

PROBABILITY AND NUMERICAL METHODS
(MATH 2202)

Time Allotted : 3 hrs

Full Marks : 70

Figures out of the right margin indicate full marks.

*Candidates are required to answer Group A and
any 5 (five) from Group B to E, taking at least one from each group.*

*Candidates are required to give answer in their own words as far as
practicable.*

Group - A
(Multiple Choice Type Questions)

Choose the correct alternatives for the following:

10 × 1 = 10

- (i) Simpson's one third rule is applicable only if the number of sub intervals is even
(a) True (b) False
- (ii) A die is tossed. If the number is odd, then the probability that it is prime, is
(a) $\frac{1}{2}$ (b) $\frac{2}{3}$ (c) $\frac{1}{3}$ (d) none.
- (iii) If A and B be two events with $P(A) = \frac{1}{3}$, $P(B) = \frac{1}{4}$ and $P(A \cup B) = \frac{1}{2}$, then $P(B/A) =$
(a) $\frac{3}{4}$ (b) $\frac{4}{3}$ (c) $\frac{1}{4}$ (d) $\frac{1}{3}$.
- (iv) A random variable X has the following $p.d.f$:
 $f(x) = k, -2 < x < 2$
 $= 0$ otherwise
Then the value of the constant k is
(a) $\frac{1}{8}$ (b) $\frac{1}{2}$ (c) $\frac{1}{4}$ (d) $\frac{1}{12}$.

B.TECH/CSE/4TH SEM /MATH 2202/2016

- (v) The mean and standard deviation of a Binomial distribution are respectively 4 and $\sqrt{\frac{8}{3}}$. The values of n and p are (where n and p are the parameters of the distribution)
- (a) $11, \frac{3}{4}$ (b) $12, \frac{2}{7}$ (c) $12, \frac{1}{3}$ (d) $11, \frac{4}{3}$
- (vi) Suppose based on live data the following two correlation coefficients were calculated.
 a = correlation coefficient between sale of cold drinks and day-temperature
 b = correlation coefficient between Production amount and price per unit of the product
Then you will expect
- (a) $a > 0, b > 0$ (b) $a > 0, b < 0$
(c) $a < 0, b > 0$ (d) $a < 0, b < 0$.
- (vii) If X is a normal variate with mean -2 and variance 25 then which one of the following is standard normal variate
- (a) $\frac{X-2}{5}$ (b) $\frac{X+2}{25}$ (c) $\frac{X-25}{2}$ (d) $\frac{X+2}{5}$.
- (viii) Among the three methods - Bisection, Newton-Raphson, Regula-Falsi - the one that converges fastest is
- (a) Bisection (b) Newton-Raphson
(c) Regula-Falsi (d) All equivalent.
- (ix) Let ∇ denote the backward difference operator and E denote the shift operator. Then $\nabla =$
- (a) $1 - E$ (b) $E - 1$ (c) $E^{-1} - 1$ (d) $1 - E^{-1}$
- (x) In Newton's backward interpolation formula, the absolute value of $s = \frac{x - x_n}{h}$ should ideally lie between
- (a) 0 and 1 (b) 0 and ∞
(c) greater than 1 (d) no restriction.

Group - B

2. (a) The table gives the distance in nautical miles of the visible horizon for the given heights in feet above the earth's surface:
- | | | | | | | | |
|---------------|-------|-------|-------|-------|-------|-------|-------|
| x = height: | 100 | 150 | 200 | 250 | 300 | 350 | 400 |
| y = distance: | 10.63 | 13.03 | 15.04 | 16.81 | 18.42 | 19.90 | 21.27 |

B.TECH/CSE/4TH SEM /MATH 2202/2016Find the values of y when(i) $x = 218$, using Newton's forward interpolation formula(ii) $x = 410$, using Newton's backward interpolation formula

- (b) Apply Runge-Kutta method to find appropriate value of y for $x = 0.2$, in steps of 0.1, if $\frac{dy}{dx} = x + y^2$, given that $y = 1$ where $x = 0$.

$$(4+3) + 5 = 12$$

3. (a) Solve the given system of equations correct upto 3 decimal places using Gauss-Seidel iteration method:

$$y - x + 10z = 35.61$$

$$x + z + 10y = 20.08$$

$$y - z + 10x = 11.19$$

- (b) Find the smallest positive root of the equation $3x^3 - 9x^2 + 8 = 0$ correct to 4 places of decimal using Newton Raphson method.

$$7 + 5 = 12$$

Group - C

4. (a) Two urns contain respectively 5 white, 7 black balls and 4 white, 2 black balls. One of the urns is selected by the toss of a fair coin and then 2 balls are drawn without replacement from the selected urn. If both balls drawn are white, what is the probability that the first urn is selected?

