Matrix Theory Assignment 1

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***Abstract*—This document contains the procedure to get image of a point in a line.**

Download the python code from the below link. Go through the README file in the reposotory. https://github.com/ankuraditya13/EE5609-Assignment-1
and latex-tikz codes from https://github.com/ankuraditya13/EE5609-Assignment-1

I. Problem

Find the image of the point $\begin{pmatrix} 3 \\ 8 \end{pmatrix} withrespecttotheline$

$$(1 \;\; 3)\mathbf{x} = 7 \tag{1}$$

II. Solution

For this problem, I am considering the general case. Let the Equation of line be ax + by = c and let the coordinates of,

$\vec{P}(given-point)be \begin{pmatrix} x1 \\ y1 \end{pmatrix}$

$\vec{Q}(image-point)be \begin{pmatrix} x2 \\ y2 \end{pmatrix}$

$\vec{R}(point-on-mirror)be \begin{pmatrix} x3 \\ y3 \end{pmatrix}$

Let vector $\vec{n} = \begin{pmatrix} a \\ b \end{pmatrix}$

Let m be the directional vector along line ax + by = c. Hence, m = (b -a)

Let m1 and m2 be the slopes of two prependicular lines,

Now, m1 = $\frac{y2-y1}{x2-x1}$ and m2 = $\frac{-a}{b}$

Now for perpendicular lines m1 m2 = -1, which in vector can be written as:

$$\vec{m}^T\vec{R} = \vec{m}^T\vec{P}; \tag{2}$$

Similarly in vector form line eqauation ax + by + c = 0 is given as,

$$\vec{n}^T\vec{Q} = c; \tag{3}$$

By property in Figure 1, the line PR bisects the mirror equation perpendicularly. Hence,

$$2\vec{Q} = \vec{P} + \vec{R} \tag{4}$$

Hence, From the equation (3) and (4)

$$\vec{n}^T\vec{R} = 2c - \vec{n}^T\vec{P} \tag{5}$$

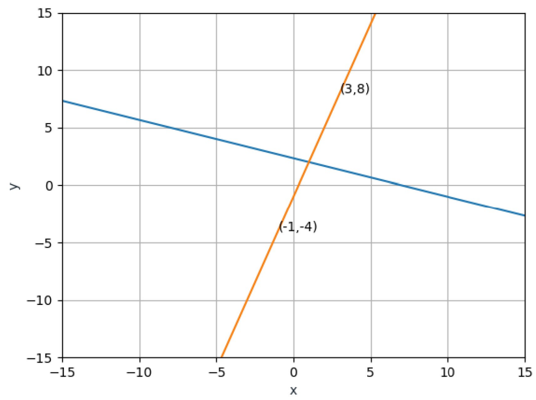

Fig. 1. Image of a point in 2D line

Now, form equation (5) and (2) we get,

$$(\vec{m} \quad \vec{n})^T\vec{R} = (\vec{m} \quad -\vec{n})^T\vec{P} + (0 \quad 2c)^T \tag{6}$$

Hence upon solving the equation for point R using the property, $(\vec{m} \quad -\vec{n}) = (\vec{m} \quad \vec{n})\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ **we get,**

$$\frac{\vec{R}}{2} = \frac{\vec{m}\vec{m}^T - \vec{n} * \vec{n}^T}{\vec{m}^T\vec{m} + \vec{n}^T * \vec{n}}\vec{P} + c\frac{\vec{n}}{||\vec{n}||^2} \tag{7}$$

Hence, substituting the value of x1 = 3, y1 = 8, a = 1, b = 3 and c = 7 we get,

$\vec{P}(given-point) = \begin{pmatrix} 3 \\ 8 \end{pmatrix}$

$\vec{m}(direction-vector) = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$

$\vec{n} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$

$Norm, ||\vec{n}|| = (a^2 + b^2)^(0.5)$

Substituting these values in equation (7) we get,

$\vec{R} = \begin{pmatrix} -1 \\ -4 \end{pmatrix}$

Hence, it is the required answer for image of $\vec{P}in - line(1 \;\; 3)x = 7.$