Chapter-3

Dielectric

3.1. Introduction

So far, we have discussed the electric field produced by a charged particle in the absence of matter (i.e., in vacuum). Now, we will consider the electrostatic problems in a dielectric medium other than in empty space.

A dielectric substance is a material in which all the electrons are firmly bound to the atoms of the material. There are no free electrons to conduct electricity. So, they under almost all conditions are insulators. Their atoms, ions or molecules may be polarized under the influence of an external electric field. Thus, **dielectric material is a special substance that under almost all conditions is insulator and has an ability to be polarized and can persist electrostatic field for a long time in it.**

• ***Examples :*** glass, mica, paper, gutta-percha, paraffin wax, air etc.

The resistivity of dielectric substances is in the range of $10^6 \Omega \cdot \text{m}$ to $10^{16} \Omega \cdot \text{m}$.

Dielectric substance plays a vital role in many electronic applications. It has been observed that no electronic circuit can be free from dielectric medium.

The dielectric substance is characterised by a parameter known as dielectric constant. The **dielectric constant (or relative permittivity) 'k' is defined as the ratio of the permittivity (ϵ) of the dielectric medium to that of free space ϵ_0** i.e., $k = \frac{\epsilon}{\epsilon_0}$

In SI unit, $\epsilon_0 = 8.854 \times 10^{-12}\ \text{F} \cdot \text{m}^{-1}$ and k is a dimensionless quantity.

Its value for different materials are given in Table 1 :

Table 1 : Dielectric constant k of different media

Key to material state : L — Liquid, S — Solid, G — Gas.

Material media	k	Temperature (°F)	State
Vacuum	1	—	—
Air (Dry) (1.005)	1	68	G
Teflon	2	75	S
Paraffin	2.3	75	S
Petroleum oil	2	75	L
Benzene	2.3	77	L

Material media	k	Temperature (°F)	State
		68	S
Polyethylene (plastic)	2.2	75	S
Polystyrene resin	2.4	75	S
Rubber (hard)	3	68	L
Water	80	75	S
Paper	2	75	S
Nylon (plastic)	2	75	S
Mica	7	75	S
Glass (Borosilicate)	4.8	75	S
Porcelains	5	75	S
Bakelite	5	75	S
Germanium Tetrachloride	2.4	75	S
Ethanol	24.3	77	L
Ammonia	25	– 104	L
Acetone	20.7	77	L
Methanol	33.1	68	L
Glycerin	47	68	L

Source : Delta Controls Corporation Website— www.deltacnt.com

3.2. Polar and Nonpolar Dielectrics

Any material media has large number of molecules. Each molecule consists of atomic nucleus and electrons. All positive charges of a molecule may be replaced by a single imaginary equivalent positive charge located at the centre of gravity of existing positive charges. Similarly, all negative charges of it may be replaced by a single imaginary equivalent negative charge located at the centre of gravity of existing negative charges.

Depending on the separation between the centre of gravity of positive charges and the centre of gravity of negative charges, the dielectrics are classified as polar (dipole) dielectrics and non-polar (neutral) dielectrics.

Polar molecules **The polar molecule [Fig. 1] is one in which the centre of gravity of the positive charges is separated from the centre of gravity of negative charges by a finite distance.** So *the polar molecule is thus an electric dipole [Fig 2] and has an intrinsic (permanent) dipole moment.* The material made up of polar molecules are called polar dielectric. Thus, *the dielectrics whose molecules have permanent dipole moment are called* ***polar dielectrics.***

• ***Examples :*** N_2O, H_2O, HCl, NH_3 and CO_2.

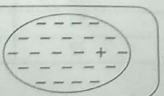

Fig. 1 ▷ Polar molecule

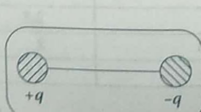

Fig. 2 ▷ Electric dipole

The diagram of polar molecule is shown in Fig. 1. In normal state (i.e., in absence of electric field), the positive and negative charges of the molecules of a polar dielectric are randomly oriented and they compensate each other. Thus, *the net (resultant) dipole moment per unit volume of the polar dielectric substance is zero even though each molecule possesses a permanent dipole moment.*

Nonpolar molecules ***The nonpolar molecule [Fig. 3] is one in which the centre of gravity of the positive charges (nuclei) coincides with the centre of gravity of negative charges*** *(electron cloud).* Thus, the nonpolar molecule has a zero electric dipole moment in absence of electric field. The material made up nonpolar molecules are called **nonpolar dielectric.**

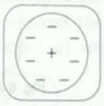

Fig. 3 ▷ Nonpolar molecule

Molecules having symmetrical structure are nonpolar. Thus, the dielec-trics whose molecules have no permanent dipole moment are called nonpolar dielectrics.

• ***Examples :*** (i) *Monoatomic molecules* like He, Ne, Ar, Xe and (ii) *mole-cules consist of two identical atoms* like H_2, N_2, O_2, Cl_2 are nonpolar. CO_2 is also a nonpolar molecule due to its linear symmetrical structure.

Special Note :

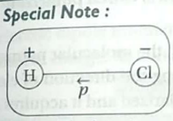

Fig. 4 ▷ Dipole nature of HCl molecule

(i) In HCl molecule the H end of the molecule is positive and the Cl end is negative. The molecule is therefore acts as a dipole having a dipole moment $\vec{p}$ directed from the Cl atom to the H atom [Fig 4].

(ii) In H_2O molecule, oxygen atom is slightly negative relative to the H atoms [Fig. 5(a)]. So the two H – O bonds in H_2O molecule will produce two dipole moments $\vec{p}_1$ and $\vec{p}_2$. In that case, the resultant dipole moment $\vec{p}$ is directed from O toward H – H base [Fig. 5(b)].

