

5. Comparison Of The Three Distribution Functions: The similarities and the differences of the three distributions can now be illustrated graphically by simultaneously plotting the occupation indices of the three distributions against the dimensionless quantity $\left(\frac{E_j - \mu}{k_B T}\right)$ as shown in Figure 4 below at a given temp-

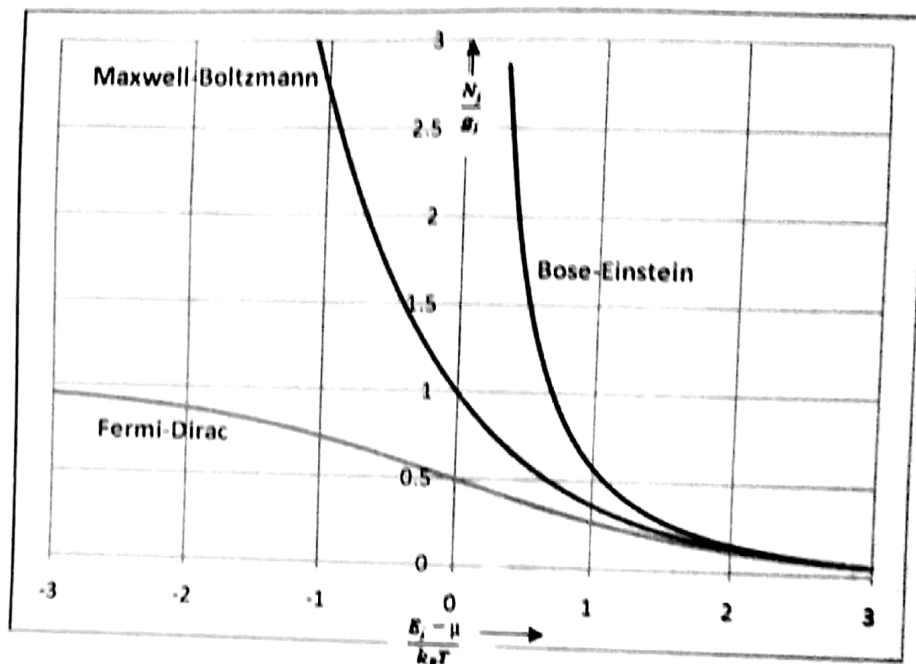


Figure 1. Plots of the Maxwell-Boltzmann, Bose-Einstein, and Fermi-Dirac distribution functions - Occupation Index $f(E_j) = N_j/g_j$ against $(E_j - \mu)/k_B T$.

erature. The energy therefore increases towards the right. (Note that technically the E_j 's are discrete energy levels.

So the ordinates of the curves in Figure 1 have meaning only for the *allowed* values of the E_j 's. In practice, however, the energy levels are closely spaced for systems of interest and essentially form a *quasi-continuum*. So the plotting of the distribution functions as continuous curves in Figure 1 is quite legitimate. This discussion will be taken up further in the following section.)

Let us now discuss some features of these distributions. The M-B distribution is a straightforward exponential function for all values of T and μ . Next, since N_j is necessarily a non-negative number (why?), the B-E distribution is only defined for positive values of the abscissa. Further since the distribution tends to infinity as the abscissa approaches zero, the chemical potential must necessarily be less than the lowest possible energy level, i.e., $\mu < E_1$. Both of these features are direct consequences of

the ' -1 ' term in the denominator. Further, the shape of the distribution curve indicates that particles like to concentrate in the energy levels whose energies (E_j) are only slightly greater than the system chemical potential μ . In the F-D distribution, because of the ' $+1$ ' term in the denominator, negative values of the abscissa is allowed but the occupation index is also restricted with a maximum value of 1. Physically, of course, this is because of the Pauli Exclusion Principle (how?). Again, the shape of the F-D distribution curve indicates that for low energies ($E_j \ll \mu$), the occupation index being close to unity, the low energy states are mostly occupied.

Both the B-E and F-D distributions approach the M-B distribution (i.e. a pure exponential decay) at higher energies with $E_j - \mu > k_B T$. That is, for the levels j with low occupation indices, (i.e., the number of particles N_j significantly lower than the degeneracy g_j) the B-E and F-D distributions begin to 'behave' like the M-B distribution with the same decay constant. Mathematically, this occurs because the exponential term in the denominator in Eqs. (31) and (32) begins to dominate at higher energies. We will discuss the F-D distribution function in some details in the following section.

for metal, Fermi-Dirac \rightarrow Numericals.

Class sheet

$g(E)$ vs E plot for $T = 0K$
 $T > 0K$ and its physical significance.

Derivation of Fermi-energy

Density of state and density of state's definition.

Density of state for photon,

$$g(p) dp = 2 \times \frac{V \times 4\pi p^2 dp}{h^3}$$

$$p = \frac{E}{c}$$

$$g(E) dE = \frac{8\pi V}{h^3} \times \frac{E^2}{c^2} \times \frac{1}{c} dE \quad \text{751} \Rightarrow$$

$$= \frac{8\pi V}{h^3 c^3} E^2 dE \quad \text{Last digit?}$$

Now, Density of state,

$$\frac{g(E) dE}{V dE} = \frac{8\pi V}{V h^3 c^3} E^2 \frac{dE}{dE} \quad \text{72008?}$$

$$G(E) = \frac{8\pi}{h^3 c^3} \times E^2 \quad \text{Last digit?}$$

Semester

1) Numericals on micro/macro states.

2) Derivation

i) Density of state (6-D Phase space)

ii). Fermi-Dirac (Distribution law).

iii). M-B

$N(E) dE \rightarrow$ given.

find:-

i) Avg. Energy

ii) Avg. Velocity

iii) Most probable speed.

iv) Planck's black body law from B-E.

v) Derivations of Fermi-energies.