Department of Mathematics
Indian Institute of Technology Guwahati

MA572: Lab Assignment 4

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1. The error function is an important special function in applied mathematics, with applications to probability theory and the solution of heat conduction problems. The formal definition of the error function is as follows.
$$erf(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2}\, dt.$$
The following table gives the values of $erf(x)$ in increments of 0.1, over the interval $[0, 1]$.

x	erf(x)
0.0	0.00000000000000
0.1	0.11246291601828
0.2	0.22270258921048
0.3	0.32862675945913
0.4	0.42839235504667
0.5	0.52049987781305
0.6	0.60385609084793
0.7	0.67780119383742
0.8	0.74210096470766
0.9	0.79690821242283
1.0	0.84270079294971

Construct the quadratic interpolating polynomial to the error function using the above data at the nodes $x_0 = 0, x_1 = 0.5$, and $x_2 = 1.0$. Plot the polynomial and the data in the table and comment on the observed accuracy.

2. Let $f(x) = e^x$. Define $p_n(x)$ to be the Newton interpolating polynomial for $f(x)$, using $n+1$ equally spaced nodes on the interval $[-1, 1]$. Thus, we are taking higher and higher degree polynomial approximations to the exponential function. Write a program that computes $p_n(x)\ for\ n = 2, 4, 8, 16, 32$, and which samples the error $f(x) - p_n(x)$ at 501 equally spaced points on $[-1, 1]$. Record the maximum error as found by the sampling, as a function of n, i.e., define E_n as
$$E_n = \max_{0 \le k \le 500} |f(t_k) - p_n(t_k)|$$
where $t_k = -1 + \frac{2k}{500}$. Print $E_2, E_4, E_8, E_{16}, E_{32}$ and plot E_n versus n.

3. Write a program to approximate the function
$$f(x) = \frac{1}{1+x^2}, \quad -5 \le x \le 5, \quad \text{(Runge's example)},$$
using the points $x_i = -5 + 10\frac{i}{8}, \quad i = 0, 1, 2, \ldots, 8$ by Lagrange's interpolating polynomial. Plot the polynomial against the exact function.