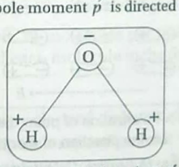

Fig. 5 (a) ▷ Dipole nature of H_2O molecule

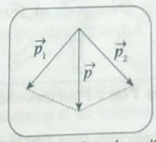

Fig. 5 (b) ▷ The direction of resultant dipole moment of H_2O molecule

3.3. Effect of Electric Field on Polar and Nonpolar Molecules; Polarization in Dielectrics

In dielectric material, electrons of the outermost shell are firmly bound with finite forces and are not able to move freely. But when an electric field is applied on a dielectric medium, we may certainly expect a displacement of these bound charges as they experience electrostatic forces. Now, we will discuss this microscopic effect of an electric field on polar and nonpolar dielectrics.

For nonpolar molecules Consider an atom of nonpolar dielectric consisting of a negative charge $-Q$ (electron cloud) and a positive charge $+Q$ (nucleus). Under no electric field condition, the positive and negative charges are so close to each other that their action becomes neutralized [Fig. 6(a)]. But when an electric field $(\vec{E})$ is applied, the positive charge is displaced locally a little from its equilibrium position in the direction of electric field $(\vec{E})$ by the force $\vec{F_1} = Q\vec{E}$ and the negative charge is displaced by a force $\vec{F_2} = Q\vec{E}$ in the opposite direction [Fig. 6(b)].

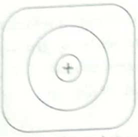

Fig. 6 (a) ▷ Nonpolar dielectric atom

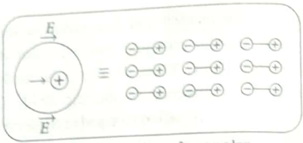

Fig. 6 (b) ▷ Polarization of nonpolar dielectric in the electric field $\vec{E}$

The displacement of these charges results in the creation of electric dipole. This phenomenon is called polarization. So the molecule acquires an induced electric dipole moment in the direction of the electric field.

The molecular phenomenon at which the molecules of a dielectric are aligned in the direction of applied electric field and produced electric dipoles is called polarization or dielectric polarization.

- **For polar molecules** In case of polar dielectric [Fig. 7(a)], the molecular permanent dipoles of it experience a torque which tends to align them along the direction of electric field [Fig. 7(b)]. Thus, the dielectrics in this state are said to be polarized and it acquires a net dipole moment.

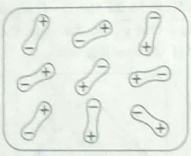

Fig. 7 (a) ▷ Polar dielectric

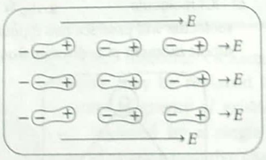

Fig. 7 (b) ▷ Polarization of polar dielectric in the direction of electric field $(\vec{E})$

We can also conclude from that above discussion that *the entire dielectric can be replaced by a configuration of vast number of tiny electric dipoles.*

3.3.1. Atoms with Specially Symmetric Charge Distribution

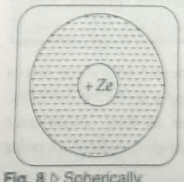

Fig. 8 ▷ Spherically symmetric charge distribution of atom

If we consider an atom of atomic number Z, its positive charge $+Ze$ is considered to be concentrated within the nucleus and acts at the centre of the atom [Fig.8]. The negative charge of the atom is $-Ze$ and it makes the atom neutral.

For a spherically symmetric atom, all these negative charge electrons are uniformly distributed over the volume of the atom. So the electrons revolving round the nucleus behaves like a **spherically symmetric cloud of negative charge** whose centre of gravity is at the centre of the atom. Now the centre of gravity of positive charge as well as negative charge of the atom will coincide and hence the atom will have **zero dipole moment**. But, the **atom with zero dipole moment is nonpolar.** Therefore, **atoms with spherically symmetric charge are nonpolar.**

3.4. Electric Dipole Moment $(\vec{p})$ and Electric Polarization Vector $(\vec{P})$

3.4.1. Electric Dipole Moment $(\vec{p})$

When an electric field is applied on a dielectric material, electric dipoles are produced. The dipole is an entity in which equal positive charge $(+q)$ and negative charge $(-q)$ are separated by a small distance $(\vec{r})$ as shown in Fig. 9.

The atomic electric dipole moment $(\vec{p})$ is defined as the product of the positive or negative charge (q) of an atom and the separation (r) between the centres of positive and negative charges.

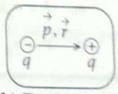

Fig. 9 ▷ The direction of dipole moment $(\vec{p})$

Thus the dipole moment,

$$\vec{p} = q\vec{r} \quad \cdots(3.4.1.1)$$

This electric dipole moment is a vector quantity and is directed from the negative charge to the positive charge. Its unit is debye (D).

$$1 \text{ debye} = 10^{-18} \text{ stat C} \cdot \text{cm} \approx (3.33) \times 10^{-30} \text{ C} \cdot \text{m}$$

3.4.2. Polarization Vector $(\vec{P})$ or Polarization Density Vector

The electric dipole moment per unit volume is called Polarization (P). It is directed from negative charge to positive charge.

If n is the number of molecules per unit volume and p is the component of the electric dipole moment of each molecule in the field direction, the polarization vector,

$$\vec{P} = n\vec{p} \quad \cdots(3.4.2.1)$$

For a continuously polarized dielectric, the polarization $\vec{p}$ at any point may be defined as the net electric dipole moment per unit volume by considering of an infinitesimally small volume Δv, with the limit taken as Δv shrinks to a point. Thus

$$\vec{P} = \lim_{\Delta v \to 0} \frac{\Delta\vec{p}}{\Delta v} \quad \cdots(3.4.2.2)$$

3.4.3. Electric Susceptibility (χ_e)

When a dielectric material is placed in an electric field, the dielectric molecules become electrically polarized. For a linear and homogeneous dielectric, the polarization vector $(\vec{p})$ is proportional to the electric field $(\vec{E})$ experienced by the molecules of the dielectric.

So, $\vec{P} = \chi\epsilon_0\vec{E}$ $\cdots(3.4.3.1)$

where χ is called the electric **susceptibility of the dielectric.**

Hence, $\chi = \dfrac{P}{\epsilon_0 E}$ $\cdots(3.4.3.2)$

Thus **the electric susceptibility of a dielectric is defined as the ratio of the polarization per unit volume to the electric intensity in the dielectric.** It is the characteristic property of a dielectric material. It has always positive value.

