① Calculate the induced dipole per unit volume of the gas if it is placed in an electric field of 6000 V/cm. Given $\alpha_{He} = 0.18\times10^{-40}\,Fm^2$ and gas density $= 2.6\times10^{25}$ atoms/m^3

Ans: $\vec{P} = ?$

$E = 6000\,V/10^{-2}m$

$\alpha = 0.18\times10^{-40}\,Fm^2$

$N = 2.6\times10^{25}$ atoms/m^3

$\vec{P} = N\vec{p} = N\alpha E = 2.6\times10^{25}\,\frac{atoms}{m^3} \times 0.18\times10^{-40}\,Fm^2 \times \frac{6000V}{10^{-2}m}$

$\therefore \vec{P} = 2.8\times10^{-10}$ C atoms/m^2

② For argon, electric polarizability is $1.43\times10^{-40}\,Fm^2$. Find the dielectric constant given gas density is $1.8\,g/cm^3$ and atomic mass of argon is 39.95 g/mol.

Ans. $\alpha = 1.43\times10^{-40}\,Fm^2$

$\chi = ?$ $\quad \varepsilon_r = ?$

$d = 1.8\times\frac{10^{-3}\,kg}{10^{-6}\,m^3}$

$amu = 39.95\times10^{-3}\,\frac{kg}{mol}$

$\vec{P} = \chi\varepsilon_0\vec{E} = N\alpha\vec{E} \Rightarrow \chi\varepsilon_0 = N\alpha$

$\chi = \frac{N\alpha}{\varepsilon_0} = \frac{dN_0}{amu}\cdot\frac{\alpha}{\varepsilon_0} = \frac{1.8\times\cancel{10^{-3}kg}}{10^{-6}m^3} \times 6.022\times10^{23}\,\frac{atoms}{\cancel{mol}} \Big/ 39.95\times\frac{\cancel{10^{-3}kg}}{\cancel{mol}} \times \frac{1.43\times10^{-40}\,Fm^2}{8.85\times10^{-12}\,\frac{s^4A^2}{m^3kg}}$

$\chi = 0.43$

$\varepsilon_r = 1+\chi = 1.43$

③ A parallel plate capacitor of area $4\times5\,cm^2$ is filled with mica ($\varepsilon_r = 6$). The distance between the plates is 1mm and the capacitor is connected to a 100V battery. Calculate

a) capacitance of the capacitor

b) free charge on the plates

c) polarized surface charge density

Repeat for $\varepsilon_r = 5$. Explain the difference in result for the two ε_r

Ans

Given

$A = 20\times10^{-4}\,m^2$

$\varepsilon_r = 6\ \&\ 5$

$d = 10^{-3}\,m$

$V_0 = 100V$

$\varepsilon_0 = 8.85\times10^{-12}\,\frac{s^4A^2}{m^3kg}$

Equations

$\vec{E} = \frac{V}{d}$

$\sigma_p = \chi\varepsilon_0 E = (\varepsilon_r - 1)\varepsilon_0 E$

$C = \frac{Q_{free}}{V}$

$\sigma_{free} = \frac{Q_{free}}{A} = \varepsilon_0\vec{E} + \sigma_p$

Case 1: battery still connected $\Rightarrow V = V_0$

$E = \frac{V_0}{d}$

$C = \frac{Q_{free}}{V} = \frac{A}{V_0}(\varepsilon_0 E + (\varepsilon_r - 1)\varepsilon_0 E) = \frac{A\varepsilon_0\varepsilon_r}{d}$

$Q_{free} = CV = \frac{A\varepsilon_0\varepsilon_r V_0}{d}$

$\sigma_p = (\varepsilon_r - 1)\varepsilon_0 E = (\varepsilon_r - 1)\varepsilon_0\frac{V_0}{d}$

	$\varepsilon_r = 5$	$\varepsilon_r = 6$
C	$8.85\times10^{-11}\,F$	$1.062\times10^{-10}\,F$
Q_{free}	$8.85\times10^{-9}\,C$	$1.062\times10^{-8}\,C$
σ_p	$3.54\times10^{-6}\,C/m^2$	$4.425\times10^{-6}\,C/m^2$

with increase in ε_r
C, Q_{free} and σ_p increase

Case 2: battery disconnected after charging $\Rightarrow V = \frac{V_0}{\varepsilon_r}$

$E = \frac{V}{d}$

$C = \frac{Q_{free}}{V} = \frac{A}{V}(\varepsilon_0 E + (\varepsilon_r - 1)\varepsilon_0 E) = \frac{A}{d}\varepsilon_0\varepsilon_r$

$Q_{free} = CV = \frac{A\varepsilon_0\cancel{\varepsilon_r}}{d}\frac{V_0}{\cancel{\varepsilon_r}} = \frac{AV_0\varepsilon_0}{d}$

$\sigma_p = (\varepsilon_r - 1)\varepsilon_0 E = \frac{(\varepsilon_r - 1)\varepsilon_0}{\varepsilon_r}\frac{V_0}{d}$

	$\varepsilon_r = 5$	$\varepsilon_r = 6$
C	$8.85\times10^{-11}\,F$	$1.062\times10^{-10}\,F$
Q_{free}	$1.77\times10^{-9}\,C$	$1.77\times10^{-9}\,C$
σ_p	$7.08\times10^{-7}\,C/m^2$	$7.375\times10^{-7}\,C/m^2$

with increase in ε_r
C and σ_p increase
Q_{free} remains constant

(4) Calculate the field strength required to reach 0.1% of the saturation value of the orientational polarization of a dipolar gas at room temperature if the dipoles have a strength of 1 Debye unit

Ans: $\% = \frac{0.1}{100}$

$T = 298\ K.$

$\vec{p}_0 = 3.33 \times 10^{-30}\ Cm.$

$k_B = 1.38 \times 10^{-23}\ J/K.$

$E = ?$

$\vec{P} = \frac{\vec{p}_0^{\,2} E}{3k_B T}$; At saturation $P = P_0$

$\frac{0.1}{100} \cancel{P_0} = \frac{P_0^{\cancel{2}} E}{3k_B T} \Rightarrow E = \frac{3k_B T}{1000\, P_0} = 3.7 \times 10^{6}\ V/m.$

(5) A hydrogen atom with Bohr radius of 0.5 Å is situated between two metal plates. 1mm apart that are connected to a 500V battery. What fraction of the atomic radius does the separation distance amount to? Estimate the voltage needed to ionize the atom

$R = 5 \times 10^{-11} m.$

$d = 10^{-3} m.$

$V_0 = 500 V.$

$\frac{\delta}{R} = ?$

$V_{ion} = ?$

$\alpha_E = 4\pi \varepsilon_0 R^3$

$E = V_0/d.$

$\delta = \frac{\alpha E}{q_E} = \frac{4\pi \varepsilon_0 R^3}{q_E} \frac{V_0}{d}.$

$\frac{\delta}{R} = \frac{4\pi \varepsilon_0 R^2}{q_E} \frac{V_0}{d} = 8.68 \times 10^{-5} \%.$

At ionization $\frac{\delta}{R} \to 1$

$\Rightarrow V_{ion} = \frac{q_E d}{4\pi \varepsilon_0 R^2} = 575\ MV$