

Assignment-4

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Abstract—This document contains the procedure to find value of $\sin 60^\circ$.

Download the python code from

<https://github.com/ankuraditya13/EE5609–Assignment4>

and latex-file codes from

<https://github.com/ankuraditya13/EE5609–Assignment4>

1 PROBLEM

Show that $\sin 60^\circ = \frac{\sqrt{3}}{2}$.

2 SOLUTION

Consider an equilateral triangle \mathbf{ABC} . Since, $\triangle \mathbf{ABC}$ is an equilateral, all of its angles are 60° . Now, The direction vector of all the sides are given as,

$$\mathbf{AB} = \|\mathbf{A} - \mathbf{B}\| \quad (2.0.1)$$

$$\mathbf{BC} = \|\mathbf{B} - \mathbf{C}\| \quad (2.0.2)$$

$$\mathbf{AC} = \|\mathbf{A} - \mathbf{C}\| \quad (2.0.3)$$

Now for an equilateral triangle,

$$\|\mathbf{A} - \mathbf{B}\| = \|\mathbf{B} - \mathbf{C}\| = \|\mathbf{A} - \mathbf{C}\| \quad (2.0.4)$$

Let, \mathbf{B} be the origin. Hence, $\mathbf{B} = 0$. Hence substituting in the equation (2.0.4) we get,

$$\|\mathbf{A}\| = \|\mathbf{C}\| = \|\mathbf{A} - \mathbf{C}\| \quad (2.0.5)$$

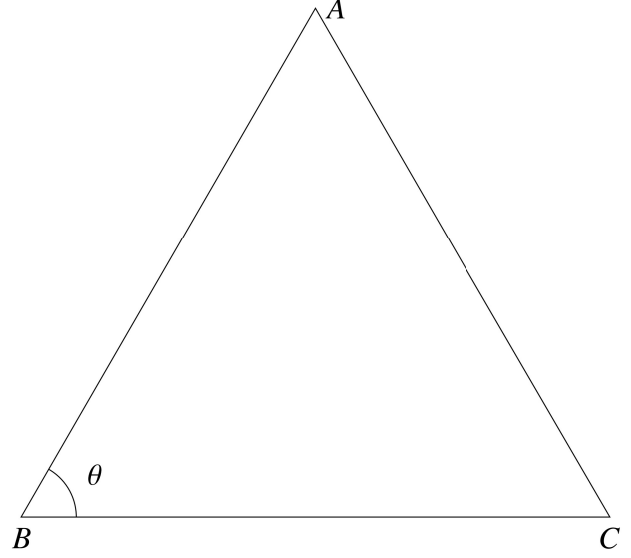
Squaring $\|\mathbf{A} - \mathbf{C}\|$ we get,

$$\|\mathbf{A} - \mathbf{C}\|^2 = \|\mathbf{A}\|^2 + \|\mathbf{C}\|^2 - 2\mathbf{A}^T \mathbf{C} \quad (2.0.6)$$

Substituting from equation (2.0.5) in above equation,

$$\Rightarrow \|\mathbf{A}\|^2 = 2\|\mathbf{A}\|^2 - 2\mathbf{A}^T \mathbf{C} \quad (2.0.7)$$

$$\Rightarrow \|\mathbf{A}\|^2 = 2\mathbf{A}^T \mathbf{C} \quad (2.0.8)$$



In above figure, taking inner products of side \mathbf{AB} and \mathbf{BC} we get,

$$(\mathbf{AB})^T \mathbf{BC} = \|\mathbf{AB}\| \|\mathbf{BC}\| \cos \theta \quad (2.0.9)$$

Substituting these results in (2.0.9) and solving for $\cos \theta$ we get,

$$\cos \theta = \frac{(\mathbf{A} - \mathbf{B})^T (\mathbf{B} - \mathbf{C})}{\|\mathbf{A} - \mathbf{B}\| \|\mathbf{B} - \mathbf{C}\|} \quad (2.0.10)$$

$$\Rightarrow \cos \theta = \frac{\mathbf{A}^T \mathbf{C}}{\|\mathbf{A}\| \|\mathbf{C}\|} \quad (2.0.11)$$

Imposing the results of (2.0.8) in (2.0.11) we get,

$$\Rightarrow \cos \theta = \frac{\mathbf{A}^T \mathbf{C}}{2\mathbf{A}^T \mathbf{C}} \quad (2.0.12)$$

$$\Rightarrow \cos \theta = \frac{1}{2} \quad (2.0.13)$$

$\therefore \theta = 60^\circ$

$$\therefore \cos 60^\circ = \frac{1}{2} \quad (2.0.14)$$

Now using the property,

$$\cos^2 \theta + \sin^2 \theta = 1 \quad (2.0.15)$$

\therefore at $\theta = 60^\circ$,

$$\implies \sin 60^\circ = \sqrt{1 - \cos^2 60^\circ} \quad (2.0.16)$$

$$\implies \sin 60^\circ = \frac{\sqrt{3}}{2}. \quad (2.0.17)$$