

Assignment-5

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Abstract—This document contains the procedure to find unknown constant value such that the equation represents pair of straight lines.

Download the python code from

<https://github.com/ankuraditya13/EE5609-Assignment5>

and latex-file codes from

<https://github.com/ankuraditya13/EE5609-Assignment5>

1 PROBLEM

Find the value of k so that following equation may represent pairs of straight lines,

$$kxy - 8x + 9y - 12 = 0 \quad (1.0.1)$$

2 SOLUTION

The general equation of second degree is given by,

$$ax^2 + 2bxy + cy^2 + 2dx + 2ey + f = 0 \quad (2.0.1)$$

In vector form the equation (2.0.1) can be expressed as,

$$\mathbf{x}^T \mathbf{V} \mathbf{x} + 2\mathbf{u}^T \mathbf{x} + f = 0 \quad (2.0.2)$$

where,

$$\mathbf{V} = \mathbf{V}^T = \begin{pmatrix} a & b \\ b & c \end{pmatrix} \quad (2.0.3)$$

$$\mathbf{u} = \begin{pmatrix} d \\ e \end{pmatrix} \quad (2.0.4)$$

Now, comparing equation (2.0.1) to (1.0.1) we get, $a = c = 0$, $b = \left(\frac{k}{2}\right)$, $d = -4$, $e = \left(\frac{9}{2}\right)$, $f = -12$. Hence, substituting these values in equation (2.0.3) and (2.0.4) we get,

$$\mathbf{V} = \mathbf{V}^T = \begin{pmatrix} 0 & \frac{k}{2} \\ \frac{k}{2} & 0 \end{pmatrix} \quad (2.0.5)$$

$$\mathbf{u} = \begin{pmatrix} -4 \\ \frac{9}{2} \end{pmatrix} \quad (2.0.6)$$

Now equation (1.0.1) represents pair of straight lines if,

$$\begin{vmatrix} \mathbf{V} & \mathbf{u} \\ \mathbf{u}^T & f \end{vmatrix} = 0 \quad (2.0.7)$$

$$\begin{vmatrix} 0 & \frac{k}{2} & -4 \\ \frac{k}{2} & 0 & \frac{9}{2} \\ -4 & \frac{9}{2} & -12 \end{vmatrix} = 0 \quad (2.0.8)$$

$$\implies k = 0, k = 6 \quad (2.0.9)$$

Substituting (2.0.9) in (1.0.1) we get,

$$6xy - 8x + 9y - 12 = 0 \quad (2.0.10)$$

$$-8x + 9y - 12 = 0 \quad (2.0.11)$$

Hence value of $k = 6$ represents pair of straight lines. Also it can be verified that the pair of lines intersect as,

$$|\mathbf{V}| = \begin{vmatrix} 0 & 3 \\ 3 & 0 \end{vmatrix} < 0 \quad (2.0.12)$$

Let the pair of straight lines is given by,

$$\mathbf{n}_1^T \mathbf{x} = c_1 \quad (2.0.13)$$

$$\mathbf{n}_2^T \mathbf{x} = c_2 \quad (2.0.14)$$

Now equating the product of equation (2.0.13) and (2.0.14) with (2.0.2) we get,

$$(\mathbf{n}_1^T \mathbf{x} - c_1)(\mathbf{n}_2^T \mathbf{x} - c_2) = \mathbf{x}^T \begin{pmatrix} 0 & 3 \\ 3 & 0 \end{pmatrix} \mathbf{x} + 2 \begin{pmatrix} -4 & \frac{9}{2} \end{pmatrix} \mathbf{x} - 12 \quad (2.0.15)$$

$$\implies n_1 * n_2 = \{0, 6, 0\} \quad (2.0.16)$$

$$c_1 n_1 + c_2 n_2 = \begin{pmatrix} 8 \\ -9 \end{pmatrix} \quad (2.0.17)$$

$$c_1 c_2 = -12. \quad (2.0.18)$$

Now the slopes of line is given by roots of polynomial,

$$cm^2 + 2bm + a = 0 \quad (2.0.19)$$

$$\Rightarrow 2bm = 0 \quad (2.0.20)$$

$$\Rightarrow m = 0 \quad (2.0.21)$$

Also

$$m_i = \frac{-b \pm \sqrt{-|V|}}{c} \quad (2.0.22)$$

$$\Rightarrow m_i = \frac{-0 \pm \sqrt{9}}{0} \quad (2.0.23)$$

$$\therefore m_1 = 0 \quad (2.0.24)$$

$$m_2 = \infty \quad (2.0.25)$$

The normal vector to the two lines is given by,

$$n_i = k_i \begin{pmatrix} -m_i \\ 1 \end{pmatrix} \quad (2.0.26)$$

$$\Rightarrow n_1 = k_1 \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (2.0.27)$$

