Assignment

Probability and Numerical Methods(MATH 2202)

Module-II & III

Probability

Fundamental Theory of Probability:

Q. 1 *The probabilities of X, Y and Z becoming the principal of a college are respectively 0.3, 0.5 and 0.2. The probabilities that "student aid fund" will introduce in the college if X, Y and Z become principle are 0.4, 0.6 and 0.1, respectively. Given that student aid fund is introduced, find the probability that Y has been appointed as the principal.*

Q. 2 *A and B are two independent witness in a case. The probability that A will speak the truth is tx and probability that B will speak the truth is y. A and B agree in a certain statement. Show that the probability that the statement is true is* $\frac{xy}{1-x-y+2xy}$.

Q. 3 *In a certain class* 25% *of the student failed in mathematics,* 15% *failed in chemistry,* 10% *failed in both mathematics and chemistry. a student is selected at random.*
(*i*) *If he failed in mathematics, what is the probability that he failed in chemistry?*
(*ii*) *If he failed in chemistry, what is the probability that he failed in mathematics?*
(*iii*) *What is the probability that he failed both in mathematics and chemistry?*
(*iv*) *What is the probability that he failed in mathematics or chemistry?*

Q. 4 *Two persons A and B play alternatively with a pair of dice. A wins if he throws 6 before B throws 7 and B wins if he throws 7 before A throws 6. If B begins, find his probability of winning.*

Q. 5 *Three urns contain respectively 1 white and 2 black balls, 2 white and 1 black balls, 2 white and 2 black balls. One ball is transferred from the first to the second urn; then one ball is transferred from the second to the third urn; finally one ball is drawn from the third urn. Find the probability that the ball is white.What will be the probability if the ball is black.*

Q. 6 *100 prizes will be given in a lottery of 10000 tickets. Find the minimum number of tickets a person has to buy in order that the probability of his wining at least one prize is greater than* $\frac{1}{2}$.

Q. 7 *In a bolt factory, machine A, B and C manufactured respectively 25%, 35% and 40% of the total.Out of their output 5%, 4% and 2% are defective bolts. A bolt is drawn at random from the product and is found to be defective. What are the probabilities that it was manufactured by machines A, B and C?*

Q. 8 *The integers* x *and* y *are chosen at random with replacement from the set of natural numbers* $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$. *Find the probability that* $|x^2 - y^2|$ *is divisible by 2.*

Q. 9 *From an urn containing* N_1 *white and* N_2 *black balls* ($N = N_1 + N_2$), *balls are successively drawn without replacement until only those of the same colour are left. Prove that the balls left are white is* N_1/N.

Q. 10 *Suppose each of three persons tosses a coin. If the outcome of one of the tosses differs from the other outcomes, then the game ends. If not, then the persons start over and retoss their coins. Assuming fair coins, what is the probability that the game will end with first round of tosses? If all three coins are biased and have probability 1/4 of landing heads, what is the probability that the game will end at the first round?*

Distribution function and Expectation:

Q. 11 *If a random variable X has the following probability mass function*

X	0	1	2	3	4	5	6	7
$P(X = x_i)$	0	K	$2K$	$2K$	$3K$	K^2	$2K^2$	$7K^2 + K$

(i) Find K.
(ii) Evaluate $P(X < 6)$,$P(X \geq 6)$, $P(X > 7)$*and* $P(0 < X < 5)$.
(iii) If $P(X \leq a) > \frac{1}{2}$, *find the minimum value of a, and*
(iv) Determine the distribution function of X
(v) Find the mean and variance of X.

Q. 12 *The distribution function F(x) of the random variable X is defined as follows*

$$F(x) = \begin{cases} A, & -\infty < x < -1; \\ B, & -1 \leq x < 0; \\ C, & 0 \leq x < 2; \\ D, & 2 \leq x < \infty. \end{cases}$$

where A, B, C and D are constants. Determine values of A, B, C and D it being given that $P(X = 0) = \frac{1}{6}$ *and* $P(X > 1) = \frac{2}{3}$.

Q. 13 *Show that*

$$f(x) = \begin{cases} x & ; 0 < x < 1 \\ k - x & ; 1 < x < 2 \\ 0 & ; otherwise \end{cases} \tag{1}$$

is a density function for a suitable value of k? Calculate the distribution function of the random variable X. Also calculate $P(\frac{1}{2} < X < \frac{3}{2})$. *Find the mean and variance of X.*

Q. 14 *The radius of a circle has distribution given by the p.d.f.*

$$f(x) = \begin{cases} 1 & ; 1 \leq x \leq 2 \\ 0 & ; otherwise \end{cases} \tag{2}$$

Find the mean and variance of the area of the circle.

Q. 15 *If the random variable X takes the values 1, 2, 3 and 4 such that*

$$2P(X = 1) = 3P(X = 2) = P(X = 3) = 5P(X = 4)$$

find the distribution function of X.

Q. 16 *If the probability density function* $f(x)$ *of a random variable X is defined by*

$$f(x) = ce^{-|x|}, -\infty < x < \infty$$

Find c. Evaluate mean and variance.

Q. 17 *Let c ba constant. Show that* $Var(c + X) = Var(X)$.

Q. 18 *Find the probability that none of three bulbs in a traffic signal will have to be replaced during the first 1500 hours of operation if the lifetime X of a bulb is a random variable with density*

$$f(x) = \begin{cases} 6[0.25 - (x - 1.5)^2] & ; 1 \leq x \leq 2 \\ 0 & ; otherwise \end{cases} \tag{3}$$

[here x is measured in multiples of 1000 hours].

