Assignment-19

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***Abstract*—This document contains the problem related to Elementary canonical forms (Hoffman Page-206, Section 6.4, Q-13)**

Download the latex-file from

https://github.com/ankuraditya13/EE5609–Assignment19

1 Problem

Let $\mathbf{V}$ be the space of n× n matrices over $\mathbf{F}$. Let $\mathbf{A}$ be a fixed n× n matrix over $\mathbf{F}$. Let T and U be the linear operators on $\mathbf{V}$ defined by

$$T(\mathbf{B}) = \mathbf{AB} \tag{1.0.1}$$

$$U(\mathbf{B}) = \mathbf{AB} - \mathbf{BA} \tag{1.0.2}$$

(a) True of False? If A is diagonalizable (over $\mathbf{F}$), then T is diagonalizable.
(b) True or False? If $\mathbf{A}$ is diagonalizable, then U is diagonalizable.

2 Theorems

2.1 Theorem 1

Let λ be a characteristic values of $\mathbf{T}$ and $\lambda \in \mathbf{F}$ and v is the corresponding characteristic vector which is a $n \times n$ matrix, then if $\mathbf{T}$ is a linear operator on finite dimensional space $\mathbf{V}$, it must be $\left|\mathbf{T} - \lambda\mathbf{I}\right| = 0$ and for $(\mathbf{T} - \lambda\mathbf{I})\,\mathbf{v}, \mathbf{v} \neq 0$.

3 Solutions

(a) Using theorem-1,

$$\mathbf{Tv} = \lambda\mathbf{v} \tag{3.0.1}$$

Now,

$$\because \mathbf{Tv} = \mathbf{Av} \tag{3.0.2}$$

$$\implies \mathbf{Av} = \lambda\mathbf{v} \tag{3.0.3}$$

$$\implies (\mathbf{A} - \lambda\mathbf{I})\,\mathbf{v} = 0 \tag{3.0.4}$$

Hence from above equation it is evident that $\mathbf{A}$ has the same characteristic values λ as $\mathbf{T}$. Hence it can be concluded that if $\mathbf{A}$ is diagonalizable (over $\mathbf{F}$) then $\mathbf{T}$ is diagonalizable, as $\mathbf{A}$ is similar to $\mathbf{T}$.

(b) Using Theorem-1,

$$\mathbf{UB} = \lambda_1\mathbf{B} \tag{3.0.5}$$

$$\implies \mathbf{AB} - \mathbf{BA} = \lambda_1\mathbf{B} \tag{3.0.6}$$

$$\implies (\mathbf{A} - \lambda_1\mathbf{I})\,\mathbf{B} = \mathbf{BA} \tag{3.0.7}$$

Hence clearly $\mathbf{A} - \lambda_1\mathbf{I}$ is not in null-space as,

$$\left|\mathbf{A} - \lambda_1\mathbf{I}\right| \neq 0 \tag{3.0.8}$$

Hence, $\mathbf{U}$ and $\mathbf{T}$ are not similar, hence statement that if $\mathbf{A}$ is diagonalizable then $\mathbf{U}$ is diagonalizable is **False**