

# Dielectric Properties of Solids

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## 1. What is the difference between metal, semiconductor and dielectric?

Ans.:

Metal	Semiconductor	Dielectric
1. Metals can conduct electricity as they have large number of free electrons.	Intrinsic (without doping) semiconductors cannot conduct electricity at zero temperature.	Dielectrics are insulators so they cannot carry electricity even in room temperature.
2. Valence and conduction bands overlaps, so metals do not have band gap.	They have band gap of the order of few electron volts.	They have large band gap, more than 10 eV.
3. Metals cannot be polarized by applying an external electric field.	Semiconductors can be partially polarized by applying an external electric field.	Dielectrics can be polarized by applying an external electric field.

## 2. What is dipole and dipole moment?

Ans. A dipole is an entity in which equal and opposite charges are separated by a small distance.

Dipole moment: The product of any one of the two charges and the distance between them is known as dipole moment.

$$\vec{p} = q \vec{dl}$$

The unit of electric dipole moment is Debye ( $1 D = 3.33 \times 10^{-30} \text{ Coulomb. Meter}$ ).

## 3. What is dielectric constant?

Ans. The ratio of capacitances with and without dielectric material within the capacitor plates is known as the dielectric constant of that particular dielectric material.

$$\frac{C}{C_0} = \epsilon_r$$

It is independent of the shape and dimension of the material or the capacitor plates and is solely a characteristic of the particular dielectric material.  $\epsilon_r$  is also called the relative permittivity or the dielectric constant of the dielectric material.

$$\epsilon = \epsilon_0 \epsilon_r$$

Where,  $\epsilon$  and  $\epsilon_0$  are called the absolute permittivity and free space permittivity respectively. The value of  $\epsilon_0$  is given by

$$\epsilon_0 = 8.85 \times 10^{-12} \text{ Farad/Meter}$$

**4. What is so special about dielectric?**

Ans. The most important property of dielectrics is their ability to become polarized under the action of an external electric field. The atoms and molecules of dielectrics are influenced by the external electric field and hence the positive parts are pushed in the direction of the electric field while the negative parts are directed oppositely from their equilibrium positions. Hence dipoles are developed and they produce a field of their own. The process of producing electric dipoles out of the natural atoms and molecules is referred as polarization.

**5. Define electric flux and flux density. What is the unit of flux density?**

Ans.

Flux: The number of electric lines of force (field lines) passing through an area is known as electric flux.

Flux density: The number of electric lines of force (field lines) passing through unit area and perpendicular to that area is known as electric flux density.

Unit: In MKS system of units, one line of force or one field line originates on each unit charge (1Coulomb). Hence a charge  $q$  will send  $q$  lines of force and this will be received by an area of  $4\pi r^2$  (considering spherical symmetry of space). The number of lines of force received per unit area is known as flux density or the electric displacement vector ( $\vec{D}$ ).

$$\vec{D} = \frac{q}{4\pi r^2} \hat{r}$$

So the unit of flux density is *Coulomb/meter<sup>2</sup>*.

**6. State the Gauss's law and verify it from the Coulomb's of electrostatics.**

Ans.

Gauss's law: It states that the normal electric flux through a closed surface is equal to the charge enclosed by the surface.

$$\Phi = \oint \vec{D} \cdot d\vec{a} = q_{\text{enclosed}}$$

From Coulomb's law the flux density is given by

$$\vec{D} = \frac{q}{4\pi r^2} \hat{r}$$

Therefore the total flux enclosed within an area is given by

$$\Phi = \oint \vec{D} \cdot d\vec{a} = \oint \frac{q}{4\pi r^2} \hat{r} \cdot d\vec{a}$$

$$= \frac{q}{4\pi} \oint \frac{\hat{r} \cdot d\vec{a}}{r^2}$$

$$\oint \frac{q}{4\pi} d\Omega$$

Where  $d\Omega$  is the differential solid angle by the area  $\vec{da}$  at the centre of the material.

