20/2/18

Probability

~~$Var(ax+b) = E(ax+b)$~~

1) $Var(ax+b) = a^2 Var(x)$

2) $E(ax+b) = aE(x) + b$

for discrete values of x

$E(ax+b) = E(ax) + E(b)$

$E(g(x)) = \sum_i g(x_i) \cdot P_i$

$E(ax+b) = \sum_i (ax_i + b) \cdot P_i = \sum_i ax_i \cdot P_i + \sum_i b P_i$

$= a\sum_i x_i P_i + b\sum_i P_i$

$= aE(x) + b$

(PROVED)

$\boxed{\because \sum_i P_i = 1}$

1) If 'x' denotes the no. of failures receiving the 1st success with probability of success 'p', then find $E(x)$.

A & B → die → throws 6 & wins & A begins. What is prob that A will win?

$\frac{1}{6} + \frac{5}{6} \cdot \frac{1}{6} + \left(\frac{5}{6}\right)^2 \cdot \frac{1}{6} + \dots \infty$

x	P_i
0	p
1	$(1-p)p$
2	$(1-p)^2 p$
3	$(1-p)^3 p$
⋮	⋮

$\therefore E(x) = 0.p + 1.p.(1-p) + 2.p(1-p)^2 + 3(1-p)^3 + \dots + \infty$

$= (1-p)p[\cancel{1(1-p)} + 2(1-p)^2 + 3(1-p)^3 + \dots + \infty]$

$(1-p) = y$

$\frac{(-2)(-2-1)}{2}$

$(1-y)^{-2} = 1 + 2y + 3y^2$

$\frac{d}{dx}(1-x)^{-1}$

$= \frac{p(1-p)}{p^2} = \frac{1-p}{p}$ (Ans) $= (-1)(-1)(1-x)^{-2}$

2) Let 'x' be a continuous random variable with p.d.f. given by $f(x) = ax, \quad 0 \leq x \leq 1$

$= a, \quad 1 \leq x \leq 2$

$= -ax + 3a, \quad 2 \leq x \leq 3$

$= 0$, elsewhere

i) Determine the const 'a'

ii) Compute $P(x \leq 1.5)$

iii) Find Cum. distribution function

$$a\frac{x^2}{2}\Big|_0^1 + ax\Big|_1^2 + \left(-\frac{ax^2}{2} + 3ax\right)_2^3 = 1$$

$$\Rightarrow \frac{a}{2} + a - \frac{5}{2}a + 3a = 1$$

$$\Rightarrow -a + 3a = 1$$

$$a = 1/2$$

(ii) $a\frac{x^2}{2}\Big|_0^1 + ax\Big|_1^{1.5}$

$$= \frac{a}{2} + \frac{a}{2} = a = \frac{1}{2}$$

26/2/18

S.C. Sir

Group Theory

Subgroup

Let, G be a group & H is a subset of G.

H is called a subgroup of G if H itself is a group under the same binary operation as that of G.

(1) Identity & inverse should lie in the set

$\{R^*, \times\}$ is a group

$R^* \to R - \{0\}$, $\times \to$ multiplication

$G = \langle R, + \rangle$

$H = \langle Q, + \rangle$

$Q \to$ set of rationals.

$\therefore$ H is a subgroup of G

$G = \langle R, + \rangle$

$H = \langle Q^*, \times \rangle$

Though both H & G are groups & $Q^* \subset R$ but H is not a subgroup of G. due to diff^n binary operations

1) Will identity element of G & H be same?

2) Will the inverse of an element in H be same as that of the element in G?

Proposition : Let H be a subgroup of a group G. Then prove that

i) the identity elements in G & H are same

ii) ~~the inverse of an element~~ for an element $a \in H$, the inverse of a in H is same as the inverse of a in G.

Proof : (i) let e_G & e_H be the identity elements in G, ~~resp~~ & H, resp.

Let, $h \in H \subseteq G$.

$\therefore h . e_H = h = e_H . h$

$h . e_G = h = e_G . h$

which implies

$h . e_H = h . e_G$

$\Rightarrow e_H = e_G$ (by left cancellation property)

ii) Let a^{-1}_G & a^{-1}_H be the inverse of a in G & H, resp.

$a \in H \subseteq G$.

operation, not necessarily product

$a . a^{-1}{}_G = e_G = a^{-1}{}_G . a$.

$a . a^{-1}{}_H = e_H = a^{-1}{}_H . a$

We know $e_G = e_H$

$\therefore a a^{-1}_G = a \; a^{-1}_H$

$\Rightarrow a^{-1}_G = a^{-1}_H$ (by left Concelation property)

—x—

Theorem : Let H be a subset of G which is a group. Then H is a subgroup iff $\forall a, b \in H$, $ab^{-1} \in H$.

Proof : Let, H be a subgroup of G and $a, b \in H$ and hence by closure property $a \in H$, $b^{-1} \in H \Rightarrow ab^{-1} \in H$.

Conversely let $a, b \in H \Rightarrow ab^{-1} \in H$.

Since $a \in H$, $aa^{-1} \in H$, i.e. $e \in H$

Now $e \in H$ & let $a \in H$. Then according to assumption $e.a^{-1} \in H$, i.e. $a^{-1} \in H$

Let, $a, b \in H$. Then $ab^{-1} \in H$, i.e. $a, b^{-1} \in H$.

Acc to assumption $\Rightarrow a(b^{-1})^{-1} \in H$, i.e. $ab \in H$

$\therefore$ H is closed.

$\because$ H is a subset of G, which is a group,
the associativity is inherited by H.

$\therefore$ H is also a group under the same binary operation as that of G.

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