Assignment - 2 : MATH 2201 / MATH 2203

1. Prove that the order of a permutation on a finite set is the l.c.m of the lengths of its disjoint cycles.

2. Prove that the number of even permutations on a finite set (containing at least two elements) is equal to the number of odd permutations on it.

3. Let $\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 6 & 4 & 7 & 5 & 2 & 3 & 1 \end{pmatrix}$ and $\beta = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 1 & 4 & 6 & 7 & 3 & 5 & 2 \end{pmatrix}$

   be elements of S_7.

   (i) Write α as a product of disjoint cycles
   (ii) Write β as a product of 2-cycles
   (iii) Is β an even permutation?
   (iv) Is α^{-1} an even permutation?

4. Let H, K be subgroups of a group G. Then prove that HK is a subgroup of G iff $HK = KH$.

5. Show that $Z(G) = \{x \in G : xg = gx \ \forall g \in G\}$ is a subgroup of G.

6. Let G be a group and $a \in G$. Prove that $C(a) = \{x \in G : xa = ax\}$ is a subgroup of G.

7. Prove that a cyclic group of finite order n has a subgroup of order d for every positive divisor d of n.

8. Prove that $(\mathbb{Q}, +)$ is a non-cyclic group.

9. Prove that the intersection of any collection of subgroups of a Group G is a subgroup of G.

10. Let $G = \langle a \rangle$ be a cyclic group of order n. Prove that

(i) If H is a subgroup of G, then $|H|$ divides $|G|$.

(ii) If m is a positive integer such that m divides n, then there exists a unique subgroup of G of order m.

11. Let G be a group of order 28. Show that G has a non-trivial subgroup.

12. Let G be a group and H be a subgroup of G. Let $a, b \in G$. Then prove that $aH = bH$ if and only if $a^{-1}b \in H$.

13. Prove that any two left cosets of ~~G~~ H in a group G have the same cardinality.

14. Prove that the order of each element in a finite group G is a divisor of $o(G)$.

15. Let G be a finite group and $a \in G$. Prove that $a^{o(G)} = e$. Hence prove Fermat's Little Theorem.

16. Prove that every group of order less than 6 is commutative.

17. Prove that $Z(G)$ is a normal subgroup of G.

18. Let H and K be finite subgroups of a group G. Then prove that $|HK| = \dfrac{|H|\,|K|}{|H \cap K|}$.

19. Let H be a subgroup of a group G such that $[G:H] = 2$. Then prove that H is a normal subgroup of G.

20. Find all subgroups of S_3. Show that union of any two nontrivial distinct subgroups of S_3 is not a subgroup of S_3.