Therefore,

$$\Phi = \frac{q}{4\pi} \cdot 4\pi = q.$$

**7. What is the relation between electric flux density and the electric field?**

Ans. From the Coulomb's law, we know that the electric field due to a point charge,  $q$  at distance  $r$  from the charge in free space is given by

$$\vec{E} = \frac{q}{4\pi\epsilon_0 r^2} \hat{r}$$

And the electric flux density is given by

$$\vec{D} = \frac{q}{4\pi r^2} \hat{r}$$

Therefore in free space,

$$\vec{D} = \epsilon_0 \vec{E}.$$

Therefore, in free space the Gauss's law can be written as in terms of the electric field,

$$\oint \vec{E} \cdot \vec{da} = \frac{q_{\text{enclosed}}}{\epsilon_0}$$

**8. Using the Gauss's law, find the electric field near a uniformly charge distributed two dimensional thin sheet.**

According to the Gauss's law

$$\oint \vec{E} \cdot \vec{da} = \frac{q}{\epsilon_0}$$

If  $q$  amount of charge is distributed at one surface of the 2D thin sheet, then the electric field will be uniform near a point of the charged surface. Hence from the above equation we can write,

$$\oint E \cdot da \cdot \cos 0^\circ = \frac{q}{\epsilon_0}$$

Since, the direction of electric field and the area are same.

As the electric field is uniform at a point near the charged surface, therefore

$$EA = \frac{q}{\epsilon_0}$$

Where,  $A$  is the area of the charged surface.

Hence,

$$E = \frac{q}{\epsilon_0 A} = \frac{\sigma}{\epsilon_0}$$

where,  $\sigma$  is the surface charge density.

For 2D metallic sheet, the charge  $q$  is distributed within two faces of the surface. In that case, Gauss's law gives,

$$EA + EA = \frac{q}{\epsilon_0}$$

$$\Rightarrow 2EA = \frac{q}{\epsilon_0}$$

$$\Rightarrow E = \frac{q}{2\epsilon_0 A} = \frac{\sigma}{2\epsilon_0}$$

**9. Find out the capacitance of a parallel plate capacitor.**

Ans. The electric field within the two capacitor plate is given by

$$E_0 = \frac{q}{2\epsilon_0 A} - \left( \frac{-q}{2\epsilon_0 A} \right) = \frac{q}{\epsilon_0 A}$$

since, one plate is has positive charge and another has negative charge.

If the separation between the two plates of the parallel plate capacitor is  $d$  then the potential difference between the two plates is given by,

$$V_0 = E_0 d = \frac{qd}{\epsilon_0 A}.$$

Therefore the capacitance of the parallel plate capacitor is given by,

$$C_0 = \frac{q}{V} = \frac{\epsilon_0 A}{d}.$$

Now if a dielectric material (dielectric constant  $= \epsilon_r$ ) is placed between the two capacitor plates, then the capacitance of the parallel plate capacitor is given by,

$$C = \frac{\epsilon A}{d}$$

Therefore,

$$\frac{C}{C_0} = \frac{\epsilon}{\epsilon_0} = \epsilon_r.$$

**10. What charge is induced in a dielectric material when it is kept within a parallel plate capacitor?**

Ans. As we know

$$\frac{C}{C_0} = \frac{V_0}{V} = \epsilon_r,$$

therefore

$$\begin{aligned} \frac{E_0}{E} &= \frac{V_0}{V} = \epsilon_r \\ \Rightarrow E &= \frac{E_0}{\epsilon_r} = \frac{q}{A \epsilon_0 \epsilon_r} \end{aligned}$$

Let the charge induced within the dielectric material is  $q'$ . Since the opposite charge is induced near a charged plate, therefore using Gauss's law we can write,

$$\oint \vec{E} \cdot d\vec{a} = \frac{q - q'}{\epsilon_0}$$

$$\Rightarrow EA = \frac{q - q'}{\epsilon_0}$$

$$\Rightarrow E = \frac{q - q'}{\epsilon_0 A}$$

Now,  $E = \frac{E_0}{\epsilon_r} = \frac{q}{A \epsilon_0 \epsilon_r}$ , therefore,

$$\frac{E_0}{\epsilon_r} = \frac{q - q'}{\epsilon_0 A}$$

$$\Rightarrow \frac{q}{A \epsilon_0 \epsilon_r} = \frac{q - q'}{\epsilon_0 A}$$

$$\Rightarrow \frac{q}{\epsilon_r} = q - q'$$

$$\Rightarrow q' = q - \frac{q}{\epsilon_r}.$$

**11. What is electric displacement vector? How is it related to the electric field and the electric polarization?**

Ans. Electric displacement Vector ( $\vec{D}$ ): It is numerically equal to the number of lines of forces (field lines) passing through unit cross sectional area perpendicular to the area, i.e. it numerically represents the electric flux density.

