Assignment-4

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***Abstract*—This document contains the procedure to find value of $\sin 60°$.**

Download the python code from

https://github.com/ankuraditya13/EE5609–Assignment4

and latex-file codes from

https://github.com/ankuraditya13/EE5609–Assignment4

1 Problem

Show that $\sin 60° = \frac{\sqrt{3}}{2}$.

2 Solution

Consider an equilateral triangle $\mathbf{ABC}$. Since, $\triangle\mathbf{ABC}$ is an equilateral, all of its angles are 60°. Now, The direction vector of all the sides are given as,

$$\mathbf{AB} = \|\mathbf{A} - \mathbf{B}\| \tag{2.0.1}$$

$$\mathbf{BC} = \|\mathbf{B} - \mathbf{C}\| \tag{2.0.2}$$

$$\mathbf{AC} = \|\mathbf{A} - \mathbf{C}\| \tag{2.0.3}$$

Now for an equilateral triangle,

$$\|\mathbf{A} - \mathbf{B}\| = \|\mathbf{B} - \mathbf{C}\| = \|\mathbf{A} - \mathbf{C}\| \tag{2.0.4}$$

In figure 0, taking inner products of side $\mathbf{AB}$ and $\mathbf{BC}$ we get,

$$\mathbf{AB} \cdot \mathbf{BC} = \|\mathbf{AB}\| \|\mathbf{BC}\| \cos\theta \tag{2.0.5}$$

Let, $\mathbf{a} = \mathbf{A} - \mathbf{B}, \mathbf{b} = \mathbf{B} - \mathbf{C}$. Hence $\mathbf{A} - \mathbf{C} = \mathbf{a} + \mathbf{b}$. $\therefore$ the angle θ between two vectors $\mathbf{AB}$ and $\mathbf{BC}$ is given by,

$$\cos\theta = \frac{\mathbf{a}^T \cdot \mathbf{b}}{\|\mathbf{a}\| \|\mathbf{b}\|} \tag{2.0.6}$$

Substituting from equation 2.0.4 to 2.0.5 we get,

$$\implies \cos\theta = \frac{1}{2} \tag{2.0.7}$$

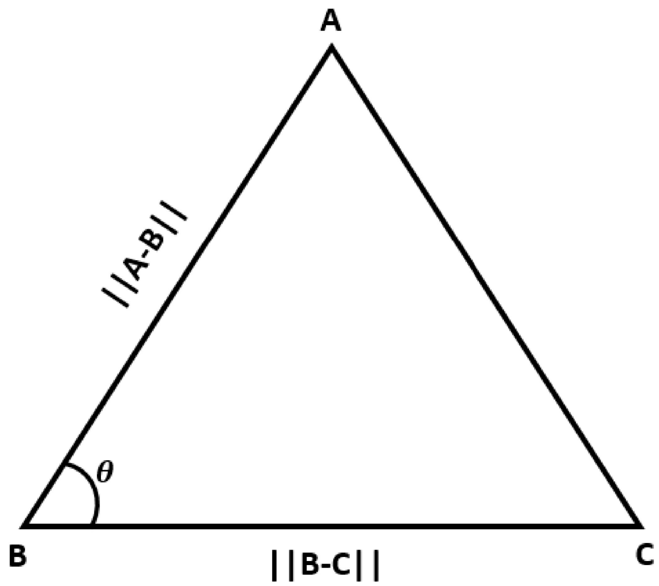

Fig. 0: Equilateral Triangle

we know that $\theta = 60°$ for an equilateral triangle

$$\therefore \cos 60° = \frac{1}{2} \tag{2.0.8}$$

Now using the property,

$$\cos^2\theta + \sin^2\theta = 1 \tag{2.0.9}$$

$\therefore$ at $\theta = 60°$,

$$\sin 60° = \sqrt{1 - \cos^2 60°} \tag{2.0.10}$$

$$\implies \sin 60° = \frac{\sqrt{3}}{2}. \tag{2.0.11}$$