GATE and miscellaneous questions

1. Two people, P and Q, decide to independently roll two identical dice, each with 6 faces, numbered 1 to 6. The person with the lower number wins. In case of a tie, they roll the dice repeatedly until there is no tie. Define a trial as a throw of the dice by P and Q. Assume that all 6 numbers on each dice are equi-probable and that all trials are independent. The probability (rounded to 3 decimal places) that one of them wins on the third trial is....... (GATE CSE 2018)

2. Consider Guwahati (G) and Delhi (D) whose temperatures can be classified as high (H), medium (M) and low (L). Let P(HG) denote the probability that Guwahati has high temperature. Similarly, P(MG) and P(LG) denotes the probability of Guwahati having medium and low temperatures respectively. Similarly, we use P(HD), P(MD) and P(LD) for Delhi. The following table gives the conditional probabilities for Delhi's temperature given Guwahati's temperature.

	HD	MD	LD
HG	0.40	0.48	0.12
MG	0.10	0.65	0.25
LG	0.01	0.50	0.49

Consider the first row in the table above. The first entry denotes that if Guwahati has high temperature (HG) then the probability of Delhi also having a high temperature (HD) is 0.40; i.e., P(HD|HG) = 0.40. Similarly, the next two entries are P(MD|HG) = 0.48 and P(LD|HG) = 0.12. Similarly for the other rows. If it is known that P(HG) = 0.2, P(MG) = 0.5, and P(LG) = 0.3, then the probability (correct to two decimal places) that Guwahati has high temperature given that Delhi has high temperature is.........(GATE CSE 2018)

3. An urn contains four balls, each ball having equal probability of being white or black. Three black balls are added to the urn. The probability that five balls in the urn are black is.....(GATE MATHS 2018)

4. Let X be the number of heads in 4 tosses of a fair coin by Person 1 and let Y be the number of heads in 4 tosses of a fair coin by Person 2. Assume that all the tosses are independent. Then the value of $P(X = Y)$ correct up to three decimal places is.......(GATE MATHS 2018)

5. Consider a sequence of tossing of a fair coin where the outcomes of tosses are independent. The probability of getting the head for the third time in the fifth toss is.......(GATE AEIE 2018)

6. Four red balls, four green balls and four blue balls are put in a box. Three balls are pulled out of the box at random one after another without replacement. The probability that all the three balls are red is.......(GATE ME 2018)

7. The arrival of customers over fixed time intervals in a bank follow a Poisson distribution with an average of 30 customers/hour. The probability that the time between successive customer arrival is between 1 and 3 minutes is.....(GATE ME 2018)[This is an example from QUEING THEORY]

8. Given that $P(A) > 0$, prove that $P(B|A) \geq 1 - \frac{P(B^c)}{P(A)}$

9. Two fair dice are thrown. What is the probability that the 2nd die falls on a higher value than the first?

10. What is the probability that a k digit number does not have any 0, 5 or 9?(GATE EE 2017)

11. Suppose that a sample of n = 1600 tires of the same type are obtained at random from an ongoing production process in which 8% of all such tires produced are defective. What is the probability that in such a sample not more than 150 tires will be defective?[Use normal approximation to binomial distribution]

12.[MIT 2014] Don't be late! Alice and Bob are trying to meet for lunch and both will arrive, independently of each other, uniformly and at random between noon and 1pm. Let A and B be the number of minutes after noon at which Alice and Bob arrive,respectively. Then A and B are independent uniformly distributed random variables on [0, 60].
(a) Find the probability that Alice arrives before 12:15 and Bob arrives between 12:30 and 12:45.
(b) Find the probability that Alice arrives less than five minutes after Bob.

13.[MIT 2016] Suppose that buses arrive are scheduled to arrive at a bus stop at noon but are always X minutes late, where X is an exponential random variable.Suppose that you arrive at the bus stop precisely at noon.

(a) Compute the probability that you have to wait for more than five minutes for the bus to arrive.
(b) Suppose that you have already been waiting for 10 minutes. Compute the probability that you have to wait an additional five minutes or more.

14.[MIT 2015] Suppose Z is a standard normal random variable and let $X = 3Z + 1$.
(a) Find $P(X \leq x)$.
(b) Recall that the probability that Z is within one standard deviation of its mean is approximately 68%. What is the probability that X is within one standard deviation of its mean?

15. Skipper is very abnormal, not that there's anything wrong with that. He doesn't fit into any of the marketing teams' models. Every day Skipper wakes up and walks to Borders Bookstore. There he flips a fair coin repeatedly until he flips his second tails. He then goes to the counter and buys 1 DVD for each head he flipped. Let R be the revenue Borders makes from Skipper each day.
What's the daily expected revenue from Skipper? What's the variance of the daily revenue from Skipper?[MIT 2006]