> **Special Note :**
>
> If $q\vec{r}$ is net dipole moment in volume Ar,
>
> $\vec{P} = \frac{q\vec{r}}{Ar}$; A is the area of the dielectric material.
>
> or, $\vec{P} = \frac{q}{A}\hat{u}_d$, u_d is a unit vector directed from $-q$ to $+q$,
>
> or, $\vec{P} = \sigma_p \hat{u}_d$, σ_p = surface charge density of polarization, $= \frac{q}{A}$
>
> or, $P = \sigma_p$ POLARIZATION = SURFACE CHARGE DENSITY

3.4.4. Atomic Polarizability (α)

The net induced dipole moment of an atom of a dielectric substance placed in an electric field is proportional to the applied electric field $(\vec{E})$ and its direction is parallel to the direction of electric field. So

$$\vec{p} \propto \vec{E}$$

or,
$$\vec{p} = \alpha\vec{E} \qquad \cdots(3.4.4.1)$$

where $\alpha\left(=\frac{\vec{p}}{\vec{E}}\right)$ is proportionality constant, called **atomic polarizability.**

Thus the atomic polarizability is the amount of induced electric dipole moment of an atom of a dielectric substance by an electric field of unit strength.

The unit of $\vec{p}$ is C · m and that of $\vec{E}$ is V · m^{-1}.

$\therefore$ Unit of $\alpha = \frac{\text{C} \cdot \text{m}}{\text{Vm}^{-1}} = \text{C} \cdot \text{V}^{-1}\text{m}^2 = \text{F} \cdot \text{m}^2$

$\because$ 1 F (farad) $= \frac{1\text{C(coulomb)}}{1\text{V(volt)}}$

We can write from equation (3.4.2.1) and equation (3.4.4.1), the polarization,

$$\vec{P} = n\alpha\vec{E} \qquad \cdots(3.4.4.2)$$

Again from equation (3.4.3.2), we know, $\chi = \frac{P}{\epsilon_0 E}$.

Substituting the magnitude of $\vec{P}$ in this equation, we get

$$\chi = \frac{n\alpha E}{\epsilon_0 E}$$

or,
$$\chi = \frac{n\alpha}{\epsilon_0} \qquad \cdots(3.4.4.3)$$

Problem 1

Calculate the induced dipole moment per unit volume of He gas if placed in a field of 6000 V · cm^{-1}. The atomic polarizability of He = 0.18×10^{-40} F · m^2 and density of He = 2.6×10^{25} atoms per m^3. Also calculate the separation between the centres of positive and negative charges.

[Pbi. U. 1991]

Solution Here, atomic polarizability of He, $\alpha = 0.18 \times 10^{-40}$ F · m^2

Electric field, $\vec{E} = 6000$ V · cm^{-1} $= 600000$ V · m^{-1}

As the density of He $= 2.6 \times 10^{25}$ atoms per m^3, the number of He atoms per m^3, $n = 2.6 \times 10^{25}$.

Now dipole moment of the atom,

$$p = \alpha E = 0.18 \times 10^{-40} \times 600000$$
$$= 1.08 \times 10^{-35} \text{ C} \cdot \text{m}$$

So the induced dipole moment per unit volume,

$$P = np = 2.6 \times 10^{25} \times 1.08 \times 10^{-35} = 2.81 \times 10^{-10} \text{ C} \cdot \text{m}^{-2}$$

The separation between the centres of positive and negative charges $(q = 2e)$ of He atom is

$$dl = \frac{p}{q} = \frac{1.08 \times 10^{-35}}{2 \times 1.6 \times 10^{-19}} = 3.37 \times 10^{-17} \text{ m}$$

Problem 2

If the density of CO_2 is 1.977 kg · m^{-3} and its susceptibility χ is 0.985×10^{-3}, find its polarizability.

Solution The molecular weight (M) of CO_2 = 44

The density (d) of CO_2 $= 1.977$ kg · m^{-3} (given)

$\therefore$ The number of molecules per cubic metre,

$$n = \frac{6.023 \times 10^{23}}{M} d = \frac{6.023 \times 10^{23}}{44} \times 1.977$$

Again dielectric susceptibility, $\chi = \frac{n\alpha}{\epsilon_0}$

$$\therefore \quad \alpha = \frac{\chi\epsilon_0}{n}$$

Putting, $\chi = 0.985 \times 10^{-3}$, $\epsilon_0 = 8.854 \times 10^{-12}$ F · m^{-1} and the value of n from above, we get

$$\alpha = \frac{8.854 \times 10^{-12} \times 0.985 \times 10^{-3}}{6.023 \times 10^{23} \times 1.977} \times 44$$
$$= 3.2226 \times 10^{-37} \text{F} \cdot \text{m}^2$$

3.5. Weakening of Electric Field within the Dielectric

Let us consider a parallel plate capacitor AB [Fig. 10]. When a uniform electric field of intensity $\vec{E_0}$ is applied across AB, one plate will acquire a positive charge $(+q)$ and the other a negative charge $(-q)$. Now if a dielectric slab is placed between the plate of the capacitors, a minute shift of negative charge occurs towards the positive plate and a shift of positive charge occurs towards the negative plate. So, dielectric slab becomes electrically polarized and its molecules become electric dipoles [Fig. 10].

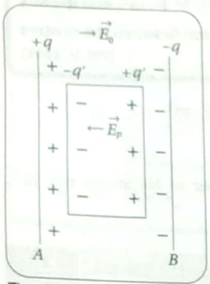

Fig. 10 ▷ Electric polarization of a dielectric placed in uniform electric field.

