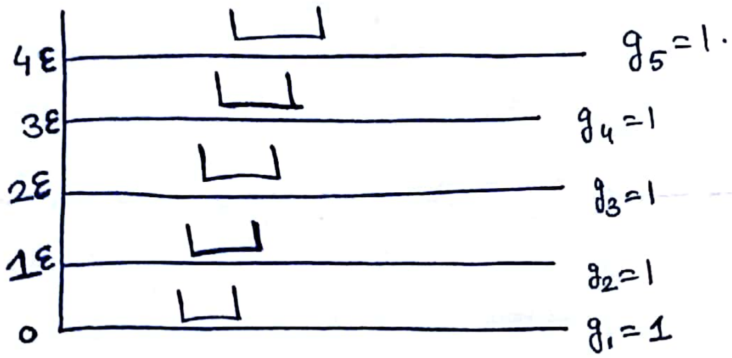
D.M SIR NUMERICAL PROBLEM

1) Three distribution particles have a total energy of 9 units. But the particles are restricted to energy levels from 0 to 4. Calculate the number of macrostates and microstate.



Case 1 :

for distinguishable Particles

Macrostates	Microstate
~~(0, 1, 8, 0, 0)~~	
~~(8, 1, 0, 0, 0)~~	
~~(3, (3, 0, 0,~~ (0, 0, 0, 3, 0)	$\rightarrow W_1 = 1$.
(0, 0, 1, 1, 1).	$\rightarrow W_2 = 6$
(2, 0, 0, 1, 0)	$\rightarrow W_3 = 3$
(0, 1, 0, 0, 2)	$\rightarrow W_4 = 3$ / 13 ways.

$\therefore$ Total number of microstates = 13

For indistinguishable Particles, Total number of microstates = 3.

$4 \times 1 + 3 \times 1$

$4 + 3 + 2$

$4 \times 2 + , 0, 0, 1 \times 1$

$3 \times 2 + 2 \times 1$

$\frac{1^3}{3!}$

$\frac{1}{3!}$ ~~$\times 3!$~~

$3! \times 1^1 \times 1^1 \times 1^1$

$1! \times 1$

1

$3! \times 1^2 \times 1^1$

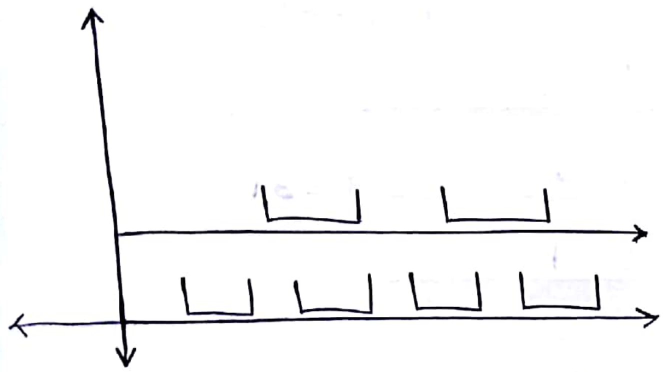
$2! \times 1! \; 1$

$3! \times 1^0 \times 1 \times 1 \times 1$

$1! \times 2!$

=

2). 8 distinguishable Particles are distributed in two compartments. The first compartment is divided into 4 cells and the second into two cells. Each cell is of equally priority probability and there is no restriction on the number of Particles that can be contained in each cell. Calculate the thermodynamic probability of :- a) the most probable State b). the macrostate (8,0).



Since the Particles are distinguishable,

Macrostates

1) (8, 0)

2). (4, 4) → which is the most probable State.

So, Total number of microstates $= W_K = \dfrac{8! \times 4^4 \times 2^4}{4! \times 4!}$

$= 4587520$ (a/c to M-B)

A/c to B-E

$W_K = {}^{4+4-1}C_4 \times {}^{2+4-1}C_4$

$= {}^7C_4 \times {}^5C_4$

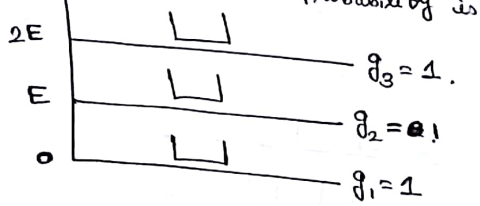
$= 175$ ways.

F.D is not applicable here.

1) For (8,0) macrostate, $W_K = \dfrac{\cancel{8!} \times 4^8 \times 2^0}{\cancel{8!}} = 65536$ (A/c to M-B)

$W_K = {}^{4+8-1}C_8 \times {}^{2+0-1}C_0 = 165$ (A/c to B-E).

