

## EXPERIMENT-9

**AIM**–Solution of differential equation using Runge-Kutta Methods

### **THEORY–**

Let an initial value problem be specified as follows:

$$\frac{dx}{dy} = f(x, y), y(0) = y_0$$

where initial value of x i.e  $x_0=0$

initial value of y i.e  $y_0=y_0$

Now choose the value of h(step height) and define :

$$K_1 = hf(x_n, y_n)$$

$$K_2 = hf\left(x_n + \frac{h}{2}, y_n + \frac{k_1}{2}\right)$$

$$K_3 = hf\left(x_n + \frac{\bar{h}}{2}, y_n + \frac{k_2}{2}\right)$$

$$K_4 = hf(x_n + \bar{h}, y_n + k_3)$$

$$y_{n+1} = y_n + k_1/6 + k_2/3 + k_3/3 + k_4/6$$

Here value of n are 0, 1, 2, 3, ... $(x - x_0)/h$ .

$$x_{n+1} = x_0 + h$$

The formula basically main objective is to find the next value  $y_{n+1}$  using current  $y_n$  plus weighted average of four increments.

- $k_1$  is the increment based on the slope at the beginning of the interval, using  $y$
- $k_2$  is the increment based on the slope at the midpoint of the interval, using  $y + hk_1/2$ .
- $k_3$  is again the increment based on the slope at the midpoint, using  $y + hk_2/2$ .
- $k_4$  is the increment based on the slope at the end of the interval, using  $y + hk_3$ .

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### ALGORITHM-

1. Start
2. Define function  $f(x,y)$
3. Read values of initial condition( $x_0$  and  $y_0$ ),  
number of steps ( $n$ ) and calculation point ( $x_n$ )
4. Calculate step size  $(h) = (x_n - x_0)/n$
5. Set  $i=0$
6. Loop
  - $k_1 = h * f(x_0, y_0)$
  - $k_2 = h * f(x_0+h/2, y_0+k_1/2)$
  - $k_3 = h * f(x_0+h/2, y_0+k_2/2)$
  - $k_4 = h * f(x_0+h, y_0+k_3)$
  - $k = (k_1+2*k_2+2*k_3+k_4)/6$
  - $y_n = y_0 + k$
  - $i = i + 1$
  - $x_0 = x_0 + h$
  - $y_0 = y_n$
  - While  $i < n$
7. Display  $y_n$  as result
8. Stop

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### CODE-

```
#include<iostream>
#include<math.h>
#define f(x,y) -2*x*pow(y,2)
using namespace std;

int main()
{
float x0, y0, xn, h, yn, k1, k2, k3, k4, k;
int i, n;
cout<<"Enter Initial Condition"<< endl;
cout<<"x0 = ";
cin>> x0;
cout<<"y0 = ";
cin >> y0;
cout<<"Enter calculation point xn = ";
cin>>xn;
cout<<"Enter the Step Size = ";
cin>>h;
n = (int)((xn - x0) / h);
/* Runge Kutta Method */
cout<<"\nx0\t\tty0\t\tty(x0)\n";
cout<<"-----\n";
for(i=0; i <=n ; i++)
{
k1 = h * (f(x0, y0));
k2 = h * (f((x0+h/2), (y0+k1/2)));
k3 = h * (f((x0+h/2), (y0+k2/2)));
k4 = h * (f((x0+h), (y0+k3)));
k = (k1+2*k2+2*k3+k4)/6;
yn = y0 + k;
cout<<x0<<"\t\t"<<y0<<"\t\t"<<yn<<"\n";
x0 = x0+h;
y0 = yn;
}
return 0;
```

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}

OUTPUT-

```
Enter Initial Condition
```

```
x0 = 0
```

```
y0 = 1
```

```
Enter calculation point xn = 0.4
```

```
Enter the Step Size = 0.2
```

x0	y0	y(x0)
0	1	0.961533
0.2	0.961533	0.862052
0.4	0.862052	0.735278