- (b) Suppose it is known that on average 5 men out of 100 and 25 women out of 10,000 are colorblind. A person is randomly selected from a city where there are roughly equal number of men and women. If the selected person is found to be colorblind then what is the probability that the selected person is male?

$$6 + 6 = 12$$

5. (a) Find the value of the constant k such that

$$f(x) = kx(1-x), 0 < x \leq 1$$

$$= 0 \text{ elsewhere}$$

is a possible probability density function. If a random variable X has *p.d.f* $f(x)$ then find $P\left(X > \frac{1}{2}\right)$ and $E(X)$.

- (b) If the life of ball bearings has the density $f(x) = ke^{-0.2x}$ if $0 \leq x \leq 10$ and 0 otherwise, then what should be the value of the constant k ?

What is the probability that a randomly chosen ball bearing will last longer than 5 units of time?

$$6 + (3+3) = 12$$

Group - D

6. (a) A car hire firm has two cars which it hires out day by day. The number of demands for a car on each day is distributed as a poisson distribution with average number of demand per day 1.5. Calculate the proportion of days on which neither car is used and the proportion of days on which some demand is refused.
($e^{-1.5} = 0.2231$)

- (b) The weight of students in a college is normally distributed with mean 40 kg and standard deviation 5 kg. Find the percentage of the students that have weight
(i) greater than 50 kg
(ii) between 38 kg and 52 kg.

It's given that

$$\left[\frac{1}{\sqrt{2\pi}} \int_{-\infty}^2 e^{-t^2/2} dt = 0.9772, \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{0.4} e^{-t^2/2} dt = 0.6554, \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{2.4} e^{-t^2/2} dt = 0.9918 \right]$$

$$(3+3) + (3+3) = 12$$

7. (a) (i) The mean and standard deviation of 20 items are found to be 10 and 2 respectively. At the time of checking it was found that one item 8 was incorrect. Calculate the mean and standard deviation if the wrong item is replaced by 12.

- (ii) Find the modal wage from the following data:

Wages (Rs.)	50-60	60-70	70-80	80-90	90-100	100-110	110-120
Employees	8	10	16	14	10	5	2

- (b) From the following results, obtain the two regression equations and estimate the yield of crops when the rainfall is 22cms., and the rainfall when the yield is 600kg:

	y (Yield in kg)	x (Rainfall in cm.)
Mean	508.4	26.7
S.D	36.8	4.6

$$(4+2) + 6 = 12$$

8. (a) The random variable X and Y have a joint probability mass function given by: $P(X = x, Y = y) = \frac{x^2 + y}{32}$ for $x = 0, 1, 2, 3$ and $y = 0, 1$.

Find

- i) the marginal probability mass function of X
- ii) the marginal probability mass function of Y
- iii) $P(X \leq 2, Y \geq 1)$

- (b) The joint probability density function of the random variables X and Y is given by:

$$f(x, y) = \begin{cases} 2 & \text{for } 0 < x < 1, 0 < y < x \\ 0 & \text{otherwise} \end{cases}$$

Find $P(X + Y < 1)$ and $E(X + Y)$

$$(2+2+2) + (3+3) = 12$$

9. (a) There are two boxes and d many balls are distributed between these two boxes. A ball is randomly selected and its box is changed. The same process is repeated again and again. Suppose the boxes are colored red and blue and let X_n be the number of balls in the red box after the above process is conducted n times.

- i) What is the name of this Markov Chain?
- ii) Is this a Birth-Death chain?
- iii) Write down its transition probability matrix.

- (b) X_1, X_2, X_3, \dots forms a Markov chain with state space $\{1, 2\}$ and the following transition probability matrix:

$$\begin{matrix} & \begin{matrix} 1 & 2 \end{matrix} \\ \begin{matrix} 1 \\ 2 \end{matrix} & \begin{bmatrix} 1/3 & 2/3 \\ 1/2 & 1/2 \end{bmatrix} \end{matrix}$$

- (i) Find $P(X_6 = 2 | X_3 = 1)$ and $P(X_4 = 2 | X_1 = 2)$
- (ii) If the Markov chain starts from the state 1 (i.e., $X_1 = 1$) then find $E(X_3)$ and $E(X_4)$.

$$(1+1+2) + (4+4) = 12$$