3). Six distinguishable particles are distributed ~~in two compartments~~ over three non-degenerate levels of energies 0, E, and 2E. Calculate the number of microstate of the system. and the energy of the distribution for which the probability is maximum.



Macrostate

Macrostates

1) (6, 0, 0).
2) (0, 0, 6)
3) (2, 2, 2)
4). (4, 2, 0)
5) (0, 2, 4)
6) (1, 2, 3)
7) (3, 2, 1).
8). (5, 1, 0)
9) (0, 1, 5).
10) (2, 4, 0)
11) (2, 1, 3)
12) (3, 1, 2).
13) (2, 3, 1).

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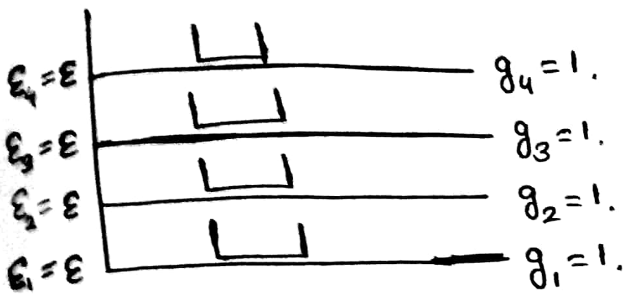
4) 3 distinguishable Particles each of which can be in one of the E, 2E, 3E, 4E energy states have total energy 6E. Find all possible numbers of distribution of all Particles in the energy states.

Distinguishable



$N = 3.$

$U = 6E.$

$U = N_1 E_1 + N_2 E_2 + \ldots$

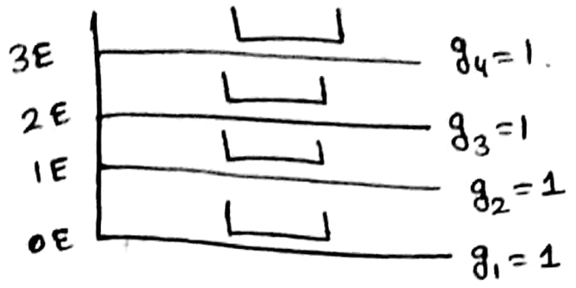
Macro State	Micro State.
1. M_1 (1, 1, 1, 0)	$\rightarrow W_1 = 6$
2. M_2 (2, 0, 0, 1).	$\rightarrow W_2 = 3$
3. M_3 (0, 3, 0, 0).	$\rightarrow W_3 = 1$ / 10 ways.

So, Total ways of arrangement = 10 ways.

For indistinguishable Particles, total number of microstates = 3. <u>Ans</u>

5). Three distinguishable Particles each of which can be in one of the ~~E, 2E, 3~~ non-degenerate states with 0, 1, 2, 3 energy of the system have total energy of 3 units. Find the microstates, if the Particles obey :- a). M-B b) F-D c) B-E Statistics

a) When the Particles obey M-B Statistics,



Macrostates

1. (1, 1, 1, 0)
2. (0, 3, 0, 0).
3. (2, 0, 0, 1).

~~$W_{M-B} = 3!$~~

Total energy = 3E.

For (1, 1, 1, 0) macrostates $\longrightarrow$ $W_{M-B} = \dfrac{3! \times 1^1 \times 1^1 \times 1^1}{1! \times 1! \times 1!}$

$= 6$ ways.

For (0, 3, 0, 0) macrostates $\longrightarrow$ $W_{M-B} = \dfrac{3! \times 1^0 \times 1^3 \times 1^0 \times 1^0}{3!}$

$= 1$ ways.

For (2, 0, 0, 1) macrostates $\longrightarrow$ $W_{M-B} = \dfrac{3! \times 1^2 \times 1^0 \times 1^0 \times 1^1}{2! \times 1!}$

$\Rightarrow 3$ ways.

So, Total number of microstates $= 6 + 3 + 1 = 10$ ways.

b) No. F-D distribution is possible as no. of Particles is greater than the degeneracy number in each energy state.

c). For B-E Statistics,
For (1, 1, 1, 0) macrostates $\longrightarrow$ $W_{B-E} = {}^{1+1-1}C_1 \times {}^{1+1-1}C_1 \times {}^{1+1-1}C_1$

$= 1$ ways.

For (0, 3, 0, 0) macrostates $\rightarrow$

$W_{B-E} = {}^{1+0-1}C_0 \times {}^{3+1-1}C_3 \times {}^{1+0-1}C_0 \times {}^{1+0-1}C_0$

$= 1$ ways.

