

Assignment: Classical Mechanics

Paper: PHYS2001

1. Write down the constraint equations for the following systems: (i) simple pendulum, (ii) simple pendulum in a car moving with constant speed 'v' along a straight line and (iii) simple pendulum in a car moving with acceleration 'f' along a straight line.
2. Choose a suitable generalized coordinate and construct the kinetic and potential energy of the systems (i), (ii) and (iii) in problem 1, hence write the Lagrangian for all the three systems described in problem 1.
3. Derive the Lagrange equations of motion for all the three systems in problem 1.
4. Write down the constraint equations for a particle slipping down an inclined plane
5. Choose a suitable generalized coordinate and construct the kinetic and potential energy of the systems in problem 4, hence write the Lagrangian for all the same.
6. Derive the Lagrange equations of motion for the particle in problem 4.
7. Classify the following constraints:
 - (i) $x \dot{x} + y \dot{y} = \text{constant}$
 - (ii) $x^2 + y^2 + z^2 = \text{constant}$
 - (iii) $(x - vt)^2 + y^2 = \text{constant}$
 - (iv) $x \dot{x} + z \dot{y} + \dot{z} = 0$
8. The kinetic energy T and potential energy V of a system are given by:
$$T = \frac{1}{2} m_1 \dot{q}_1^2 + \frac{1}{2} m_2 \dot{q}_2^2 + \frac{1}{2} m_3 \dot{q}_3^2$$
$$V = \frac{1}{2} k_1 q_1^2 + \frac{1}{2} k_2 q_2^2 + k q_1 q_2$$
 - (a) Construct the Lagrangian.
 - (b) Calculate the components of generalized momentum.
 - (c) Find out the cyclic coordinates (if any).
 - (d) Identify the corresponding conserved momenta.
 - (e) Calculate the components of generalized force.
 - (f) Derive Lagrange equation of motion.
 - (g) What is the dimension of the configuration space?
 - (h) Show that the system admits a Hamiltonian.
 - (i) Construct the Hamiltonian.
 - (j) Construct the Hamilton's equations of motion.
 - (k) What is the dimension of phase space?
9. If Lagrangian of the system is independent of the time, show that the Hamiltonian of the system is conserved.
10. If a coordinate is cyclic in L show that it is also cyclic in H.