Assignment on Quantum Mechanics (PHYS 2001) By AD and SDG

1) If the wave function of a quantum particle is given by,

$\psi(x,t) = A\, e^{-\left(\sqrt{cm}\ \ {x^2}/{2\hbar}\right)} e^{\frac{-it\sqrt{{c}/{m}}}{2}}$, where *c* and *m* are constants.

Find the value of ***A*** from normalization condition. 3

2) Find the probability current density for a free particle. 4

3) Find the value of x at which the probability density of finding the particle is maximum in a 1-D infinite potential well. [Consider ground state (n=1)]. 2

4) Find the eigenfunction of the momentum operator $\widehat{p_x} = -i\hbar d/dx$ corresponding to the eigenvalue p_x *3*

5) Prove that : 3x3

$$i)\quad [\widehat{x^n},\quad \widehat{p_x}] = ni\hbar\widehat{x^{n-1}}$$

ii) $\left[\widehat{L_y},\widehat{L_z}\right] = i\hbar\widehat{L_x}$

$$iii)\ and\ \left[\widehat{L_z},\widehat{L_x}\right] = i\hbar\widehat{L_y}$$

6) The ground state and 1st excited state wave function of an atom are ψ_0 and ψ_1 respectively with corresponding energies being E_0 and E_1. If the system has a 40% probability of being found in the ground state and 60% probability of being found in the 1st excited state.

i) What is the general wave function of the atom? 3

ii) Compute the average energy of the atom? 3

7) Prove that the eigenfunctions of an 1-D infinite potential well are orthogonal to each other.
[**Hint**: $\frac{2}{L}\int_0^L \sin {m\pi x}/{L}\ \sin {n\pi x}/{L}\, dx = 0, for\ m \neq n$
$and = 1\ for\ m = n$] 4

8) The wave function of a particle is given by,

$$\psi(x) = \begin{cases} x(x-1), & 0 \le x \le 1 \\ 0, & otherwise \end{cases}$$

i) Calculate the probability of finding the particle in the region $0 \le x \le 0.5$.

ii) What is the average position of the particle?

iii) What will be the maximum probability of finding the particle within the given range. 2+3+2

9) Eigenfunction of a particle in 1-D infinite potential well of length L is given by, $\varphi_n(x) = \sqrt{2/L}\ \sin n\pi x/L$. For the ground state eigenfunction (n=1) show that i) $\langle x^2 \rangle = L^4/6\pi^2$, ii) $\langle x \rangle = 1/2$, iii) $\langle p_x \rangle = 0$ and $\langle p_x^2 \ \rangle = \pi^2\hbar^2/L^2$

[**Hint:** ii) $\langle x \rangle = \frac{2}{L}\int_0^L \sin \pi x/L\ x\ \sin \pi x/L\ dx = \frac{2}{L}\int_0^L x sin^2 \pi x/L\ dx$

Or, $\langle x \rangle = \frac{1}{L}\int_0^L x(1 - \cos 2\pi x/L) = \frac{1}{2}$.] 3X4

10) Write down the energy eigen-value and normalized energy eigen-function in case of a 3-D infinite cubic potential well of dimension L and find the normalization constant. Hence prove that the 1st excited state has 3-fold degeneracy. 1+1+3

QUESTIONS ON SUPERCONDUCTOR(PHYS2011)

1. What do you mean by superconductivity? Describe the two types of superconductors? 2+3
2. Define London penetration depth? 2
3. Calculate the number density of electrons in a material for which London penetration depth is $\lambda = 0.5 \times 10^{-8} m$.(take $\mu_0 = 4\pi \times 10^{-7}$ in SI unit) 3
4. Write down the name of two devices where property of superconductivity is used. 2

1. What is the critical magnetic field for a superconductor? How does it vary with temperature? 2+2
2. Write the mathematical expression for how the critical magnetic field for a superconductor vary with temperature. Draw the graph. Explain it. 2+2+2
3. What are the two basic properties of a superconductor? Show that a superconductor behaves as a diamagnetic material. 2+3
4. Establish London equation of superconductivity in terms of magnetic field induction and hence describe Meissner effect? 4+3
5. Lead (Pb) gets transition to its superconducting state at 7.20 K. If its critical magnetic field at 0K is 0.08k, calculate its critical magnetic field at $-271^0\ C$