Super Conductivity

[2nd Yr, PHYS2001, by SDG]

H. Kamerling Onnes, successfully could liquefy helium in 1908. Superconductivity was discovered in 1911 in the Leiden laboratory of Kamerlingh Onnes when a so-called "blue boy" (local high school student recruited for the tedious job of monitoring experiments) noticed that the resistivity of mercury (Hg) metal vanished abruptly at about 4.2 K, a phase transition to a zero resistance state. The experiment was repeated with many other metals and alloys. **The phenomenon is called superconductivity** and the temperature, which is material specific, below which this transition takes place is called its **critical temperature.**

Although phenomenological models with predictive power were developed in the 30's and 40's, the microscopic mechanism underlying superconductivity was not discovered until 1957 by Bardeen Cooper and Schrieffer. Superconductors have been studied intensively for their fundamental interest and for the promise of technological applications which would be possible if a material which superconducts at room temperature were discovered. Until 1986, critical temperatures (T_c 's) at which resistance disappears were always less than about 23K. In 1986, Bednorz and Mueller published a paper, subsequently recognized with the 1987 Nobel prize, for the discovery of a new class of materials which currently include members with T_c 's of about 135K.

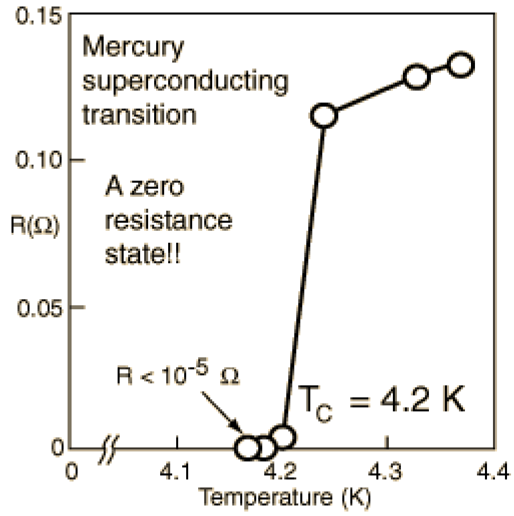


Fig 1: Zero resistance of Hg as temperature decreased to 4.2K

Meissner and Robert Ochsenfeld discovered that superconductor also expels magnetic field from their interior and behaves exactly as diamagnetic substance. Superconducting properties are best described by BCS theory that takes quantum aspects into its consideration.

Meissner effect

In 1933, Walter Meissner and Robert Ochsenfeld discovered a magnetic phenomenon that showed that superconductors are not just perfect conductors. They found that when a bulk super conducting material makes the transition from the normal to the superconducting state when cooled in **longitudinal magnetic field below its critical temperature**, the **lines of induction B are pushed out of the bulk body** of the superconductor. The phenomenon is called **Meissner effect**.

Figure 2 illustrates a thought experiment that highlights this difference. Imagine that both the ideal conductor and superconductor are above their critical temperature, $\boldsymbol{T_c}$. That is, they both are in a normal conducting state and have electrical resistance. A magnetic field, $\boldsymbol{B_a}$, is then applied. This results in the field penetrating both materials. Both samples are then cooled so that the ideal conductor now has zero resistance. It is found that the superconductor expels the magnetic field from inside it, while the ideal conductor maintains its interior field. Note that energy is needed by the superconductor to expel the magnetic field. This energy comes from the exothermic superconducting transition. Switching off the field induces currents in the ideal conductor that prevent changes in the magnetic field inside it – by Lenz's law. However, the superconductor returns to its initial state, i.e. no magnetic field inside or outside it.

