(1) A coin is tossed → I want to find no. of tumbles in air

$S = \{1, 2, 3, 4, \ldots\}$

No. of tumbles is always positive integer

~~I want to find~~

→ S.C. Sir   26/3/18

Cosets:

Let, G be a group & H is a subgroup and H is a subgroup of G. Let $a \in G$. Then

$aH = \{ah : h \in H\}$ is known as left coset of H in G.

$Ha = \{ha : h \in H\}$ is known as right coset of H in G

Eg: Let $G = \langle \mathbb{Z}, + \rangle = \{\ldots -1, 0, 1, 2, \ldots\}$

$H = \langle 3\mathbb{Z}, + \rangle = \{\ldots -3, 0, 3, 6, \ldots\}$

Here left cosets are

$0H = \{0 + h : h \in H\} = H$ → group

$1H = \{1 + h : h \in H\} = \{\ldots -2, 1, 4, 7, \ldots\}$ → Not a group: zero is absent → identity element

$2H = \{2 + h : h \in H\} = \{\ldots -1, 2, 5, 8, \ldots\}$

$3H = H$

No. of left cosets in G = no. of elements in G / no. of elements in each coset = no. of elements in H.

Any two cosets picked from G → either they will be same or not.

Proposition: Let G be a group and H is a subgroup of G. Let $h \in H$. Then $\boxed{hH = H}$

Proof: Let, $\boxed{x \in hH}$ set

Then $x \in hh_1$ for some $\boxed{h_1 \in H}$

$\therefore x = hh_1 \in H$ (closure property in H)

$\therefore hH \subseteq H \rightarrow (1)$

Now let $y \in H$

Then for ~~some $a \in H$~~ an unique $a \in H$

$y = ha$

$\hookrightarrow y \in hH$

Remember $ax = b$ has unique solution

$\boxed{H \subseteq hH}$

$\hookrightarrow (2)$

from (1) & (2)

$\boxed{hH = H}$ Proved

Proposition: Let G be a group and H be a subgroup of G. Let $a \notin G-H$. then

$aH \cap H = \Phi$

Proof: Let $p \in aH \cap H$

Then $p \in aH$ & $p \in H$

Since $p \in aH$, $p = ah_1$ for some $h_1 \in H$

Since $p \in H$, $p = h_2$ for some h_2 in H.

$\therefore ah_1 = h_2$

$\Rightarrow a = h_2 . h_1^{-1}$ Now, $h_1 \in H$ $\hookrightarrow$ group $\therefore h_1^{-1} \in H$

$\therefore \boxed{a \in H} \rightarrow$ contradiction $\because a \notin G-H$

$\therefore \boxed{H \cap aH = \phi}$ (Proved)

<u>Proposition</u>: Let G be a group and H is a subgroup of G. Then any two left (right) cosets of H in G are either same or disjoint.

<u>Proof</u>: Let, aH, bH be two cosets of H in G

Let, $p \in aH \cap bH$,

Then $p = ah_1$ for some $h_1 \in H$

& also $p = bh_2$ for some $h_2 \in H$

$\therefore ah_1 = bh_2$

$\Rightarrow a = bh_2 h_1^{-1}$

$b = ah_1 h_2^{-1}$

Let, $x \in aH$

Then $x \in aH$

Then $x = ah_3$, for some $h_3 \in H$

$= bh_2 h_1^{-1} h_3$

$= bh_4$, for some $\boxed{h_4 \in H}$

$\in bH$

$\therefore \boxed{aH \subseteq bH} \to (1)$

Let, $y \in bH$

$y = bh_5$, $h_5 \in H$

$= ah_1 h_2^{-1} h_5$

$= ah_6$, $h_6 \in H$

$\therefore y \in aH$

$\therefore \boxed{bH \subseteq aH} \to (2)$ $\therefore \boxed{aH = bH}$

$\therefore$ Any 2 cosets will ~~have~~ be either same or have no element in common.

Proposition: Any two left (right) cosets of H in G have the same cardinality.

Proof: Let aH & bH be 2 left cosets of H in G

$\boxed{\text{Establish a bijective Mapping betn } aH \text{ \& } bH}$

↳ Procedure to count no. of elements in a set.

One to One & Onto are possible only if 2 sets have same no. of elements.

$A \to B$	Proof for Onto
$f(x) = f(y)$	$\forall y \in B$
$\Rightarrow x = y$	$\exists f(x) = y$
Proof for One-One	

Let, $f : aH \to bH$ is defined as

$$f(ah) = bh, \ h \in H$$

Let, $f(ah_1) = f(ah_2)$

$$\Rightarrow bh_1 = bh_2$$

$$\Rightarrow h_1 = h_2 \text{ (Left Cancellation property)}$$

$$\Rightarrow ah_1 = ah_2$$

One-One proved.

Let, $bh \in bH$

Then by ~~$bh_1 = ah_1$~~ defn of f $\exists$ ah, such that

$$f(ah) = bh.$$

Onto Mapping

$\therefore f$ is bijective

$$|aH| = |bH|$$

Lagrange's Theorem

Let G be a finite group & H is a subgroup of G. Then $o(H)$ divides $o(G)$

Disjoint Cosets of H in G divide G into Equivalance classes/parts.

$|G| = n$

$|H| = m$.

No. of cosets of H in G = $|G| = n$
No. of elements in each coset $= |H| = m$

Suppose k no. of disjoint cosets.

$\boxed{mk = n}$

Proof Let, H be a subgroup of a finite group 'G'. $o(G) = n$.

Let us Consider the set of all distinct left (right) cosets of H in G.

Since G contains a finite no. of elements the no. of distinct left (right) cosets of H is finite.

Then there exists elements $a_1, a_2, \ldots a_k$ elements in G such that $a_1H, a_2H, a_3H, \ldots$ etc are the disjoint left cosets of H in G.

Also each of the left cosets are having same no. of elements, since $H = eH$ is also a left coset, each of the Disjoint cosets $o(H) = m$ (say)

$\therefore o(G) = \sum_{i=1}^{k} o(a_iH)$

Quotient is no. of Disjoint cosets

$\Rightarrow \boxed{n = mk} \Rightarrow \boxed{m|n}$ (Proved)

K → no. of Disjoint left (right) cosets of H in G is called index of H in G or $[G : H]$

—×—

Prove : Any group of prime order is cyclic

Proof: Let $O(G) = p$. $a \in G$

$H = \{a, a^2, a^3, \ldots a^k\}$ → subgroup of G

↳ Distinct elements.

$a^k = e$

$\therefore O(H) \mid O(G)$, $O(H) = k$, $O(G) = p$

Now $O(G)$ is prime

So either $k = 1$ or $k = p$

$k = 1$: H contains only identity element

(E) Every subgroup of a group of order 6 is a cyclic group

Reason : $O(G) = 6$, $O(H) = 2$ or 3 ← Prime nos

—×—

Probability (3 B Ma'am)

X \ Y	1	2	3	$P(X = x_i)$
1	0.1	0.1	0.1	$P(X=1) = 0.3$
2	0.1	0.2	0.1	$P(X=2) = 0.4$
3	0.1	0.1	0.1	$P(X=3) = 0.3$

$E(X) = 1 \times 0.3 + 2 \times 0.4 + 3 \times 0.3 = 2$

$E(X+Y) = (1+1)(0.1) + (1+2)(0.1) + (1+3)(0.1)$
$+ (2+1)(0.1) + (2+2)(0.2) + (2+3)(0.1)$
$+ (3+1)(0.1) + (3+2)(0.1) + (3+3)(0$