Assignment-12

Ankur Aditya - EE20RESCH11010

***Abstract*—This document contains the problem related to basis and dimensions.(Hoffman:- Page-49,Q-12)**

Download the latex-file codes from

https://github.com/ankuraditya13/EE5609−Assignment12

1 Problem

Prove that the space of all **m***x***n** matrices over the field **F** has dimension mn, by exhibiting a basis for this space.

2 Solution

Let **M** be the space of all $\mathbf{m} \times \mathbf{n}$ matrices. Let, $\mathbf{M}_{ij} \in \mathbf{M}$ be,

$$\mathbf{M}_{ij} = \begin{cases} 0 & m \neq i, n \neq j \\ 1 & m = i, n = j \end{cases} \tag{2.0.1}$$

For example,

$$\mathbf{M}_{12} = \begin{pmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 0 \end{pmatrix}_{mxn} \tag{2.0.2}$$

$$\tag{2.0.3}$$

Let $\mathbf{A} \in \mathbf{M}$ given as,

$$\mathbf{A} = \begin{pmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{pmatrix}_{mxn} \tag{2.0.4}$$

Now clearly,

$$\mathbf{a}_{11} = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} \begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 \end{pmatrix} \tag{2.0.5}$$

$$\implies \mathbf{a}_{11} = \mathbf{AM}_{11} \tag{2.0.6}$$

$$\therefore \mathbf{A} = \sum_{i=1}^{m} \sum_{j=1}^{n} a_{ij} M_{ij} \tag{2.0.7}$$

$\implies$ $\mathbf{M}_{ij}$ span **M**. Also from equation (2.0.7), **A**= 0 if and only if all elements are zero, that is,

$$\mathbf{A} = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} = \begin{pmatrix} 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 \end{pmatrix} \tag{2.0.8}$$

$$\implies a_{ij} = 0 \tag{2.0.9}$$

Hence, $\mathbf{M}_{ij}$ are linearly independent as well. Hence, $\mathbf{M}_{ij}$ constitutes a basis for **M**. and number of elememts in basis are mn. Hence dimension of space of all mxn matrices **M** is mn. **Hence Proved.**