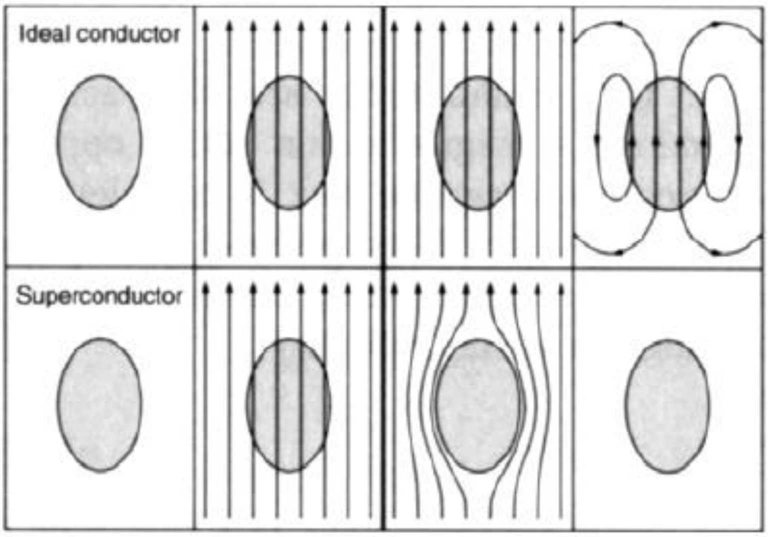


Fig 2: Meissner effect

We have,

$\boldsymbol{B} = \mu_0(\boldsymbol{H} + \boldsymbol{M})$ where, **H** and **M** are the **magnetising field and intensity of magnetization** respectively and μ_0 is the free space permeability.

Since, **B** is **zero inside** the superconducting bulk material, we have, **susceptibility,**

$\chi = \frac{\boldsymbol{M}}{\boldsymbol{H}}$ **= -1,** showing diamagnetic property.

Superconducting materials exhibit the following unusual behaviours:

1. **Zero resistance**. Below a material's Tc , the DC electrical resistivity ρ is really zero, not just very small. This leads to the possibility of a related effect,

2. **Persistent currents**. If a current is set up in a superconductor with multiply connected topology, e.g. a torus, then it will flow forever without any driving voltage. (In practice experiments have been performed in which persistent currents flow for several years without signs of degrading).

3. **Perfect diamagnetism**. A superconductor expels a weak magnetic field nearly completely from its interior (screening currents flow to compensate the field within a surface layer of a few 100 or 1000 A, and the field at the sample surface drops to zero over this layer).

Thus zero resistivity and diamagnetism are the two basic characteristics of superconductors. (Important)

London Equation

Let us first consider the magnetic penetration depth in the context of the conventional London theory at T=0. If a magnetic field is applied to a superconductor which is initially in zero field, the magnetic field is a function of time. According to the Maxwell equation

$\boldsymbol{\nabla X E} = -\frac{\partial \boldsymbol{B}}{\partial \boldsymbol{t}}$, the time-varying magnetic field gives rise to an electric field.

In a normal metal this will induce eddy currents, but in a superconductor the **E**-field will give rise to persistent currents (*i.e.* supercurrents). The induced supercurrents will in turn generate a magnetic field of their own which opposes the applied magnetic field.

If the applied magnetic field is weak, the flux is totally screened from the bulk of the superconductor. This phenomenon is often described as ``perfect diamagnetism". From Newton's law, the equation of motion for a superconducting carrier with mass *m* and charge -*e* in the presence of an electric field **E** is

$$\text{F}= m\frac{d\mathbf{v_s}}{dt} = -e\boldsymbol{E} \qquad (1)$$

where $\boldsymbol{v_s}$ is the velocity of the superconducting carrier.

The field-induced supercurrent density is given by

$$\boldsymbol{J_s} = -n_s e \boldsymbol{v_s} \qquad \textbf{(2)}$$

where $\boldsymbol{n_s}$ is the local density of superconducting carriers.

(1) and (2) implies,

$$\frac{d\boldsymbol{J}_s}{dt} = \frac{n_s e^2}{m}\boldsymbol{E} \qquad \textbf{(3)}$$

Taking curl on both the sides,

$$\boldsymbol{\nabla}\mathbf{x}\frac{d\boldsymbol{J}_s}{dt} = \boldsymbol{\nabla}\mathbf{x}\left(\frac{n_s e^2}{m}\boldsymbol{E}\right)$$

Since we are within non-relativistic electrodynamics regime, space and time derivative can be interchanged.