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For $(2,0,0,1)$ macrostate,

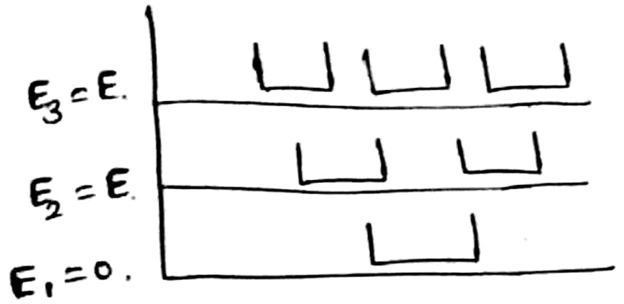
$$W_{B\text{-}E} = {}^{1+2-1}C_2 \times {}^{1+0-1}C_0 \times {}^{1+0-1}C_0 \times {}^{1+1-1}C_1$$

$$= 1 \times 1 \times 1 \times 1$$

$$= 1 \text{ ways.}$$

So, total number of microstates $= 1+1+1 = 3$ ways.

6) 4 Particles are distributed in 3 energy levels having energies 0, E, 3E so that the total energy is 4E. If the levels are degenerate with degeneracy 1, 2, 3 respectively, find out the macrostates and the corresponding microstates for M-B Particles and B-E Particles.



$N = 4.$

$U = 4E.$

M-B

$M_1 = (0,4,0) \longrightarrow W_1 = N! \times \prod_i \frac{g_i^{N_i}}{N_i!}$

$= \frac{\cancel{4!} \times 1^0 \times 2^4 \times 3^0}{\cancel{4!}}$

$= 16$ ways.

$M_2 = (2,1,1) \longrightarrow W_2 = 4! \times \frac{1^2 \times 2^1 \times 3^1}{2! \times 1! \times 1!}$

$= 72$ ways.

So, total number of microstates $= 72 + 16 = 88$ ways.

F-D

$g_i \gg N_i$.

for this given data, no F-D Statistics is Possible.

B-E

$M_1\ (0,4,0) \longrightarrow W_1 = {}^{1+0-1}C_0 \times {}^{2+4-1}C_4 \times {}^{3+0-1}C_0$

$= 5$ ways.

$M_2\ (2,1,1) \longrightarrow W_2 = {}^{1+2-1}C_2 \times {}^{2+1-1}C_1 \times {}^{3+1-1}C_1$ ways

$= 6$ ways.

So, total no. of microstates $= 6 + 5 = 11$ ways.

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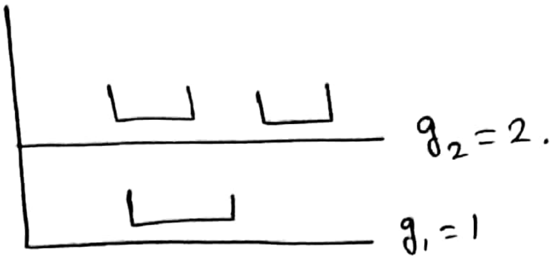
7). 2 Particles are distributed into two energy levels with degeneracy 1, 2 respectively. Find the most probable state of the distribution of the Particle in the system, if the Particles obey a) M-B b) F-D c) B-E Statistics.



When the Particles are distinguishable,

For M-B Statistics

Macrostates

$M_1 \rightarrow (2,0)$

$M_2 \rightarrow (1,1)$

$M_3 \rightarrow (0,2)$.

Microstates

For (2,0) macrostates,

$$W_{M-B} = \frac{\cancel{2!} \times 1^2 \times 2^0}{\cancel{2!}} = 1.$$

For (1,1) macrostates,

$$W_{M-B} = \frac{2! \times 1^1 \times 2^1}{1! \times 1!} = 4$$

For (0,2) macrostates

$$W_{M-B} = \frac{\cancel{2!} \times 1^0 \times 2^2}{\cancel{2!}} = 4$$

∴ Total number of microstates according to M-B $= 4+4+1$
$= 9$

For F-D Statistics

For (2,0) macro state → No F-D distribution possible.

For (1,1) macrostate → ${}^1C_1 \times {}^2C_1 = 2$

For (0,2) macro state → ${}^1C_0 \times {}^2C_2 = 1$

So, Total $= 2+1 = 3$ ways.

For B-E Statistics

For (2,0) macrostate $\Rightarrow W_1 = {}^{1+2-1}C_2 \times {}^{2+0-1}C_0 = 1$

For (1,1) macrostate $\Rightarrow W_2 = {}^{1+1-1}C_1 \times {}^{2+1-1}C_1 = 2.$

For (0,2) macrostate $\Rightarrow W_3 = {}^{1+0-1}C_0 \times {}^{2+2-1}C_2 = 6$

So, Total number of ways = 6+2+1 = 9 ways.