Q. 19 *Let a random variable X have the distribution*

$$P(X = 0) = P(X + 2) = p \text{ and } P(X = 1) = 1 - 2p, \text{ for } 0 \leq p \leq 1/2$$

For what value of p, Var(X) is maximum.

Some Special Type Of Distributions:

Q. 20 *Trains on a certain line run every half an hour between midnight and six in the morning. What is the probability that a man entering the station at a random time during this period will have to wait for at least 20 minutes.*

Q. 21 *A defective die is thrown ten times independently. The probability that an even number will appear 5 times is twice the probability that an even number will appear 4 times. What is the probability that odd face appear in each of ten throws.*

Q. 22 *A random variable X follows binomial distribution with mean 4 and variance* $\sqrt{2}$. *Find the probability of assuming nonzero value of the variable.*

Q. 23 *A car-hire firm has two cars which it hires out on a daily basis. The number of demands for a car on each day is distributed as a Poisson distribution with average number of demand per day 1.5. Calculate the proportion of days on which neither of the car is used and the proportion of days on which some demand is refused.[Given that* $e^{-1.5} = 0.2231$ *]*

Q. 24 *The waiting time (in minutes) for customers at a drive-in bank is an exponentially distributed random variable. The average (mean) time a customer waits is 4 minutes. What is the probability that a customer waits for more than 5 minutes?*

Q. 25 *A fair coin is tossed 400 times. using normal approximation to binomial distribution, find the probability of obtaining*
(i) exactly 200 heads
(ii)between 190 and 210 heads both inclusive.
[*Given that the area under standard normal curve between* $z = 0$ *and* $z = 0.05$ *is* 0.0199 *and between* $z = 0$ *and* $z = 1.05$ *is* 0.3531]

Q. 26 *In a normal distribution* 41% *of the items are below 55 and* 7% *are above 64. Find the mean and standard deviation.*
Given $P(0 < Z < 1.34) = 0.43, P(-0.22 < Z < 0) = 0.09$

Q. 27 *Suppose that during rainy season, on a tropical island, the length of shower has an exponential distribution with average length of shower* $\frac{1}{2}$ *mins. What is the probability that a shower will last more than three minutes? If a shower has already lasted for 2 minutes, what is the probability that it will last for at least one more minute?*

Q. 28 *If X is normally distributed with mean 3 and standard deviation 2, find c such that* $P(X > c) = 2P(X \leq c)$.
Given that $\int\limits_{-\infty}^{0.43} \phi(t)dt = \frac{1}{3}$.

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Q. 29 *The time required to assemble a piece of machinery is a random variable having approximately a normal distribution with* $\mu = 13.2$ *minutes and* $\sigma = 3$ *minutes. What are the probabilities that the assembly of a piece of machinery of this kind will take (a) at least 11.1 minutes (b) anywhere from 10.35 to 16.05 minutes.*

Q. 30 *A point P is chosen at random on a straight line segment AB of length* $2a$. *Find the probability that the area of the rectangle AP, PB will exceed* $\frac{a^2}{2}$.

Statistics

1. Compute the arithmetic mean and standard deviation for the following data

Score	4-5	6-7	8-9	10-11	12-13	14-15
Frequency	4	10	20	15	8	3

2. The following are the scores of 10 students studied for a mathematics test and their scores on the test :

Hours of study(x)	4	9	10	14	4	7	12	22	1	17
Test score(y)	31	58	65	73	37	44	60	91	21	84

Find the equation of least square line approximate of the test score on the numbers of hour studied. Also predict the average test score of a student who studied 14 hours for the test.

3. Calculate the coefficient of correlation and obtain the lines of regression for the following data.

X	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
Y	**9**	**8**	**10**	**12**	**11**	**13**	**14**	**16**	**15**

Obtain an estimate of y which should correspond on the average to x=6.2.

4. From an ordinary frequency table from the following table

Marks	**Below of 10**	**Below of 20**	**Below of 30**	**Below of 40**	**Below of 50**
No. of student	**3**	**8**	**17**	**20**	**22**

Hence find the median and mode.

5. If x=4y+5 and y=kx+4 are two regression equations of x on y and y on x respectively, obtain the interval in which k lies.

6. In the following data two class frequency are missing :

Class interval	**100-110**	**110-120**	**120-130**	**130-140**	**140-150**	**150-160**	**160-170**	**170-180**	**180-190**	**190-200**
Frequency	**4**	**7**	**15**	**?**	**40**	**?**	**16**	**10**	**6**	**3**

Total number of frequencies is 150 and the median is 146.25. Find out the missing frequencies.

7. **The two regression equations of the variables x and y are $x=19.13-0.87y$ and $y=11.64-0.50x$. Find (i) mean of x's (ii) mean of y's and (iii)the correlation coefficient between x and y.**

8. **For two variables x and y the equations of two regression lines are $x+4y+3=0$ and $4x+9y+5=0$. Identity which one is 'of y on x'. Find the means of x and y. Find the correlation coefficient between x and y. Estimate the values of x when $y=1.5$.**
9. **Find the missing frequency of the following data if its mode is 25.**

Class interval	0-10	10-20	20-30	30-40	40-50	50-60	60-70
Frequency	14	22	27	?	23	20	15

10. **The expenditure of 1000 families is given below:**

Expenditure (Rs.)	40-59	60-79	80-99	100-119	120-139
Frequency	50	?	500	?	50

The median and mean for the distribution are both Rs. 87.50. Calculate the missing frequencies.