When, a dielectric material is placed within the two plates of a parallel plate capacitor, then the induced charge ( $q'$ ) within the dielectric due to polarization is given by,

$$q' = q - \frac{q}{\epsilon_r}.$$

Therefore,

$$q = \frac{q}{\epsilon_r} + q'$$

$$\Rightarrow \frac{q}{A} = \epsilon_0 \left( \frac{q}{A \epsilon_0 \epsilon_r} \right) + \frac{q'}{A}.$$

In the above equation  $\frac{q}{A}$  represents the number of field lines passing through unit area in

SI system of units and  $\left( \frac{q}{A \epsilon_0 \epsilon_r} \right)$  represents the electric field ( $E$ ) within the dielectric and

$\frac{q'}{A}$  represents the induced charge per unit area which is also known as electric polarization ( $P$ ). Hence we may write,

$$D = \epsilon_0 E + P$$

In vector form, we may write,

$$\vec{D} = \epsilon_0 \vec{E} + \vec{P}$$

## 12. What is dielectric susceptibility? How is it related to the dielectric constant?

Ans.

Susceptibility ( $\chi$ ): It represents how easily and up to what extent a dielectric material can be polarized.

In free space, i.e. when there is no dielectric, then the polarization  $\vec{P} = 0$ . Therefore,

$$D = \epsilon_0 E.$$

But when, there is dielectric, then

$$D = \frac{q}{A} = \frac{\epsilon_0 \epsilon_r q}{\epsilon_0 \epsilon_r A} = \epsilon_0 \epsilon_r E = \epsilon E.$$

Therefore, we may write,

$$\epsilon E = \epsilon_0 E + P$$

$$\Rightarrow P = \epsilon E - \epsilon_0 E$$

$$\Rightarrow P = \epsilon_0 \epsilon_r E - \epsilon_0 E$$

$$\Rightarrow P = \epsilon_0 E (\epsilon_r - 1)$$

$$\therefore \chi = \frac{P}{\epsilon_0 E} = (\epsilon_r - 1).$$

### 13. What is polarization? How is it related to the electric dipole moment?

Ans. When an electric field is applied to a dielectric material, charge separation occurs. For example, the bound electrons may be displaced with respect to the nucleus opposite to the direction of the applied electric field. As a net effect a dipole is created and this process is known as polarization.

The electric dipole moment is defined as,

$$\vec{p} = q \vec{dl}$$

where,  $\vec{dl}$  is the separation between the two charges and a vector pointing from positive to negative charge.

Polarization: The total dipole moment per unit volume within a dielectric is called the polarization of the dielectric.

$$\vec{P} = \frac{\sum_{i=0}^n \vec{p}_i}{V}$$

where,  $n$  is the total number of dipoles and  $V$  is the volume of the dielectric. If we consider, there are  $N$  number of dipoles per unit volume of the dielectric and the average dipole moment of each of the dipole moment is  $p$ , then we may write the polarization as,

$$\vec{P} = N \vec{p}.$$

### 14. What are the ways by which a material can be polarized?

Ans. Dielectric polarization is nothing but the displacement of the charges under the action of the electric field to which they are subjected. An external electric field can induce dipole moment or polarize a dielectric material via four basic mechanisms.

(i) Electronic polarization.

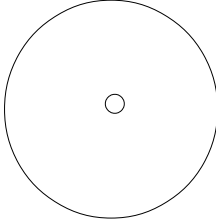
(ii) Ionic polarization.

(iii) Orientational polarization.

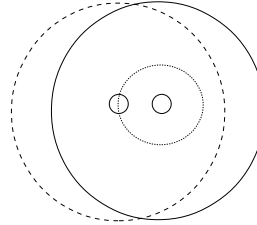
(iv) Interfacial or space charge polarization.

15. Describe electronic polarization, find out the electronic polarizability and establish the relation between polarization and polarizability.

Ans.



