Matrix Theory Assignment 1

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***Abstract*— This document contains the procedure to get image of a point in a line.**

Download the python code from the below link. Go through the README file in the reposotory. https://github.com/ankuraditya13/EE5609-Assignment-1
and latex-tikz codes from https://github.com/ankuraditya13/EE5609-Assignment-1

I. Problem

Find the image of the point $\begin{pmatrix} 3 \\ 8 \end{pmatrix} withrespecttotheline$

$$(1 \quad 3)\mathbf{x} = 7 \tag{1}$$

II. Solution

For this problem, I am considering the general case. Let the Equation of line be a*x + b*y = c and let the coordinates of,

P(given-point) be $\begin{pmatrix} x1 \\ y1 \end{pmatrix}$

$Q(image - point)be \begin{pmatrix} x2 \\ y2 \end{pmatrix}$

$R(point - on - mirror)be \begin{pmatrix} x3 \\ y3 \end{pmatrix}$

Let vector n = $\begin{pmatrix} a \\ b \end{pmatrix}$

Let m be the directional vector along line a*x + b*y = c. Hence, m = (b -a)

Let m1 and m2 be the slopes of two prependicular lines,

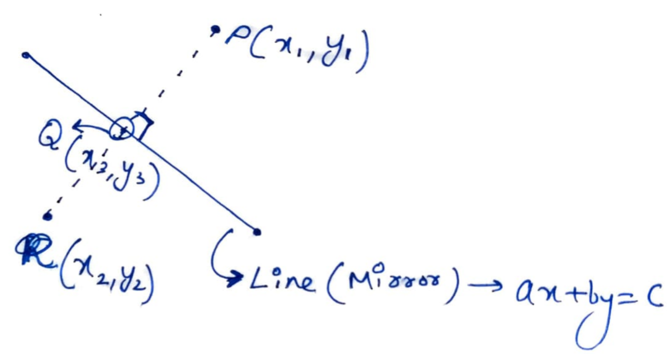
Now, m1 = $\frac{y2-y1}{x2-x1}$ and m2 = $\frac{-a}{b}$

Now for perpendicular line m1 * m2 = -1, which in vector can be written as:

$$m^T * R = m^T * P; \tag{2}$$

Similarly in vector form line equation a*x + b*y + c = 0 is given as,

$$n^T * Q = c; \tag{3}$$



By property in Figure 1, the line PR bisects the mirror equation perpendicularly. Hence,

$$2 * Q = P + R \tag{4}$$

Hence, From the equation (3) and (4)

$$n^T * R = 2 * c - n^T * P \tag{5}$$

Now, form equation (5) and (2) we get,

$$(m \quad n)^T * R = (m \quad -n)^T * P + (0 \quad 2*c)^T \tag{6}$$

Hence upon solving the equation for point R using the property, (m -n) = (m n) * $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ **we get,**

$$\frac{R}{2} = \frac{m * m^T - n * n^T}{m^T * m + n^T * n} * P + c * \frac{n}{||n||^2} \tag{7}$$

Hence, substituting the value of x1 = 3, y1 = 8, a = 1, b = 3 and c = 7 we get,

P(given-point) = $\begin{pmatrix} 3 \\ 8 \end{pmatrix}$

$m(direction - vector) = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$

$n = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$

$Norm, ||n|| = (a^2 + b^2)^(0.5)$

Substituting these values in equation (7) we get,

R = $\begin{pmatrix} -1 \\ -4 \end{pmatrix}$

Hence, it is the required answer for image of P in line(1 3) x =7.