Assignment-16

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***Abstract*—This document contains the problem related to Eigenvalue and Eigenvectors (UGC-June-2017 Maths Q-78)**

Download the latex-file from

https://github.com/ankuraditya13/EE5609−Assignment16

1 Problem

Consider the matrix

$$A(x) = \begin{pmatrix} 1+x^2 & 7 & 11 \\ 3x & 2x & 4 \\ 8x & 17 & 13 \end{pmatrix}; x \in \mathbf{R}. \tag{1.0.1}$$

Then,

a) A(x) has eigenvalue 0 for some $x \in \mathbf{R}$.
b) 0 is not an eigenvalue of A(x) for any $x \in \mathbf{R}$.
c) A(x) has eigenvalue 0 $\forall x \in \mathbf{R}$.
d) A(x) is invertible $\forall x \in \mathbf{R}$.

2 Solution

Let $\lambda = 0$ be an eigenvalue. Hence,

$$\left|A - \lambda I\right| = 0 \tag{2.0.1}$$

$$\implies \left|A\right| = 0 \tag{2.0.2}$$

$$\implies \left|A\right| = \begin{vmatrix} 1+x^2 & 7 & 11 \\ 3x & 2x & 4 \\ 8x & 17 & 13 \end{vmatrix} = 0 \tag{2.0.3}$$

Performing row reduction we get,

$$\begin{vmatrix} 1+x^2 & 7 & 11 \\ 0 & \frac{2x^3-19x}{1+x^2} & \frac{4x^2-33x+4}{1+x^2} \\ 0 & 0 & \frac{26x^3-244x^2+538x-68}{2x^3-19x} \end{vmatrix} = 0 \tag{2.0.4}$$

$$\implies 26x^3 - 244x^2 + 538x - 68 = 0 \tag{2.0.5}$$

$$\implies x_1 = 6.01, x_2 = 3.23, x_3 = 0.13 \tag{2.0.6}$$

In other words at these values of x, eigenvalue $\lambda = 0$. Hence option (b) is eliminated. Now for these values of x, $\left|A\right| = 0$. Hence it is not invertible $\forall x \in \mathbf{R} \implies$ option (d) is eliminated. Now, if one of the eigenvalue is 0 for A(x), then $\left|A(x)\right| = 0 \forall x \in \mathbf{R}$. But from equation (2.0.6) we have concluded that $\left|A\right| = 0$ only for $x_1 = 6.01, x_2 = 3.23, x_3 = 0.13$. hence, option (b) is also eliminated. Hence the correct option is (a).