Hence, by using Maxwells' equation,

$$\frac{d(\boldsymbol{\nabla}\mathbf{x}\boldsymbol{J}_s)}{dt} = \frac{n_s e^2}{m}(\boldsymbol{\nabla}\mathbf{x}\boldsymbol{E}) = -\frac{n_s e^2}{m}\frac{\partial \boldsymbol{B}}{\partial t}$$

Therefore,

$$(\boldsymbol{\nabla}\mathbf{x}\boldsymbol{J}_s) = -\frac{n_s e^2}{m}\boldsymbol{B} \qquad (4)$$

$$(\boldsymbol{\nabla}\mathbf{x}\boldsymbol{J}_s) = -\frac{n_s e^2}{m}\boldsymbol{\nabla}\mathbf{x}\boldsymbol{A}$$

Implies,

$$\boldsymbol{J_s} = -\frac{\boldsymbol{n_s e^2}}{\boldsymbol{m}}\boldsymbol{A} \qquad \textbf{(5)}$$

(4) and (5) are called London first equation in two different forms.

From the differential form of **Ampere circuital law**, $\boldsymbol{\nabla} \times \boldsymbol{B} = \mu_0 \boldsymbol{J}$ (6)

Taking curl on both the sides, $\boldsymbol{\nabla}\mathbf{X}(\boldsymbol{\nabla}\mathbf{X}\boldsymbol{B}) = \mu_0(\boldsymbol{\nabla}\mathbf{X}\boldsymbol{J})$ (7)

using (4), $\boldsymbol{\nabla}\mathbf{X}(\boldsymbol{\nabla}\mathbf{X}\boldsymbol{B}) = -\boldsymbol{\mu_0}\frac{\boldsymbol{n_s e^2}}{\boldsymbol{m}}\boldsymbol{B}$

$$\boldsymbol{\nabla}(\boldsymbol{\nabla}.\boldsymbol{B}) - \nabla^2\boldsymbol{B} = -\mu_0\frac{\boldsymbol{n_s}e^2}{m}\boldsymbol{B}$$

$$\nabla^2\boldsymbol{B} = \mu_0\frac{\boldsymbol{n_s}e^2}{m}\boldsymbol{B}$$

$$\nabla^2\boldsymbol{B} = \frac{1}{{\lambda_L}^2}\boldsymbol{B} \qquad \text{where, } = \sqrt{\frac{m}{\mu_0\,\boldsymbol{n_s}e^2}}$$

hence, $\boldsymbol{B} = \boldsymbol{B_0}\boldsymbol{e}^{-x/\lambda_L}$ **(8)**

Where $\boldsymbol{\lambda}$ is called the **London penetration depth**. $\boldsymbol{B_0}$ **is** the magnetic flux density at the surface of the bulk superconducting material. **Equation (8) is called second London equation**.

The penetration depth is defined as the distance below the bulk material where the magnetic field reduces to 1/e of its value at the surface. This explains Meissner effect. Typical penetration depth varies from 50 to 500 nm.

Variation of London Penetration depth with Temperature:

Equation (8) was derived for **T= 0 K**. **At nonzero temperature the behaviour of λ_L can be approximated by incorporating the two-fluid model of Gorter and Casimir**. In this model the electron system is assumed to contain a superconducting component with electron density $\boldsymbol{n_s}$, and a normal component with an electron density $\boldsymbol{n_n}$. The total electron density $\boldsymbol{n = n_s + n_n}$ becomes $\boldsymbol{n = n_s \; at \; T = 0 \; K \; and \; n = n_n \; for \; T > T_c}$, where $\boldsymbol{T_c}$ is the superconducting transition temperature. For arbitrary temperature, Gorter and Casimir found that good agreement with early experiments could be obtained if one assumes that

$$\boldsymbol{n_s(T) = n\left[1 - \left(\frac{T}{T_c}\right)^4\right]}$$

when this expression is combined with Eq. (8) the penetration depth is given by,

$$\boldsymbol{\lambda_L(T) = \lambda_L(0)\left[1 - \frac{T^4}{T_c^4}\right]^{-1/2}} \quad \dots\dots\dots.(\mathbf{9}),$$

where, $\boldsymbol{\lambda_L(0)} = \sqrt{\frac{m}{\mu_0 \, n_s e^2}}$ is the London Penetration depth at T = 0 K and T_c is the critical temperature above which the substance becomes normal and the magnetic field penetrates the whole specimen.