As a result of polarization, negative charge appears on one face of the slab and an equal but opposite charge on the other face. This produces an electric field of intensity E_p. It will be induced within the dielectric slab that opposes the external field $\vec{E_0}$. The resultant electric field $\vec{E}$ within the dielectric,

$$\vec{E} = \vec{E_0} - \vec{E_p} \qquad \cdots(3.5.1)$$

But the induced electric field $(\vec{E}_P)$ is always smaller than applied electric field $(\vec{E_0})$. Thus, **when a dielectric is placed in an electric field, the field with in the dielectric is reduced.**

3.6. Few Fundamental Results of Electrostatics

3.6.1. Electric Field and Electric Field Intensity

An electric charge experiences a force in the vicinity of a system of charges. *The region over which an electric charge experiences electric force is called an electric field and its strength at any point is defined as the force experienced per unit positive test charge placed at that point. If F is the force experienced by a test charge q placed at a point in an electric field, the electric field intensity at that point is given by*

$$\vec{E} = \frac{\vec{F}}{q} \qquad \cdots(3.6.1.1)$$

The direction of the electric field intensity is that of the force $\vec{F}$ exerted on a positive charge.

Electric field intensity of a point charge Let Q be a point charge placed at O [Fig. 11]. To find the intensity $\vec{E}$ of the electric field for due to the charge Q at a point P at a distance r, a test charge $+q$ at P is placed. Thus, following Coulomb's law, force experienced by the test charge q is

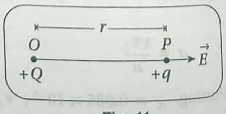

Fig. 11

$$F = \frac{Qq}{4\pi\epsilon r^2} \qquad \cdots(3.6.1.2)$$

where ϵ is the permittivity or *dielectric constant of the medium* in which the charges are placed. For vacuum, $\epsilon = \epsilon_0 = 8.854 \times 10^{-12}$ farad · metre^{-1}.

∴ The electric field intensity in a medium of dielectric constant ϵ is

$$\vec{E} = \frac{\vec{F}}{q} = \frac{Q}{4\pi\epsilon r^2}\hat{n} \qquad \cdots(3.6.1.3)$$

where $\hat{n}$ is the unit vector along the direction of electric field intensity.

3.6.2. Electric Flux Density $(\vec{D})$

We know an electric charge is considered to give rise to electric flux that streaming away from the charge. **The electric flux density $(\vec{D})$ is defined as the number of limes of force crossing normally per unit area of a surface.** So the electric flux density at a distance r from a point charge of magnitude Q is

$$\vec{D} = \frac{Q}{4\pi r^2}\hat{n} \qquad \cdots(3.6.2.1)$$

where $4\pi r^2$ is the surface area of a sphere of radius r.

We can write the relation between electric field intensity $(\vec{E})$ and electric flux density $(\vec{D})$ for an isotropic medium from equations (3.6.1.3) and equation (3.6.2.1) as

$$\vec{D} = \epsilon\vec{E} \qquad \cdots(3.6.2.2)$$

or, $$\vec{D} = k\epsilon_0\vec{E} \qquad \cdots(3.6.2.3)$$

[∵ dielectric constant of any substance, $k = \frac{\epsilon}{\epsilon_0}$]

> **Special Note :**
> In an isotropic medium ϵ is independent of the direction of measurement. If ϵ depends upon the direction of measurement, then $\vec{D}$ and $\vec{E}$ should not be parallel and then ϵ would be a tensor.

3.6.3. Gauss' Law in Dielectrics

It states that the total electric flux ϕ emitting from a closed surface is equal to the total charge (Q) enclosed by the surface.

If the charges enclosed by the surface are $Q_1, Q_2, Q_3, \cdots, Q_n$, this law is expressed as

$$\phi = \int_S \vec{E}\cdot d\vec{S} = \frac{\sum_{i=1}^{n} Q_i}{\epsilon} \quad \text{or,} \quad \int_S \epsilon\vec{E}\cdot d\vec{S} = \sum_{i=1}^{n} Q_i$$

or, $$\int_S \vec{D}\cdot d\vec{S} = \sum_{i=1}^{n} Q_i \qquad \cdots(3.6.3.1)$$

where $\vec{D}$ represents the electric flux density, S is the area of the closed surface and $d\vec{S}$ is a small vector area on the surface.

3.7. Homogeneity, Linearity and Isotropy

A medium is said to be **homogeneous** if its **physical characteristics** (i.e., permittivity, mass, density, molecular structure) **are same at all points**. So, its physical characteristics is independent on the space co-ordinates (x, y, z). If the **physical characteristics** of the medium **depend on the space co-ordinates**, the medium is called **inhomogeneous medium**. The atmosphere is an example of a homogeneous medium as its permittivity varies with altitude.

A medium is said to be **linear** if its **permittivity** (ϵ) **does not change with the applied electric field** (E). If it does not remain constant, the medium is said to be non-linear. The flux density $\vec{D}\ (= \epsilon\vec{E})$ is constant in a linear medium.

A material is said to be **isotropic** if its properties (like permittivity) **does not change with direction**. If it is **not constant**, the material is said to be **anisotropic medium**. Basically crytalline materials are anisotropic.

We can conclude from the above discussions that **a dielectric material (where $\vec{D} = \epsilon\vec{E}$ is applicable) becomes linear if ϵ (permittivity) does not change with the applied electric field, homogeneous if ϵ does not change from point to point and isotropic if ϵ does not change with direction.**

3.8. Relation between Electric Flux Density ($\vec{D}$), Electric Field ($\vec{E}$) and Polarization ($\vec{P}$) in a Dielectric Medium

Suppose an isotropic, homogeneous dielectric slab of small thickness t and large face area A is placed between the plates of a parallel plate capacitor [Fig. 12]. When an external electric field of intensity $\vec{E}_0$ is applied across the two plates of the capacitor, one plate will acquire a positive charge of $+q$ and the other a negative charge of $-q$. But the dielectric slab, becomes polarized due to this applied electric field on the capacitor.