Without applying  
an electric field



With applying an  
electric field

This type of polarization has been explained with the help of rare gas atoms, in which it is assumed that the interaction among the atoms is negligible. Here the nucleus of charge  $+Ze$  is surrounded by an electronic density of charge  $-Ze$  distributed in a sphere of radius  $R$ . The electronic charge density  $\rho$  is given by,

$$\rho = \frac{-Ze}{\frac{4}{3}\pi R^3} = -\frac{3}{4} \frac{Ze}{\pi R^3}$$

When the system is subjected to an external electric field of intensity  $E$ , the nucleus and the electrons experience Lorentz force of magnitude  $ZeE$  in opposite directions. Therefore the nucleus and the electron density are pulled apart. As they are pulled apart a Coulomb force develops between them which tends to counter the displacement and hence the actual magnitude of the displacement is of the order of  $10^{-17}$  meter for a field of  $30 \text{ kV/m}$ . Let the small displacement be  $x$ . To calculate the induced dipole moment in the atomic model, it is firstly assumed that only the electron density is displaced by the field.

Thus the Lorentz force  $= -ZeE$ .

$$\text{Coulomb force} = Ze \times \frac{\text{Charge enclosed in the sphere of radius } x}{4\pi\epsilon_0 x^2}.$$

$$\text{The charge enclosed} = \frac{4}{3}\pi x^3 \rho = \frac{4}{3}\pi x^3 \left( -\frac{3}{4} \frac{Ze}{\pi R^3} \right) = -\frac{Ze x^3}{R^3}.$$

$$\text{Hence the Coulomb force} = \frac{Ze}{4\pi\epsilon_0 x^2} \left( \frac{-Ze x^3}{R^3} \right) = -\frac{Z^2 e^2 x}{4\pi\epsilon_0 R^3}.$$

In equilibrium position, the two forces are equal, i.e.,



$$-ZeE = \frac{-Z^2 e^2 x}{4\pi\epsilon_0 R^3}$$

$$\Rightarrow x = \frac{4\pi\epsilon_0 R^3 E}{Ze}.$$

Therefore, the displacement of electronic density is proportional to the electric field  $E$ .

Thus, the two charges  $+Ze$  and  $-Ze$  are now separated by a distance  $x$  constitute an induced electric dipole moment ( $p_e$ ) given by,

$$p_e = Zex = \frac{Ze \cdot 4\pi\epsilon_0 R^3 E}{Ze} = 4\pi\epsilon_0 R^3 E.$$

$$\therefore p_e \propto E$$

$$\therefore p_e = \alpha_e E,$$

where,  $\alpha_e = 4\pi\epsilon_0 R^3$  is called the electronic polarizability. Thus, the induced electric dipole moment is proportional to the applied electric field. The dipole moment per unit volume is called electronic polarization ( $P_e$ ).

$$P_e = N p_e = N \alpha_e E,$$

where,  $N$  is the number of atoms per unit volume.

Relation between polarization and polarizability:

We know,

$$P_e = \epsilon_0 E (\epsilon_r - 1).$$

Therefore,

$$\epsilon_0 E (\epsilon_r - 1) = N \alpha_e E$$

$$\therefore \chi = \epsilon_r - 1 = \frac{N \alpha_e}{\epsilon_0}.$$

## 16. What is the microscopic mechanism of ionic polarization?

When atoms form molecules, electronic polarization is still possible, but there may be an additional polarization due to a relative displacement of the atomic components of the molecule in the presence of the electric field. Consider a  $NaCl$  molecule, it is composed of  $Na^{+\ddot{\cdot}}$  and  $Cl^{-\ddot{\cdot}}$  ions and their distance is, say,  $r_0$ . It is evident that such a molecule has

a inbuilt dipole moment equal to  $r_0 e$  which exists even before the field applied. Such molecules are said to be polar. Note, all the molecules are not polar, since an essential requirement is that the centre of positive charge and the centre of negative charge should not coincide and this not true for all the molecules for example, an oxygen molecule ( $O_2:O=O$ ). We can estimate the permanent dipole moment of an  $NaCl$  molecule by assuming an interatomic separation of  $0.1\text{ nm}$ ; the dipole moment is then of the order of  $10^{-29}\text{ C.m}$ .

When an electric field is applied to the molecule, the  $Na^{+}$  and the  $Cl^{-}$  ions are displaced in opposite directions until the ionic binding forces stop the process, thus increasing the dipole moment.