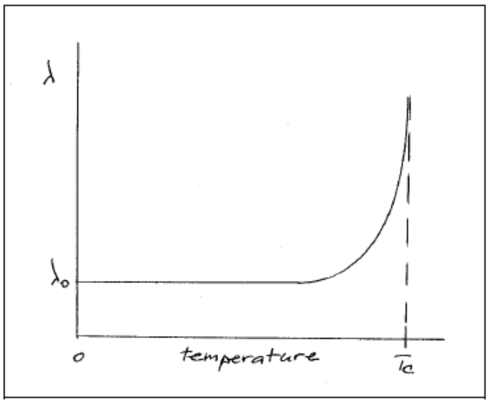


Fig 3: Variation of London penetration depth with temperature

Variation of Critical Magnetic Field with Temperature:

If a superconductor is placed in a magnetic field and the field is kept increasing, then at a particular magnetic field depending on its temperature the superconductor loses its state of superconductivity. This magnetic field is called **critical magnetic field**.
The empirical formula that fits the variation of critical magnetic field that can destroy the superconductivity varies with temperature as,

$$B_c \approx B_c(0)\left[1-\left(\frac{T}{T_c}\right)^2\right] \qquad (10)$$

where $\boldsymbol{B_c}\,(\mathbf{0})$ **and** $\boldsymbol{T_c}$ **are the critical magnetic field at 0K and the critical temperature for the material.**

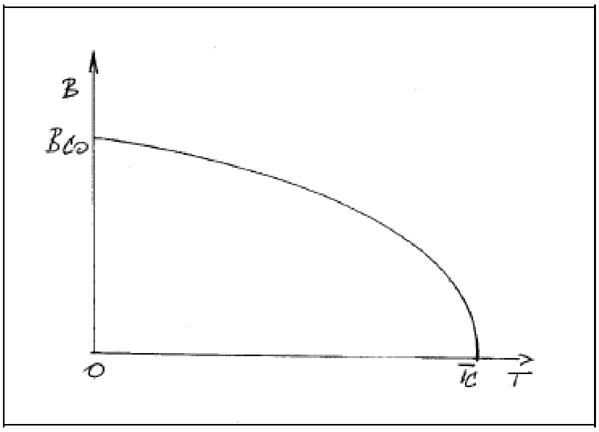


Fig 3: Variation of Critical Magnetic field with temperature

Types of Superconductors: (Important)

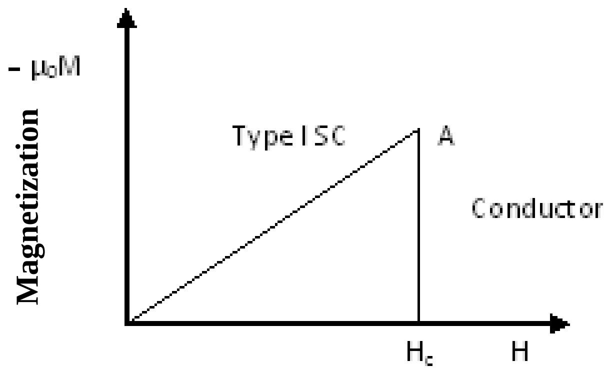
Depending upon their behavior in an external magnetic field, superconductors are divided into two types:

a) **Type I** superconductors and b) **Type II** superconductors

Let us discuss them one by one:

1) **Type I superconductors**:

a)Type I superconductors are those superconductors which **loose their superconductivity very easily or abruptly when placed in the external magnetic field**. As you can see from the graph of intensity of magnetization (M) versus applied magnetic field (H), when the Type I superconductor is placed in the magnetic field, it suddenly or easily looses its superconductivity at critical magnetic field (Hc) (point A).



