Assignment-4

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***Abstract*—This document contains the procedure to find value of $\sin 60°$.**

Download the python code from

https://github.com/ankuraditya13/EE5609–Assignment4

and latex-file codes from

https://github.com/ankuraditya13/EE5609–Assignment4

1 Problem

Show that $\sin 60° = \frac{\sqrt{3}}{2}$.

2 Solution

Consider an equilateral triangle **ABC**. Since, $\triangle\mathbf{ABC}$ is an equilateral, all of its angles are 60°. Now, The direction vector of all the sides are given as,

$$\mathbf{AB} = \|\mathbf{A} - \mathbf{B}\| \tag{2.0.1}$$

$$\mathbf{BC} = \|\mathbf{B} - \mathbf{C}\| \tag{2.0.2}$$

$$\mathbf{AC} = \|\mathbf{A} - \mathbf{C}\| \tag{2.0.3}$$

Now for an equilateral triangle,

$$\|\mathbf{A} - \mathbf{B}\| = \|\mathbf{B} - \mathbf{C}\| = \|\mathbf{A} - \mathbf{C}\| \tag{2.0.4}$$

Let, **B** be the origin.Hence, $\mathbf{B} = 0$. Hence substituting in the equation (2.0.4) we get,

$$\|\mathbf{A}\| = \|\mathbf{C}\| = \|\mathbf{A} - \mathbf{C}\| \tag{2.0.5}$$

Squaring $\|\mathbf{A} - \mathbf{C}\|$ we get,

$$\|\mathbf{A} - \mathbf{C}\|^2 = \|\mathbf{A}\|^2 + \|\mathbf{C}\|^2 - 2\mathbf{A}^T\mathbf{C} \tag{2.0.6}$$

Substituting from equation (2.0.5) in above equation,

$$\implies \|\mathbf{A}\|^2 = 2\|\mathbf{A}\|^2 - 2\mathbf{A}^T\mathbf{C} \tag{2.0.7}$$

$$\implies \|\mathbf{A}\|^2 = 2\mathbf{A}^T\mathbf{C} \tag{2.0.8}$$

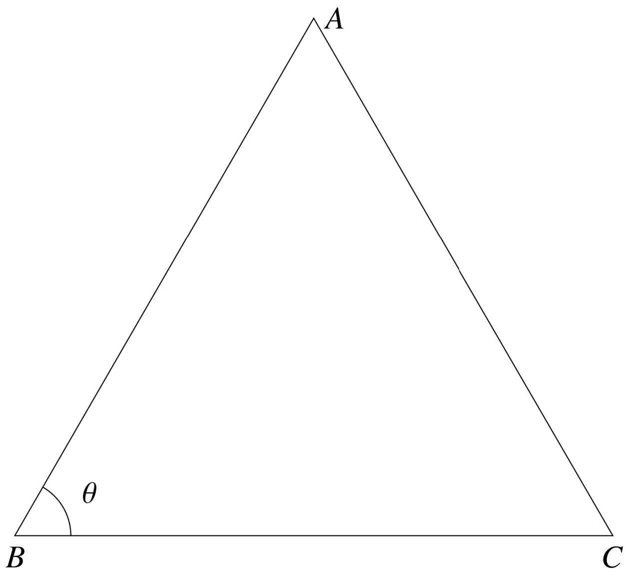

Fig. 1: Equilateral Triangle

In figure 1, taking inner products of side **AB** and **BC** we get,

$$(\mathbf{AB})^T\mathbf{BC} = \|\mathbf{AB}\|\,\|\mathbf{BC}\|\cos\theta \tag{2.0.9}$$

Substituting these results in (2.0.9) and solving for $\cos\theta$ we get,

$$\cos\theta = \frac{(\mathbf{A} - \mathbf{B})^T(\mathbf{B} - \mathbf{C})}{\|\mathbf{A} - \mathbf{B}\|\,\|\mathbf{B} - \mathbf{C}\|} \tag{2.0.10}$$

$$\implies \cos\theta = \frac{\mathbf{A}^T\mathbf{C}}{\|\mathbf{A}\|\,\|\mathbf{C}\|} \tag{2.0.11}$$

Imposing the results of (2.0.8) in (2.0.11) we get,

$$\implies \cos\theta = \frac{\mathbf{A}^T\mathbf{C}}{2\mathbf{A}^T\mathbf{C}} \tag{2.0.12}$$

$$\implies \cos\theta = \frac{1}{2} \tag{2.0.13}$$

$\therefore \theta = 60°$

$$\therefore \cos 60° = \frac{1}{2} \tag{2.0.14}$$

Now using the property,

$$\cos^2\theta + \sin^2\theta = 1 \tag{2.0.15}$$

$\therefore$ at $\theta = 60°$,

$$\implies \sin 60° = \sqrt{1 - \cos^2 60°} \tag{2.0.16}$$

$$\implies \sin 60° = \frac{\sqrt{3}}{2}. \tag{2.0.17}$$