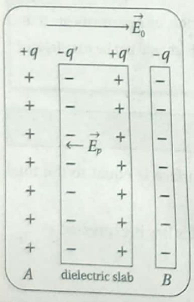

Fig. 12 ▷ The direction of induced electric field $(\vec{E}_p)$ due to polarization of dielectric material

Due to the polarization of dielectric slab, let $-q'$ and $+q'$ be the bound charges induced on the end faces of the slab producing their own electric field intensity $(\vec{E}_p)$. Thus, *the dipole moment of the whole slab is $q't$ and its volume is At.* The magnitude of electric polarization $|\vec{P}|$ is defined as the dipole moment per unit volume.

$$\therefore \quad |\vec{P}| = \frac{q't}{At} \quad \text{or,} \quad |\vec{P}| = \frac{q'}{A}$$

$$\text{or,} \quad |\vec{P}| = \sigma_p \qquad \cdots(3.8.1)$$

where $\sigma_p = \frac{q'}{A}$ = surface *density of polarization charges* on the dielectric (i.e. surface density of induced charge).

The induced electric field $(\vec{E}_p)$ due to the polarization of charges of dielectric material always opposes the external electric field $\vec{E}_0$. So the resultant electric field within the dielectric,

$$\vec{E} = \vec{E}_0 - \vec{E}_p \qquad \cdots(3.8.2)$$

If σ_p is the surface density of polarization (induced) charges on the dielectric slab, the magnitude of electric field due to polarization,

$$|\vec{E}_p| = \frac{\sigma}{\epsilon_0} \qquad \cdots(3.8.3)$$

Similarly, if σ is the surface density of charges on the capacitor plate, $|\vec{E}_0| = \frac{\sigma}{\epsilon_0}$

Now we can write from equation (3.8.2),

$$|\vec{E}| = \frac{\sigma}{\epsilon_0} - \frac{\sigma_p}{\epsilon_0} \quad \text{or,} \quad \epsilon_0|\vec{E}| = \sigma - \sigma_p \quad \text{or,} \quad \sigma = \sigma_p + \epsilon_0|\vec{E}| \qquad \cdots(3.8.4)$$

But we know, $\sigma_p = |\vec{P}|$.

Now we can write from equation (3.8.4),

$$\sigma = P + \epsilon_0 E \qquad \cdots(3.8.5)$$

The *quantity* $(\epsilon_0 E + P)$ within the dielectric is known as **'electric displacement vector'** (or 'displacement flux density vector') and it is denoted by $\vec{D}$.

As E and P are vectors, D will also be a vector. Thus the equation (3.8.5) can be written as

$$\vec{D} = \epsilon_0\vec{E} + \vec{P} \qquad \cdots(3.8.6)$$

This equation gives the relation between the three electric vectors $\vec{E}$, $\vec{P}$ and $\vec{D}$.

In **free space** (or vacuum), there is no polarization i.e., $\vec{P} = 0$. Thus, we can write **electric displacement vector** $(\vec{D})$ **in free space**

$$\vec{D} = \epsilon_0\vec{E} \qquad \cdots(3.8.7)$$

Relation between dielectric constant (k) and electric susceptibility (χ)

We know the electric displacement vector, $\vec{D} = \epsilon\vec{E} = \epsilon_0 k\vec{E}$ and the electric polarization vector, $\vec{P} = \chi\epsilon_0\vec{E}$. Substituting the values of $\vec{D}$ and $\vec{P}$ in equation (3.8.6), we get

$$\epsilon_0 k\vec{E} = \epsilon_0\vec{E} + \chi\epsilon_0\vec{E}$$

$$\text{or,} \quad k = 1 + \chi \qquad \cdots(3.8.8)$$

Relation between electric polarization ($\vec{P}$) and dielectric constant (k) of a dielectric medium

We know, $\vec{D} = \epsilon_0\vec{E} + \vec{P}$ or, $\epsilon\vec{E} = \epsilon_0\vec{E} + \vec{P}$ or, $k\epsilon_0\vec{E} = \epsilon_0\vec{E} + \vec{P}$

$$\text{or,} \quad \vec{P} = (k-1)\epsilon_0\vec{E} \qquad \cdots(3.8.9)$$

This equation is known as **macroscopic relation for polarization.**

Relation between applied electric field ($\vec{E_0}$) and net electric field ($\vec{E}$) within the dielectric

We can write from equation (3.8.2), the net electric field within the dielectric

$$\vec{E} = \vec{E_0} - \vec{E_p} \quad \text{or,} \quad \vec{E} = \vec{E_0} - \frac{\vec{P}}{\epsilon_0} \quad \text{or,} \quad \vec{E} = \vec{E_0} - \frac{\chi \epsilon_0 \vec{E}}{\epsilon_0}$$

$$\text{or,} \quad \vec{E}(1+\chi) = \vec{E_0} \quad \text{or,} \quad \vec{E} = \frac{E_0}{1+\chi}$$

$$\text{or,} \quad \boxed{\vec{E} = \frac{\vec{E_0}}{k}} \qquad \cdots (3.8.10)$$

Relation between dielectric constant (k) of a dielectric material and atomic polarizability (α)

We know that,

$$k = 1+\chi \quad \text{or,} \quad k = 1 + \frac{n\alpha}{\epsilon_0} \quad \left[\because \chi = \frac{n\alpha}{\epsilon_0} \text{ from equation (3.4.4.3)}\right]$$

$$\text{or,} \quad \boxed{k - 1 = \frac{n\alpha}{\epsilon_0}} \qquad \cdots (3.8.11)$$

where n is the number of molecules per unit volume.

Problem 1

A dielectric slab of flat surface with relative permittivity 3 is disposed with its surface normal to a uniform field of flux density 1.6 $C \cdot m^{-2}$. The slab occupies a volume of 0.08 cubic metre and is uniformly polarized. Determine : (i) the polarization in the slab and (ii) the total dipole moment of slab.

Solution Here, dielectric constant $k = 3$, flux density $D = 1.6 \ C \cdot m^{-2}$ and volume of the slab $V = 0.08 \ m^3$.

(i) We know that polarization,

$$P = (k-1)\epsilon_0 E = (3-1) \cdot \frac{D}{k} \quad [\because D = k\epsilon_0 E] \quad = 2 \times \frac{1.6}{3} = 1.066 \ C \cdot m^{-2}$$

(ii) Polarization $(P) = \dfrac{\text{dipole moment of the slab } (p)}{\text{volume of the slab } (V)}$

$\therefore$ Total dipole moment $= P \times V = 1.066 \times .08 = 0.085 \ C \cdot m$

Problem 2

If the diameter of an argon atom is 3.84×10^{-10} m, find its polarizability at S.T.P. Given dielectric constant of argon = 1.00044.

