear classifiers (Perceptrons)
- Bottom figure: X axis is number of rounds, Y axis is training error

$t = 2$

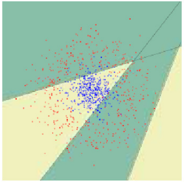


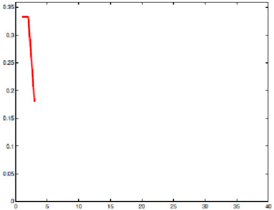


AdaBoost: Round 3

- After round 3, our ensemble has 3 linear classifiers (Perceptrons)
- Bottom figure: X axis is number of rounds, Y axis is training error

$t = 3$

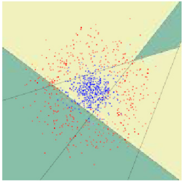


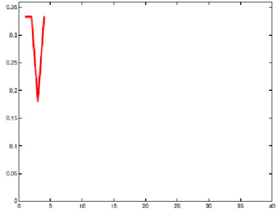


AdaBoost: Round 4

- After round 4, our ensemble has 4 linear classifiers (Perceptrons)
- Bottom figure: X axis is number of rounds, Y axis is training error

$t = 4$

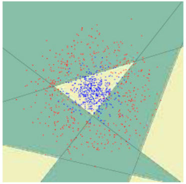


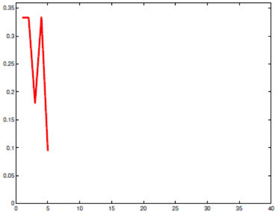


AdaBoost: Round 5

- After round 5, our ensemble has 5 linear classifiers (Perceptrons)
- Bottom figure: X axis is number of rounds, Y axis is training error

$t = 5$

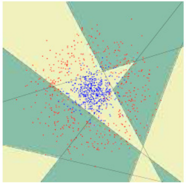


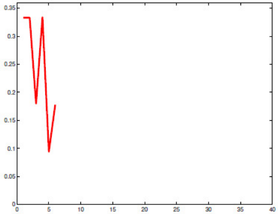


AdaBoost: Round 6

- After round 6, our ensemble has 6 linear classifiers (Perceptrons)
- Bottom figure: X axis is number of rounds, Y axis is training error

$t = 6$

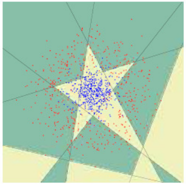


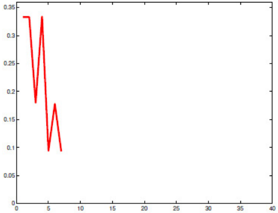


AdaBoost: Round 7

- After round 7, our ensemble has 7 linear classifiers (Perceptrons)
- Bottom figure: X axis is number of rounds, Y axis is training error

$t = 7$

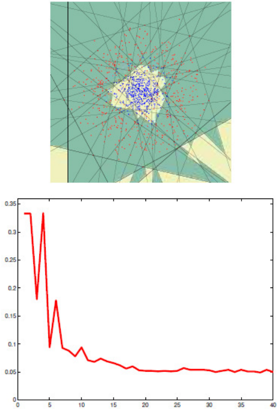




AdaBoost: Round 40

- After round 40, our ensemble has 40 linear classifiers (Perceptrons)
- Bottom figure: X axis is number of rounds, Y axis is training error

$t = 40$



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where $w_d = \sum_{t:i_t=d} 2\alpha_t s_t$ and $b = -\sum_t \alpha_t s_t$

Boosting: Some Comments

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- Boosting in general can perform badly if some examples are outliers

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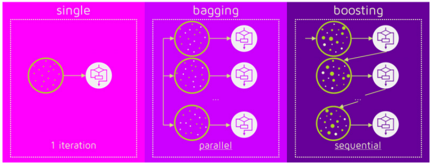

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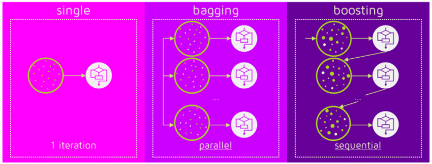

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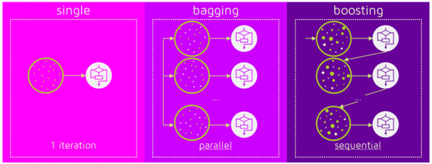

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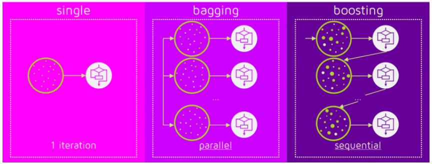

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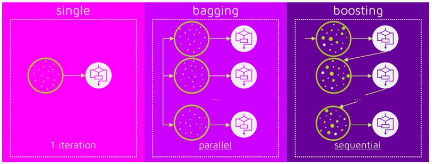

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- Bagging usually performs better than boosting if we don't have a high bias and only want to reduce variance (i.e., if we are overfitting)