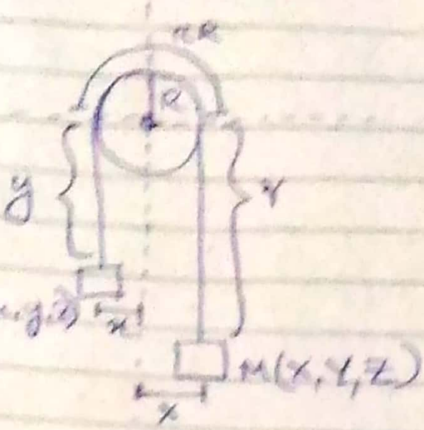
Physics (1st Internal solved)

1) (a)



(i) There a particle m, M

i.e. N=2

Now,

constraint relatn.

$z=0$ — ①

$Z=0$ — ②

$x=$ constant — ③

$X=$ constant — ④

and, $Y+y+\pi R=0$

$\Rightarrow Y+y=C$ — ⑤

All 5 of these relatn are constraint relatn for the system and they are holonomic constraints

means the constraints are only depended on the co-ordinates and time. (1st order derivate of time only)

(ii) degree of freedom $= 3N-K$

$= 3(2)-5 = 1$

→ A co-ordinate q_j (explicit functn of j^{th} coordinate) is called cyclic iff the momentum corresponding to it is conserved

Mathematically,

~~q_j $\frac{\partial L}{\partial q_j}$~~, $q_j \rightarrow$ cyclic coordinate

$$\Rightarrow \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right) - \frac{\partial L}{\partial q_j} = 0 \quad \text{[Lagrange 2nd Kind]}$$

$$\therefore \text{iff } \frac{\partial L}{\partial q_j} = 0$$

$$\Rightarrow \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right) = 0$$

$$\Rightarrow \frac{dP_j}{dt} = 0 \rightarrow (P_j \text{ is conserved})$$

momentum of q_j^{th} coordin

Q) Derive time independent schrodinger eq^n from dependent

If interaction environment $\longrightarrow$ time independent
then, probability (particle) $\longrightarrow$ " "

Let $V = V(x)$ only, set $\psi(x,t) = \psi(x) \cdot \phi(t)$

$$\frac{\partial \psi}{\partial t} = \frac{\partial(\psi(x) \cdot \phi(t))}{\partial t} = \psi(x)\, \phi'(t).$$

$$\frac{\partial^2 \psi}{\partial x^2} = \psi''(x)\, \phi(t)$$

Substituting in time dependent schrodinger Eq^n

$$\Rightarrow i\hbar\, \psi(x)\, \phi'(t) = -\frac{\hbar^2}{2m} \psi''(x)\, \phi(t) + V(x)\, \psi(x) \cdot \phi(t)$$

$$\Rightarrow \frac{i\hbar\, \phi'(t)}{\phi(t)} = \frac{-\hbar^2}{2m} \frac{\psi''(x)}{\psi(x)} + V(x) = G \text{ constant.}$$

$$\Rightarrow i\hbar\, \phi'(t) = G\, \phi(t). \quad (1)$$

Now,

$$\Rightarrow i\hbar \frac{\partial \psi(x,t)}{\partial t} = E\psi(x,t)$$

$$\Rightarrow i\hbar\, \cancel{\psi(x)} \cdot \phi'(t) = E\, \cancel{\psi(x)}\, \phi(t) \quad (2)$$

$\therefore$ from (1) & (2)

$E = G$

$$\Rightarrow \frac{-\hbar^2}{2m} \frac{\psi''(x)}{\psi(x)} + \frac{V(x)}{\psi(x)} = E \cdot \cancel{\psi(x)})$$

$$\Rightarrow \boxed{\frac{-\hbar^2}{2m} \psi''(x) + V(x) \cdot \psi(x) = E \cdot \psi(x)}$$

3D

$$\Rightarrow \frac{-\hbar^2}{2m} \nabla^2 \psi(x,y,z) + V(x,y,z)\, \psi(x,y,z) = E\, \psi(x,y,z)$$

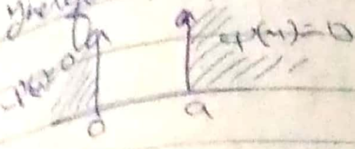
Solving (2) $\phi(t) = e^{-iEt/\hbar}$

$$\rightarrow \psi(x,t) = \psi(x)\, e^{-iEt/\hbar}$$

Particle in a 1D box (Infinite square well Potential)

$V(x) = 0$, for $0 \leq x \leq a$
$= \infty$, otherwise.

therefore, particles lies between 0 to a



$$E\psi(x) = -\frac{\hbar^2}{2m}\psi''(x) + V(x)\psi(x)$$

$\therefore$ at $0 \leq x \leq a$ $V(x) = 0$.

