(4) Prove that $\mathbb{Z}_8$ is not a homomorphic image of $\mathbb{Z}_{15}$.

If $\mathbb{Z}_8$ is a homomorphic image of $\mathbb{Z}_{15}$,

then $\phi(\mathbb{Z}_8)$ is a subgroup of $\mathbb{Z}_{15}$ for some mapping $\phi : \mathbb{Z}_8 \to \mathbb{Z}_{15}$

For a non trivial homomorphism $|\phi(\mathbb{Z}_8)| = 8$. Also $|\mathbb{Z}_{15}| = 15$

But $8 \nmid 15$. $\Rightarrow \phi(\mathbb{Z}_8) \not\leq \mathbb{Z}_{15}$ [By Lagrange Theorem of groups]

Thus, $\mathbb{Z}_8$ is not a homomorphic image of $\mathbb{Z}_{15}$.

⑳ Find all solutions of the equation $x^2+x-6=0$ in the ring $\mathbb{Z}_{14}$ by factoring

$x^2+x-6 = (x+3)(x-2) = 0$ (given) $\therefore$ roots $= \bar{2}, \bar{4}, \bar{9}, \bar{11}$ mod 14

㉑ Find all solution of the equation $x^3-2x^2-3x=0$ in the ring $\mathbb{Z}_{12}$

$x^3-2x^2-3x = x(x+1)(x-3)=0$ (given) $\therefore$ roots $= 0, 3, 5, 8, 9, 11$ mod 12.