8). A system has 7 particles arranged in two compartments. The 1st compartment has 8 cells and the second has ~~3~~ 10 cells. , all cells are of equal size. Calculate the number of microstates in the macrostates (3, 4), when the particles are fermions and bosons.

9). A system has two particles, each one of them can be in one of three quantum states. Find the ~~corresponding~~ possible number of microstates of the system according to the three statistics.

10). Find the fermi-energy at $T = 0K$ for Sodium, Given that density of Sodium $= 0.97 \times 10^3\ kg/m^3$, atomic weight $= 23$ and Avogadro's number $= 6.023 \times 10^{26}\ kg/mol$.

$$E_F = \frac{h^2}{2m}\left(\frac{3N}{8\pi V}\right)^{2/3}$$

where $$\frac{N}{V} = \frac{N_0 \rho}{W}$$

$$= \frac{6.023 \times 10^{26} \times 0.97 \times 1000}{23}$$

$$= 2.54 \times 10^{28} \text{ electrons}/m^3.$$

$$E_F = \frac{(6.626 \times 10^{-34})^2}{2 \times 9.11 \times 10^{-31}} \times \left(\frac{3 \times 2.54 \times 10^{28}}{8\pi}\right)^{2/3}.$$

$$= \frac{5.047 \times \cancel{10^{-19}}}{1.6 \times \cancel{10^{-19}}}\ eV. \quad = 3.154\ eV.$$

~~3.154 eV (Ans)~~

$\therefore E_f = 3.154\ eV.$ (Ans)

11). The fermi energy of Sodium at $T = 0K$ is $3.1\ eV$. Find its Value for aluminium given that the free electron density of Aluminium is approximately 8 times that in Na.

$$E_F(0) = 3.1\ eV.$$

$$\left(\frac{N}{V}\right)_{Aluminium} = \frac{6.023 \times 10^{26} \times 8 \times 0.97 \times 1000}{27}$$

$$= 1.73 \times 10^{29}$$

$$\therefore \; E_F = \frac{(6.626\times10^{-34})^2}{2\times9.11\times10^{-31}} \times \left(\frac{3}{8\pi}\times1.731\times10^{29}\right)^{2/3}$$

$$= 11.34 \text{ eV}.$$

$$\therefore \; E_F (\text{Al at } T = 0K) = 11.34 \text{ eV}. \;\; (\text{Ans})$$

12). Find the fermi energy at T=0K for Cu, given that, $\rho = 8.96\times10^3$ kg/mol., atomic weight = 63.5, $N = 6.023\times10^{26}$/kg-mol.

$$\left(\frac{N}{V}\right) = \frac{N_0\,\rho}{W}$$

where $\rho \rightarrow$ Density

$N_0 \rightarrow$ Avogadro's number.

$W \rightarrow$ Atomic weight.

$$= \frac{6.023\times10^{26}\times8.96\times10^{3}}{63.5}$$

$$= \cancel{84}\; 8.4985\times10^{28}.$$

SUB GROUP TOTAL	1,466,035.70	0.00
SUB TOTAL	1,466,035.70	0.00
CHARGES	54,301.00	0.00
SUB GROUP TOTAL	54,301.00	0.00
SUB TOTAL	54,301.00	0.00
GRAND TOTAL	66,136,069.27	0.00
		66,136,069.27

$$\therefore E_f = \frac{h^2}{2m}\left(\frac{3N}{8\pi V}\right)^{2/3}.$$

$$= \frac{(6.626\times10^{-34})^2}{2\times9.11\times10^{-31}} \times \left(\frac{3}{8\pi}\times 8.4985\times10^{28}\right)^{2/3}.$$

$$= \frac{1.1292\times10^{-18}}{1.6\times10^{-19}}\ eV.$$

$$= 7.057\ eV.$$

$\therefore E_f = 7.057\ eV$ | for cu at $T=0K$.