$$\Rightarrow -\frac{\hbar^2}{2m}\psi''(x) = E\psi(x).$$

$$\Rightarrow \psi''(x) = -\frac{2mE}{\hbar^2}\psi(x) \qquad 2mE/\hbar^2 = K^2.$$

$$\Rightarrow \frac{\partial^2\psi(x)}{\partial x^2} = -K^2\psi(x).$$

$$\Rightarrow \psi(x) = A\cos Kx + B\sin Kx . \quad [\text{sol}^n]$$

Boundary condition,

At, $x = 0$, $\psi(x) = 0$, $\Rightarrow$ ~~$\psi(0) = 0 = A$~~ $\boxed{A = 0}$

$x = a$, $\psi(x) = 0$

$\Rightarrow$ $B \sin Ka = 0$

$B \neq 0$ $\because$ then infinite sol^n will be there

$\therefore \sin Ka = 0$ $\Rightarrow Ka = n\pi$

$\Rightarrow$ $\boxed{K = n\pi/a}$

Now, $\psi(x) = B\sin\left(\frac{n\pi x}{a}\right)$, $n = \pm1, \pm2, \pm3 \cdots$ $\boxed{n \neq 0}$

Also, $K^2 = 2mE/\hbar^2$ $\Rightarrow \frac{2mE}{\hbar^2} = \frac{n^2\pi^2}{a^2}$.

$\Rightarrow E_n = \frac{n^2\pi^2\hbar^2}{2ma^2}$ $\longrightarrow$ Energy is discrete

we know,

$$\Rightarrow \int_0^a |\psi(x)|^2 dx = 1 \quad \Rightarrow \int_0^a B^2 \sin^2\left(\frac{n\pi x}{a}\right) dx = 1.$$

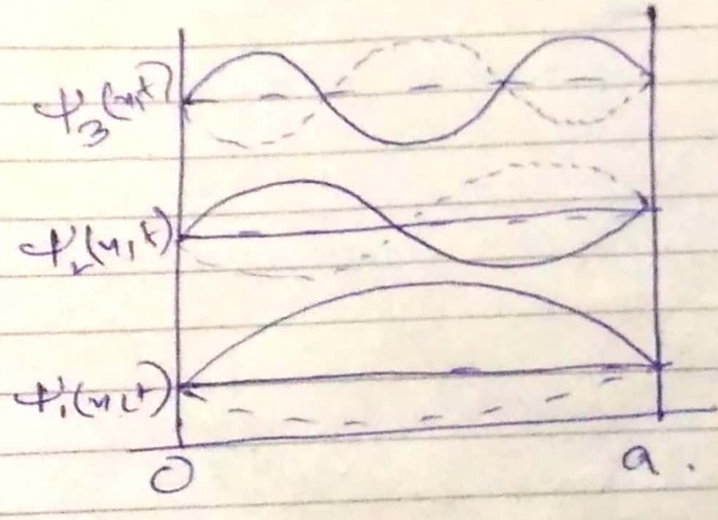
$$\Rightarrow B = \pm\sqrt{\frac{2}{a}}$$

$$\therefore \psi_n(x) = \sqrt{\frac{2}{a}}\sin\left(\frac{\pi n x}{a}\right)$$

NOW, $\psi_n(x) \rightarrow$ space part only.

$\therefore \psi_n(x,t) = \psi_n(x)\, e^{-iEt/\hbar}$

$$\Rightarrow \boxed{\psi_n(x,t) = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi x}{a}\right) e^{-iEt/\hbar}}$$



$\rightarrow$ {stationary wave}

$$P_1(x) = \psi_1^*(x) \cdot \psi_1(x)$$
$$= \frac{2}{a} \sin^2\left(\frac{n\pi x}{a}\right)$$

2(b) i) The Electric Displacement ($\vec{D}$)

It is mathematically defined as $\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$ — (1)

→ Polarisability

→ Electric field

Now, for a "Linear isotropic" dielectric,

$\vec{P} = \chi \varepsilon_0 \vec{E}$ — (2), where $\vec{P}$ & $\vec{E}$ ~~have~~ have same direction

substituting (2) in (1) we get,

$\Rightarrow \vec{D} = \varepsilon_0 \vec{E} + \chi \varepsilon_0 \vec{E}$

$\Rightarrow \vec{D} = \varepsilon_0 \vec{E}(1+\chi)$ $\Rightarrow \vec{D} = \varepsilon_0 \varepsilon_r \vec{E}$ — (3)

Now $\vec{D}$ & $\vec{E}$ are in same directⁿ

∴ since in same directⁿ we can (3) as

$D = \varepsilon_0 \varepsilon_r E.$

~~Now, for a parallel plate~~

~~Actual answer begins here~~

(a) Now, for a parallel plate capacitor without dielectric

$P = 0$, & $\boxed{E = \frac{\sigma_{free}}{\varepsilon_0}}$

Then, eq (1) becomes $\rightarrow D = \varepsilon_0 E + P = \frac{\sigma_{free}}{\varepsilon_0} \times \varepsilon_0 + 0$

$\Rightarrow \boxed{D = \sigma_{free}}$

(b) for a parallel plate capacitor "with" dielectric

Again as D

from (3)

$\Rightarrow D = \varepsilon_0 \varepsilon_r E$, $E = \frac{\sigma_{free}}{\varepsilon_0 \varepsilon_r}$