Solution The number of argon atom per unit volume

$$n = \frac{6.023 \times 10^{23}}{22.4 \times 10^{-3}} = 2.7 \times 10^{25}$$

$\therefore$ Atomic polarizability

$$\alpha = \frac{\epsilon_0(k-1)}{n} = \frac{8.854 \times 10^{-12} \times (1.00044 - 1)}{2.7 \times 10^{25}} \simeq 1.443 \times 10^{-34} \ F \cdot m^2$$

Problem 3

Find the electric field and polarization in each region of a parallel plate capacitor shown in the following figure [Fig. 13] containing three dielectrics of relative permittivities 1, 2 and 3. The charge density (σ_s) on each of the plates of the capacitor is 20μC $\cdot m^{-2}$.

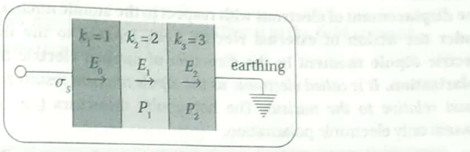

Fig. 13

Solution The electric flux density D in all the regions is same and equal to σ_s. Now, the electric field in free space region ($k_1 = 1$) is

$$E_0 = \frac{D}{\epsilon_0} = \frac{20 \times 10^{-6}}{8.854 \times 10^{-12}} V \cdot m^{-1} = 2.26 \times 10^6 \ V \cdot m^{-1}$$

Similarly,

$$E_1 = \frac{E_0}{k_2} = \frac{2.26 \times 10^6}{2} = 1.13 \times 10^6 \ V \cdot m^{-1}$$

$$E_2 = \frac{E_0}{k_2} = \frac{2.26 \times 10^6}{3} = 0.75 \times 10^6 \ V \cdot m^{-1}$$

We know there is no polarization in free space region ($k_1 = 1$). Therefore, the polarizations in other two regions are

$$P_1 = (E_0 - E_1)\epsilon_0 \quad \left[\because E_1 = E_0 - \frac{P_1}{\epsilon_0}\right]$$
$$= (2.26 \times 10^6 - 1.13 \times 10^6) \times 8.854 \times 10^{-12} \ C \cdot m^{-2}$$
$$= 10 \times 10^{-6} \ C \cdot m^{-2} = 10 \mu C \cdot m^{-2}$$

Similarly,

$$P_2 = (E_0 - E_2)\epsilon_0 = (2.26 \times 10^6 - 0.75 \times 10^6) \times 8.854 \times 10^{-12} \ C \cdot m^{-2}$$
$$= 13.37 \times 10^{-6} \ C \cdot m^{-2} = 13.37 \ \mu C \cdot m^{-2}$$

3.9. Types of Polarization[1]

There are several types of polarization. Each of them is explained by its intrinsic physical mechanism. But the three basic types of polarization are—

1. **electronic polarization**
2. **ionic polarization**
3. **orientational or dipolar polarization.**

Here we will discuss only the electronic polarization.

[1] The detail discussions are given in **Appendix 2**.

3.9.1. Electronic Polarization

When a dielectric is placed in an electric field, the electron cloud is displaced relative to the nuclei of the atoms forming the molecules. As a result, an induced electric dipole moment in the direction of applied electric field is produced. This phenomenon is called electronic polarization.

The displacement of electrons with respect to the atomic nucleus of a dielectric material under the action of external electric field, leading to the development of induced electric dipole moment in the direction of applied electric field is called electronic polarization. *It is called electronic as the dipole moment results from a shift of the electron cloud relative to the nucleus.* The homopolar dielectrics (i.e., without ionic structure) possess only electronic polarization.

Relation between electronic polarizability and atomic radius In order to make a relationship between electronic polarizability and atomic radius, we consider an atom of atomic number Z and of radius a [Fig. 14]. It is assumed that the charge cloud is of uniform density and is distributed in a sphere of radius a. So the charge density is given as $\rho = \dfrac{-Ze}{\frac{4}{3}\pi a^3}$.

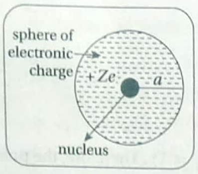

Fig. 14 ▷ An atom without any field

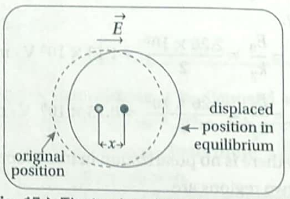

Fig. 15 ▷ Electronic polarization under the influence of electric field $\vec{E}$

When this atom is subjected to an electric field $\vec{E}$, the nucleus and the electron cloud shift with respect to one another by a distance x [Fig 15].

The force (*Lorentz force*) along the direction of electric field,

$$F_1 = (Ze)E \qquad \cdots(3.9.1.1)$$

The *magnitude of coulomb attractive force* (F_2) between the nucleus and electron cloud is

$$F_2 = \frac{Ze \times \text{(charge enclosed in the sphere of radius } x)}{4\pi\epsilon_0 x^2}$$

$$= \frac{Ze\left(\frac{4}{3}\pi x^3 \cdot \rho\right)}{4\pi\epsilon_0 x^2} = -\frac{Z^2e^2x}{4\pi\epsilon_0 a^3} \qquad \left[\because \rho = \frac{-Ze}{\frac{4}{3}\pi a^3}\right]$$

At equilibrium, the nucleus is balanced and hence, **the total force on the nucleus must be zero.** Therefore,

$$ZeE = +\frac{Z^2e^2x}{4\pi\epsilon_0 a^3}$$

or, $$x = \frac{4\pi\epsilon_0 a^3 E}{Ze} \qquad \cdots(3.9.1.2)$$

This implies **the displacement of electron cloud is proportional to the applied electric field** E.

