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Assignment-5

Ankur Aditya - EE20RESCH11010

Abstract—This document contains the procedure to find unknown constant value such that the equation represents pair of straight lines.

Download the python code from

https://github.com/ankuraditya13/EE5609—Assignment5

and latex-file codes from

https://github.com/ankuraditya13/EE5609—Assignment5

1 Problem

Find the value of k so that following equation may represent pairs of straight lines,

$$kxy - 8x + 9y - 12 = 0 ag{1.0.1}$$

2 Solution

The general equation of second degree is given by,

$$ax^2 + 2bxy + cy^2 + 2dx + 2ey + f = 0$$
 (2.0.1)

In vector from the equation (2.0.1) canb be expressed as,

$$\mathbf{x}^T \mathbf{V} \mathbf{x} + 2\mathbf{u}^T \mathbf{x} + f = 0 \tag{2.0.2}$$

where,

$$\mathbf{V} = \mathbf{V}^T = \begin{pmatrix} a & b \\ b & c \end{pmatrix} \tag{2.0.3}$$

$$\mathbf{u} = \begin{pmatrix} d \\ e \end{pmatrix} \tag{2.0.4}$$

Now, comparing equation (2.0.1) to (1.0.1) we get, a = c = 0, $b = \left(\frac{k}{2}\right)$, d = -4, $e = \left(\frac{9}{2}\right)$, f = -12. Hence, substituting these values in equation (2.0.3) and (2.0.4) we get,

$$\mathbf{V} = \mathbf{V}^T = \begin{pmatrix} 0 & \frac{k}{2} \\ \frac{k}{2} & 0 \end{pmatrix} \tag{2.0.5}$$

$$\mathbf{u} = \begin{pmatrix} -4\\ \frac{9}{2} \end{pmatrix} \tag{2.0.6}$$

Now equation (1.0.1) represents pair of straight lines if,

$$\begin{vmatrix} \mathbf{V} & \mathbf{u} \\ \mathbf{u}^T & f \end{vmatrix} = 0 \tag{2.0.7}$$

$$\begin{vmatrix} 0 & \frac{k}{2} & -4 \\ \frac{k}{2} & 0 & \frac{9}{2} \\ -4 & \frac{9}{2} & -12 \end{vmatrix} = 0 \tag{2.0.8}$$

$$\implies k = 0, k = 6 \tag{2.0.9}$$