Assignment-5

Ankur Aditya - EE20RESCH11010

***Abstract*—This document contains the procedure to find unknown constant value such that the equation represents pair of straight lines.**

Download the python code from

```
https://github.com/ankuraditya13/EE5609−
    Assignment5
```

and latex-file codes from

```
https://github.com/ankuraditya13/EE5609−
    Assignment5
```

1 Problem

Find the value of k so that following equation may represent pairs of straight lines,

$$kxy - 8x + 9y - 12 = 0 \tag{1.0.1}$$

2 Solution

The general equation of second degree is given by,

$$ax^2 + 2bxy + cy^2 + 2dx + 2ey + f = 0 \tag{2.0.1}$$

In vector from the equation (2.0.1) canb be expressed as,

$$\mathbf{x}^T\mathbf{V}\mathbf{x} + 2\mathbf{u}^T\mathbf{x} + f = 0 \tag{2.0.2}$$

where,

$$\mathbf{V} = \mathbf{V}^T = \begin{pmatrix} a & b \\ b & c \end{pmatrix} \tag{2.0.3}$$

$$\mathbf{u} = \begin{pmatrix} d \\ e \end{pmatrix} \tag{2.0.4}$$

Now, comparing equation (2.0.1) to (1.0.1) we get, a = c = 0, b = $\left(\frac{k}{2}\right)$, d = -4, e = $\left(\frac{9}{2}\right)$, f = -12. Hence, substituting these values in equation (2.0.3) and (2.0.4) we get,

$$\mathbf{V} = \mathbf{V}^T = \begin{pmatrix} 0 & \frac{k}{2} \\ \frac{k}{2} & 0 \end{pmatrix} \tag{2.0.5}$$

$$\mathbf{u} = \begin{pmatrix} -4 \\ \frac{9}{2} \end{pmatrix} \tag{2.0.6}$$

Now equation (1.0.1) represents pair of straight lines if,

$$\begin{vmatrix} \mathbf{V} & \mathbf{u} \\ \mathbf{u}^T & f \end{vmatrix} = 0 \tag{2.0.7}$$

$$\begin{vmatrix} 0 & \frac{k}{2} & -4 \\ \frac{k}{2} & 0 & \frac{9}{2} \\ -4 & \frac{9}{2} & -12 \end{vmatrix} = 0 \tag{2.0.8}$$

$$\implies k = 0, k = 6 \tag{2.0.9}$$

Substituting (2.0.9) in (1.0.1) we get,

$$6xy - 8x + 9y - 12 = 0 \tag{2.0.10}$$

$$-8x + 9y - 12 = 0 \tag{2.0.11}$$

Hence value of k = 6 represents pair of straight lines. Also it can be verified that the pair of lines intersect as,

$$\left|\mathbf{V}\right| = \begin{vmatrix} 0 & 3 \\ 3 & 0 \end{vmatrix} < 0 \tag{2.0.12}$$

Let the pair of straight lines is given by,

$$\mathbf{n_1}^T\mathbf{x} = c1 \tag{2.0.13}$$

$$\mathbf{n_2}^T\mathbf{x} = c2 \tag{2.0.14}$$

Now equating the product of equation (2.0.13) and (2.0.14) with (2.0.2) we get,

$$(\mathbf{n_1}^T\mathbf{x} - c1)(\mathbf{n_2}^T\mathbf{x} - c2) = \tag{2.0.15}$$

$$\mathbf{x}^T\begin{pmatrix} 0 & 3 \\ 3 & 0 \end{pmatrix}\mathbf{x} + 2\begin{pmatrix} -4 & \frac{9}{2} \end{pmatrix}\mathbf{x} - 12 \tag{2.0.16}$$

$$\implies n_1 * n_2 = \{0, 6, 0\} \tag{2.0.17}$$

$$c_1n_1 + c_2n_2 = \begin{pmatrix} 8 \\ -9 \end{pmatrix} \tag{2.0.18}$$

$$c_1c_2 = -12. \tag{2.0.19}$$

Now the slopes of line is given by roots of polynomial,

$$cm^2 + 2bm + a = 0 \tag{2.0.20}$$

$$\implies 2bm = 0 \tag{2.0.21}$$

$$\implies m = 0 \tag{2.0.22}$$

Also

$$m_i = \frac{-b \pm \sqrt{-|V|}}{c} \tag{2.0.23}$$

$$\implies m_i = \frac{-0 \pm \sqrt{9}}{0} \tag{2.0.24}$$

$$\therefore m_1 = 0 \tag{2.0.25}$$

$$m_2 = \infty \tag{2.0.26}$$

The normal vector to the two lines is given by,

$$n_i = k_i \begin{pmatrix} -m_i \\ 1 \end{pmatrix} \tag{2.0.27}$$

$$\implies n_1 = k_1 \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{2.0.28}$$