So the *induced electronic dipole moment*

$$p = (Ze)x$$

or, $$\vec{p} = 4\pi\epsilon_0 a^3\vec{E} \qquad \left[\because x = \frac{4\pi\epsilon_0 a^3 E}{Ze}\right] \qquad \cdots(3.9.1.3)$$

Thus, **the induced electronic dipole moment is proportional to the field strength.**

Therefore the **electronic polarizability,**

$$\alpha_e = \frac{\vec{p}}{\vec{E}}$$

or, $$\alpha_e = 4\pi\epsilon_0 a^3\vec{E} \qquad \cdots(3.9.1.4)$$

If n is the number of atoms per cubic metre, the electronic polarization can be written from equation (3.4.2.1) as

$$\vec{P}_e = n\vec{p}$$

or, $$\vec{P}_e = n\alpha_e\vec{E} \qquad \cdots(3.9.1.5)$$

or, $$\vec{P}_e = n(4\pi\epsilon_0 a^3)\vec{E} \qquad \cdots(3.9.1.6)$$

Relation between dielectric constant (k) and radius (a) of the atom

We know from macroscopic relation given in equation (3.8.10)

$$\vec{P}_e = (k-1)\epsilon_0\vec{E} \qquad \cdots(3.9.1.7)$$

or, $n\alpha_e\vec{E} = (k-1)\epsilon_0\vec{E}$ [Substituting the value of $\vec{P}_e$ from equation (3.9.1.5)]

or, $$n\alpha_e = (k-1)\epsilon_0 \qquad \cdots(3.9.1.8)$$

This equation gives the relation between dielectric constant and electronic polarizability of a dielectric material.

Now substituting the value of α_e in equation (3.9.1.8) we get,

$$n4\pi\epsilon_0 a^3 = (k-1)\epsilon_0 \qquad [\because \alpha_e = 4\pi\epsilon_0 a^3]$$

or, $$k - 1 = 4\pi n a^3$$

or, $$k = 1 + 4\pi n a^3 \qquad \cdots(3.9.1.9)$$

Problem 1

If the radius of hydrogen atom is 0.053 nm, find its electronic polarizability.

Solution The radius of hydrogen atom, $a = 0.053 \text{ nm} = 0.53 \times 10^{-10}$ m

$\therefore$ Electronic polarizability of hydrogen,

$$\alpha_e = 4\pi\epsilon_0 a^3 = 4 \times 3.14 \times (8.85 \times 10^{-12}) \times (0.53 \times 10^{-10})^3 \text{ F}\cdot\text{m}^2$$

$$= 1.65 \times 10^{-41} \text{ F}\cdot\text{m}^2$$

Problem 2

The dielectric constant (k) of a rare gas of atomic number 2 at 0°C and 1 atmospheric pressure is 1.0000684. Find [i] the radius of electron cloud (atomic radius) and [ii] the displacement (i.e., shift) of its nucleus and electron cloud with respect to one another due to an external electric field of $10^6\ V \cdot m^{-1}$. Given : the number of atoms per cubic metre is 2.7×10^{25}.

Solution Here n = number of atoms / m^3 = 2.7×10^{25} of the rare gas

E = applied electric field on the dielectric material = $10^6\ V \cdot m^{-1}$

k = dielectric constant = 1.0000684

[i] We know from the relation between dielectric constant (k) and atomic radius (a) of the atom—

$$k = 1 + 4\pi n a^3$$

$$\text{or,}\quad a^3 = \frac{k-1}{4\pi n} = \frac{6.84 \times 10^{-9}}{4 \times 3.14 \times 2.7 \times 10^{25}} = 20.17 \times 10^{-36}$$

$$\therefore\quad a = 2.72 \times 10^{-12}\ m$$

[ii] The displacement of its nucleus and electron cloud,

$$x = \frac{4\pi\epsilon_0 a^3}{Ze} E$$

$$= \frac{4 \times 3.14 \times 8.854 \times 10^{-12} \times 20.17 \times 10^{-36} \times 10^6}{2 \times 1.6 \times 10^{-19}}$$

$$\simeq 7 \times 10^{-21}\ m$$

3.10. Dielectric Strength

The theory of dielectrics discussed so far is assumed for ideal dielectric. **If the electric field applied on a dielectric material is sufficiently large, it begins to pull electrons completely out of the molecules and then becomes conducting.** When the dielectric becomes conducting, a **dielectric breakdown** is said to be occurred.

The maximum electric field intensity that a dielectric can tolerate without breakdown is called dielectric strength of a material. The dielectric strength for some common dielectric materials are given in the following table.

Table 2 : Dielectric strength of few dielectric materials

Material	Dielectric strength ($MV \cdot m^{-1}$)
Air	3
Rubber	21
Glass	30
Mica	20 - 70
Backelite	25

Exercise

Multiple Choice Questions

1. In dielectric substance, electrons are —
Ⓐ firmly bound Ⓑ loosely bound Ⓒ none of them **Ans.** Ⓐ

2. The electrical conductivity of an ideal dielectric is —
Ⓐ infinity Ⓑ very large Ⓒ zero **Ans.** Ⓒ

3. In isotropic dielectric medium, the directions of polarization (P) and electric field ($\vec{E}$) are —
Ⓐ in same direction
Ⓑ in opposite direction
Ⓒ changing with time **Ans.** Ⓒ

4. The net electric field ($\vec{E}$) within the dielectric placed in a uniform electric field $\vec{E}_0$ is given by —
Ⓐ $\vec{E}_0 - \vec{P}$ Ⓑ $\vec{E}_0 - \frac{\vec{P}}{\epsilon_0}$ Ⓒ $\vec{E} + \frac{\vec{P}}{\epsilon_0}$
where $\vec{P}$ is the electric polarization. **Ans.** Ⓑ

5. The relation between $\vec{D}$ (flux density), $\vec{E}$ (electric field) and $\vec{P}$ (electric polarization) is —
Ⓐ $\vec{D} = \epsilon_0\vec{E} - \vec{P}$ Ⓑ $\vec{D} = \epsilon_0\vec{P} + \vec{E}$ Ⓒ $\vec{D} = \epsilon_0\vec{E} + \vec{P}$ **Ans.** Ⓒ