$$n_2 = k_2 \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (2.0.28)$$

Also,

$$k_1 k_2 = 6 \quad (2.0.29)$$

Let $k_1 = 2$ and $k_2 = 3$

$$\Rightarrow n_1 = \begin{pmatrix} 0 \\ 2 \end{pmatrix} \quad (2.0.30)$$

$$n_2 = \begin{pmatrix} 3 \\ 0 \end{pmatrix} \quad (2.0.31)$$

We verify obtained n_1 and n_2 using Toeplitz matrix,

$$n_1 * n_2 = \begin{pmatrix} 0 & 0 \\ 2 & 0 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} 2 \\ 0 \end{pmatrix} \begin{pmatrix} 3 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 6 \\ 0 \end{pmatrix} \quad (2.0.32)$$

Hence (2.0.16) and (2.0.32) are same. Hence verified.

Now substituting it in (2.0.17) we get,

$$c_2 \begin{pmatrix} 0 \\ 2 \end{pmatrix} + c_1 \begin{pmatrix} 3 \\ 0 \end{pmatrix} = \begin{pmatrix} 8 \\ -9 \end{pmatrix} \quad (2.0.33)$$

Solve using Row reduction Technique we get,

$$\Rightarrow \begin{pmatrix} 3 & 0 & 8 \\ 0 & 2 & -9 \end{pmatrix} \quad (2.0.34)$$

$$\xleftrightarrow{R_1 \leftarrow R_1/3} \begin{pmatrix} 1 & 0 & 8/3 \\ 0 & 2 & -9 \end{pmatrix} \quad (2.0.35)$$

$$\xleftrightarrow{R_2 \leftarrow R_2/2} \begin{pmatrix} 1 & 0 & 8/3 \\ 0 & 1 & -9/2 \end{pmatrix} \quad (2.0.36)$$

$$\Rightarrow c_1 = \frac{8}{3} \quad (2.0.37)$$

$$c_2 = \frac{-9}{2} \quad (2.0.38)$$

substituting the values of c_1 , c_2 and equation

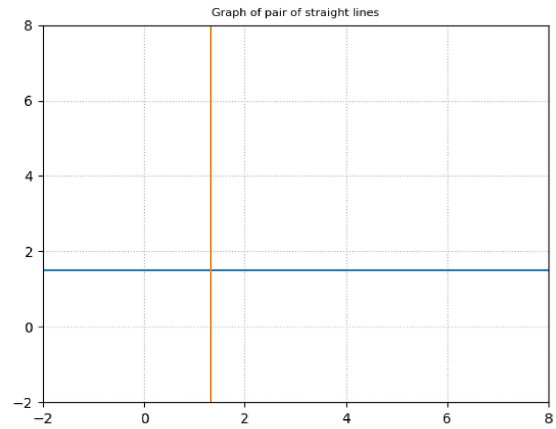


Fig. 0: Intersection of 2 lines

(2.0.30) and (2.0.31) to equation (2.0.13) and (2.0.14) we get equation of two straight lines.

$$\Rightarrow \begin{pmatrix} 0 & 2 \end{pmatrix} \mathbf{x} = \frac{8}{3} \quad (2.0.39)$$

$$\begin{pmatrix} 3 & 0 \end{pmatrix} \mathbf{x} = \frac{-9}{2} \quad (2.0.40)$$

Hence the equation of pair of straight lines are,

$$\left((0 \ 2)\mathbf{x} - \frac{8}{3}\right)\left((3 \ 0)\mathbf{x} - \frac{-9}{2}\right) = 0 \quad (2.0.41)$$

Hence, Plot of the equation (2.0.41) is shown in Figure.0 Now for value of $k = 0$ does not represent pair of straight lines.as,

$$|\mathbf{V}| = \begin{vmatrix} 0 & 0 \\ 0 & 0 \end{vmatrix} \neq 0 \quad (2.0.42)$$

Hence, Plot of the equation $(-8 \ 9)\mathbf{x} = 12$ is shown in figure 0,

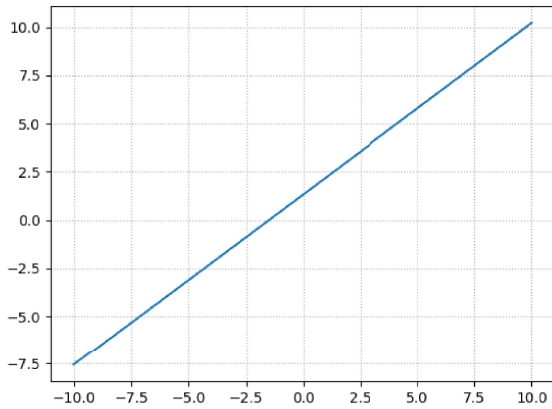


Fig. 0: Intersection of 2 lines