$\Rightarrow D = \frac{\varepsilon_0 \varepsilon_r \times \sigma_{free}}{\varepsilon_0 \varepsilon_r}$ $\Rightarrow \boxed{D = \sigma_{free}}$

(ii) $\alpha_e = 1.43 \times 10^{-40}$ Fm^2

$\rho = 1.8 \times 10^{+6} \times 10^{-3}$ kg/m^3.

a.m.u $\rightarrow$ 39.95×10^{-3} kg/mol.

$$V = \frac{39.95 \times 10^{-3}}{1.8 \times 10^{+6} \times 10^{-3}} \frac{m^3}{mol} = 22.194 \times 10^{-6}\ m^3/mol.$$

1 mole/m^3 $\longrightarrow$ 6.022×10^{23} atoms/m^3.

$$\rightarrow \frac{1}{22.194 \times 10^{-6}} \text{ mole}/m^3 \longrightarrow \frac{6.022 \times 10^{23} \times 10^{6}}{22.194} \text{ atoms}/m^3$$

$$\Rightarrow N \doteq .271 \times 10^{29} \text{ atoms}/m^3$$

$$\chi = \frac{N\alpha_e}{\varepsilon_0} = \frac{.271 \times 10^{29} \times 1.43 \times 10^{-40}}{8.85 \times 10^{-12}}$$

$$\chi = 0.438 \ F/m$$

$\therefore$ $\varepsilon_r = 1 + \chi = 1.438$ F/m $\longrightarrow$ Ans

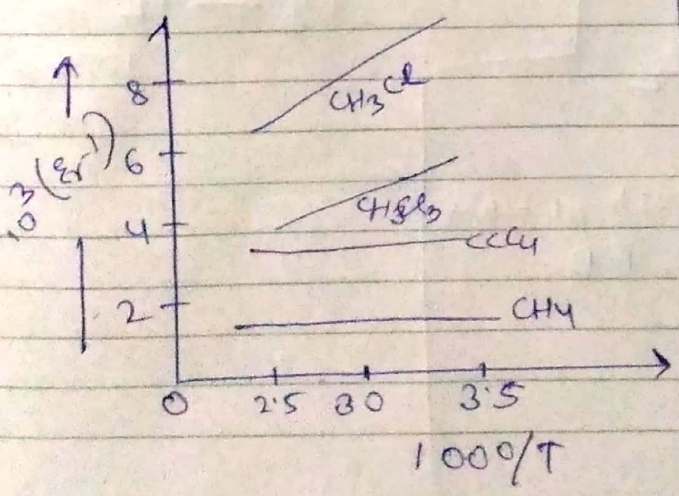
3) (a) suppose $\vec{E}$ – applied Electric field,

$N \rightarrow$ No. of gas molecules per unit volume

$\alpha_e, \alpha_i, \alpha_0$ are the respective electronic, ionic & oriental polarizabilities of the gas molecules

$$\vec{P} = \alpha_p \vec{E} = (\alpha_e + \alpha_i + \alpha_0)\vec{E} = \left(\alpha_e + \alpha_i + \frac{P_0^2}{3kT}\right)\vec{E}$$

$$\Rightarrow \varepsilon_r - 1 = \frac{N}{\varepsilon_0}\left(\alpha_e + \alpha_i + \frac{P_0^2}{3kT}\right)$$



we see CH_3Cl decreases with increase in temperature.

and

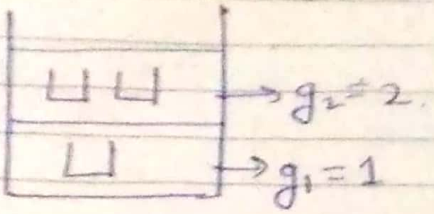
CH_4 remain constant with increase in temperature.

3(b)(i) $\omega_{MB} = N! \prod_i \frac{(g_i)^{N_i}}{N_i!}$

$\omega_{F.D} = \prod_i {}^{g_i}C_{N_i}$

$\omega_{BE} = \prod_i {}^{g_i+N_i-1}C_{N_i}$

(ii)



$N = 3$

Macrostates possible

1) (0, 3)
2) (3, 0)
3) (2, 1)
4) (1, 2)

F.D

1) (0, 3) $\because g_i \ngtr N_i$ Not possible.
2) (3, 0) → ×
3) (2, 1) → ×
4) (1, 2) → ✓

$W = {}^1C_1 \times {}^2C_2$

$= 1$

↳ (1, 2) ↳ most probable for F.D.

$W_{M.B}$ (~~for~~)

1) (0, 3)

$\omega_1 = \frac{3! (1)^0 (2)^3}{0! \, 3!}$

$= 8$

$\omega_2(3,0) = \frac{3! (1)^3 (2)^0}{3! \, 0!}$

$= 1$

$\omega_3(2,1) = \frac{3! \times (1)^2 (2)^1}{2! \, 1!}$

$= 6$

$\omega_3(1,2) = \frac{3! (1)^1 (2)^2}{1! \, 2!}$

$= 12.$

$\therefore \omega_3 \to 12$ most probable state for M.B