6. The susceptibility (χ) and dielectric constant (k) are related by the equation —
Ⓐ $k = 1 - \chi$ Ⓑ $k = 1 + \chi$ Ⓒ $\chi = 1 + k$ **Ans.** Ⓑ

7. If the radius of hydrogen atom is a, its electronic polarizability α is —
Ⓐ $4\pi\epsilon_0 a^2$ Ⓑ $4\pi a^3$ Ⓒ $4\pi\epsilon_0 a^3$ **Ans.** Ⓒ

8. If the radius of a rare gas of atomic number Z is a, its dielectric constant k is —
Ⓐ $1 - 4\pi n a^3$ Ⓑ $1 + 4\pi n a^3$ Ⓒ $4\pi n a^3$ **Ans.** Ⓑ
where n = number of atoms per unit volume.

9. In an isotropic medium, electric field intensity ($\vec{E}$) and electric flux density ($\vec{D}$) is related by —
Ⓐ $\vec{D} = \epsilon\vec{E}$ Ⓑ $\vec{D} = \epsilon_0\vec{E}$ Ⓒ $\vec{D} = \frac{\vec{E}}{\epsilon}$ **Ans.** Ⓐ
where ϵ = absolute permittivity of the medium and ϵ_0 = permittivity of vacuum.

10. In a linear dielectric medium, its permittivity—

Ⓐ changes with applied electric field

Ⓑ does not change with applied field

Ⓒ changes with time

Ans. Ⓑ

11. The dielectric constant of a conductor is—

Ⓐ 0 Ⓑ 1 Ⓒ ∞

Ans. Ⓒ

12. The net dipole moment of a polar dielectric in absence of electric field is—

Ⓐ ∞ Ⓑ 0 Ⓒ 1

Ans. Ⓑ

13. The dipole moment of a nonpolar dielectric in absence of electric field is—

Ⓐ ∞ Ⓑ 0 Ⓒ 1

Ans. Ⓑ

14. If P represents the polarization vector, then $\vec{\nabla} \cdot \vec{P} = -\rho$, where ρ is the density of—

Ⓐ free charge

Ⓑ bound charge

Ⓒ free charge at the boundary of the dielectric

Ⓓ sum of free and bound charge

Ans. Ⓑ **[WBUT 2011]**

15. Dimension of $\mu_0\varepsilon_0$ (symbols with their physical significance) is—

Ⓐ $L^{-2}T^{-2}$ Ⓑ LT^{-1}

Ⓒ $L^{-2}T^{-1}$ Ⓓ $L^{-2}T^{2}$

Ans. Ⓓ **[WBUT 2011; WBUT 2012]**

16. The electronic polarizability (α_e) of an atom is related to its radius (R) as—

Ⓐ $\alpha_e \alpha R^3$ Ⓑ $\alpha_e \alpha R^2$

Ⓒ $\alpha_e \alpha R$ Ⓓ $\alpha_e \alpha R^0$

Ans. Ⓐ **[WBUT 2012]**

17. The electric dipole moment of a particle (atom or molecule) per unit polarizing electric field is termed as—

Ⓐ polarization Ⓑ polarizability

Ⓒ net dipole moment Ⓓ susceptibility

Ans. Ⓑ **[WBUT 2012]**

18. Dielectrics are substances which are—

Ⓐ semiconductor Ⓑ conductor

Ⓒ insulator Ⓓ none of these

Ans. Ⓒ **[WBUT 2013]**

19. The dimension of $\frac{1}{\mu_0\varepsilon_0}$ is—

Ⓐ $L^{-2}T^{-2}$ Ⓑ $L^{-2}T^{2}$

Ⓒ LT^{-1} Ⓓ $L^{2}T^{-2}$

Ans. Ⓓ

20. The relation between electronic polarizability and atomic radius for monoatomic gas is—

Ⓐ $\alpha_e \propto a^3$ Ⓑ $\alpha_e \propto \frac{1}{a^3}$

Ⓒ $\alpha_e \propto a^2$ Ⓓ $\alpha_a \propto a^{\frac{3}{2}}$

Ans. Ⓐ

Short Answer Type Questions

1. Define dielectric substance? What is meant by polarization in the context of dielectrics? [See Articles 3.1 and 3.3]

2. Define atomic polarizability? Establish a relation between polarization and atomic polarizability. [See Article 3.4.4]

3. Explain the term susceptibility, dielectric constant and permittivity of the medium. Prove that electric flux density in an isotropic medium $\vec{D} = \epsilon\vec{E}$, where ϵ is absolute permittivity of the medium and E is electric field intensity. [See Article 3.6.2]

4. Prove that electric polarization $(\vec{P})$ is equal to the surface charge density of polarization. [See equation (3.8.1) of Article 3.8]

5. Show that when a dielectric is placed in an electric field, the field within the dielectric becomes weaker than the original field. [See Article 3.5]

6. Establish the relation between electric susceptibility (χ) and dielectric constant of a dielectric medium. [See equation (3.8.8) of Article 3.8]

7. Establish the relation between electric polarization (P) and dielectric constant (k) of a dielectric medium. [See equation (3.8.10) of Article 3.8]

8. Define electronic polarization. Prove that the induced electronic dipole moment is proportional to the field strength. [See Article 3.9]

Long Answer Type Questions

1. [a] What do you understand by a dielectric and the dielectric constant of a material? [See Article 3.1]

[b] Distinguish between nonpolar and polar molecules. Give examples. [See Article 3.2]

[c] Explain electric polarization of matter. [See Article 3.3]

[d] The relative permittivity of argon at 0°C and at 1 atm pressure is 1.000435. Find the polarizability of the argon atom. [Ans $1.43 \times 10^{-40}\ F \cdot m^2$]

2. [a] Explain the behaviour of a dielectric placed in an electrostatic field. [See Article 3.5]

[b] Derive the relation $k = 1 + \chi$, where k is dielectric constant and χ is electric susceptibility. [See Article 3.8]