

Assignment-6

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Abstract—This document contains the procedure to find the equation of tangent to parabola.

Download the python code from

<https://github.com/ankuraditya13/EE5609-Assignment6>

and latex-file codes from

<https://github.com/ankuraditya13/EE5609-Assignment6>

1 PROBLEM

Find the equation of the tangent to the curve,

$$y = \sqrt{3x-2} \quad (1.0.1)$$

which is parallel to the line,

$$(4 \ 2)\mathbf{x} + 5 = 0 \quad (1.0.2)$$

2 SOLUTION

The equation (1.0.1) can be written as,

$$y^2 - 3x + 2 = 0 \quad (2.0.1)$$

Comparing it with standard equation,

$$ax^2 + 2bxy + cy^2 + 2dx + 2ey + f = 0 \quad (2.0.2)$$

$\therefore a = b = e = 0, d = \frac{-3}{2}, c = 1, f = 2.$

$$\therefore \mathbf{V} = \begin{pmatrix} a & b \\ b & c \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \quad (2.0.3)$$

$$\therefore \mathbf{u} = \begin{pmatrix} d \\ e \end{pmatrix} = \begin{pmatrix} \frac{-3}{2} \\ 0 \end{pmatrix} \quad (2.0.4)$$

$$\text{Now, } |V| = \begin{vmatrix} 0 & 0 \\ 0 & 1 \end{vmatrix} = 0 \quad (2.0.5)$$

\Rightarrow that the curve is a parabola. Now, finding the eigen values corresponding to the \mathbf{V} ,

$$|\mathbf{V} - \lambda \mathbf{I}| = 0 \quad (2.0.6)$$

$$\begin{vmatrix} -\lambda & 0 \\ 0 & 1 - \lambda \end{vmatrix} = 0 \quad (2.0.7)$$

$$\Rightarrow \lambda = 0, 1. \quad (2.0.8)$$

Calculating the eigenvectors corresponding to $\lambda = 0, 1$ respectively,

$$\mathbf{V}\mathbf{x} = \lambda\mathbf{x} \quad (2.0.9)$$

$$\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} = 0 \Rightarrow \mathbf{p}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (2.0.10)$$

$$\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} = \mathbf{x} \Rightarrow \mathbf{p}_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (2.0.11)$$

Now by eigen decomposition on \mathbf{V} ,

$$\mathbf{V} = \mathbf{P}\mathbf{D}\mathbf{P}^T \quad (2.0.12)$$

$$\text{where, } \mathbf{P} = (\mathbf{p}_1 \mathbf{p}_2) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad (2.0.13)$$

$$\mathbf{D} = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \quad (2.0.14)$$

Hence equation (2.0.12) becomes,

$$\mathbf{V} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad (2.0.15)$$

$$\mathbf{V} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \quad (2.0.16)$$

Now the tangent to parabola is parallel to the line equation (1.0.2), Hence the direction vectors (\mathbf{m}) and normal (\mathbf{n}) vectors are,

$$\mathbf{m} = \begin{pmatrix} 1 \\ -2 \end{pmatrix} \quad (2.0.17)$$

$$\mathbf{n} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \quad (2.0.18)$$

Now, the equation for the point of contact for the parabola is given as,

$$\begin{pmatrix} \mathbf{u}^T + \kappa \mathbf{n}^T \\ \mathbf{v} \end{pmatrix} \mathbf{q} = \begin{pmatrix} -f \\ \kappa \mathbf{n} - \mathbf{u} \end{pmatrix} \quad (2.0.19)$$

$$\text{where, } \kappa = \frac{\mathbf{p}_1^T \mathbf{u}}{\mathbf{p}_1^T \mathbf{n}} = \frac{-3}{4} \quad (2.0.20)$$

Hence substituting the values of (2.0.20), (2.0.18), (2.0.12) and (2.0.4) in equation (2.0.19) we get,

$$\begin{pmatrix} -3 & \frac{-3}{4} \\ 0 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{q} = \begin{pmatrix} -2 \\ 0 \\ \frac{-3}{4} \end{pmatrix} \quad (2.0.21)$$

Solving for \mathbf{q} by removing the zero row and representing (2.0.21) as augmented matrix and then converting the matrix to echelon form,

$$\Rightarrow \begin{pmatrix} -3 & \frac{-3}{4} & -2 \\ 0 & 1 & \frac{-3}{4} \end{pmatrix} \xrightarrow{R_1 \leftarrow (-\frac{R_1}{3})} \begin{pmatrix} 1 & \frac{1}{4} & \frac{2}{3} \\ 0 & 1 & \frac{-3}{4} \end{pmatrix} \quad (2.0.22)$$

$$\xrightarrow{R_1 \leftarrow R_1 - \frac{1}{4}R_2} \begin{pmatrix} 1 & 0 & \frac{41}{48} \\ 0 & 1 & \frac{-3}{4} \end{pmatrix} \quad (2.0.23)$$

Hence from equation (2.0.23) it can be concluded that the point of contact is,

$$\mathbf{q} = \begin{pmatrix} \frac{41}{48} \\ \frac{-3}{4} \end{pmatrix} \quad (2.0.24)$$

Now \mathbf{q} is a point on the tangent. Hence, the equation of the line can be expressed as

$$\mathbf{n}^T \mathbf{x} = c \quad (2.0.25)$$

where c is,

$$c = \mathbf{n}^T \mathbf{q} = \begin{pmatrix} 2 & 1 \end{pmatrix} \begin{pmatrix} \frac{41}{48} \\ \frac{-3}{4} \end{pmatrix} = \frac{23}{24} \quad (2.0.26)$$

Hence equation of tangent to the curve (1.0.1) parallel to (1.0.2) is given by substituting the value of c and \mathbf{n} from equation (2.0.26) and (2.0.18) respectively to the equation (2.0.25),

$$\Rightarrow \begin{pmatrix} 2 & 1 \end{pmatrix} \mathbf{x} = \frac{23}{24} \quad (2.0.27)$$

Figure 0 verifies that the $\begin{pmatrix} 2 & 1 \end{pmatrix} \mathbf{x} = \frac{23}{24}$ is a tangent to parabola $y = \sqrt{3x - 2}$

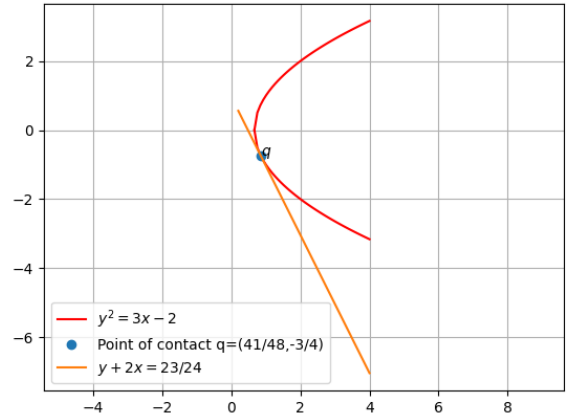


Fig. 0: Tangent to parabola $y = \sqrt{3x - 2}$