Assignment-4

Ankur Aditya - EE20RESCH11010

***Abstract*—This document contains the procedure to find value of** $\sin 60°$**.**

Download the python code from

https://github.com/ankuraditya13/EE5609−Assignment4

and latex-file codes from

https://github.com/ankuraditya13/EE5609−Assignment4

1 Problem

Show that $\sin 60° = \frac{\sqrt{3}}{2}$.

2 Solution

Consider an equilateral triangle **ABC**. Since, $\triangle\mathbf{ABC}$ is an equilateral, all of its angles are 60°. Now, The direction vector of all the sides are given as,

$$\mathbf{AB} = \|\mathbf{A} - \mathbf{B}\| \tag{2.0.1}$$

$$\mathbf{BC} = \|\mathbf{B} - \mathbf{C}\| \tag{2.0.2}$$

$$\mathbf{AC} = \|\mathbf{A} - \mathbf{C}\| \tag{2.0.3}$$

Now for an equilateral triangle,

$$\|\mathbf{A} - \mathbf{B}\| = \|\mathbf{B} - \mathbf{C}\| = \|\mathbf{A} - \mathbf{C}\| \tag{2.0.4}$$

Let, **B** be the origin.Hence, $\mathbf{B} = 0$. Hence substituting in the equation (2.0.4) we get,

$$\|\mathbf{A}\| = \|\mathbf{C}\| = \|\mathbf{A} - \mathbf{C}\| \tag{2.0.5}$$

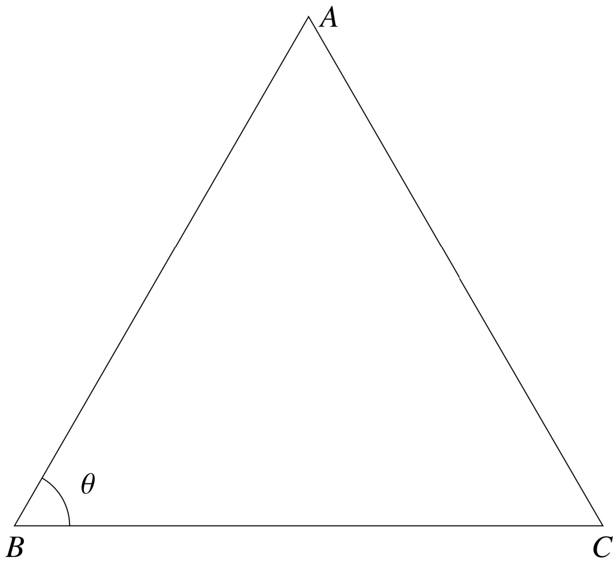
Squaring $\|\mathbf{A} - \mathbf{C}\|$ we get,

$$\|\mathbf{A} - \mathbf{C}\|^2 = \|\mathbf{A}\|^2 + \|\mathbf{C}\|^2 - 2\mathbf{A}^T\mathbf{C} \tag{2.0.6}$$

Substituting from equation (2.0.5) in above equation,

$$\implies \|\mathbf{A}\|^2 = 2\|\mathbf{A}\|^2 - 2\mathbf{A}^T\mathbf{C} \tag{2.0.7}$$

$$\implies \|\mathbf{A}\|^2 = 2\mathbf{A}^T\mathbf{C} \tag{2.0.8}$$



Now by Lagrange's Identity, cross product of vectors **AB** and **BC** can be represented in terms of inner products of **AB** and **BC**,

$$\|\mathbf{AB} \times \mathbf{BC}\|^2 = \|\mathbf{AB}\|^2 \|\mathbf{BC}\|^2 - (\mathbf{AB} \cdot \mathbf{BC})^2 \tag{2.0.9}$$

$$\implies (\mathbf{AB} \cdot \mathbf{BC})^2 = \|\mathbf{AB}\|^2 \|\mathbf{BC}\|^2 - \|\mathbf{AB} \times \mathbf{BC}\|^2 \tag{2.0.10}$$

$$\implies (\mathbf{AB} \cdot \mathbf{BC})^2 = \|\mathbf{AB}\|^2 \|\mathbf{BC}\|^2 - \|\mathbf{AB}\|^2 \|\mathbf{BC}\|^2 \sin^2\theta \tag{2.0.11}$$

$$\implies (\mathbf{AB} \cdot \mathbf{BC})^2 = \|\mathbf{AB}\|^2 \|\mathbf{BC}\|^2 (1 - \sin^2\theta) \tag{2.0.12}$$

Imposing the condition that **B** is at origin,

$$\implies \mathbf{A}^T\mathbf{C} = \|\mathbf{A}\| \|\mathbf{C}\| \sqrt{1 - \sin^2\theta} \tag{2.0.13}$$

Substituting the results of (2.0.8) in (2.0.13) and solving for $\sqrt{1 - \sin^2\theta}$ we get,

$$\sqrt{1 - \sin^2\theta} = \frac{\mathbf{A}^T \cdot \mathbf{C}}{\|\mathbf{A}\| \|\mathbf{C}\|} \tag{2.0.14}$$

From equation (2.0.5), $\|\mathbf{A}\| = \|\mathbf{C}\|$.

$$\implies \sqrt{1 - \sin^2\theta} = \frac{\mathbf{A}^T \cdot \mathbf{C}}{\|\mathbf{A}\|^2} \tag{2.0.15}$$