13) The numbers of conduction electron per cc is ~~24.22~~ 24.2×10^{22} in Beryllium and 0.91×10^{22} in cesium. If the fermi-energy of conduction electrons in Beryllium is 14.44 eV, calculate that in cesium.

$$\left.\frac{N}{V}\right|_{Beryllium} = 24.2\times10^{22}. \qquad \left.\frac{N}{V}\right|_{cesium} = 0.91\times10^{22}.$$

$$\left.E_F\right|_{Beryllium} = 14.44\ eV.$$

Beryllium

$$\therefore 14.44\times1.6\times10^{-19} = \frac{h^2}{2m}\left(\frac{3}{8\pi}\left(\frac{N}{V}\right)\right)^{2/3}.$$

$$14.44\times1.6\times10^{-19} = \frac{h^2}{2m}\left(\frac{3}{8\pi}\times 24.2\times10^{22}\right)^{2/3}.$$

$$\frac{h^2}{2m} = 2.45\times10^{-33}.$$

$$\therefore \left.E_f\right|_{cesium} = \frac{h^2}{2m}\left(\frac{3}{8\pi}\times0.91\times10^{22}\right)^{2/3}.$$

$$= \frac{2.588\times10^{-19}}{1.6\times10^{-19}}\ eV$$

$$= 1.62\ eV.$$ Ans.

14) Fermi energy of conduction electron in silver is 5.48 eV, calculate the number of such electron per cc.

$$E_f = 5.48\,eV = 5.48 \times 1.6 \times 10^{-19}\,J.$$

$\therefore$ We have to find, $\frac{N}{V}$.

$$\therefore E_f = \frac{h^2}{2m}\left(\frac{3}{8\pi}\frac{N}{V}\right)^{2/3}$$

$$5.48 \times 1.6 \times 10^{-19} = \frac{h^2}{2m} \times \left(\frac{3}{8\pi}\right)^{2/3} \times \left(\frac{N}{V}\right)^{2/3}.$$

$$3.6387 \times 10^{18} = \left(\frac{3}{8\pi}\right)^{2/3} \times \left(\frac{N}{V}\right)^{2/3}.$$

$$1.5 \times 10^{19} = \left(\frac{N}{V}\right)^{2/3}.$$

$$\Rightarrow \ln(1.5 \times 10^{19}) = \frac{2}{3} \ln\left(\frac{N}{V}\right).$$

$$\Rightarrow \frac{44.155 \times 3}{2} = \ln\left(\frac{N}{V}\right).$$

$$\frac{N}{V} = 5.814 \times 10^{28} \text{ per cc.}$$ {Ans}

15) Calculate the occupation probability at 2KT units of energy above the fermi Energy E_F.

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16). Assuming that in tungsten (at. Wt. = 183.8, density = 19.3 gm/cc), there are two free-electron per atom. Calculate the fermi energy and electron density.

$$\cancel{\frac{N}{V} = 2\times 6.023\times 10^{28}}$$

$$\frac{N}{V} = \frac{2\times 6.023\times 10^{26}\times 19.3\times 1000}{183.8} \text{ electrons}/m^3$$

$$= 1.509\times 10^{28} \text{ electrons}/m^3.$$

$$\cancel{\mathcal{E}_f = \frac{h^2}{2m}\left(\frac{3}{8\pi}\times(1.264\times 10^{23})\right)^{2/3}}$$

$$\mathcal{E}_f = \frac{h^2}{2m}\left(\frac{3}{8\pi}\times(1.509\times 10^{28})\right)^{2/3}$$

$$= 1.472\times 10^{-18} \text{ J}$$

$$= \frac{1.472\times 10^{-18}}{1.6\times 10^{-19}} \text{ eV}$$

$$= 9.2 \text{ eV}.$$

$\therefore \mathcal{E}_f = 9.2$ eV.

Electron density $= \frac{N}{V} = 1.509\times 10^{28}$ electrons$/m^3$

17). Find the average velocity of electrons at 0°c in a metal having 3×10^{22} electrons per cm^3.

$$\bar{V} = \frac{3}{4} v_F.$$

$\frac{N}{V} = 3\times 10^{22}$ electrons$/cm^3$
$= 3\times 10^{28}$ electrons$/m^3$

$$V_F = \frac{h}{m}\left(\frac{3N}{8\pi V}\right)^{1/3}$$

$$= \frac{6.626\times 10^{-34}}{9.11\times 10^{-31}}\left(\frac{3}{8\pi}\times 3\times 10^{28}\right)^{1/3}$$

$$= 1.1127\times 10^{6} \text{ m/sec.}$$

$\therefore$ Average velocity, $\overline{V_F} = \frac{3}{4} V_F$

$= \frac{3}{4} \times 1.1127 \times 10^6$ m/sec.

$= 8.345 \times 10^5$ m/sec.

$\therefore$ Average velocity $= 8.345 \times 10^5$ m/sec.

18). Consider a free electron gas at 0K, and show that the de-Broglie wavelength associated with an electron is given by $\lambda_F = 2\left(\frac{\pi}{3n_0}\right)^{1/3}$, where, $n_0 =$ concentration of electron.