# Assignment-7

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***Abstract*—This document contains the procedure to find the equation of tangent to parabola.**

Download the python code from

https://github.com/ankuraditya13/EE5609−Assignment7

and latex-file codes from

https://github.com/ankuraditya13/EE5609−Assignment7

## 1 Problem

Find the QR Decomposition of matrix,

$$\mathbf{A} = \begin{pmatrix} 2 & -6 \\ 1 & -2 \end{pmatrix} \tag{1.0.1}$$

## 2 Solution

Let $c_1$ and $c_2$ be the column vectors of given matrix $\mathbf{A}$

$$c_1 = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \tag{2.0.1}$$

$$c_2 = \begin{pmatrix} -6 \\ -2 \end{pmatrix} \tag{2.0.2}$$

We can express the matrix $\mathbf{A}$ as,

$$\mathbf{A} = \mathbf{QR} \tag{2.0.3}$$

Where, $\mathbf{Q}$ is an orthogonal matrix given as,

$$\mathbf{Q} = \begin{pmatrix} \mathbf{u_1} & \mathbf{u_2} \end{pmatrix} \tag{2.0.4}$$

and $\mathbf{R}$ is an upper triangular matrix given as,

$$\mathbf{R} = \begin{pmatrix} k_1 & r_1 \\ 0 & k_2 \end{pmatrix} \tag{2.0.5}$$

Now, we can express $\alpha$ and $\beta$ as,

$$c_1 = k_1\mathbf{u_1} \tag{2.0.6}$$

$$c_2 = r_1\mathbf{u_1} + k_2\mathbf{u_2} \tag{2.0.7}$$

$$\text{where, } k_1 = \|c_1\| = \sqrt{2^2 + (1^2} = \sqrt{5} \tag{2.0.8}$$

Solving equation (2.0.6) for $\mathbf{u_1}$ ,

$$\mathbf{u_1} = \frac{c_1}{k_1} = \frac{1}{\sqrt{5}}\begin{pmatrix} 2 \\ 1 \end{pmatrix} \tag{2.0.9}$$

$$\text{Now, } r_1 = \frac{\mathbf{u_1}^T c_2}{\|\mathbf{u_1}\|^2} \tag{2.0.10}$$

$$\implies \frac{\frac{1}{\sqrt{5}}\begin{pmatrix} 2 & 1 \end{pmatrix}\begin{pmatrix} -6 \\ -2 \end{pmatrix}}{1} \tag{2.0.11}$$

$$\text{Hence, } r_1 = -\frac{-14}{\sqrt{5}} \tag{2.0.12}$$

$$\mathbf{u_2} = \frac{c_2 - r_1\mathbf{u_1}}{\|c_2 - r_1\mathbf{u_1}\|} \tag{2.0.13}$$

$$\implies \frac{\begin{pmatrix} -6 \\ -2 \end{pmatrix} - \left(\frac{-14}{\sqrt{5}}\right)\left(\frac{1}{\sqrt{5}}\begin{pmatrix} 2 \\ 1 \end{pmatrix}\right)}{\left\|\begin{pmatrix} -6 \\ -2 \end{pmatrix} - \left(-\frac{14}{\sqrt{5}}\frac{1}{\sqrt{5}}\begin{pmatrix} 2 \\ 1 \end{pmatrix}\right)\right\|} \tag{2.0.14}$$

$$\implies \mathbf{u_2} = \frac{1}{\sqrt{5}}\begin{pmatrix} -1 \\ 2 \end{pmatrix} \tag{2.0.15}$$

$$\text{Now, } k_2 = u_2^T c_2 \tag{2.0.16}$$

$$\implies \frac{1}{\sqrt{5}}\begin{pmatrix} -1 & 2 \end{pmatrix}\begin{pmatrix} -6 \\ -2 \end{pmatrix} \tag{2.0.17}$$

$$\implies k_2 = \frac{2}{\sqrt{5}} \tag{2.0.18}$$

Hence substituting the values of unknown parameter from equations (2.0.8), (2.0.18), (2.0.9), (2.0.15) and (2.0.12) to equation (2.0.4) and (2.0.5) we get,

$$\mathbf{Q} = \frac{1}{\sqrt{5}}\begin{pmatrix} 2 & -1 \\ 1 & 2 \end{pmatrix} \tag{2.0.19}$$

$$\mathbf{R} = \begin{pmatrix} \sqrt{5} & \frac{-14}{\sqrt{5}} \\ 0 & \frac{2}{\sqrt{5}} \end{pmatrix} \tag{2.0.20}$$