Applied Magnetic field

After Hc, the Type I superconductor will become conductor.

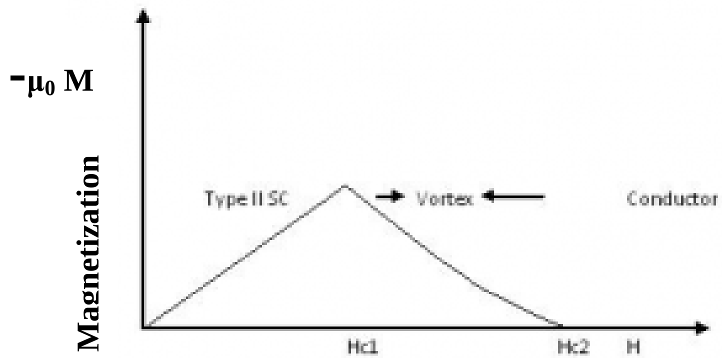
b). Type I superconductors are also known as **soft superconductors** because of this reason that is they loose their superconductivity easily.

c) Type I superconductors perfectly obey **Meissner effect.**

d) Example of Type I superconductors: Aluminum (Hc = 0.0105 Tesla), Zinc (Hc = 0.0054)

2) **<u>Type II superconductors</u>**:

a). Type II superconductors are those superconductors **which loose their superconductivity gradually but not easily or abruptly when placed in the external magnetic field**. As you can see from the graph of intensity of magnetization (M) versus applied magnetic field (H), when the Type II superconductor is placed in the magnetic field, it gradually looses its superconductivity. Type II superconductors start to loose their superconductivity at lower critical magnetic field (Hc1) and completely loose their superconductivity at upper critical magnetic field (Hc2).



Applied Magnetic field

b) The state between the lower critical magnetic field (Hc1) and upper critical magnetic field (Hc2) is known as **vortex state or intermediate state**.

After Hc2, the Type II superconductor will become conductor.

c). Type II superconductors are also known as **hard superconductors** because of this reason that is they loose their superconductivity gradually but not easily.

c) Type II superconductors obey **Meissner effect** but not completely.

d) Example of Type II superconductors: NbN (Hc = 8 x 10^6 Tesla), $Babi_3$ (Hc = 59 x 10^3 Tesla)

e) Application of Type II superconductors: Type II superconductors are used for strong field superconducting magnets.

Uses of superconductors:

Superconducting electromagnets finds its use to produce the large magnetic fields required in the world's largest **particle accelerators**, in **MRI machines** used for diagnostic imaging of the human body, in **magnetically levitated trains** and in superconducting magnetic energy storage systems.

But at the other extreme superconductors are used in **SQUID** (superconducting quantum interference device) magnetometers, which can measure the tiny magnetic fields associated with electrical activity in the brain etc.

Suggestive Questions: **(Important)**

1. What do you mean by superconductivity? What are the two types of superconductors?
2. What is the critical magnetic field for a superconductor? How does it vary with temperature?
3. Write the mathematical expression for how the critical magnetic field for a superconductor vary with temperature. Draw the graph.
4. What are the two basic properties of a superconductor? Show that a superconductor behaves as a diamagnetic material.
5. Establish London equation of superconductivity in terms of magnetic field induction and hence describe Meissner effect?
6. Define London penetration depth?
7. Lead (Pb) gets transition to its superconducting state at 7.20 K. If its critical magnetic field at 0K is 0.08k, calculate its critical magnetic field at $-271^0\,C$
8. Calculate the number density of electrons in a material for which London penetration depth is $\lambda = 0.5 \times 10^{-8} m$.(take $\mu_0 = 4\pi \times 10^{-7}$in SI unit)