

Assignment-10

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Abstract—This document contains the proof of the property $\mathbf{C}=\mathbf{AB}-\mathbf{BA}$, iff $\text{Trace}(\mathbf{C}) = 0$

Download the python code from

<https://github.com/ankuraditya13/EE5609-Assignment9>

and latex-file codes from

<https://github.com/ankuraditya13/EE5609-Assignment9>

Substituting equation (2.0.7) to (2.0.2) we getm

$$\implies \text{Trace}(\mathbf{C}) = \text{Trace}(\mathbf{AB}) - \text{Trace}(\mathbf{BA}) = 0 \quad (2.0.8)$$

Hence, Proved

1 PROBLEM

Let,

$$\mathbf{C} = \begin{pmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{pmatrix} \quad (1.0.1)$$

be a 2×2 matrix. We inquire when it is possible to find 2×2 matrices \mathbf{A} and \mathbf{B} such that $\mathbf{C}=\mathbf{AB} - \mathbf{BA}$. Prove that such matrices can be found if and only if $C_{11} + C_{22}=0$.

2 SOLUTION

We have to find,

$$\text{Trace}(\mathbf{C}) = C_{11} + C_{22} = \text{Trace}(\mathbf{AB} - \mathbf{BA}) \quad (2.0.1)$$

$$\implies \text{Trace}(\mathbf{C}) = \text{Trace}(\mathbf{AB}) - \text{Trace}(\mathbf{BA}) \quad (2.0.2)$$

We know that,

$$\text{Trace}(\mathbf{AB}) = \sum_{i=1}^2 (\mathbf{AB})_{ii} \quad (2.0.3)$$

$$\implies \sum_{i=1}^2 \sum_{j=1}^2 a_{ij} b_{ji} \quad (2.0.4)$$

$$\implies \sum_{j=1}^2 \sum_{i=1}^2 b_{ji} a_{ij} \quad (2.0.5)$$

$$\implies \text{Trace}(\mathbf{AB}) = \sum_{j=1}^2 \mathbf{BA}_{jj} \quad (2.0.6)$$

$$\implies \text{Trace}(\mathbf{AB}) = \text{Trace}(\mathbf{BA}) \quad (2.0.7)$$