Again, it is found that this induced dipole moment is proportional to the applied field and an ionic polarizability ( $\alpha_i$ ) is introduced to account for the increase, giving,

$$p_i = \alpha_i E$$

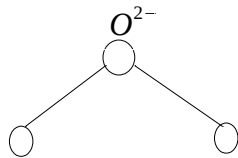
So, the combined effect of both the polarization is given by,

$$P = P_e + P_i = N(\alpha_e + \alpha_i)E.$$

### 17. What is the mechanism of orientational polarization?

This type of polarization occurs in polar substances. The existence of a permanent dipole moment is purely a matter of molecular geometry. The permanent molecular dipoles in such materials can rotate about their axis of symmetry to align with an applied field which exerts a torque on them. The additional polarization effect is accounted by an orientational polarizability term  $\alpha_0$ .

The orientational polarizability,  $\alpha_0$ , is temperature dependent. Since higher the temperature, the greater is the thermal agitation and lower is  $\alpha_0$ .



### 18. Discuss the Langevin's theory of polarization in dielectrics.

When a polar dielectric is subjected to an electric field the individual dipoles experience torques which tend to align them in the direction of the electric field. If the field is very strong, we may assume all the dipoles to be aligned completely and the polarization may be achieved the saturation value,

$$P_s = Np$$

where,  $N$  is the number of molecules per unit volume. This electric field will also produce induced dipole effects. For the moment, we will ignore this contribution. The deviation from saturation value of polarization is due to the thermal energy of the molecules, which produces random dipole orientations. Statistical mechanics gives the probability of finding a molecule with a particular molecular energy  $W$  at temperature  $T$  is proportional to  $e^{\frac{-W}{k_B T}}$ .

Here the system is subjected to a field  $E$  applied along  $x$  direction. The potential energy associated with each dipole may be obtained as follows:

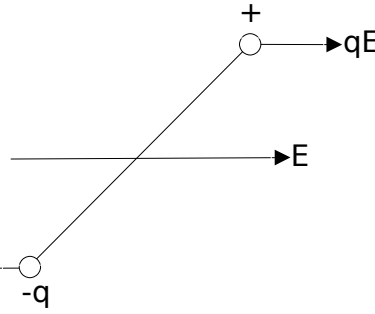
$$\text{Torque of the couple } (\vec{T}) = \vec{r} \times \vec{F}$$

$$\Rightarrow T = qEl \sin \theta.$$

Therefore, potential energy

is given by,

$$W = \int_{90^\circ}^{\theta} T d\theta = \int_{90^\circ}^{\theta} qEl \sin \theta d\theta = -pE \cos \theta$$



since,  $p = ql$ , the dipole moment.

But, the number of molecules ( $dN$ ) having inclinations between  $\theta$  and  $\theta + d\theta$  per unit volume with energy vale  $W$  is proportional to  $\left(e^{\frac{-W}{k_B T}}\right) \sin \theta d\theta$ , i.e.

$$dN \propto \left(e^{\frac{-pE \cos \theta}{k_B T}}\right) \sin \theta d\theta$$

$$\Rightarrow dN = C \left(e^{\frac{-pE \cos \theta}{k_B T}}\right) \sin \theta d\theta,$$

where,  $C$  is the proportionality constant.

Let  $a = \frac{pE}{k_B T}$ , then

$$dN = C e^{-a \cos \theta} \sin \theta d\theta$$

Therefore, the total number of molecules ( $N$ ) having inclinations between 0 to  $\pi$  is given by,

$$N = \int_0^\pi C \left(e^{-a \cos \theta}\right) \sin \theta d\theta.$$

Let,  $x = \cos\theta$ ,  $\Rightarrow dx = -\sin\theta d\theta$ ,

$$\therefore N = \int_1^{-1} C(e^{-ax})(-dx) = C \int_{-1}^1 (e^{-ax}) dx = C \frac{e^a - e^{-a}}{a}$$

$$\Rightarrow C = \frac{aN}{e^a - e^{-a}}.$$

Therefore, the resolved components of all the dipole moments inclined at an angle  $\theta$  will be,

$$dP_o = dNp \cos\theta.$$

Hence the total dipole moment due to all  $N$  molecules lying between 0 to  $\pi$  contained in unit volume gives orientational polarization,

$$P_o = \int dP_o = \int_0^\pi dNp \cos\theta = \int_0^\pi p \cos\theta C e^{a \cos\theta} d\theta.$$