$$n_2 = k_2 \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{2.0.29}$$

Also,

$$k_1k_2 = 6 \tag{2.0.30}$$

Let $k_1 = 2$ and $k_2 = 3$

$$\implies n_1 = \begin{pmatrix} 0 \\ 2 \end{pmatrix} \tag{2.0.31}$$

$$n_2 = \begin{pmatrix} 3 \\ 0 \end{pmatrix} \tag{2.0.32}$$

We verify obtained n_1 and n_2 using Toeplitz matrix,

$$n_1 * n_2 = \begin{pmatrix} 0 & 0 \\ 2 & 0 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} 2 \\ 0 \end{pmatrix} \begin{pmatrix} 3 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 6 \\ 0 \end{pmatrix} \tag{2.0.33}$$

Hence (2.0.17) and (2.0.33) are same. Hence verified.

Now substituting it in (2.0.18) we get,

$$c_2 \begin{pmatrix} 0 \\ 2 \end{pmatrix} + c_1 \begin{pmatrix} 3 \\ 0 \end{pmatrix} = \begin{pmatrix} 8 \\ -9 \end{pmatrix} \tag{2.0.34}$$

Solve using Row reduction Technique we get,

$$\implies \begin{pmatrix} 3 & 0 & 8 \\ 0 & 2 & -9 \end{pmatrix} \tag{2.0.35}$$

$$\xleftrightarrow{R_1 \leftarrow R_1/3} \begin{pmatrix} 1 & 0 & 8/3 \\ 0 & 2 & -9 \end{pmatrix} \tag{2.0.36}$$

$$\xleftrightarrow{R_2 \leftarrow R_2/2} \begin{pmatrix} 1 & 0 & 8/3 \\ 0 & 1 & -9/2 \end{pmatrix} \tag{2.0.37}$$

$$\implies c_1 = \frac{8}{3} \tag{2.0.38}$$

$$c_2 = \frac{-9}{2} \tag{2.0.39}$$

substituting the values of c_1, c_2 and equation

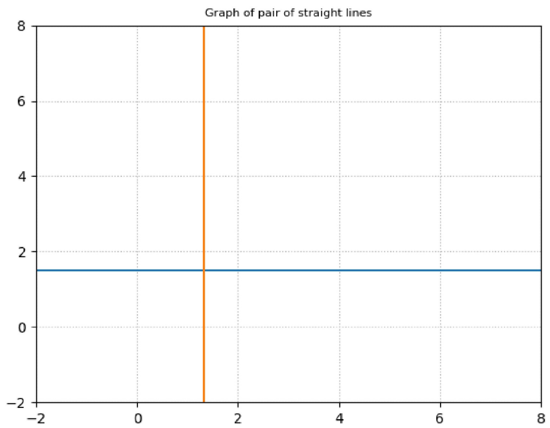

Fig. 0: Intersection of 2 lines

(2.0.31) and (2.0.32) to equation (2.0.13) and (2.0.14) we get equation of two straight lines.

$$\implies \begin{pmatrix} 0 & 2 \end{pmatrix} \mathbf{x} = \frac{8}{3} \tag{2.0.40}$$

$$\begin{pmatrix} 3 & 0 \end{pmatrix} \mathbf{x} = \frac{-9}{2} \tag{2.0.41}$$

Hence the equation of pair of straight lines are,

$$\left(\begin{pmatrix}0 & 2\end{pmatrix}\mathbf{x} - \frac{8}{3}\right)\left(\begin{pmatrix}3 & 0\end{pmatrix}\mathbf{x} - \frac{-9}{2}\right) = 0 \tag{2.0.42}$$

Hence, Plot of the equation (2.0.42) is shown in Figure.0 Now for value of k = 0 does not represent pair of straight lines.as,

$$\left|\mathbf{V}\right| = \begin{vmatrix}0 & 0\\0 & 0\end{vmatrix} \not< 0 \tag{2.0.43}$$

Hence, Plot of the equation $\begin{pmatrix}-8 & 9\end{pmatrix}\mathbf{x} = 12$ is shown in figure 0,

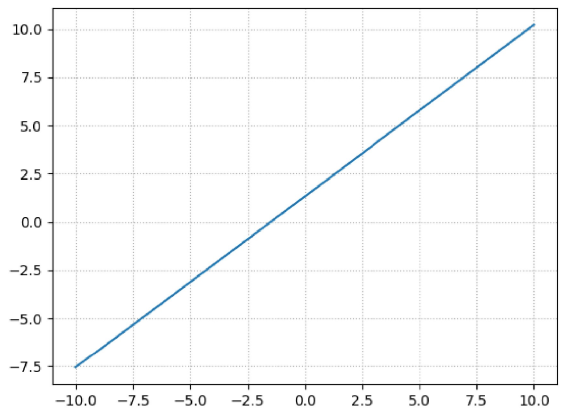

Fig. 0: Intersection of 2 lines