Matrix Theory Assignment 1

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***Abstract*—This document contains the procedure to get image of a point in a line.**

Download the python code from the below link. Go through the README file in the reposotory.

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https://github.com/ankuraditya13/EE5609−
    Assignment−1
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1 Problem

Find the image of the point $\begin{pmatrix} 3 \\ 8 \end{pmatrix}$ with respect to the line

$$\begin{pmatrix} 1 & 3 \end{pmatrix}\mathbf{x} = 7 \tag{1.0.1}$$

2 Solution

For this problem, I am considering the general case. Let the Equation of line be ax + by = c and let the coordinates of,

$\mathbf{P}$(given point) = $\begin{pmatrix} x1 \\ y1 \end{pmatrix}$

$\mathbf{Q}$(image point) = $\begin{pmatrix} x2 \\ y2 \end{pmatrix}$

$\mathbf{R}$(point on mirror) = $\begin{pmatrix} x3 \\ y3 \end{pmatrix}$

Let vector $\mathbf{n}$ = $\begin{pmatrix} a \\ b \end{pmatrix}$

Let m be the directional vector along the line
ax + by = c hence, m = $\begin{pmatrix} b & -a \end{pmatrix}$

Let m1 and m2 be the slopes of two prependicular lines,

Now, m1 = $\frac{y2-y1}{x2-x1}$ and m2 = $\frac{-a}{b}$

Now for perpendicular lines m1m2 = -1, which in vector can be written as:

$$\mathbf{m}^T\mathbf{R} = \mathbf{m}^T\mathbf{P} \tag{2.0.1}$$

Similarly in vector form line eqauation ax + by + c = 0 is given as,

$$\mathbf{n}^T\mathbf{Q} = c \tag{2.0.2}$$

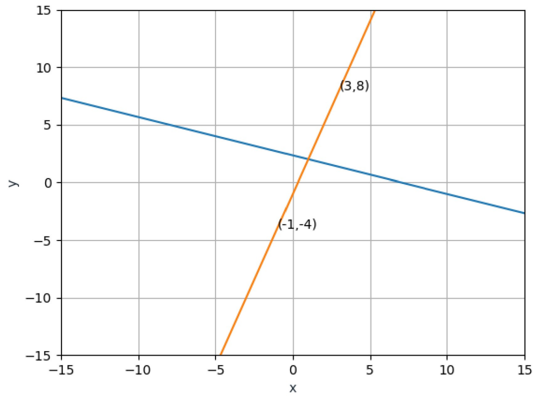

Fig. 0: Image of a point in 2D line

By property in Figure 0, the line PR bisects the mirror equation perpendicularly. Hence,

$$2\mathbf{Q} = \mathbf{P} + \mathbf{R} \tag{2.0.3}$$

Hence, From the equation (2.0.3) and (2.0.4)

$$\mathbf{n}^T\mathbf{R} = 2c - \mathbf{n}^T\mathbf{P} \tag{2.0.4}$$

Now, form equation (2.0.5) and (2.0.2) we get,

$$\begin{pmatrix} \mathbf{m} & \mathbf{n} \end{pmatrix}^T \mathbf{R} = \begin{pmatrix} \mathbf{m} & -\mathbf{n} \end{pmatrix}^T \mathbf{P} + \begin{pmatrix} 0 \\ 2c \end{pmatrix} \tag{2.0.5}$$

Hence upon solving the equation for point R using the property, $\begin{pmatrix} \mathbf{m} & -\mathbf{n} \end{pmatrix} = \begin{pmatrix} \mathbf{m} & \mathbf{n} \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ **we get,**

$$\frac{\mathbf{R}}{2} = \frac{\mathbf{m}\mathbf{m}^T - \mathbf{n}\mathbf{n}^T}{\mathbf{m}^T\mathbf{m} + \mathbf{n}^T\mathbf{n}}\mathbf{P} + c\frac{\mathbf{n}}{\|\mathbf{n}\|^2} \tag{2.0.6}$$

Hence, substituting the value of x1 = 3, y1 = 8, a = 1, b = 3 and c = 7 we get,

$\mathbf{P}$(given point) = $\begin{pmatrix} 3 \\ 8 \end{pmatrix}$

$\mathbf{m}$ (direction vector) $= \begin{pmatrix} 3 \\ -1 \end{pmatrix}$

$\mathbf{n} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$

Norm, $|||\mathbf{n}||| = \sqrt[2]{a^2 + b^2}$

Substituting these values in equation (2.0.6) we get,

$$\mathbf{R} = \begin{pmatrix} -1 \\ -4 \end{pmatrix} \tag{2.0.7}$$

Hence, it is the required answer for image of $\mathbf{P}$ in line $\begin{pmatrix} 1 & 3 \end{pmatrix} \mathbf{x} = 7$.