Let  $x = \cos\theta \Rightarrow dx = -\sin\theta d\theta$ .

$$\therefore P_o = Cp \int_1^{-1} x e^{ax} (-dx) = \frac{pN}{e^a - e^{-a}} \int_{-1}^1 x e^{ax} dx$$

$$= \frac{pN}{e^a - e^{-a}} \left[ \left[ x e^{ax} \right]_{-1}^1 - \int_{-1}^1 e^{ax} dx \right]$$

$$= \frac{pN}{e^a - e^{-a}} \left\{ (e^a + e^{-a}) - \frac{(e^a - e^{-a})}{a} \right\}$$

$$= pN \left\{ \frac{(e^a + e^{-a})}{e^a - e^{-a}} - \frac{1}{a} \right\} = pNL(a),$$

where,  $L(a) = \left\{ \frac{(e^a + e^{-a})}{e^a - e^{-a}} - \frac{1}{a} \right\}$  is known as Langevin's function.

In the above equation,  $pN$  represents the dipole moment per unit volume when all the molecules are turned into the direction of the applied electric field and hence it gives the saturation value of  $P_o$  say  $P_s$ . This  $P_s$  is a constant depending upon the nature of the gas.

Thus,

$$\frac{P_o}{P_s} = L(a) = \left( \coth(a) - \frac{1}{a} \right).$$

When  $a \left( \frac{pE}{k_B T} \right)$  becomes large, that is, at very low temperature the function reaches a saturation corresponding to a maximum alignment of the dipoles along the field direction, i.e. at very high field and low temperature.

So, at large  $a$ ,  $L(a) \rightarrow 1$ , and so,  $P_o = P_s = pN$ .

The second characteristic of the curve is important for many practical purposes. That is for high temperature or small field,

$a = \frac{pE}{k_B T} \ll 1$  and in such case,

$$L(a) = \left( \coth(a) - \frac{1}{a} \right) = \left( \frac{e^a + e^{-a}}{e^a - e^{-a}} - \frac{1}{a} \right)$$

$$= \frac{\left( 1 + a + \frac{a^2}{2!} + \frac{a^3}{3!} + \dots \right) + \left( 1 - a + \frac{a^2}{2!} - \frac{a^3}{3!} + \dots \right)}{\left( 1 + a + \frac{a^2}{2!} + \frac{a^3}{3!} + \dots \right) - \left( 1 - a + \frac{a^2}{2!} - \frac{a^3}{3!} + \dots \right)} - \frac{1}{a}$$

$$= \frac{2 + 2\frac{a^2}{2!}}{2a + 2\frac{a^3}{3!}} - \frac{1}{a} = \frac{1 + \frac{a^2}{2} - 1 - \frac{a^2}{6}}{a(1 + \frac{a^2}{6})} = \frac{2a^2}{6a(1 + \frac{a^2}{6})} \approx \frac{a}{3}$$

Neglecting the higher powers of  $a$ .

$$\therefore \frac{P_o}{P_s} = L(a) \approx \frac{a}{3}.$$

The initial part of the curve (i.e. for small values of  $a$ ) is sensibly linear and coincides with the tangent of the curve at the origin whose slope is  $\frac{1}{3}$ .

$$\therefore P_o = P_s \left( \frac{a}{3} \right) = pN \frac{pE}{3k_B T} = \frac{p^2 NE}{3k_B T}.$$

Now,  $P_o = \alpha_o NE$  with  $\alpha_o = \frac{p^2}{3k_B T}$ . i.e. the orientational polarization is inversely proportional to the absolute temperature  $T$  and directly proportional to the applied electric field  $E$ . Thus,

$$L(a) = \frac{P_o}{P_s} = \frac{a}{3}$$

$$\Rightarrow \frac{L(a)}{a} = \frac{1}{3}.$$

Now, the total polarization for the polyatomic gas is given by,

$$P = P_e + P_i + P_o = NE(\alpha_e + \alpha_i + \alpha_o)$$

$$\therefore \epsilon_0(\epsilon_r - 1) = N(\alpha_e + \alpha_i) + \frac{Np^2}{3k_B T}$$

$$\therefore \chi = (\epsilon_r - 1) = \frac{N}{\epsilon_0} \left[ \alpha_e + \alpha_i + \frac{p^2}{3k_B T} \right]$$

### 17. What do you mean by interfacial or space charge polarization?

Ans. Let's consider a dielectric medium placed between two electrodes. When there is no applied field, the positive and the negative charges are not separated and there are fixed number of charges.

