Assignment-10

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***Abstract*—This document contains the proof of the property C=AB-BA, iff tr(C) = 0.**

Download the python code from

https://github.com/ankuraditya13/EE5609−Assignment9

and latex-file codes from

https://github.com/ankuraditya13/EE5609−Assignment9

1 Problem

Let,

$$\mathbf{C} = \begin{pmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{pmatrix} \tag{1.0.1}$$

be a 2×2 matrix. We inquire when it is possible to find 2×2 matrices $\mathbf{A}$ and $\mathbf{B}$ such that $\mathbf{C}$=$\mathbf{AB} - \mathbf{BA}$. Prove that such matrices can be found if and only if $C_{11} + C_{22}$=0.

2 Solution

We have to find,

$$tr\left(\mathbf{C}\right) = C_{11} + C_{22} = tr\left(\mathbf{AB} - \mathbf{BA}\right) \tag{2.0.1}$$

$$\implies tr\left(\mathbf{C}\right) = tr\left(\mathbf{AB}\right) - tr\left(\mathbf{BA}\right) \tag{2.0.2}$$

We know that,

$$tr\left(\mathbf{AB}\right) = \sum_{i=1}^{2} (\mathbf{AB})_{ii} \tag{2.0.3}$$

$$\implies \sum_{i=1}^{2}\sum_{j=1}^{2} a_{ij}b_{ji} \tag{2.0.4}$$

$$\implies \sum_{j=1}^{2}\sum_{i=1}^{2} b_{ji}a_{ij} \tag{2.0.5}$$

$$\implies tr\left(\mathbf{AB}\right) = \sum_{j=1}^{2} \mathbf{BA}_{jj} \tag{2.0.6}$$

$$\implies tr\left(\mathbf{AB}\right) = tr\left(\mathbf{BA}\right) \tag{2.0.7}$$

Substituting equation (2.0.7) to (2.0.2) we get

$$\implies tr\left(\mathbf{C}\right) = tr\left(\mathbf{AB}\right) - tr\left(\mathbf{BA}\right) = 0 \tag{2.0.8}$$

Hence, Proved