If an electric field is now applied, the charges are separated and migrate towards the electrodes. The positive charges accumulated near the negative electrode while the negative charges are accumulated near the positive electrode. Therefore a dipole moment is induced due to displacement of the ions. Then the induced dipole moment per unit volume gives the induced polarization. This polarization is known as interfacial polarization. It is usually neglected in the calculation of total polarization, since it is very small.

## Numerical Problems

1. If the susceptibility of  $CO_2$  is  $0.985 \times 10^{-3}$ , calculate the total polarizability of  $CO_2$ , if its density is  $1.997 \text{ kg/m}^3$ .

Molecular weight of  $CO_2 = 44 \text{ g} = 0.044 \text{ kg}$ .

Therefore, number of molecules per unit volume,

$$N = \frac{1.997 \times N_A}{0.044} = \frac{1.997 \times 6.022 \times 10^{23}}{0.044} = 273.317 \times 10^{23} / \text{m}^3.$$

Where,  $N_A$  is the Avogadro number.

$$\chi = \frac{N \alpha_e}{\epsilon_0}$$

$$\Rightarrow \alpha_e = \frac{\chi \epsilon_0}{N} = \frac{0.985 \times 10^{-3} \times 8.85 \times 10^{-12}}{273.317 \times 10^{23}} \text{ F m}^2$$

2. Calculate the electronic polarizability of Argon atom having radius  $10^{-10} m$ .

$$\alpha_e = 4\pi\epsilon_0 R^3 = 4 \times 3.14 \times 8.85 \times 10^{-12} \times (10^{-10})^3 F m^2.$$

3. A solid dielectric has electronic polarizability of  $10^{-40} F m^2$ , find out the dielectric constant of the material if there are  $10^{22} atoms/m^3$ .

$$\epsilon_r - 1 = \frac{N\alpha_e}{\epsilon_0}.$$

4. Calculate the value of radius of the electron orbit of Helium atom and hence the permittivity for  $2.7 \times 10^{25} atoms/m^3$ . [Given, the electronic polarizability of He =  $0.98 \times 10^{-40} F m^2$ ].

5. Consider a parallel plate capacitor without dielectric with its plate having equal and opposite charges. The capacitor is not connected to any circuit. What is the electric field ( $E_0$ ) between the plates of the capacitor? Now insert a dielectric between the plates so that it completely fills the region between the plates. Derive an expression for the electric field between the plates in the presence of the dielectric and express it in terms of  $E_0$ .

6. Derive an expression for the electronic polarizability using a simple spherical model of an atom.

7. Consider a gas of polar molecules where the magnitude of the permanent dipole moment per molecule is  $1 D$ . Calculate the field strength required at  $T = 297 K$  so that the average dipole moment per molecule is 0.07 times the saturation value.

Orientational polarization is given by,

$$P_o = \frac{p^2 NE}{3k_B T}.$$

Saturation polarization is given by,

$$P_s = Np$$

Now,

$$P_o = 0.07 \times P_s$$

$$\therefore \frac{p^2 NE}{3k_B T} = 0.07 \times Np$$

$$\Rightarrow E = 0.07 \times \frac{3k_B T}{p}.$$

Where,  $p = 1 D = 3.33 \times 10^{-33} Cm$

8. Calculate the induced dipole moment per unit volume of He gas if it is placed in an electric field of 6000 V/cm. The atomic polarizability of He is  $0.18 \times 10^{-40} \text{ F m}^2$  and gas density is  $2.6 \times 10^{25}$  atoms per  $\text{m}^3$ .

$$E = 6000 \frac{\text{V}}{\text{cm}} = 6000 \frac{\text{V}}{0.01 \text{ m}} = 600000 \frac{\text{V}}{\text{m}}$$

$$P_e = N \alpha_e E = 2.6 \times 10^{25} \times 0.18 \times 10^{-40} \text{ F m}^2 \times 600000 \frac{\text{C}}{\text{m}^2}$$